Lectures ?–?: Numerical Methods

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Last updated May 7, 2015

1 Mathematics Review

1.1 Derivatives

A derivative is the change in a function ($f(x)$) with respect to the change in the independent variable (x) as the interval (Δx) approaches 0:

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{1}$$

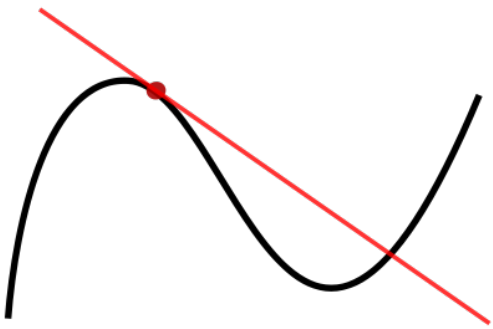

Figure 1: The graph of a function, drawn in black, and a tangent line to that function, drawn in red. The slope of the tangent line is equal to the derivative of the function at the marked point. (Text from https://en.wikipedia.org/wiki/Derivative on 2015.05.07; borrowing it because I couldn't find a better way to say it!)

@TODO: Create a better figure for derivative definition and finite difference, with shown Δx

1.2 Integrals

Integration An indefinite integral

$$\int_a^b f(x)\,dx = F(b) - F(a) \tag{2}$$

$$\int_a^b f(x)\,dx \tag{3}$$

Figure 2: This "S" in the Berlin–Wannsee station sign is written in the old style. It is used for integrals to stand for "sum", as the area under a curve can be imagined to be a sum ($\sum$) of every infinitessimally thin column of space under a curve.

1.3 Taylor series

The Taylor series of any complex function $f(x)$ (i.e., $f(x) \in \mathbb{C}$) that is infinitely differentiable at a point x_0 approximates that function as a power series:

$$f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x - x_0)^3 + \cdots \tag{4}$$

Or, in the more-compact summation notation, this is:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}\,(x - x_0)^n \tag{5}$$

where n is the number of the derivative of f.

dot products
cross products
vectors
tensors
Meshes

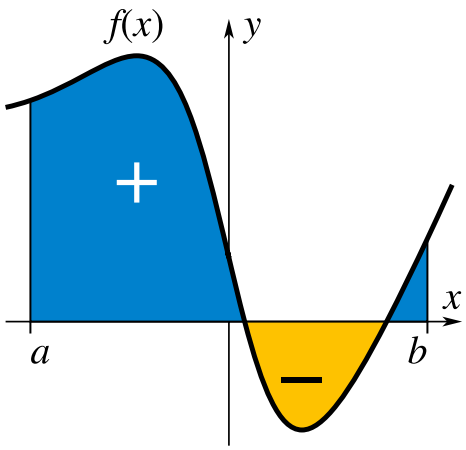

Figure 3: An integral is the sum of all area under a curve. (Contributed to Wikimedia Commons by User:KSmrq)

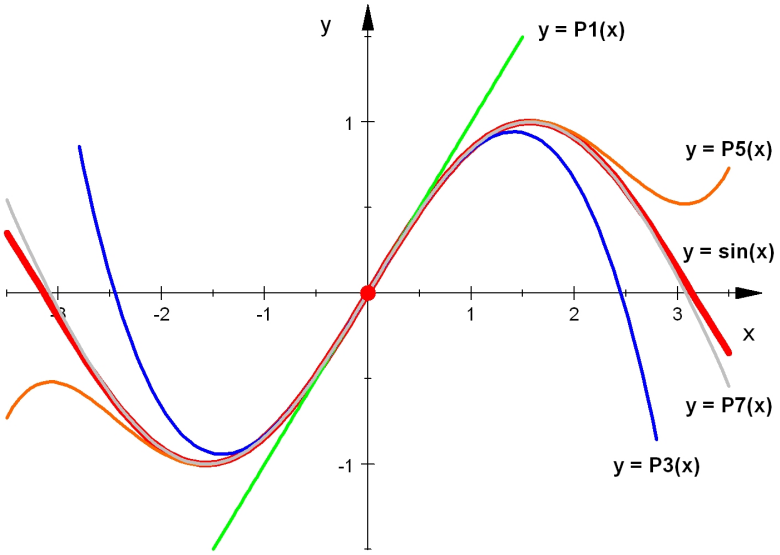

Figure 4: Increasing orders of the derivative n, denoted Pn, where $n = 1, 2, 3, \cdots$, show the increasing approximation of a sum of polynomials to the sine function.

2 Finite difference

2.1 Discretization

You may have wondered why I covered derivatives and Taylor series right after one another

Finite difference, finite element, and spectral

Forward difference and "implicit" on both structured and unstructured meshes

Stencils

Linearizing equations (is this the right word? turning them into a set of linear equations)

Differential equations and linear algebra review

Include examples