FTDS // Eigendecomposition

Hacktiv8 DS Curriculum Team

Phase 0 Learning Materials

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Contents

- Basic understanding of eigen vector and value
- Able to finding eigen vector and value using Python
- Able to reconstruct a matrix using Python

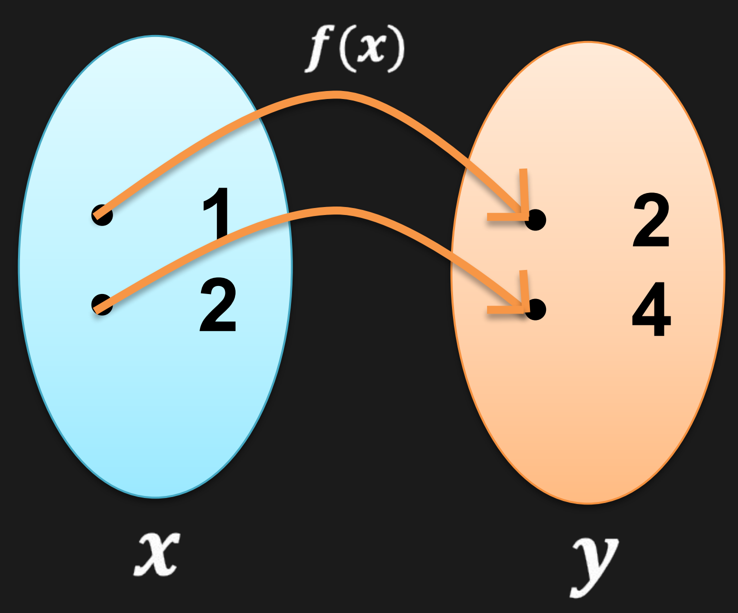
We can take an analogy of function y=f(x) to understand linear transformation.

Real Space

$$f: X \to Y;\ f(x) = y$$

Example:

$$x = [1,2]$$
$$f(x) = 2x$$

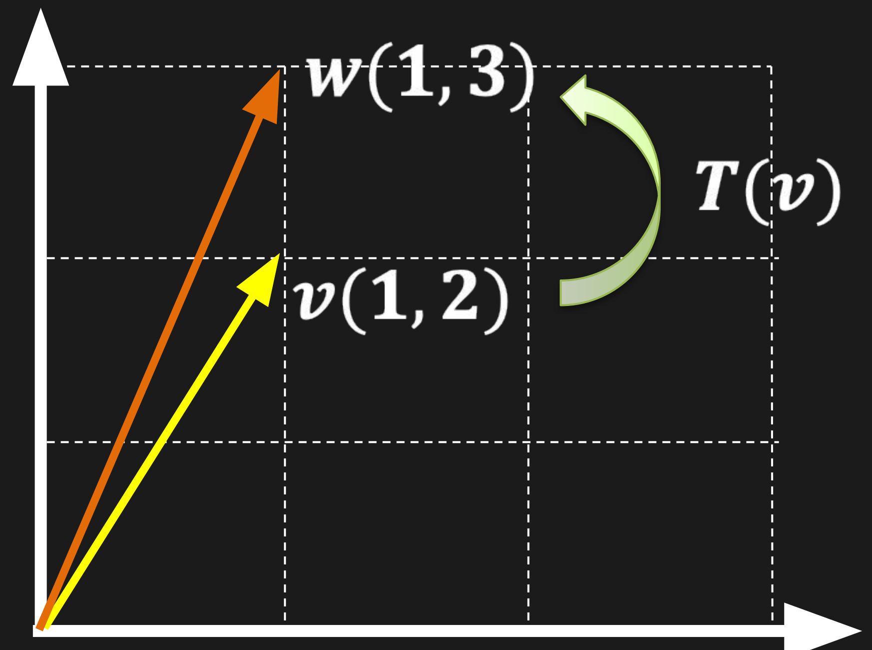


Vector Space

$$T: \boldsymbol{V} \to \boldsymbol{W};\ T(\boldsymbol{v}) = \boldsymbol{w}$$

Example:

$$\boldsymbol{v} = \begin{bmatrix} x = 1 \\ y = 2 \end{bmatrix}, T = \begin{bmatrix} x \\ x + y \end{bmatrix}$$



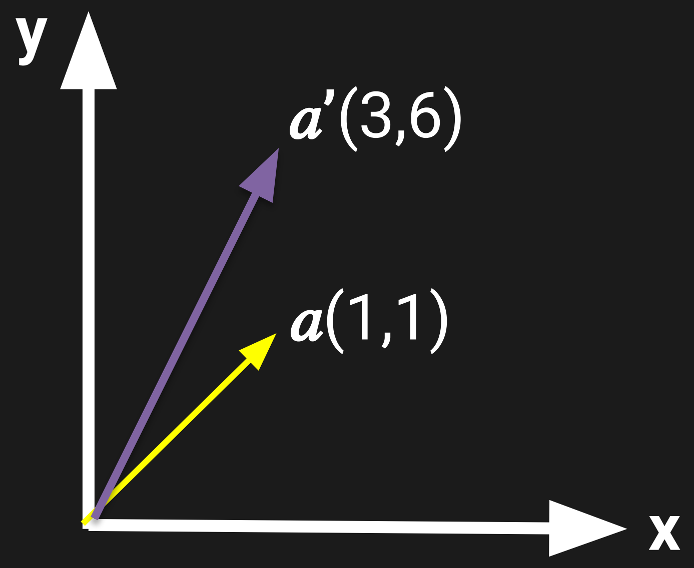
Let's look at another example of linear transformation

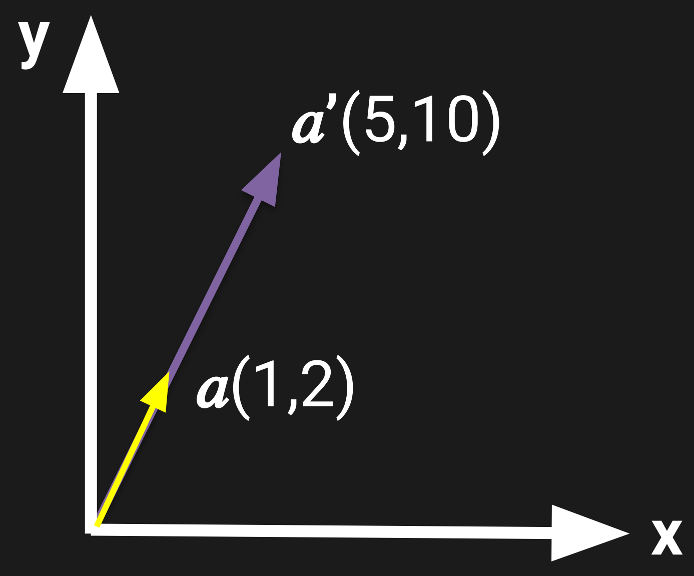
$$T: \boldsymbol{V} \rightarrow \boldsymbol{W}$$

Let $T(\boldsymbol{v}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \boldsymbol{v}$, $\boldsymbol{v}$ is a vector. Two vectors $\boldsymbol{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ will be transformed.

$$\boldsymbol{a}' = T(\boldsymbol{a}) = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\boldsymbol{b}' = T(\boldsymbol{b}) = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$





$$\boldsymbol{b}' = \begin{bmatrix} 5 \\ 10 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5\boldsymbol{b}$$

$$T(\boldsymbol{v}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \boldsymbol{v}, \boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax = \lambda x$$

x: Eigen vector
λ: Eigen value

Eigen vector is a vector that be scaled only without rotated.

Nilai eigen is a scale magnitude of the vector.

How to find eigen vector and value of a matrix?

$$\boldsymbol{Ax} = \lambda\boldsymbol{x}$$

$$\boldsymbol{Ax} = \lambda I\boldsymbol{x}$$

$$\boldsymbol{Ax} - \lambda I\boldsymbol{x} = \boldsymbol{0}$$

$$(A - \lambda I)\boldsymbol{x} = \boldsymbol{0}$$

$$det(A - \lambda I) = 0$$

From this we will find the possible eigen values, then we can find the eigen vectors for each eigen value.

Finding eigen vectors and values means that you are doing the eigendecomposition. The other words, eigendecomposition is a process to decompose a matrix into eigen vectors and eigen values.

Eigendecomposition on Code

```
# eigendecomposition
from numpy import array
from numpy.linalg import eig
# define matrix
A = array([ [1, 2, 3],
        [4, 5, 6],
        [7, 8, 9]])
# factorize
values, vectors = eig(A)
print(values)
print(vectors)
```

Output:

```
[ 1.61168440e+01 -1.11684397e+00 -9.75918483e-16]

[[-0.23197069 -0.78583024 0.40824829]
 [-0.52532209 -0.08675134 -0.81649658]
 [-0.8186735 0.61232756 0.40824829]]
```

Along you find the eigen vectors and values of a matrix, you split a matrix into eigen vectors and values as well. However, how do we reconstruct our matrix into the origin version?

$$A = V \cdot diag(\Lambda) \cdot V^{-1}$$

Λ is a vector that contains the possible eigen values
V is a matrix that contains the eigen vectors

```
# Reconstruct a matrix
from numpy import diag
from numpy.linalg import eig,inv

values, vectors = eig(A)
B=vectors.dot(diag(values)).dot(inv(vectors))
```

A

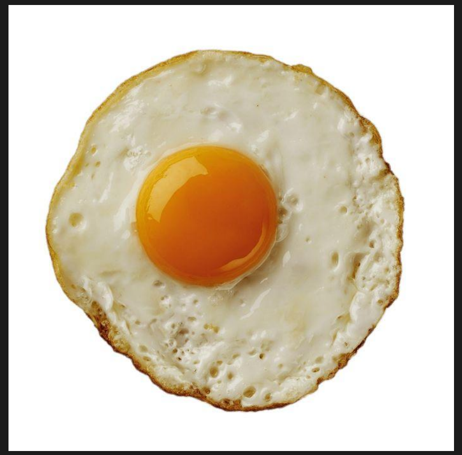
$$Ax = \lambda x$$

Eigendecomposition can be analogized by a "Nasi Goreng"

x

λ=50

30



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10

45

40

10

Eigen vectors and values are widely used in computer science. In data science itself, the eigen concept is very useful for the Principal Component Analysis. Additionally, the eigen also implemented in Google Search Engine.

Search a keyword

find webpages related to the keyword and calculate the incoming link from neighboring pages

Store into matrix

$$Ax = \lambda x$$

Eigendecomposition

$$\lambda = 1$$

$$v = (0.65, 0.5, 0.95)$$

Take eigen vector for λ = 1. The largest value is the webpage that you will see on the top of search values.