Given:

X: dense $B \times IC \times M \times N$ input

val_W, idx_W: sparse representation of the sparse filter (dense shape $OC \times IC \times K \times K$) with nnz non-zero values, with indices of type (out-channel, in-channel, row, column)

O: dense $B \times OC \times OM \times ON$ output ($OM = M - K + 1, ON = N - K + 1$)

$\frac{\partial L}{\partial O}$: dense $B \times OC \times OM \times ON$ errors

Targets:

dense $B \times IC \times M \times N$ tensor $\frac{\partial L}{\partial X}$

sparse tensor $val_{\frac{\partial L}{\partial W}}, idx_{\frac{\partial L}{\partial W}}$ with nnz elements

Algorithm:

$\frac{\partial L}{\partial X} \leftarrow B \times IC \times M \times N$ zero tensor

$idx_{\frac{\partial L}{\partial W}} \leftarrow$ copy of idx_W

$val_{\frac{\partial L}{\partial W}} \leftarrow$ zero list of length nnz

for $0 \leq si < nnz$ **do**

    $(oc, ic, i, j) \leftarrow idx_W[si]$

    $v \leftarrow val_W[si]$

    for $0 \leq b < B$ **do**

        for $0 \leq p < OM$ **do**

            for $0 \leq q < ON$ **do**

                $o \leftarrow \frac{\partial L}{\partial O}[b, oc, p, q]$

                add $o \times v$ to $\frac{\partial L}{\partial X}[b, ic, p + i, q + j]$

                add $o \times X[b, ic, p + i, q + j]$ to $val_{\frac{\partial L}{\partial W}}[si]$

            end for

        end for

    end for

end for