Assignment 7

# Largest Hub

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Calculate the expected maximum degree for the undirected networks listed in Table 4.1 of Barabasi's book 'Network Science', except Power Grid. Assume ($k_{\min}$) is 1 for all networks. Explain your reasoning.

## 1 Maximum Degree

Following the *Continuum Formalism*, the minimum degree is given by

$$k_{\max} = k_{\min} \cdot N^{\frac{1}{\gamma-1}}$$

### 1.1 Discrete Formalism

We can also consider the *Discrete Formalism*, which is described by

$$P(\mathbf{k} = k) = \frac{k^{-\gamma}}{\sum_{q=k_{\min}}^{\infty} q^{-\gamma}} = \frac{k^{-\gamma}}{\zeta(\gamma, k_{\min})}$$

Where $\zeta$ is the Hurwitz zeta function. From the definition, $k_{\max}$ is the first value such that $P(\mathbf{k} \geq k_{\max}) \leq \frac{1}{N}$, but

$$\begin{aligned} P(\mathbf{k} \geq k_{\max}) &= \sum_{k=k_{\max}}^{\infty} P(\mathbf{k} = k) \\ &= \frac{1}{\zeta(\gamma, k_{\min})} \sum_{k=k_{\max}}^{\infty} k^{-\gamma} \\ &= \frac{\zeta(\gamma, k_{\max})}{\zeta(\gamma, k_{\min})} \end{aligned}$$

So $k_{\max}$ is found as the smallest integer such that

$$\frac{\zeta(\gamma, k_{\max})}{\zeta(\gamma, k_{\min})} \leq \frac{1}{N}$$

Since the graph is simple, $k_{\max} < N$, then we can use the Bisection Method with range $[1, N]$ to find the solution numerically.

## 2 Results

| Network | $N$ | $\gamma_{in}$ | $\gamma_{out}$ | Continuum | | Discrete | |
|---|---|---|---|---|---|---|---|
| | | | | $k_{maxin}$ | $k_{maxout}$ | $k_{maxin}$ | $k_{maxout}$ |
| Internet | 192244 | 3.42 | 3.42 | 152.55 | 152.55 | 107 | 107 |
| WWW | 325729 | 2.00 | 2.31 | 325729.00 | 16154.27 | 325730 | 13146 |
| Mobile-Phone Calls | 36595 | 4.69 | 5.01 | 17.25 | 13.74 | 13 | 11 |
| Email | 57194 | 3.43 | 2.03 | 90.73 | 41570.65 | 64 | 40396 |
| Science Collaboration | 23133 | 3.35 | 3.35 | 71.96 | 71.96 | 51 | 51 |
| Actor Network | 702388 | 2.12 | 2.12 | 166019.12 | 166019.12 | 150043 | 150043 |
| Citation Network | 449673 | 3.03 | 4.00 | 609.09 | 76.61 | 431 | 54 |
| E. Coli Metabolism | 1039 | 2.43 | 2.90 | 128.68 | 38.70 | 101 | 29 |
| Protein Interactions | 2018 | 2.89 | 2.89 | 56.06 | 56.06 | 41 | 41 |

Table 1: $k_{max}$ for the Scale-Free networks on Table 4.1, using the Continuum and Discrete Formalisms.