NUMERICAL METHODS

Lecture 0

Dr. P V Ramana

Detailed Syllabus

SOLUTION OF EQUATIONS :

- Error analysis; the roots of nonlinear equations, solutions of large system of linear equations. Pascal's triangle for one, two and three dimensions, Newton's divided differences. Numerical interpolation functions geometrical functions and their stiffness matrices properties.

INTERPOLATION, APPROXIMATION AND EIGENVALUE PROBLEMS:

- Open and Bracket methods; Fixed point iteration: x=g(x) method; Newton's method; Direct and Iterative method-Gauss Seidel method, Inverse of a matrix by Gauss Jordon method; Eigen value of matrix by power method and by Jacobi method for symmetric matrix.

NUMERICAL DIFFERENTIATION AND INTEGRATION:

- Numerical integration by trapezoidal and Simpson's rules; Differentiation using interpolation formulae; Romberg's method; One, Two and Three point Gaussian quadrature formulae. Advanced numerical linear algebra and related numerical methods. Decompositions and SVD factorizations; stability and accuracy of numerical algorithms. Nonlinear ordinary differential equations & partial differential equations. Nonlinear optimization, FFTs, and simulation using Mento Carlo Method.

References:

[Erwin *Kreyszig*](#), Advanced *Engineering Mathematics,* Wiley publications, Delhi.

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- Bau III, David, and Lloyd N. Trefethen. *Numerical Linear Algebra*. Philadelphia, PA: Society for Industrial and Applied Mathematics, 1997. ISBN: 9780898713619.
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- Sankara Rao K, "Numerical Methods for Scientists and Engineers", 3rd Edition, Printice Hall of India Private Ltd, New Delhi, 2007.
- F B Hildebrand, Introduction to Numerical Analysis, Dover publications, Newyork.
- Gerald, C. F. and Wheatley, P.O., "Applied Numerical Analysis", 6th Edition, Pearson Education, Asia, New Delhi, 2006.
- Grewal, B.S. and Grewal,J.S., " Numerical methods in Engineering and Science", 6th Edition, Khanna Publishers, New Delhi, 2004.
- Bai, et al. *Templates for the Solution of Algebraic Eigenvalue Problems: a Practical Guide*. Philadelphia, PA: Society for Industrial and Applied Mathematics, 2000.
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Weightage

20%

Attendance & Assignments

20%

Mid – Term Exam 1

20%

Mid – Term Exam 2

40%

Final End Exam

fppt.com

Grading

GRADE	MARKS RANGE
AA	>95
AB	84-95
BB	73-83
BC	64-72
CC	56-63
CD	48-55
D	40-47
F	<40

Standard Gaussian Distribution

Methods

Analytical Methods

$EI\, d^2 u / dx^2 = M$, $EI\, d^4 u / dx^4 = w$

$D\,(\delta^4 u / \delta x^4 + 2\, \delta^4 u / \delta x^2 \delta y^2 + \delta^4 u / \delta y^4) = w$

$D\,(\delta^4 u / \delta x^4 + \delta^4 u / \delta y^4 + \delta^4 u / \delta z^4 + 2\, \delta^4 u / \delta x^2 \delta y^2 + 2\, \delta^4 u / \delta y^2 \delta z^2 + 2\, \delta^4 u / \delta x^2 \delta z^2) = w$

Numerical Methods

Finite Element/Strip/Difference/Volume Methods

Boundary Element/Knot/Particle Methods

Domain Decomposition / Neumann & Neumann Methods

Experimental Methods

Can't Simulate

Skilled persons required

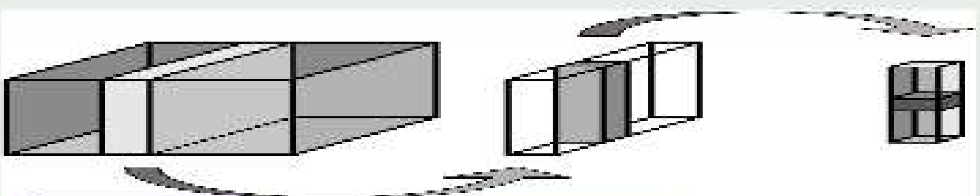
Analytical Methods

- $EI\, d^2 u/ dx^2 = M$
- $EI\, d^4 u/ dx^4 = w$
- $D\,(\delta^4 u/ \delta x^4 + 2\, \delta^4 u/ \delta x^2\delta y^2 + \delta^4 u/ \delta y^4) = w$
- $D\,(\delta^4 u/ \delta x^4 + \delta^4 u/ \delta y^4 + \delta^4 u/ \delta z^4 + 2\, \delta^4 u/ \delta x^2\delta y^2 + 2\, \delta^4 u/ \delta y^2\delta z^2 + 2\, \delta^4 u/ \delta x^2\delta z^2) = w$

$$EI(x)\left(\frac{d^2u}{dx^2}\right) = f(x); \quad EI(x,y)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = f(x,y)$$

$$EI(x,y,z)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = f(x,y,z)$$

Three to One Dimensional



Analytical Versus Numerical Analysis

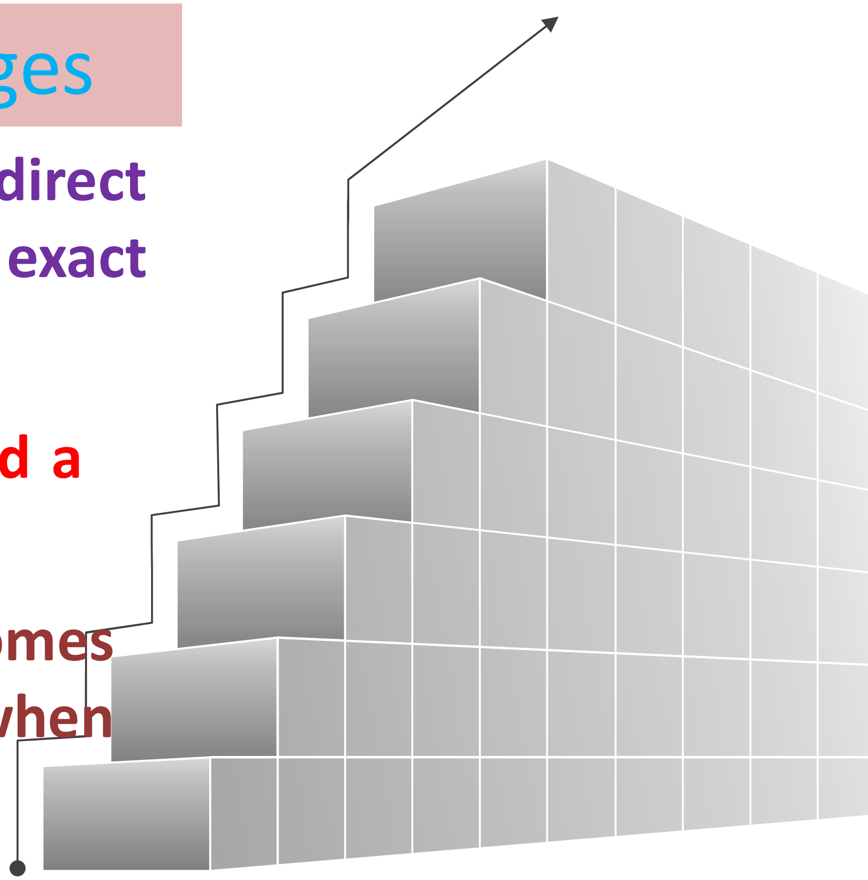
Analytical Methods

Advantages and Disadvantages

❖ Analytical techniques provide direct solution and will result in an exact solution, if one exists.

❖ Its usually require less time to find a solution.

❖ The solution procedure becomes considerably more complex when constraints are involved.



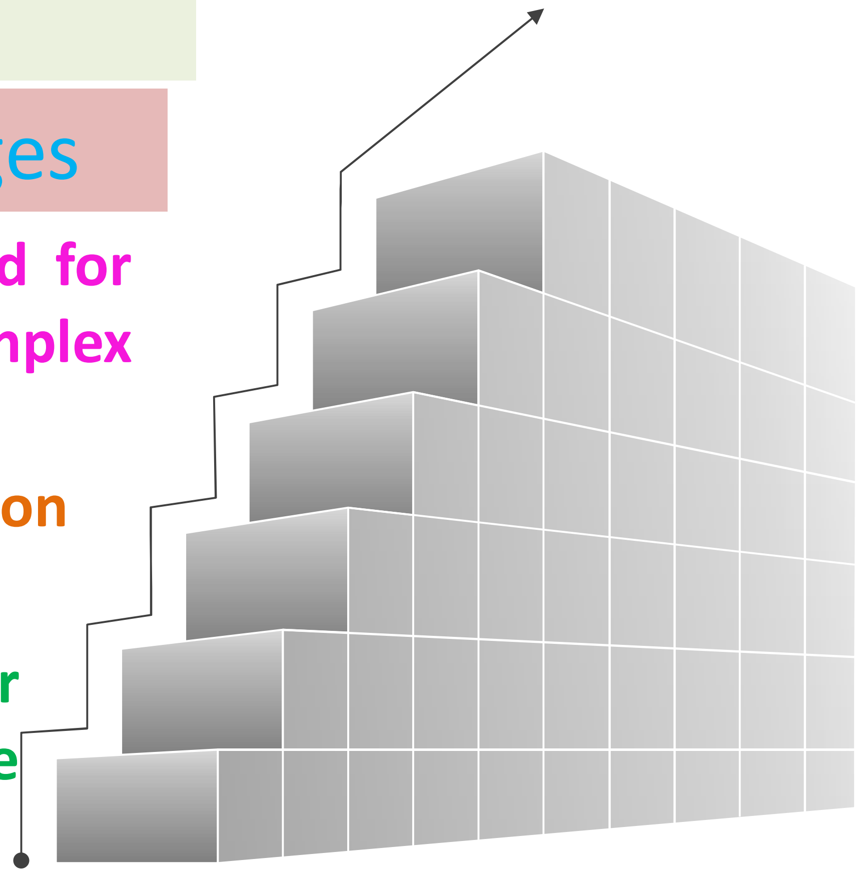
Numerical Methods

- Finite Element/Strip/Difference/Volume Methods
- Boundary Element/Knot/Particle Methods
- Domain Decomposition / Neumann & Neumann Methods

Analytical Versus Numerical Analysis

Numerical Methods

Advantages and Disadvantages



- **Numerical techniques can be used for functions that have moderately complex structure.**
- **It is easy to include constraints on the unknowns in the solution.**
- **Its require a considerable number of *iterations in order to* approach the true solution.**
- **The solution usually is not exact, and therefore it is necessary to provide *initial estimates of the* unknowns**

Numerical Analysis in Engineering

The numerical methods deal with engineering problems that can be solved by hand or computers.

The numerical methods can be effectively demonstrated in cases dealing with complex problems for which analytical solutions cannot be obtained or hand calculations cannot be made.

The common engineering problems are used to demonstrate the computational procedures

The use of any computational method, or numerical, without the proper understanding of the limitations and shortcomings can have serious consequences.

Numerical Methods

Numerical Analysis in Engineering

Numerical Methods

➢ When using numerical methods, the user should be aware of their:

1. **Computational details**
2. **Their limitations**
3. **Shortcomings, and**
4. **Strength**

➢ Numerical methods should not be used as black boxes with input and output or as numerical recipes.

Engineering Design & Analysis

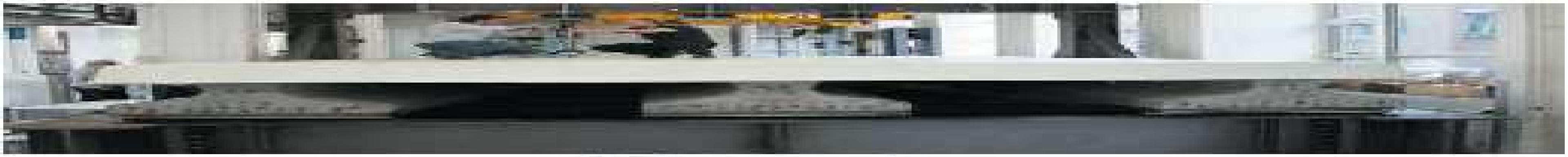
Design of Engineering Systems

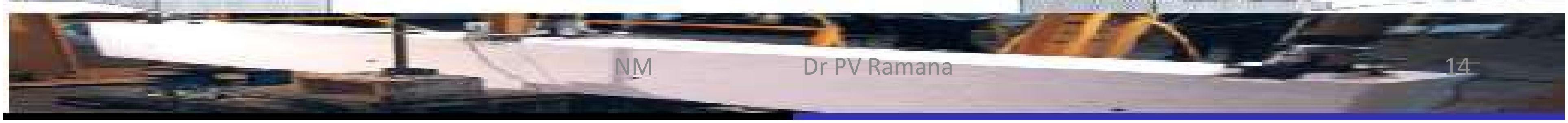
- Design of engineering systems is usually a trade-off between maximizing safety and minimizing cost.

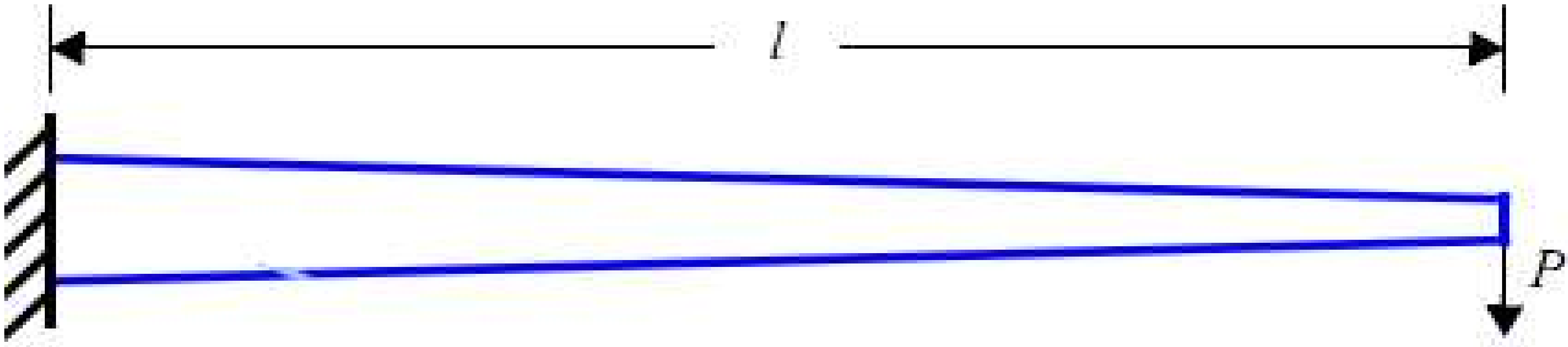
- A design procedure that can accomplish both of these objective is highly desirable, but also difficult.

Experimental Methods

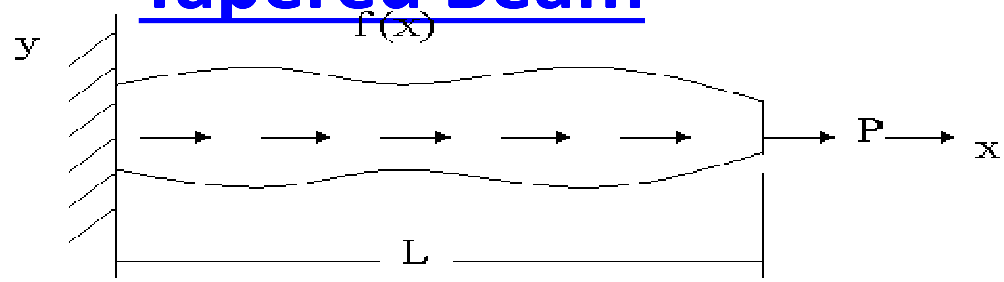
- Can’t Simulate
- Skilled persons required

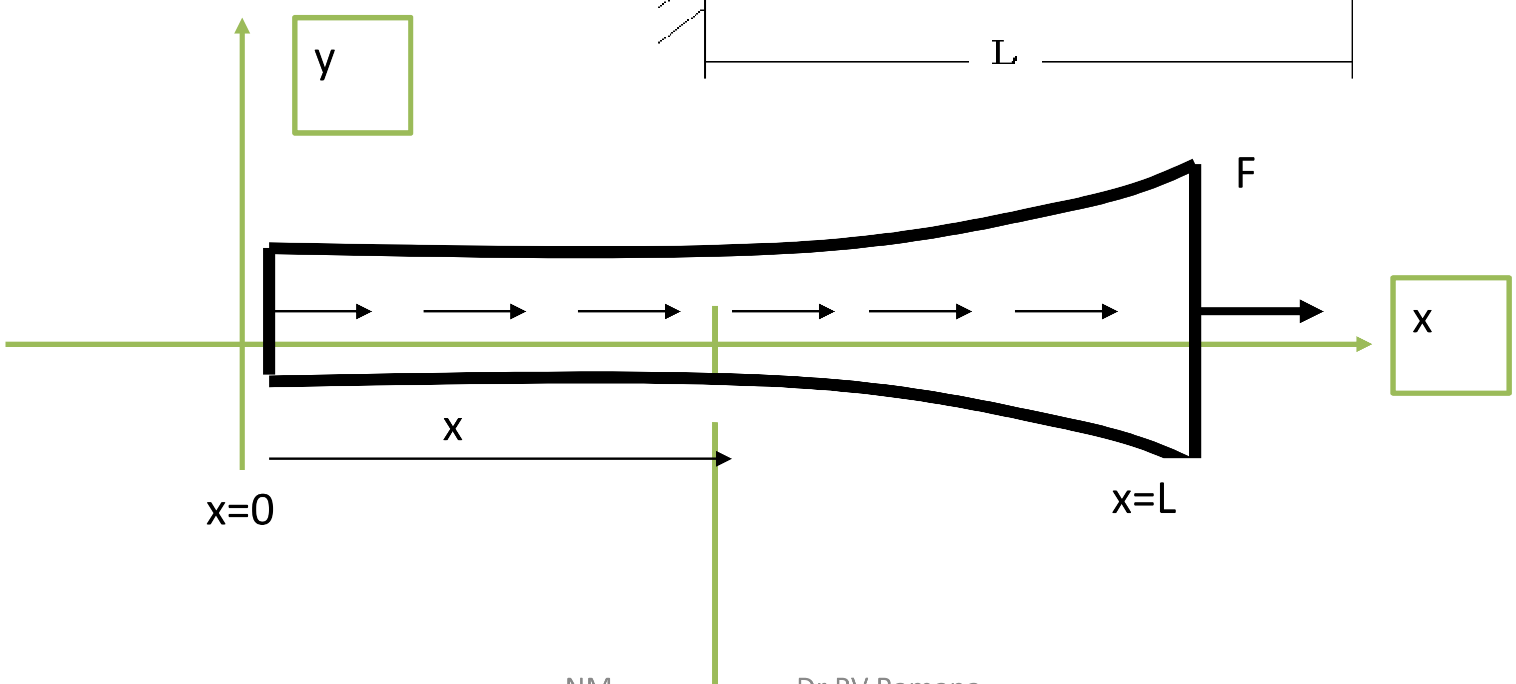


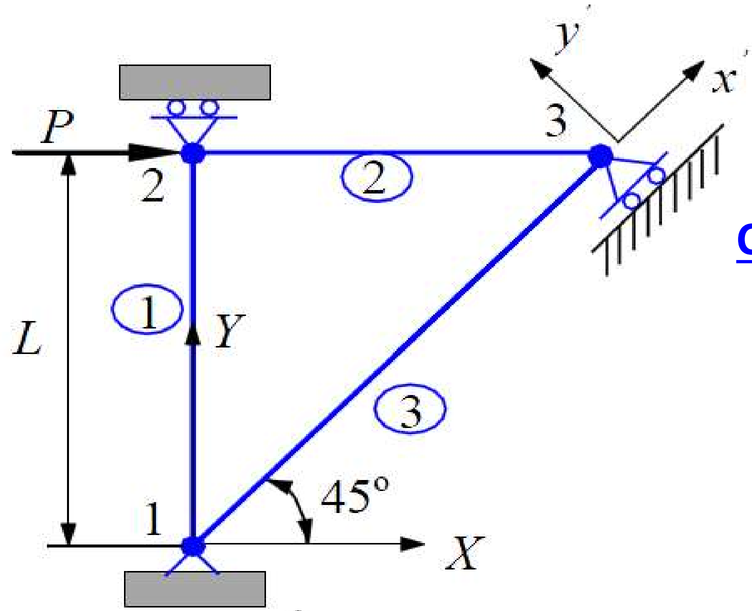




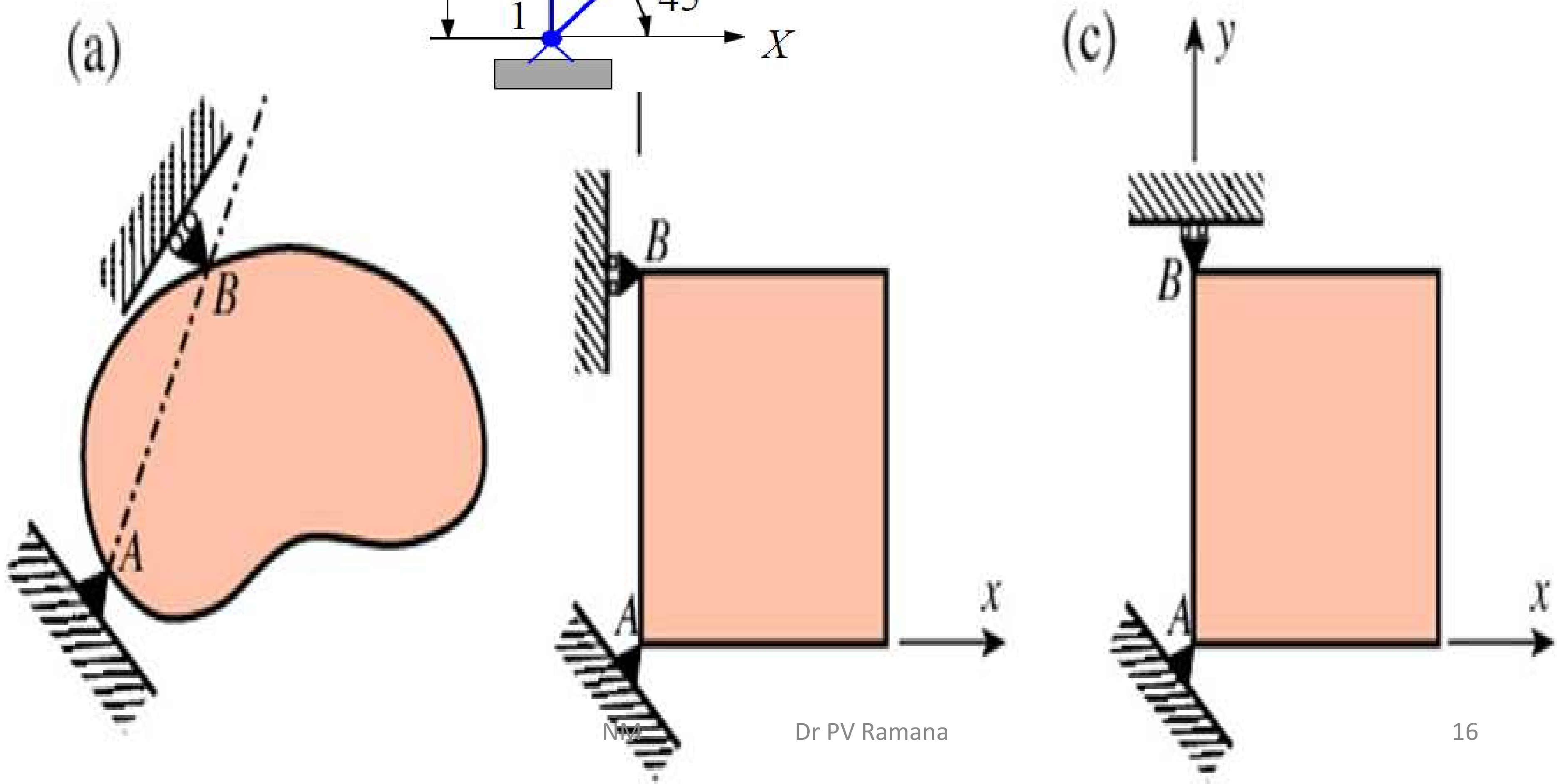
Tapered Beam







Complex Boundary Conditions



What do we need?

Basic Needs in the Numerical Methods:

Practical:

Can be computed in a reasonable amount of time.

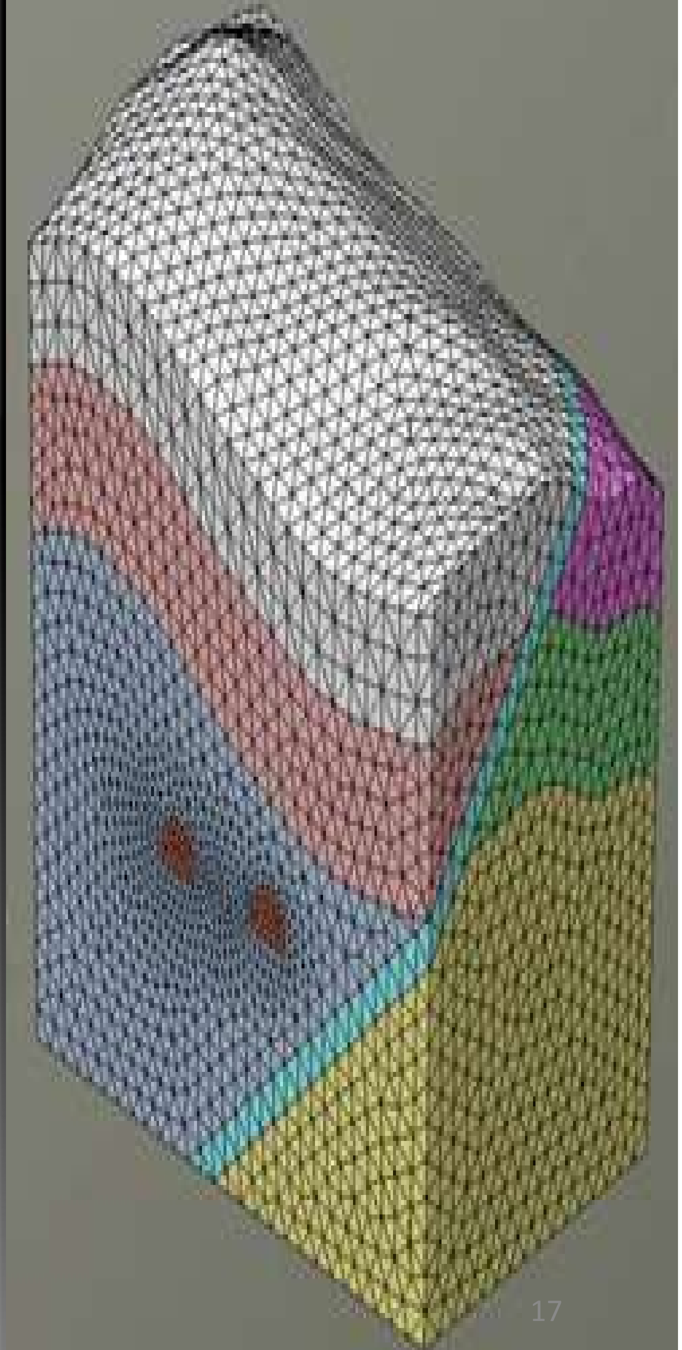
Accurate:

Good approximate to the true value,

Information about the approximation error (Bounds, error order,…).

Solution of Linear Equation y = mx + c

$$4x + 3 = 0$$



Solution of Nonlinear Equations

- Some simple equations can be solved analytically:

$$x^2 + 4x + 3 = 0$$

$$\text{Analytic solution} \quad roots = \frac{-4 \pm \sqrt{4^2 - 4(1)(3)}}{2(1)}$$

$$x = -1 \;\; and \;\; x = -3$$

- Many other equations have no analytical solution:

$$\left.\begin{aligned} x^9 - 2x^2 + 5 = 0 \\ x = e^{-x} \end{aligned}\right\} \text{No analytic solution}$$

Find the minimum of the following function both analytically and numerically:

$$y = x^2 - 3x + 2$$

Analytical Solution:

$$\frac{dy}{dx} = 2x - 3 = 0 \qquad \Rightarrow \; x = \frac{3}{2} = 1.5$$

Therefore, $y_{\min} = (1.5)^2 - 3(1.5) + 2 = -0.25$