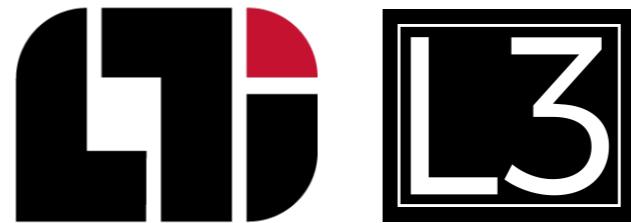


CS11-711 Advanced NLP

Learned Representations

Sean Welleck

Carnegie
Mellon
University



<https://cmu-l3.github.io/anlp-fall2025/>

<https://github.com/cmu-l3/anlp-fall2025-code>

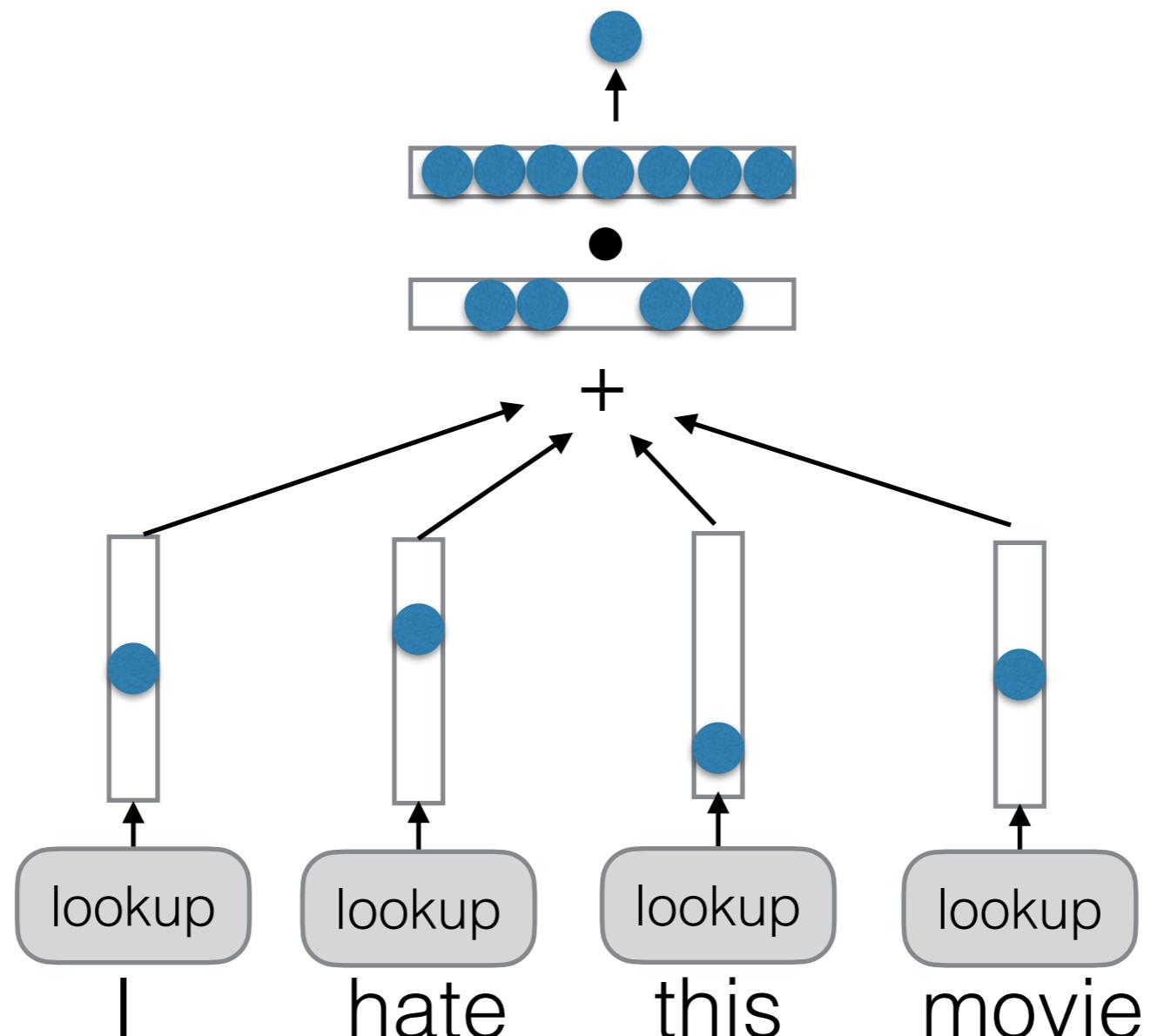
Recap

- Goal: learn a good scoring function $s_\theta(x, y)$
 - => good probabilistic models $p_\theta(y | x) \propto s_\theta(x, y)$
- Three key ingredients
 - **Modeling/Parameterization:** how s_θ (or p_θ) is implemented (e.g., the architecture)
 - **Learning:** setting the parameters θ using supervision
 - **Inference:** making a decision after learning
- We saw an example *classification* model based on:
 - Bag-of-words and word identities
 - Structured perceptron learning
 - A simple inference algorithm

Today's lecture

- We will still focus on classification: $g(x) \rightarrow \{1, 2, \dots, K\}$
- We will go over fundamentals that underlie any state-of-the-art NLP system:
 - Continuous representations of subwords
 - Parameterization based on neural networks
 - Learning by optimizing a loss function with back propagation and gradient descent

Recap: Bag of Words (BoW)



Features: sum of 1-hot vectors

Weights: learned

Bag of Words: Symptoms

- Handling of *conjugated or compound words*
 - I **love** this move -> I **loved** this movie

Subword
Models

- Handling of *word similarity*
 - I **love** this move -> I **adore** this movie

Word
Embeddings

- Handling of *combination features*
 - I **love** this movie -> I **don't love** this movie
 - I **hate** this movie -> I **don't hate** this movie

Neural
Networks

- Handling of *sentence structure*
 - It has an interesting story, **but** is boring overall

Sequence
Models

Subword Models

Basic Idea

- Split less common words into multiple **subword tokens**

the companies are expanding

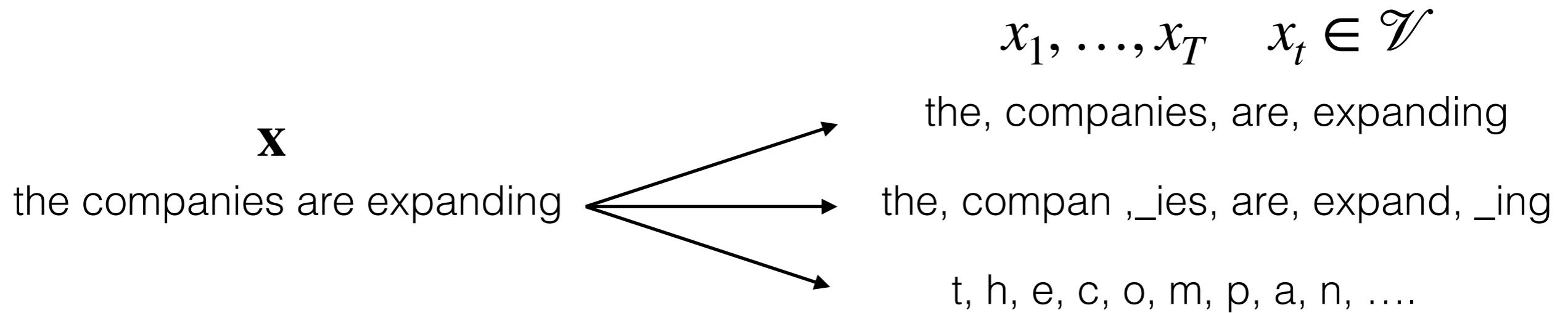


the **compan _ies** are **expand _ing**

- Benefits:
 - **Share parameters** between subwords
 - Reduce parameter size, **save compute+memory**

Core problem: tokenization

- Map text into a sequence of discrete **tokens** from a **vocabulary**



- We want a vocabulary \mathcal{V} that is:
 - Expressive**: represent any text (English, Japanese, code, ...)
 - Efficient**
 - Not too large**: larger vocabulary means more parameters to learn/store
 - Not too small**: smaller vocabulary means longer inputs

Core problem: tokenization

- Demo: <https://tiktokenizer.vercel.app/>

Tiktokenizer

Add message

元気ですかHello, how are you
123456789425217423
def foo(x):
 return None

gpt-4o

Token count
24

元気ですかHello, how are you
123456789425217423
def foo(x):
 return None

The screenshot shows the Tiktokenizer interface. On the left, a dark blue button says 'Add message'. Below it, there are three text input fields containing Japanese text, a string of digits, and a Python function definition. On the right, a light blue box displays the tokens for the first message, colored by category: '元' is yellow, '気ですか' is blue, 'Hello,' is orange, 'how' is cyan, 'are' is purple, and 'you' is light blue. The total token count is shown as 24. The model used is 'gpt-4o', indicated by a dropdown menu at the top right.

Idea 1: UTF-8

- Tokenize text as UTF-8 bytes

元気ですか。Hello!

Unicode string



```
utf = "元気ですか。Hello!".encode("utf-8")  
print([x for x in utf])  
✓ 0.0s  
[229, 133, 131, 230, 176, 151, 227, 129, 167, 227, 129, 153, 227, 129, 139, 227, 128, 130, 72, 101, 108, 108, 111, 33]
```

UTF-8

(Vocabulary = 256 byte choices)

- **Expressive:** any Unicode string (Japanese, English, Latex, ...)
- **Vocabulary is too small:** sequences are very long (inefficient)

Idea 2: Byte Pair Encoding

- **Key idea:** merge the most common token pairs into new tokens
 - Start with a base vocabulary (e.g., UTF-8) and a training set
 - Repeat:
 - Find the token pair that occurs most often
 - Introduce a new token and replace the token pair

```
training_text = """Hello, world!
Here is some example text to test
the BPE algorithm. It is not very
interesting, but it will do the job.
"""

pair: ('e', ' ') freq: 5
merging ('e', ' ') into a new token 256

pair: ('t', ' ') freq: 5
merging ('t', ' ') into a new token 257

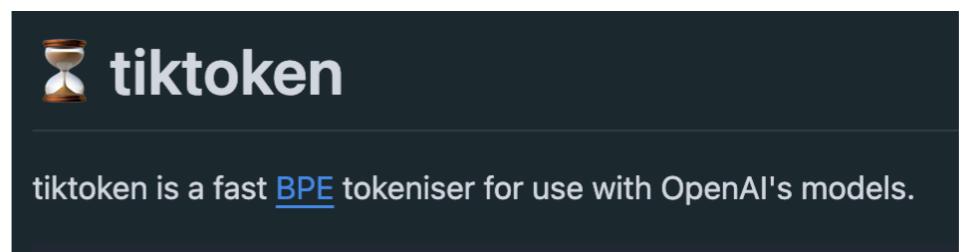
pair: ('e', 'r') freq: 3
merging ('e', 'r') into a new token 258

pair: ('t', 'h') freq: 3
merging ('t', 'h') into a new token 259

pair: ('l', 'l') freq: 2
merging ('l', 'l') into a new token 260
```

Practical tools: tiktoken

- Load pre-existing OpenAI vocabularies (e.g., GPT-2, GPT-4)
- Tokenize and decode text



```
# !pip install tiktoken
import tiktoken

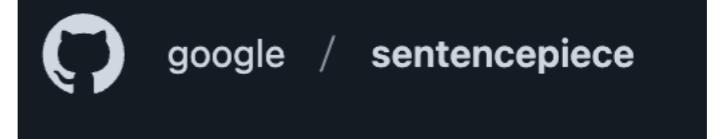
enc = tiktoken.get_encoding("gpt2")
print(enc.encode("Hello, こんにちは"))

enc = tiktoken.get_encoding("cl100k_base")
print(enc.encode("Hello, こんにちは"))

] ✓ 0.0s
[15496, 11, 23294, 241, 22174, 28618, 2515, 94, 31676]
[9906, 11, 220, 90115]
```

Practical tools: SentencePiece

- Also supports *training* a tokenizer
- Uses *Unicode* as the base vocabulary
- *byte_fallback=True*: tokenize as UTF-8 bytes when a Unicode character is out-of-vocabulary



```
ids = sp.encode("hello, こんにちは マラソ マラソン marathon")
print(ids)

print([sp.id_to_piece(idx) for idx in ids])

[1298, 295, 1339, 1353, 1333, 1534, 1457, 1366, 1793, 1373, 1333, 329, 1407, 584, 964]
['_he', 'll', 'o', ',', '_', 'こ', 'ん', 'に', 'ち', 'は', '_', 'マラ', 'ソ', '_マラソン', '_marathon']
```

Subword Considerations

- **Vocabulary depends on the BPE training data:**
 - Under-represented languages: merged less, hence longer sequences
 - *Work-around:* upsample under-represented languages
- **Inconsistent numbers:** 123 -> “123” vs. 927 -> “92” “7”
 - *Work-around:* Hand-defined rules, e.g. never group digits together

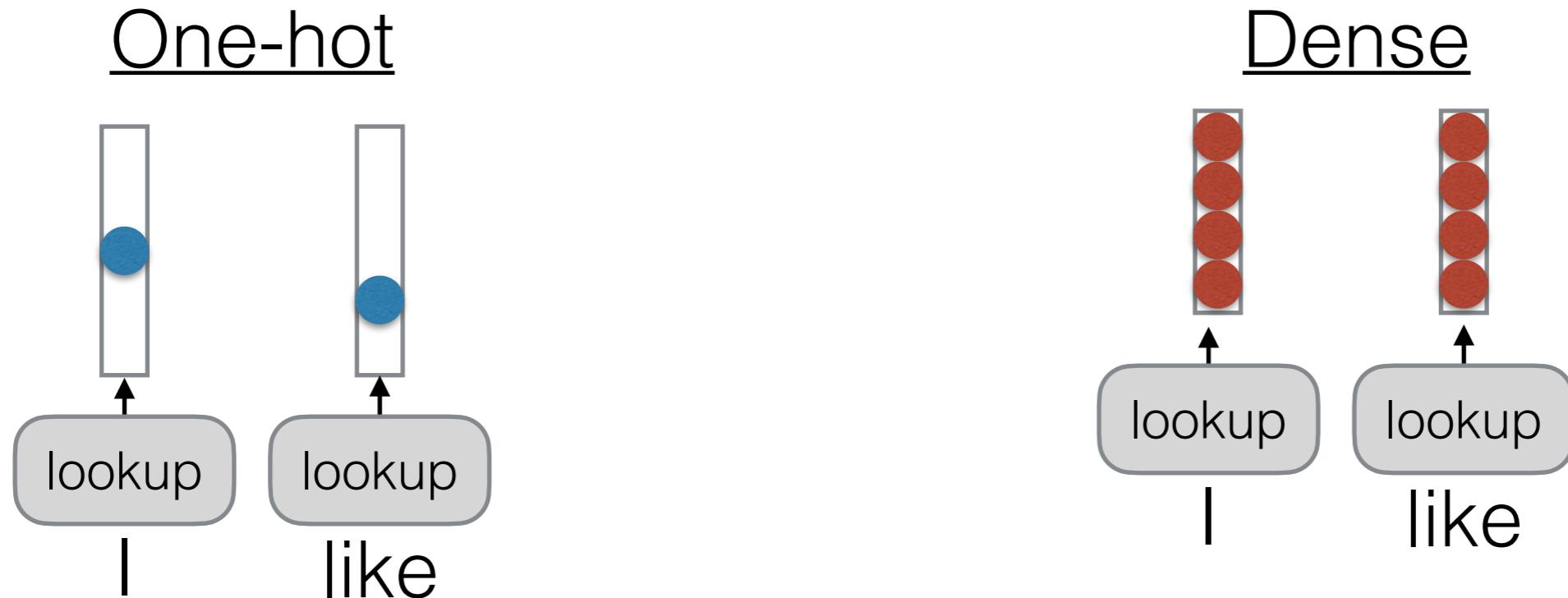
Recap

- Tokenization and subword models
 - Represent sequences as tokens determined based on frequency
- **Next:** Token embeddings

Continuous Word Embeddings

Basic Idea

- Previously: **one-hot** vectors (*sparse*)
- Continuous embeddings: *dense* vectors in $\mathbb{R}^{d_{emb}}$



$$x_t : [0, \dots, 1, \dots, 0] \in \{0, 1\}^V$$

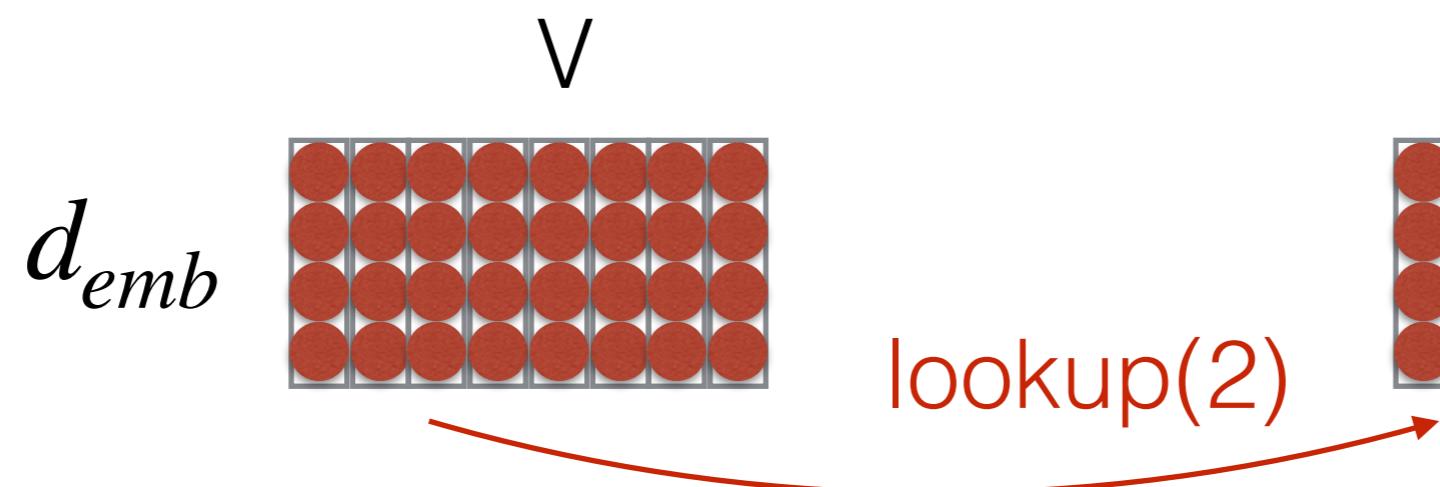
V : vocabulary size

$$x_t : [0.2, -1.3, \dots, 0.6] \in \mathbb{R}^{d_{emb}}$$

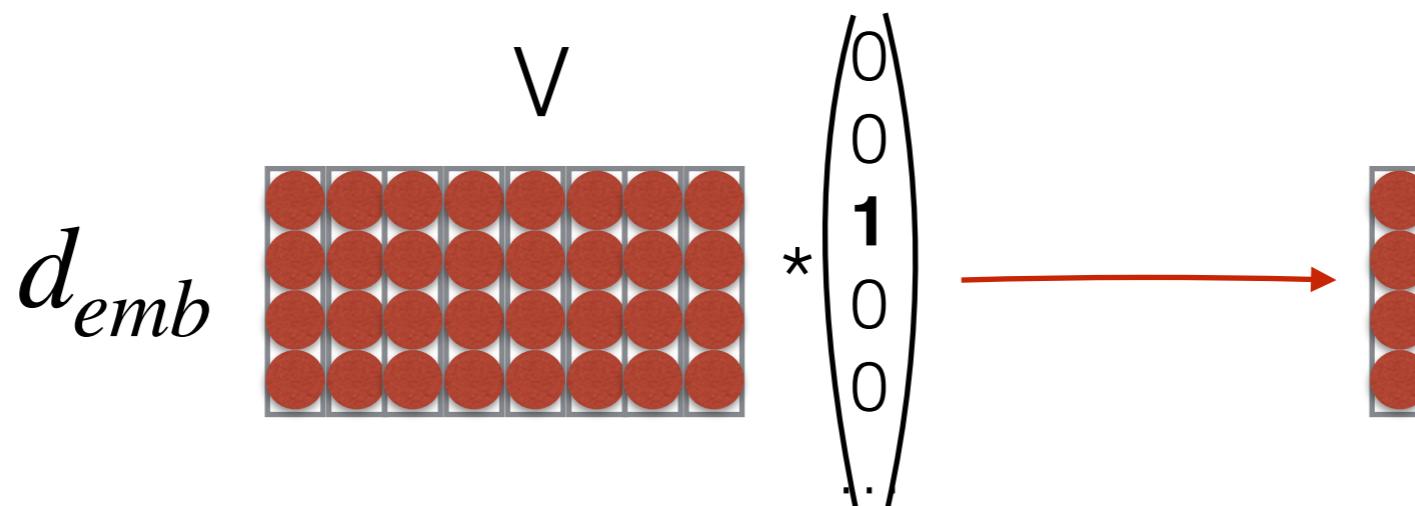
d_{emb} : “embedding dimension”

Embedding Layer

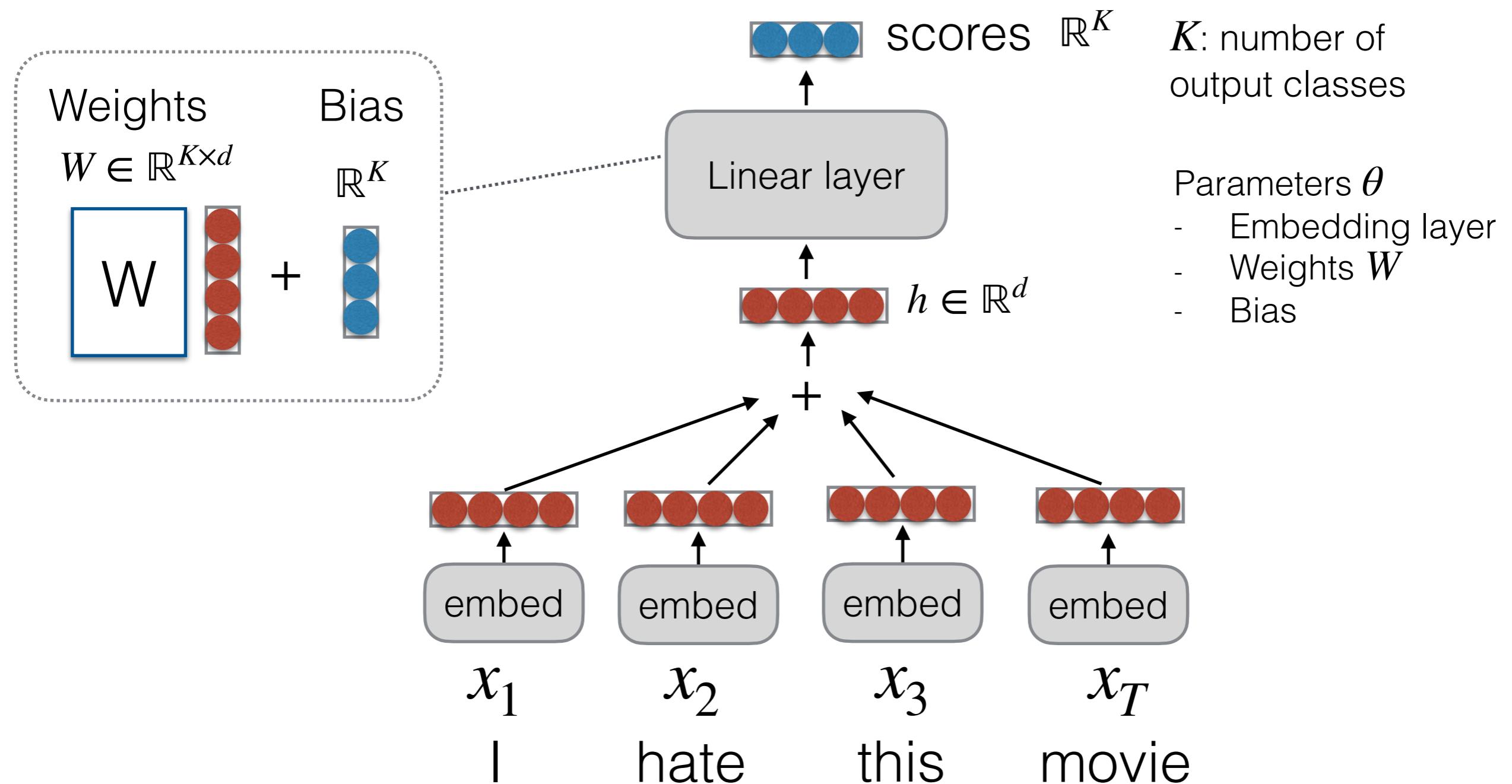
- Embedding layer: matrix with a row/column for each vocabulary token. “Lookup”: select a row/column.



- Equivalent to multiplying by a one-hot vector



Continuous Bag of Words (CBOW)



In Code

```
class Embedding(nn.Module):
    def __init__(self, vocab_size, emb_size):
        super(Embedding, self).__init__()
        self.weight = nn.Parameter(torch.randn(vocab_size, emb_size))
        self.vocab_size = vocab_size

    def forward(self, x):
        xs = torch.nn.functional.one_hot(x, num_classes=self.vocab_size).float()
        return torch.matmul(xs, self.weight)
```

In practice, implemented in libraries (e.g., nn.Embedding)

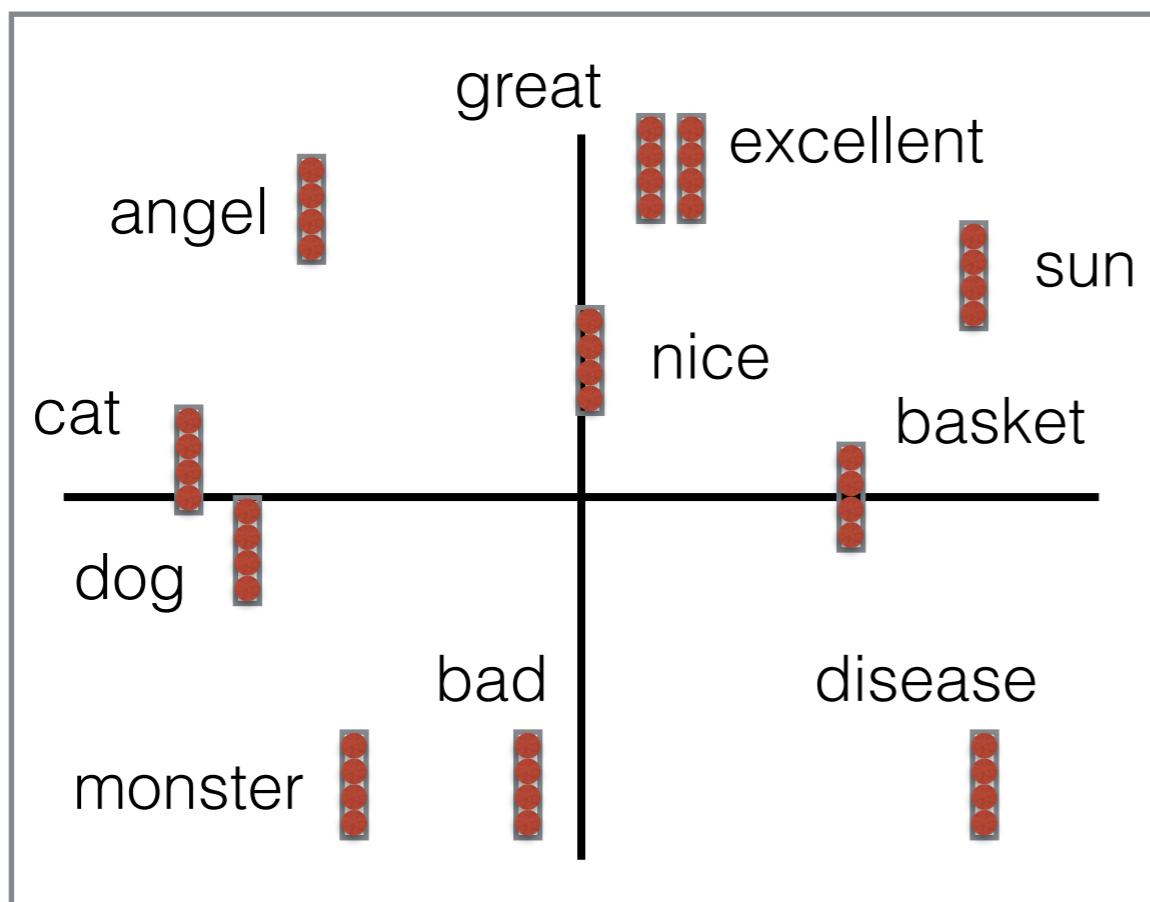
In Code

```
class CBow(torch.nn.Module):
    def __init__(self, vocab_size, num_labels, emb_size):
        super(CBow, self).__init__()
        self.embedding = nn.Embedding(vocab_size, emb_size)
        self.output_layer = nn.Linear(emb_size, num_labels)

    def forward(self, tokens):
        emb = self.embedding(tokens)      # [len(tokens) x emb_size]
        emb_sum = torch.sum(emb, dim=0) # [emb_size]
        h = emb_sum.view(1, -1)         # [1 x emb_size]
        out = self.output_layer(h)       # [1 x num_labels]
        return out
```

What do Our Vectors Represent?

- No guarantees, but we hope that:
 - Words that are **similar** are **close** in vector space
 - Each vector element is a **feature**



Shown in 2D, but
in reality we use
512, 1024, etc.

Recap

- Tokenization and subword models
 - Represent sequences as tokens determined based on frequency
- Token embeddings
 - Represent tokens as learned continuous vectors
- **Next:** Neural networks

Neural Network Features

Motivation: combination features

I don't love this movie

A diagram illustrating the concept of combination features. It shows two statements on the left, each pointing to a five-point rating scale on the right. The first statement is "I don't love this movie" and the second is "There's nothing I don't love about this movie". Each statement has an arrow pointing to a vertical list of five ratings: "very good" (green), "good" (green), "neutral" (black), "bad" (red), and "very bad" (red). The "very good" and "good" options are in green, while "neutral", "bad", and "very bad" are in red.

very good
good
neutral
bad
very bad

There's nothing I don't
love about this movie

A diagram illustrating the concept of combination features. It shows two statements on the left, each pointing to a five-point rating scale on the right. The first statement is "I don't love this movie" and the second is "There's nothing I don't love about this movie". Each statement has an arrow pointing to a vertical list of five ratings: "very good" (green), "good" (green), "neutral" (black), "bad" (red), and "very bad" (red). The "very good" and "good" options are in green, while "neutral", "bad", and "very bad" are in red.

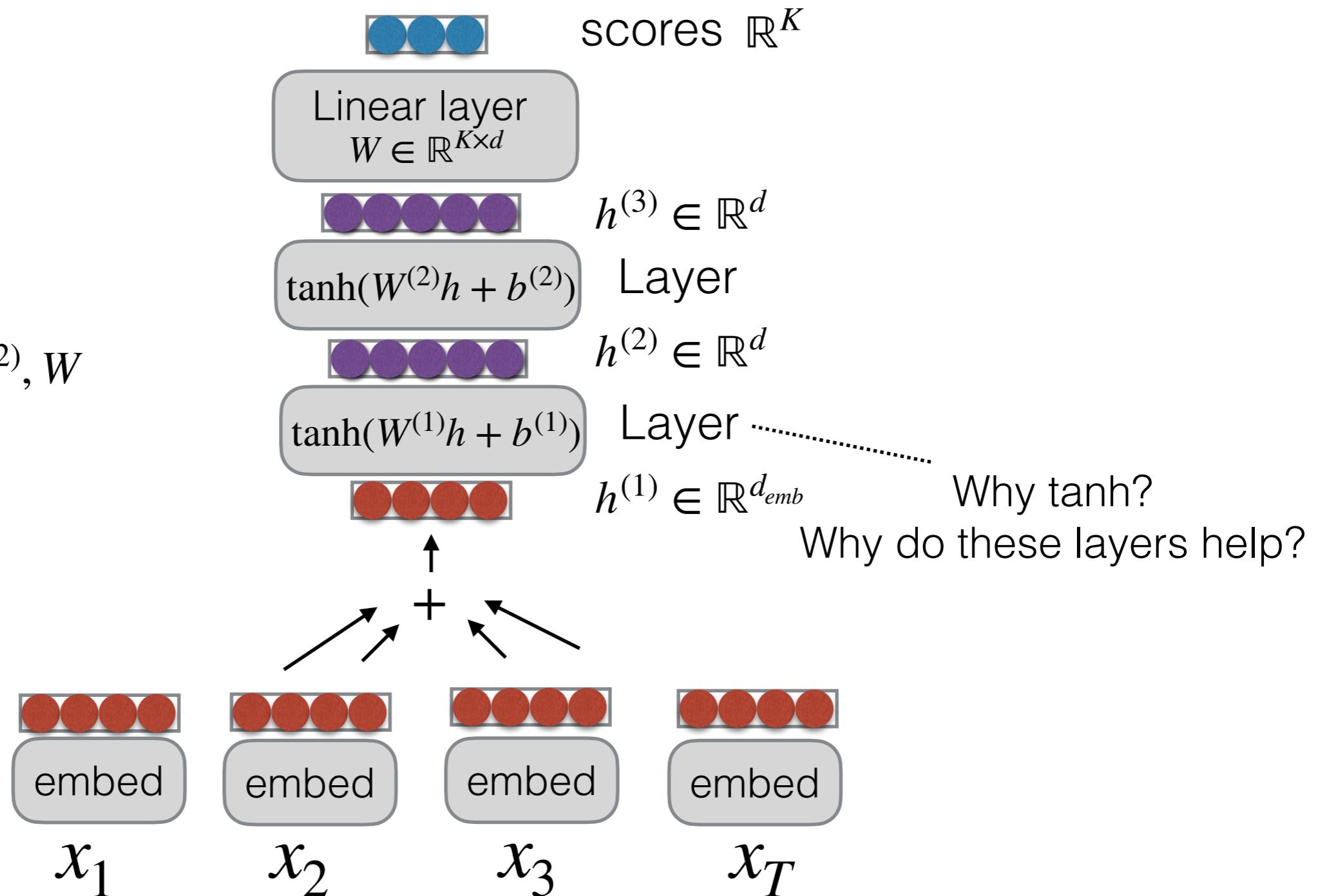
very good
good
neutral
bad
very bad

Deep CBoW

K : number of output classes

Parameters θ

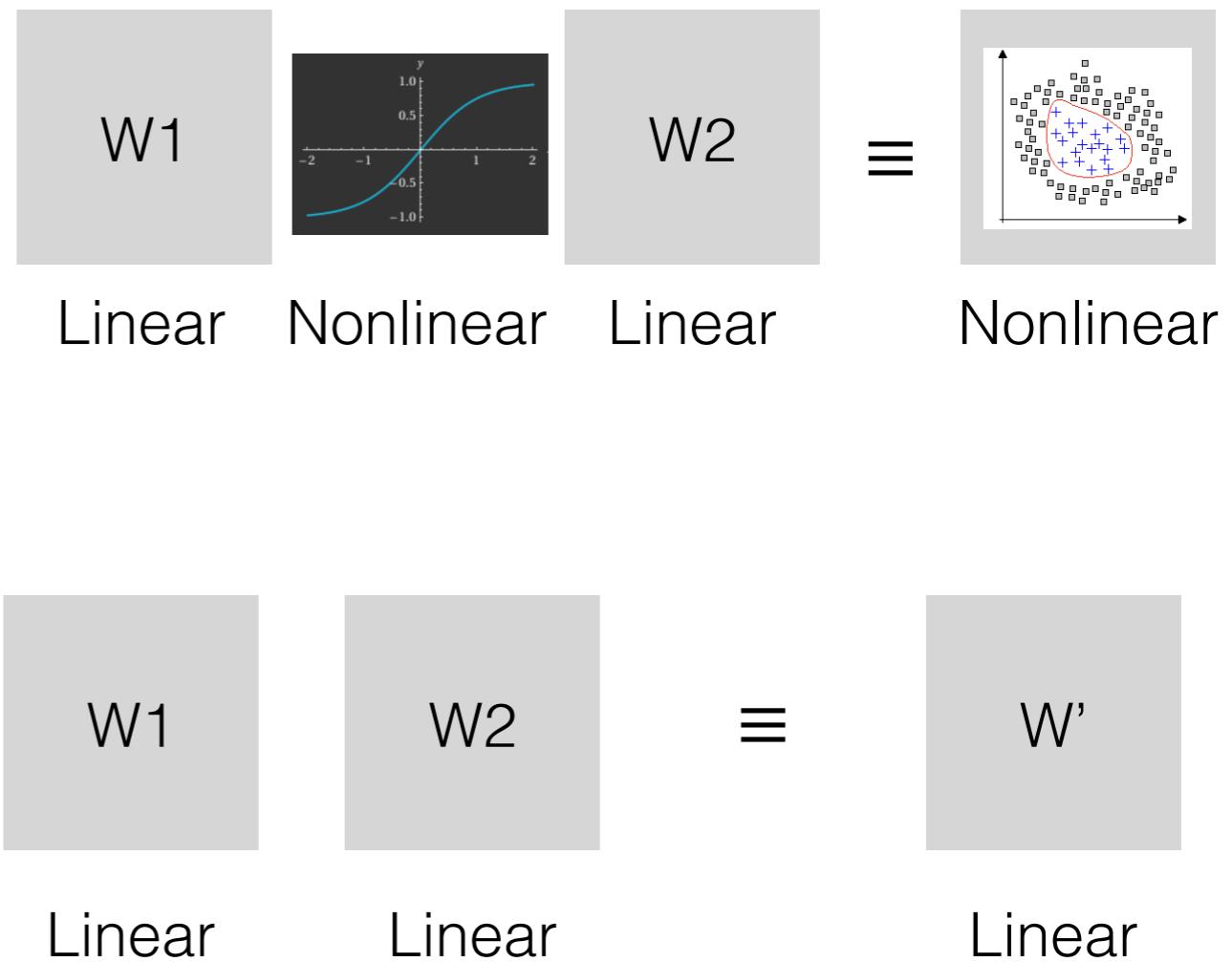
- Embedding layer
- Weights $W^{(1)}, W^{(2)}, W$
- Biases



Nonlinearities

$$\tanh(W^*h + b)$$

- *Activation functions* such as \tanh introduce *nonlinearity*
 - Non-linearities allow the neural network to model more complex patterns
 - Without activation functions, stacking matrices collapses to a linear transformation



Other activation functions: sigmoid, ReLU, GELU, see [PyTorch list](#)

Deep CBoW In Code

```
class DeepCBoW(torch.nn.Module):
    def __init__(self, vocab_size, num_labels, emb_size, hid_size):
        super(DeepCBoW, self).__init__()
        self.embedding = nn.Embedding(vocab_size, emb_size)
        self.linear1 = nn.Linear(emb_size, hid_size)      # New addition
        self.output_layer = nn.Linear(hid_size, num_labels)

    def forward(self, tokens):
        emb = self.embedding(tokens)
        emb_sum = torch.sum(emb, dim=0)
        h = emb_sum.view(1, -1)
        h = torch.tanh(self.linear1(h))    # New addition
        out = self.output_layer(h)
        return out
```

(One hidden-layer version)

What do Our Vectors Represent?

- We can learn feature combinations
 - E.g., a node in the second layer might be “feature 1 AND feature 5 are active”
 - E.g. capture things such as “not” AND “hate”
- We can learn nonlinear transformations of the previous layer’s features

Recap

- Tokenization and subword models
 - Represent sequences as tokens determined based on frequency
- Token embeddings
 - Represent tokens as learned continuous vectors
- Neural networks
 - Learn complex, non-linear feature functions
- **Next:** Training neural network models

Training neural network models

Training neural network models

- We use *gradient descent*
 - Write down a *loss function*
 - Calculate gradients of the loss function with respect to the parameters
 - Move the parameters in the direction that reduces the loss function

Example Loss: Binary Cross entropy

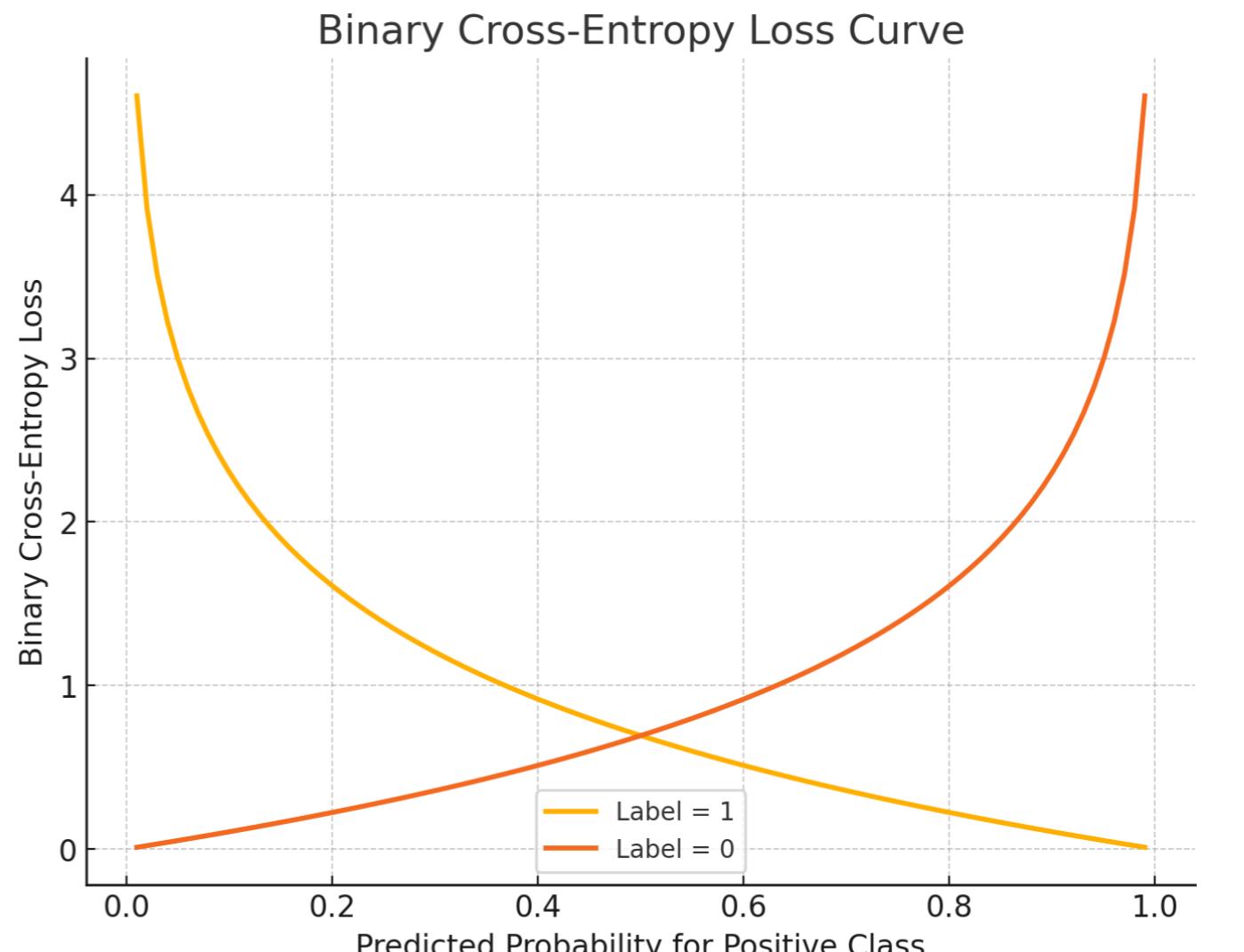
- Example task: classify tweets as positive (1) or negative (0)

- Model outputs a probability $p \in [0,1]$ for the positive class

- Use a *sigmoid*:

$$\text{Sigmoid}(s) = \sigma(s) = \frac{1}{1 + \exp(-s)}$$

- Ground truth label $y \in \{0,1\}$



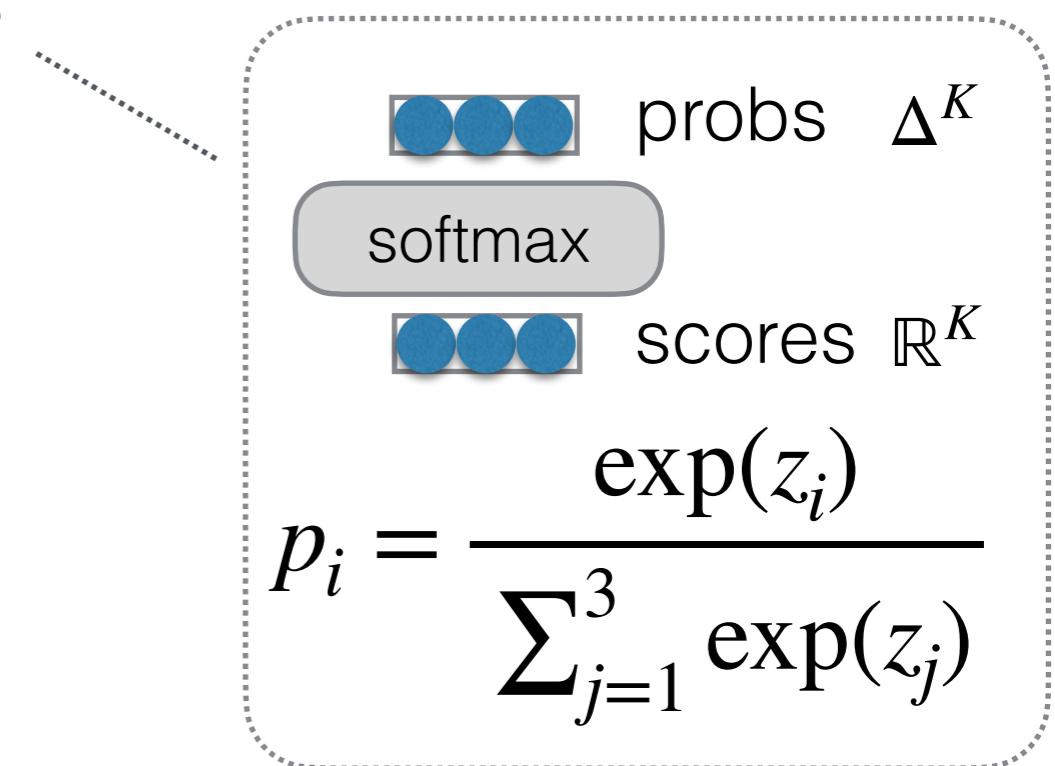
$$L_{\text{BCE}} = -y \log(p) - (1 - y) \log(1 - p)$$

Cross entropy loss (multi-class)

- Example task: classify tweets as positive (2), neutral (1), or negative (0)

$$L_{CE} = - \sum_{i=1}^3 y_i \log(p_i)$$

- Given a training example (x, y)
- Model outputs a probability vector
 - E.g. $p = [0.2, 0.5, 0.3]$
- Ground truth label: one-hot vector
 - E.g. $y = [0, 0, 1]$



Cross entropy loss (multi-class)

$$L_{CE} = - \sum_{i=1}^K y_i \log(p_i)$$

- Model assigns **high probability** to correct class:
 - $p_i \approx 1 \implies \log p_i \approx 0 \implies \text{small loss}$
- Model assigns **low probability** to correct class:
 - $p_i \approx 0 \implies \log p_i \approx -\infty \implies \text{large loss}$

Where does cross entropy loss come from?

- Minimize the KL Divergence between two distributions:

$$\begin{aligned} \underset{p_2}{\min} \text{KL}(p_1, p_2) &= \underset{p_2}{\min} - \sum_x p_1(x) \log \left(\frac{p_2(x)}{p_1(x)} \right) \\ &\equiv \underset{p_2}{\min} \sum_x -p_1(x) \log p_2(x) + p_1(x) \log p_1(x) \\ &\equiv \underset{p_2}{\min} - \sum_x p_1(x) \log p_2(x) \end{aligned}$$

- In our example:

- $p_1 = [0,0,1]$, and $p_2 = [0.2,0.5,0.3]$

Cross entropy loss (in code)

```
def ce_loss(logits, target):
    log_probs = torch.nn.functional.log_softmax(logits, dim=1)
    loss = -log_probs[:, target]
    return loss
```

Implemented in standard libraries, e.g. nn.CrossEntropyLoss

Training neural network models

- We use *gradient descent*
 - Write down a *loss function*
 - ***Calculate gradients of the loss function with respect to the parameters***
 - Move the parameters in the direction that *reduces the loss function*

Calculating gradients

- $p = \underbrace{\sigma(wx + b)}_z$, where $\sigma(x) = \frac{1}{1 + \exp(-x)}$
- $L = -y \log p - (1 - y) \log(1 - p)$
- $\frac{\partial L}{\partial w} = \frac{\partial L}{\partial p} \frac{\partial p}{\partial z} \frac{\partial z}{\partial w}$
- $\frac{\partial L}{\partial p} = -\frac{y}{p} + \frac{1 - y}{1 - p}$
$$= \frac{p - y}{p(1 - p)}$$
- $\frac{\partial p}{\partial z} = p(1 - p)$
- $\frac{\partial z}{\partial w} = x$
- Multiplying the three terms, we get $\frac{\partial L}{\partial w} = (p - y)x$

Coming up soon:
gradient computation
handled automatically

Training neural network models

- We use *gradient descent*
 - Write down a *loss function*
 - Calculate gradients of the loss function with respect to the parameters
 - **Move the parameters in the direction that reduces the loss function**

Optimizing Parameters

- Standard stochastic gradient descent does

$$g_t = \frac{\nabla_{\theta_{t-1}} \ell(\theta_{t-1})}{\text{Gradient of Loss}}$$

$$\theta_t = \theta_{t-1} - \frac{\eta g_t}{\text{Learning Rate}}$$

- There are many other optimization options! (e.g., see Ruder 2016 in the references.)

In Code

Loss
Optimizer

```
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(model.parameters(), lr=5e-4)

for EPOCH in range(10):
    random.shuffle(train)
    train_loss = 0.0
    start = time.time()
    model.train()
    for x, y in train:
        x = torch.tensor(x, dtype=torch.long)
        y = torch.tensor([y])
        logits = model(x)
        loss = criterion(logits, y)
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()
```

Compute loss

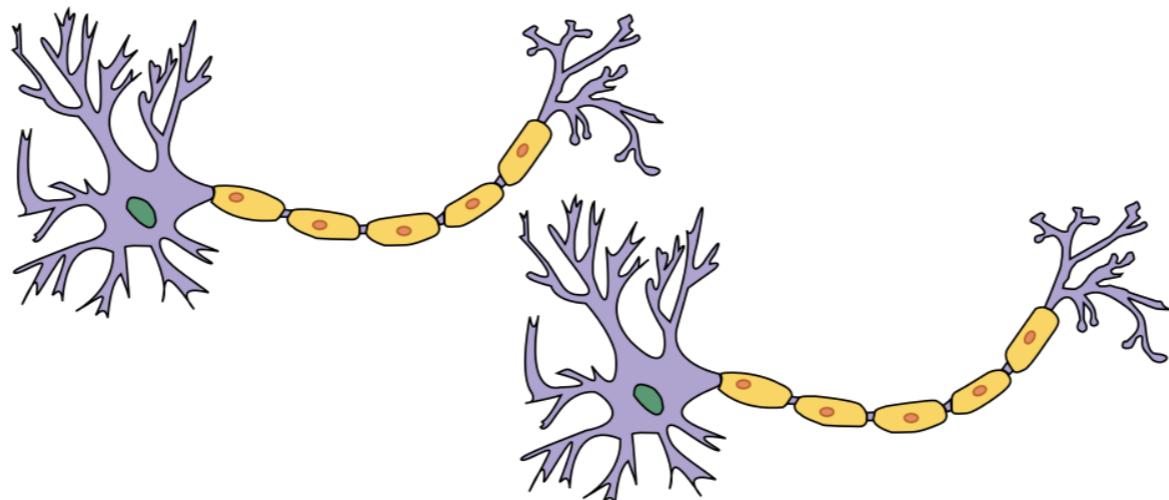
Compute gradients

Update parameters

What is a Neural Net?: Computation Graphs

“Neural” Nets

Original Motivation: Neurons in the Brain



Current Conception: Computation Graphs

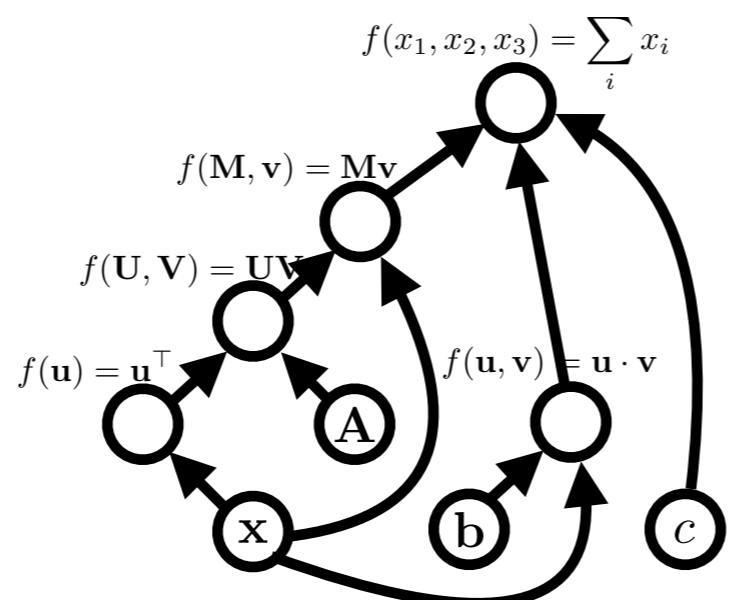


Image credit: Wikipedia

expression:

x

graph:

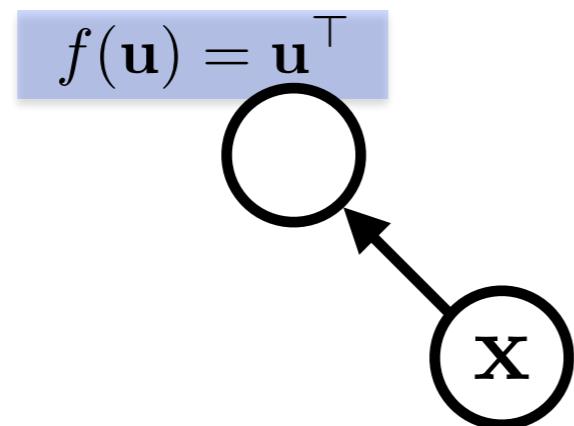
A **node** is a {tensor, matrix, vector, scalar} value



An **edge** represents a function argument. They are just pointers to nodes.

A **node** with an incoming **edge** is a **function** of that edge's tail node.

A **node** knows how to compute its value and the *gradient with respect to each input, here* $\frac{\partial f(\mathbf{u})}{\partial \mathbf{u}}$



$$\frac{\partial F}{\partial \mathbf{u}} = \frac{\partial F}{\partial f(\mathbf{u})} \frac{\partial f(\mathbf{u})}{\partial \mathbf{u}}$$

Incoming
gradient

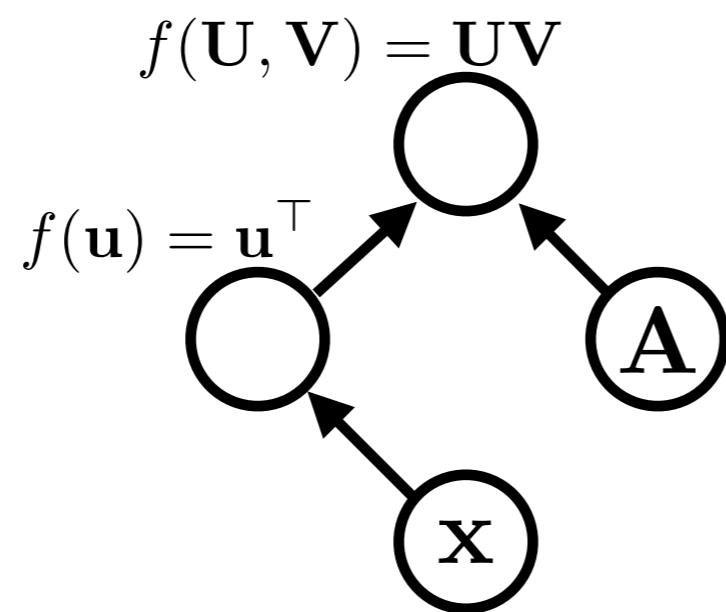
Local
Gradient

expression:

$$\mathbf{x}^\top \mathbf{A}$$

graph:

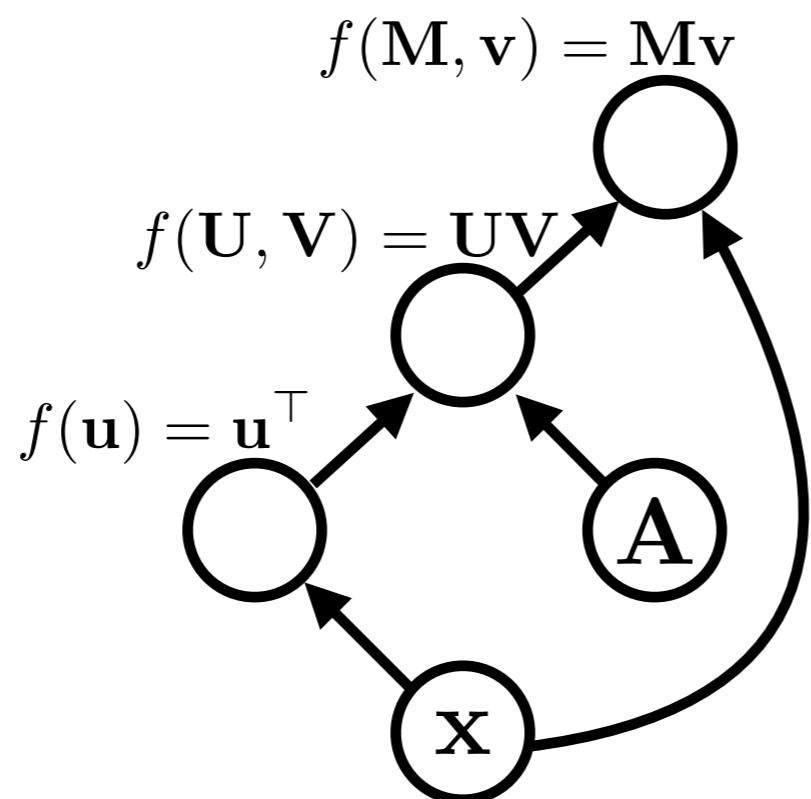
Functions can be nullary, unary, binary, ... n -ary. Often they are unary or binary.



expression:

$$\mathbf{x}^\top \mathbf{A} \mathbf{x}$$

graph:

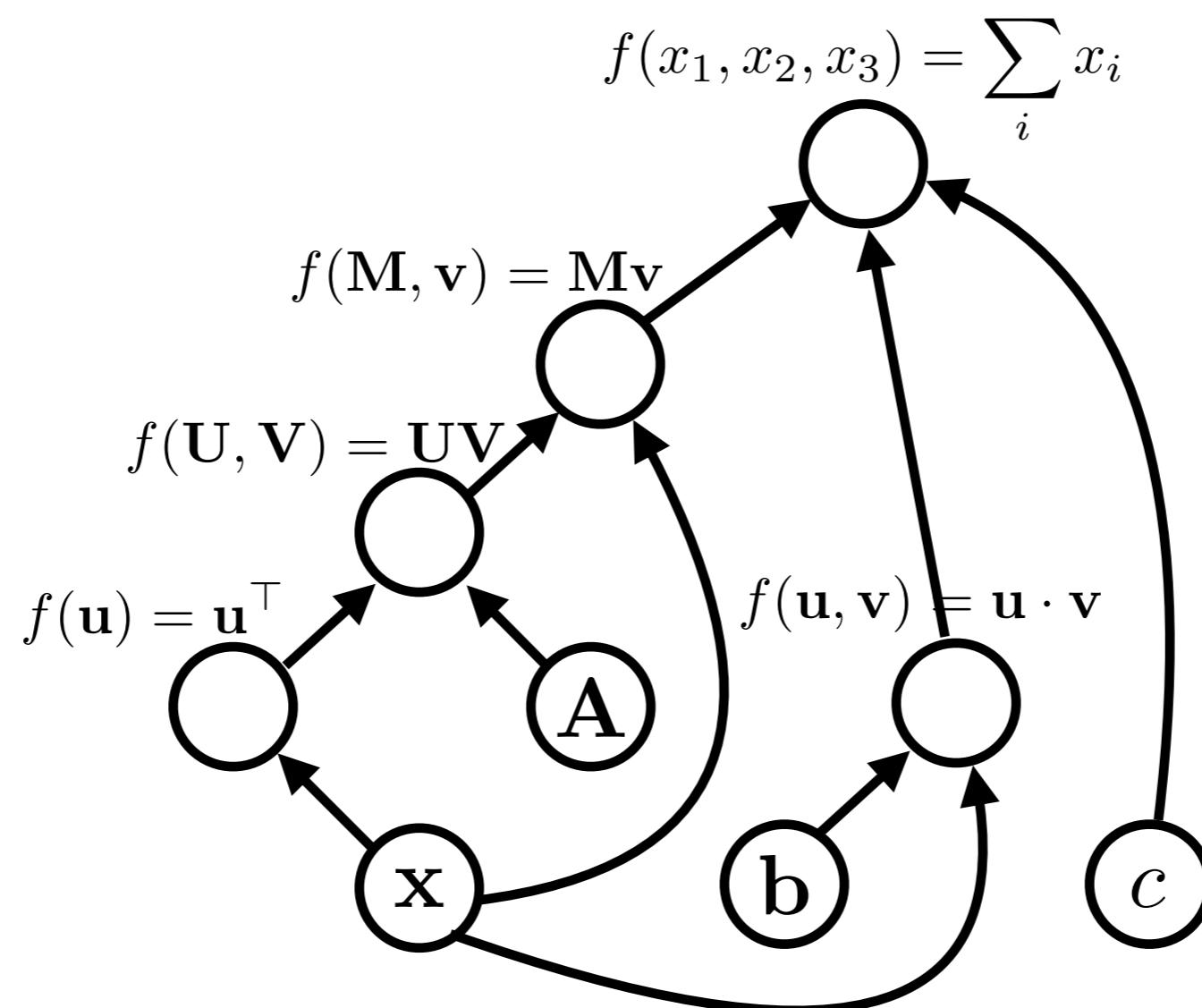


Computation graphs are directed and acyclic (in DyNet)

expression:

$$\mathbf{x}^\top \mathbf{A} \mathbf{x} + \mathbf{b} \cdot \mathbf{x} + c$$

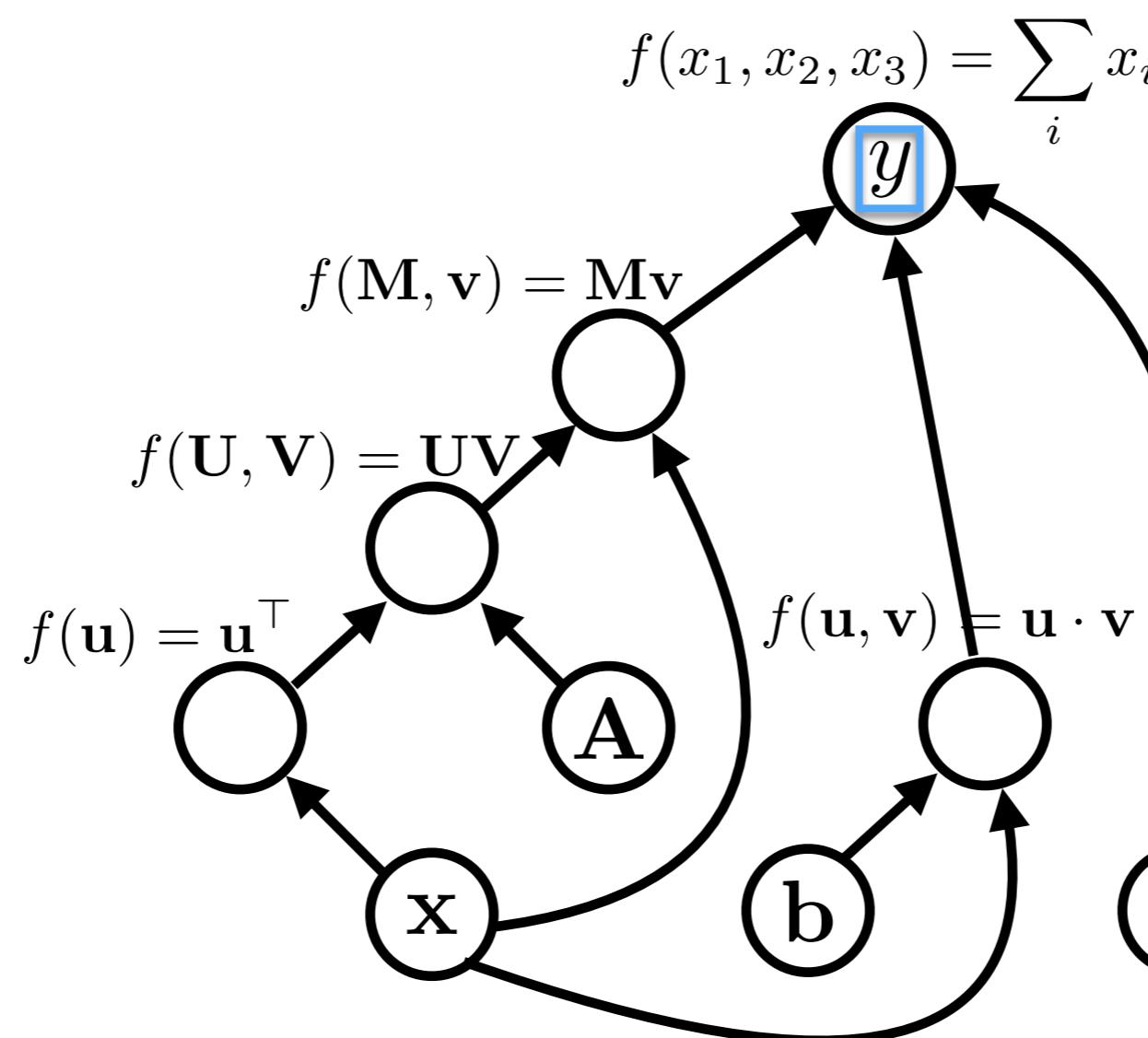
graph:



expression:

$$y = \mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b} \cdot \mathbf{x} + c$$

graph:



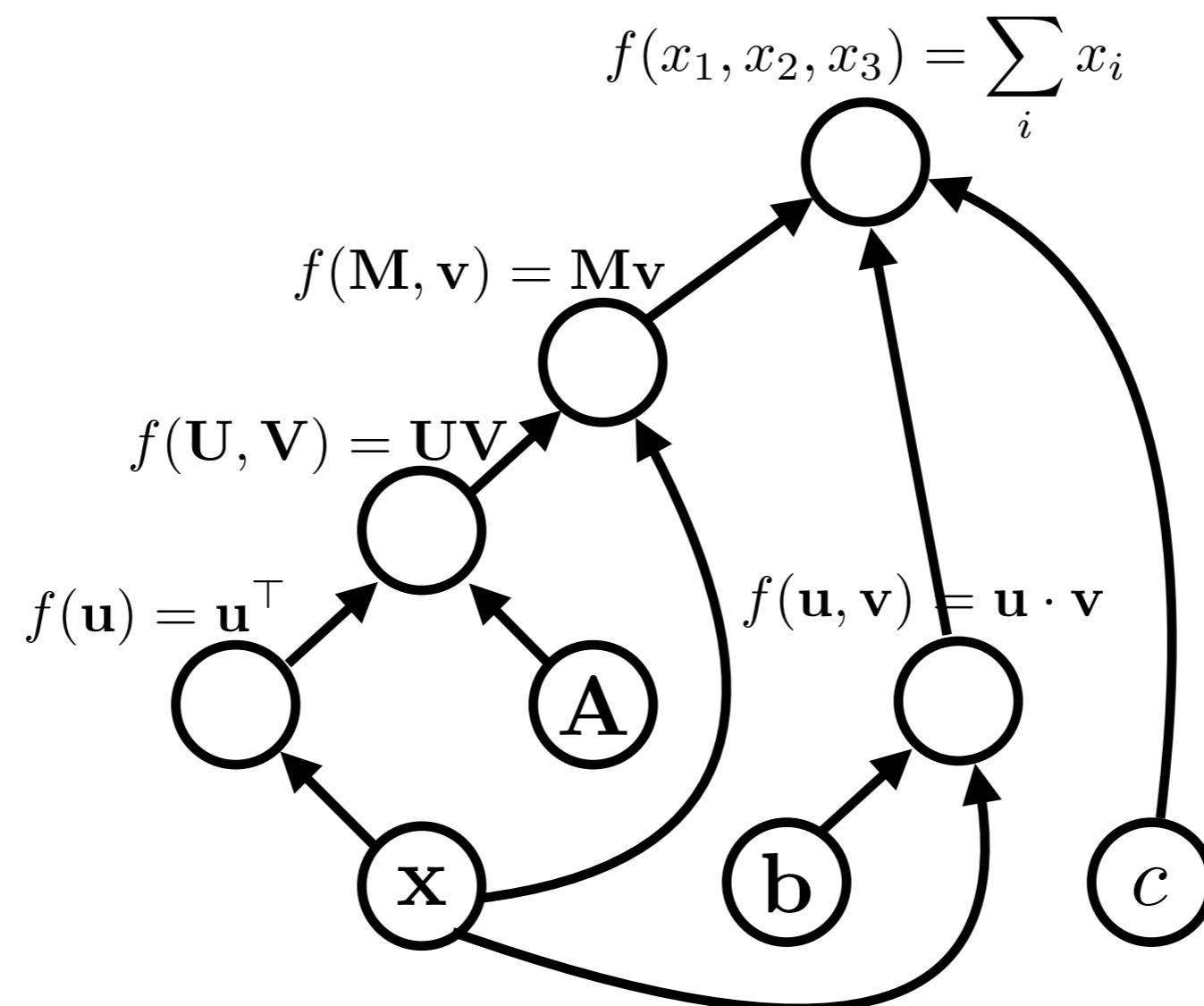
variable names are just labelings of nodes.

Algorithms (1)

- **Graph construction**
- **Forward propagation**
 - In topological order, compute the **value** of the node given its inputs

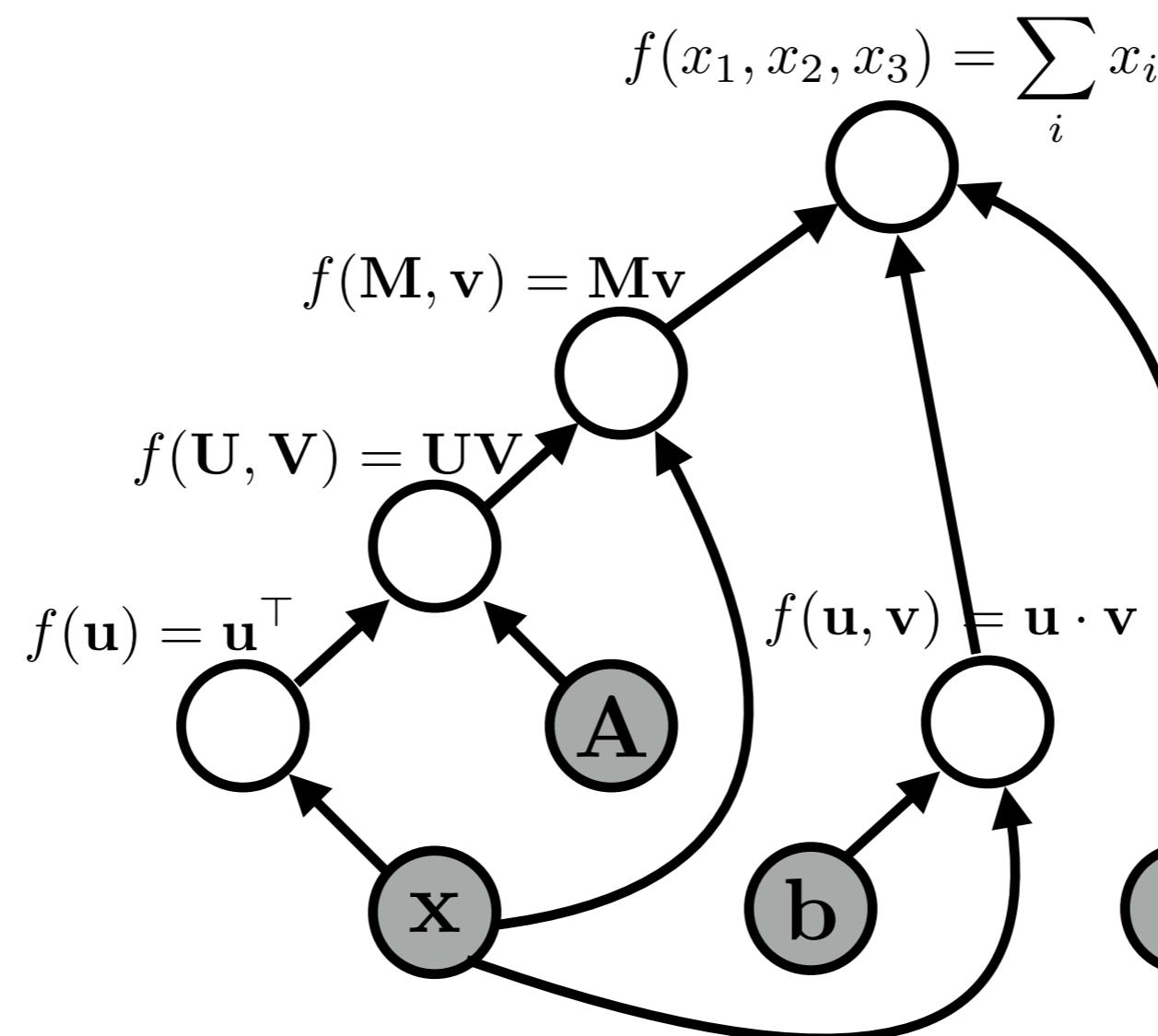
Forward Propagation

graph:



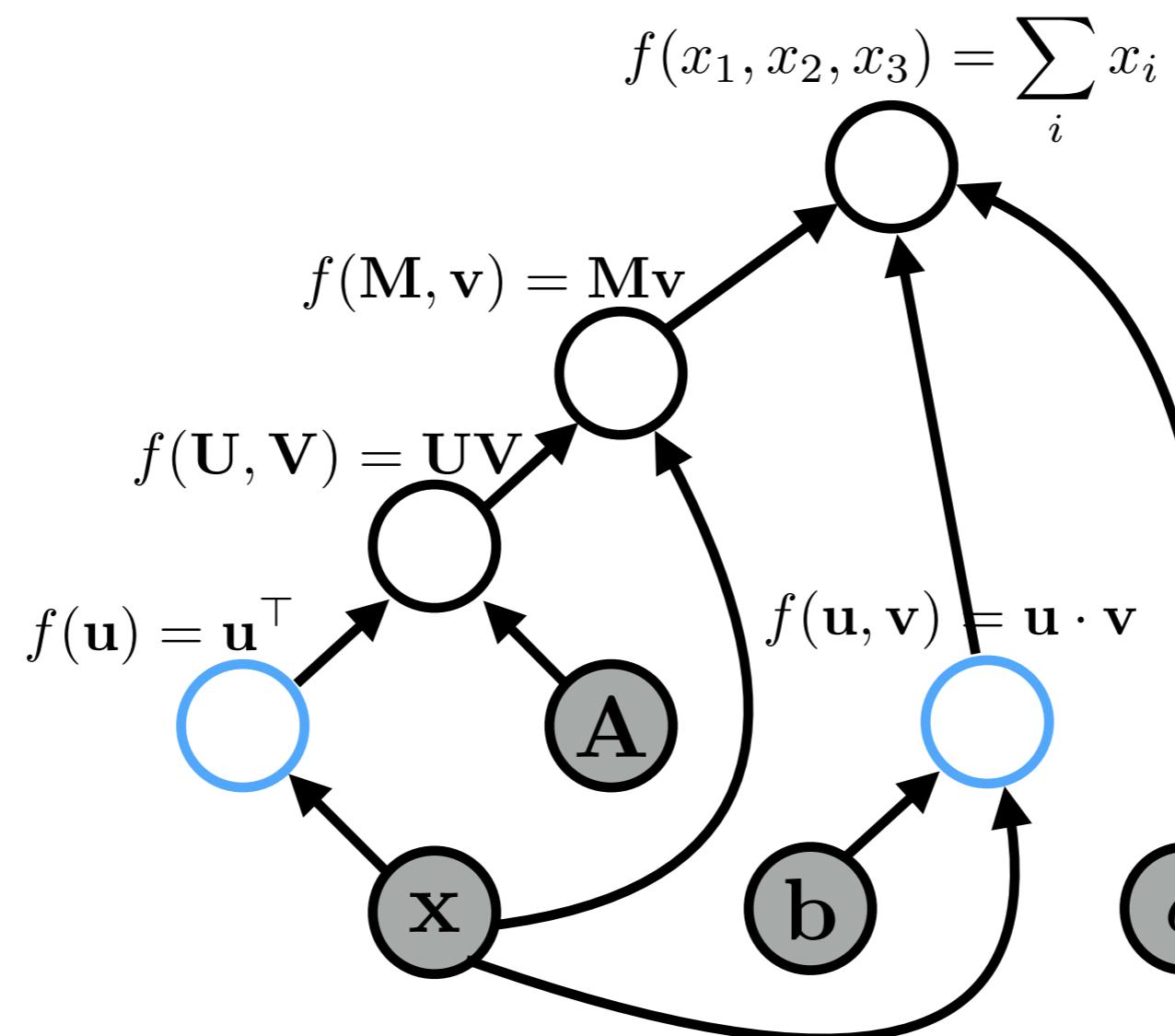
Forward Propagation

graph:



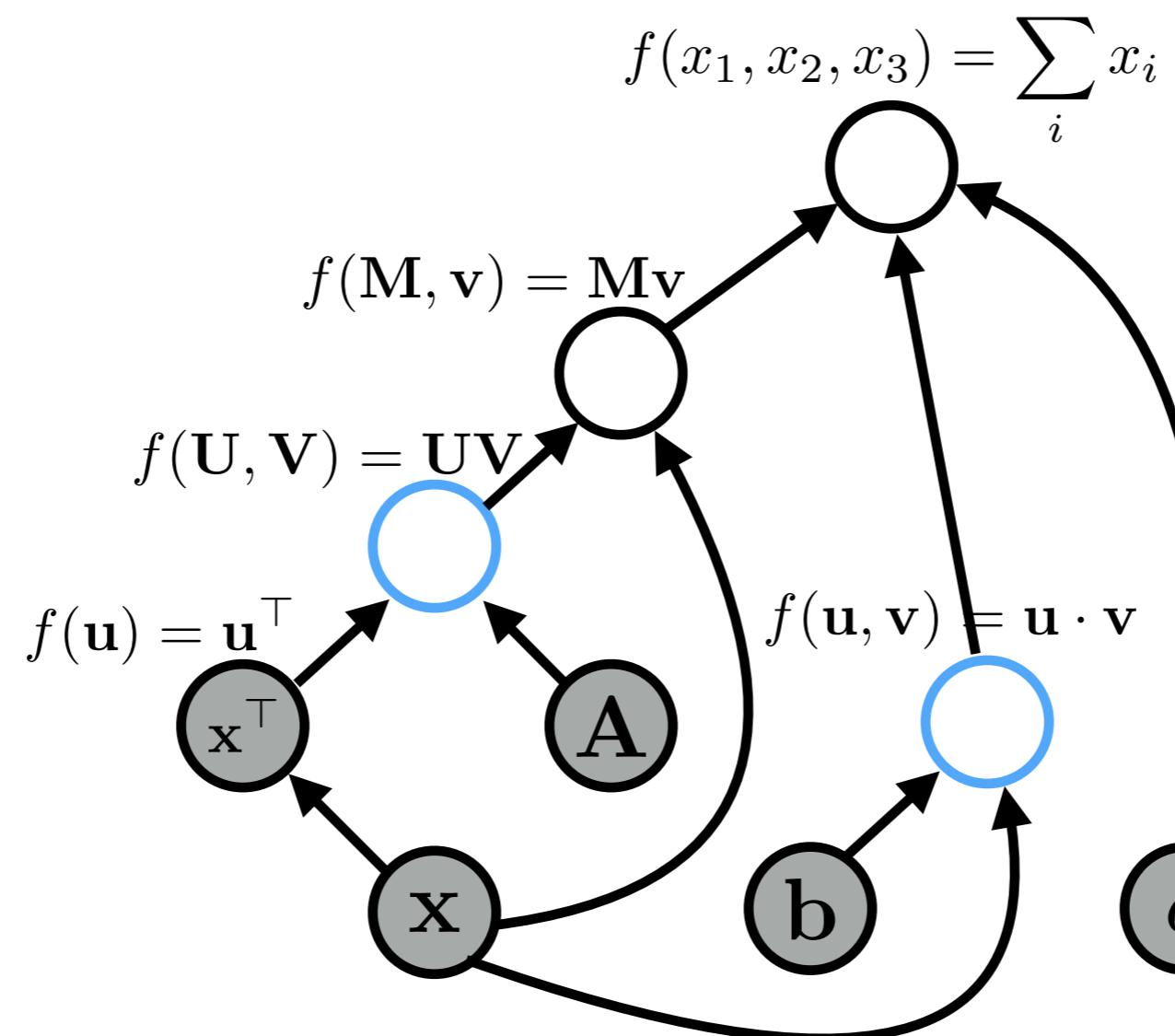
Forward Propagation

graph:



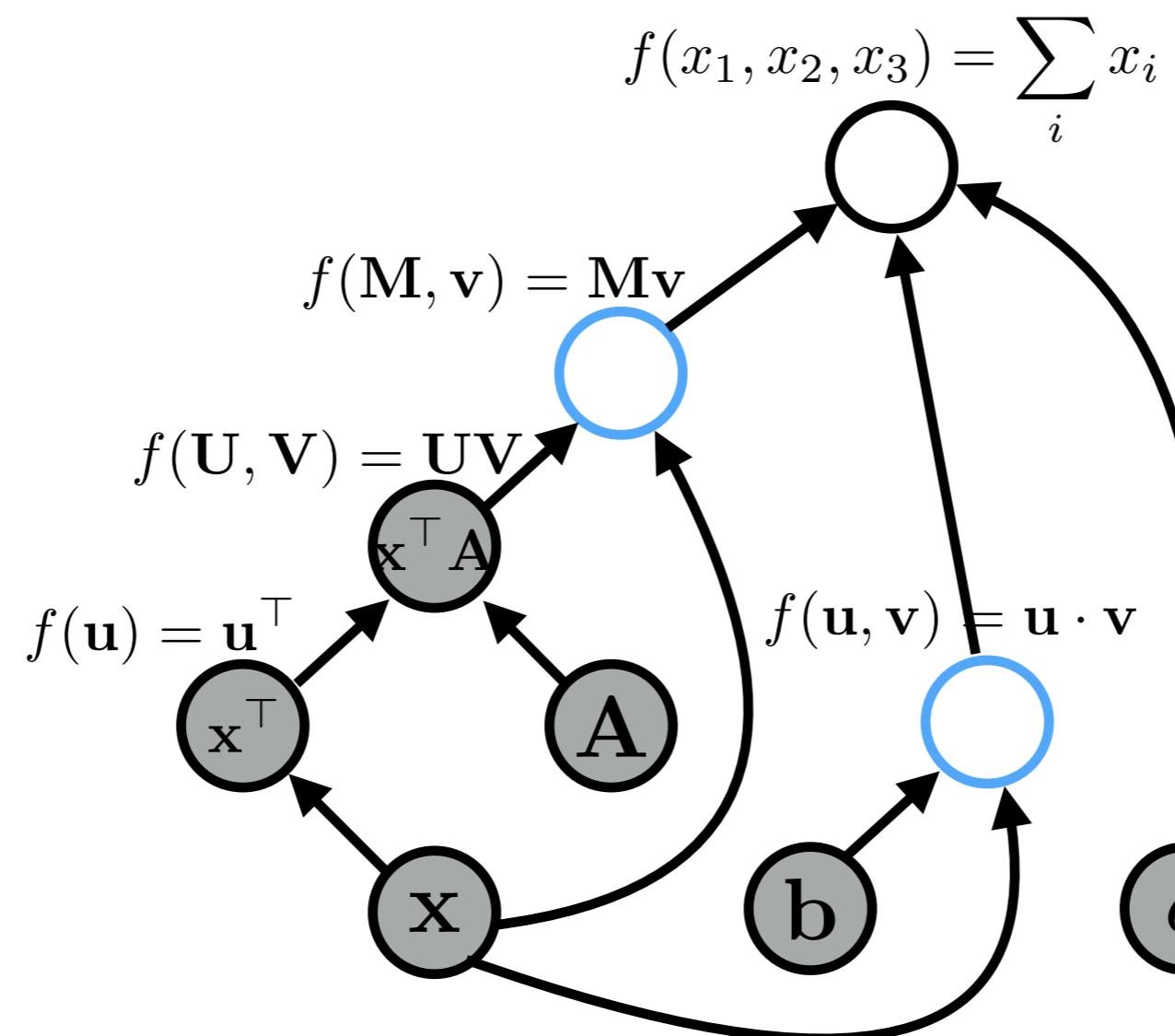
Forward Propagation

graph:



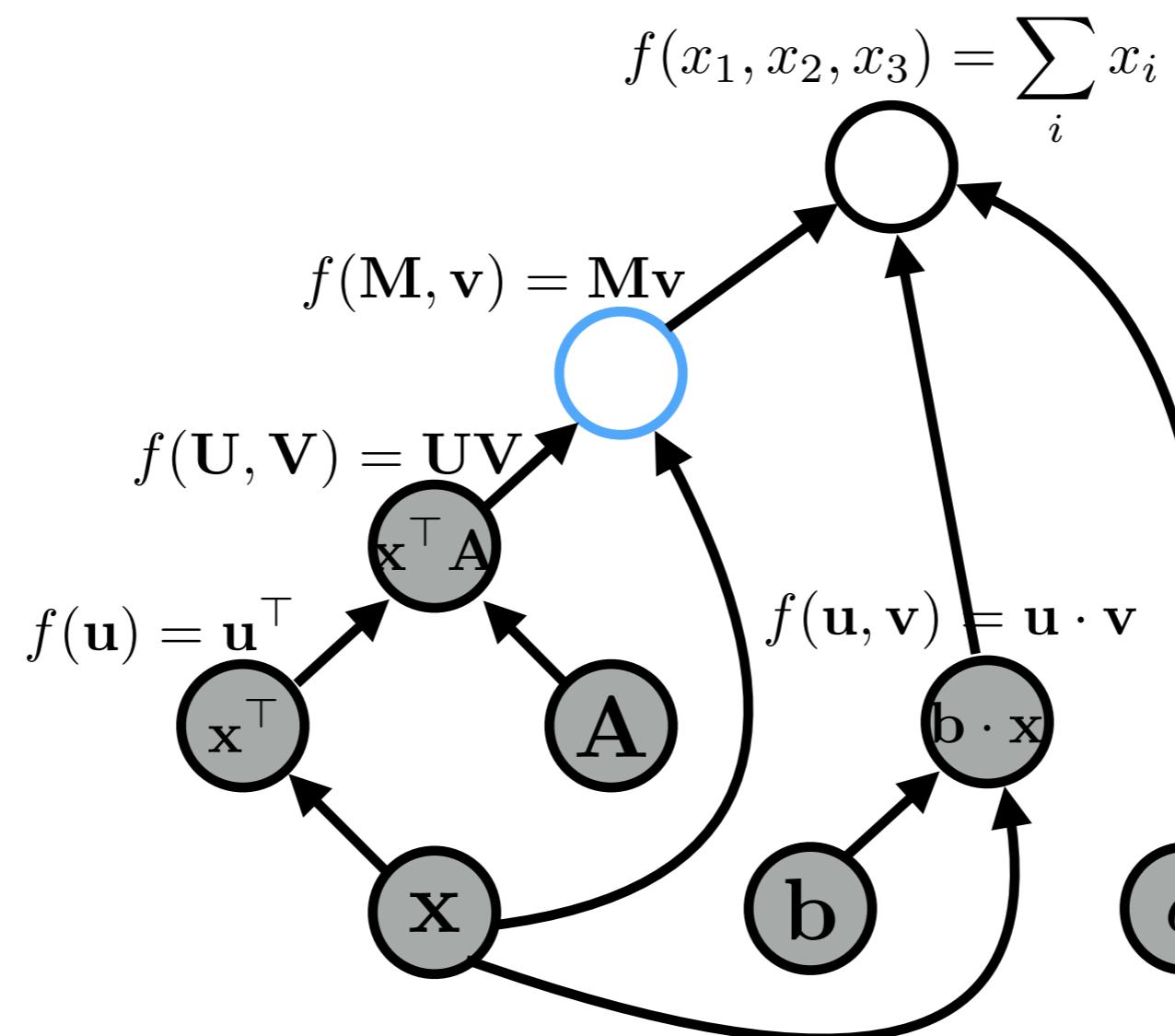
Forward Propagation

graph:



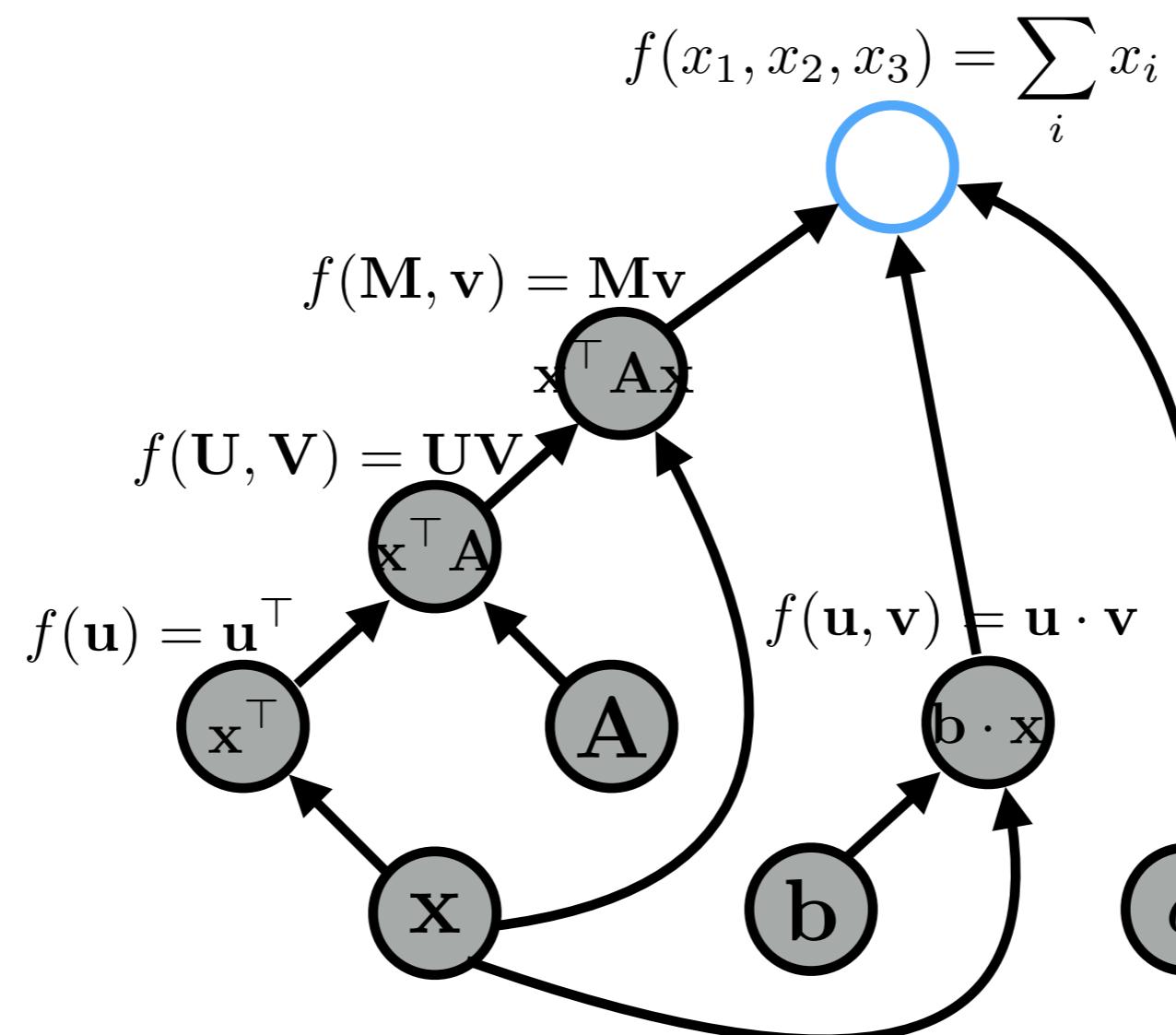
Forward Propagation

graph:



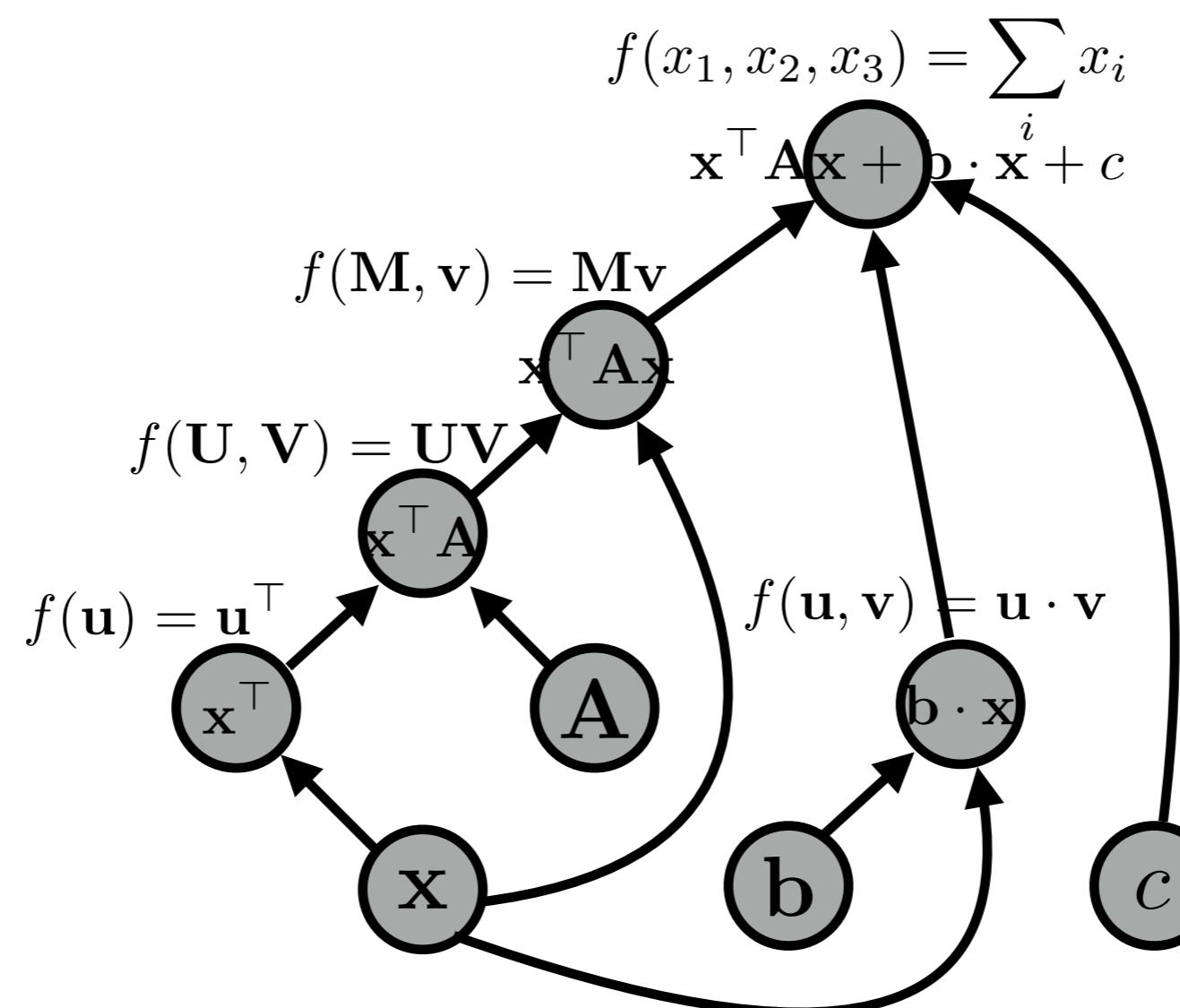
Forward Propagation

graph:



Forward Propagation

graph:

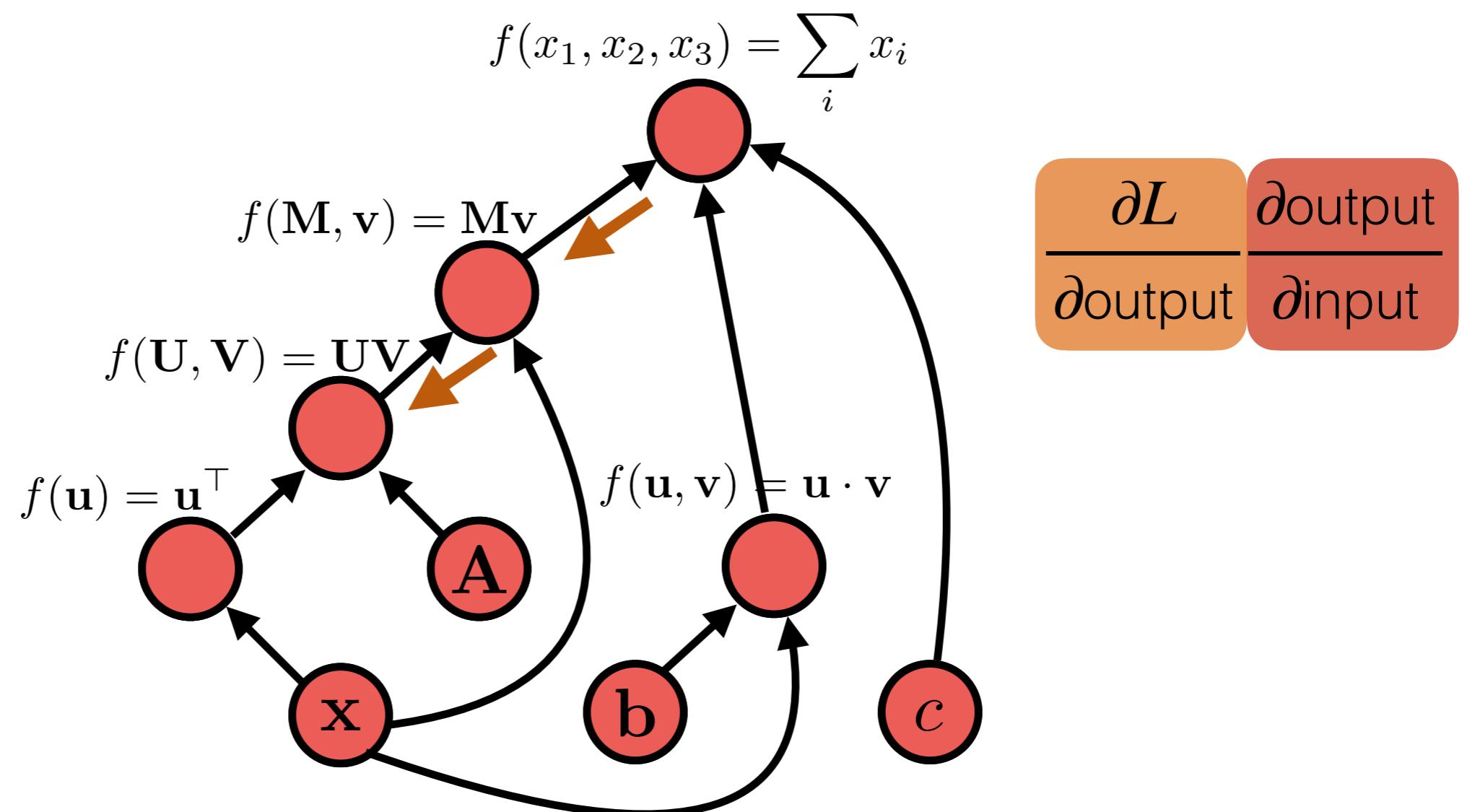


Algorithms (2)

- **Back-propagation:**
 - Process examples in reverse topological order
 - Calculate the gradients of the parameters with respect to the final value (usually a loss function)
- **Parameter update:**
 - Move the parameters in the direction of this gradient
$$W -= \alpha * \frac{dL}{dW}$$

Back Propagation

graph:



Basic Process in Neural Network Frameworks

- Create a model
- For each example
 - **create a graph** that represents the computation you want
 - **calculate the result** of that computation
 - if training, perform **back propagation and update**

Concrete Implementation

Neural Network Frameworks



Developed by FAIR/Meta

Most widely used in NLP

Favors dynamic execution

More flexibility

Most vibrant ecosystem

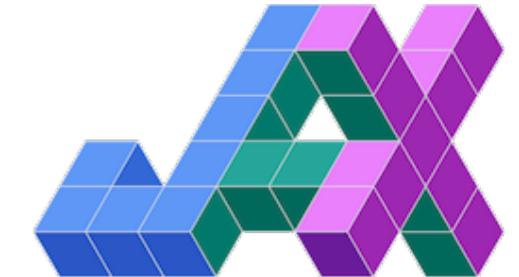


Developed by Google

Used in some NLP projects

Favors definition+compilation

Conceptually simple parallelization



Code Example

- Classify tweets as positive, negative, or neutral
- BoW, CBoW, DeepCBoW

```
# Classify an example with our trained model
tweet = "I'm learning so much in advanced NLP!"
tokens = torch.tensor(sp.encode(tweet), dtype=torch.long)
scores = model(tokens)[0].detach()
predict = scores.argmax().item()
label_to_text[predict]
```

[131]

```
...    'positive'
```

Recap

- Tokenization and subword models
 - Represent sequences as tokens determined based on frequency
- Token embeddings
 - Represent tokens as learned continuous vectors in \mathbb{R}^d
- Neural networks
 - Learn complex, non-linear feature functions
- Training a neural network
 - Choose a loss, construct a differentiable graph, take gradients

Thank you!