Geometric Tools for Computer Graphics - MIRI **08/01/2019**

Name:

Please justify your answers!

1) (3 points) Consider a fixed circle C centered at $(0, a)$, with $a > 0$, and tangent to the axis Ox at the origin O. For each line ℓ through O, let Q be the intersection point of ℓ and C, and let A be the intersection point of ℓ and the line parallel to Ox and tangent to C. Finally, let P be the intersection point of the vertical line through A and the horizontal line through Q. The *locus* of all points P obtained through this procedure from all possible lines ℓ is called *Witch of Agnesi (versiera di Agnesi)*.

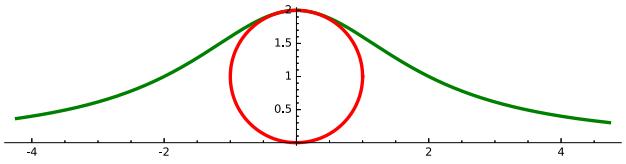

Figura 1: Witch of Agnesi.

(a) Using the polar angle t of the line ℓ, verify that the following is a parameterization of the curve:

$$\left.\begin{aligned} x &= 2a\frac{\cos t}{\sin t} \\ y &= 2a\sin^2 t \end{aligned}\right\}, \quad t \in (0, \pi).$$

(b) Obtain the implicit equation of the curve.

(c) Prove that the point $P = (-2a, a)$ belongs to the curve. Considering the parameterization given in (a), which value of the parameter t gives the point P? Prove that the tangent vector to the curve at the point P is not perpendicular to the position vector of P.

(d) For $a = 3$, compute the position of the center of a circle of radius 2 tangent to the curve at point P and located above the curve. Give the cartesian equation of such a circle.

(e) Using change of reference systems, give a parametrization of the *Witch* associated to a circle of radius 5 and tangent to the line $x - 3y = 1$ at point $(4, 1)$, and located above the line.

2) (2 points) Consider the sphere S of radius 4 centered at the origin and the point $Q = (1, 1, \sqrt{2})$.

(a) Check that Q belongs to S. Which is the cartesian equation of S? Give a parameterization of S.

(b) Describe the motion taking S to a new position in which the point Q moves to point $(3, 0, 0)$, and S is tangent to the plane $x - \sqrt{3}y - 2z - 3 = 0$ at $(3, 0, 0)$. Give a parameterization of the resulting sphere.