Assignment 12

Star Network

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Consider a star network, where a single node is connected to $N-1$ degree-one nodes. Assume that N is much larger than 1. Your goal is to compute the degree correlation coefficient of this network as a function of N, using Formulas 7.11 and 7.12 in Box 7.2 of the adopted book, following the steps:

(a) compute the numerator of 7.11;

(b) compute the denominator of 7.11 using 7.12;

(c) divide the result of (a) by the result of (b).

1 The Network

Looking at figure 1, we can clearly that every link is connected to a node of degree 1 and the central node, with degree $N-1$. Therefore, $e_{1,N-1} = 1$ when $N > 1$ and $e_{i,j} = 0$ for any other distinct pair i, j.

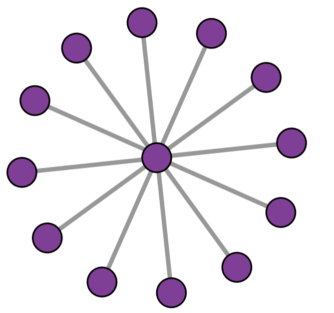

Figure 1: Star network with $N = 13$ nodes.

From this, we get

$$q_1 = \sum_{j=1}^{N-1} e_{1,j} = e_{1,N-1} = 1$$

Similarly, we reach $q_{N-1} = e_{1,N-1} = 1$, such that

$$e_{1,N-1} = 1 \cdot 1 = q_1 \cdot q_{N-1}$$

For $i \neq 1$ and $i \neq N-1$, however,

$$q_i = \sum_{j=1}^{N-1} e_{i,j} = \sum_{j=1}^{N-1} 0 = 0 = e_{i,k}\ , \text{ for all degrees } k$$

Lastly,

$$\begin{aligned} e_{1,j} &= 0 = 1 \cdot 0 = q_1 \cdot q_j\ , \text{ for } j \neq N-1 \\ e_{i,N-1} &= 0 = 0 \cdot 1 = q_i \cdot q_{N-1}\ , \text{ for } i \neq 1 \end{aligned}$$

Then, $e_{i,j} = q_i q_j$ for all degrees i, j.

2 Degree Correlation Coefficient

The Degree Correlation Coefficient r is equivalent to the Pearson correlation coefficient for network degrees and is given by

$$r = \sum_{jk} \frac{jk\left(e_{j,k} - q_j q_k\right)}{\sigma^2} = \frac{1}{\sigma^2} \sum_{jk} jk\left(e_{j,k} - q_j q_k\right) \tag{7.11}$$

Where σ^2 is comparable to the variance and is given by

$$\sigma^2 = \sum_k k^2 q_k - \left(\sum_k k q_k\right)^2 \tag{7.12}$$

2.1 Numerator

The numerator, here represented as C, is

$$\begin{aligned} C = \sigma^2 r &= \sum_{jk} jk \left(e_{j,k} - q_j q_k\right) \\ &= \sum_{jk} jk \cdot 0 \\ &= 0 \end{aligned}$$

2.2 Denominator

$$\begin{aligned} \sigma^2 &= \sum_k k^2 q_k - \left(\sum_k k q_k\right)^2 \\ &= q_1 + (N-1)^2 q_{N-1} - \left(q_1 + (N-1) q_{N-1}\right)^2 \\ &= (N-1)^2 - (N-1) \\ &= (N-2)(N-1) \end{aligned}$$

2.3 Result

Finally, we can compute the degree correlation coefficient as

$$r = \frac{C}{\sigma^2} = \frac{0}{(N-2)(N-1)} = 0$$

This indicates that star networks are neutral.