Biomedical Signals - Lab 3

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November 15, 2025

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Spectral Estimation in Stationary Signals

1 Introduction

1.1 Stages of Sleep

Sleep scoring is based on characteristic patterns in the electroencephalogram (EEG), electro-oculogram (EOG), and electromyogram (EMG). The modern American Academy of Sleep Medicine (AASM) manual recognizes four sleep stages (N1, N2, N3, R) plus Wakefulness, updating the older Rechtschaffen and Kales (R and K) system.

1. **Wake (W)**: Defined by more than 50% scorable **alpha activity** (8 to 13 Hz) in the EEG. EOG shows blinking and rapid movements, and submental EMG tone is high.

2. **Stage N1** (Light/Transitional Sleep): A transitional state characterized by low-voltage, mixed-frequency **theta activity** (4 to 7 Hz). **Slow Rolling Eye Movements (SREMs)** are evident, but neither sleep spindles nor K complexes are present.

3. **Stage N2** (Intermediate Sleep): Accounts for up to 50% of total sleep time. Defined by the first appearance of **Sleep Spindles** (11 to 16 Hz) and **K Complexes**.

4. **Stage N3** (Slow Wave Sleep, SWS): Deepest sleep stage, combining R and K stages 3 and 4. Defined by **Slow Wave Activity (SWA)** or Delta activity (0.5 to 2 Hz) with an amplitude $\geq 75\mu$V occupying more than 20% of the epoch.

5. **Stage R** (REM/Paradoxical Sleep): Characterized by low-amplitude, mixed-frequency EEG activity, **Rapid Eye Movements (REMs)** in the EOG leads, and the **lowest muscle tone (atonia)** in the chin EMG. It typically occurs 90 to 120 minutes after sleep onset.

1.2 Defined Frequency Bands

For the quantitative analysis of the EEG signal across sleep stages, the Power Spectral Density (PSD) is segmented into the following frequency bands. The power contained within these ranges correlates directly to distinct mental or sleep states.

- **Low Delta ($0.5 - 2$ Hz):** A sub-band of Delta often associated with the deepest stages of non-REM sleep (N3) and slow wave activity.
- **Delta (δ, $0.5 - 4$ Hz):** The hallmark of deep sleep (SWS/N3). Characterized by high amplitude and slow activity.
- **Theta (θ, $4 - 8$ Hz):** Predominant activity during the transition into sleep and light sleep stages (N1 and N2).
- **Alpha (α, $8 - 13$ Hz):** Associated with quiet, relaxed wakefulness, typically seen over the posterior scalp with eyes closed.
- **Sigma (σ, $11 - 16$ Hz):** This range encompasses the **Sleep Spindles**, which are a defining feature of Stage N2 sleep.
- **Beta (β, $16 - 35$ Hz):** High-frequency activity linked to active concentration, complex thought, and wakefulness.

1.3 Power Spectral Density (PSD) Methods

Power Spectral Density (PSD) estimation determines how the power of a signal is distributed over frequency. The methods fall into two main categories: Non-Parametric (based on the Fourier Transform) and Parametric (based on model fitting).

Non-Parametric Methods

These methods compute the PSD directly from the signal data without assuming an underlying generating model.

- **Periodogram:** The most basic PSD estimator. It is calculated as the squared magnitude of the Discrete Fourier Transform (DFT) of the signal. While simple, it typically suffers from high variance (noisy results) and poor spectral leakage due to implicit rectangular windowing.
- **Periodogram + Hamming Window (HW):** Applies a non-rectangular window (Hamming) to the time-domain signal before computing the DFT. This improves the estimate by smoothing the data boundaries, which significantly **reduces spectral leakage** (smearing of energy across frequency bins) but slightly **worsens frequency resolution**.
- **Welch Periodogram:** A technique to reduce the high variance of the raw Periodogram. The signal is segmented into overlapping sections, each segment is windowed, and the PSDs of all segments are calculated and then averaged together. This results in a much smoother, more stable (lower variance) PSD estimate.

Parametric (AR Model) Methods

These methods assume the signal is the output of an Autoregressive (AR) system driven by white noise. The PSD is calculated from the estimated coefficients (parameters) of the AR model, offering much higher frequency resolution, especially with short data records. All use the **Burg method** for coefficient estimation.

- **Burg Method (Low/High Order):** The AR model order is chosen arbitrarily low (e.g., $P = 2$) or high (e.g., $P = 50$). A low order yields a very smooth, low-resolution PSD, while a very high order can capture noise components and become unstable.
- **Burg Method (AIC):** Selects the model order (P) using the **Akaike Information Criterion (AIC)**. AIC favors models that minimize estimation error while penalizing complexity, but it tends to slightly overestimate the true order.
- **Burg Method (MDL):** Selects the model order (P) using the **Minimum Description Length (MDL)** criterion. MDL imposes a stricter penalty on model complexity than AIC, resulting in a more parsimonious (lower) order, which is often considered the most stable and reliable estimate for short data sets.

2 Feasibility Assessment

2.1 a) Compare the PSD functions for the channels Fz, Cz and Pz obtained in all 7 spectral methods in a figure (consider only the epoch 1).

Epoch 1: corresponds to the first 5 seconds

- First, we compute the PSD for the last 19 EEG channels, for the first 5 seconds (epoch 1).
- Then select from the third channel (this included) until the last one, in total 29 channels. This is because the first two correspond to the two ocular channels.

```
fs = 100; % Sampling frequency (Hz)
N = 5*fs; % samples in one 5-seconds epoch

% Compute all PSD from the 19 EEG channels (ephoch 1)
% We select the first 5 seconds epoch and the last 19EEG channels:
eegi = signals(1:N,3:21); % From 1 to 500 samples (5 seconds)
```

Then, it is important to filter the signal. As frequency bands come from 0.5 Hz to 35 Hz, it is convenient to remove the very low frequency components associated mainly with the movement and other brain activities different to delta rhythms. By doing this it will permit to estimate better the PSD.

```
% Apply high-pass filter
[b,a] = ellip(6,0.5,40,.4/(fs/2),'high');
eeg = filtfilt(b,a,eegi);
```

With the data filtered, we will proceed to compute the PSD using the 7 spectral methods, defined in 1.3.

```
% Method 1 - Periodogram, considering a FFT with 1000 points:
[Pxx,f]=periodogram(eeg,[],1000,fs,'onesided');

% Method 2 - Periodogram using a Hamming window
[PxxH,f]=periodogram(eeg,hamming(500),1000,fs,'onesided');

% Method 3 - averaged Welch periodogram
number_segments = 4;
overlap = 0.5;
samples_segment =
   floor(length(eeg)/(number_segments-(number_segments-1)*overlap));
[PxxW,f]=pwelch(eeg,samples_segment,floor(samples_segment*overlap),1000,fs,'onesid

% Method 4 - Burg method with low order of 2
```

```
[Parb2,f]= pburg(eeg,2,1000,fs,'onesided');

% Method 5 - Burg method with high order of 50
[Parb50,f]= pburg(eeg,50,1000,fs,'onesided');

% Method 6 and 7 - AIC and MDL, respectively
nch = size(eeg, 2);
nnaic = zeros(nch, 1);
nnmdl = zeros(nch, 1);

for n = 1:nch
 data1= iddata(eeg(1:N/2, n),[],fs);
 data2= iddata(eeg(N/2 + 1: N,n),[], fs);
 V1=arxstruc(data1,data2, (1:60));
 nnaic(n)=selstruc(V1,'aic');
 nnmdl(n)=selstruc(V1,'mdl');
end

% Method 6 (AIC)
naic = round(mean(nnaic));
[Parbaic,f] = pburg(eeg,naic,1000,fs,'onesided');

% Method 7 (MDL)
nmdl=round(mean(nnmdl));
[Parbmdl,f]= pburg(eeg,nmdl,1000,fs,'onesided');
```

Listing 1: Code used for running the different Power Spectral Methods.

We now select the channels indicated (Fz, Cz and Pz). Since the data we are working with ("eeg") has already 19 channels (the two first, corresponding to the ocular channels, have been removed previously, variable "eegi"), we can assign the following correspondences between channelsa and rows from "eegi":

- Fz: 5th channel
- Cz: 10th channel
- Pz: 15th channel

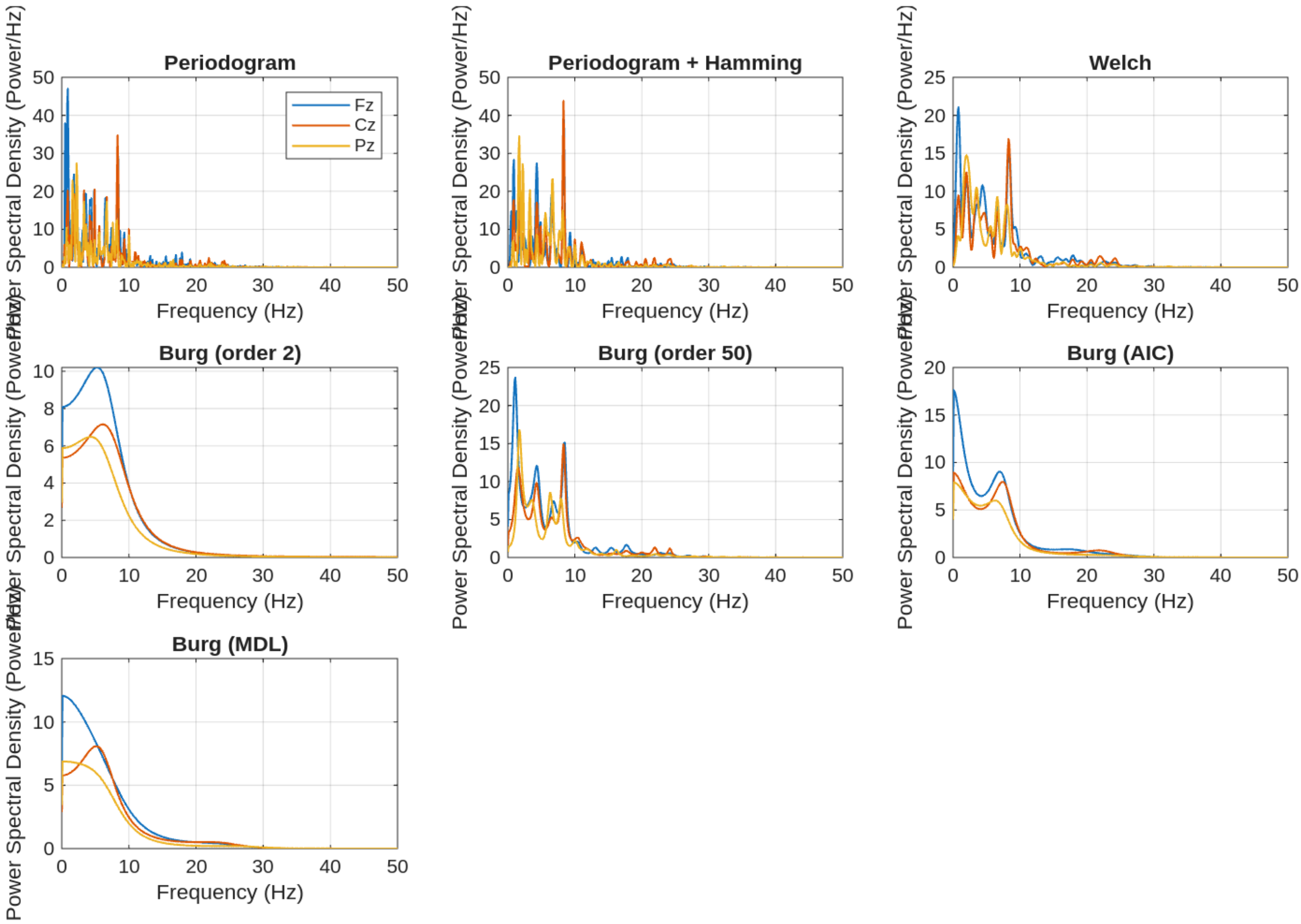


Figure 1: 7 PSD methods applied to the epoch 1 of S1 data.

2.2 b) Compare the topograms of all 7 spectral methods in a figure for each relative power (indicate the max value considered in each of both figures).

The frequency bands and their corresponding limits are the ones indicated in 1.2. Next, we present the code used for calculating the PSD methods for the defined bands.

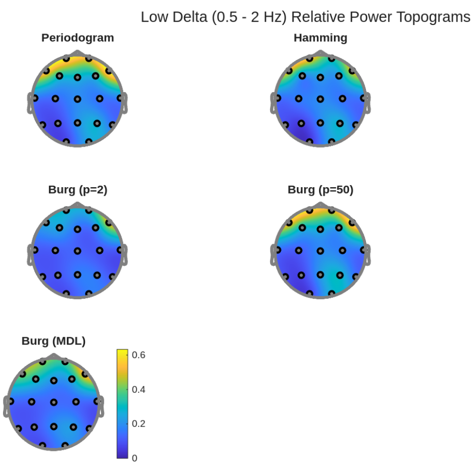
```
% Define the 7 spectral methods and their names
PSD_methods = {Pxx, PxxH, PxxW, Parb2, Parb50, Parbaic, Parbmdl};
method_names = {'Periodogram', 'Hamming', 'Welch', 'Burg (p=2)', 'Burg (p=50)', 'Burg (AIC)', 'Burg (MDL)'};
num_methods = length(PSD_methods);
num_channels = size(eeg, 2);

% Define all 6 frequency bands and their limits
Band_Limits = {
    'Low Delta (0.5 - 2 Hz)', [0.5, 2.0];
    'Delta (2 - 4 Hz)',       [2.0, 4.0];
    'Theta (4 - 7.5 Hz)',     [4.0, 7.5];
    'Alpha (7.5 - 12 Hz)',    [7.5, 12.0];
    'Sigma (12 - 15 Hz)',     [12.0, 15.0];
```

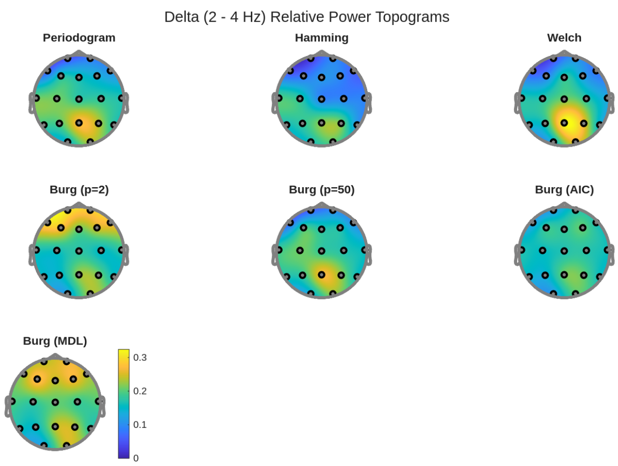
```
    'Beta (15 - 35 Hz)',      [15.0, 35.0];
};

num_bands = size(Band_Limits, 1);

% Total frequency range for normalization
f_min_total = 0.5; % Hz
f_max_total = 30.0; % Hz
idx_total = find(f >= f_min_total & f <= f_max_total); % f: frequency
    vector

for band_idx = 1:num_bands

    % --- Get Current Band Info ---
    band_name = Band_Limits{band_idx, 1};
    f_min_band = Band_Limits{band_idx, 2}(1);
    f_max_band = Band_Limits{band_idx, 2}(2);

    disp(['Processing band: ', band_name]);

    % Find frequency indices for the current band
    idx_band = find(f >= f_min_band & f <= f_max_band);

   % Initialize matrix to store relative power for all methods
      (Rows=Methods, Cols=Channels)
   % Relative power calculated as before
    Relative_Power_Matrix = zeros(num_methods, num_channels);

    for m = 1:num_methods
        current_Pxx = PSD_methods{m};

        for ch = 1:num_channels
            % Absolute power
            absolute_power = sum(current_Pxx(idx_band, ch));

            % Total power
            total_power = sum(current_Pxx(idx_total, ch));

            % Relative power
            Relative_Power_Matrix(m, ch) = absolute_power / total_power;
        end
    end

    % --- Determine Common Limits for plotting ---
    % Find the overall maximum relative power for consistent color scaling
    max_relative_power = max(Relative_Power_Matrix(:));
    common_limits = [0, max_relative_power];

    figure;
    sgtitle([band_name, ' Relative Power Topograms'], 'FontSize', 14);

```

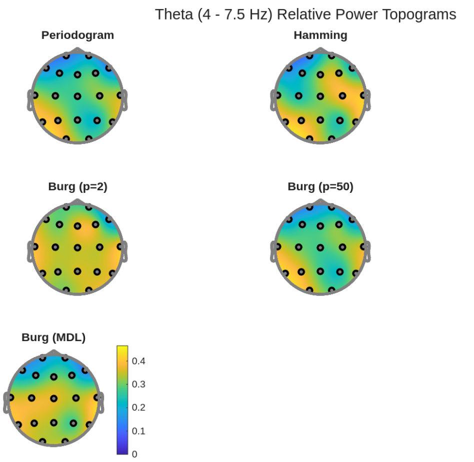
```
    for m = 1:num_methods
        subplot(3, 3, m);
        draw_topogram(Relative_Power_Matrix(m, :)', common_limits);

        title(method_names{m});

        % Add a colorbar only to the last plot (Method 7) for reference
        if m == num_methods
            colorbar('Location', 'EastOutside');
        end
    end

end
```

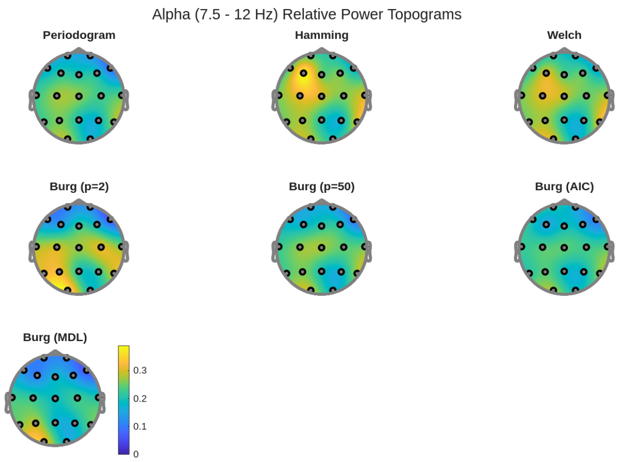
Listing 2: Matlab code used for running the different PSD for each frequency band.

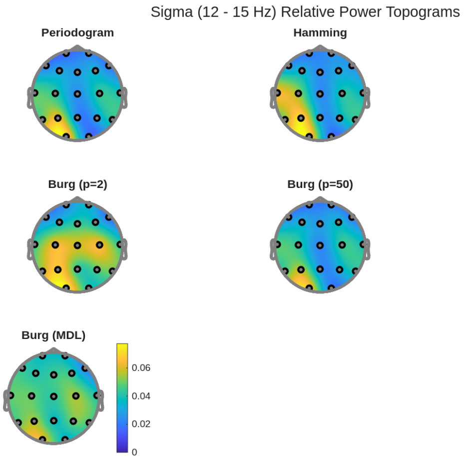
2.3 Results

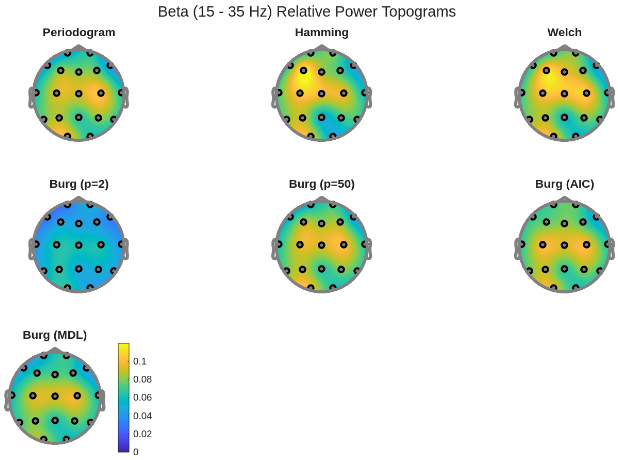












It can be observed that we have satisfied the requirement to use the saem limits [min, max] across all seven topograms in a signle figure, where min is 0 and max corresponds to the maximum value for all channesl and all methods in the figure.

2.4 Discussion

Feasibility

For the dominant bands—**Low Delta, Delta, Theta, and Alpha**—all seven estimators (non-parametric Periodogram, Hamming, and Welch, along with the parametric Burg methods) produced **highly similar spatial distributions**. For instance, the Alpha band consistently showed a posterior-occipital maximum, while the **Delta band** (2-4 Hz) was also strongly localized posteriorly. This high level of agreement across various algorithms **confirms the feasibility and robustness** of the calculated relative power distributions for these major frequency components.

Can we observe the same tendency in all frequency bands during deeper sleep stages?

When examining the "tendency" across all six frequency bands, it is clear that **the same topographic pattern is not observed**. The maximum relative power shifts significantly depending on the band: Low Delta, Theta, and Beta activity were concentrated primarily in the anterior **(frontal)** regions of the scalp, whereas, the Delta (2-4 Hz), Alpha, and Sigma activity localized to the **posterior (occipital/parietal) and central** areas.

Is there any band more affected than others by the estimator selection (methods)?

The **Sigma band (12-15 Hz)** was the **most affected** because its low power (max 0.06) makes the estimate volatile. Non-parametric methods like **Welch** create a **diffuse** pattern due to smoothing. In contrast, **parametric Burg methods** (AIC/MDL) offer **superior frequency resolution** and successfully reveal the **sharper, localized peaks** of the sleep spindles. This difference proves that estimator choice is critical for accurately localizing narrow-band, low-magnitude features.

3 Changes in Relative Power during Sleep

To observe changes of each relative power in frequency band during sleep (comparing the five sleep stages in a figure with the same max value for the five topograms which must be indicated). You can select only two spectral methods: selecting one of them with similar performance in most methods and another different to observe the influence of the estimator in checking these changes (each method in separated figures). Select the Welch periodogram and AR with order selected by MDL criteria.

Welch Periodogram vs AR MDL

In this exercise we will:

- Compare the power changes across all stages of sleep
- Both for Welch Periodogram and AR MDL

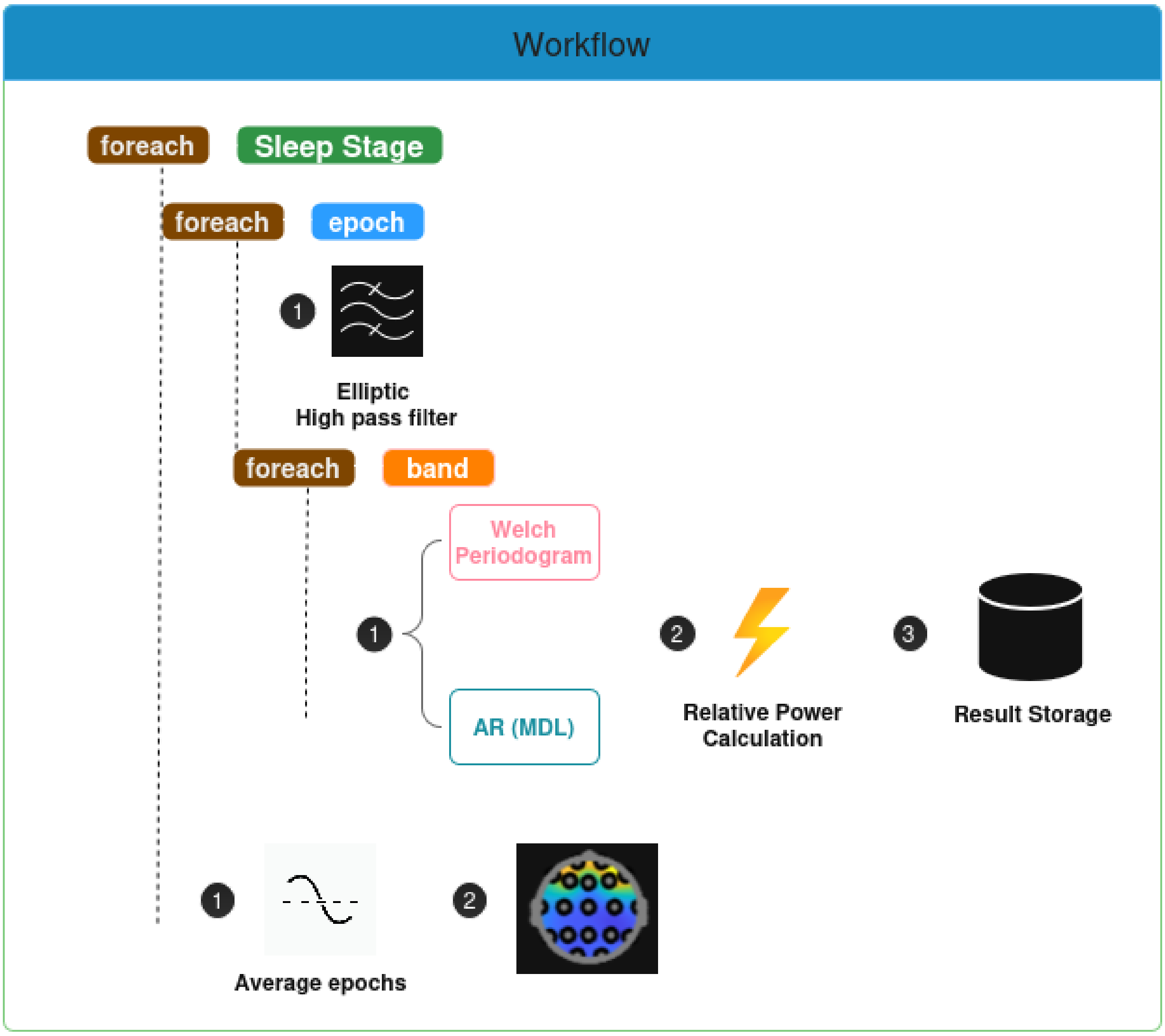


Figure 5: Workflow done to achieve the exercise.

With that logic, we implemented the following code.

```
for i = 1:num_stages
    stage_name = stages{i};
    fprintf('  Processing Stage: %s\n', stage_name);

    % Load Data
    file_name = sprintf('%s%s.mat', data_path, stage_name);
    data = load(file_name);
    signals = data.signals;

    % Temporary storage for all epochs within this stage (Channels x
       Epochs)
    stage_power_welch = struct();
    stage_power_ar = struct();
    for b = 1:num_bands
        stage_power_welch.(band_names{b}) = zeros(num_channels,
           num_epochs);
        stage_power_ar.(band_names{b}) = zeros(num_channels, num_epochs);
    end

    % Loop through Epochs
    for j = 1:num_epochs

        % Extract Epoch
        start_idx = (j-1) * nsamples_epoch + 1;
        end_idx = j * nsamples_epoch;
        eeg_epoch_raw = signals(start_idx:end_idx, eeg_channel_indices);

        % Filter Epoch
        eeg_epoch_filtered = filtfilt(b_filter, a_filter, eeg_epoch_raw);

        % Method 1: Welch
        [PxxW, f_welch] = pwelch(eeg_epoch_filtered, samples_segment,
           overlap_samples, nfft, fs, 'onesided');
        rel_power_w = calculate_relative_powers(PxxW, f_welch, bands);

        % Method 2: AR (MDL)
        avg_nmdl = get_ar_mdl_order(eeg_epoch_filtered, fs, max_ar_order);
        [PxxAR, f_ar] = pburg(eeg_epoch_filtered, avg_nmdl, nfft, fs,
           'onesided');
        rel_power_ar = calculate_relative_powers(PxxAR, f_ar, bands);

        % Store results for this epoch
        for b = 1:num_bands
            stage_power_welch.(band_names{b})(:, j) =
               rel_power_w.(band_names{b});
            stage_power_ar.(band_names{b})(:, j) =
               rel_power_ar.(band_names{b});
        end
    end
```

```

    % Average Epochs
    for b = 1:num_bands
        results_welch.(band_names{b})(:, i) =
           mean(stage_power_welch.(band_names{b}), 2);
        results_ar_mdl.(band_names{b})(:, i) =
           mean(stage_power_ar.(band_names{b}), 2);
    end

end
```

Listing 3: Code used for batch processing both PSD methods on different stages of sleep for different frequency bands.

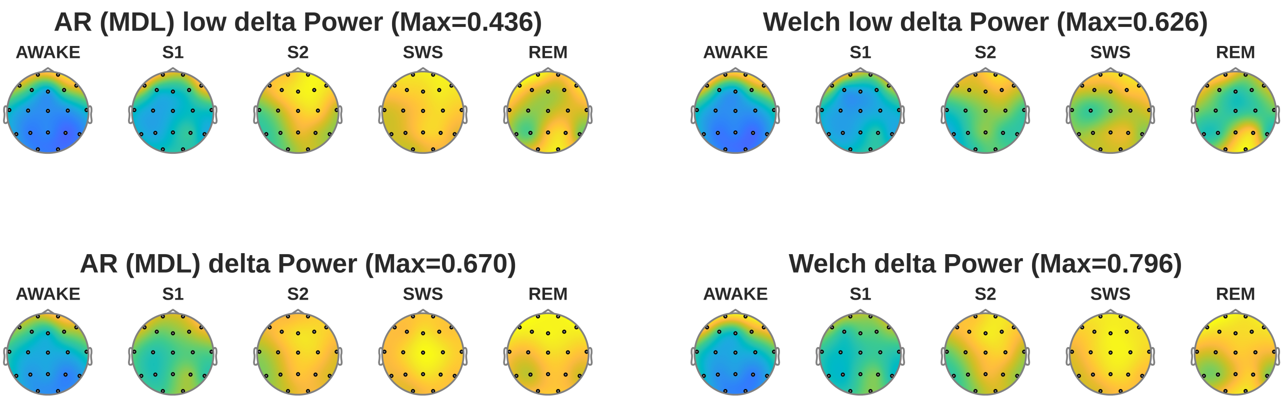
Once processed all the data, we plotted the results with the following code:

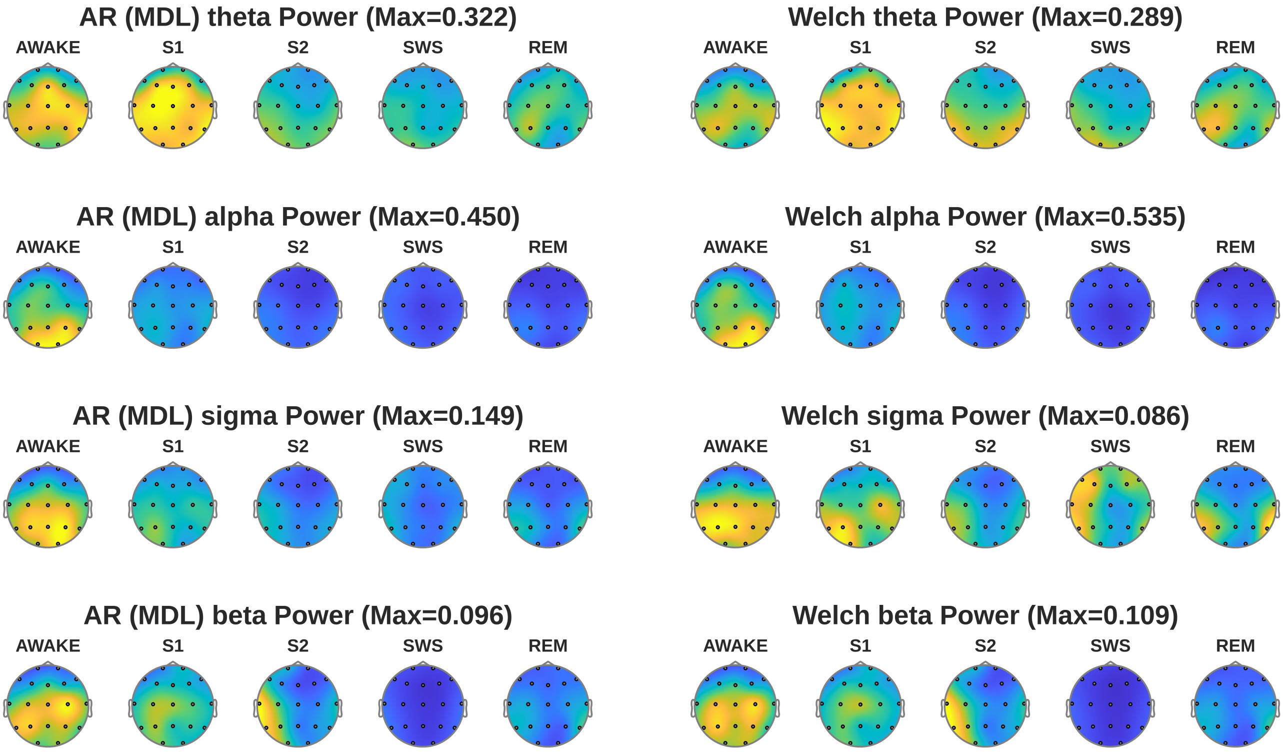
```
for b = 1:num_bands
    band_name = band_names{b};

    % Welch results
    plot_comparison_topograms(results_welch.(band_name), stages,
       band_name, 'Welch', ['report/img/topogram_welch_', band_name,
       '.png']);

    % AR(MDL) results
    plot_comparison_topograms(results_ar_mdl.(band_name), stages,
       band_name, 'AR (MDL)', ['report/img/topogram_ar_', band_name,
       '.png']);
end
```

Listing 4: Customized plotting function for laying topogram plots horizontally.

3.1 Results

After applying the defined workflow to the 5 stages of sleep, we obtained the following results for the previously defined bands.





3.2 Discussion

The obtained result match very well with what is described in the literature. Showing a clear correlation with Power Sprectral Density and activity in each specific case. In our opinion, the stage S1 (N1) is very interesting because appears to have oscillations from both stages (sleep/awake) and has activity of all the bands except from alpha band.

Both Welch Periodogram and AR (MDL) PSD methods perform very similarly. However, by observing topogram representations of PSD power/electrode, we think that the AR method is more prone to saturate (giving less visual evidence of differences across electrodes), whereas the Welch method seems more balanced.