Lab 5 - Matched Filtering

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November 2025

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1 Template Matching

1.1 Background and Template Extraction

The objective of this section is the detection of **Spike-and-Wave Complexes (SWC)** in EEG signals. An SWC is characterized by a sharp spike followed by a slow wave (approx. 3 Hz). To perform template matching, a representative SWC was manually extracted from channel **C3** within the time interval of 0.61s to 0.83s.

Figure 1 illustrates the original EEG signal from channel C3 and the isolated template. The template clearly exhibits the sharp negative deflection followed by the slower positive recovery wave typical of this complex.

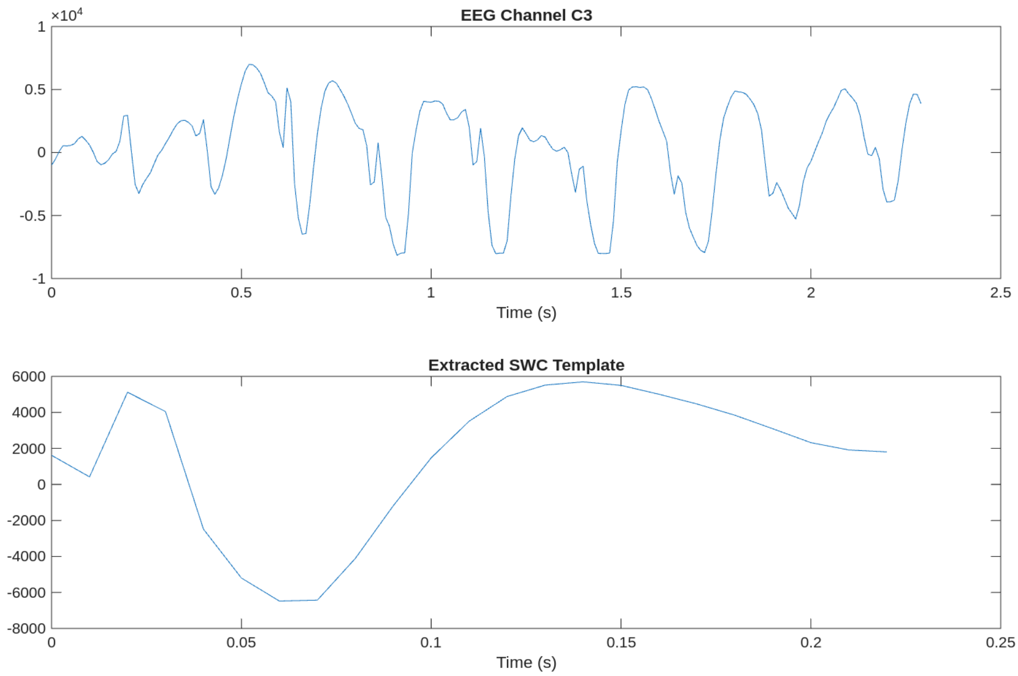

Figure 1: Original EEG signal (top) and the extracted SWC template (bottom).

1.2 Methodology

We utilized the **cross-correlation function (CCF)** to measure the similarity between the template and the continuous EEG signals. The process involved:

1. Computing the cross-correlation between the signal and the template.
2. Normalizing the result to identify relative peaks effectively.
3. Applying a threshold to detect local maxima corresponding to SWC occurrences.
4. Correcting the time delay, as the cross-correlation peak corresponds to the alignment lag, effectively marking the end of the template match.

The following MATLAB code snippet demonstrates the core logic applied to each channel:

```
% Cross-Correlation
[corr_raw, lags] = xcorr(signal, template);

% Normalize
corr_norm = corr_raw / max(abs(corr_raw));

% Peak Detection
threshold = 0.2;
[pks, locs] = findpeaks(corr_norm, 'MinPeakHeight', threshold);

% Map lags to time for plotting
lag_time = lags/fs;
```

Listing 1: Cross-correlation and Peak Detection

1.3 Results and Discussion

The template matching procedure was applied to the source channel (**C3**), a spatially close channel (**C4**), and a farther channel (**P4**).

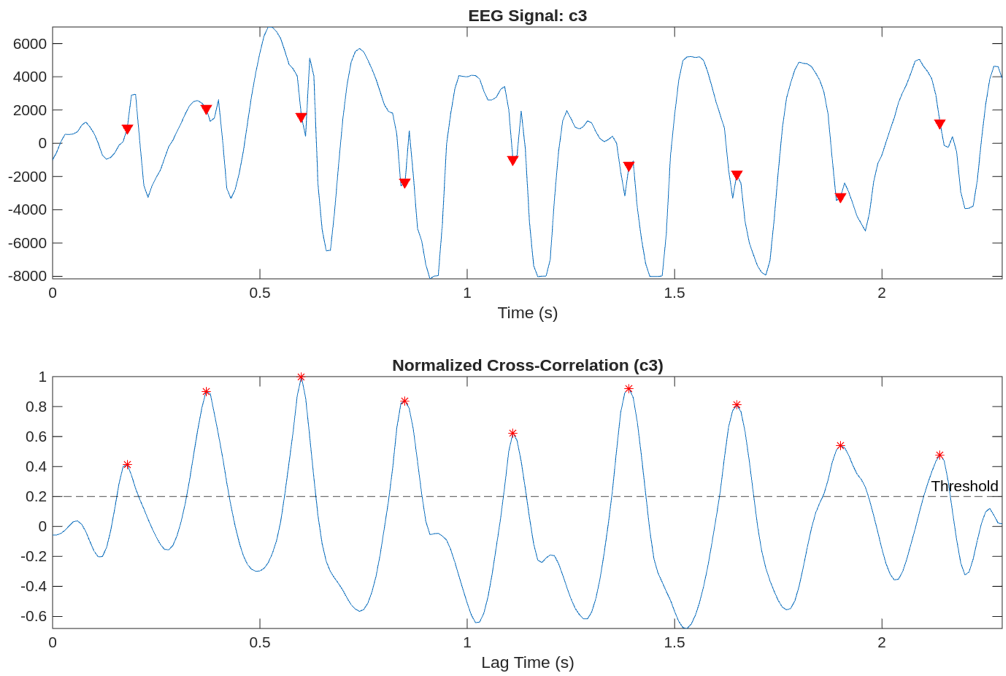

Figure 2: Detection results for Channel C3 (Source). Top: Signal with markers; Bottom: Normalized CCF.

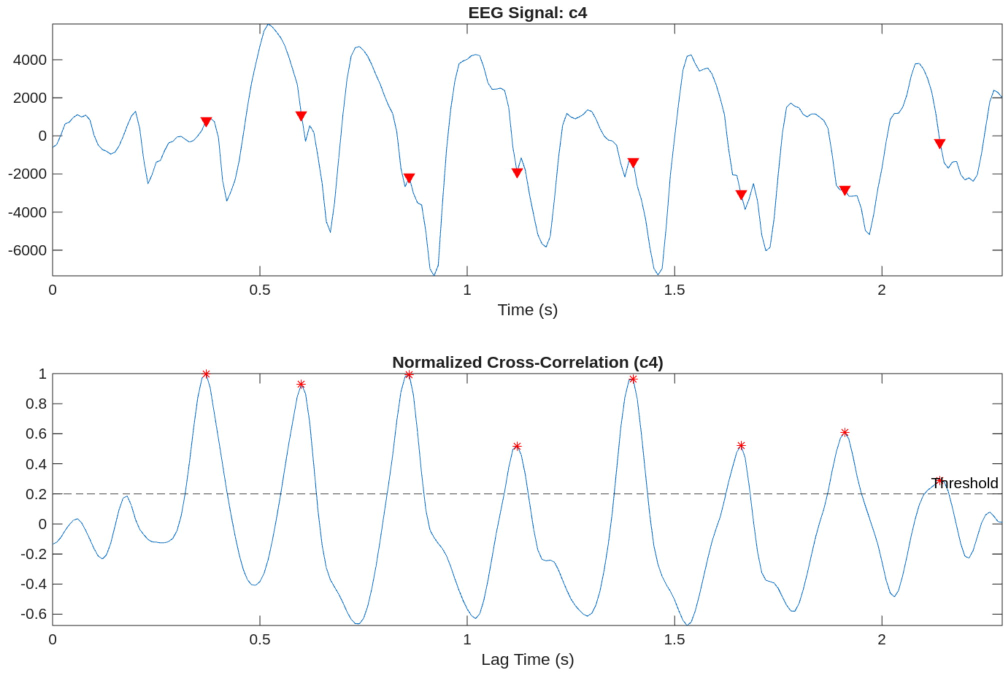

Figure 3: Detection results for Channel C4.

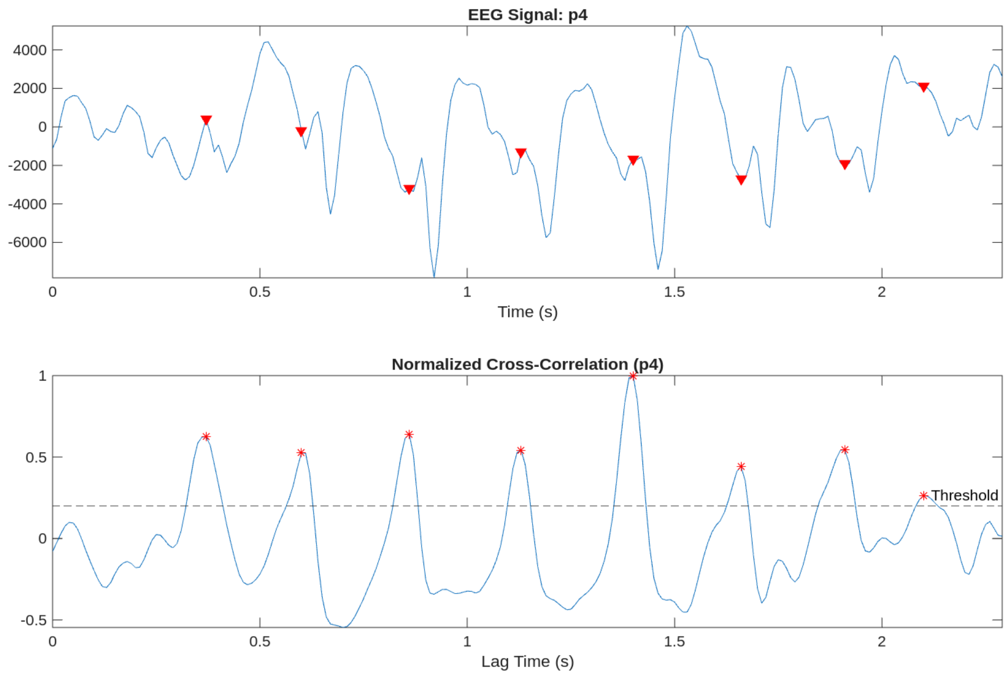

Figure 4: Detection results for Channel P4.

1.3.1 Analysis of Detection

As observed in Figure 2, the correlation for **C3** reaches a maximum of 1.0 (at the location where the template was extracted). The periodic nature of the SWC is clearly visible in the CCF, with multiple peaks exceeding the 0.2 threshold.

Figure 3 shows the results for **C4**. The signal morphology is highly similar to C3, resulting in distinct correlation peaks. This suggests that the SWC activity is synchronized across the central region of the brain.

Figure 4 displays the results for the parietal channel **P4**. Although the waveform is slightly different due to spatial distance, the template matching algorithm remains robust. The normalized correlation still yields distinct peaks, allowing for the correct identification of the SWC events.

In all cases, the red markers on the EEG signals (top subplots) confirm that the algorithm successfully located the epileptic complexes, demonstrating that **template matching** is an effective method for event detection even when the signal morphology varies slightly across channels.

2 Matched Filtering

2.1 Impulse Response Derivation

The matched filter is the optimal linear filter for maximizing the Signal-to-Noise Ratio (SNR) in the presence of stochastic noise. The impulse response $h(n)$ of a matched filter is the time-reversed version of the template signal $s(n)$.

Figure 5 displays the template extracted in the previous section and the derived impulse response. As expected, $h(n)$ is a mirrored version of $s(n)$.

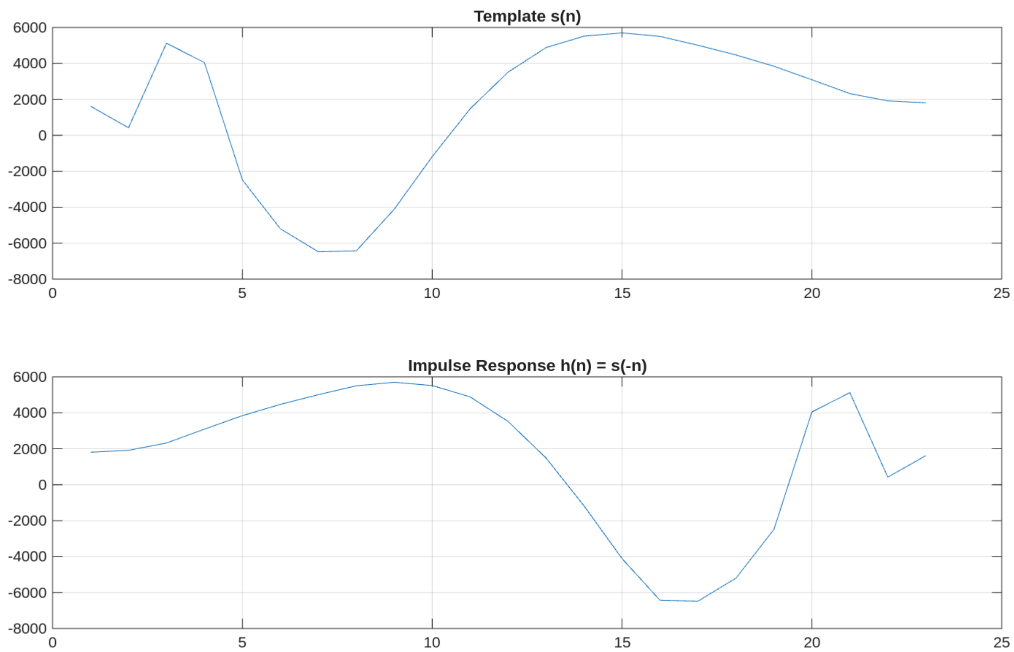

Figure 5: Top: The SWC template $s(n)$. Bottom: The matched filter impulse response $h(n) = s(-n)$.

2.2 Linear Matched Filtering (Unnormalized)

We first applied the matched filter using the standard linear convolution method. The filter coefficients were defined as the reversed template ($b = h(n)$) and the denominator $a = 1$.

```
% Derive Impulse Response
h_t = fliplr(template);

% Apply FIR Filter
matched_output = filter(h_t, 1, signal);
```

Listing 2: Linear Matched Filtering Implementation

Figure 6 shows the result for channel C3. While the filter successfully highlights the periodic nature of the SWCs, the output amplitude is extremely high (in the order of 10^8). This occurs because the standard convolution accumulates the product of the signal and template energies (effectively $K = 1$). This unbounded output makes it difficult to set a universal detection threshold.

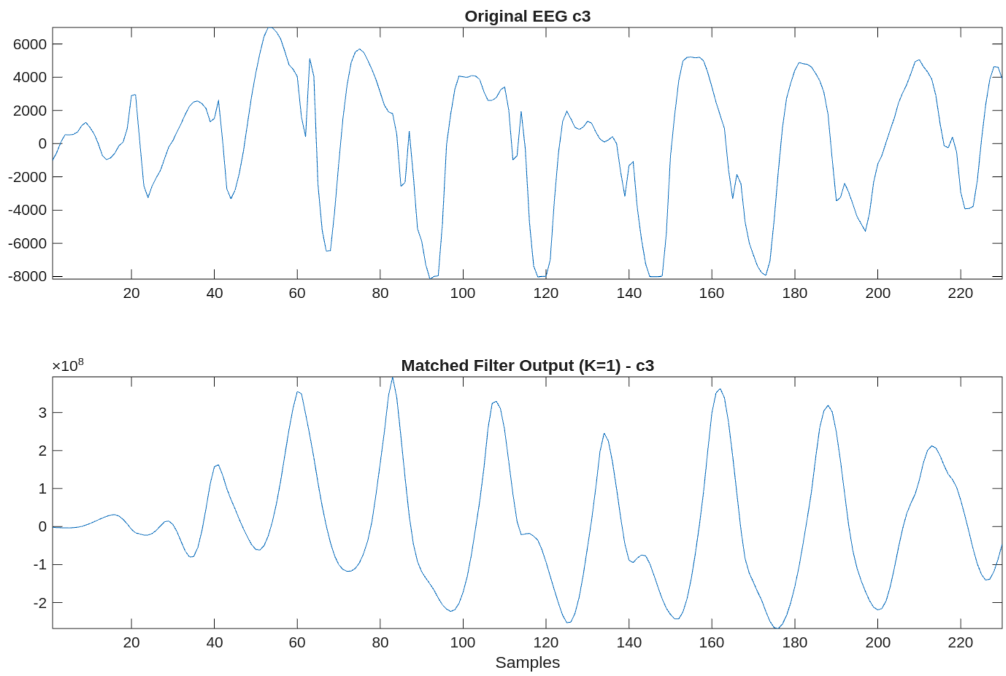

Figure 6: Unnormalized matched filter output for C3. Note the large amplitude scale.

2.3 Normalized Matched Filtering (Time-Variant K)

To bound the output between -1 and 1 (similar to the normalized cross-correlation), we implemented a normalization factor K. Since the energy of the EEG signal changes over time, K must be **time-variant**.

The normalization factor at each sample is derived from the geometric mean of the template energy ($E_{template}$) and the local signal energy (E_{local}).

```
% Calculate Template Energy
E_template = sum(template.^2);

% Calculate Local Signal Energy
ones_filter = ones(size(template));
E_local = filter(ones_filter, 1, signal.^2);

```

```
% Apply Normalization
normalization_factor = sqrt(E_template * E_local);
y_normalized = y_raw ./ normalization_factor;
```

Listing 3: Normalized Filtering with Time-Variant K

2.4 Results and Discussion

The normalized matched filter was applied to channels C3, C4, and P4. A threshold of 0.2 was used to detect the Spike-and-Wave Complexes.

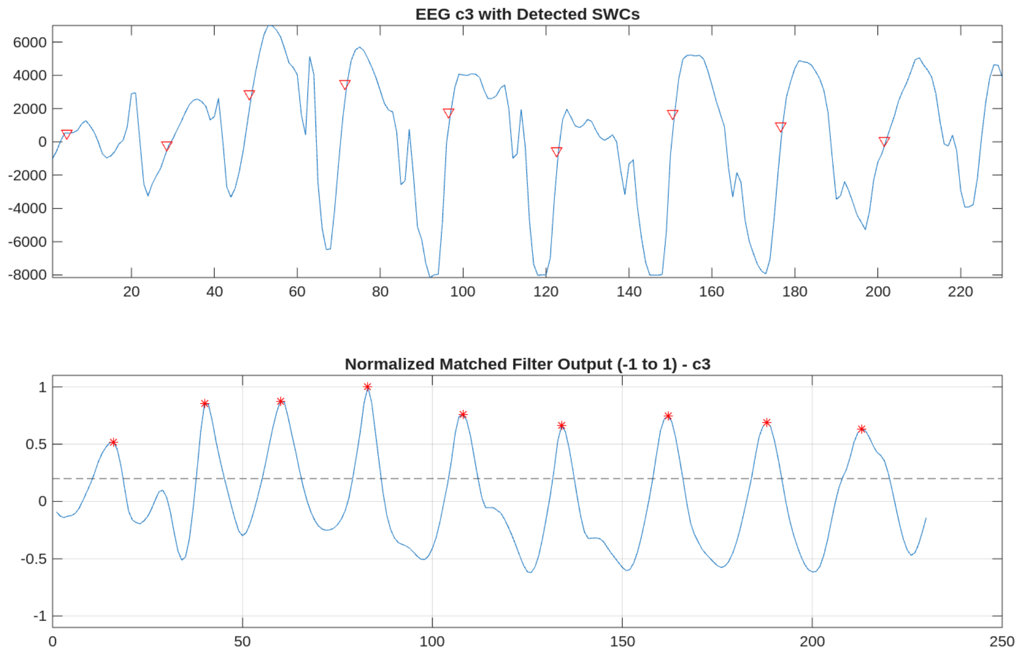

Figure 7: Normalized Matched Filter output for Channel C3.

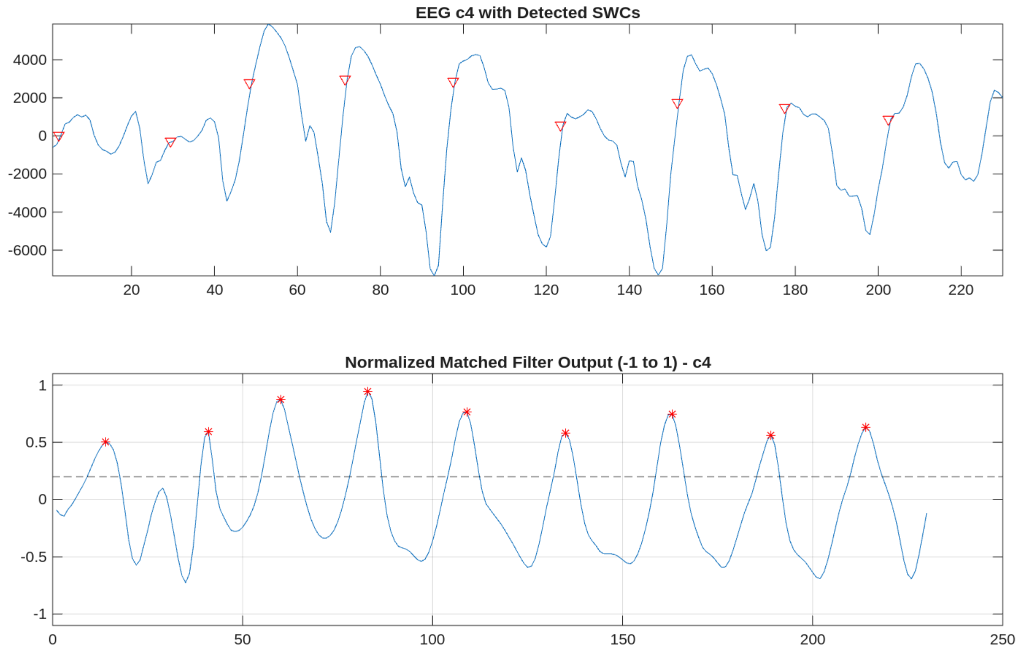

Figure 8: Normalized Matched Filter output for Channel C4.

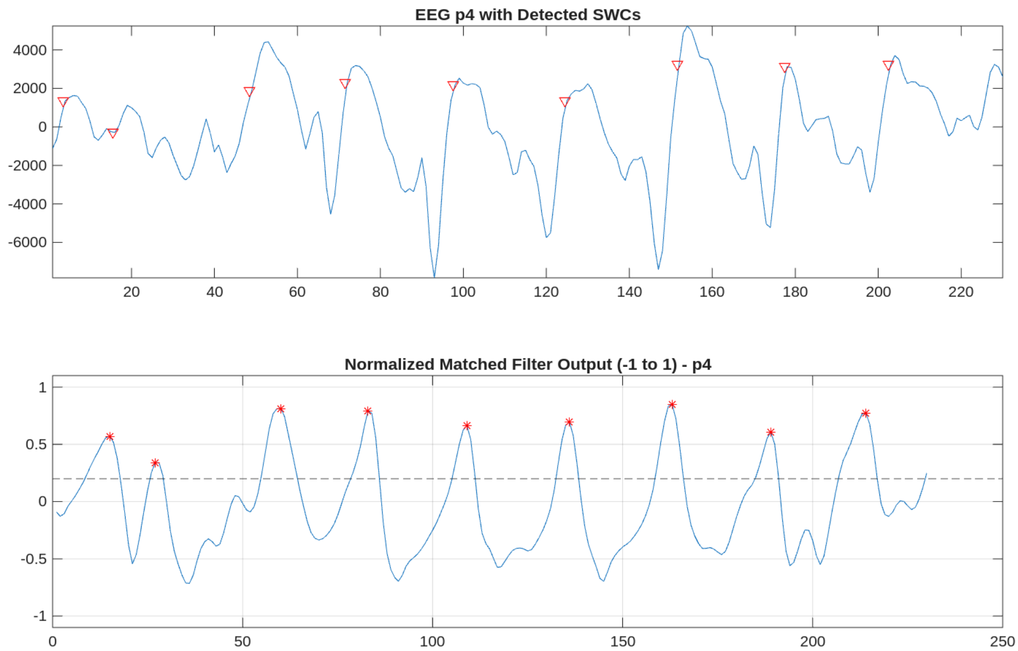

Figure 9: Normalized Matched Filter output for Channel P4.

2.4.1 Analysis

The results demonstrate the equivalence between **normalized cross-correlation** (Section 2) and **normalized matched filtering** (Section 3).

- **C3 (Source):** Figure 7 shows a perfect correlation of 1.0 at the template extraction point. The detected peaks align perfectly with the epileptic complexes.
- **C4 (Close):** As seen in Figure 8, the matched filter is highly robust, detecting SWCs with correlation values consistently above 0.5.
- **P4 (Far):** Figure 9 shows that even in the parietal region, where the signal morphology differs slightly, the matched filter successfully identifies the events.

The matched filter approach, when properly normalized, provides a robust detection mechanism. The primary difference between this and the method in Section 2 is computational: Section 2 used ‘xcorr‘ (global computation), while Section 3 used ‘filter‘ (causal convolution), which is more suitable for real-time processing applications.

2.5 Noise Robustness Analysis

2.5.1 Methodology

To evaluate the robustness of the matched filter, we introduced additive stochastic noise to the EEG signal (Channel C3). We utilized White Gaussian Noise (AWGN) to simulate varying environmental conditions and sensor noise.

The signal-to-noise ratio (SNR) was varied from **20 dB** (clean) down to **-5 dB** (very noisy). The matched filter was applied to each noisy instance to observe if the Spike-and-Wave Complexes (SWCs) remained detectable.

The following code snippet demonstrates the injection of noise and the subsequent filtering:

```matlab
% Define SNR levels to test
snr_values = [20, 15, 10, 5, 0, -5];

for k = 1:length(snr_values)
    current_snr = snr_values(k);

    % Add White Gaussian Noise
    % 'measured' ensures the noise power is relative to the signal power
    noisy_signal = awgn(c3_data, current_snr, 'measured');

    % Apply Normalized Matched Filter
    y_raw = filter(h_t, 1, noisy_signal);
    E_local = filter(ones(size(template)), 1, noisy_signal.^2);
    y_norm = y_raw ./ sqrt(E_template * E_local);
end
```

Listing 4: Adding Noise and Filtering

2.5.2 Results and Discussion

Figure 10 illustrates the degradation of the input signal (left column) and the corresponding filter output (right column) as the SNR decreases.

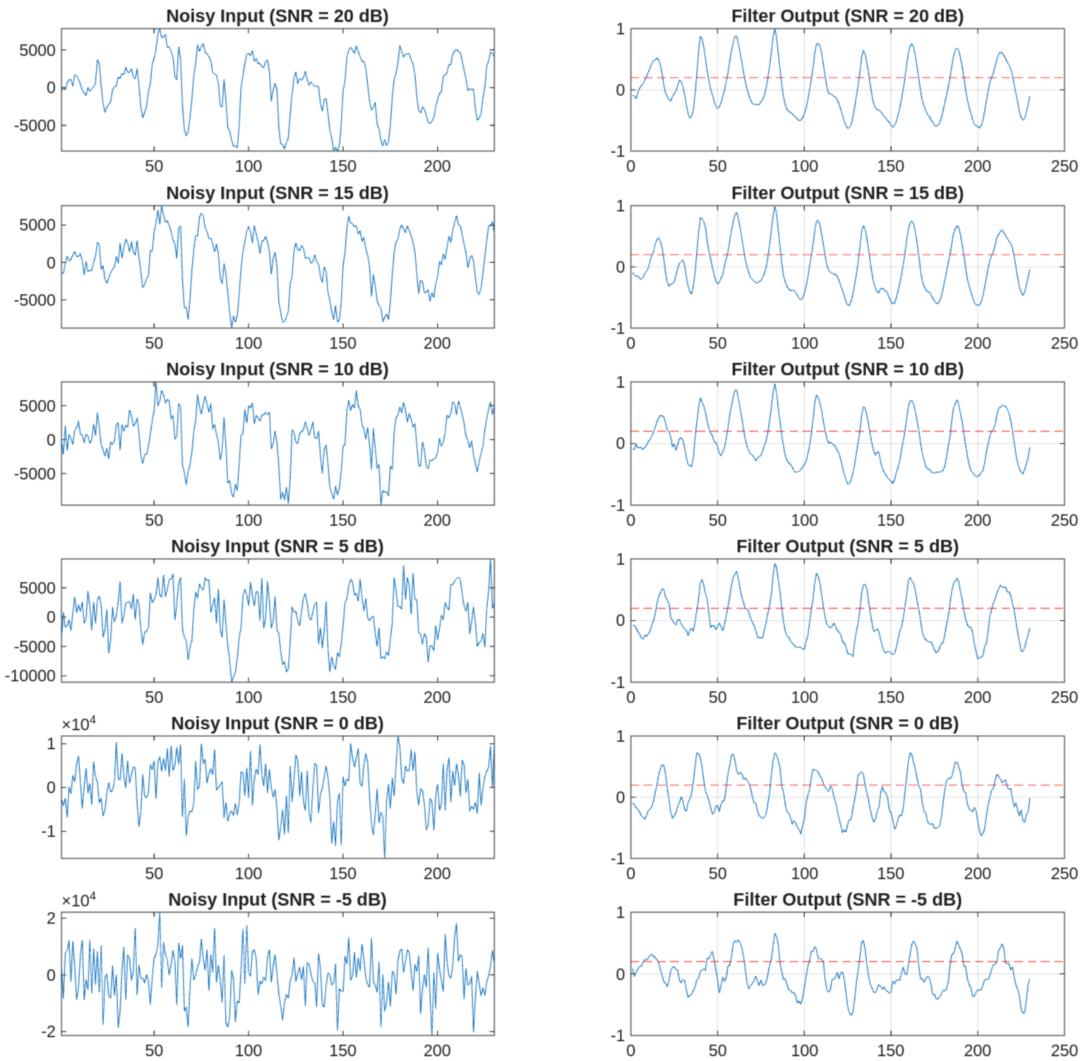

Figure 10: Left: Noisy Input Signal at decreasing SNRs. Right: Matched Filter Output.

The results demonstrate the **optimality** of the matched filter in maximizing SNR:

- **High SNR (20 dB - 10 dB):** The SWCs remain clearly visible in the time domain. The matched filter output is extremely clean, with peaks approaching 1.0. Detection is trivial in this range.
- **Critical SNR (0 dB):** At 0 dB, the power of the noise is equal to the power of the signal. In the time domain (left), the SWC morphology is heavily obscured. However, the matched filter (right) successfully suppresses the uncorrelated noise while amplifying the correlated template pattern. The peaks remain distinct and well above the detection threshold.
- **Low SNR (-5 dB):** At this level, the noise is stronger than the signal. Visually identifying the complex in the raw signal is challenging. Remarkably, the matched filter output still reveals periodic peaks corresponding to the SWC locations. Although the "noise floor" in the correlation output rises, the peaks generally remain distinguishable above the 0.2 threshold.

Conclusion: The matched filter proves to be highly robust. Because it performs a correlation integration, zero-mean Gaussian noise tends to average out over the duration of the template, whereas the coherent signal adds up constructively. This allows for reliable detection of epileptic events even in environments with significant background noise.