Isabel Navazo

Pere-Pau Vázquez

Scientific Visualization 2020 – 2021

FUNDAMENTAL BACKGROUND

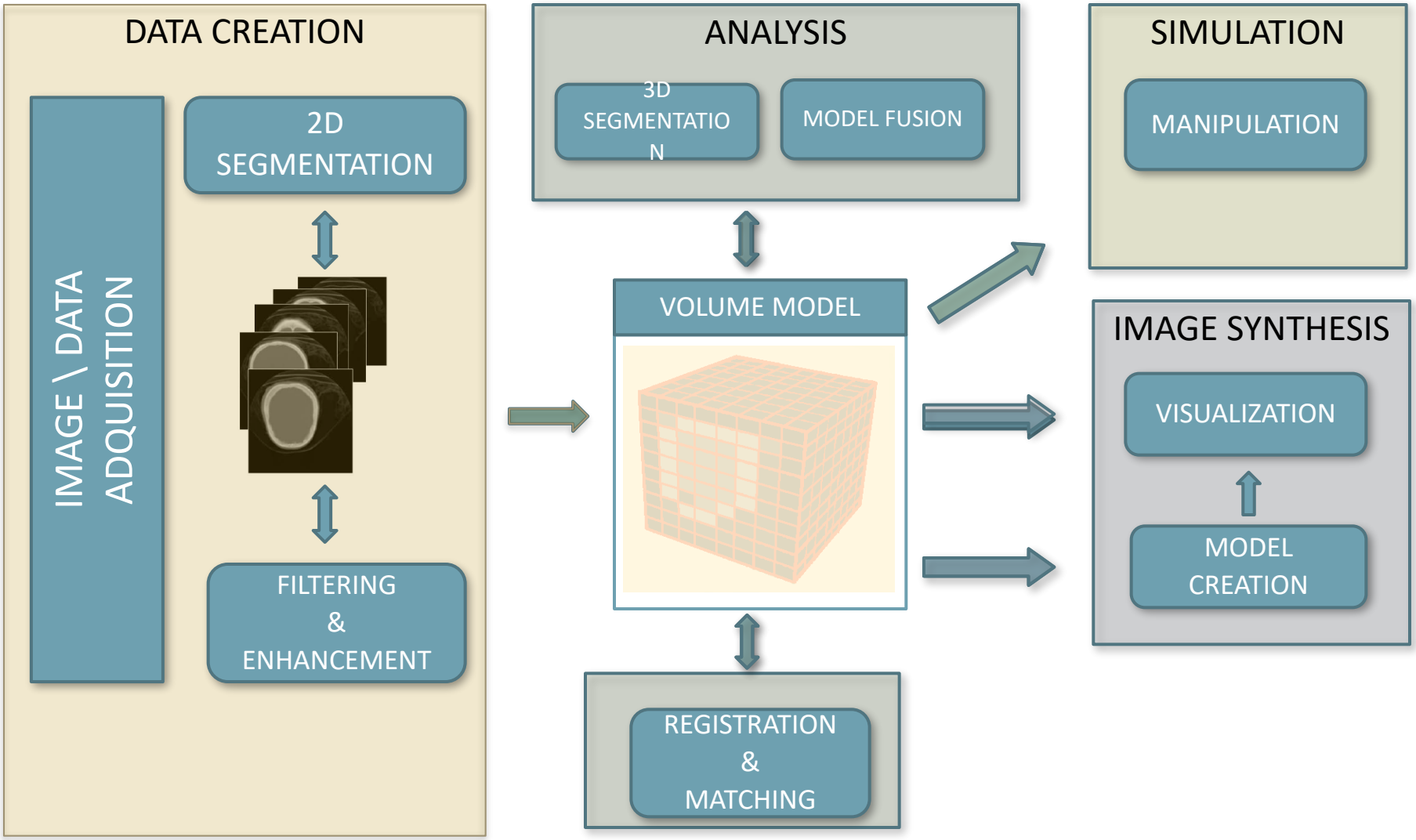
About the slides

- Contents coming from different sources, including:
  - Christof Rezk-Salama, Markus Hadwiger, Patric Ljung, Timo Ropinski (EG Volume Rendering Course, 2006), Travis Gorking, Eva Monclús …

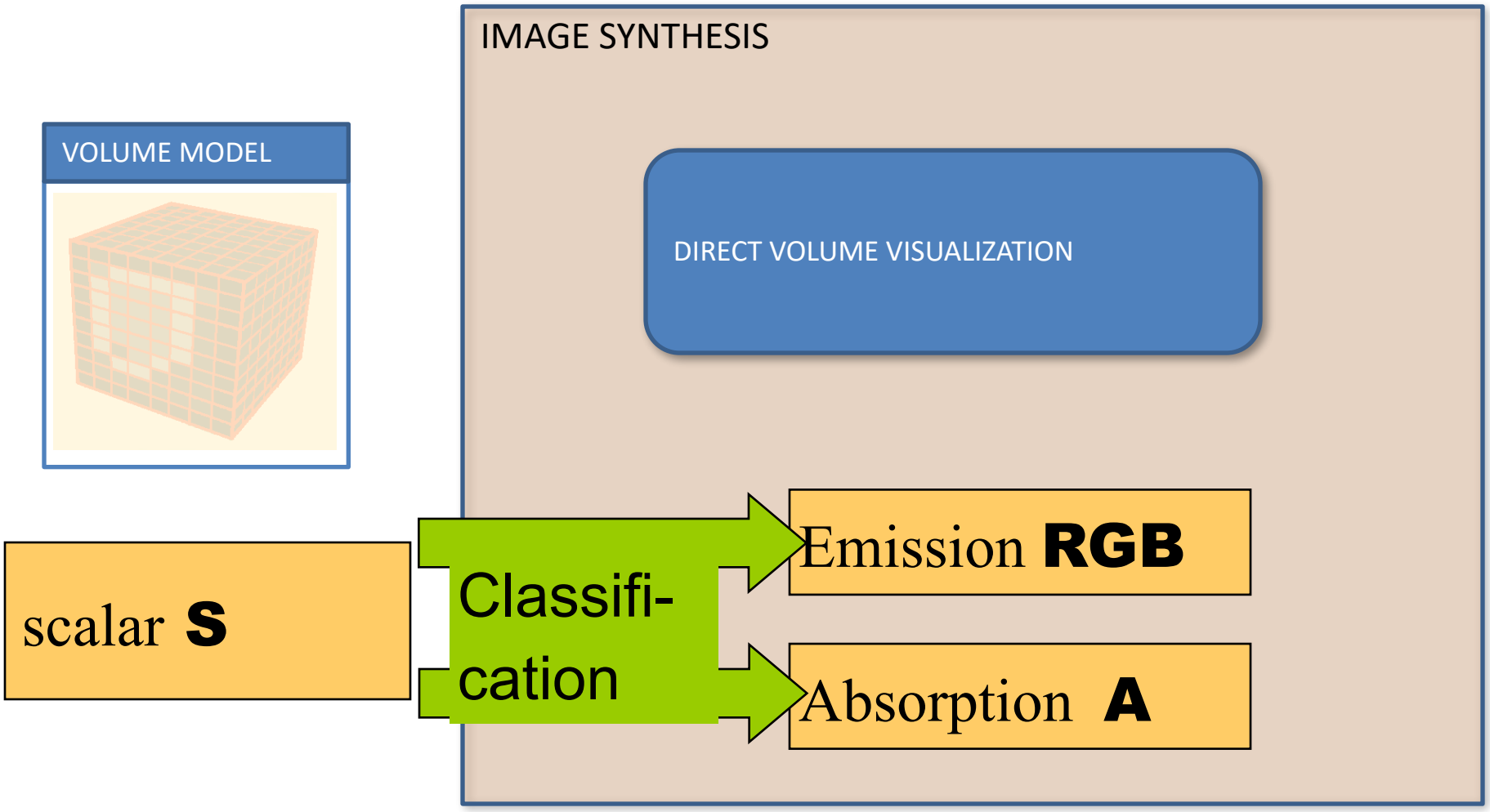
Outline

- Volume rendering equation
- Basic Approaches: Direct volume rendering, 2D and 3D textures

Volume Graphics Pipeline

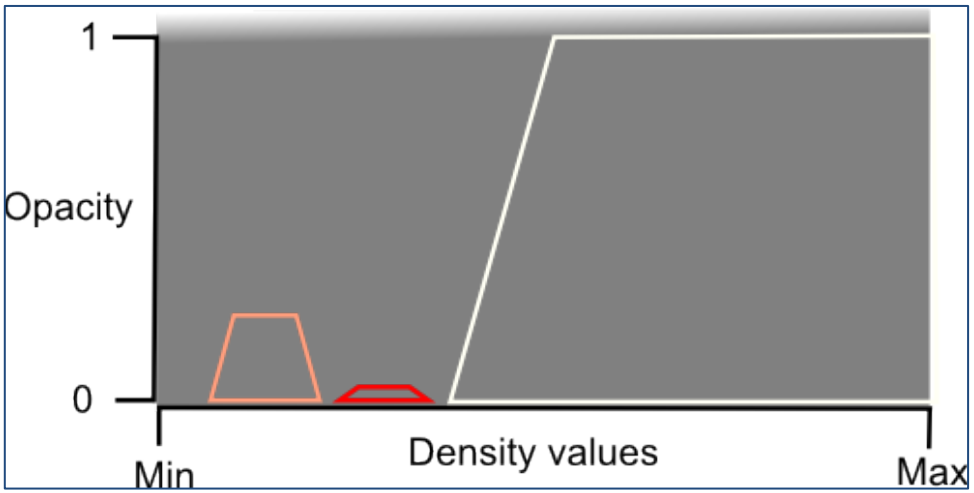


Direct Volume Rendering



Classification

- Specifies the visual appearance of volume data: the Transfer Function



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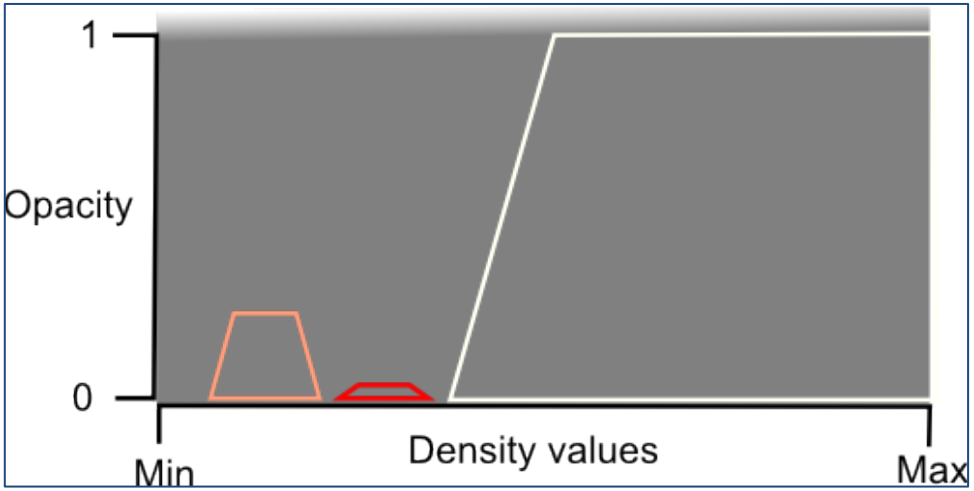

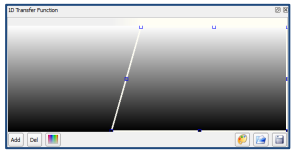

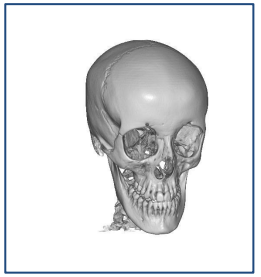

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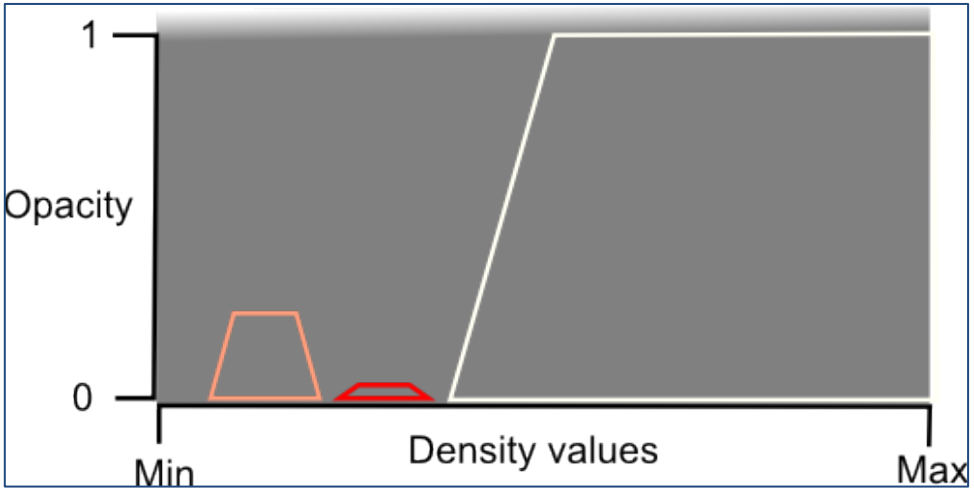

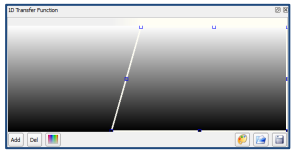

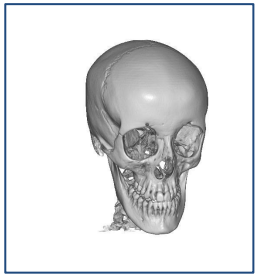

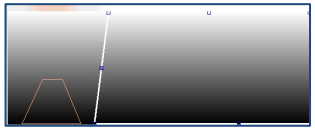

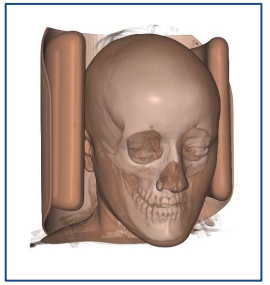

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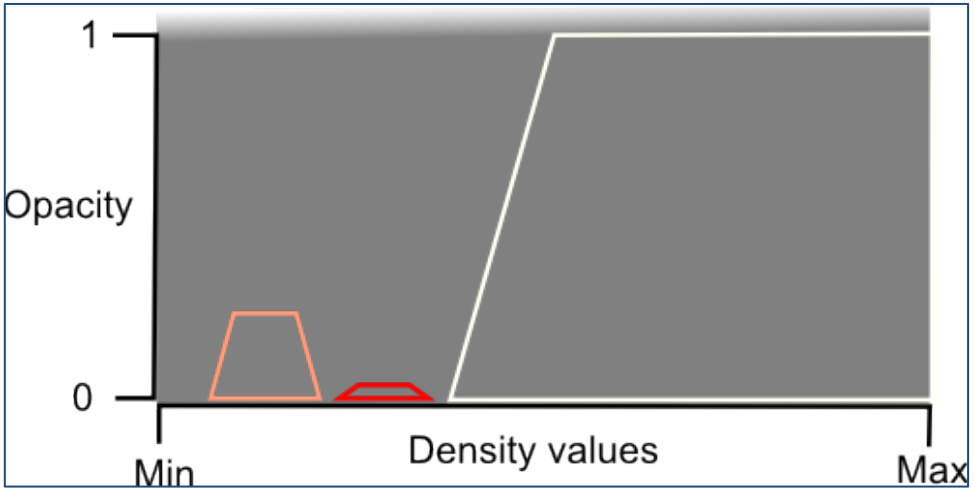
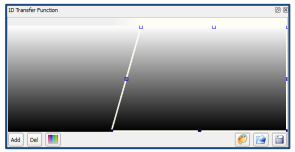
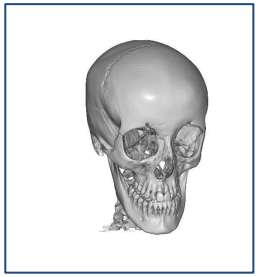
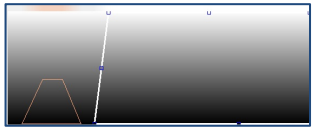
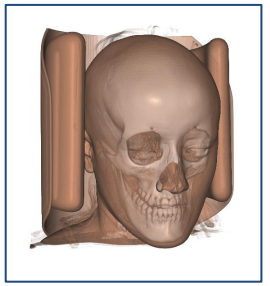
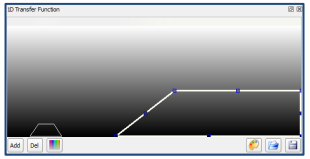


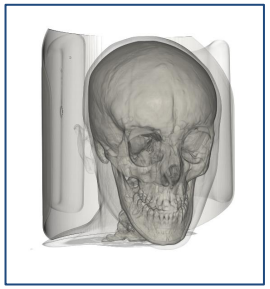








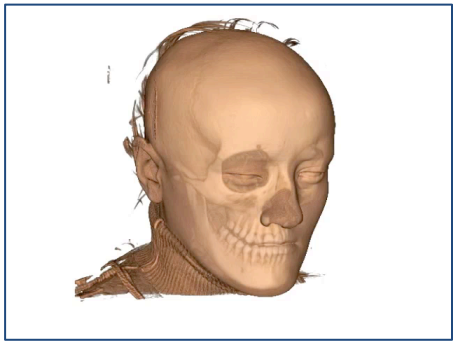




Classification

- Some considerations: pros & cons

- FT gives good results in many applications
- Focuses on “materials” not on anatomy structures
    - Some structures has the same “material” (for acquisition devises features)
    - It is not possible “identify” all structures
    - Fuzzy boundaries (densities)
- *Not semantic information only visual perception*
- Sometimes is difficult to design the needed TF



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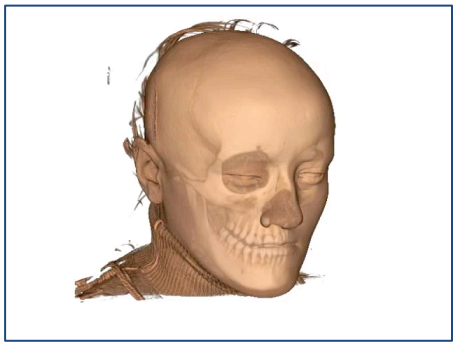

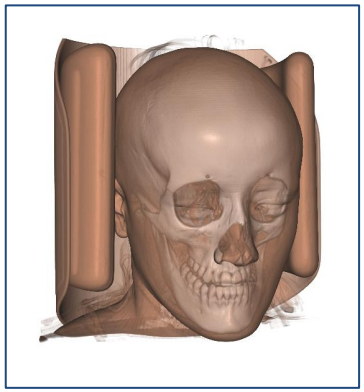

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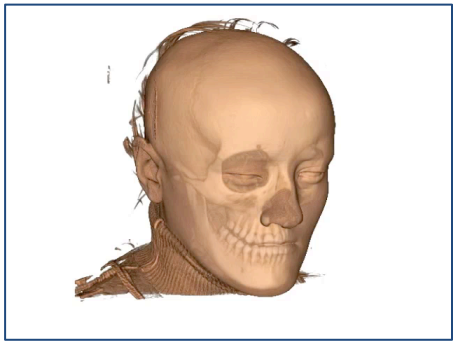
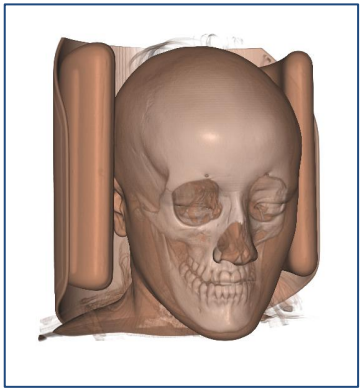
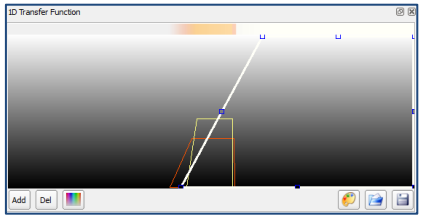
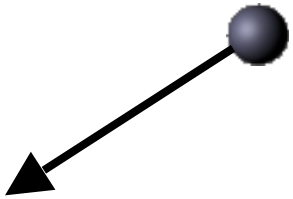






Physical Model of Radiative Transfer



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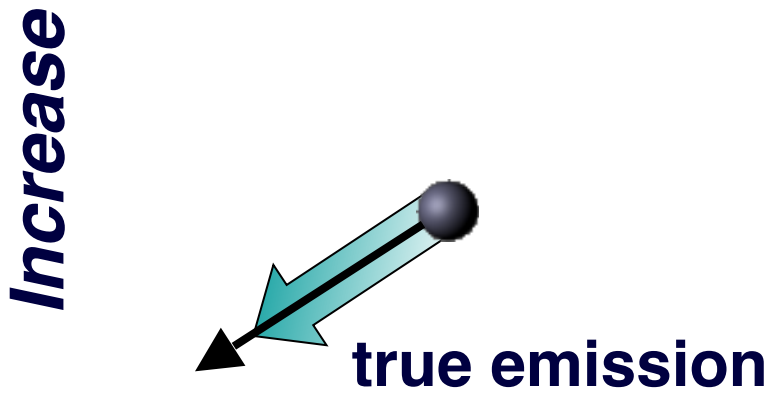

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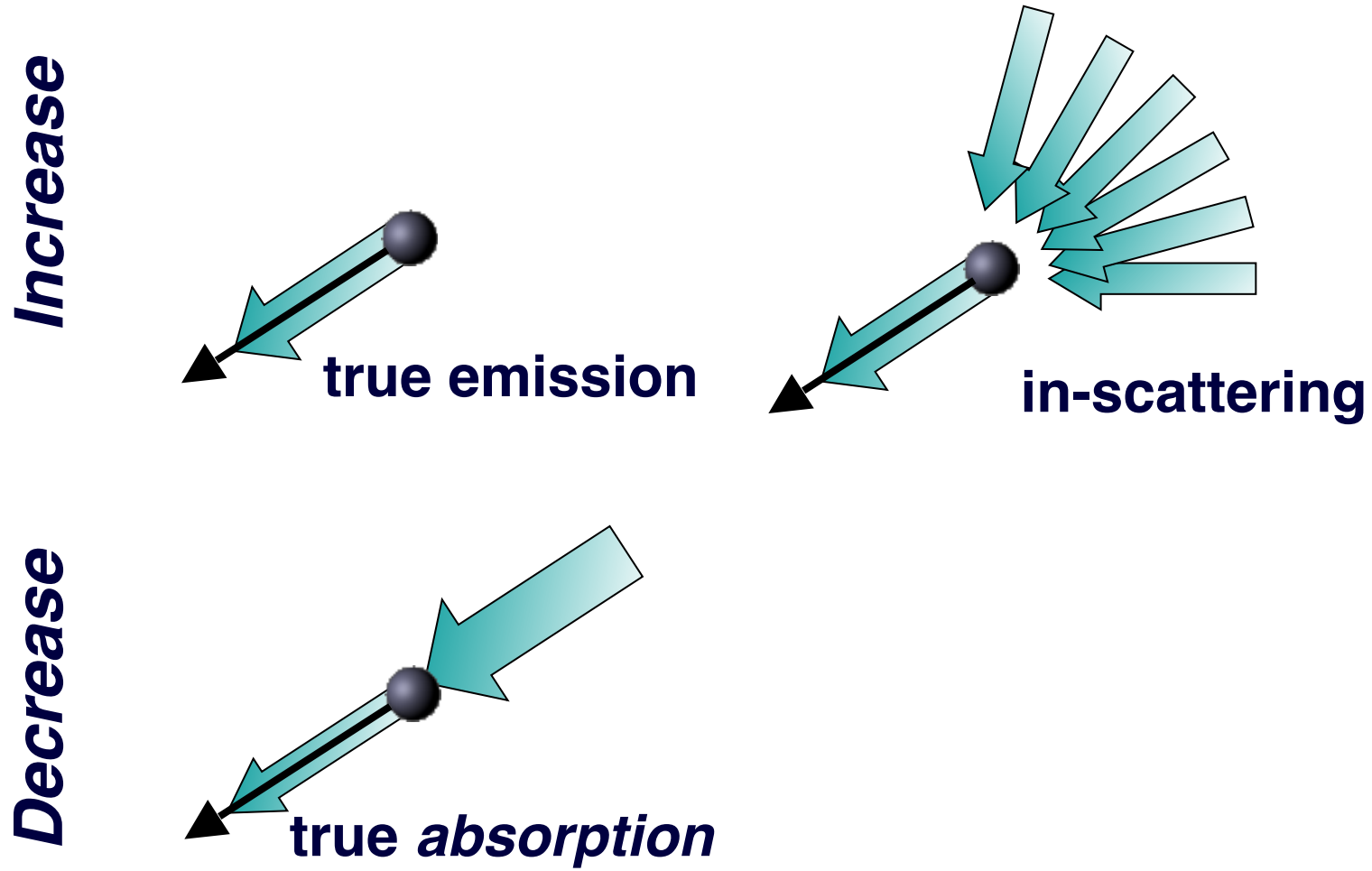

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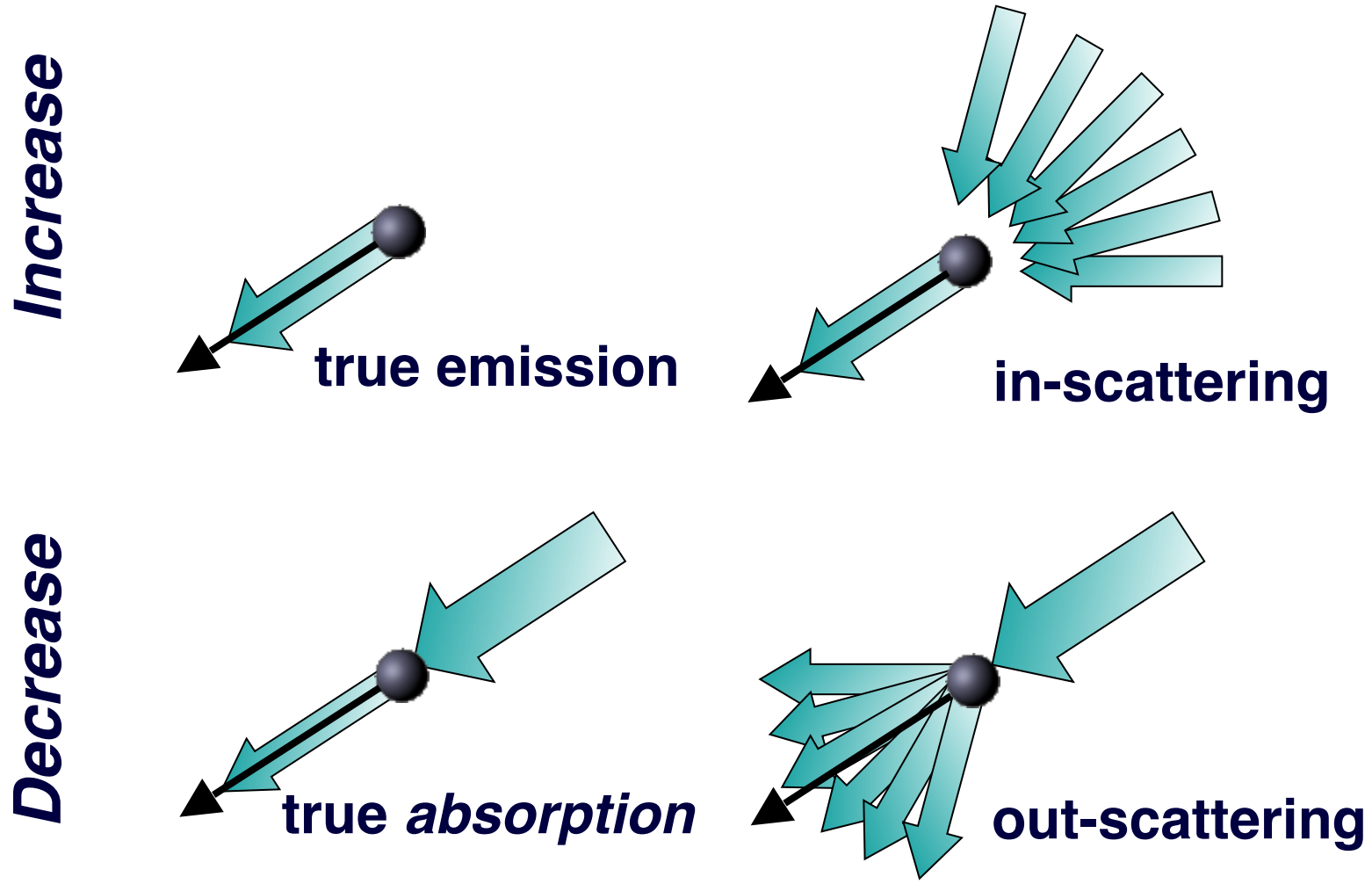

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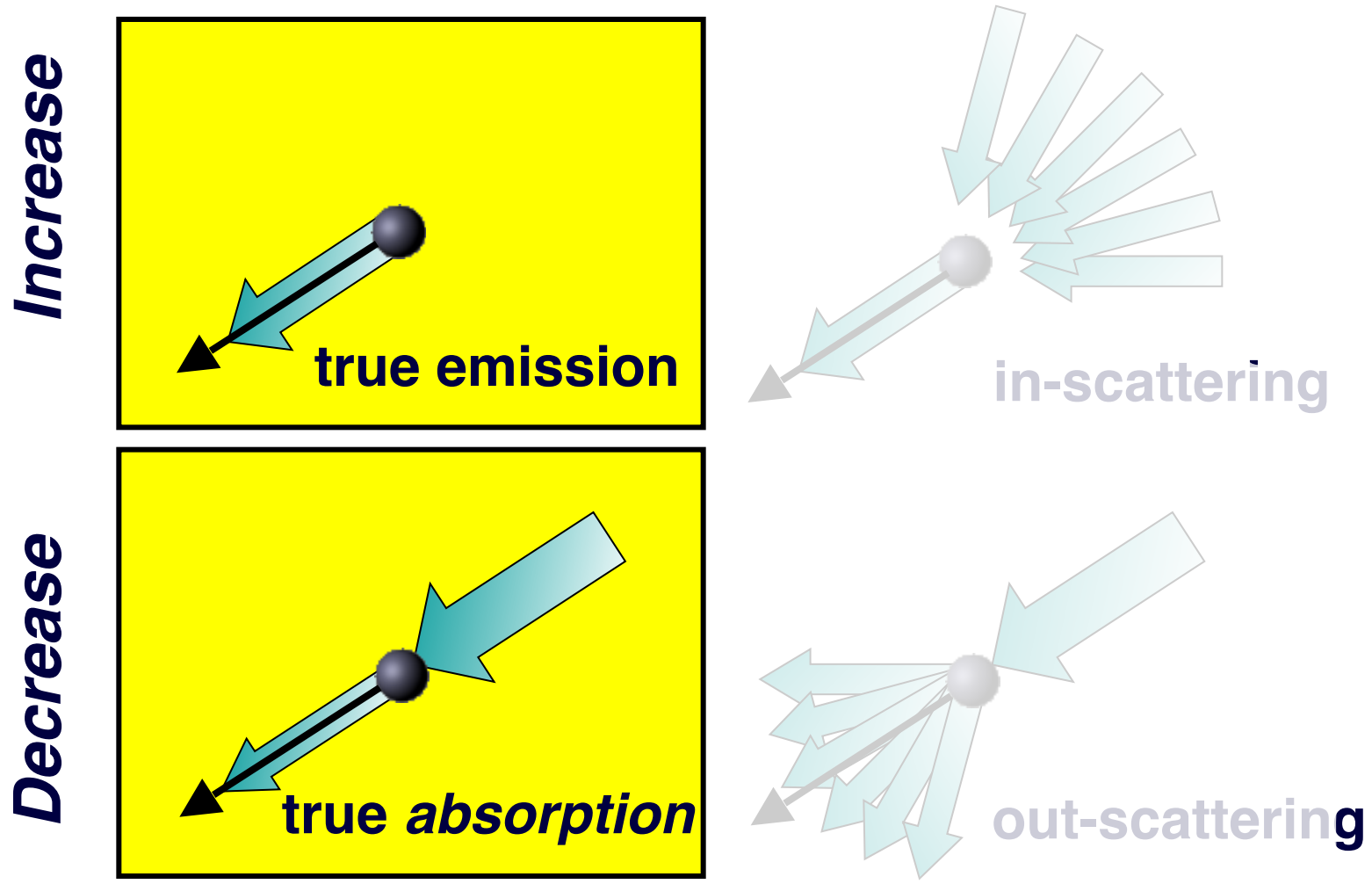

Ray Integration

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

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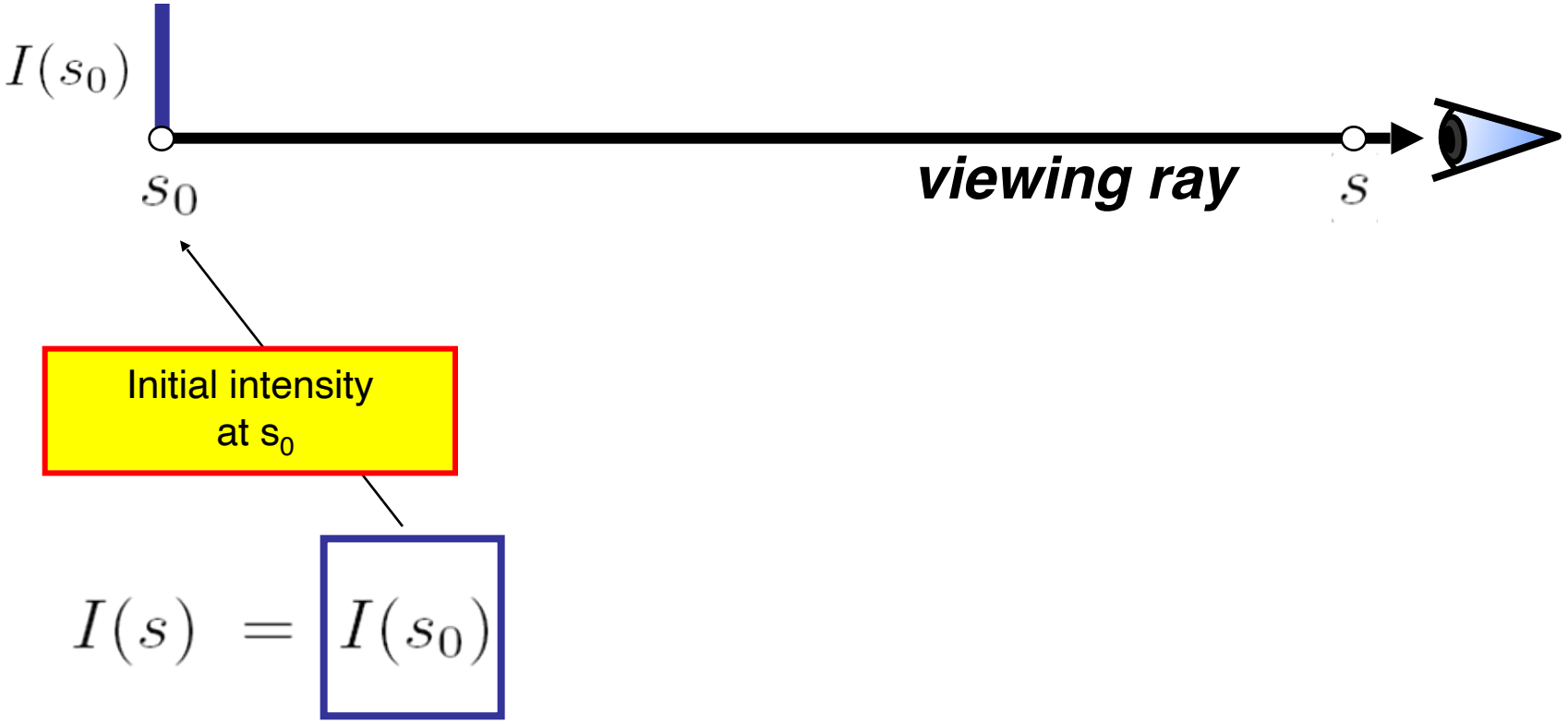

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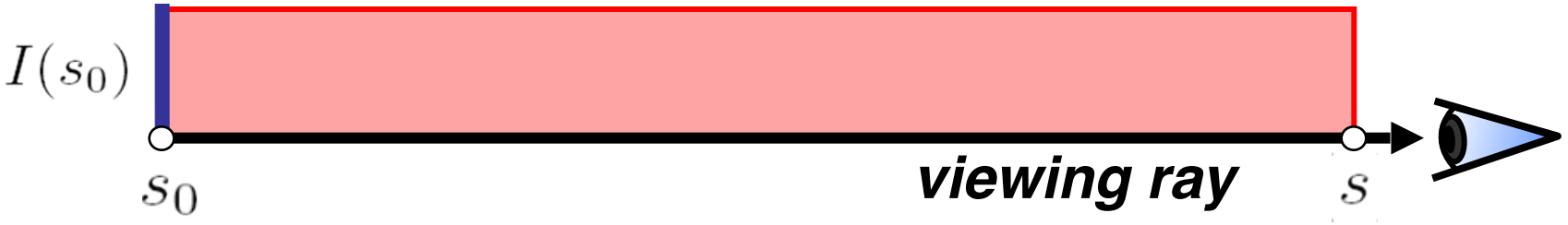

Initial intensity at s_0

$$I(s) = I(s_0)$$

Without absorption all the initial radiant energy would reach the point ***s.***

Ray Integration

How do we determine the radiant energy along the ray?

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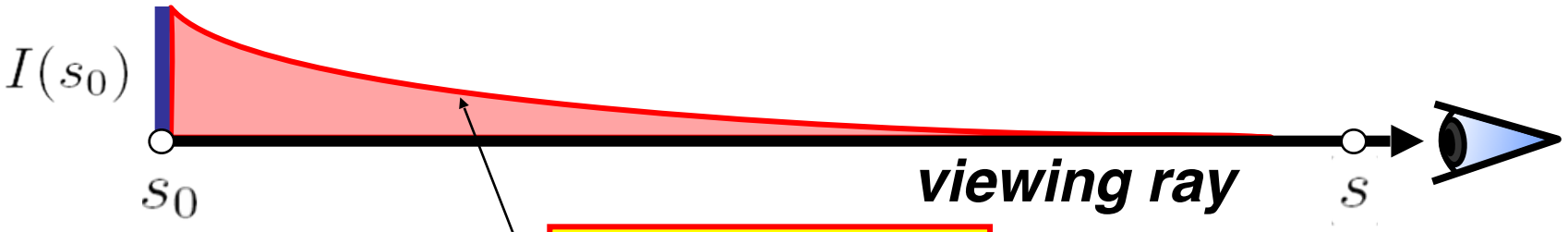

Absorption along the ray segment s_0 - s

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)}$$

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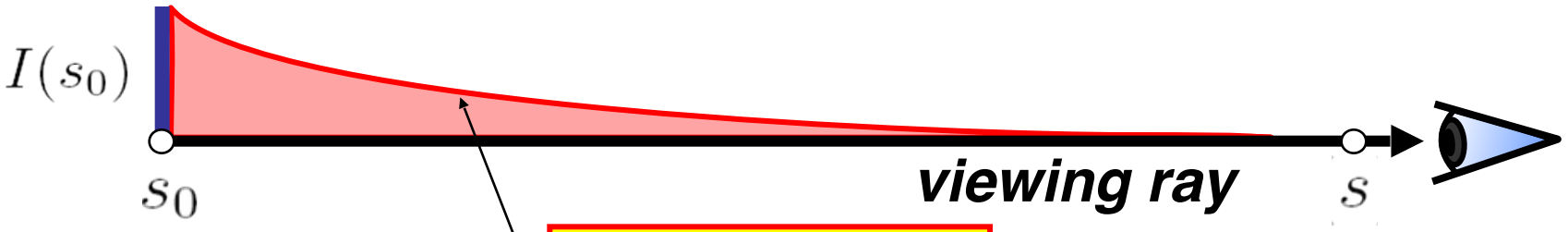

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transparency

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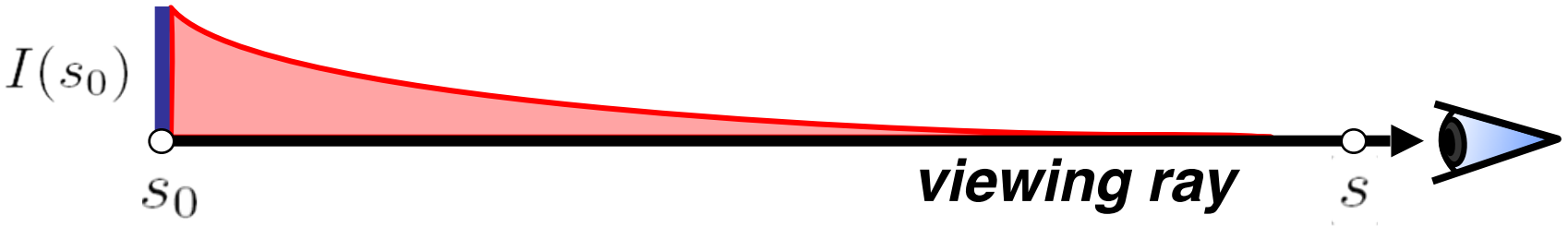

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transparency

Extinction τ

Absorption κ

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

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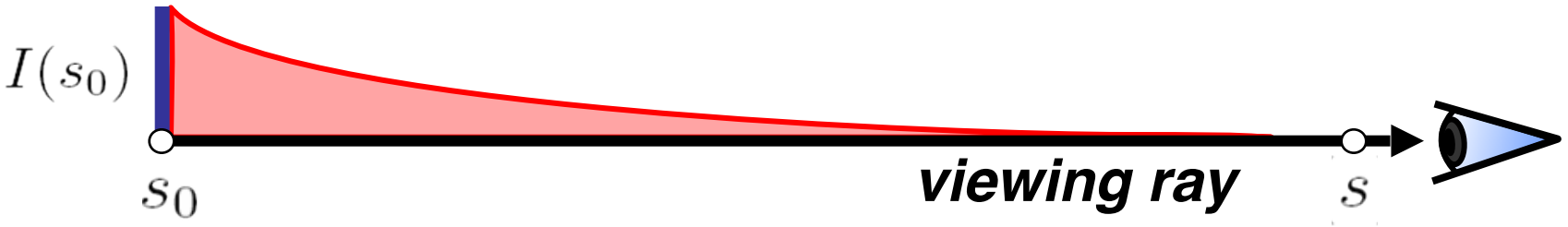

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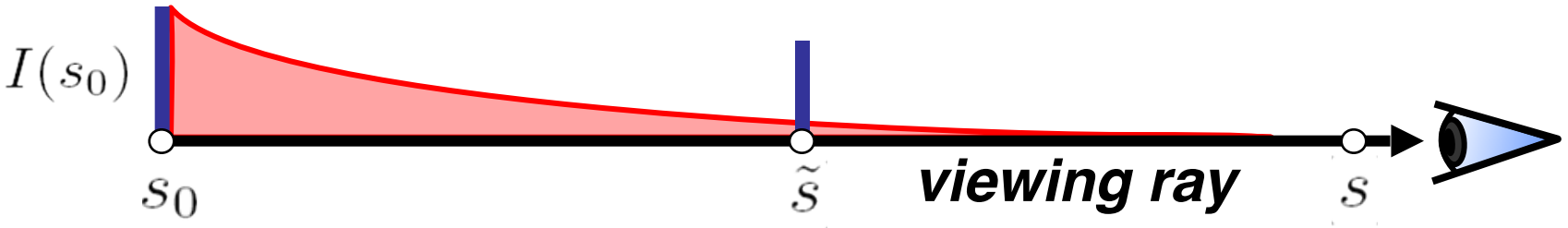

One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)}$$

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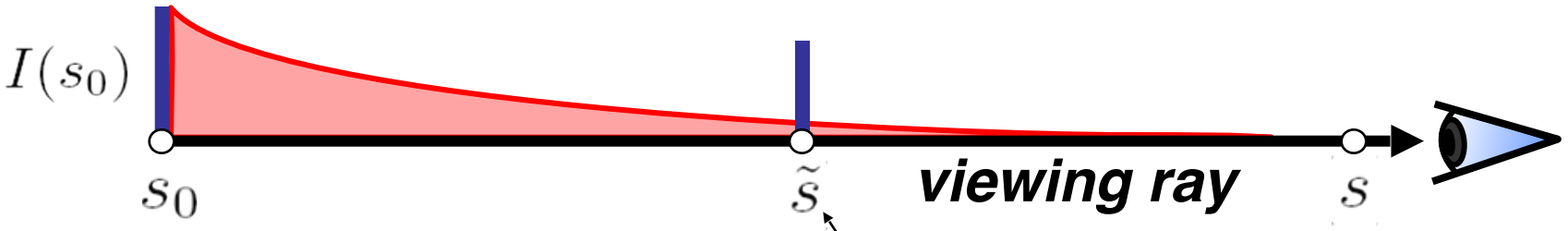

One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

Active emission at point $\tilde{s}$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + q(\tilde{s})$$

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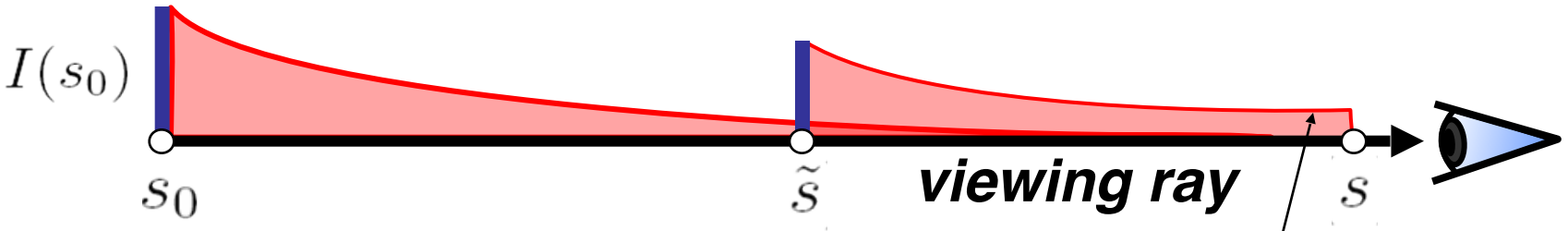

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Absorption along the distance $s - \tilde{s}$

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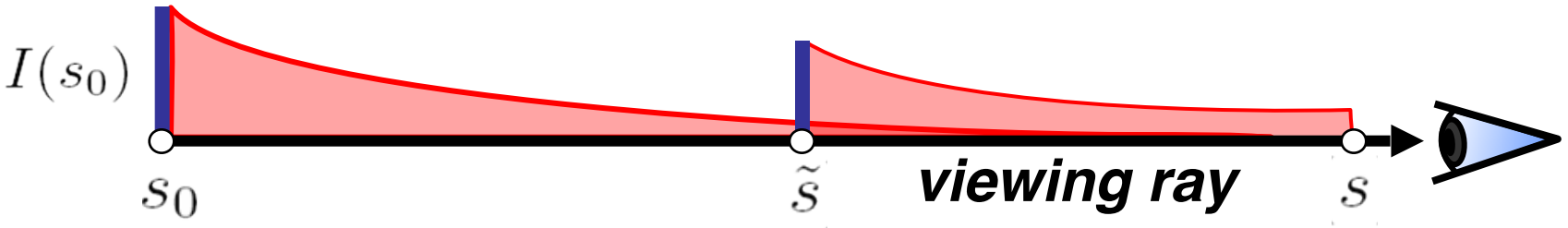

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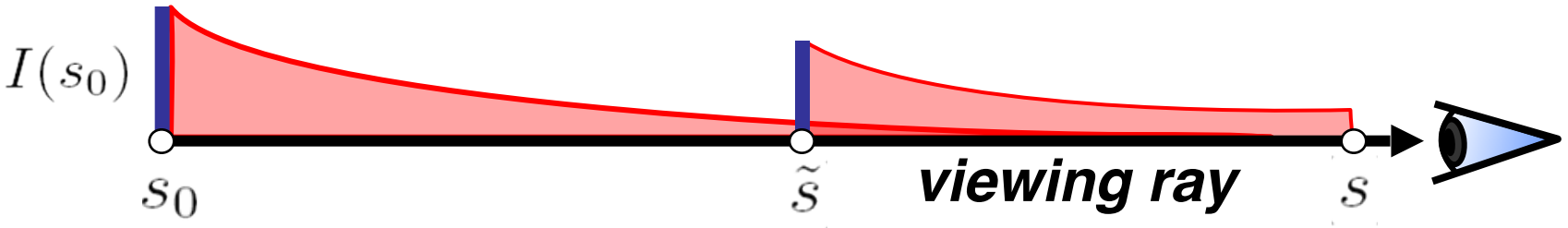

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Ray Casting

$$I(s) = + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

- Software Solution

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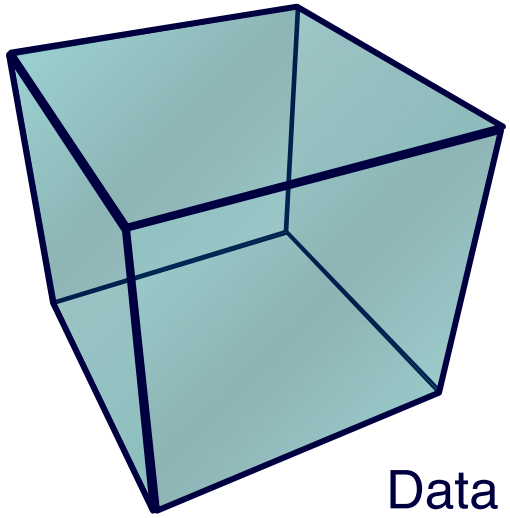

Data Set

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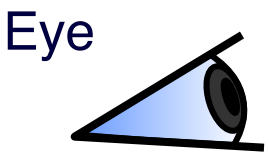

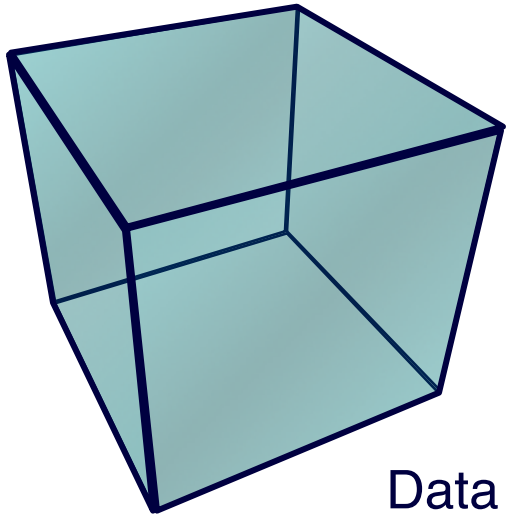

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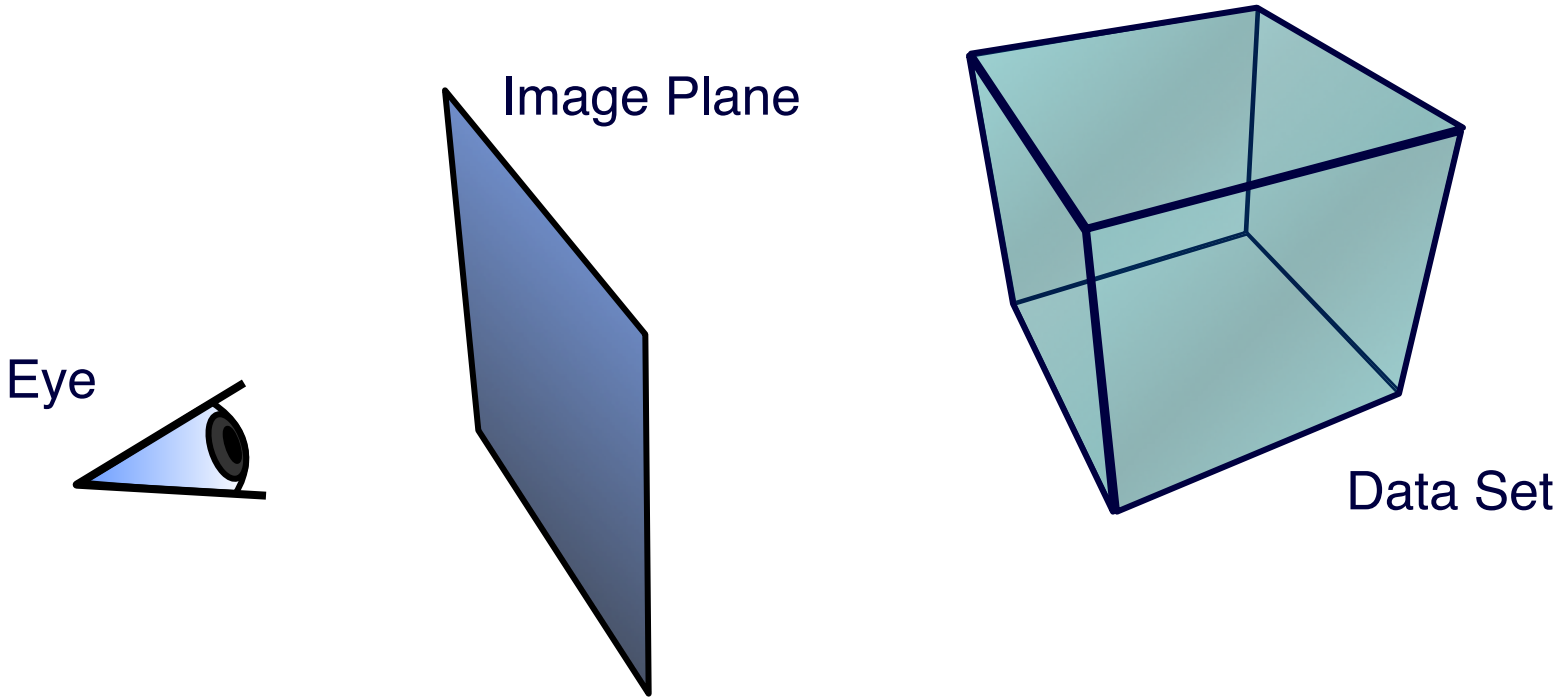
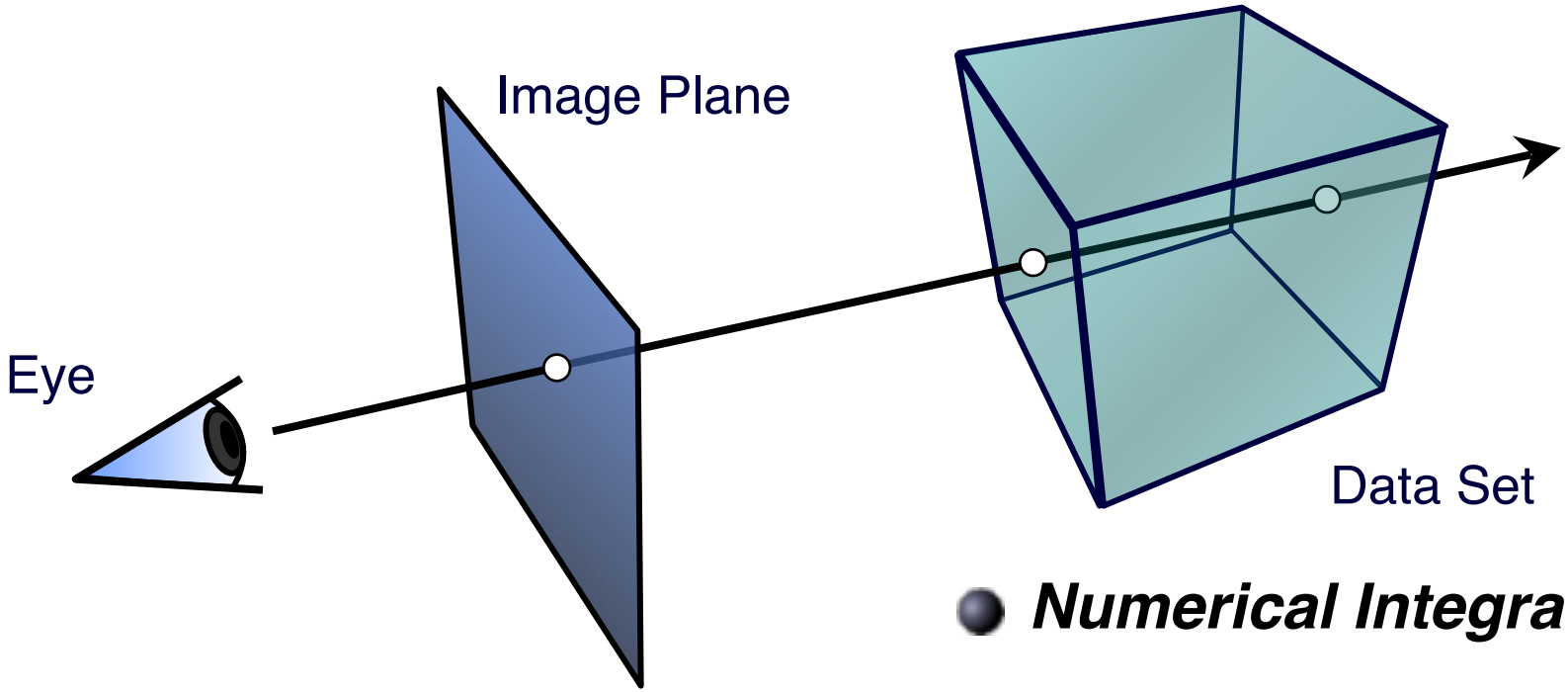
- Software Solution



Ray Casting

$$I(s) = + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

- Software Solution



- ***Numerical Integration***

Ray Casting

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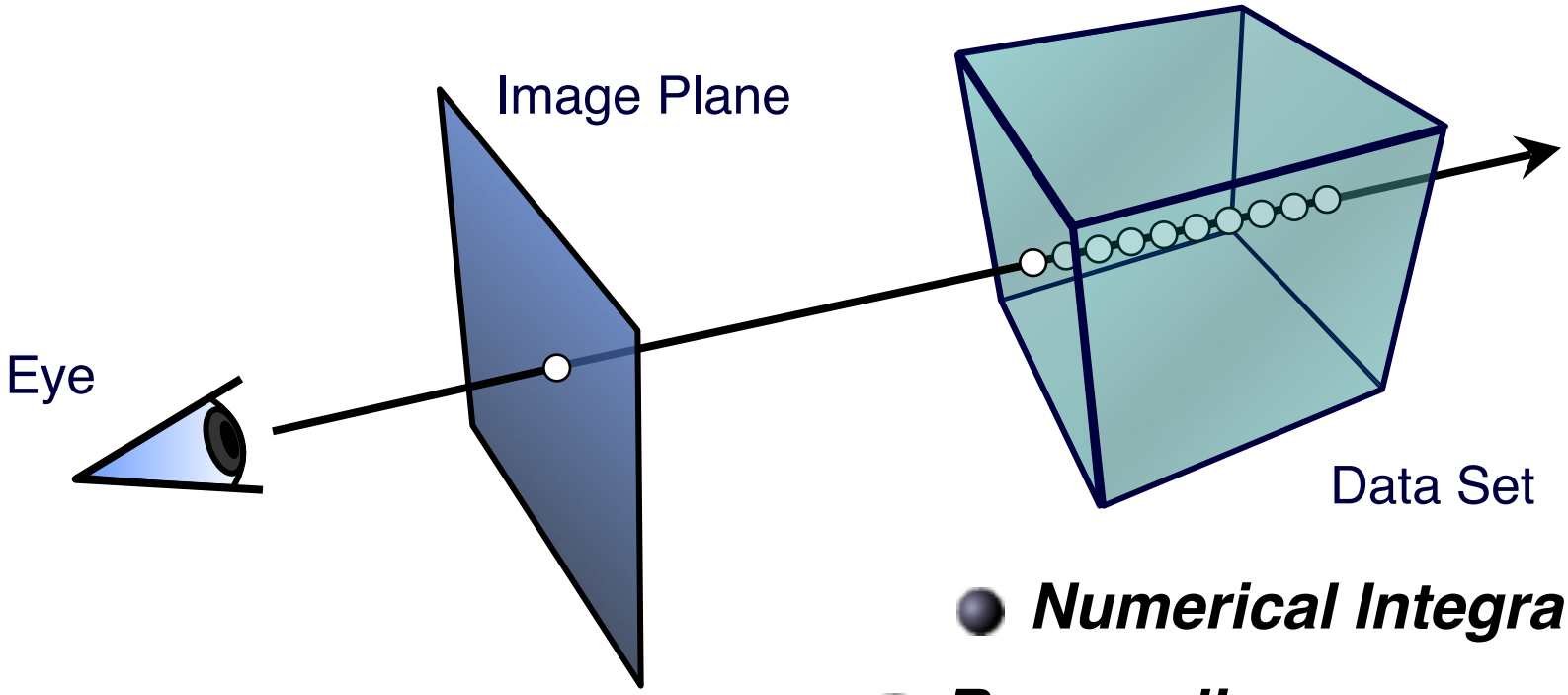
- Software Solution



- ***Numerical Integration***
- ***Resampling***

Numerical Solution

$$I(s) = + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

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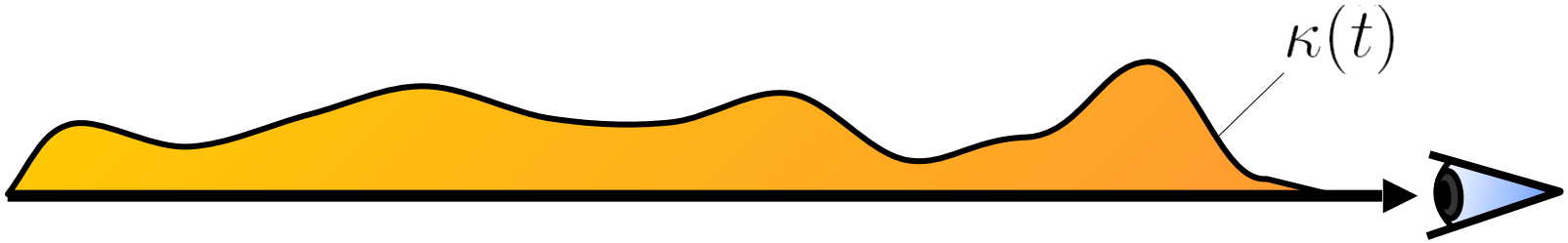

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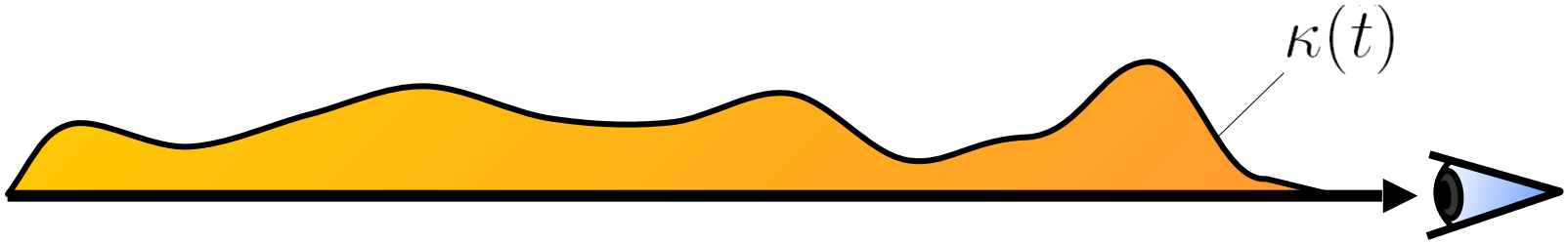

Extinction: $\tau(0,t) = \int_0^t \kappa(\hat{t})\, d\hat{t}$

Numerical Solution

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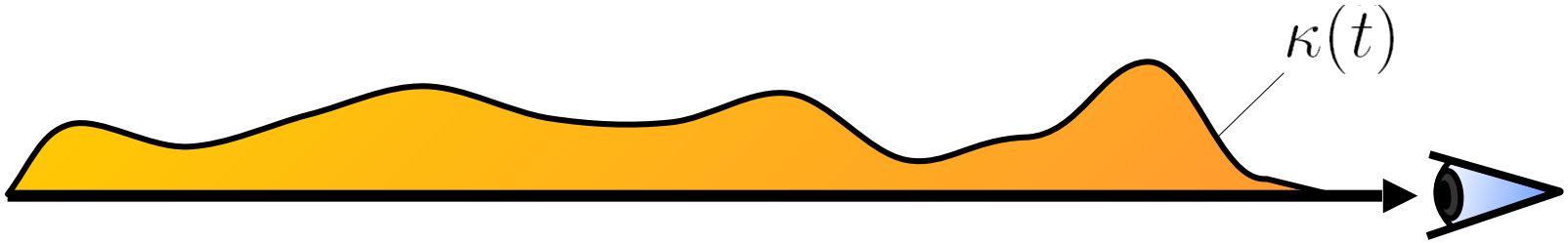

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Approximate Integral by Riemann sum:

$$\tau(0,t) \approx$$

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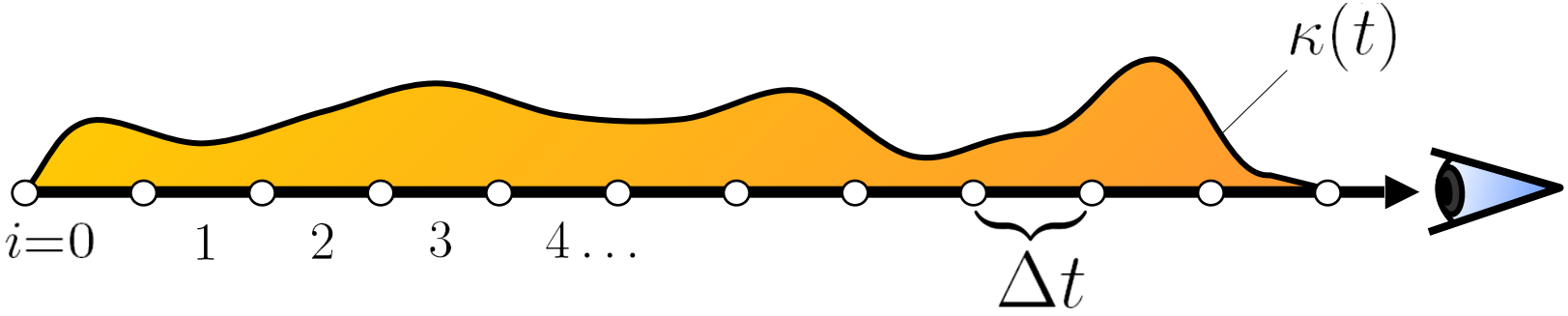

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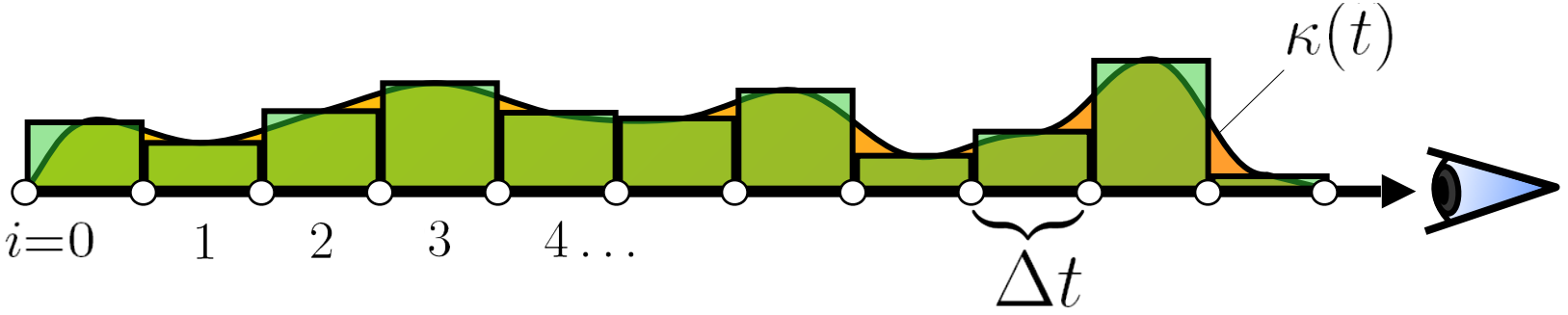

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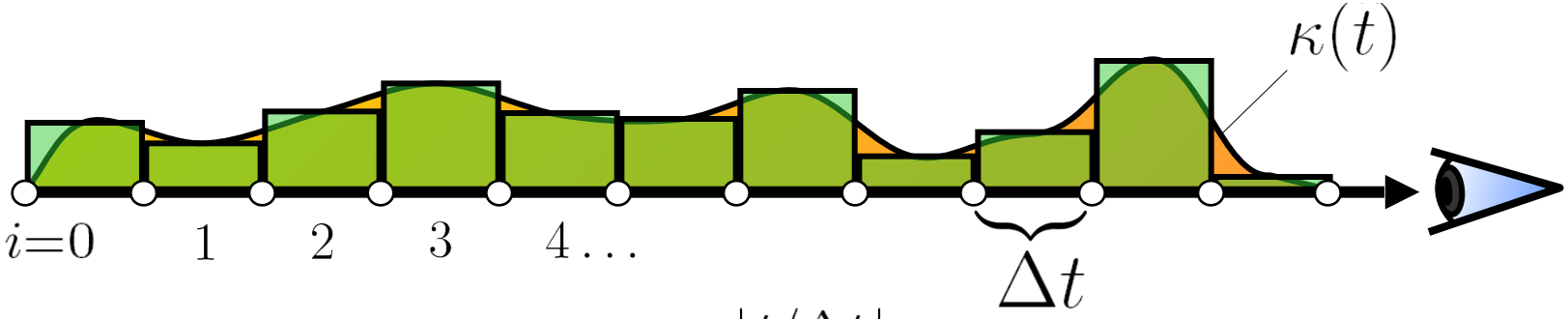

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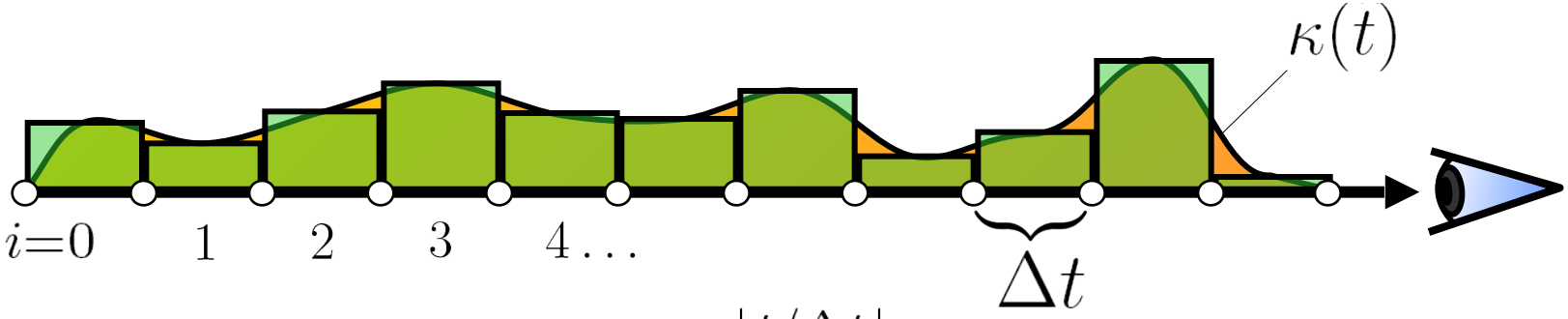

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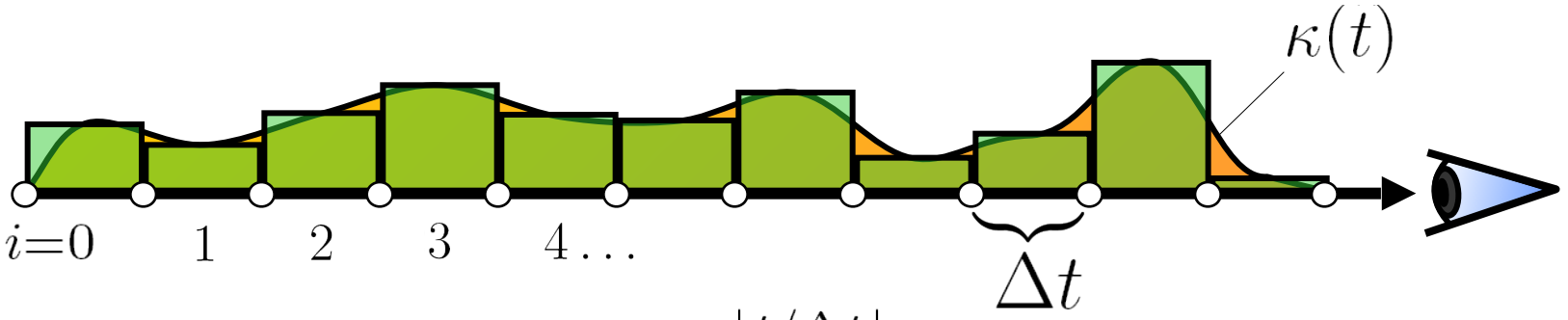

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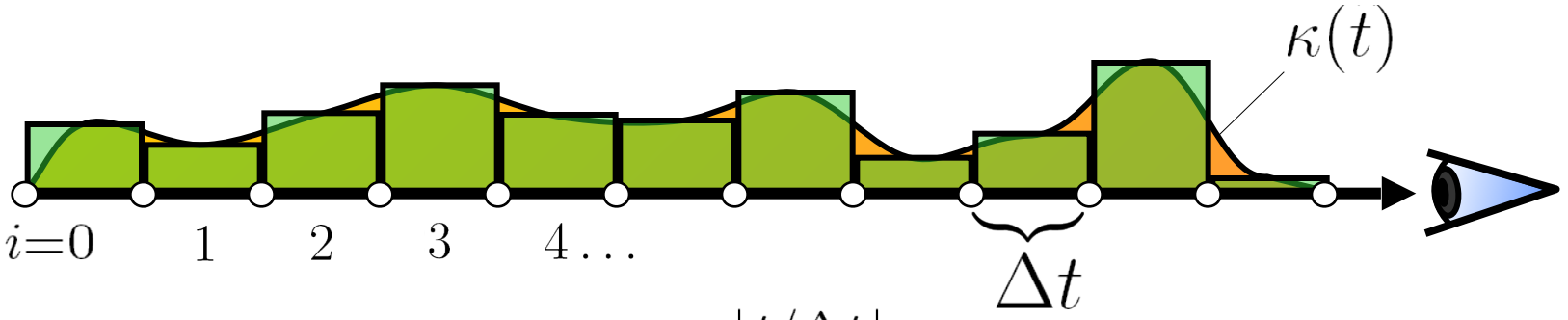

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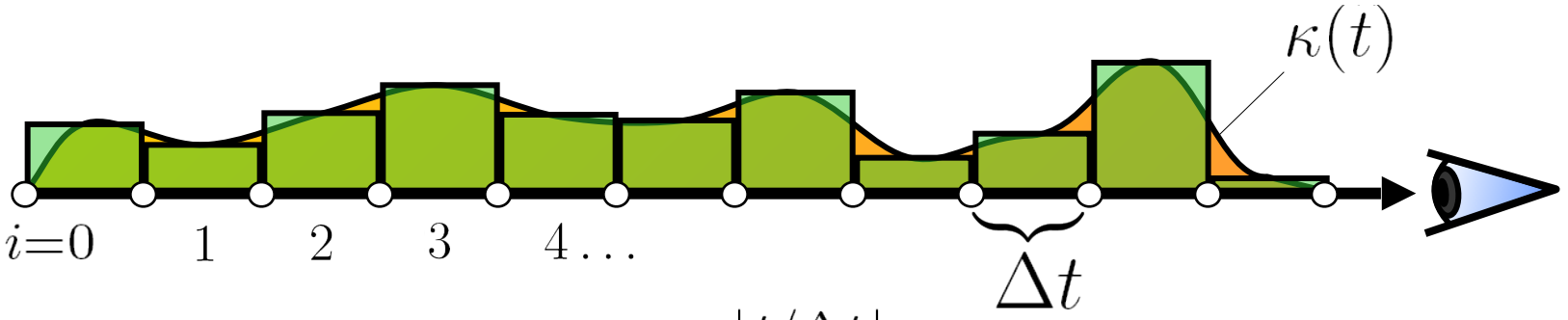

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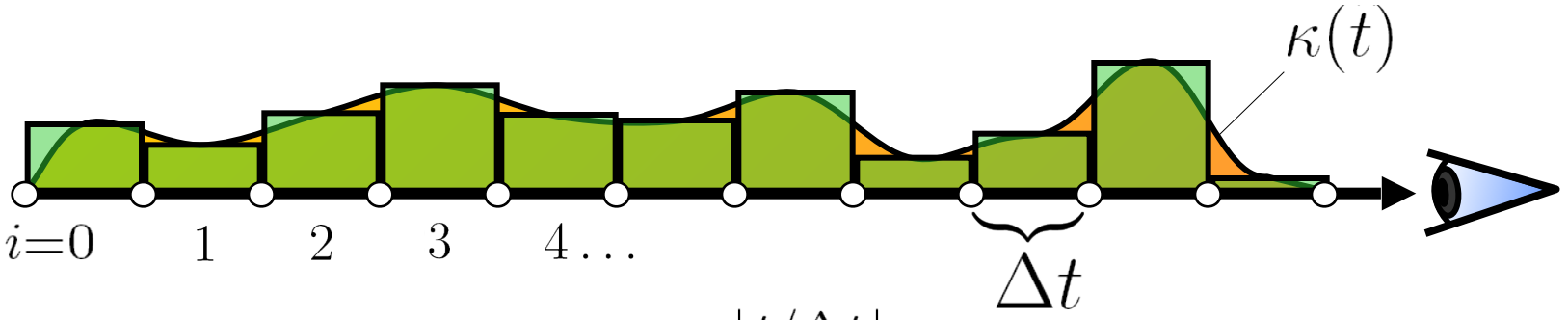

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Now we introduce opacity:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

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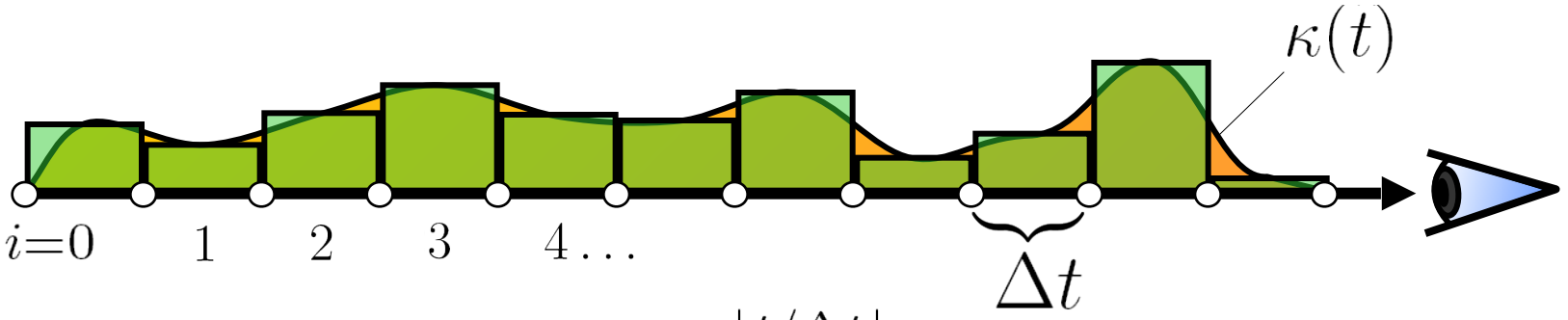

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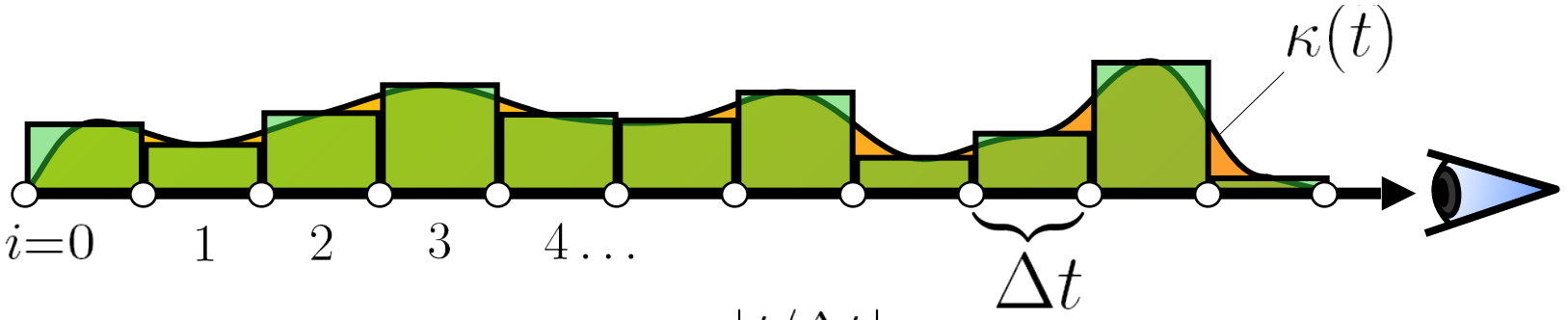

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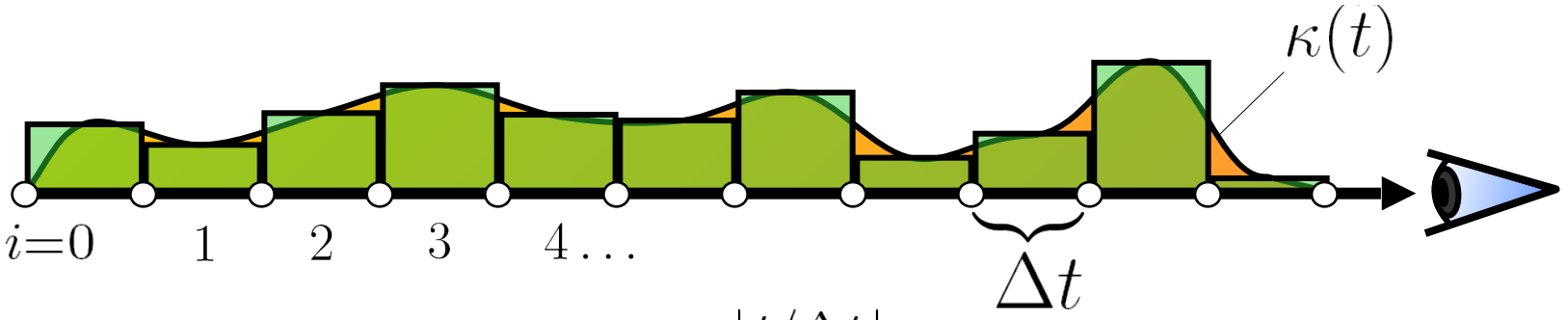

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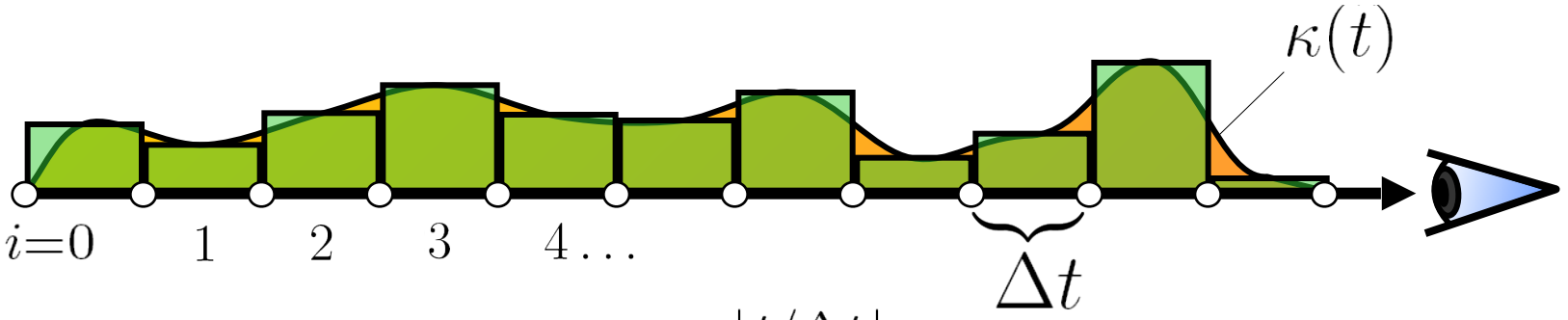

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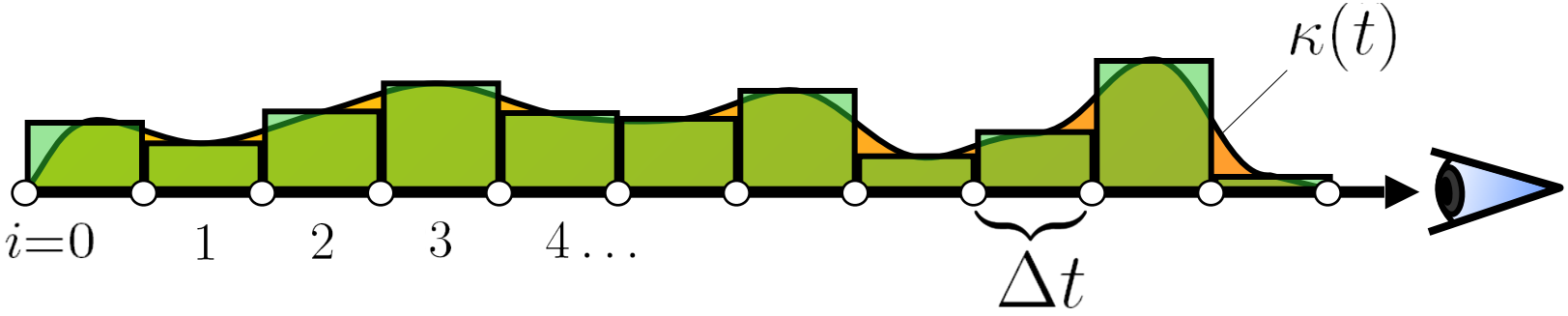

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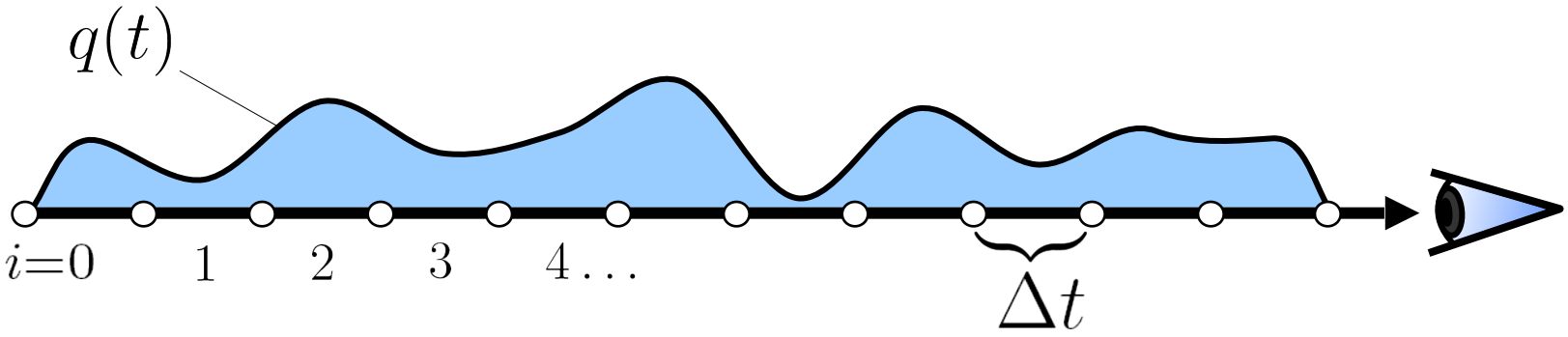

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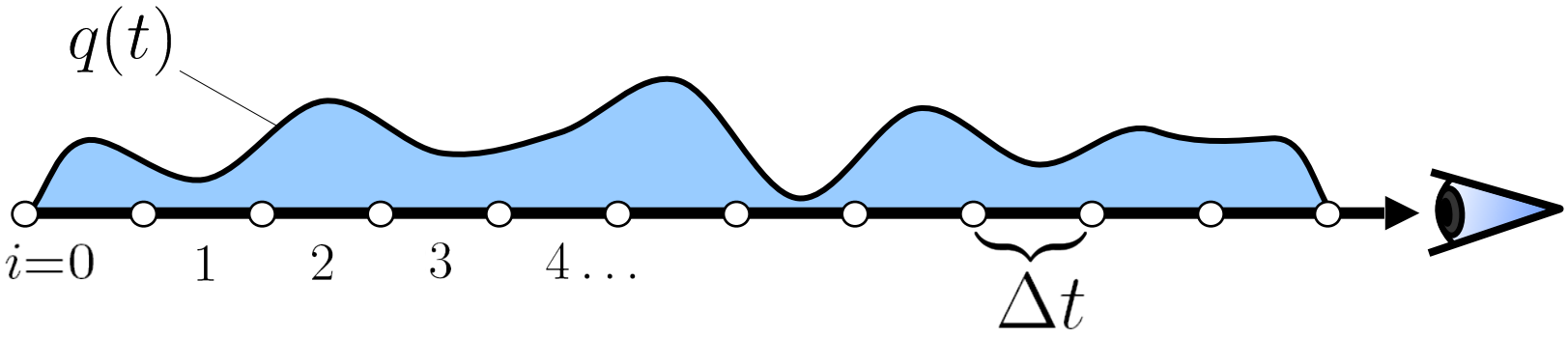

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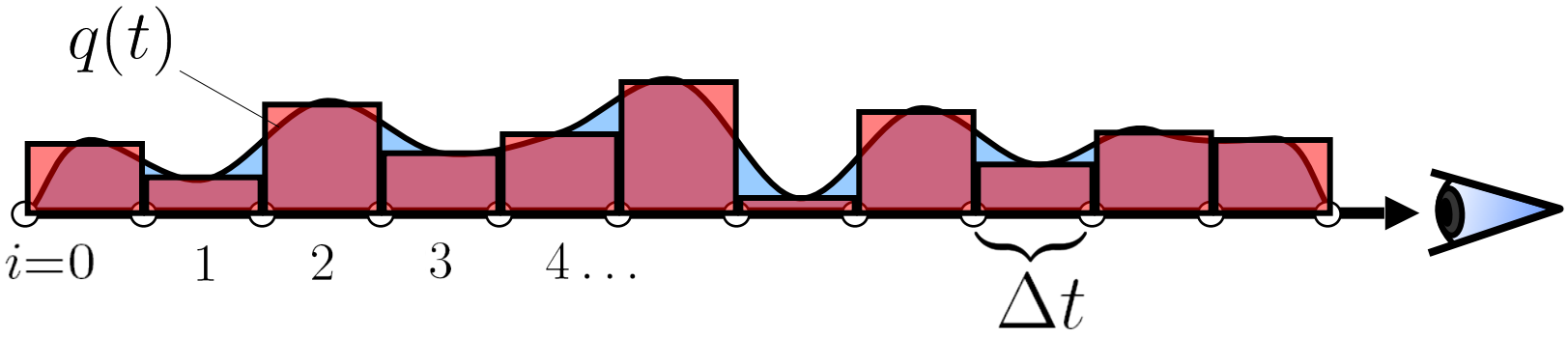

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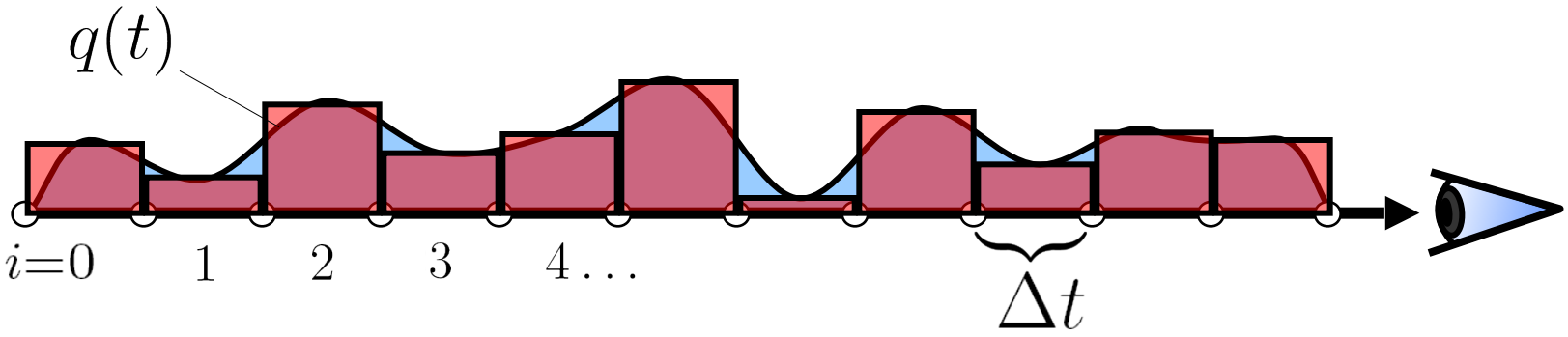

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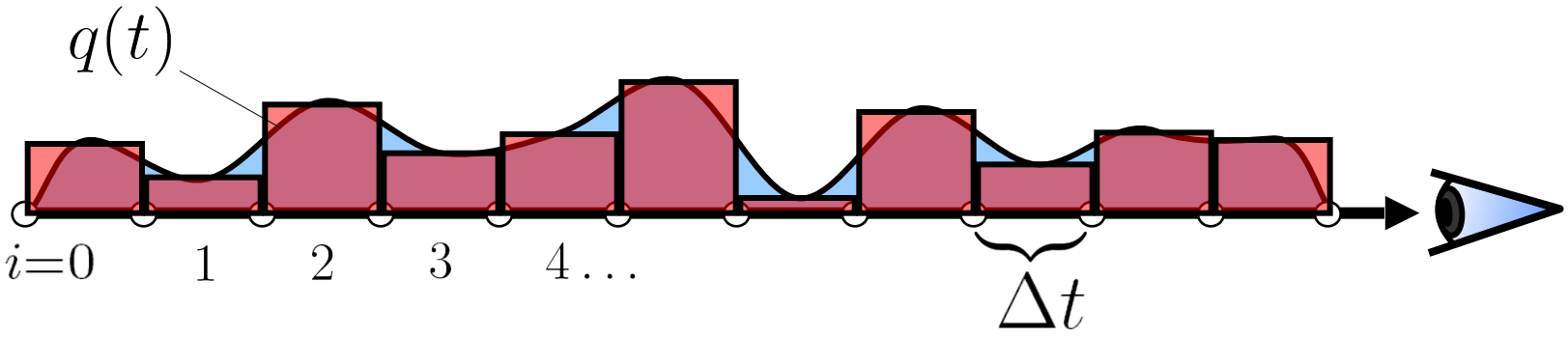

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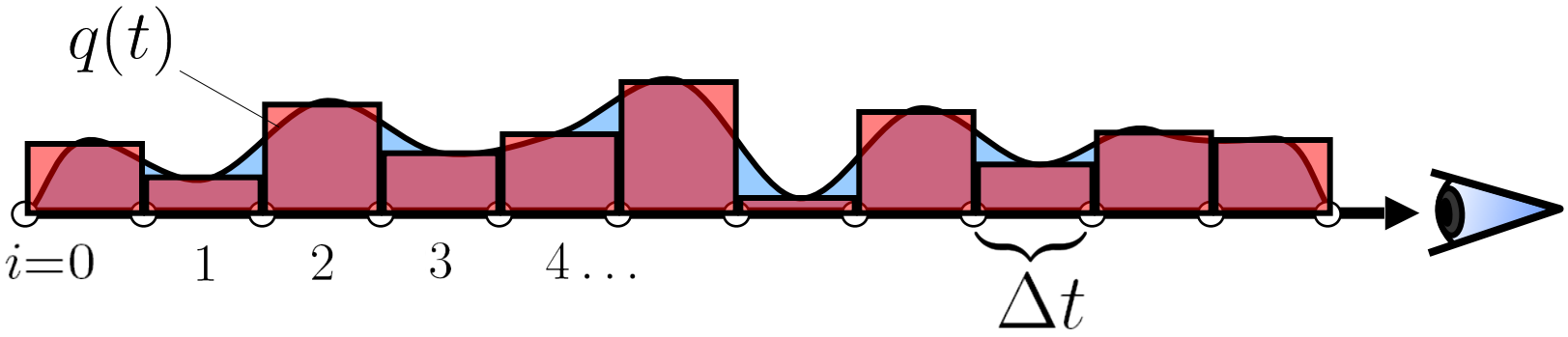

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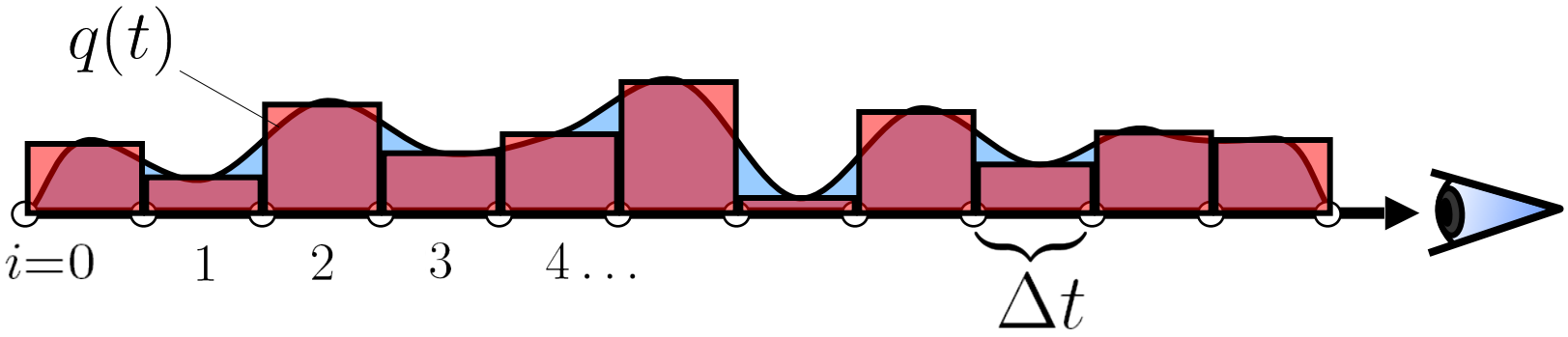

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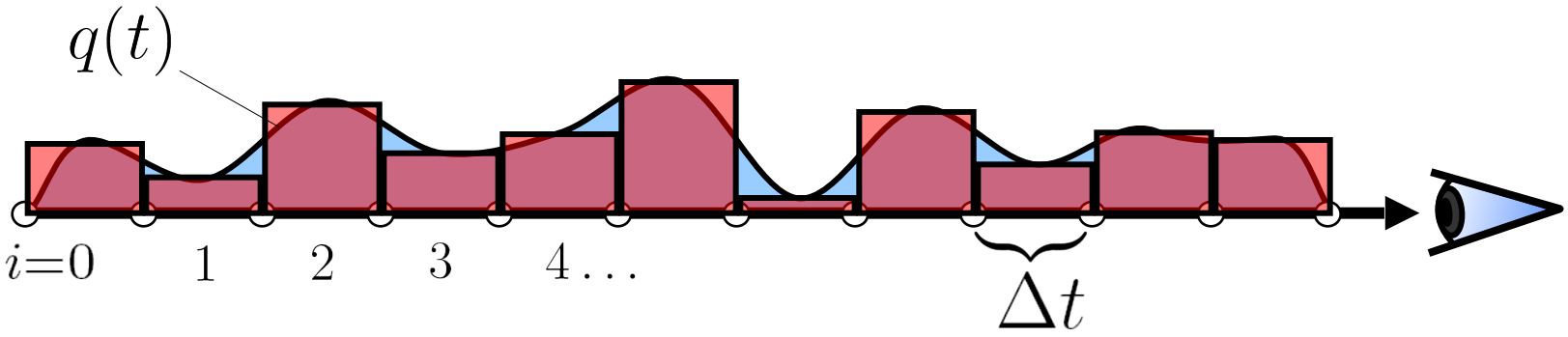

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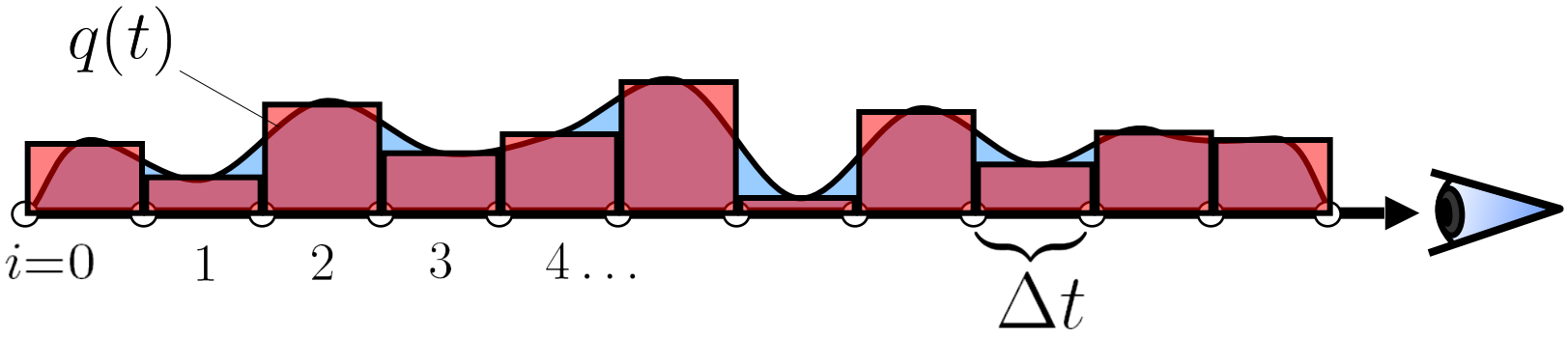

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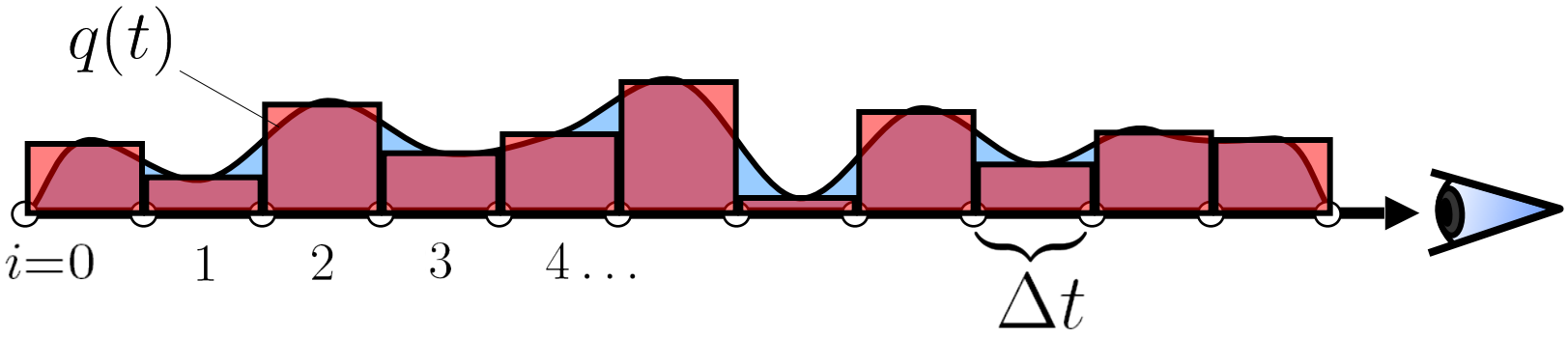

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Radiant energy observed at position ***i***

Numerical Solution

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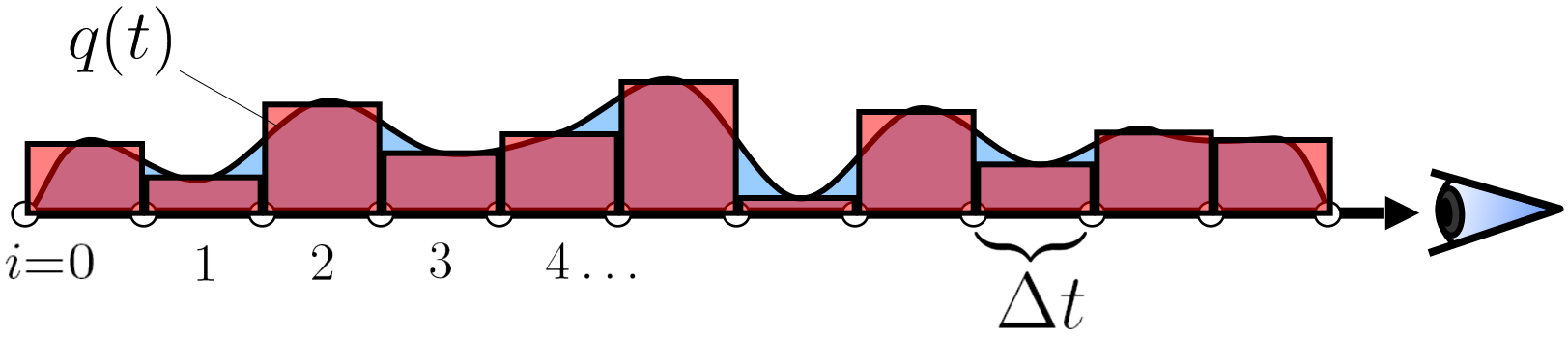

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Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Numerical Solution

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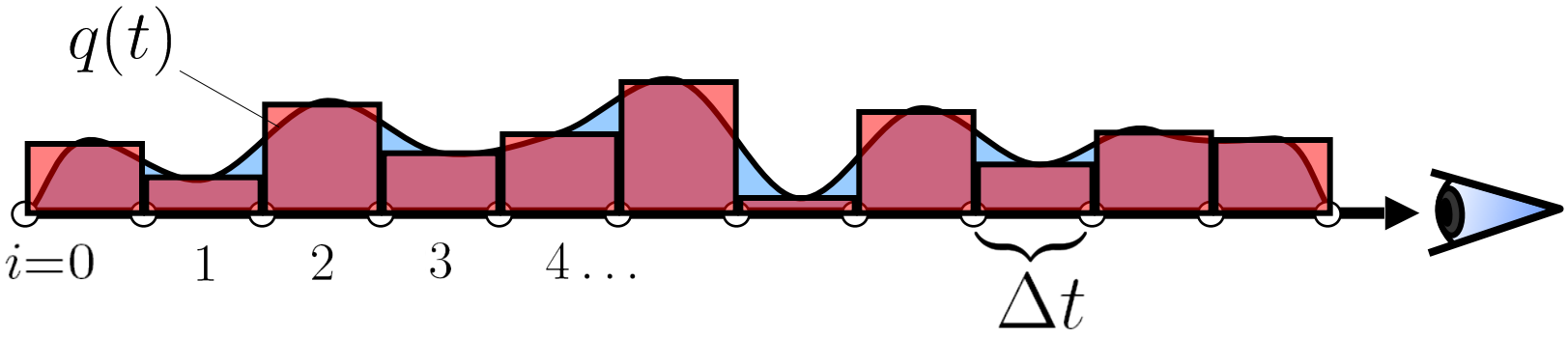

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed recursively

$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Radiant energy observed at position ***i–1***

Numerical Solution

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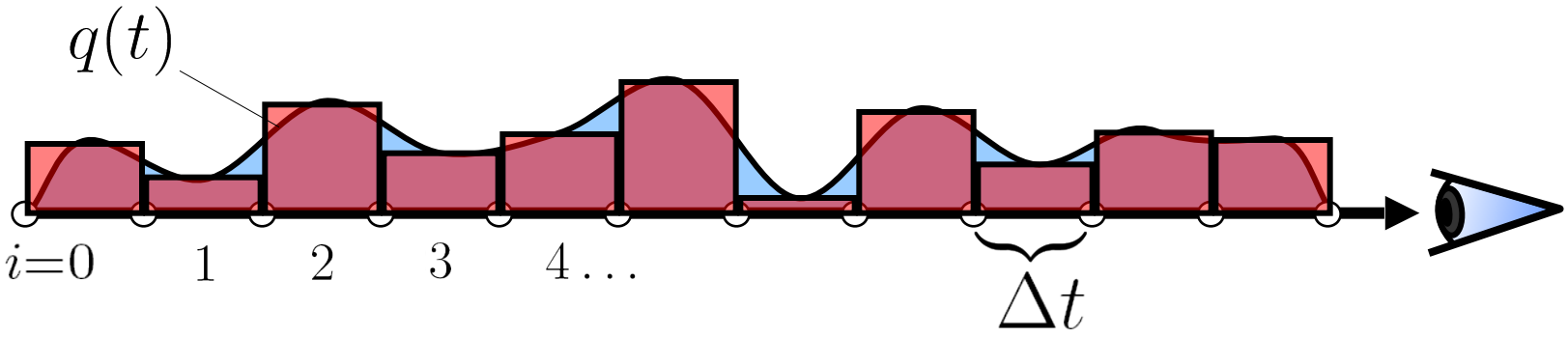

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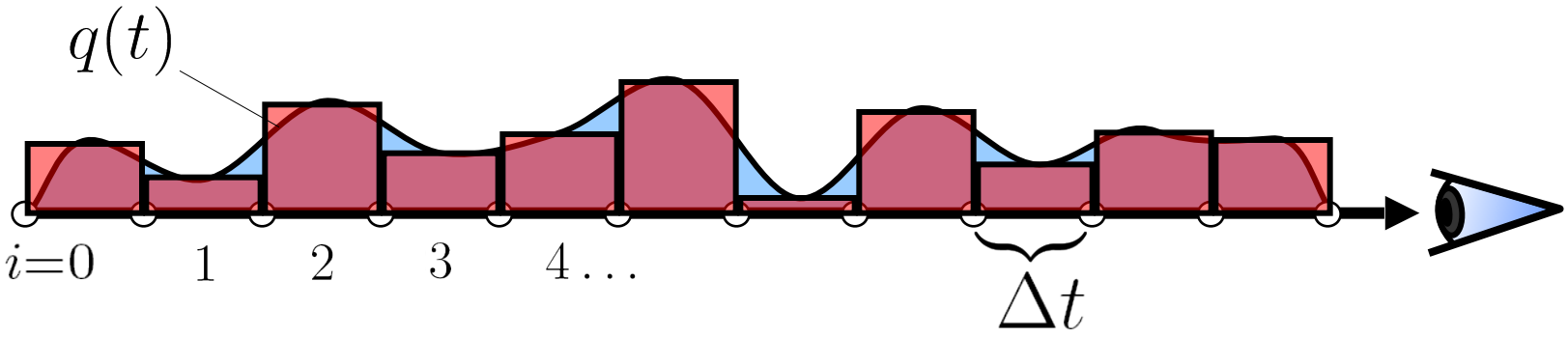
Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Absorption at position ***i***

Radiant energy observed at position ***i–1***

Numerical Solution



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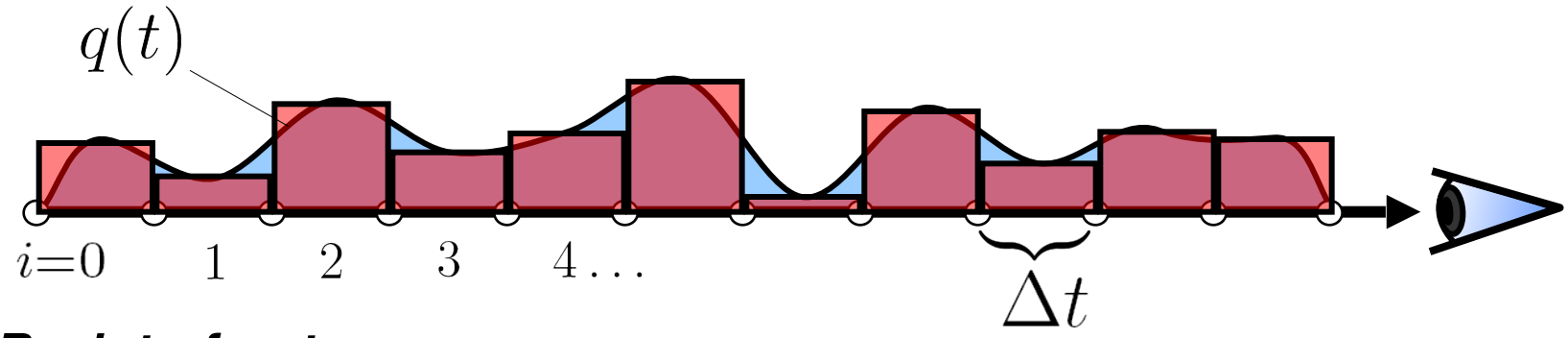
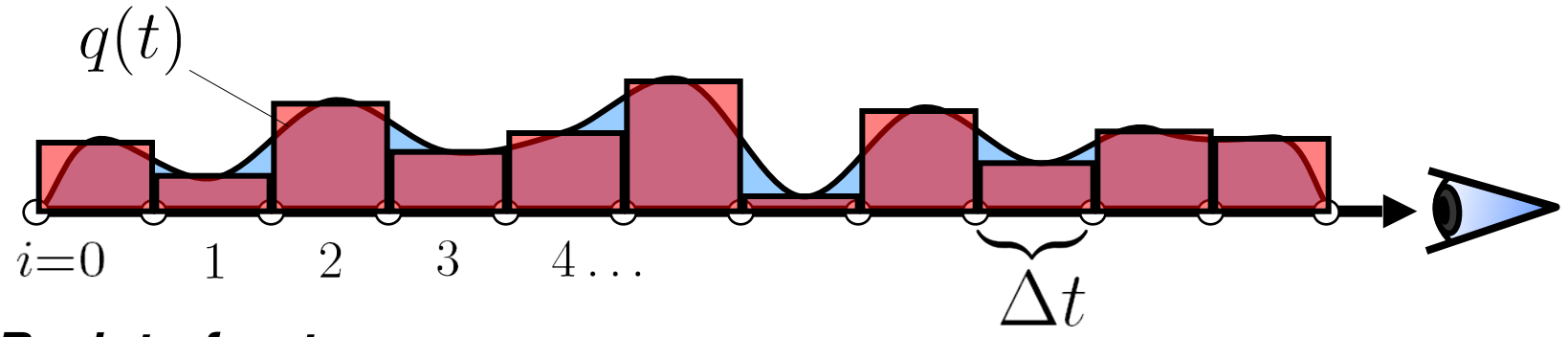


Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Numerical Solution



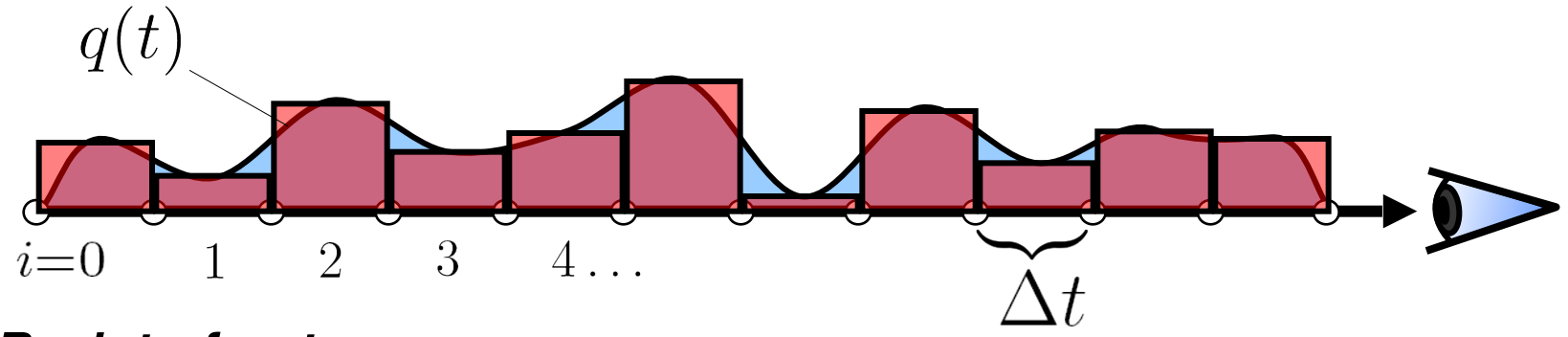
Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Numerical Solution



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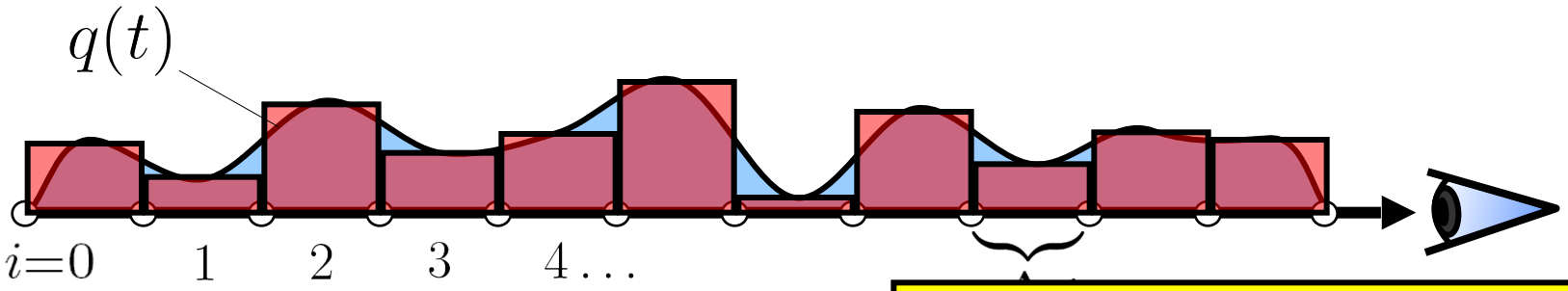
Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1 -$$

Early Ray Termination:

Stop the calculation when

$$A'_i \approx 1$$

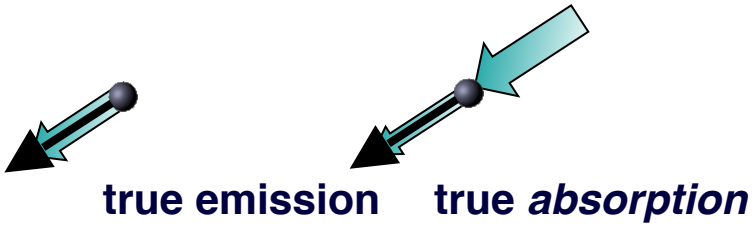
Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$

$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Summary

- Emission Absorption Model



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- Numerical Solutions

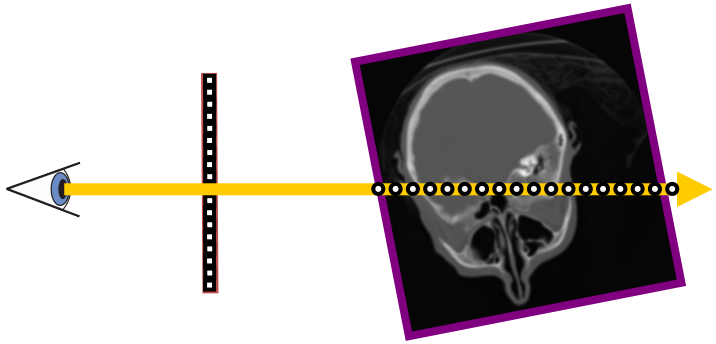
Back-to-front iteration

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Front-to-back iteration

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Direct Volume Rendering Pipeline



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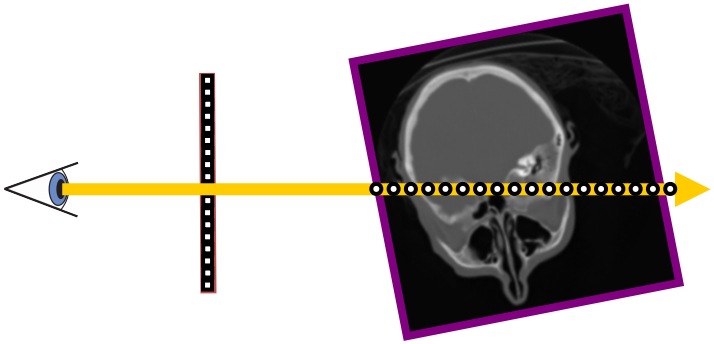

SAMPLING → CLASSIFICATION & ILLUMINATION → COMPOSITING

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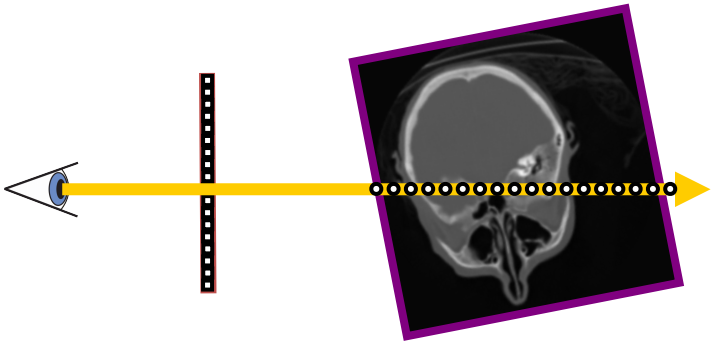

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CLASSIFICATION & ILLUMINATION → SAMPLING

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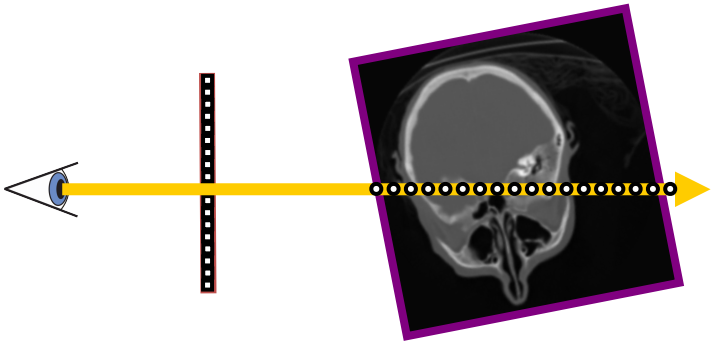

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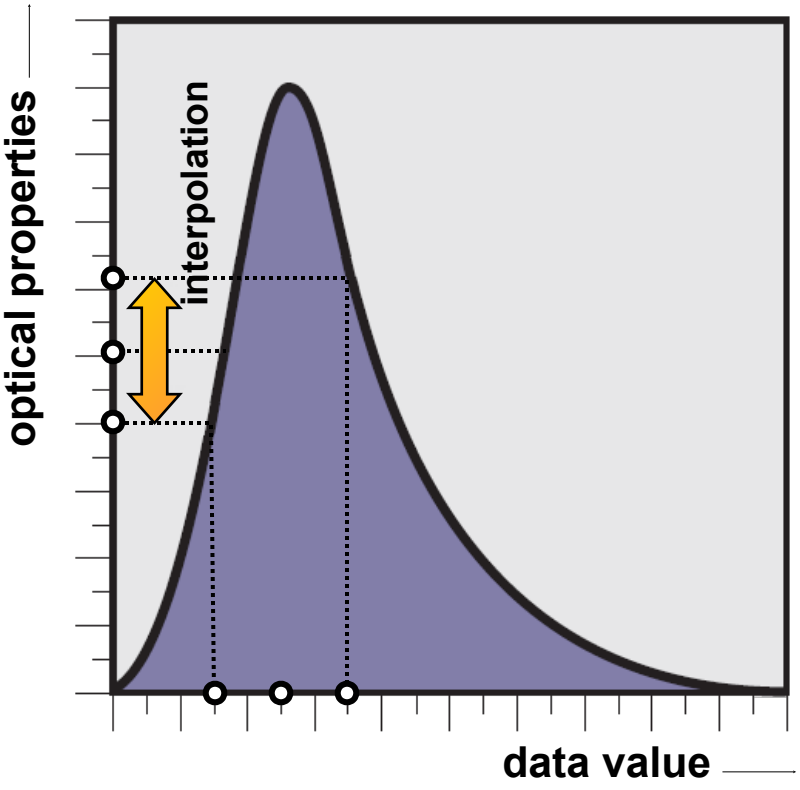
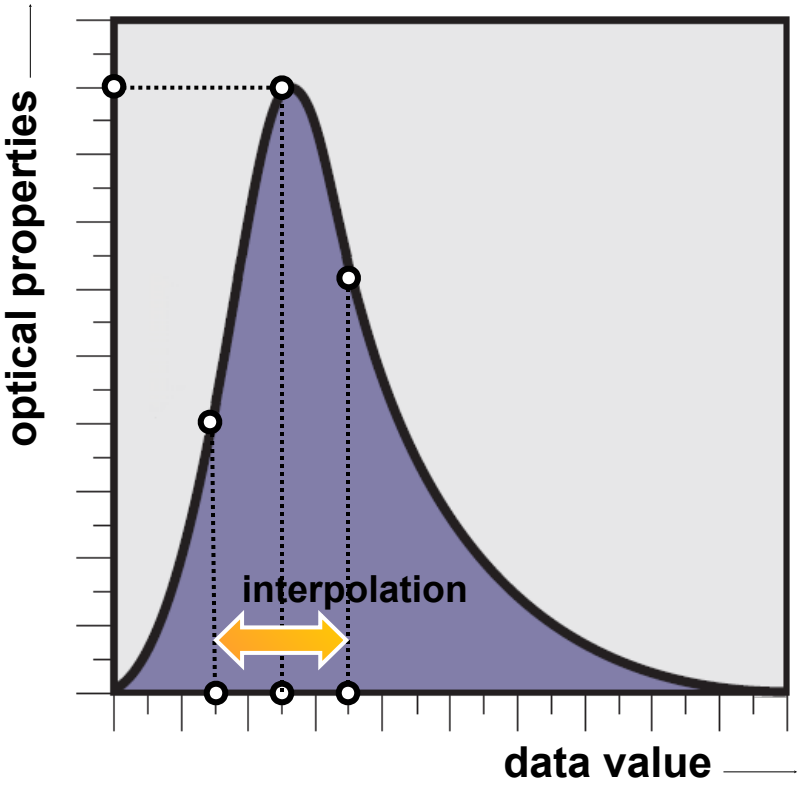
CLASSIFICATION & ILLUMINATION → SAMPLING → COMPOSITING

Pre- vs Post-Interpolative Classification

PRE-INTERPOLATIVE



POST-INTERPOLATIVE

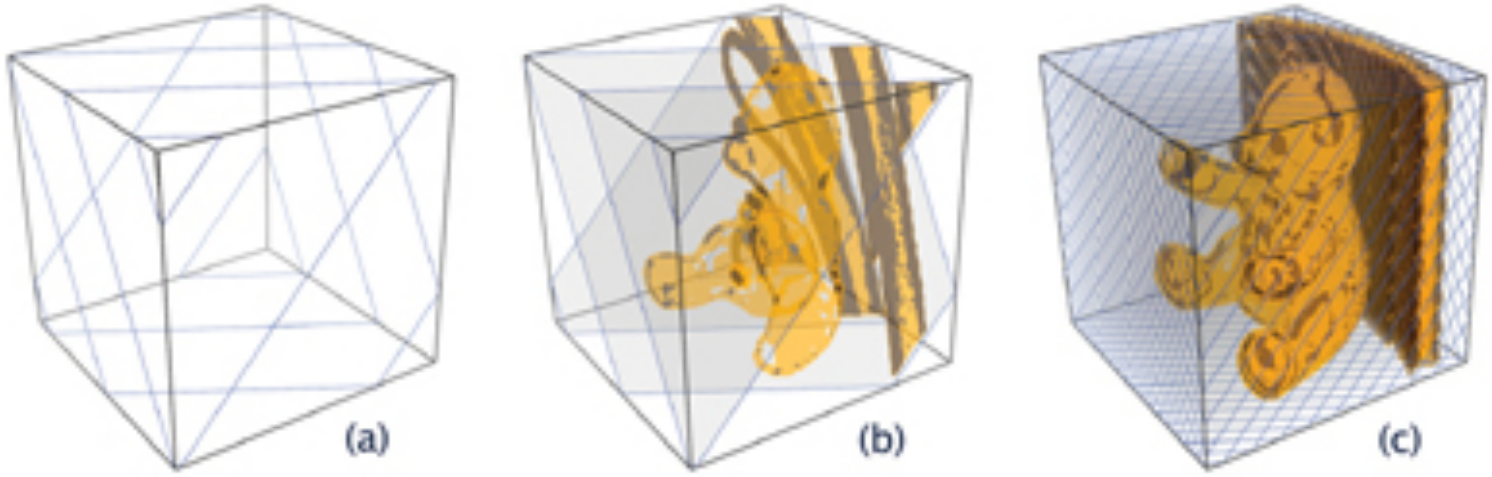


Algorithms for DVR

- OBJECT SPACE
  - Splatting, cell-projection (Westover'90, Chen'04)
    - project voxels to the image plane
    - spherical kernel
    - back-to-front
    - uniform regular grids
  - 2d- slicing (2D textures)
  - 3d- slicing (3D textures)
- IMAGE SPACE
  - Ray Casting
- FOURIER/WAVELET Domain

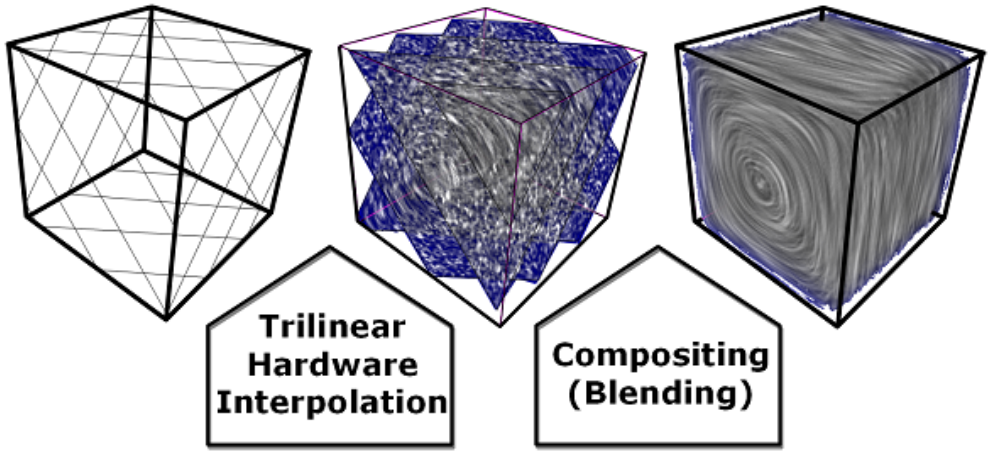
Slice-Based Volume Rendering (SBVR)

- No volumetric primitive in graphics API
- Proxy geometry - polygon primitives as slices through volume
- Texture polygons with volumetric data
- Draw slices in sorted order – back-to-front
- Use fragment shader to perform compositing (blending)



Volumetric Data

- Voxel data sent to GPU memory as
  - Stack of 2D textures
  - 3D texture
- Leverage graphics pipeline



Instructions for setting up 3D texture in OpenGL

- http://gpwiki.org/index.php/OpenGL_3D_Textures

Proxy Geometry

- Slices through 3D voxel data
- 3D voxel data = 3D texture on GPU
- Assign texture coordinate to every slice vertex
  - CPU or vertex shader

Texture-based Approaches

- No volumetric hardware-primitives!

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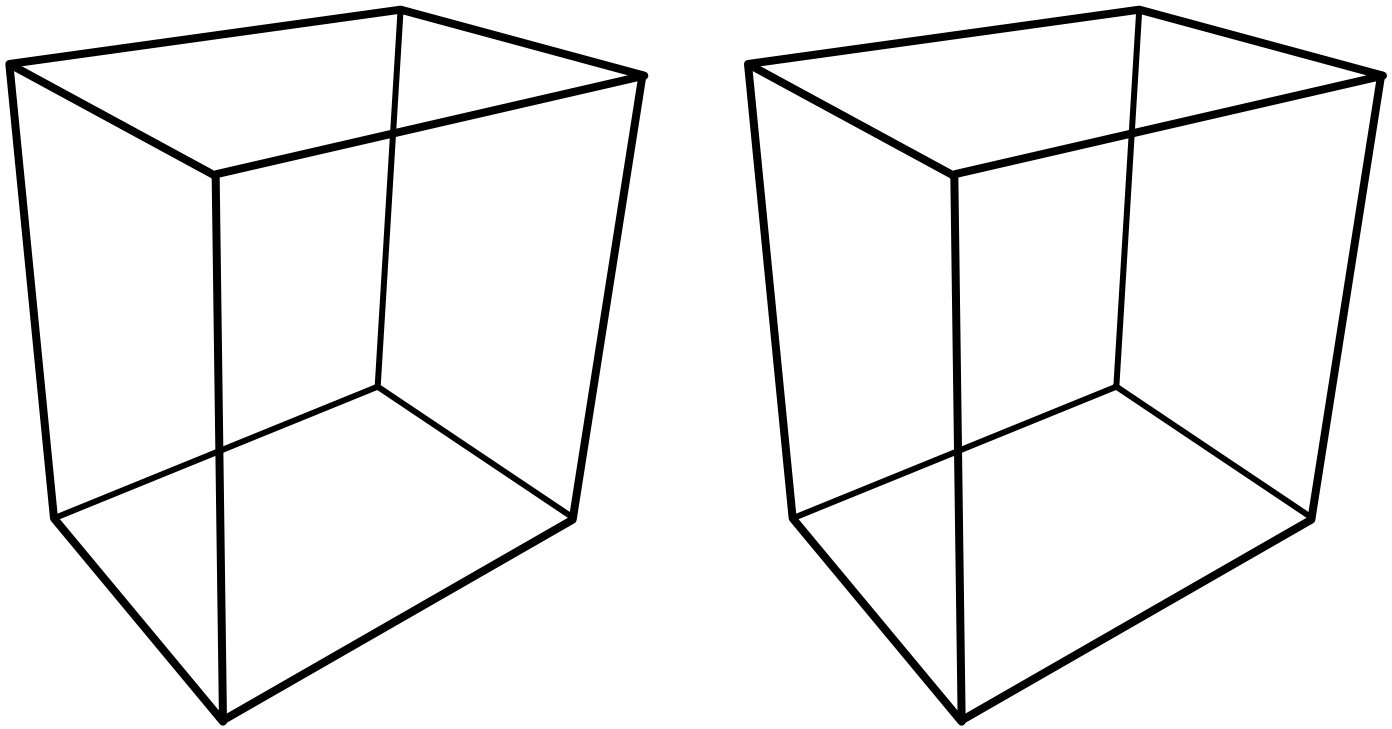
- ⇨ Proxy geometry (Polygonal Slices)



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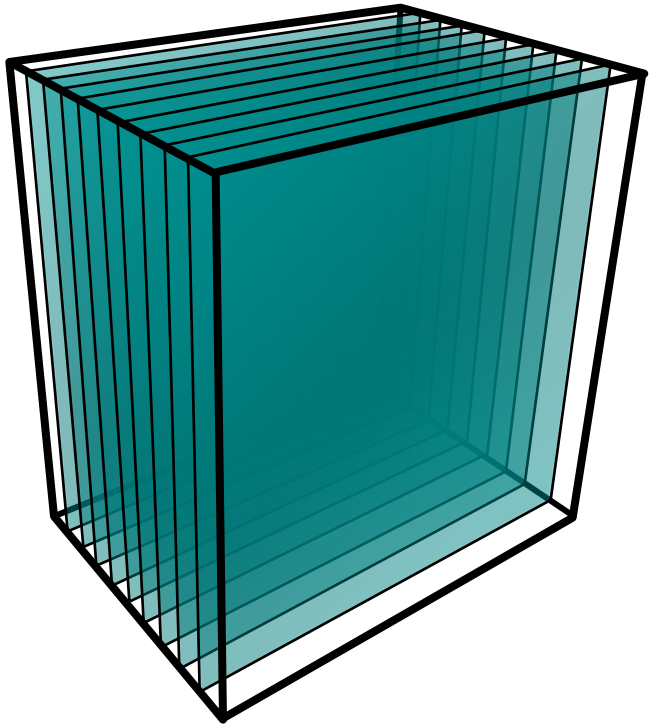

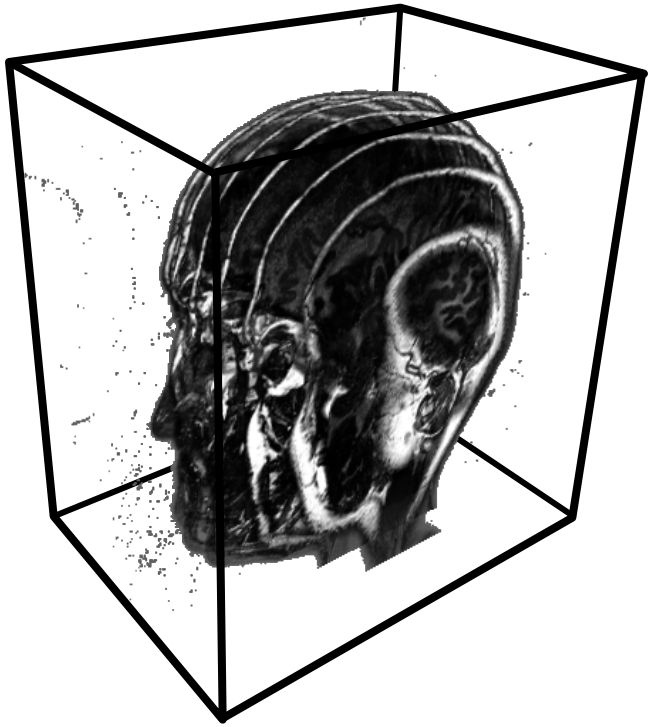

2D Textures

- Draw the volume as a stack of 2D textures
  Bilinear Interpolation in Hardware

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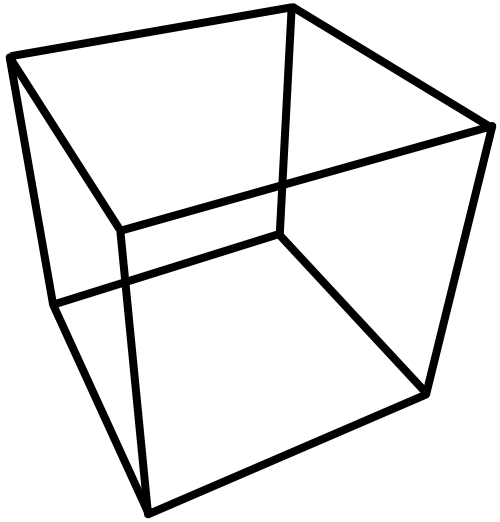
⇨ Decompostition into axis/object-aligned slices



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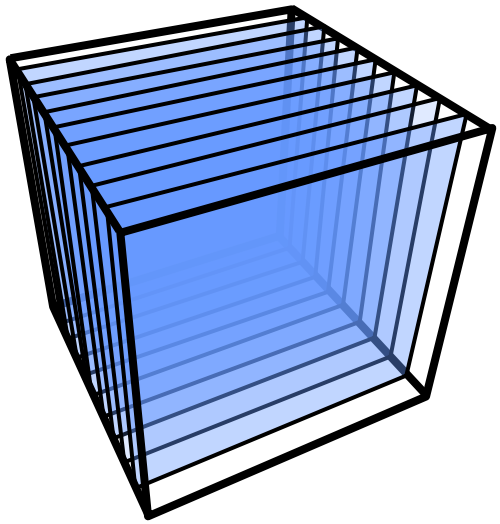

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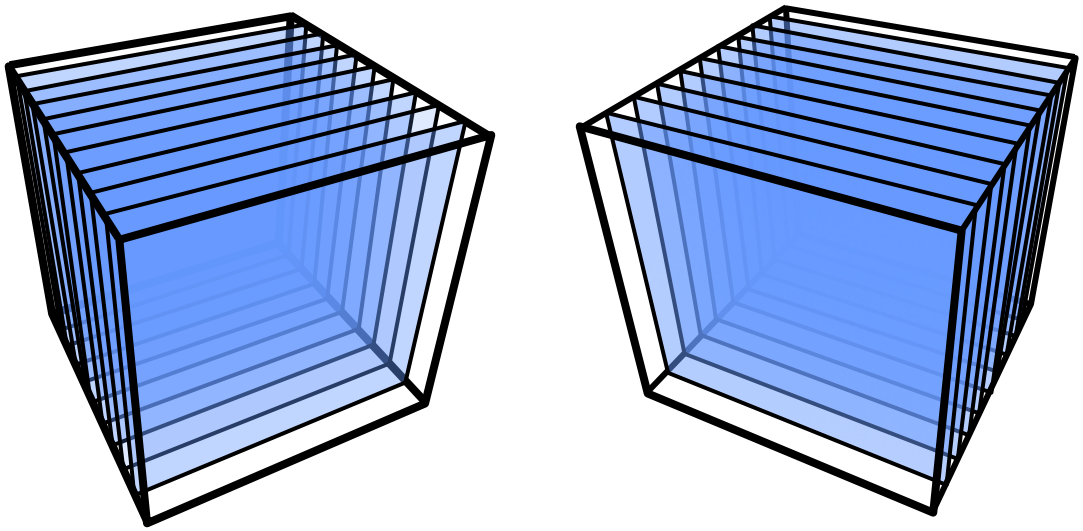

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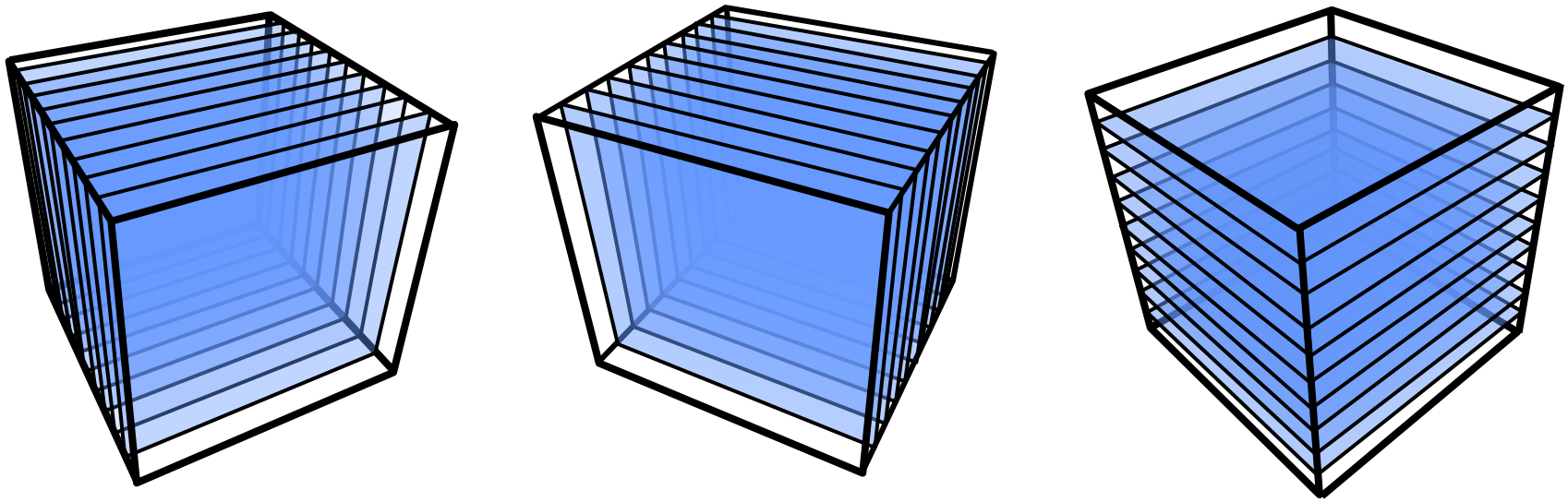

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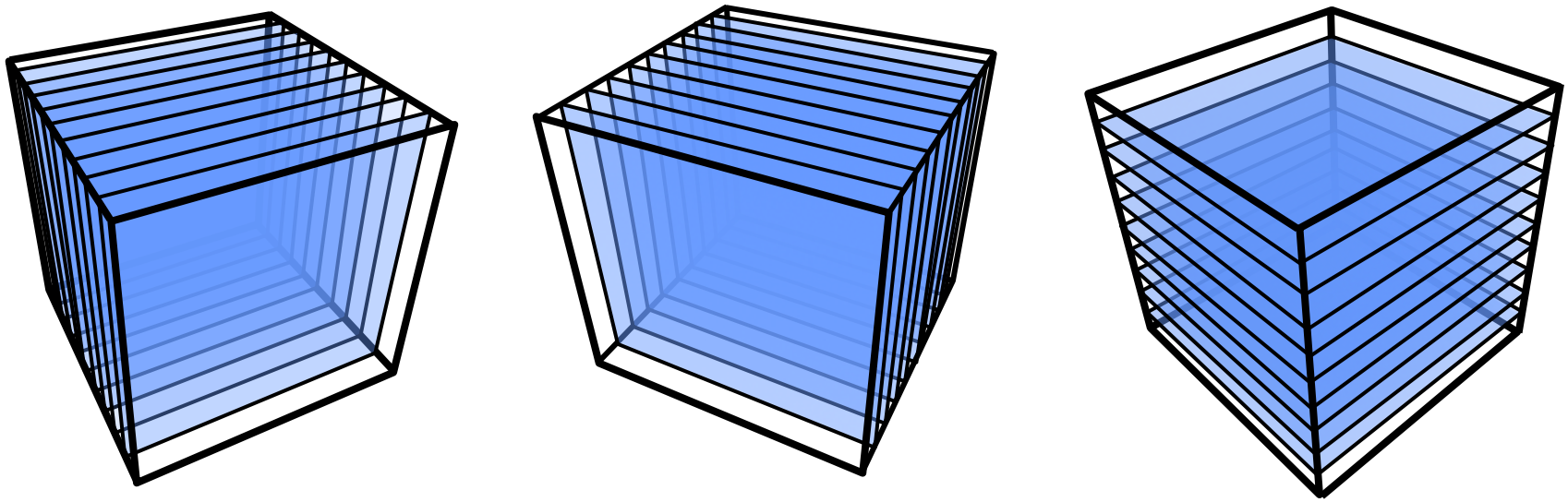
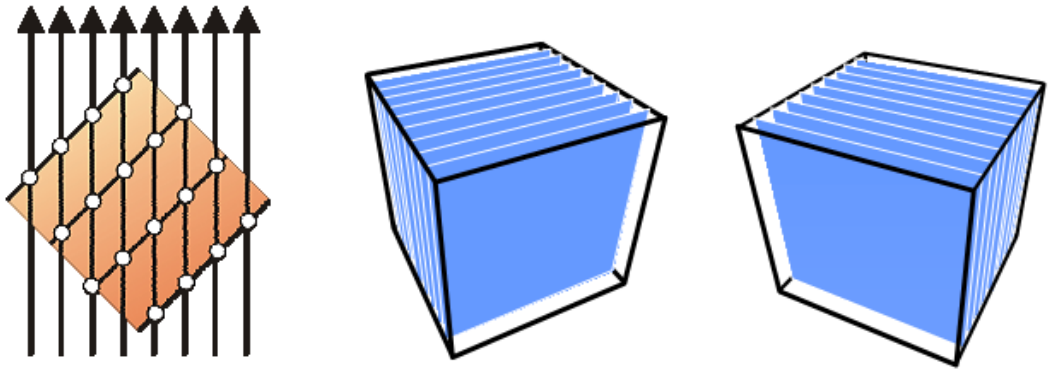


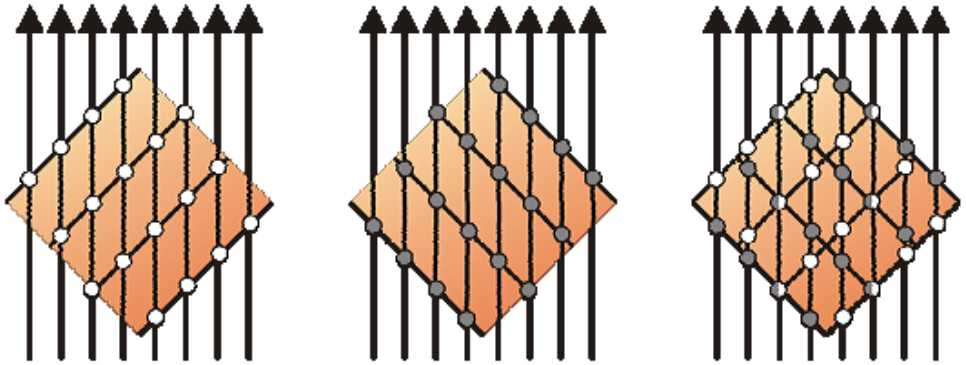
- 3 copies of the data set in memory

Proxy Geometry

- Object-Aligned Slices
  - Fast and simple
  - Three stacks of 2D textures – x, y, z principle directions

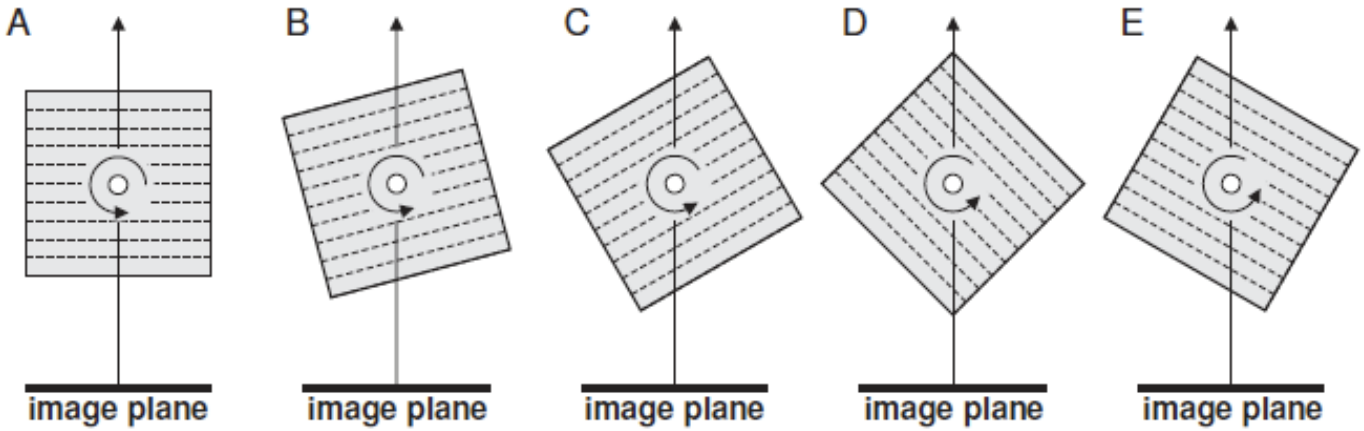


  - Texture stack swapped based on closest to viewpoint



Implementation

- Prepare the texture images for the 3 stacks of slices
- The slicing direction = minimizes the angle between the slice normal and the viewing direction
- Send to draw the slices/polygons with their correct bounded textures.
- Back-to-front composition:



2D Textures: Drawbacks. Flickering

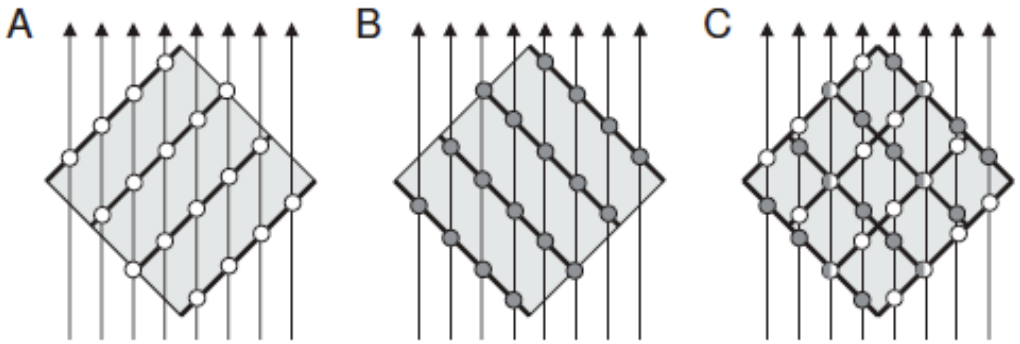


Figure 3.5: The location of sampling points changes abruptly (C), when switching from one slice stack (A), to the next (B).

Proxy Geometry

- Issues with Object-Aligned Slices
  - 3x memory consumption
    - Data replicated along 3 principle directions
  - Change in viewpoint results in stack swap
    - Image popping artifacts
    - Lag while downloading new textures
  - Sampling distance changes with viewpoint
    - Intensity variations as camera moves

2D Textures: Drawbacks. Sampling rate is inconsistent

- Sampling rate is inconsistent

2D Textures: Drawbacks. Sampling rate is inconsistent

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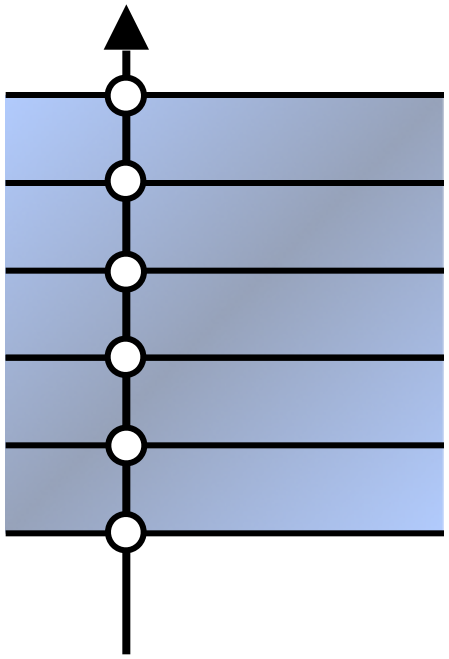

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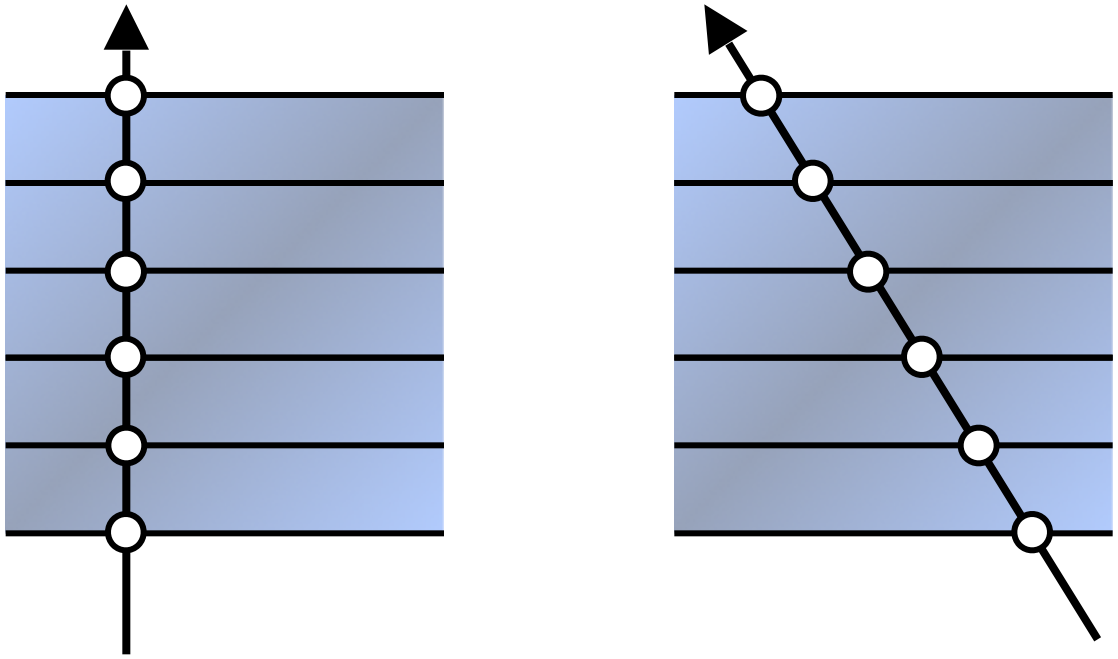

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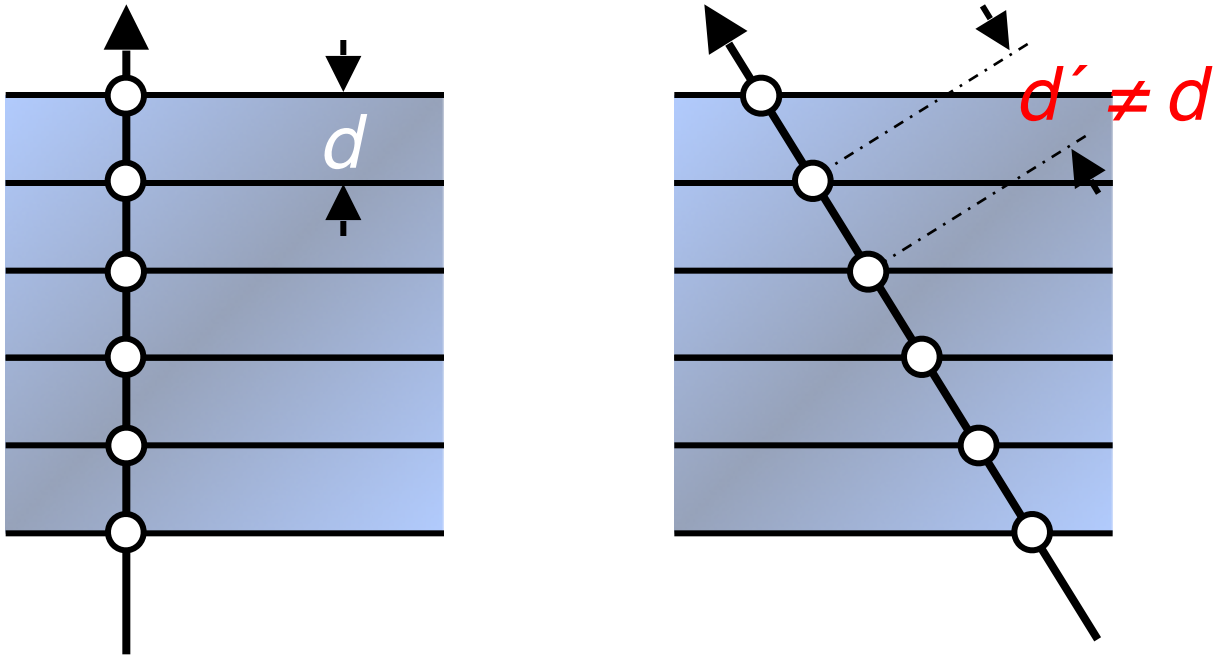
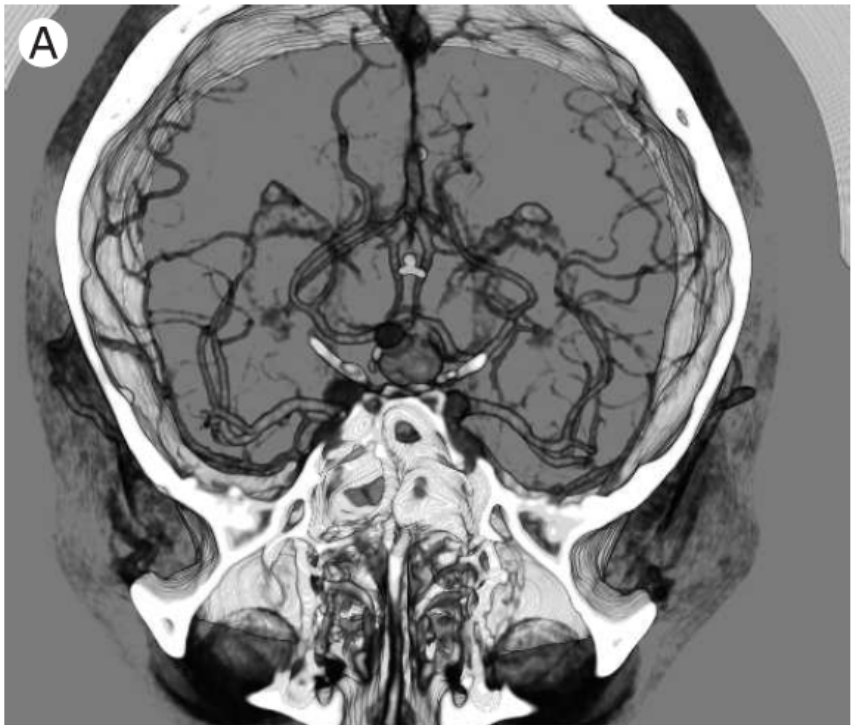
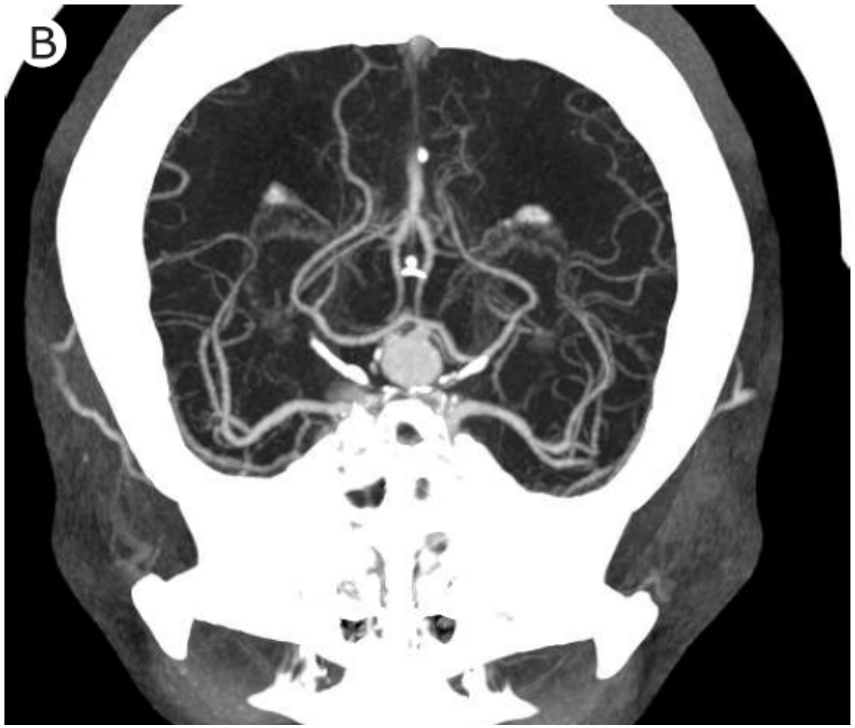


- Emission/absorption slightly incorrect
- ***Super-sampling on-the-fly impossible***

Compositing



Emission/Absorption



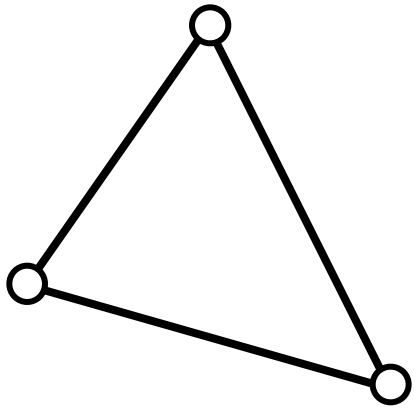
Maximum Intensity Projection

2D Textures: Drawbacks. Sampling rate is inconsistent

- Numerical Integration assumes constant length between samples.
- Distances between samples = distances between slices with respect to the view direction.
- Distance between slices is fixed, and color and opacity are computed for each texture assuming these fixed distances.

3D Textures

3D Textures



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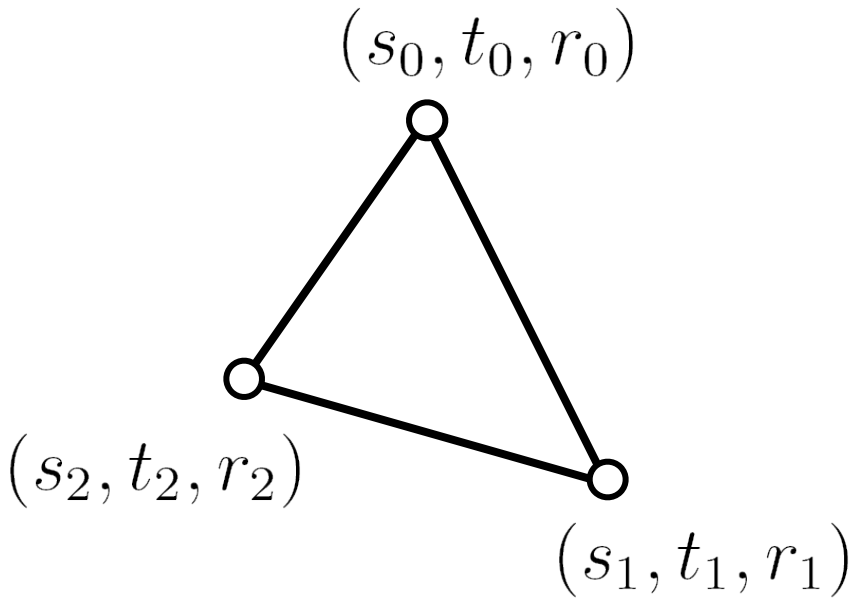

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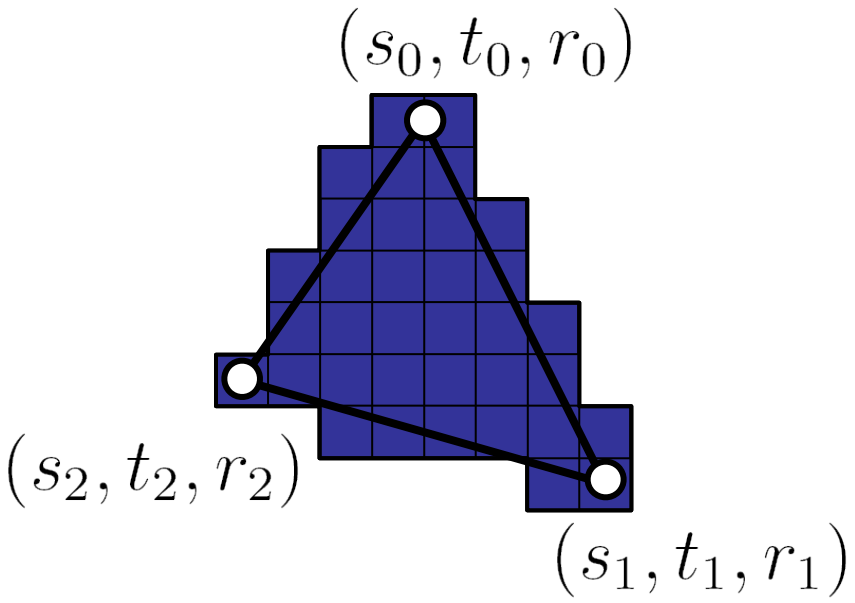

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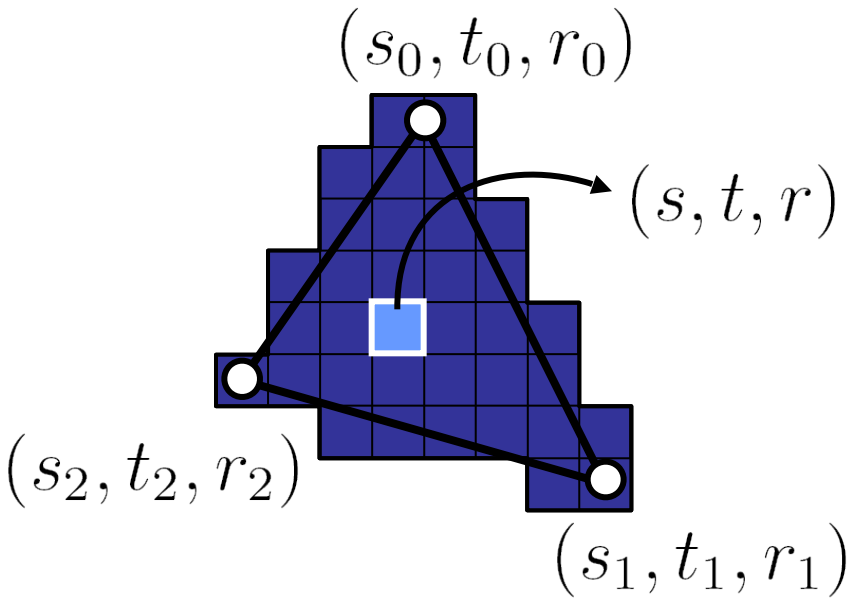
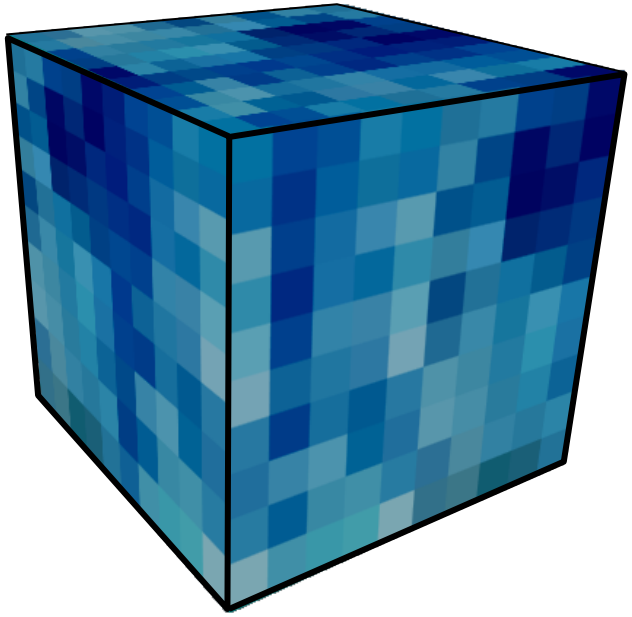
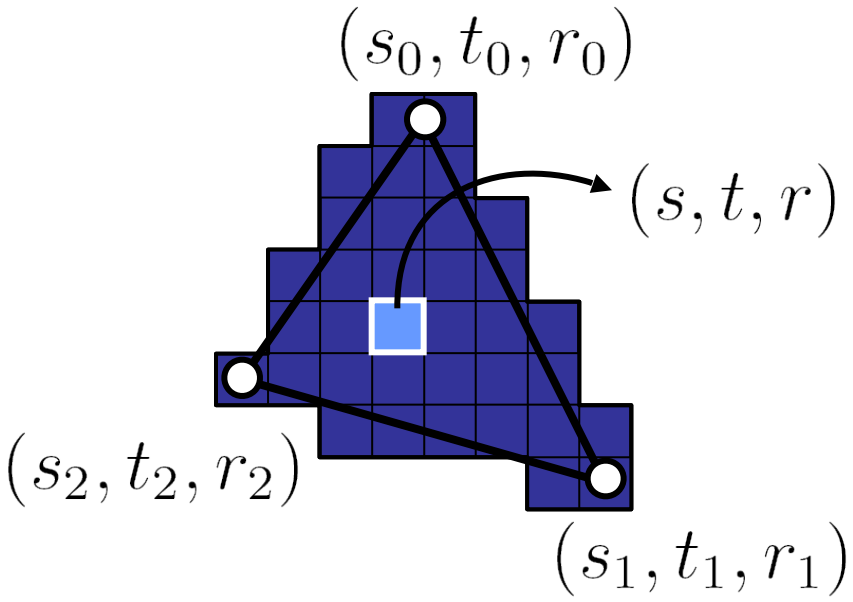
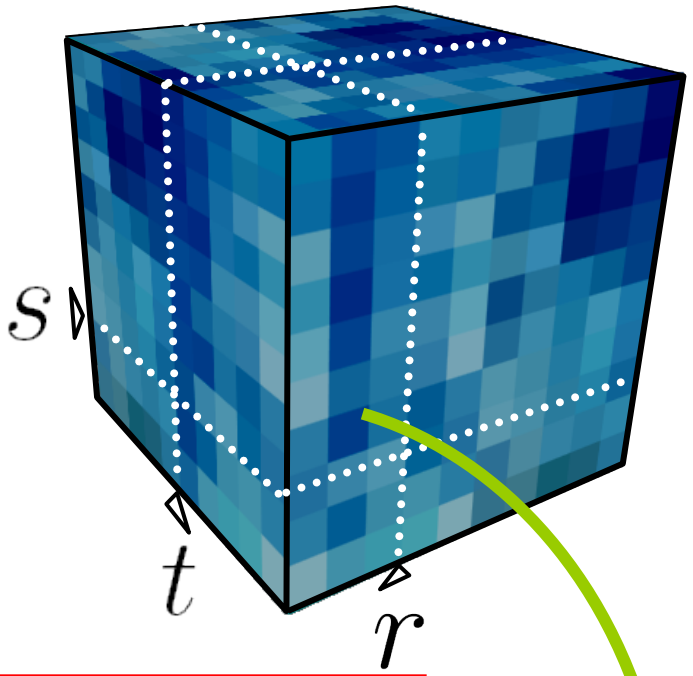




3D Textures





Don't be confused: *3D textures are not volumetric rendering primitives!*

Only ***planar polygons*** *are supported as rendering primitives.*

R G B A

3D Textures

3D Texture: Volumetric Texture Object

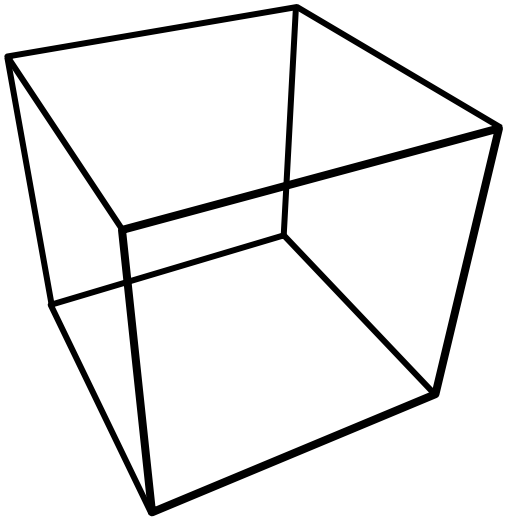
- Trilinear Interpolation in Hardware

3D Textures

3D Texture: Volumetric Texture Object

- Trilinear Interpolation in Hardware

⇨ Slices parallel to the image plane



3D Textures

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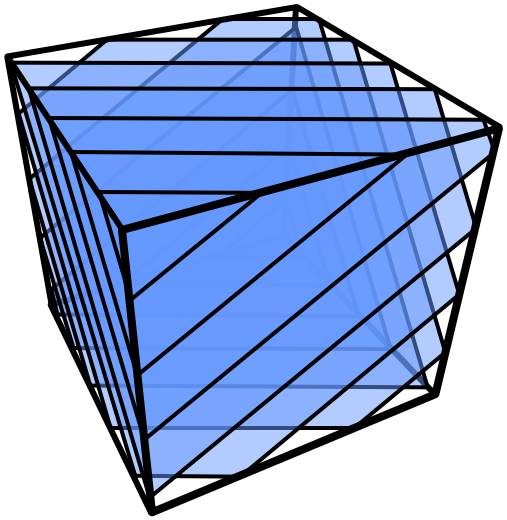
- Trilinear Interpolation in Hardware

⇨ Slices parallel to the image plane



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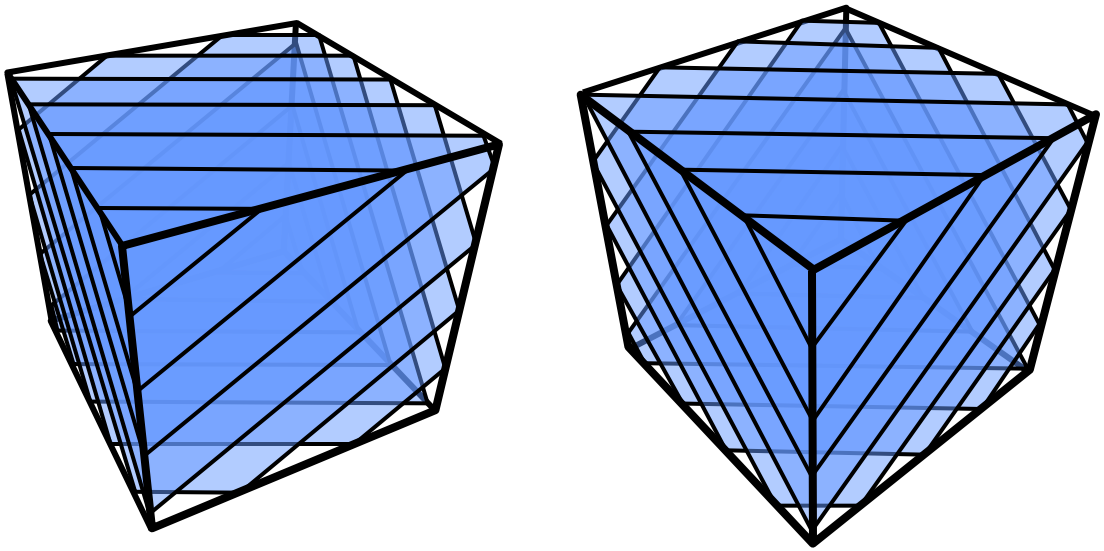

3D Textures

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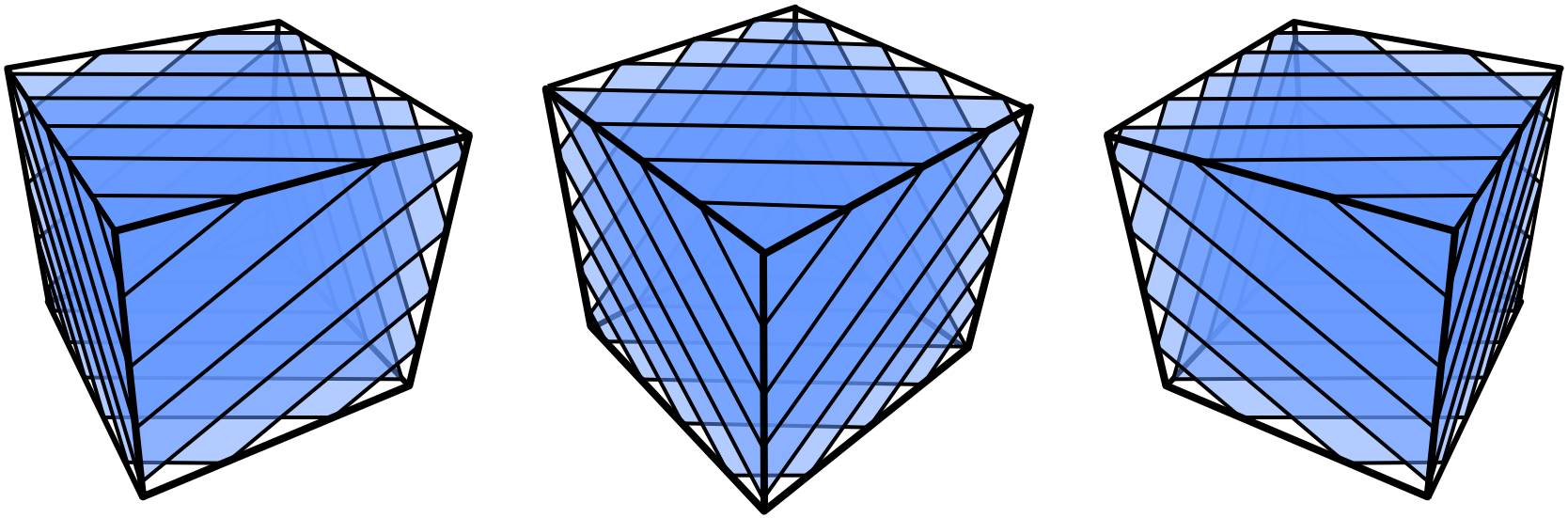
⇨ Slices parallel to the image plane



3D Textures

3D Texture: Volumetric Texture Object

- Trilinear Interpolation in Hardware

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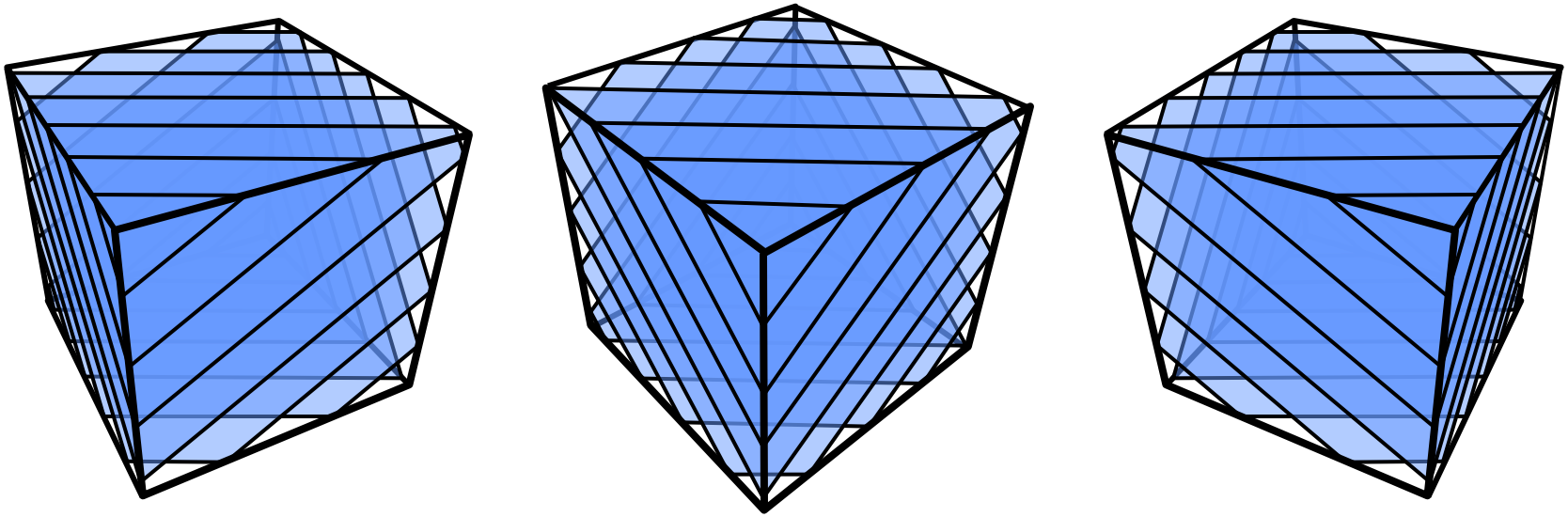


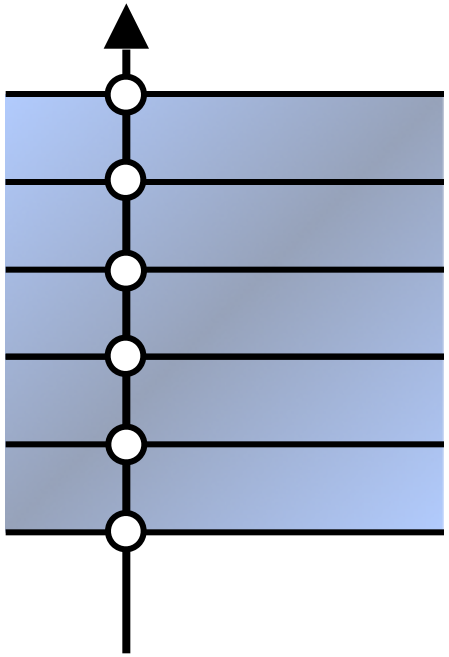
- One large texture block in memory

Resampling via 3D Textures

- ***Sampling rate is constant***

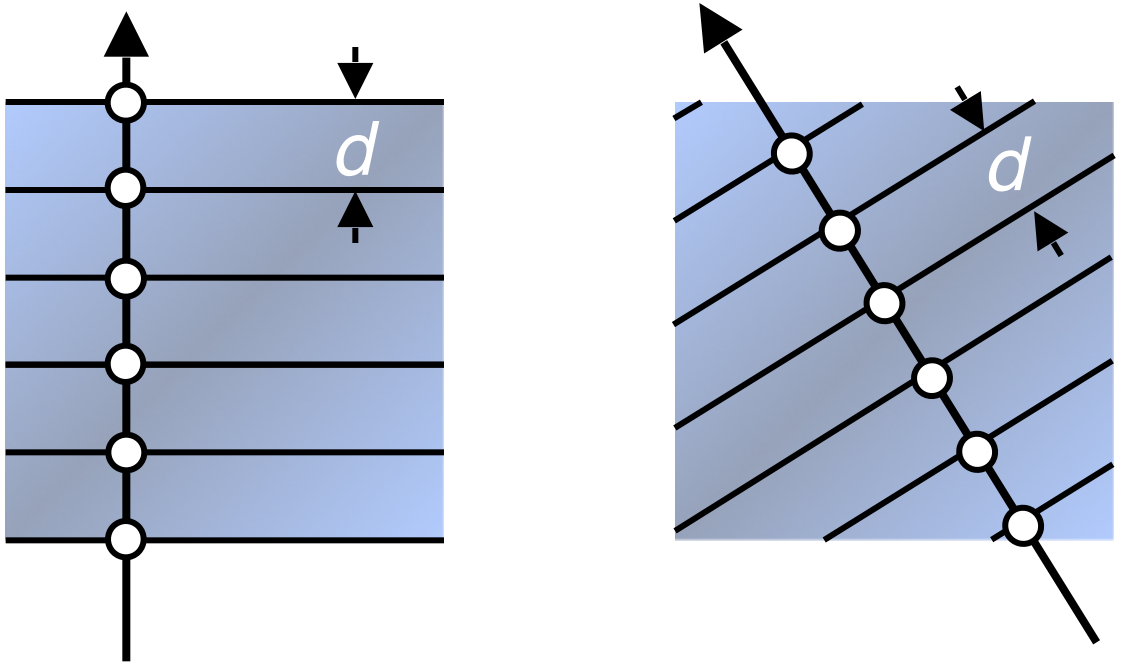
Resampling via 3D Textures

- ***Sampling rate is constant***



Resampling via 3D Textures

- ***Sampling rate is constant***



Resampling via 3D Textures

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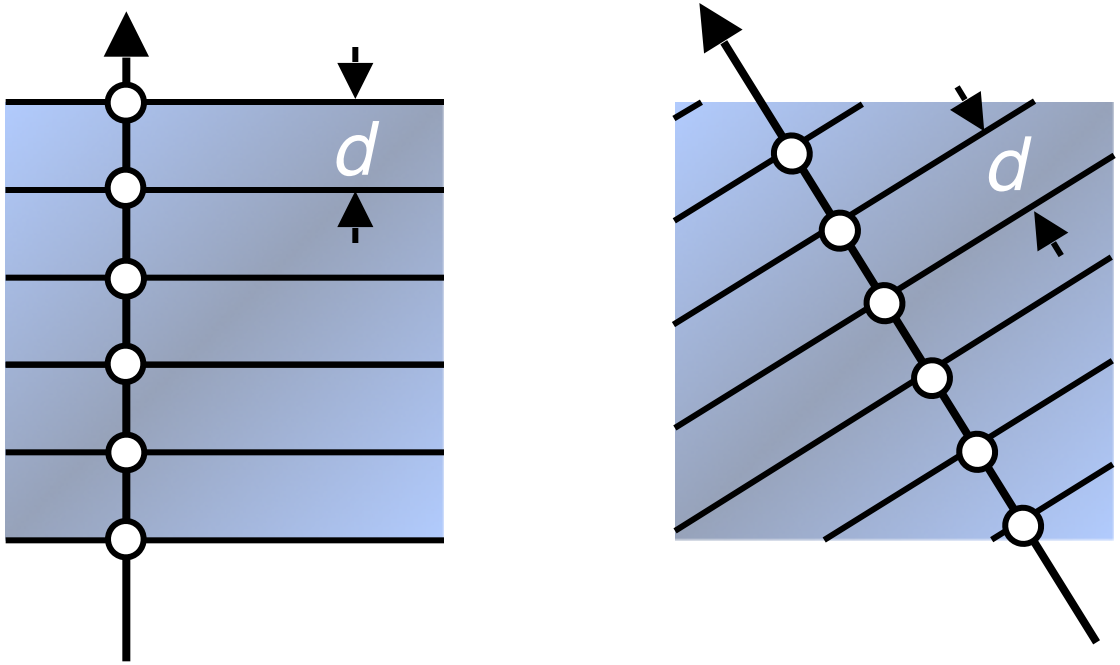
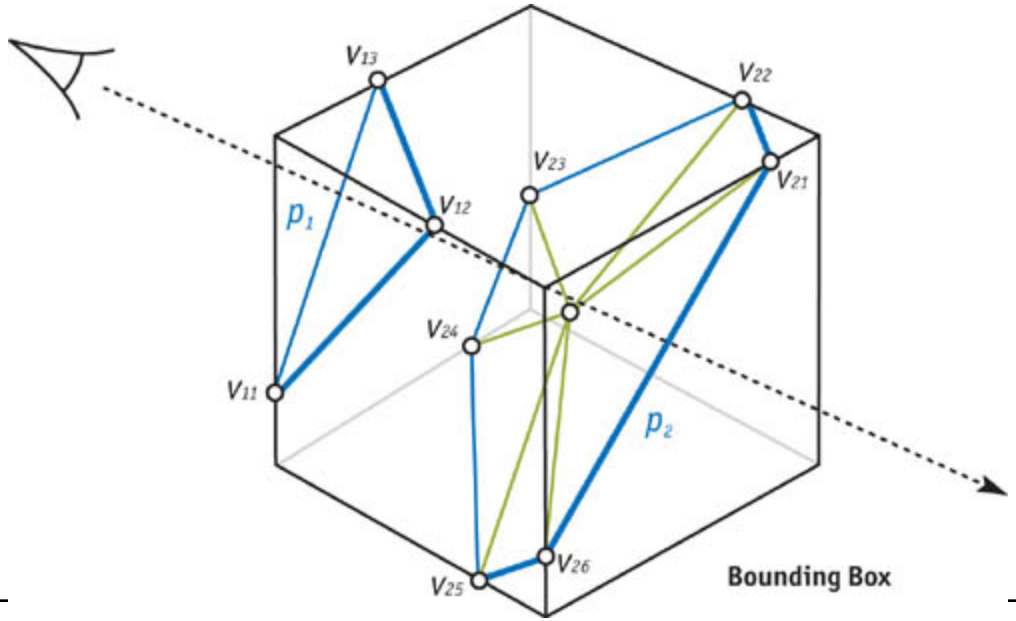


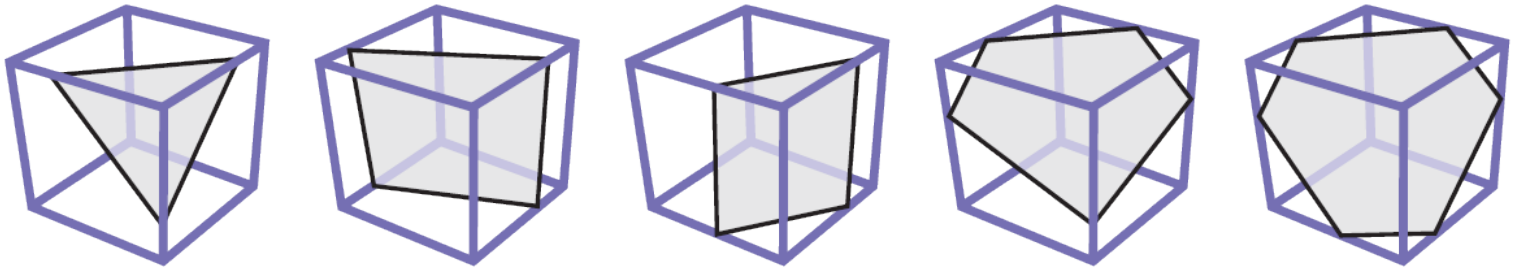
- Supersampling by increasing the number of slices

Proxy Geometry

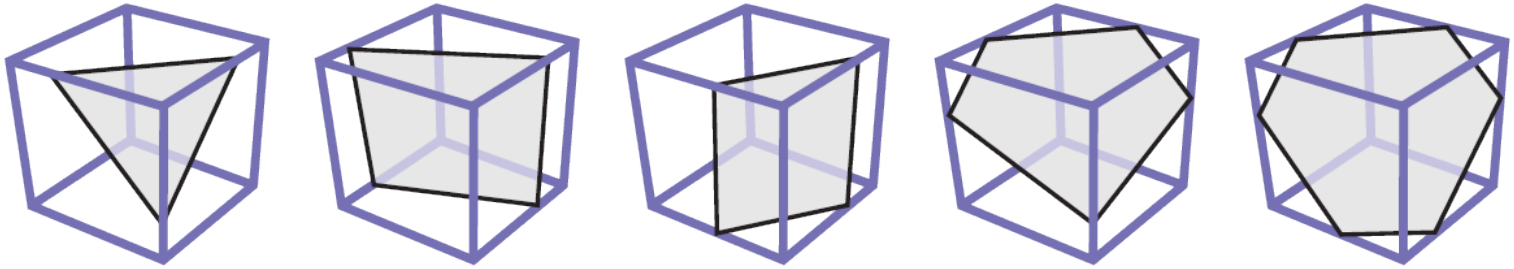
- View-Aligned Slices Algorithm
  - Intersect slicing planes with bounding box
  - Sort resulting vertices in (counter)clockwise order
  - Construct polygon primitive from centroid as triangle fan



Cube-Slice Intersection

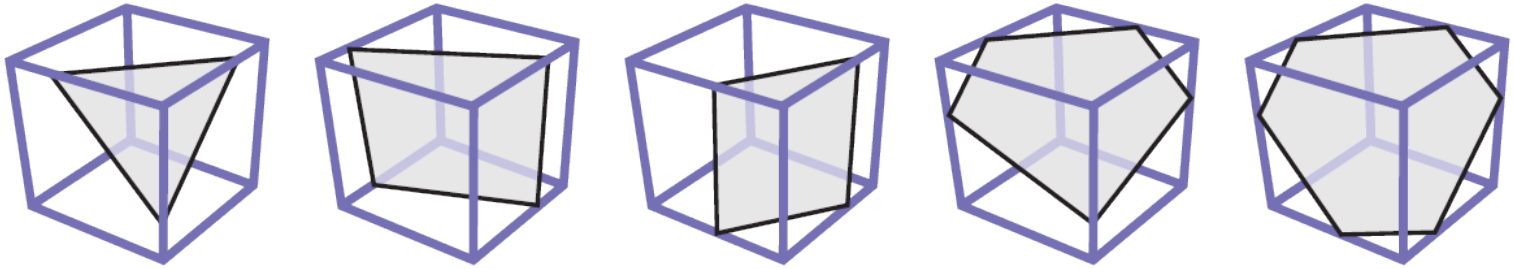


Cube-Slice Intersection



We can compute it in a vertex program

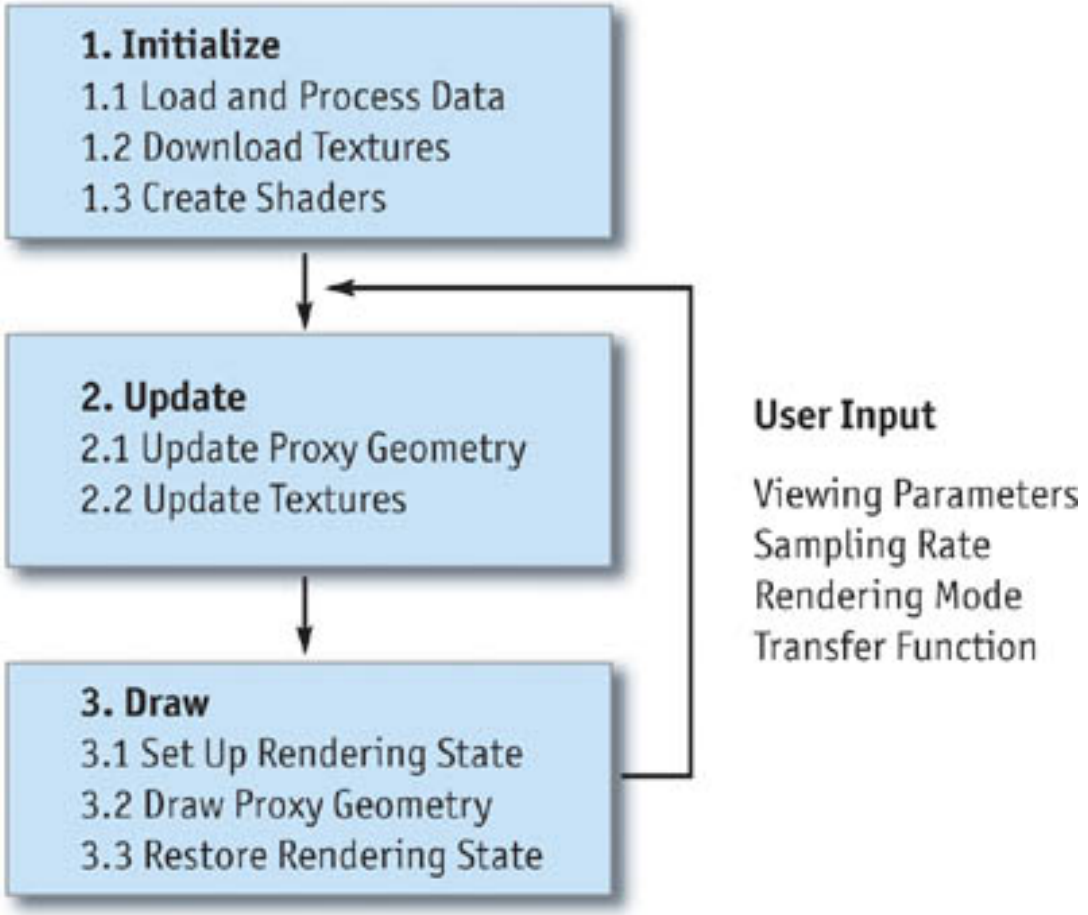
Cube-Slice Intersection



We can compute it in a vertex program

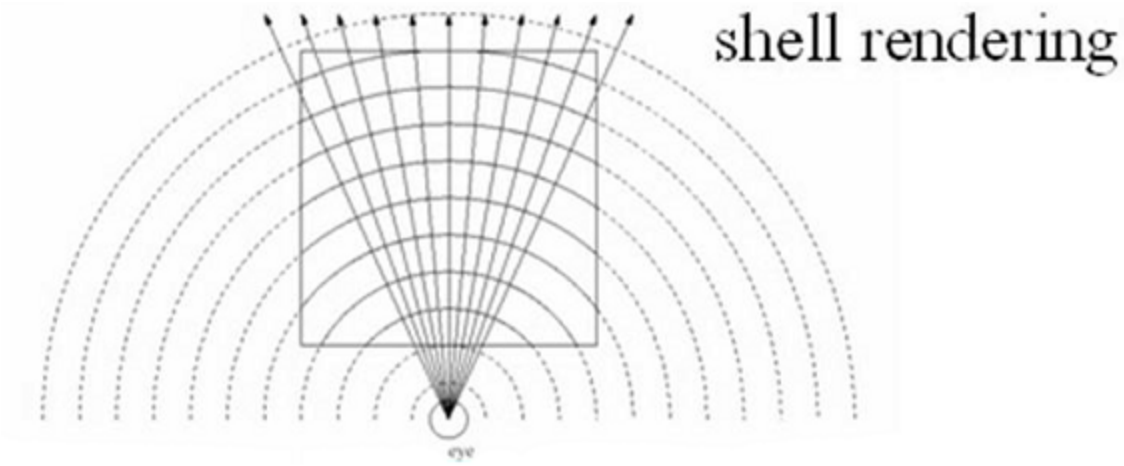
We can rotate the texture and having a fix set of object oriented planes.

Sliced-Based Volume Rendering Steps



Proxy Geometry

- Spherical Shells
  - Best replicates volume ray casting
  - Impractical – complex proxy geometry



Rendering Proxy Geometry

- Compositing
  - Over operator – back-to-front order

$$\hat{C}_i = C_i + (1 - A_i)\hat{C}_{i+1}$$

$$\hat{A}_i = A_i + (1 - A_i)\hat{A}_{i+1}$$

  - Under operator – front-to-back order

$$\hat{C}_i = (1 - \hat{A}_{i-1})C_i + \hat{C}_{i-1}$$

$$\hat{A}_i = (1 - \hat{A}_{i-1})A_i + \hat{A}_{i-1}$$

Rendering Proxy Geometry

- Compositing = Color and Alpha Accumulation Equations
- Easily implemented using hardware alpha blending
  - Over
    - Source = 1
    - Destination = 1 - Source Alpha
  - Under
    - Source = 1 - Destination Alpha
    - Destination = 1

Simple Volume Rendering Fragment Shader

```
void main(   uniform float3 emissiveColor,
             uniform sampler3D dataTex,
             float3 texCoord : TEXCOORD0,
             float4 color : COLOR)
{
   float a = tex3D(texCoord, dataTex); // Read 3D data
   texture color = a * emissiveColor; // Multiply by
   opac
}
```

Fragment Shader with Transfer Function

```
void main(   uniform sampler3D dataTex,
             uniform sampler1D tfTex,
             float3 texCoord : TEXCOORD0,
             float4 color : COLOR
         )
{
   float v = tex3d(texCoord, dataTex); // Read 3D data
   color = tex1d(v, tfTex); // transfer function
}
```

Local Illumination

- Blinn-Phong Shading Model

$$I = k_a + I_L k_d (\hat{l} \cdot \hat{n}) + I_L k_s (\hat{h} \cdot \hat{n})^N$$

Resulting = Ambient + Diffuse + Specular

Local Illumination

- Blinn-Phong Shading Model

$$I = k_a + I_L k_d (\hat{l} \cdot \hat{n}) + I_L k_s (\hat{h} \cdot \hat{n})^N$$

Resulting = Ambient + Diffuse + Specular

- Requires surface normal vector
  - Whats the normal vector of a voxel?

Local Illumination

- Blinn-Phong Shading Model

$$I = k_a + I_L k_d (\hat{l} \cdot \hat{n}) + I_L k_s (\hat{h} \cdot \hat{n})^N$$

Resulting = Ambient + Diffuse + Specular

- Requires surface normal vector
  - Whats the normal vector of a voxel? **Gradient**
  - Central differences between neighboring voxels

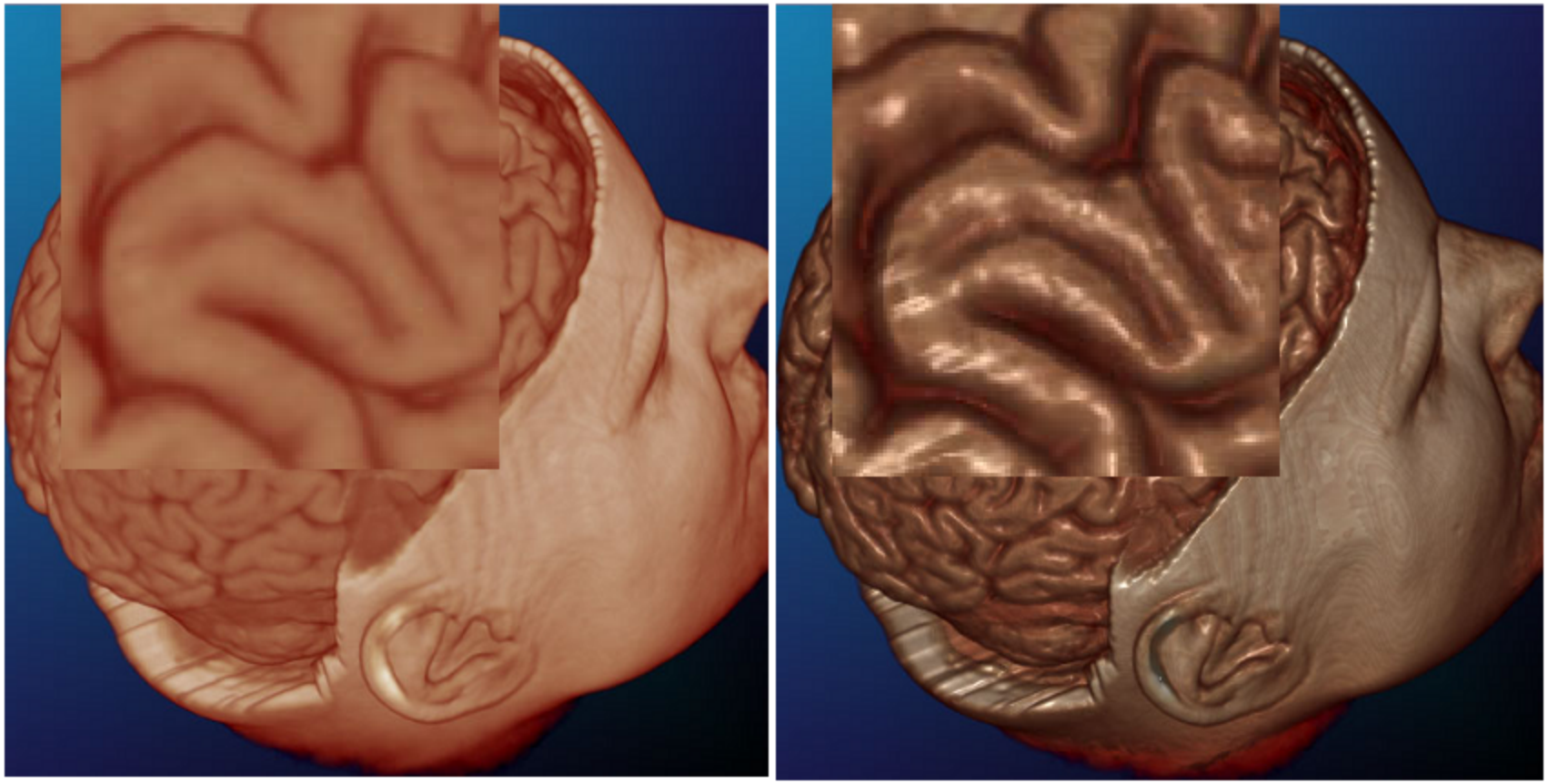
$$grad(I) = \nabla I = \frac{(right - left)}{2x}, \frac{(top - bottom)}{2x}, \frac{(front - back)}{2x}$$

Local Illumination

- Compute on-the-fly within fragment shader
  - Requires 6 texture fetches per calculation
- Precalculate on host and store in voxel data
  - Requires 4x texture memory
  - Pack into 3D RGBA texture to send to GPU

Local Illumination

- Improve perception of depth
- Amplify surface structure

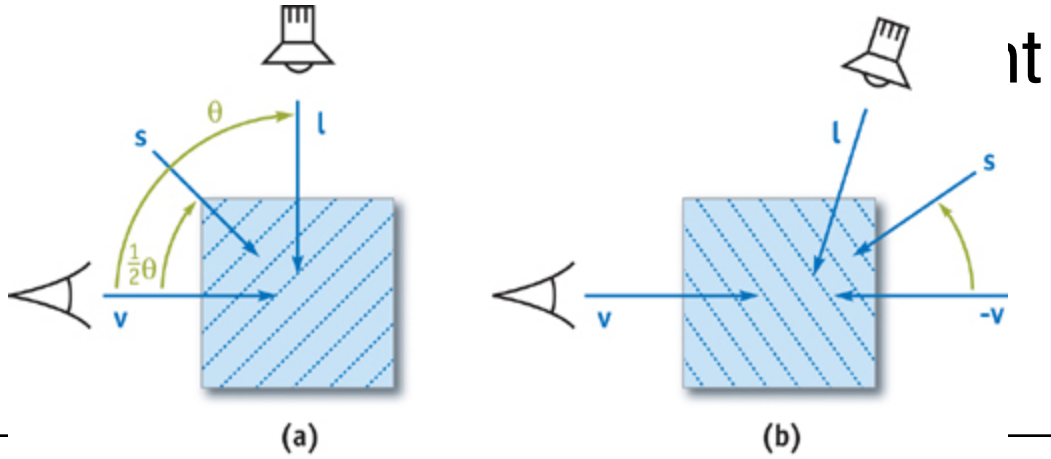


Volumetric Shadows on GPU

- Light attenuated from light's point of view
- CPU – Precomputed Light Transfer
  - Secondary raymarch from sample to light source
- GPU
  - Two-pass algorithm
  - Modify proxy geometry slicing
  - Render from both the eye and the light's POV
    - Two different frame buffers

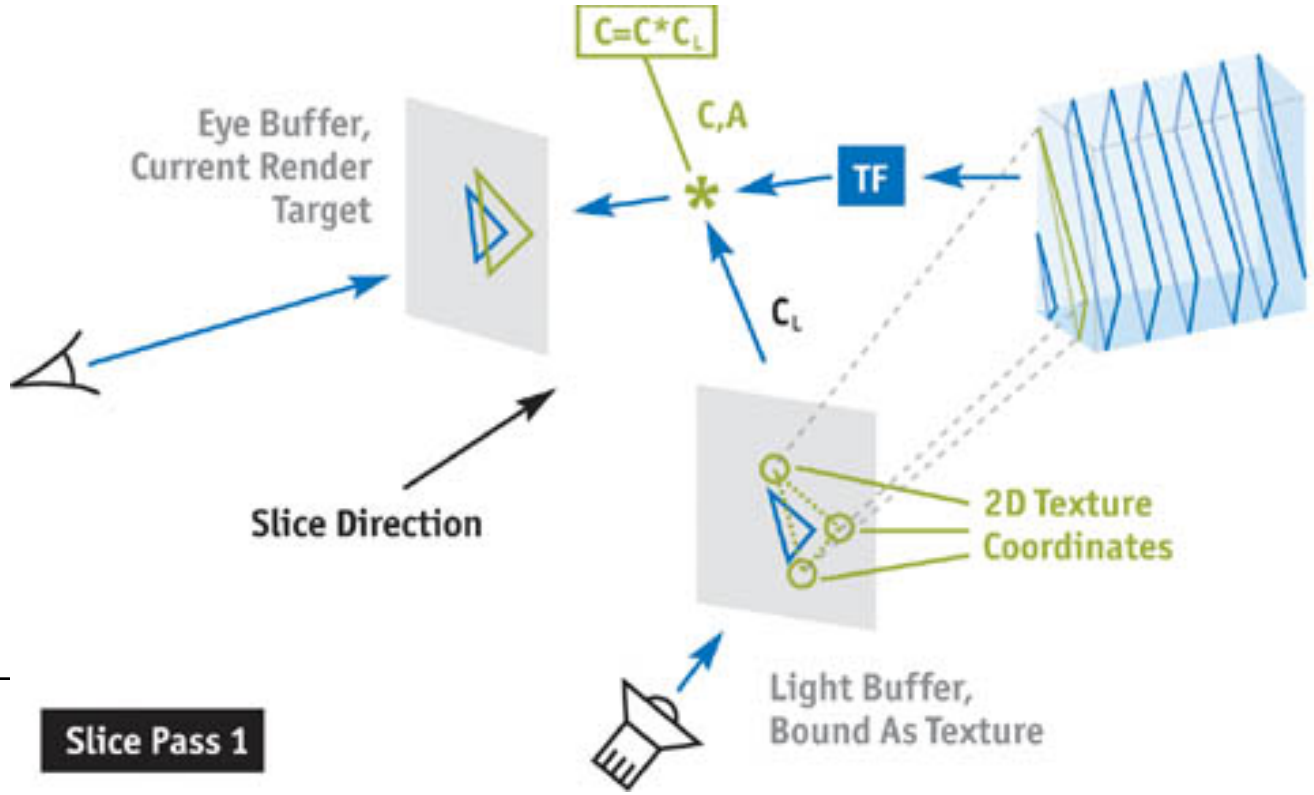
Two Pass Volume Rendering with Shadows

- Slice axis set half-way between view and light directions
  - Allows each slice to be rendered from eye and light POV
- Render order for light – front-to-back
- Render order for eye – (a) front-to-back



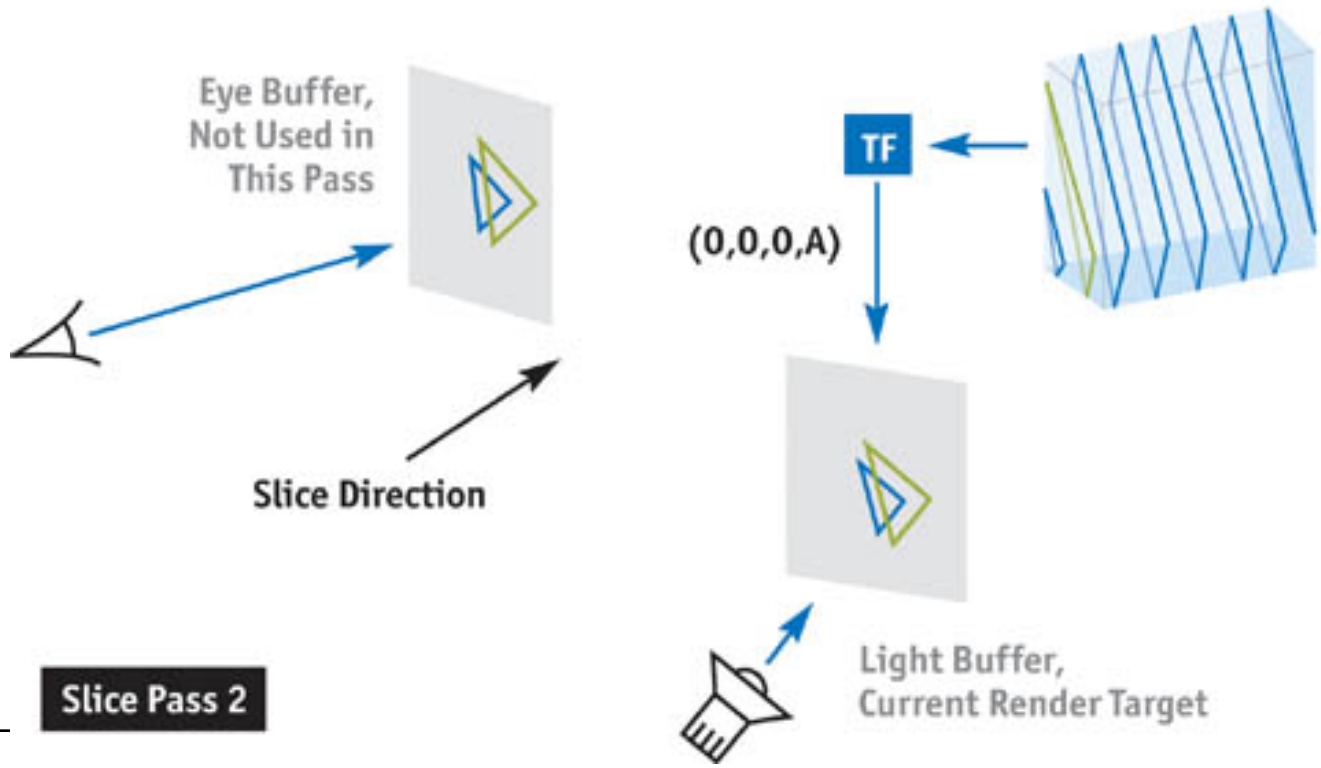
First Pass

- Render from eye
- Fragment shader
  - Look up light color from light buffer bound as texture
  - Multiply material color * light color

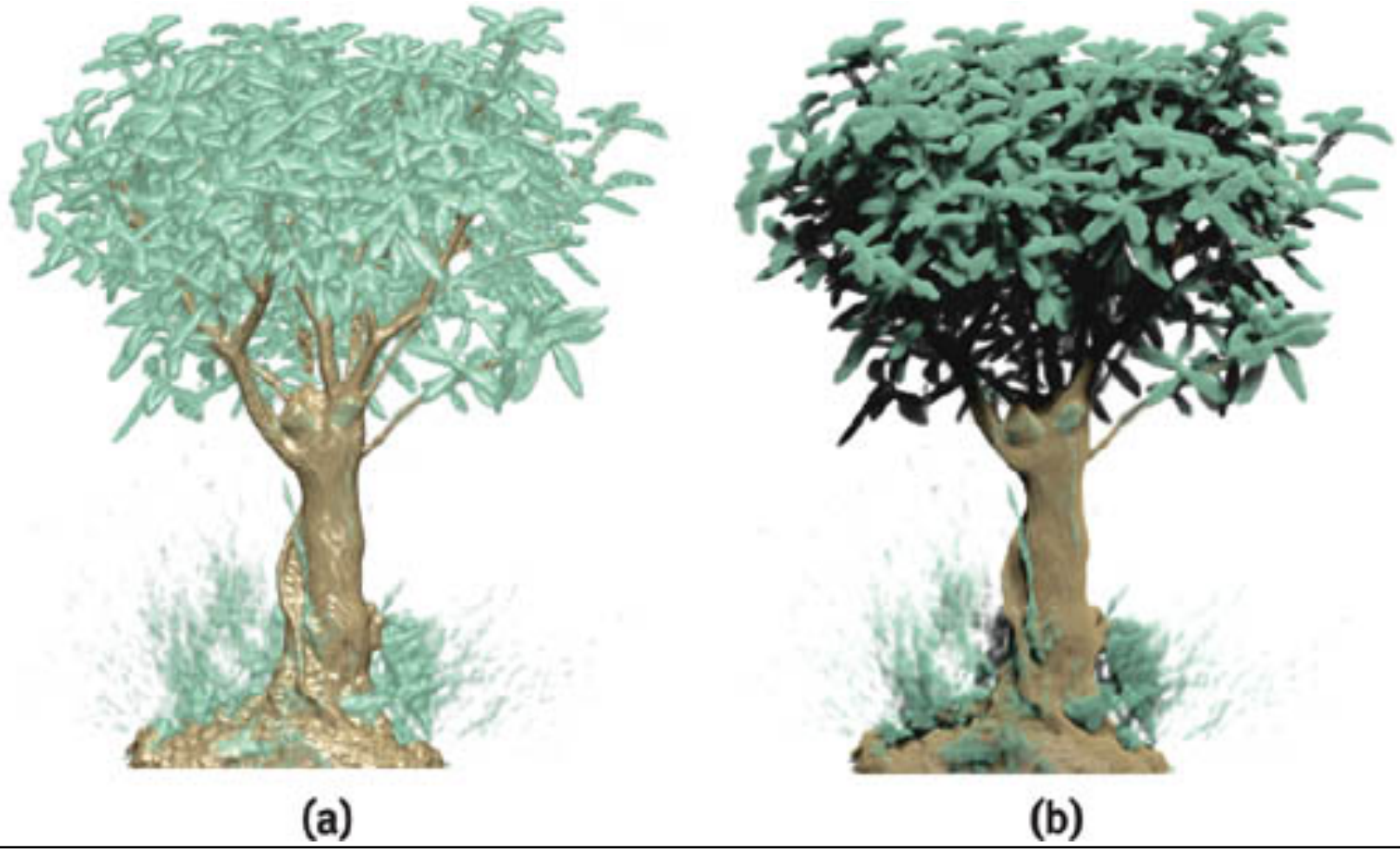


Second pass

- Render from light
- Fragment shader
  - Only blend alpha values – light transmissivity

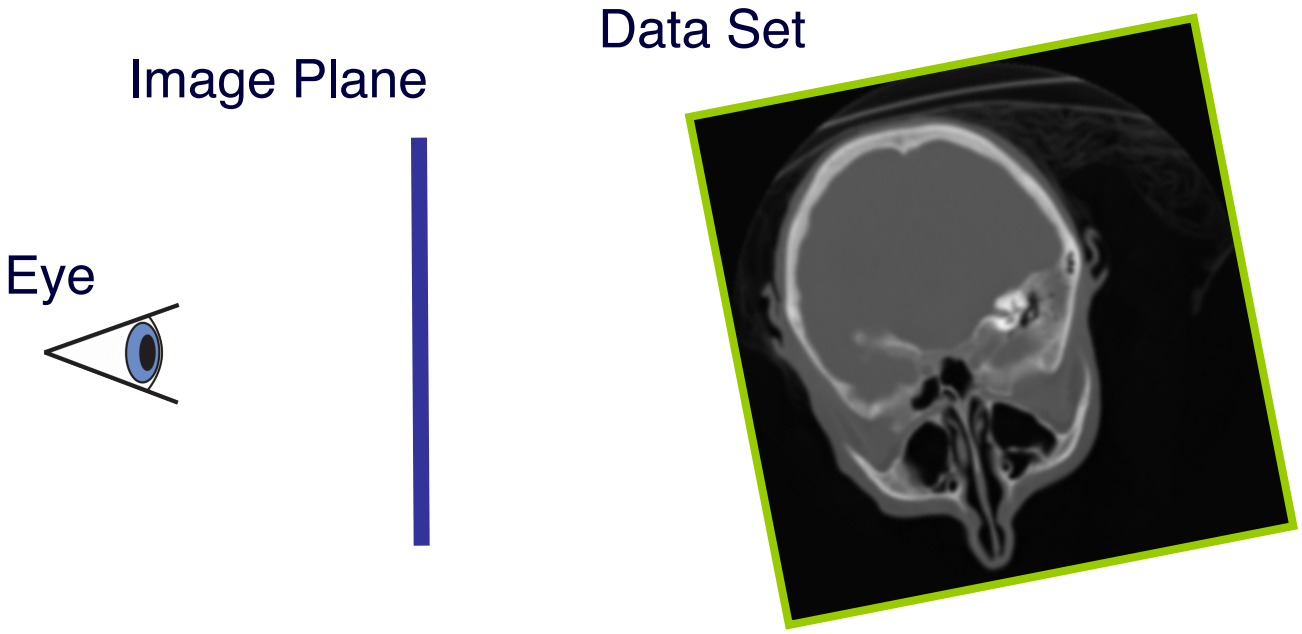


Volumetric Shadows

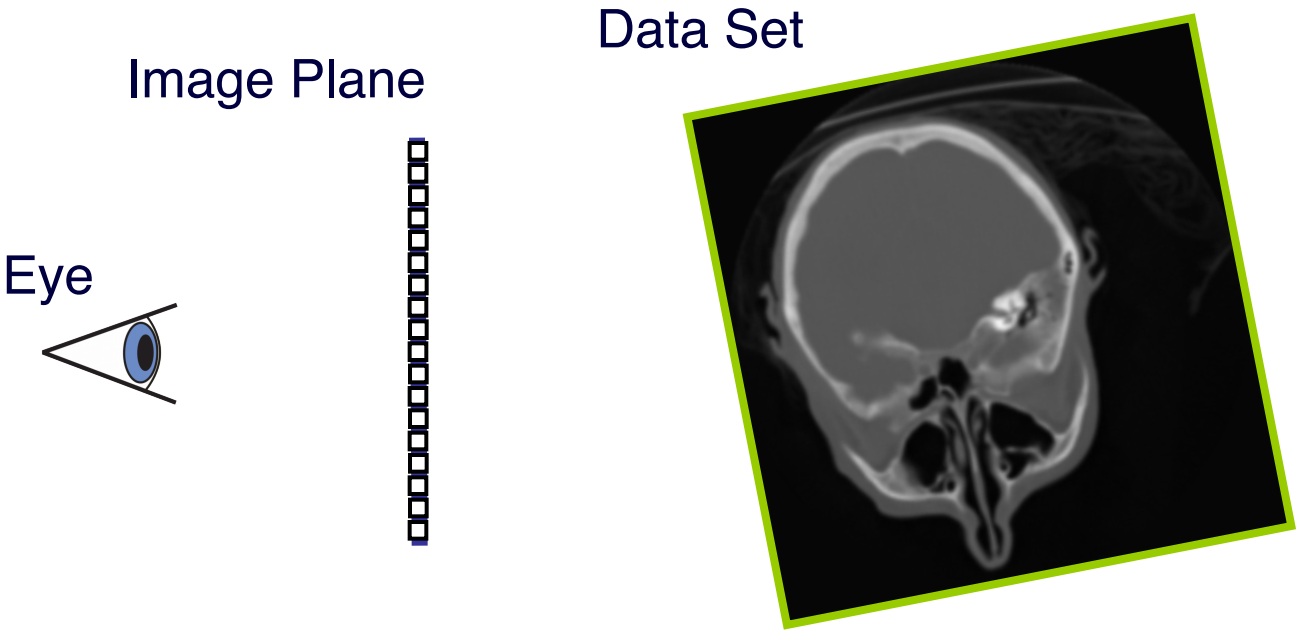


(a) (b)

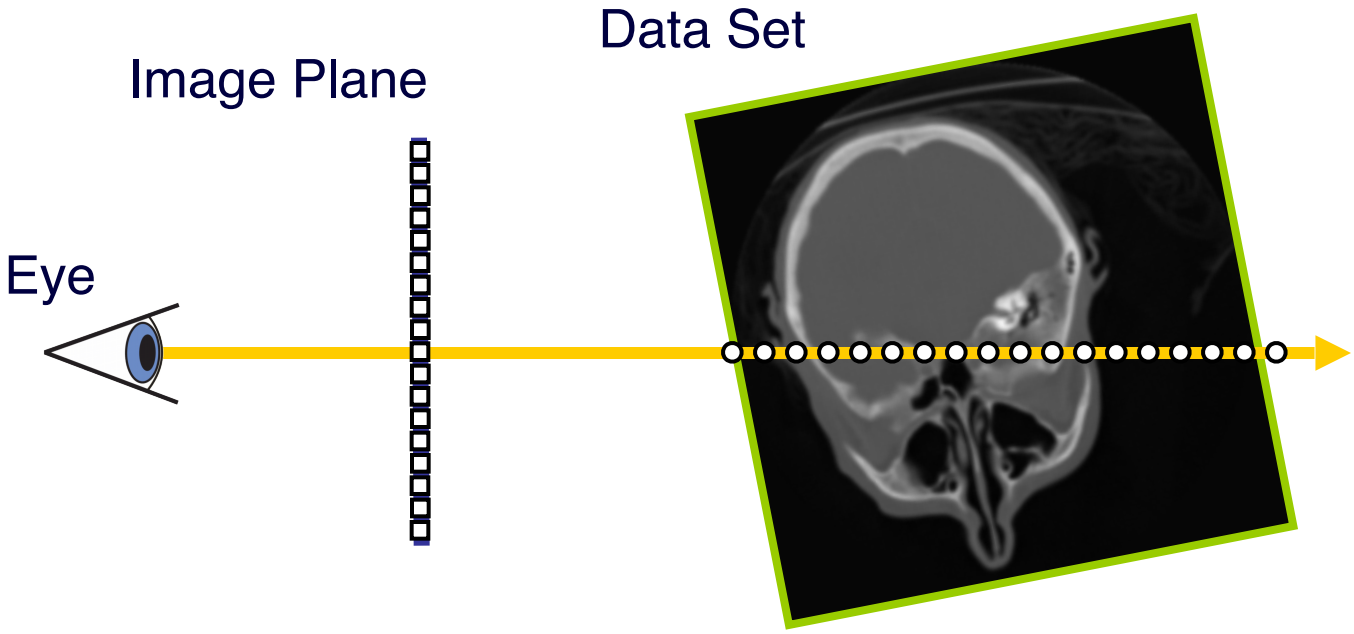
Ray-casting



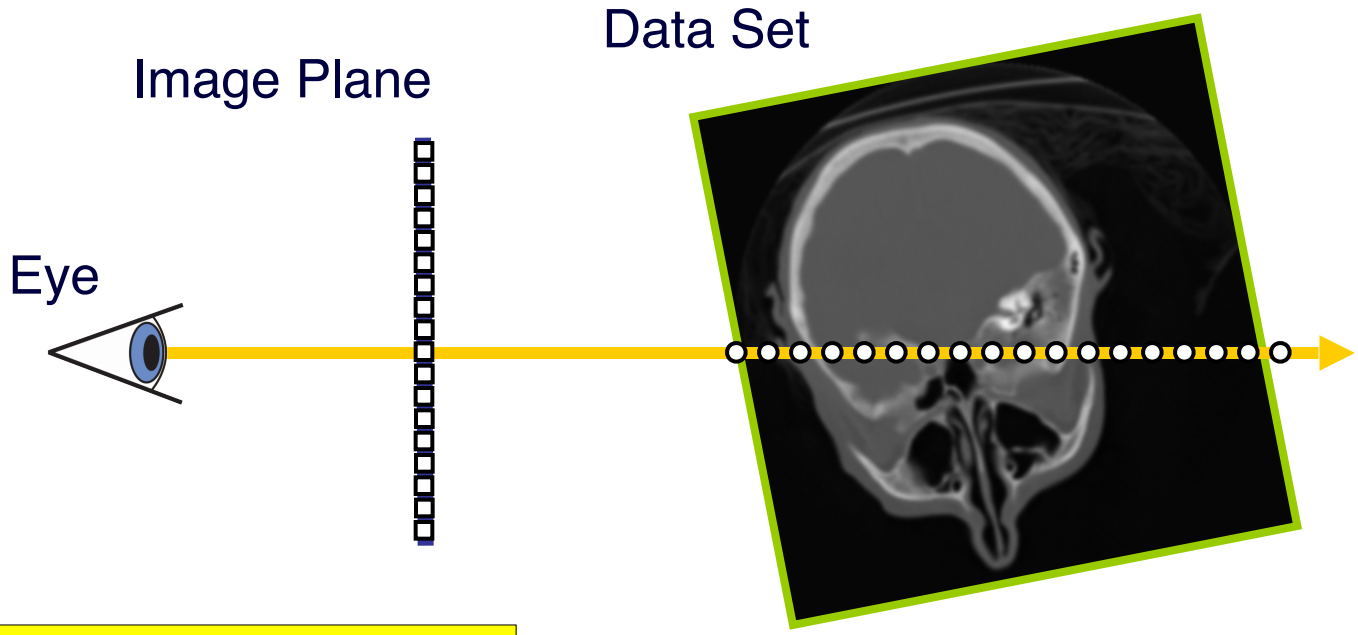
Ray-casting



Ray-casting



Ray-casting

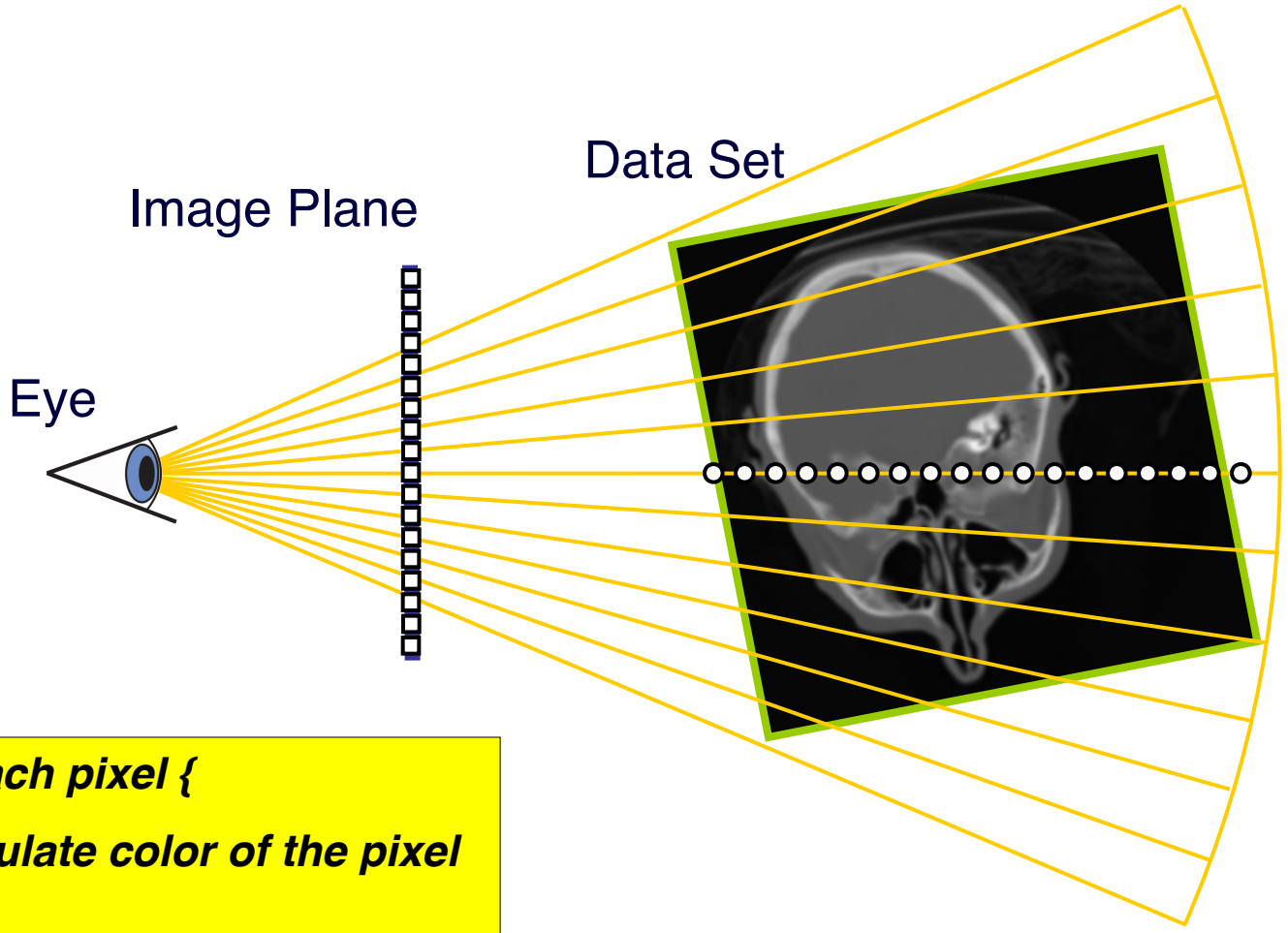


For each pixel {

calculate color of the pixel

}

Ray-casting



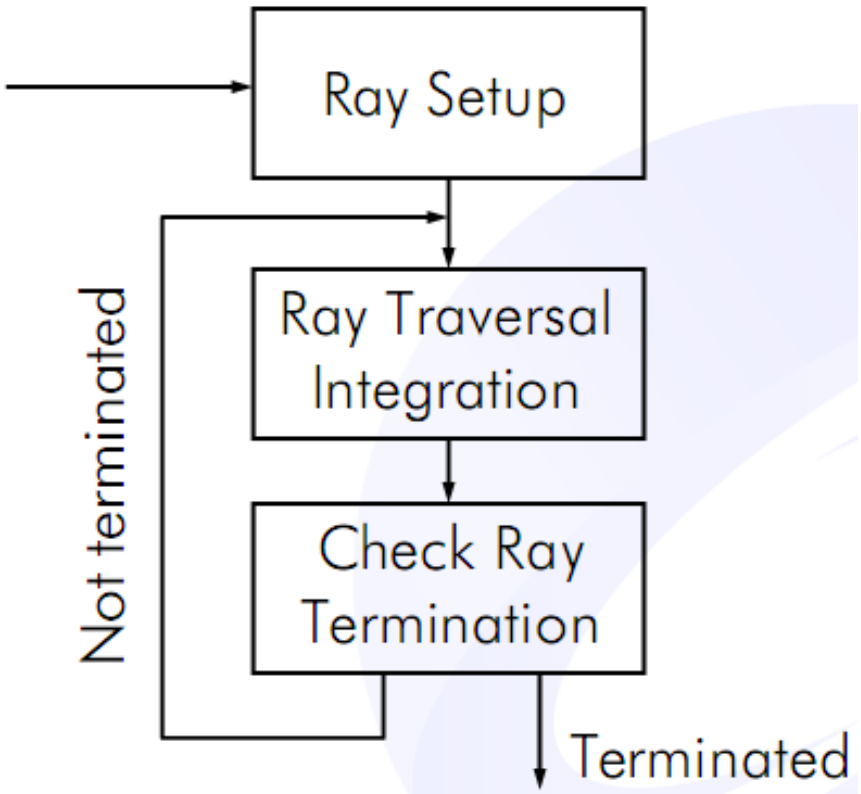
For each pixel {

calculate color of the pixel

}

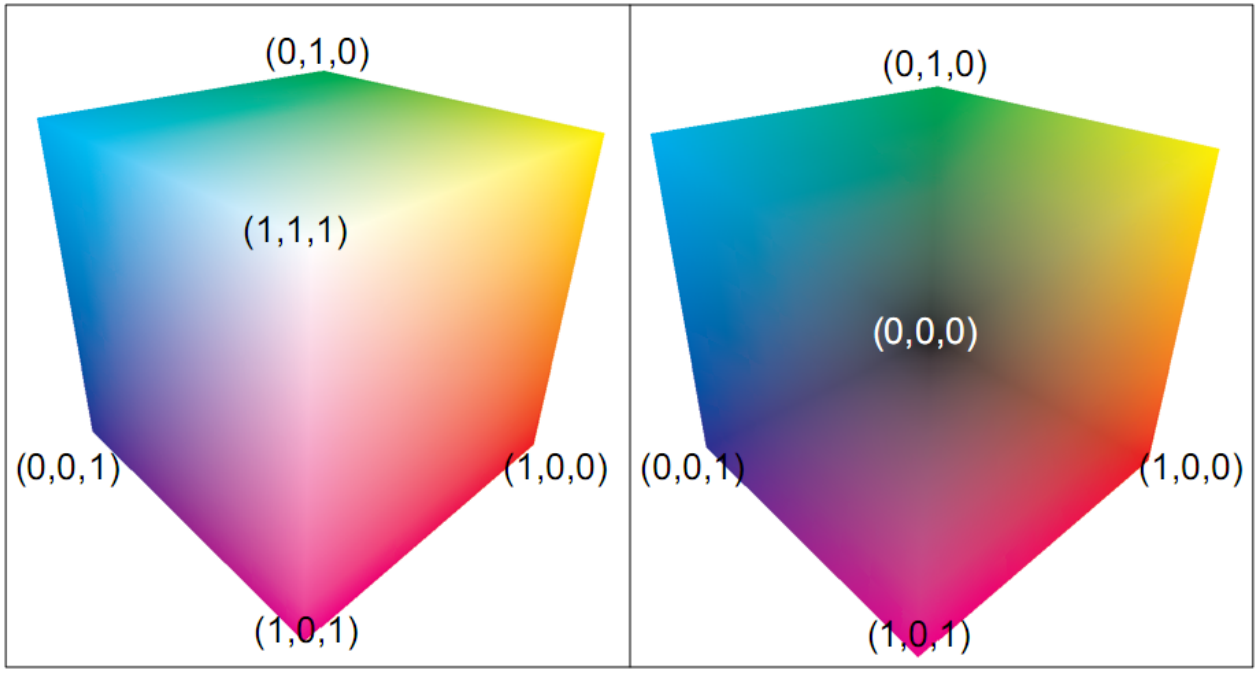
Volume Raycasting on GPU

- Multi-pass algorithm
- Initial passes
  - Precompute ray directions and lengths
- Additional passes
  - Perform raymarching in parallel for each pixel
  - Split up full raymarch to check for early termination



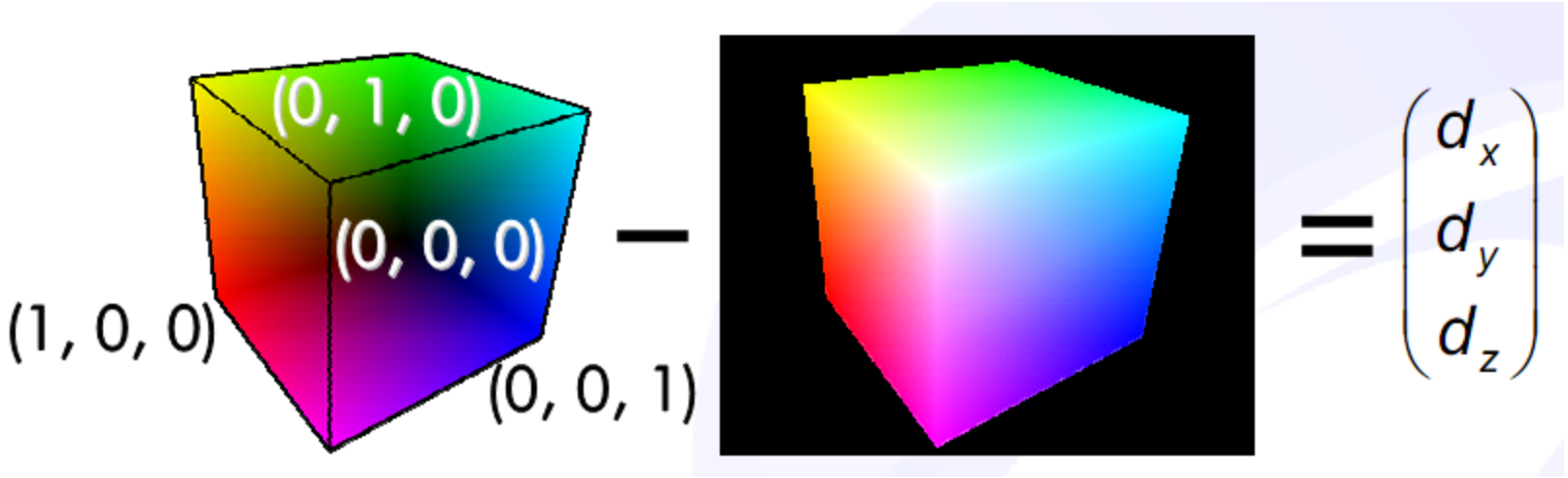
Step 1: Ray Direction Computation

- Ray direction computed for each pixel
- Stored in 2D texture for use in later steps
- Pass 1: Front faces of volume bounding box
- Pass 2: Back faces of volume bounding box
- Vertex color components encode object-space principle directions



Step 1: Ray Direction Computation

- Subtraction blend two textures
- Store normalized direction – RGB components
- Store length – Alpha component



Fragment Shader Ray Marching

- DIR[x][y] – ray direction texture
  - 2D RGBA values
- P – per-vertex float3 positions, front of volume bounding box
  - Interpolated for fragment shader by graphics pipeline
- s – constant step size
  - Float value
- d – total raymarched distance, s x #steps
  - Float value

Fragment Shader Raymarching

- DIR[x][y] – ray direction texture
  - 2D RGBA values
- P – per-vertex float3 positions, front of volume bounding box
  - Interpolated for fragment shader by graphics pipeline
- s – constant step size
  - Float value
- d – total raymarched distance, s x #steps
  - Float value

- Parametric Ray Equation

$$r = P + d \cdot DIR[x][y]$$

  - r – 3D texture coordinates used to sample voxel data

Fragment Shader Ray Marching

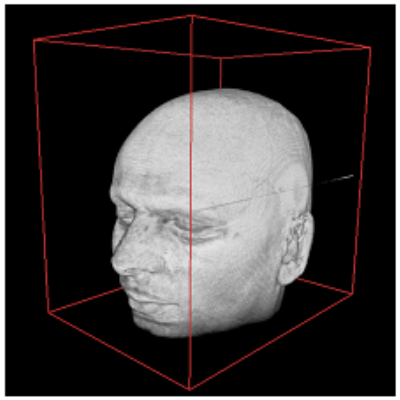
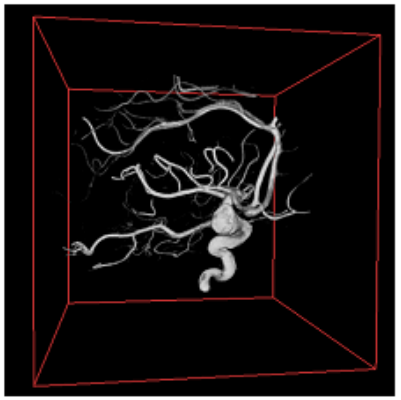
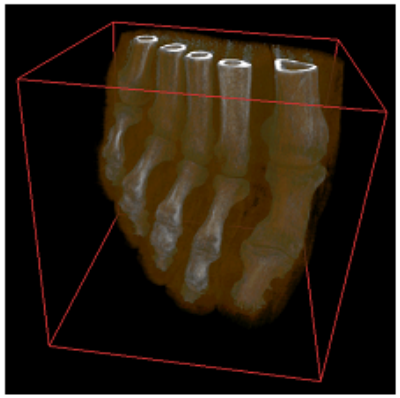
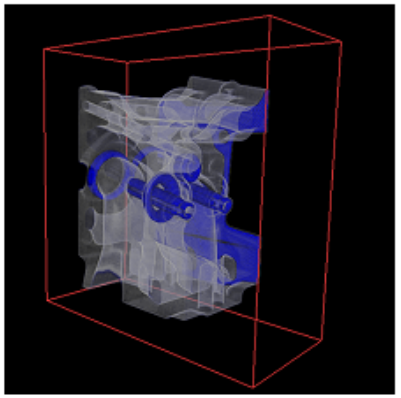
- Ray traversal procedure split into multiple passes
  - M steps along ray for each pass
  - Allows for early ray termination, optimization
- Optical properties accumulated along M steps
  - Simple compositing/blending operations
  - Color and alpha(opacity)
- Accumulation result for M steps blended into 2D result texture
  - Stores overall accumlated values between multiple passes
- Intermediate Pass – checks for early termination
  - Compare opacity to threshold
  - Check for ray leaving bounding volume

Optimizations

- Early Ray Termination
  - Compare accumulated opacity against threshold
- Empty Space Skipping
  - Additional data structure encoding empty space in volume
  - Oct-tree
  - Encode measure of empty within 3D texture read from fragment shader
  - Raymarching fragment shader can modulate sampling distance based on empty space value

Performance and Limitations

- More physically-based than slice-based volume rendering
  - Guarantees equal sampling distances
- Does not incorporate volumetric shadows
- Reduced number of fragment operations
  - Fragment programs made more complex
- Optimizations work best for non-opaque data sets
  - Early ray termination and empty space skipping can be applied

	SBVR	RC	RC-α	RC-β
Head	7.1	4.8	14.4	23.4
Aneurism	7.1	4.8	8.9	19.6
Foot	7.1	4.8	10.2	17.7
Engine	10.4	6.3	6.3	13.6

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Pere-Pau Vázquez

Scientific Visualization 2019 – 2020

FUNDAMENTAL BACKGROUND