IN4320 Machine Learning

Exercise Computational Learning Theory: Boosting

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a.

Let $x \geq 0$. then $e^{-x} \geq (1-x)$.

Proof:

Note that $e^{-x} \geq 0$ and $e^0 = 1$, so that the result is trivial for $x \geq 1$ and $x = 0$. We assume $x \in (0,1)$. Then we have, using the Taylor series representation of e^{-x}:

$$
\begin{aligned}
e^{-x} - (1-x) &= \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} - (1-x) \\
&= \sum_{n=2}^{\infty} \frac{(-x)^n}{n!} \\
&= \sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!} - \frac{x^{(2n+1)}}{(2n+1)!} \\
&= \sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!} \left(1 - \frac{x}{2n+1}\right) \\
&\geq 0
\end{aligned}
$$

Where in the last step it was used that each term of the sum is positive for $x \in (0,1)$.