NUMERICAL METHODS

Lecture 2

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Open Methods

- Simple Fixed-Point Iteration
- Newton-Raphson Method
- Secant Methods
- MATLAB function: fzero

Open Methods

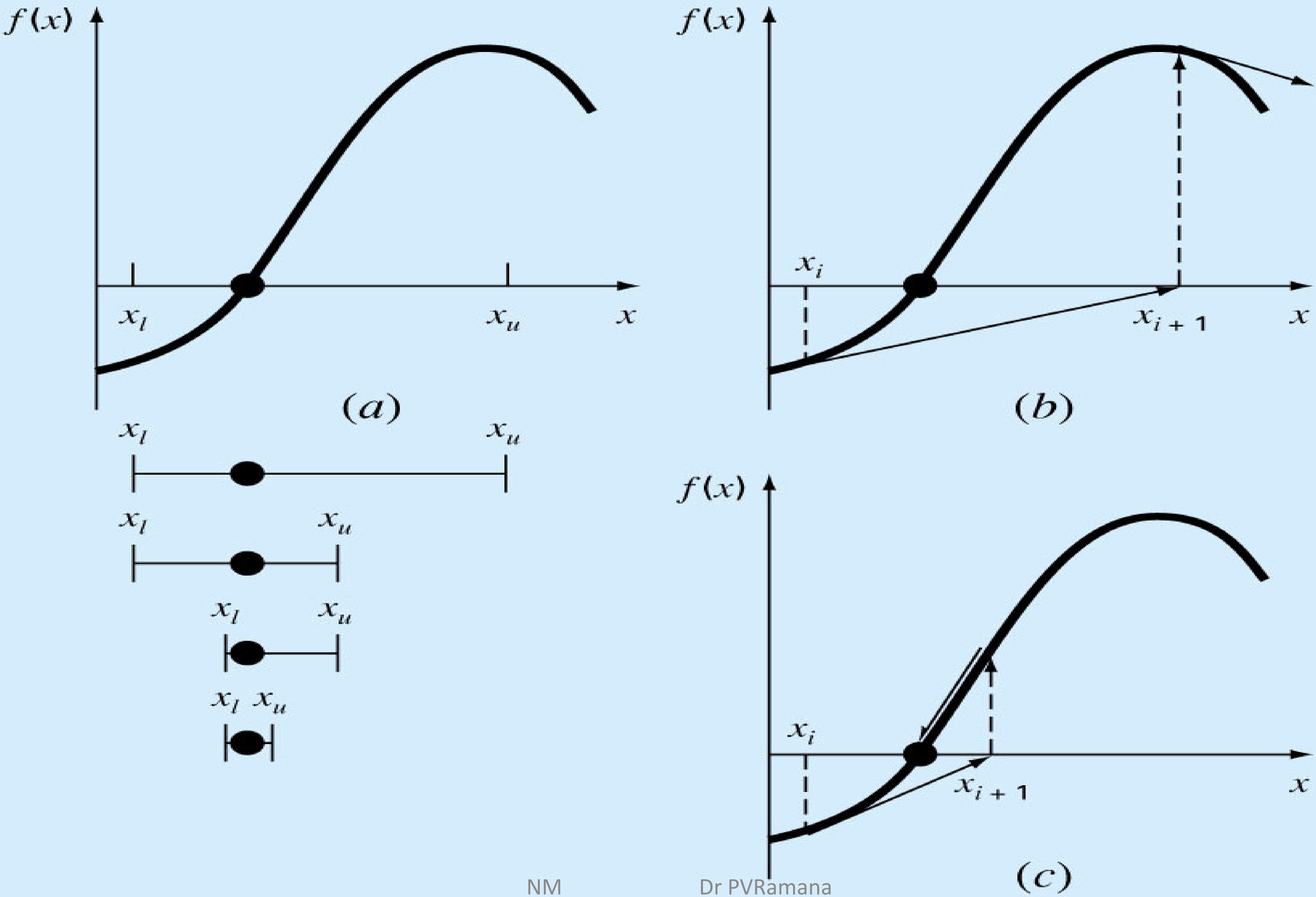
There exist <u>open methods</u> which do not require bracketed intervals

Newton-Raphson method, Secant Method, Muller's method, fixed-point iterations

First one to consider is the fixed-point method

Converges <u>faster </u>but <u>not necessary converges</u>

Bracketing and Open Methods



$x = g(x)$, $f(x) = x - g(x) = 0$

Steps of Fixed-Point Iteration

- **Step 1: Guess x_0 and calculate $y_0 = g(x_0)$**
- **Step 2: Let $x_1 = g(x_0)$**
- **Step 3:Examining if x_1 is the solution of $f(x) = 0$**
- **Step 4. If not, repeat the iteration, $x_0 = x_1$**

Fixed-Point Iteration

First open method is fixed point iteration

Idea: rewrite original equation $f(x) = 0$ into another form $x = g(x)$

Use iteration $x_{i+1} = g(x_i)$ to find a value that reaches convergence

Example:

$$x^2 - 2x + 3 = 0 \quad \Rightarrow \quad x = \frac{x^2 + 3}{2}$$

$$\sin x = 0 \quad \Rightarrow \quad x = \sin x + x$$

Simple Fixed-point Iteration

- Rearrange the function so that x is on the left-hand side of the equation:

$$f(x) = 0 \quad \Rightarrow \quad g(x) = x$$

$$x_k = g(x_{k-1}) \quad x_o \text{ is given}, \quad k = 1, 2, \ldots$$

- Bracketing methods are "convergent" if you have a bracket to start with
- Fixed-point methods may sometimes "converge", depending on the starting point (initial guess) and how the function behaves.

EXAMPLE: 1

$$f(x) = x^2 - x - 2 = 0$$

Rewrite it as : $x = g(x)$

$$x = x^2 - 2$$

or

$$x = \sqrt{x + 2}$$

or

$$x = 1 + \frac{2}{x}$$

M

Find a root of $x^4 - x - 10 = 0$

Consider $\mathbf{g1(x) = 10 / (x^3-1)}$ and the fixed point iterative scheme $\mathbf{x_{i+1} = 10 / (x_i^3 - 1)}$, $\mathbf{i = 0, 1, 2, \ldots}$
let the initial guess $\mathbf{x_0}$ be **2.0**

i	0	1	2	3	4	5	6	7	8
x_i	2	1.429	5.214	0.071	-10.004	-9.978E-3	-10	-9.99E-3	-10

Consider another function $\mathbf{g2(x) = (x + 10)^{1/4}}$ and the fixed point iterative scheme
$\mathbf{x_{i+1} = (x_i + 10)^{1/4}}$, $\mathbf{i = 0, 1, 2, \ldots}$ let the initial guess $\mathbf{x_0}$ be **1.0, 2.0 and 4.0**

i	0	1	2	3	4	5	6
x_i	1.0	1.82116	1.85424	1.85553	1.85558	1.85558	
x_i	2.0	1.861	1.8558	1.85559	1.85558	1.85558	
x_i	4.0	1.93434	1.85866	1.8557	1.85559	1.85558	1.85558

That is for **g2** the iterative process is converging to **1.85558 with** any initial guess.

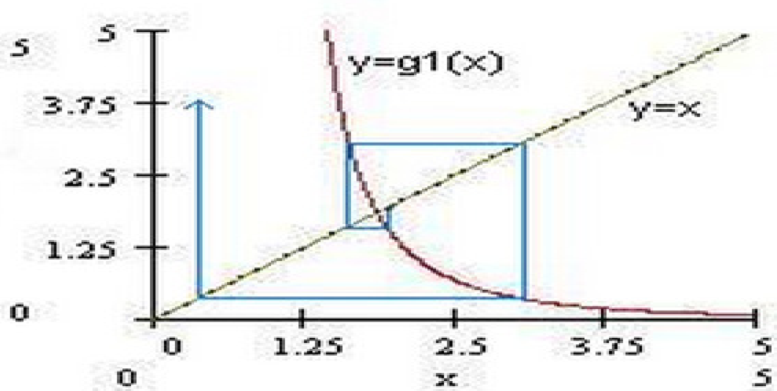


Fig g1

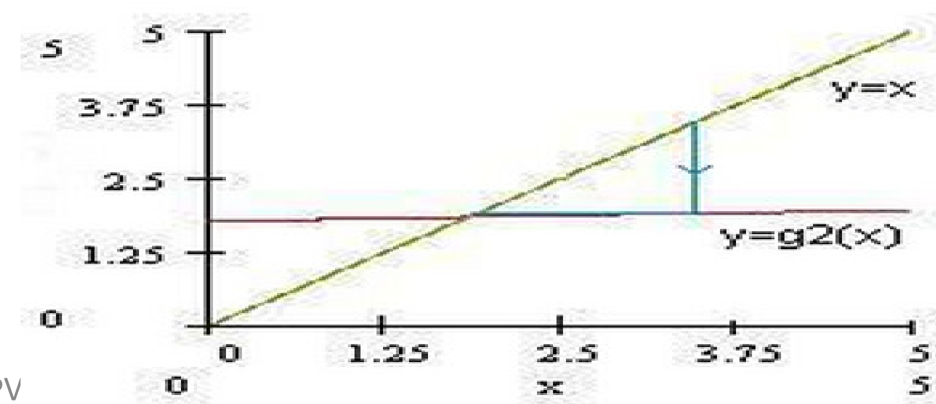
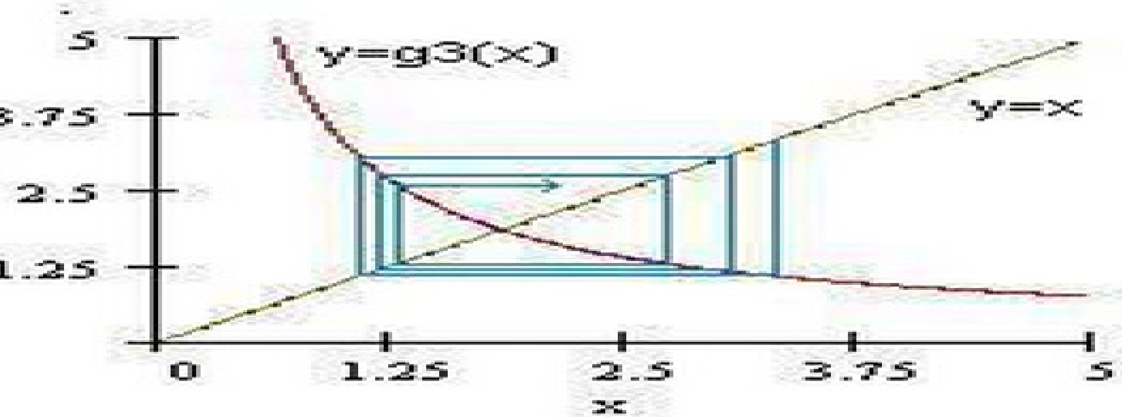


Fig g2

Consider **$g3(x) = (x+10)^{1/2}/x$** and the fixed point iterative scheme **$x_{i+1} = (x_i+10)^{1/2}/x_i$, $i = 0, 1, 2, \ldots$** let the initial guess **x_0** be **1.8**,

i	0	1	2	3	4	5	6	...	98
x_i	1.8	1.9084	1.80825	1.90035	1.81529	1,89355	1.82129	...	1.8555



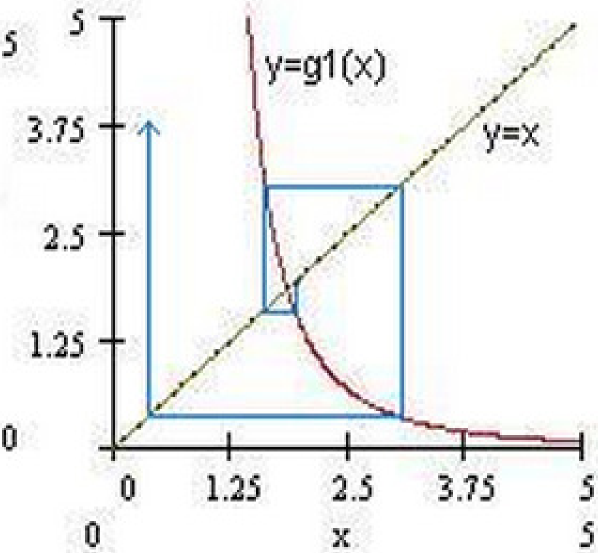


Fig g1

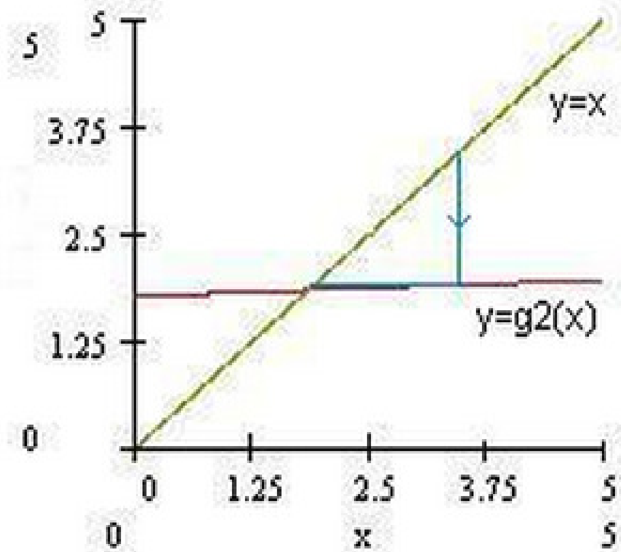


Fig g2

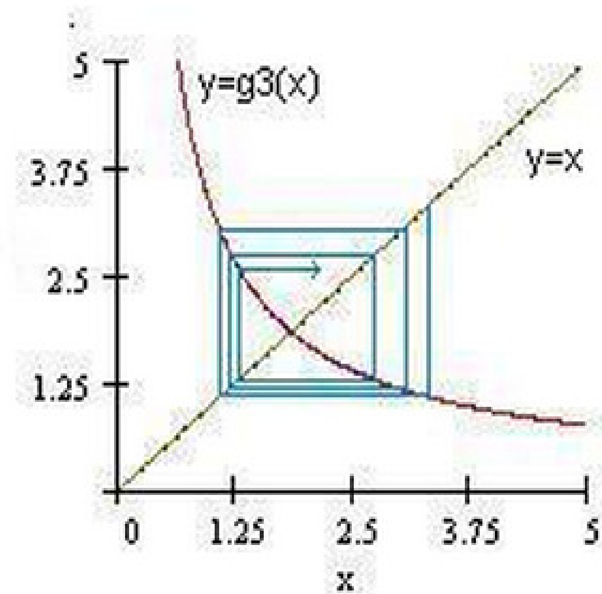
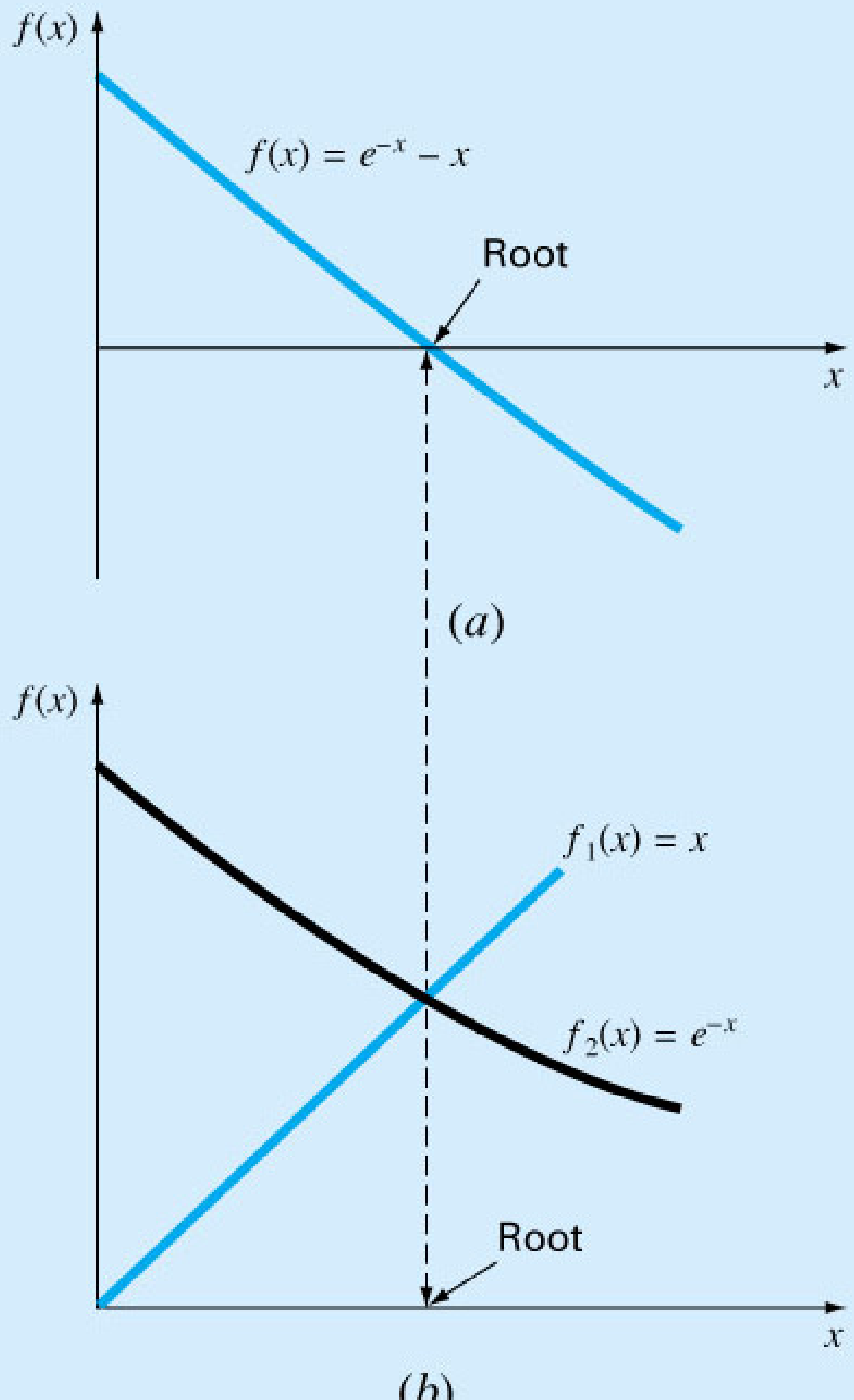


Fig g3

Simple Fixed-Point Iteration

Two Alternative Graphical Methods



$$f(x) = 0$$

$$f_1(x) = f_2(x)$$

$$f(x) = f_1(x) - f_2(x) = 0$$

Fixed-Point Iteration

Convergent

Divergent