Introduction to Robotics

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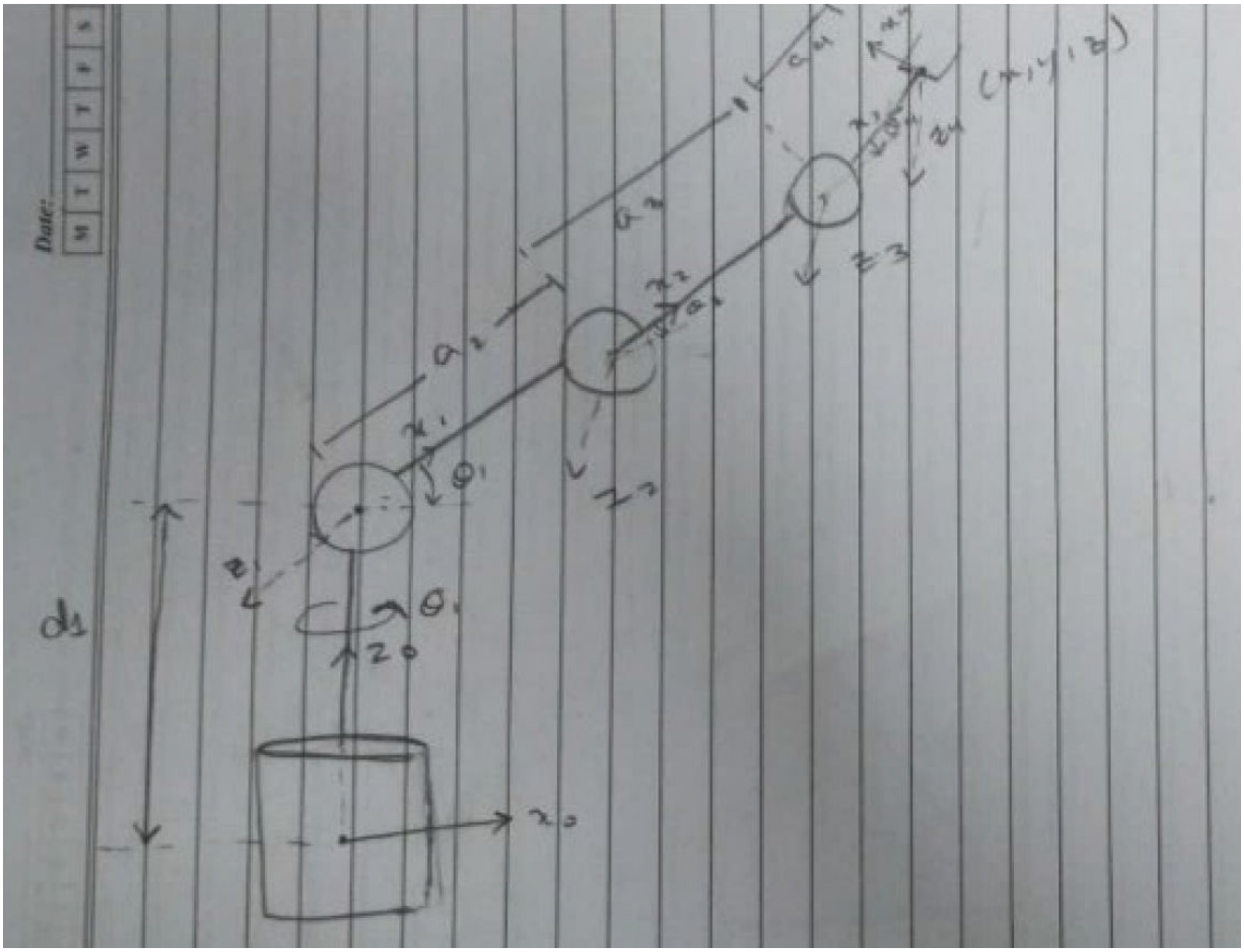
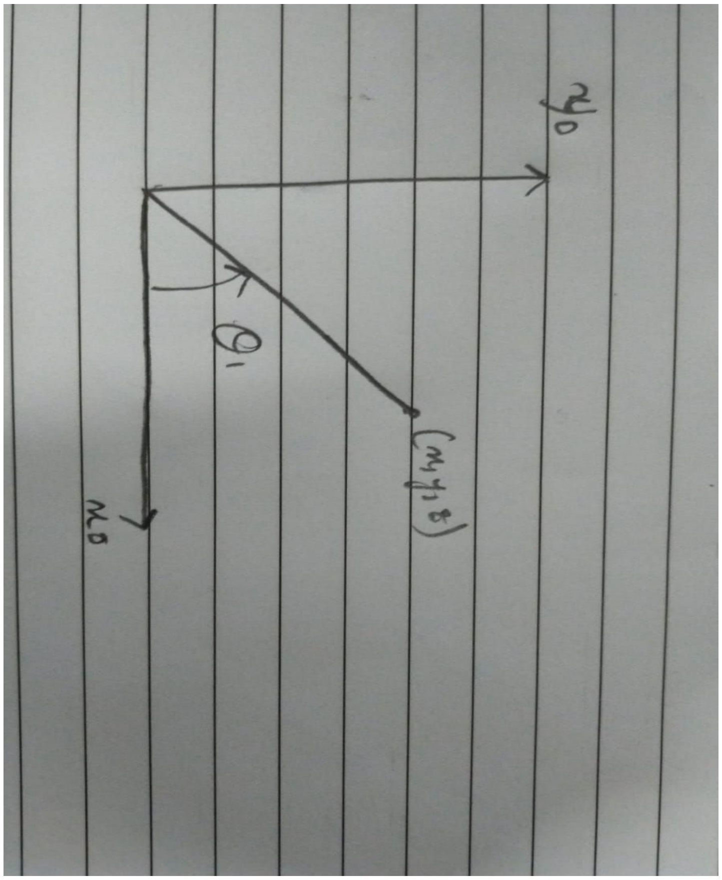
Inverse Kinematics Solution



Projecting the robot on the $x_0 - y_0$ plane



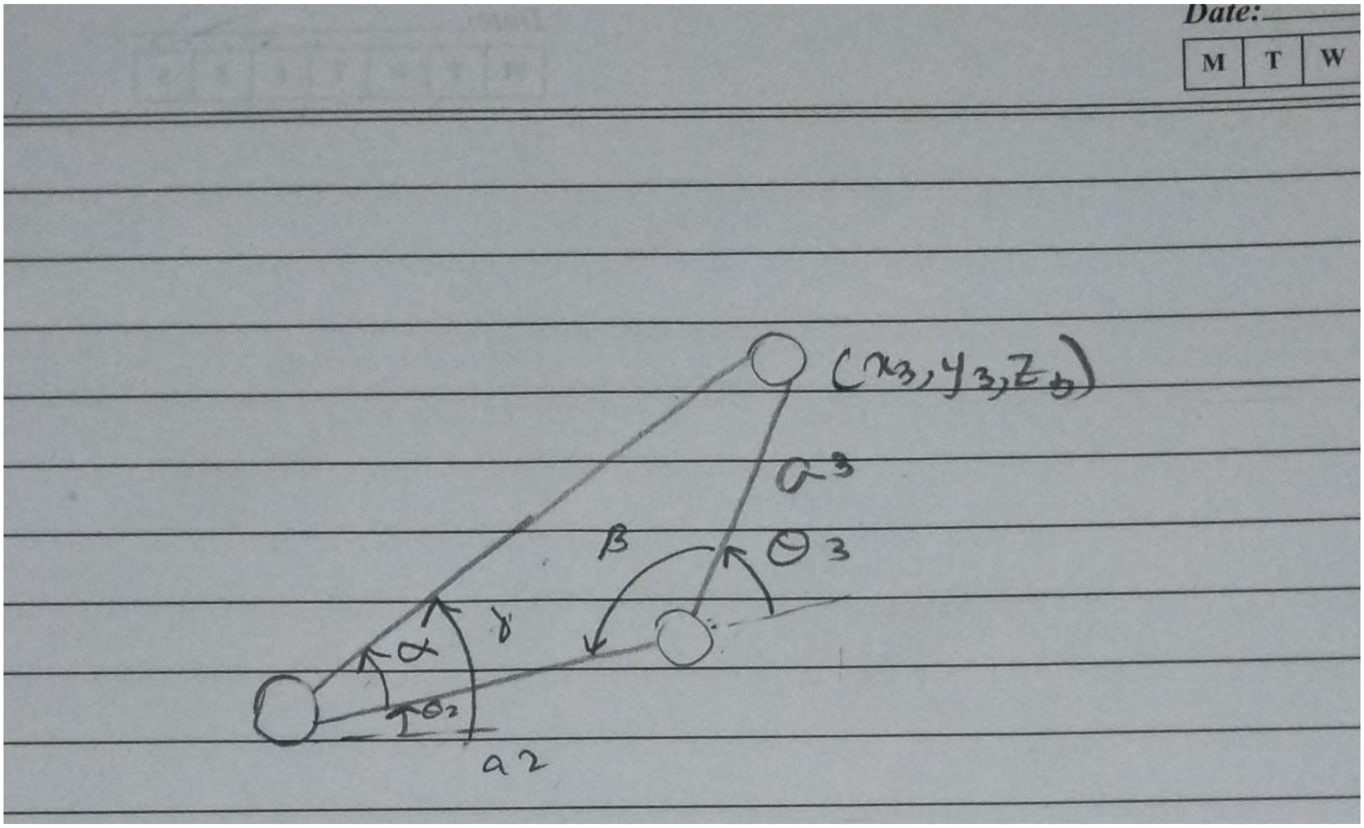
$\tan\theta_1 = \frac{y}{x}$

$\theta_1 = \arctan\frac{y}{x}$

$\theta_1 = \arctan\frac{y}{x} + \pi$

$x = x_3 + a_4\cos(\theta_1 + \theta_2 + \theta_3) \rightarrow x_3 = x - a_4\cos(\phi)$
similarly
$y_3 = y - a_4\sin(\phi)$

Now project the robot on the $x_1 - y_1$



In the above picture the coordinates x_3, y_3, z_3 represent the coordinates of frame {1}

Using the cosine law on the above figure

$$\beta = \cos^{-1}\left(\frac{a_2^2 + a_3^2 - r^2}{2a_2^2 a_3^2}\right) \ni r^2 = x_3^2 + y_3^2$$

$$\alpha = \cos^{-1}\left(\frac{a_2^2 + r^2 - a_3^2}{2a_2^2\sqrt{(x_3^2 + y_3^2)}}\right) \ni r^2 = x_3^2 + y_3^2$$

Now we have to find x_3, y_3, z_3 :

$${}^{1}y_3 = z - d_1 - a_4 \sin\phi$$

$${}^{1}x_3 = \sqrt{(x^2 + y^2)} - a_4 \cos(\phi)$$

Since, $\theta_2 = \gamma \pm \alpha$

$$\theta_2 = \tan^{-1}\left(\frac{z-d_1-a_4\sin\phi}{\sqrt{x^2+y^2}-a_4\cos\phi}\right) \pm \cos^{-1}\left[\frac{a_2^2-a_3^2\left(\sqrt{x^2+y^2}-a_4\cos\phi\right)^2+\left(z-d_1-a_4\sin\phi\right)^2}{2a_2\sqrt{\left(\sqrt{x^2+y^2}-a_4\cos\phi\right)^2+\left(z-d_1-a_4\sin\phi\right)^2}}\right)$$

$$\theta_3 = \pi \pm \beta,\ \beta - \pi$$

$$\theta_3 = \pi - \cos^{-1}\left[\frac{a_2^2+a_3^2-\left(\sqrt{x^2+y^2}-a_4\cos\phi\right)^2-\left(z-d_1-a_4\sin\phi\right)^2}{2a_2a_3}\right)$$

or

$$= \cos^{-1}\left[\frac{a_2^2+a_3-\left(\sqrt{x^2+y^2}-a_4\cos\phi\right)^2-\left(z-d_1-a_4\sin\phi\right)^2}{2a_2a_3}\right)-\pi$$

$$\theta_4 = \phi - \theta_2 - \theta_3$$

Solution set

$$\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{bmatrix} = \begin{bmatrix} \tan^{-1}(y/x) & \tan^{-1}(y/x) & \pi+\tan^{-1}(y/x) & \pi+\tan^{-1}(y/x) \\ \pi-\beta & \beta-\pi & -(\pi-\beta) & -(\beta-\pi) \\ \gamma-\alpha & \gamma+\alpha & \pi-(\gamma-\alpha) & \pi-(\gamma+\alpha) \\ \underbrace{\phi-\theta_2-\theta_3}_{\text{Solution 1}} & \underbrace{\phi-\theta_2-\theta_3}_{\text{Solution 2}} & \underbrace{\phi-\theta_2-\theta_3}_{\text{Solution 3}} & \underbrace{\phi-\theta_2-\theta_3}_{\text{Solution 4}} \end{bmatrix}$$

Task 2

Link	a_i	α_i	d_i	θ_i
1	0	90	14 cm	θ_1
2	10.5 cm	0	0	θ_2
3	10.5cm	0	0	θ_3
4	7.5 cm	0	0	θ_4

Table 1: Joint DH Parameter Table

Find Joint Angles

```
function q = findJointAngles(x,y,z,phi)
    %Making PLiminary calculations
    q = zeros(4,4);
    a4 = 7.5; d1 = 14; a2 = 10.5; a3 = 10.5;
    x13 = a4*cos(phi) - sqrt(x^2 + y^2);
    y13 = a4*sin(phi) - z + d1;
    r = sqrt(x13^2 + y13^2);
    alpha = acos((a2^2 + (r^2-a3^2))/(2*a2*r));
    beta = acos((a2^2 + a3^2 - r^2)/(2*a2*a3));

    % Making the solution set
    % Solution Set 1
    theta1 = atan(y/x); % Theta 1 (1)
    theta3 = pi - beta;
    gamma = atan2(z - d1 -a4*sin(phi),( sqrt(x^2 + y^2)) -a4*cos(phi));
    theta2 = gamma - alpha;
    theta4 = phi - theta2 - theta3;
    q(1,:) = [theta1, theta2, theta3, theta4];

    % Solution Set 2
    theta1 = atan(y/x); % Theta 1 (1)
    theta = -(pi - beta);
    gamma = atan(y13/x13);
    theta2 = gamma + alpha;
    theta4 = phi - theta2 - theta3;
    q(2,:) = [theta1, theta2, theta3, theta4];

    % Solution Set 3
    theta1 = atan(y/x) + pi; % Theta 1 (2)
    theta3 = -(pi - beta);
    gamma = atan(y13/x13);
    theta2 = pi - (gamma - alpha);
    theta4 = phi - theta2 - theta3;
    q(3,:) = [theta1, theta2, theta3, theta4];

    % Solution Set 4
    theta1 = atan(y/x) + pi; % Theta 1 (2)
    theta3 = pi - beta;
    gamma = atan(y13/x13);
    theta2 = pi - (gamma + alpha);
    theta4 = phi - theta2 - theta3;
    q(4,:) = [theta1, theta2, theta3, theta4];
end
```

Find Optimal Solutions

```
function lst = findOptimalSolution(x, y, z, phi)
    lst = [0 0 0];
    current_angles = getCurrentPose();
    current_angles(1) = current_angles(1) + pi/2;
    current_angles(2) = current_angles(2) + pi/2;
    current_angles = current_angles(:,1:4);
    IK_sol = findJointAngles(x, y, z, phi);
    %Euclidean Distance
    d1 = sum(abs(current_angles - real(IK_sol(1,:)))); % Delta 1
    d2 = sum(abs(current_angles - real(IK_sol(2,:)))); % Delta 2
    d3 = sum(abs(current_angles - real(IK_sol(3,:)))); % Delta 3
    d4 = sum(abs(current_angles - real(IK_sol(4,:)))); % Delta 4
    d = min([d1, d2, d3, d4]); % Extract Min
    if d == d1 && checkJointLimits(IK_1) && isValid(real(IK_sol(1,:)))
        lst = IK_1;
    elseif d == d2 && checkJointLimits(IK_2) && isValid(real(IK_sol(2,:)))
        lst = IK_2;
    elseif d == d3 && checkJointLimits(IK_3) && isValid(real(IK_sol(3,:)))
        lst = IK_3;
    elseif d == d4 && checkJointLimits(IK_4) && isValid(real(IK_sol(4,:)))
        lst = IK_4;
    end
end
```

Helper Functions

Check Joint Limits

```
function out = checkJointLimits(jointAngles)
    new_theta = zeros(1,4); % Array for JointAngles Mapped to [-pi, pi]
    offset = [-pi/2 -pi/2 0 0];
    out = 1;
    for i=1:4
        if i == 1 || i == 2
          new_theta(i) = jointAngles(i) - pi/

        elseif i == 3 || i ==4
          new_theta(i) = jointAngles(i)
    end
        new_theta(i) = mod(new_theta(i)+pi, 2*pi) - pi;
        % Condition for invalid input angle
        if (new_theta(i) > 5*pi/6 || new_theta(i) < -5*pi/6)
            out = 0;
            break;
        end
    end
end
```

Is valid

```
function out = isValid(jointAngles)
    out = 0;
    if isreal(jointAngles(1)) && isreal(jointAngles(2)) && isreal(jointAngles(3)) && ...
    isreal(jointAngles(4))
    out = 1;
    end
end
```

Task 5.4

```
End_actual = zeros(5,4);
End_actual(1,:) = [17 15 10 pi/2];
End_actual(2,:) = [-17 15 3 pi/2];
End_actual(3,:) = [17 -15 2 -pi/2];
End_actual(4,:) = [13 15 10 pi/2];
End_actual(5,:) = [-17 -10 15 pi/2];



End_Measured
```

```
End_Measured = 5×4
   17.2194   15.2449   10.1380    1.8200
  -16.8092   15.2228    3.3399    2.0507
   17.3828  -14.6768    2.3275   -1.4006
   13.3976   15.3547   10.0813    1.8634
  -16.9066   -9.6227   15.0595    1.6827
```

```


Error = End_actual - End_Measured;
Error = sqrt(sum(Error.^2));
Error
```

```
Error = 1×4
    0.6307    0.6944    0.5020    0.6477
```

Observations:

b)The Error compared to lab 4 is lesser since we are minimizing the change in choint angle as well.