NUMERICAL METHODS

$$U^{n+1} = U^n + \Delta t\, f(U^n)$$

$$\frac{\partial v}{\partial t} + V \cdot \nabla v = \nabla \cdot (k \nabla v) + g(v)$$

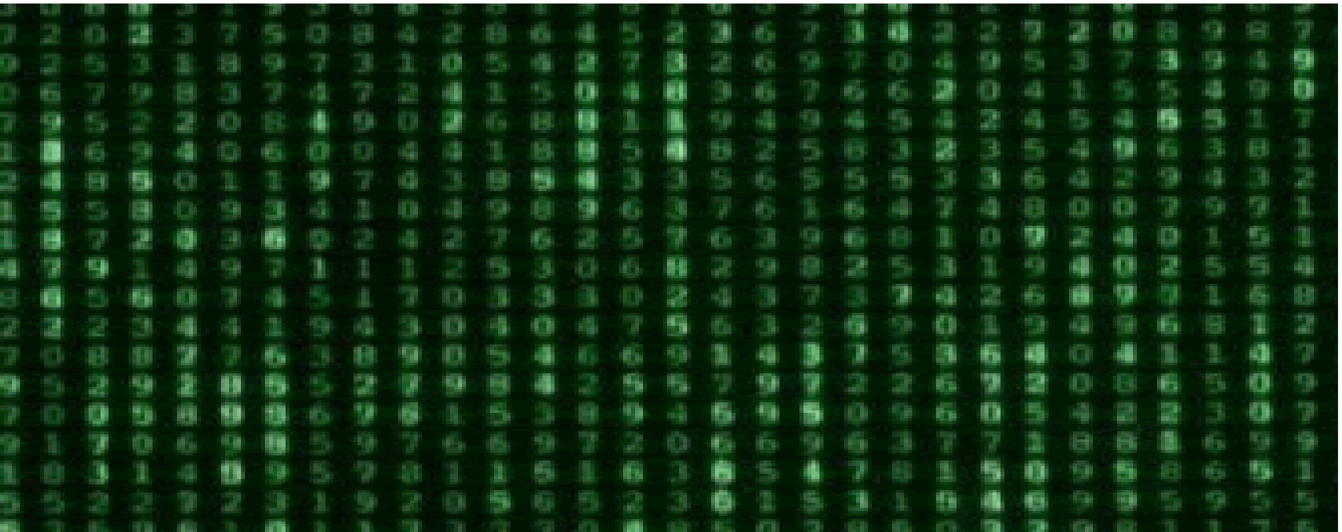
$$(\lambda + \mu)\nabla(\nabla \cdot u) + \mu \nabla^2 u = \alpha(3\lambda + 2\mu)\nabla T - \rho b$$

Lecture 3

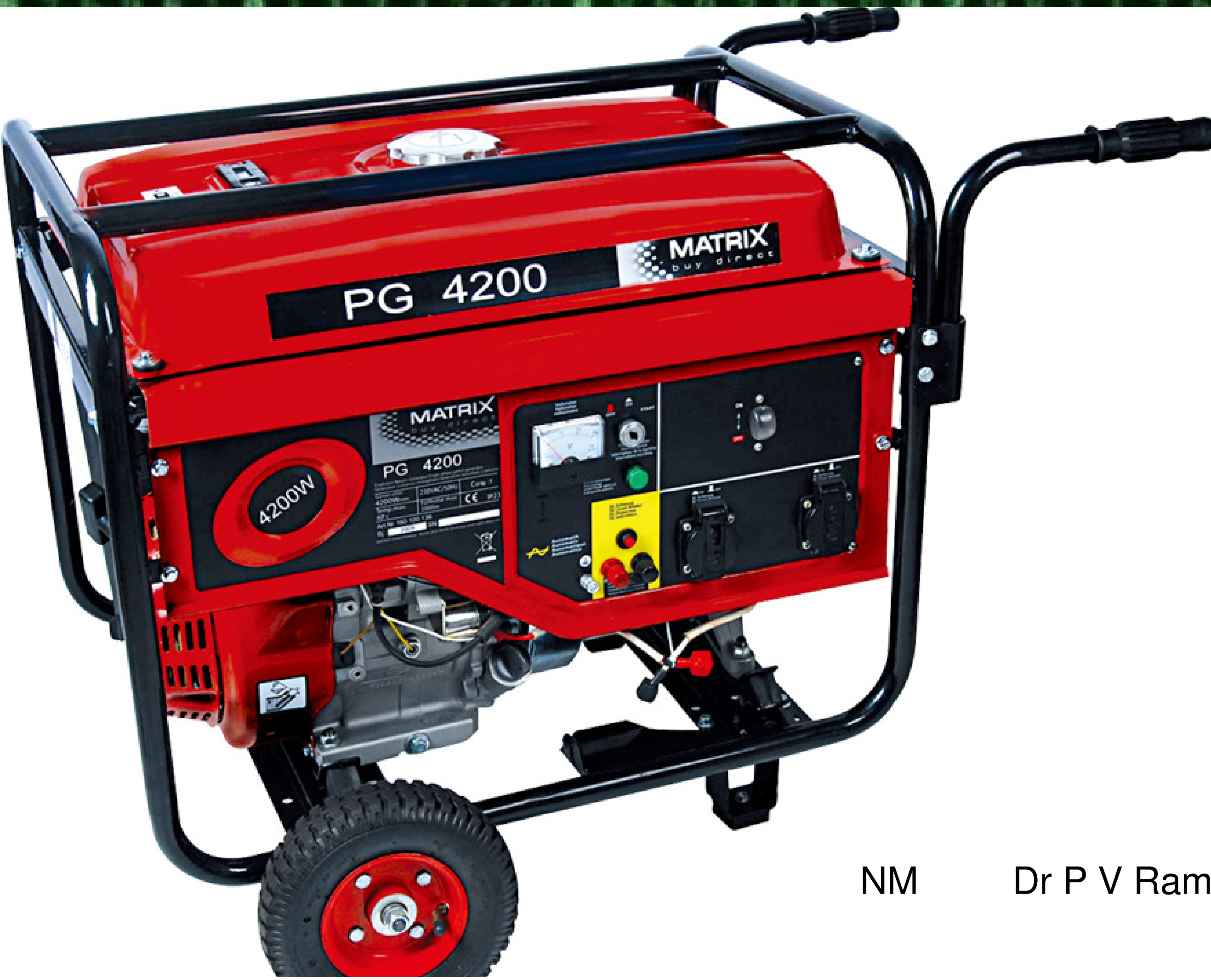
$$\rho\left(\frac{\partial u}{\partial t} + V \cdot \nabla u\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x$$

$$\nabla^2 u = f$$

Dr. P V Ramana

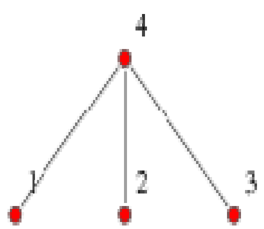


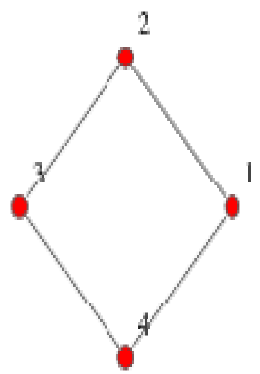
$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

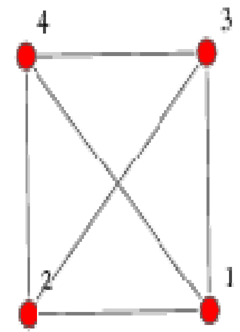


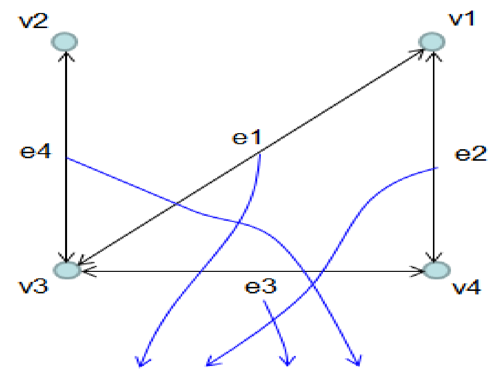
$$\begin{bmatrix} 0.4 & 0.3 & 0.3 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.3 & 0.2 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.4 & 0.3 \\ 0.1 & 0.1 & 0.3 & 0.3 & 0.5 \end{bmatrix}$$

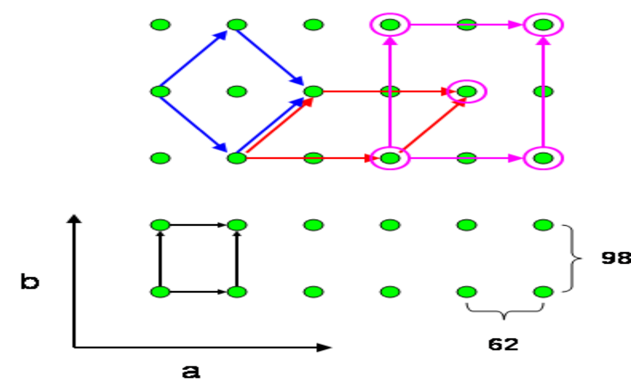
GOAL ANNUAL TURNOVER	£60,000											
		QUARTER ONE			QUARTER TWO			QUARTER THREE			QUARTER FOUR	
QUARTERLY TURNOVER		£18,000			£20,000			£12,000			£10,000	
	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC	JAN	FEB	MARCH
MONTHLY TURNOVER	£6,000	£6,000	£7,000	£7,000	£7,000	£6,000	£5,500	£4,500	£2,000	£3,000	£3,000	£4,000
SERVICE												
Group training	£600	£600	£600	£800	£800	£800	£700	£500	£0	£600	£600	£600
One-off Bootcamps	£500	£500	£700	£1,000	£1,000	£800	£700	£450				£500
Fitness holidays	£2,000	£2,000	£2,500	-		-		-	-	-	-	-
Pre & post natal	£200	£200	£200	£300	£300	£200	£400	£350		£200	£200	£200
Personal training	£2,500	£2,500	£2,800	£3,700	£4,700	£4,000	£3,500	£3,000	£1,800	£2,000	£2,000	£2,500
Online coaching	£200	£200	£200	£200	£200	£200	£200	£200	£200	£200	£200	£200
MONTHLY TOTAL	£6,000	£6,000	£7,000	£6,000	£7,000	£6,000	£5,500	£4,500	£2,000	£3,000	£3,000	£4,000
ANNUAL TOTAL												£60,000











$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{array}{c|cccc} & e1 & e2 & e3 & e4 \\ \hline v1 & 1 & 1 & 0 & 0 \\ v2 & 0 & 0 & 0 & 1 \\ v3 & 1 & 0 & 1 & 1 \\ v4 & 0 & 1 & 1 & 0 \end{array}$$

e1 and e2 are connected with v1. Both of them get in (+1)

e4 is connected with v2 . It gets in (+1)

e1,e3 and e4 are connected with v3. All of them get in (+1)

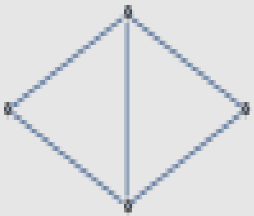
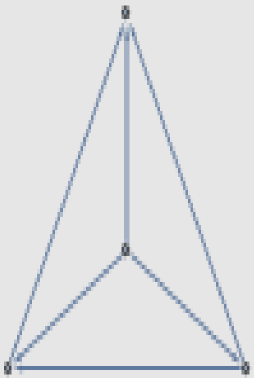
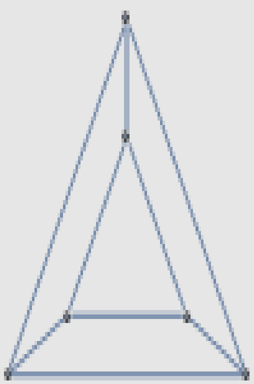
e2 and e3 are connected with v4. Both of them get in (+1)

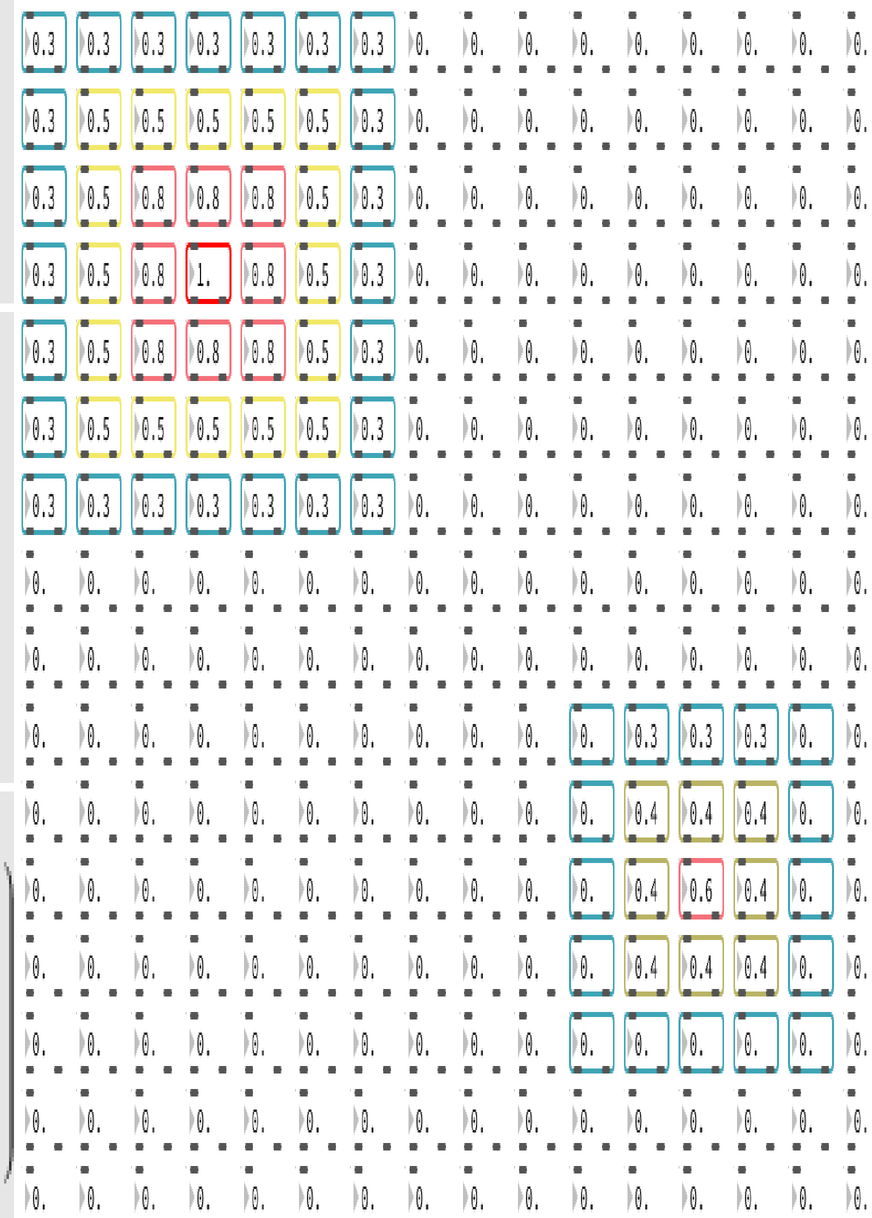
e4 gets into v3 (+1) and gets into v2 (+1)

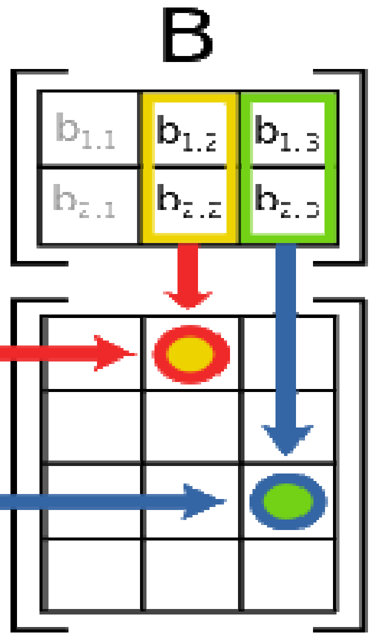
e3 gets into v4 (+1) and gets into v3 (+1)

e2 gets into v4 (+1) and gets into v1 (+1)

e1 gets into v1 (+1) and gets into v3 (+1)

Graph	Adjacency Matrix	Incidence Matrix	Kirchhoff Matrix
	$\begin{pmatrix} 0&1&1&0 \\ 1&0&1&1 \\ 1&1&0&1 \\ 0&1&1&0 \end{pmatrix}$	$\begin{pmatrix} 1&1&0&0&0 \\ 1&0&1&1&0 \\ 0&1&1&0&1 \\ 0&0&0&1&1 \end{pmatrix}$	$\begin{pmatrix} 2&-1&-1&0 \\ -1&3&-1&-1 \\ -1&-1&3&-1 \\ 0&-1&-1&2 \end{pmatrix}$
	$\begin{pmatrix} 0&1&1&1 \\ 0&0&1&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix}$	$\begin{pmatrix} -1&-1&-1&0&0&0 \\ 1&0&0&-1&-1&0 \\ 0&1&0&1&0&-1 \\ 0&0&1&0&1&1 \end{pmatrix}$	$\begin{pmatrix} 3&-1&-1&-1 \\ 0&3&-1&-1 \\ 0&0&3&-1 \\ 0&0&0&3 \end{pmatrix}$
	$\begin{pmatrix} 0&1&1&1&0&0 \\ 1&0&1&0&1&0 \\ 1&1&0&0&0&1 \\ 1&0&0&0&1&1 \\ 0&1&0&1&0&1 \\ 0&0&1&1&1&0 \end{pmatrix}$	$\begin{pmatrix} 1&1&1&0&0&0&0&0&0 \\ 1&0&0&1&1&0&0&0&0 \\ 0&1&0&1&0&1&0&0&0 \\ 0&0&1&0&0&0&1&0&1 \\ 0&0&0&0&1&0&1&1&0 \\ 0&0&0&0&0&1&0&1&1 \end{pmatrix}$	$\begin{pmatrix} 3&-1&-1&-1&0&0 \\ -1&3&-1&0&-1&0 \\ -1&-1&3&0&0&-1 \\ -1&0&0&3&-1&-1 \\ 0&-1&0&-1&3&-1 \\ 0&0&-1&-1&-1&3 \end{pmatrix}$





ABB^T

$$B = \begin{bmatrix} 2 & 2 & 3 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \\ -3 & 4 & 0 \\ 1 & -3 & 4 \end{bmatrix} \qquad B^T = \begin{bmatrix} 2 & -2 & 3 & -3 & 1 \\ 2 & -1 & 4 & 4 & -3 \\ 3 & 2 & 1 & 0 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 & 0 & 3 & -4 \\ 3 & -2 & 2 & 1 & 0 \\ -1 & 3 & -1 & -2 & -4 \\ 3 & 3 & -4 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 29 & -12 \\ 13 & 20 & 7 \\ -9 & -5 & -14 \\ -20 & 1 & 3 \end{bmatrix} = AB$$

$$\begin{bmatrix} 8 & -39 & 83 & 137 & -142 \\ 87 & -32 & 126 & 41 & -19 \\ -70 & -5 & -61 & 7 & -50 \\ -29 & 45 & -53 & 64 & -11 \end{bmatrix} = ABB^T$$

UID No.	Protagonists	Occupation	Monthly Income	Without Wal-Mart Monthly Expenditure on Kirana	Monthly Savings
1	Gupta	Kirana store owner	Rs. 364	Rs. 237	Rs. 127
2	Mambani	Big Business Owner	Rs. 1,000	Rs. 650	Rs. 350
3	Sharma	Works for Mambani	Rs. 400	Rs. 260	Rs. 140
4	Verma	Works for Mambani	Rs. 350	Rs. 228	Rs. 123
5	Gokhale	Works for Mambani	Rs. 300	Rs. 195	Rs. 105
6	Reddy	Works for Mambani	Rs. 250	Rs. 163	Rs. 88
7	Ghose	Works for Mambani	Rs. 200	Rs. 130	Rs. 70
8	Kumar	Works for Mambani	Rs. 150	Rs. 98	Rs. 53
9	Singh	Works for Gupta	Rs. 100	Rs. 65	Rs. 35
10	ABC	NREGA/Unemployed	Rs. 50	Rs. 33	Rs. 18
Sub-Total			Rs. 2,800	Rs. 1,820	Rs. 980
Grand Total			**Rs.3,164**	**Rs.2,057**	**Rs.1,107**

Table 1: **COST STRUCTURE FOR AN ONLINE GROCERY START-UP THAT EVENTUALLY SHUT DOWN**

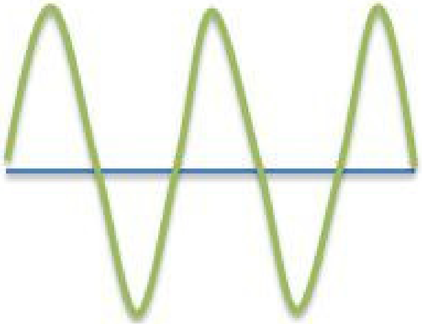
Deliveries/Day	500	1,000	2,000
Basket size (₹)	1,000	1,400	1,500
Gross margin (%)	17	19	22
Gross margin (₹)	170	266	330
Warehousing cost (₹)	25	25	20
Delivery van (₹)	102	84	68
Delivery van (%)	10.2	6	4.5
Manpower cost/delivery (₹)	145	125	110
Manpower (%)	14.5	8.9	7.3
Other expenses (₹)	10	14	14
Contribution/delivery (₹)	-112	18	118

Convenience Store Inventory List

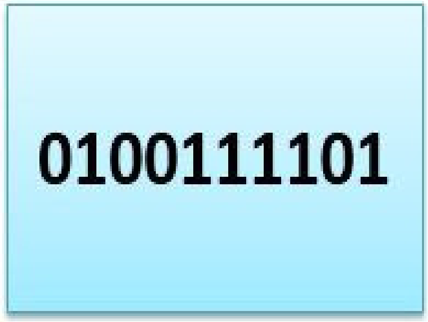
Inventory ID	Name	Description	Unit Price	Quantity in Stock	Inventory Value	Reorder Level	Reorder Time in Days
IN0001	Item 1	Desc 1	$51.00	25	$1,275.00	29	13
IN0002	Item 2	Desc 2	$93.00	132	$12,276.00	231	4
IN0003	Item 3	Desc 3	$57.00	151	$8,607.00	114	11
IN0004	Item 4	Desc 4	$19.00	186	$3,534.00	158	6
IN0005	Item 5	Desc 5	$75.00	62	$4,650.00	39	12
IN0006	Item 6	Desc 6	$11.00	5	$55.00	9	13
IN0007	Item 7	Desc 7	$56.00	58	$3,248.00	109	7
IN0008	Item 8	Desc 8	$38.00	101	$3,838.00	162	3
IN0009	Item 9	Desc 9	$59.00	122	$7,198.00	82	3
IN0010	Item 10	Desc 10	$50.00	175	$8,750.00	283	8
IN0011	Item 11	Desc 11	$59.00	176	$10,384.00	229	1
IN0012	Item 12	Desc 12	$18.00	22	$396.00	36	12
IN0013	Item 13	Desc 13	$26.00	72	$1,872.00	102	9
IN0014	Item 14	Desc 14	$42.00	62	$2,604.00	83	2
IN0015	Item 15	Desc 15	$32.00	46	$1,472.00	23	15
IN0016	Item 16	Desc 16	$90.00	96	$8,640.00	180	3
~~IN0017~~	~~Item 17~~	~~Desc 17~~	~~$97.00~~	~~57~~	~~$5,529.00~~	~~98~~	~~12~~
IN0018	Item 18	Desc 18	$12.00	6	$72.00	7	13
IN0019	Item 19	Desc 19	$82.00	143	$11,726.00	164	12
IN0020	Item 20	Desc 20	$16.00	124	$1,984.00	113	14
IN0021	Item 21	Desc 21	$19.00	112	$2,128.00	75	11
IN0022	Item 22	Desc 22	$24.00	182	$4,368.00	132	15

	A	B	C	D	E	F
1	Master Ingredient List	Type (from drop down menu)	Brand	Price	Size	Per unit
2	can red beans	Canned Good	Walmart	0.5	15.5	$0.0323
3	spaghetti sauce	Canned Good	Hunts	1.08	26.5	$0.0408
4	can diced tomatoes big	Canned Good	Walmart	0.97	28	$0.0346
5	tomato soup	Canned Good	Walmart	0.58	10.8	$0.0537
6	chili seasoning	Misc	Walmart	0.5	1	$0.5000
7	1 lb ground beef	Freezer	Walmart	3.72	1	$3.7200
8	corn chips	Snacks	Santitas	2	16	$0.1250
9	mushroom soup	Canned Good	Walmart	0.92	10.5	$0.0876
10	onion soup mix	Misc	Walmart	0.86	1	$0.8600
11	can black beans	Canned Good	Walmart	0.48	15.2	$0.0316

	KINDLE	KOBO	FLIPKART	ROCKSTAND
Harry Potter Series	Rs. 3,995	Rs. 3,821	n/a	n/a
To Kill a Mockingbird	Rs. 242	Rs. 536	Rs. 102	n/a
The Lord of the Rings	Rs. 376	Rs. 2073	Rs. 376	n/a
The Hobbit	Rs. 207	Rs. 694	n/a	n/a
Pride and Prejudice	Free	Free	Free	Free
The Bible	Free	Free	Free	Free
The Hitchhikers Guide to the Galaxy	Rs. 209	Rs. 260	Rs. 224	n/a
The Hunger Games Trilogy	Rs. 1,088	Rs. 1,873	n/a	n/a
The Catcher in the Rye	n/a	n/a	n/a	n/a
The Chronicles of Narnia	Rs. 2,487	Rs. 2,487	Rs. 2,637	n/a
The Great Gatsby	Free	Rs. 49	Rs. 10	Rs. 25
1984	Rs. 49	Rs. 61	Rs. 10	Rs. 33
Little Women	Free	Rs. 61	Rs. 47	Free
Jane Eyre	Free	Free	Rs. 47	Free
The Stand	Rs. 270	Rs. 327	Rs. 299	n/a
Gone With the Wind	Rs. 49	Rs. 61	Rs. 60	Rs. 33
A Wrinkle in Time	Rs. 225	Rs. 286	Rs. 333	n/a
The Handmaid's Tale	Rs. 253	Rs. 588	Rs. 253	n/a
The Lion, the Witch, and the Wardrobe	Rs. 178	Rs. 387	Rs. 461	n/a
The Alchemist	Rs. 76	Rs. 404	Rs. 77	n/a

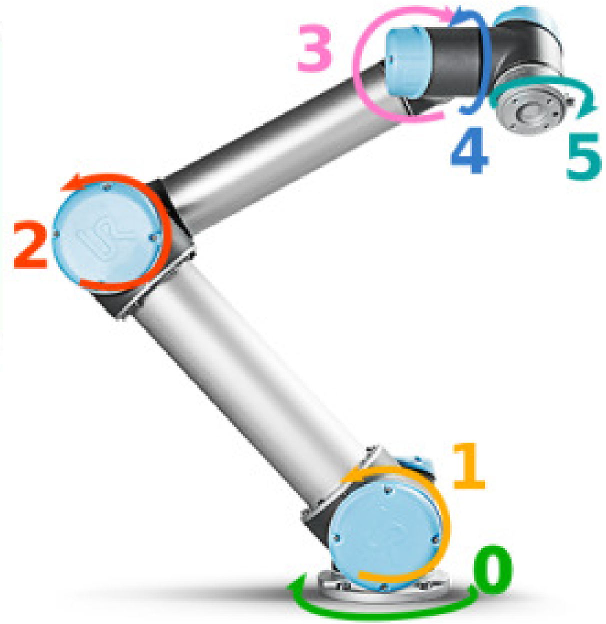


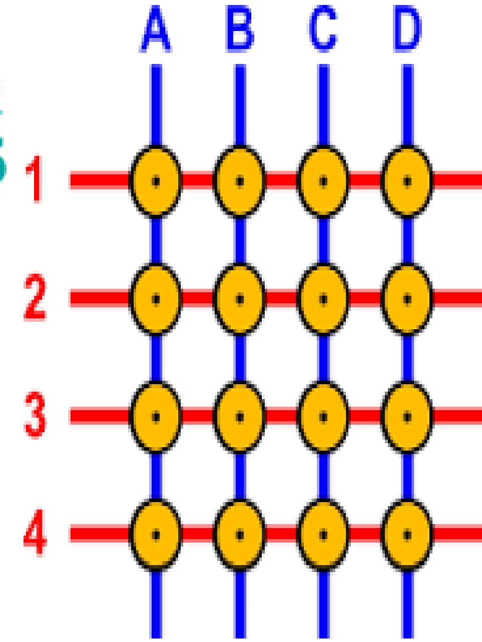
Vs



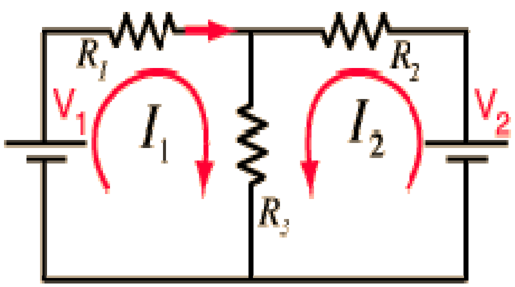
Analog
Signal

Digital
Signal

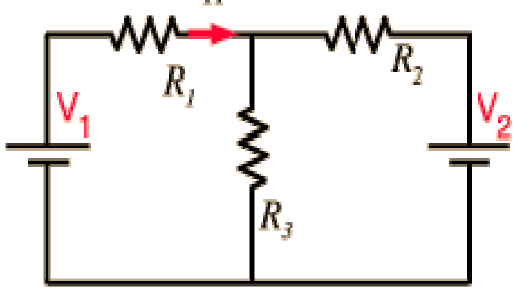


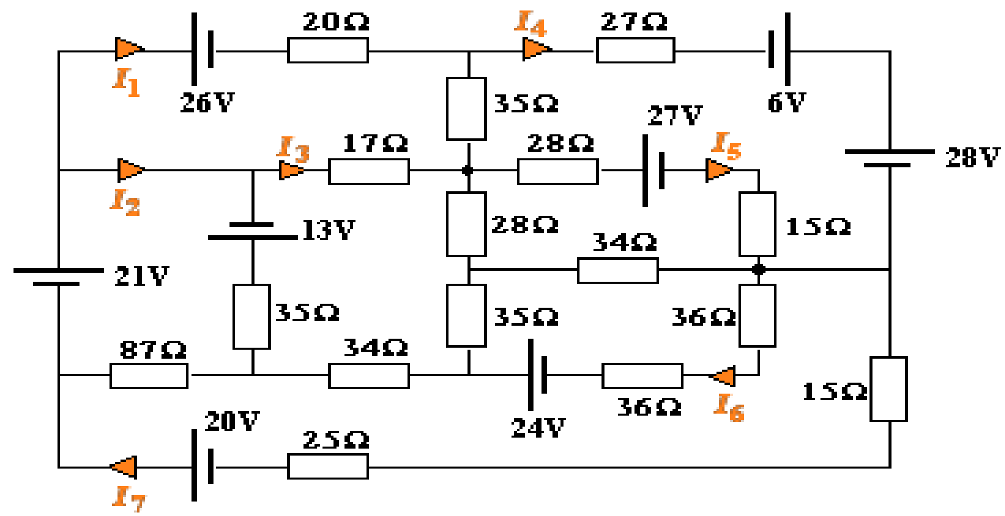


$I_1 = I_{11} + I_{12}$



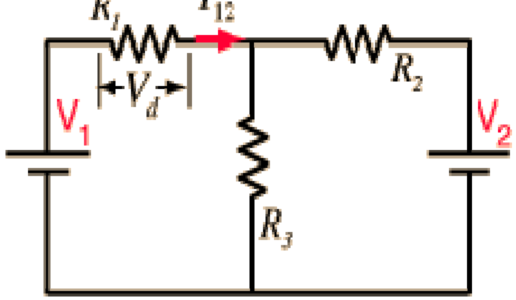
I_{11}





$$I_{11} = \frac{V_1}{R_1 + R_2 || R_3} \qquad I_{12} = \frac{-V_d}{R_1}$$

I_{12}



$$V_d = \frac{R_1 || R_3 V_2}{R_2 + R_1 || R_3}$$

$R_1 || R_3$ means the parallel resistance of R_1 and R_3 .

Ohms Law Formulas				
Known Values	Resistance (R)	Current (I)	Voltage (V)	Power (P)
Current & Resistance	---	---	$V = I \times R$	$P = I^2 \times R$
Voltage & Current	$R = \frac{V}{I}$	---	---	$P = V \times I$
Power & Current	$R = \frac{P}{I^2}$	---	$V = \frac{P}{I}$	---
Voltage & Resistance	---	$I = \frac{V}{R}$	---	$P = \frac{V^2}{R}$
Power & Resistance	---	$I = \sqrt{\frac{P}{R}}$	$V = \sqrt{P \times R}$	---
Voltage & Power	$R = \frac{V^2}{P}$	$I = \frac{P}{V}$	---	---

Solutions

- *Every system of linear equations has either no solutions, exactly one solution, or infinitely many solutions.*

- A general system of two linear equations: (Figure1.1.1)

$$a_1x + b_1y = c_1 \text{ } (a_1, b_1 \text{ not both zero})$$
$$a_2x + b_2y = c_2 \text{ } (a_2, b_2 \text{ not both zero})$$

  – **Two lines may be parallel -> <u>no solution</u>**
  – **Two lines may intersect at only one point -> <u>one solution</u>**
  – **Two lines may coincide -> <u>infinitely many solution</u>**

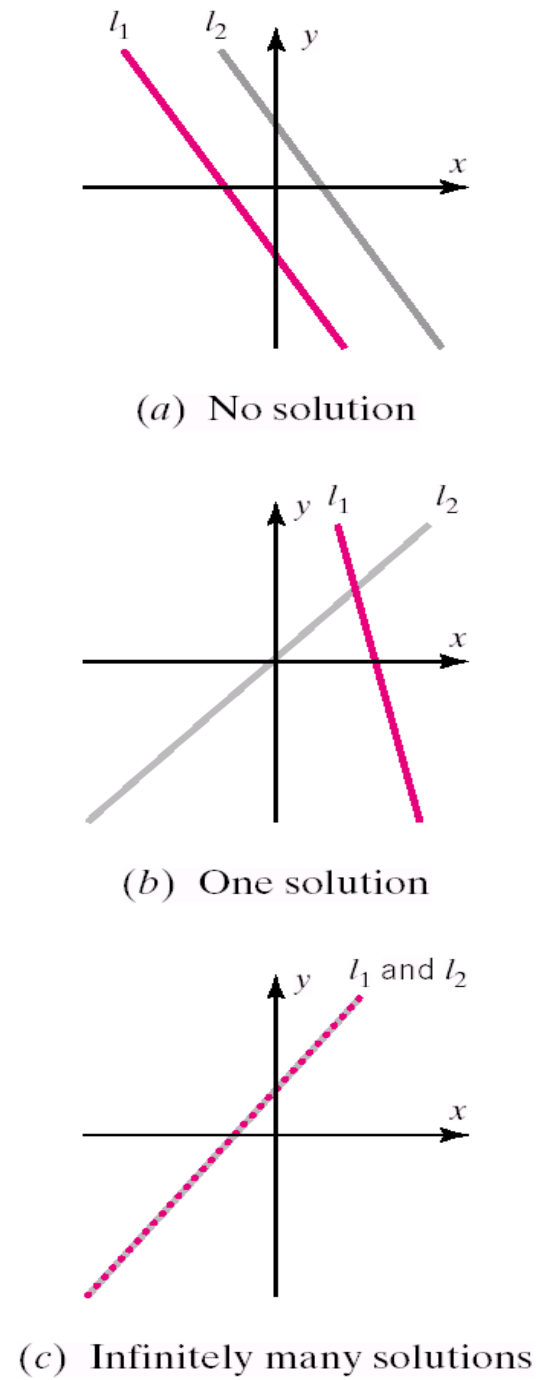
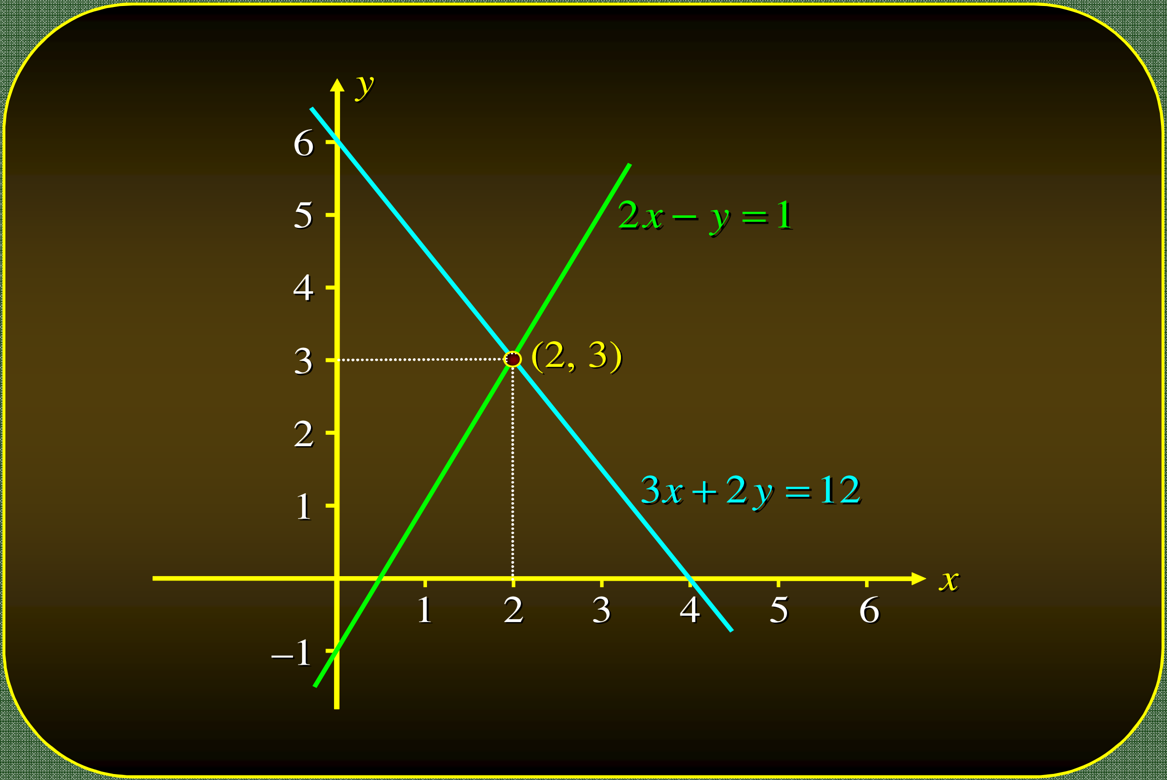


Figure 1.1.1

Systems of Linear Equations and Matrices

- Systems of Linear Equations:
  - An Introduction
  - Unique Solutions
  - Underdetermined and Overdetermined Systems
- Matrices
- Multiplication of Matrices
- The Inverse of a Square Matrix

Systems of Linear Equations: An Introduction



Systems of Equations

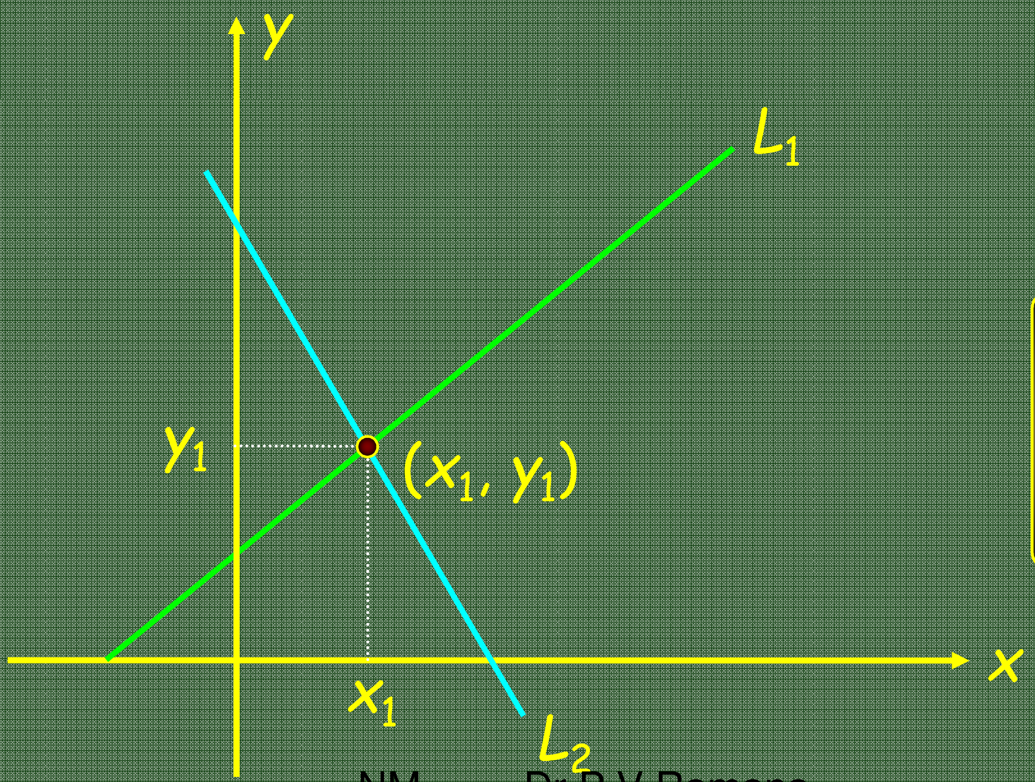
- Recall that a system of two linear equations in two variables may be written in the general form

$$\begin{aligned} ax + by &= h \\ cx + dy &= k \end{aligned}$$

  where a, b, c, d, h, and k are real numbers and neither a and b nor c and d are both zero.
- Recall that the graph of each equation in the system is a straight line in the plane, so that geometrically, the solution to the system is the point(s) of intersection of the two straight lines L_1 and L_2, represented by the first and second equations of the system.

Systems of Equations

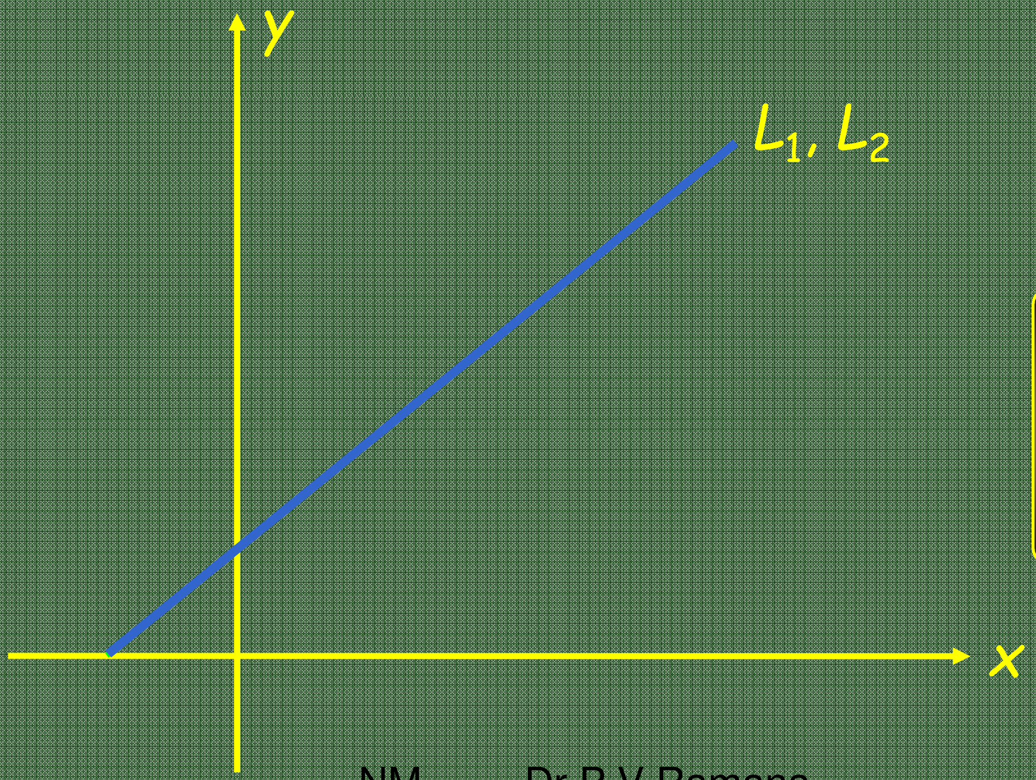
- Given the two straight lines L_1 and L_2, one and only one of the following may occur:
  1. L_1 and L_2 intersect at exactly one point.



Unique solution
(x_1, y_1)

Systems of Equations

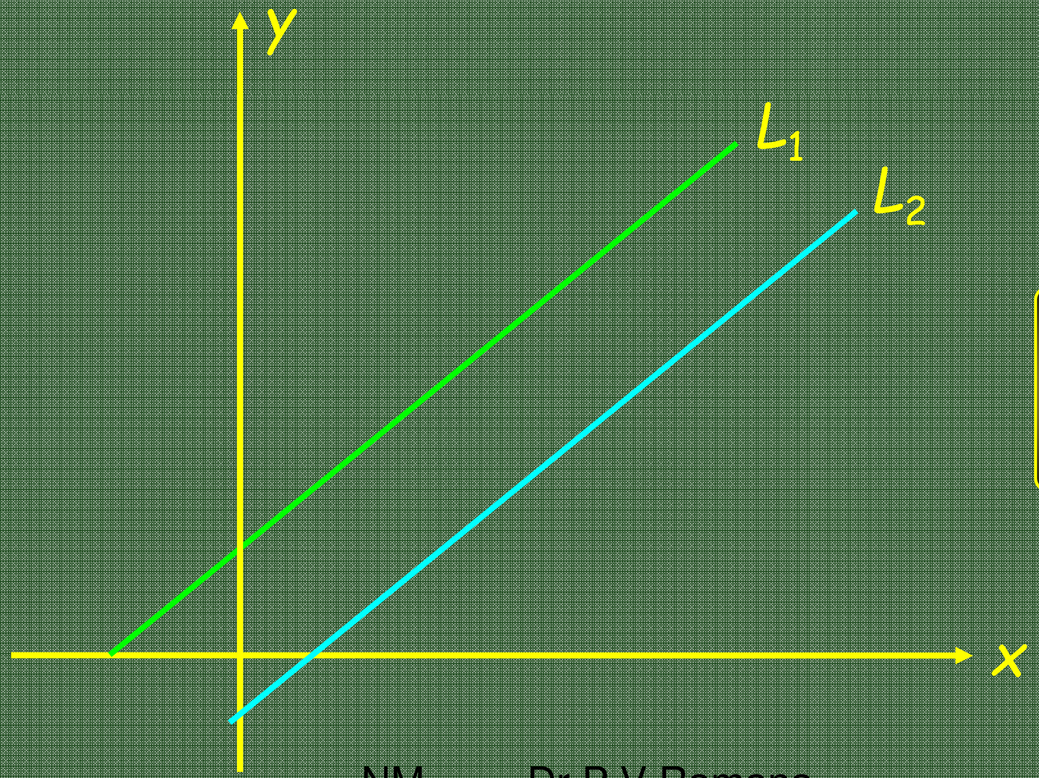
- Given the two straight lines L_1 and L_2, one and only one of the following may occur:
  2. L_1 and L_2 are coincident.



Infinitely many solutions

Systems of Equations

- Given the two straight lines L_1 and L_2, one and only one of the following may occur:
  3. L_1 and L_2 are parallel.



No solution

Example:

A System of Equations With Exactly One Solution

- Consider the system

$$2x - y = 1$$
$$3x + 2y = 12$$

- Solving the first equation for *y* in terms of *x*, obtain

$$y = 2x - 1$$

- Substituting this expression for *y* into the second equation yields

$$3x + 2(2x - 1) = 12$$
$$3x + 4x - 2 = 12$$
$$7x = 14$$
$$x = 2$$

Example:

A System of Equations With Exactly One Solution

- Finally, substituting this value of x into the expression for y obtained earlier gives

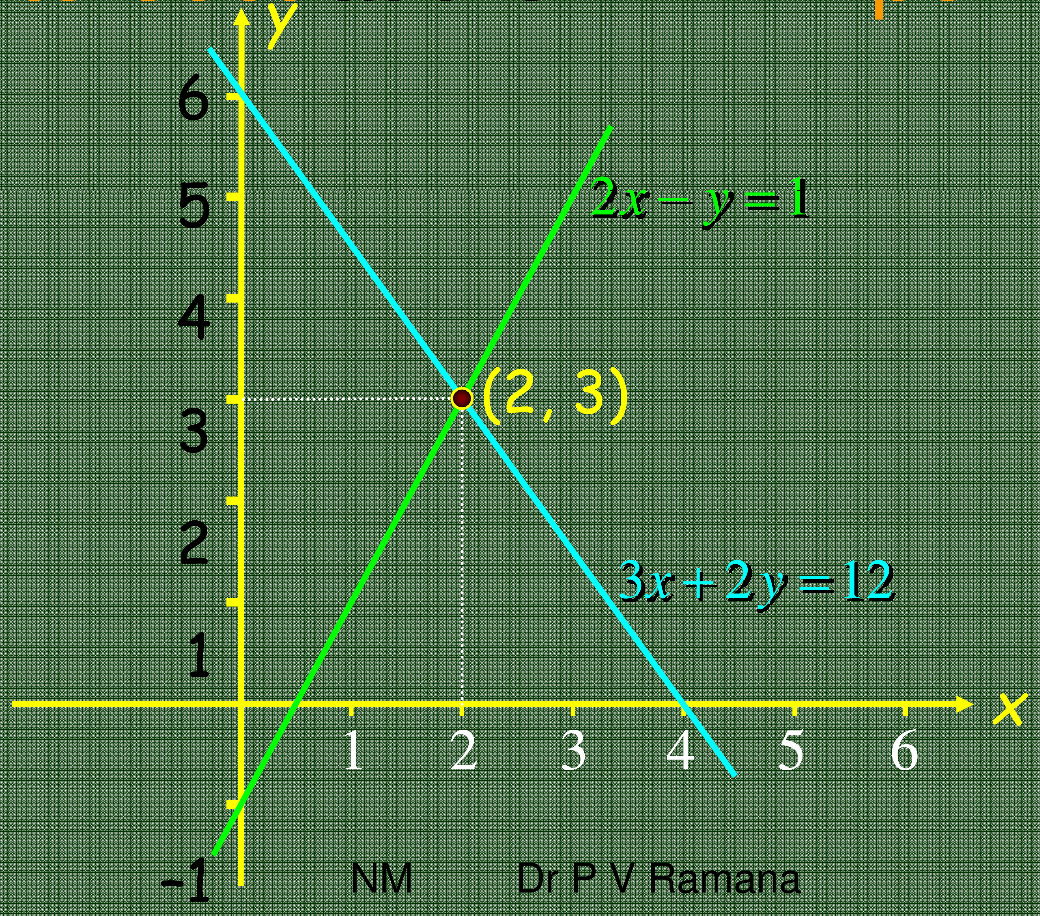
$$\begin{aligned} y &= 2x - 1 \\ &= 2(2) - 1 \\ &= 3 \end{aligned}$$

- Therefore, the unique solution of the system is given by $x = 2$ and $y = 3$.

Example:

A System of Equations With Exactly One Solution

- Geometrically, the two lines represented by the two equations that make up the system intersect at the point (2, 3):



Example:

$$2x - y = 1$$
$$3x + 2y = 12$$

A System of Equations With Infinitely Many Solutions

- Consider the system

$$2x - y = 1$$
$$6x - 3y = 3$$

- Solving the first equation for *y* in terms of *x*, obtain

- Substituting this expression for *y* into the second equation yields

$$y = 2x - 1$$
$$6x - 3(2x - 1) = 3$$
$$6x - 6x + 3 = 3$$
$$0 = 0$$

which is a true statement.

- This result follows from the fact that the second equation is equivalent to the first.

Example:

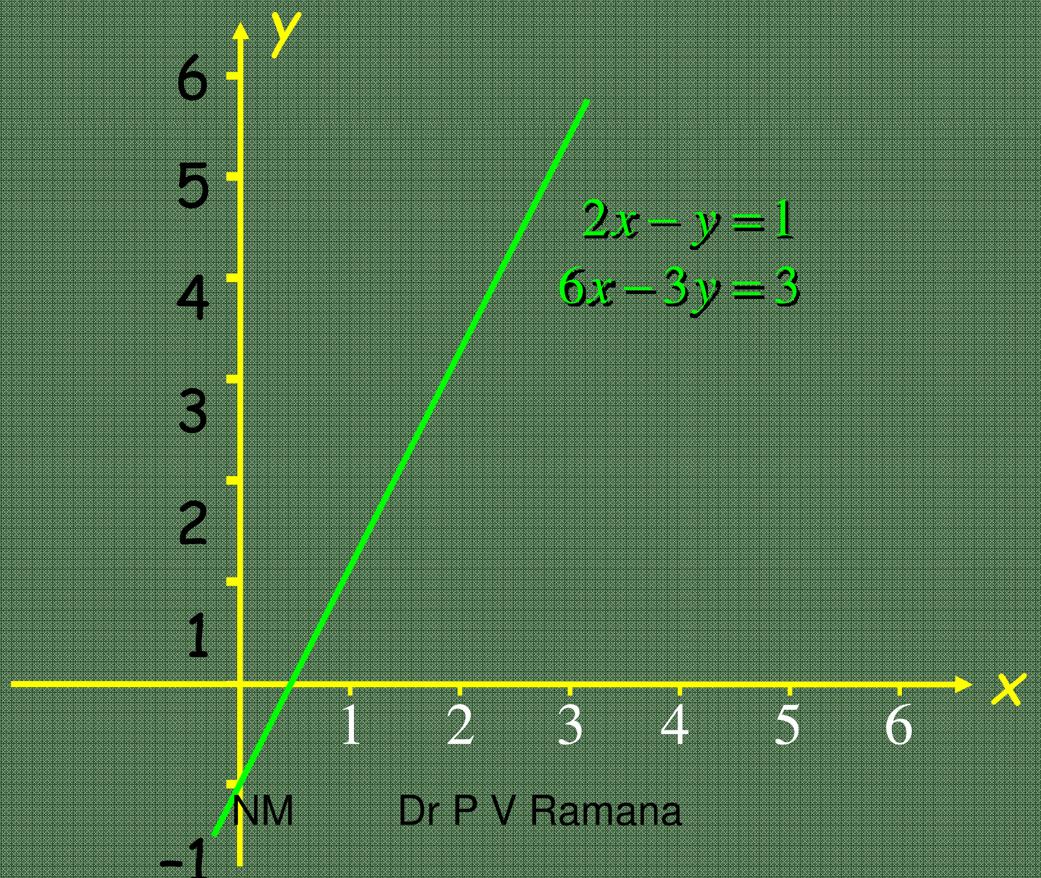
A System of Equations With Infinitely Many Solutions

- Thus, any order pair of numbers (x, y) satisfying the equation $y = 2x - 1$ constitutes a solution to the system.
- By assigning the value t to x, where t is any real number, find that $y = 2t - 1$ and so the ordered pair $(t, 2t - 1)$ is a solution to the system.
- The variable t is called a parameter.
- *For example:*
  - Setting $t = 0$, gives the point $(0, -1)$ as a solution of the system.
  - Setting $t = 1$, gives the point $(1, 1)$ as another solution of the system.

Example:

A System of Equations With Infinitely Many Solutions

- Since t represents any real number, there are infinitely many solutions of the system.
- Geometrically, the two equations in the system represent the same line, and all solutions of the system are points lying on the line:



Example:

$$\begin{aligned} 2x - y &= 1 \\ 3x + 2y &= 12 \end{aligned}$$

A System of Equations That Has No Solution

- Consider the system

$$\begin{aligned} 2x - \quad y &= 1 \\ 6x - 3y &= 12 \end{aligned}$$

- Solving the first equation for y in terms of x, obtain

$$y = 2x - 1$$

- Substituting this expression for y into the second equation yields

$$\begin{aligned} 6x - 3(2x - 1) &= 12 \\ 6x - 6x + 3 &= 12 \\ 0 &= 9 \end{aligned}$$

  which is clearly impossible.
- Thus, there is no solution to the system of equations.

Example:

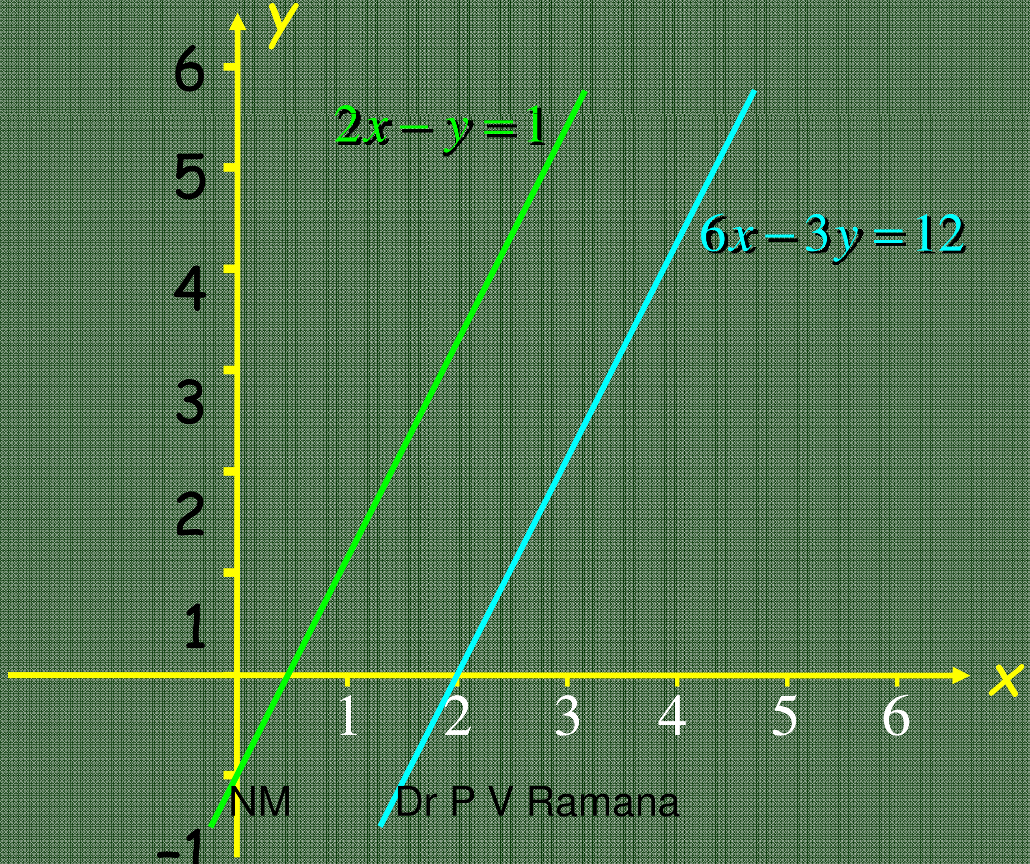
A System of Equations That Has No Solution

- To interpret the situation geometrically, cast both equations in the slope-intercept form, obtaining

$y = 2x - 1$ and $y = 2x - 4$

which shows that the lines are parallel.

- Graphically:



Introduction

Roots of a single equation: $f(x) = 0$

A general set of equations:
- n equations,
- n unknowns.

$$\begin{cases} f_1(x_1, x_2, \ldots x_n) = 0 \\ f_2(x_1, x_2, \ldots x_n) = 0 \\ \vdots \qquad\qquad\qquad \vdots \\ f_n(x_1, x_2, \ldots x_n) = 0 \end{cases}$$