Quiz

Tiago de Paula Alves (187679)
tiagodepalves@gmail.com

July 15, 2022

For each question, give the answer and a short justification.

2021-013

Choose the option that contains the derivative of $f(x) = g(x)h(x)$, where $g(x) = x^3$ and $h(x) = (1 + \ln x)^2$.

A. $3x^2 + 2(1 + \ln x)/x$

B. $x^2(1 + \ln x)(5 + \ln x)$

C. $x^2(1 + \ln x)(5 + 3 \ln x)$

D. $x^2(1 + \ln x)(5 + 3 \ln x + 2x)$

E. None of the above

Original idea by: Maria Tejada Begazo

Answer: C

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x} f(x) &= \frac{\mathrm{d}g(x)}{\mathrm{d}x} h(x) + g(x) \frac{\mathrm{d}h(x)}{\mathrm{d}x} && \text{(by Product Rule)} \\
&= \frac{\mathrm{d}x^3}{\mathrm{d}x} (1 + \ln x)^2 + x^3 \frac{\mathrm{d}(1 + \ln x)^2}{\mathrm{d}x} \\
&= 3x^2 (1 + \ln x)^2 + x^3 \cdot 2(1 + \ln x) \frac{\mathrm{d}(1 + \ln x)}{\mathrm{d}x} && \text{(by Chain Rule)} \\
&= 3x^2 (1 + \ln x)^2 + 2x^3 (1 + \ln x) \frac{\mathrm{d}1}{\mathrm{d}x} + 2x^3 (1 + \ln x) \frac{\mathrm{d} \ln x}{\mathrm{d}x} \\
&= 3x^2 (1 + \ln x)^2 + 0 + 2x^3 (1 + \ln x) \frac{1}{x} \\
&= x^2 (1 + \ln x) \cdot (3(1 + \ln x) + 2) \\
&= x^2 (1 + \ln x)(5 + 3 \ln x)
\end{aligned}
$$

2021-018

Suppose that we want to build an **undirected network** with **200 nodes** and **600 links**. What will the *average degree* of this network be?

A. 5

B. 6

C. 7

D. 8

E. None of the above

Original idea by: Hismael Costa

Answer: B

$$\begin{aligned}
\langle k \rangle &= \frac{1}{|V|} \sum_{v \in V} \deg(v) \\
&= \frac{1}{N} \cdot 2|E| && \text{(by the Handshake Lemma)} \\
&= \frac{2L}{N} \\
&= \frac{2 \times 600}{200} \\
&= 6
\end{aligned}$$

2021-036

Consider a Random Network such that $N = 1000$ and $p = 0.004$. We need to break apart its giant component, and the only way to do that is by randomly removing nodes. Under these conditions, how many nodes should we remove at minimum to have a good chance of achieving our goal?

A. 750

B. 720

C. 840

D. 810

E. None of the above

Original idea by: José Nascimento

Answer: A

Using the Binomal distribution, we have

$$\langle k \rangle = p(N-1) = 3.996$$

And

$$\langle k^2 \rangle = p(1-p)(N-1) + p^2(N-1)^2 = 19.948$$

Therefore, the critical threshold as given by the Molloy-Reed criterion is

$$\begin{aligned} f_c &= 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1} \\ &= 1 - \frac{1}{\frac{19.948}{3.996} - 1} \\ &= 0.7495 \end{aligned}$$

Then the giant component should start to break apart after $f_c \cdot N = 749.5$ nodes are removed.

2021-062

Given two networks G1 and G2 generated by the Barabási-Albert model with $N = 1000$ and $N = 100$ nodes, respectively, find out which network likely has the smallest diameter. Also, give their expected diameters, rounded to two decimal places.

A. G1 has the smallest diameter, 6.29, while G2 has diameter 6.64

B. G2 has the smallest diameter, 3.02, while G1 has diameter 3.57

C. G1 has the smallest diameter, 3.22, while G2 has diameter 5.44

D. G2 has the smallest diameter, 5.31, while G1 has diameter 5.44

E. None of the above

Original idea by: Victor Antonio Menuzzo

Answer: B

For a Barabási-Albert network G with $m \geq 2$ there is a real number $\epsilon > 0$ such that the diameter $d_{\max}(G)$ of the network is bounded by

$$(1-\epsilon)\frac{\ln N}{\ln \ln N} \leq d_{\max}(G) \leq (1+\epsilon)\frac{\ln N}{\ln \ln N}$$

Béla Bollobás and Oliver Riordan. "The Diameter of a Scale-Free Random Graph". In: *Combinatorica* 24 (July 2004), pp. 5–34. DOI: `10.1007/s00493-004-0002-2`. URL: `https://www.math.cmu.edu/users/af1p/Teaching/INFONET/Papers/PowerLaw/swdiam.pdf`

Therefore, if we assume $m > 1$, we can approximate $d_{\max}(G) \approx \ln N / \ln \ln N$. Then

$$d_{\max}(\mathrm{G1}) \approx \frac{\ln N(\mathrm{G1})}{\ln \ln N(\mathrm{G1})} = \frac{\ln 1000}{\ln \ln 1000} = \frac{6.907}{1.933} = 3.574$$
$$d_{\max}(\mathrm{G2}) \approx \frac{\ln N(\mathrm{G2})}{\ln \ln N(\mathrm{G2})} = \frac{\ln 100}{\ln \ln 100} = \frac{4.605}{1.527} = 3.015$$

2022-082

Considering that the evolution of a random network could be classified into four topologically distinct regimes, analyze the information below about a random network with N=1000 during different stages of its life.

Stage I: p = 0.01

Stage II: p = 0.0036

Stage III: p = 0.001

Stage IV: p = 0.0007

Stage V: p = 0.007

Please, select the option that better represents the stage's topological regimes:

A. **Stage I:** Connected
Stage II: Critical
Stage III: Subcritical
Stage IV: Critical
Stage V: Connected

B. **Stage I:** Connected
Stage II: Connected
Stage III: Critical
Stage IV: Subcritical
Stage V: Connected

C. **Stage I:** Connected
Stage II: Critical
Stage III: Subcritical
Stage IV: Subcritical
Stage V: Supercritical

D. **Stage I:** Connected
Stage II: Supercritical
Stage III: Critical
Stage IV: Subcritical
Stage V: Connected

E. None of the above

Original idea by: Felipe Crispim da Rocha Salvagnini

Answer: D

Note that

$$\langle k \rangle = \frac{2\langle L \rangle}{N} = \frac{2 \cdot pL_{\max}}{N} = \frac{2p}{N} \cdot \frac{N(N-1)}{2} = p(N-1)$$

With that, we can fill the following table to analyze each stage:

Stage	p	$\langle k \rangle = 999p$	Comp. to 1	Comp. to $\ln N \approx 6.907$	Regime
I	0.01	9.99	$\langle k \rangle > 1$	$\langle k \rangle > 7 > \ln 1000$	Connected
II	0.0036	3.5964	$\langle k \rangle > 1$	$\ln 1000 > 6 > 3.5964$	Supercritical
III	0.001	0.999	$\langle k \rangle \approx 1$	—	Critical or Sub
IV	0.0007	0.6993	$1 > \langle k \rangle$	—	Subcritical
V	0.007	6.993	$\langle k \rangle > 1$	$\langle k \rangle \approx \ln 1000$	Super or Connected

Since Stage II can only be Supercritical the only viable option is D, which matches the other regimes.

2022-094

A cell has an initial amount of l_0 liters of water inside it. After one hour has passed, the amount of water is $3l_0/4$. If the rate of water usage for the cell's physiological functions is inversely proportional to the amount of water in the cell, find an equation that allows you to calculate the amount $L(t)$ of water in the cell at any point t in time.

Note: Always consider positive values. Consider time measured in hours, and volume in liters.

A. $L(t) = l_0\sqrt{16 - 7t}/4$

B. $L(t) = l_0(4 - t)/4$

C. $L(t) = l_0\sqrt{-7t^2/16 + 1}/4$

D. $L(t) = l_0(t^2 + 16)/4$

E. None of the above

Original Idea by Rómulo Condori

Answer: A

By the problem description,

$$\frac{\partial}{\partial t}L(t) \propto \frac{1}{L(t)}$$

That is, for some real k,

$$\begin{aligned} \frac{\partial L}{\partial t} &= k\frac{1}{L} \\ k &= L\frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}$$

If we integrate in t, we get

$$\begin{aligned} \int_0^t k\,\mathrm{d}t &= \int_0^t L\frac{\partial L}{\partial t}\,\mathrm{d}t \\ kt &= \int_{l_0}^{L(t)} L\,\mathrm{d}L \\ kt &= \left.\frac{L^2}{2}\right|_{l_0}^{L(t)} \\ kt &= \frac{L(t)^2 - l_0^2}{2} \end{aligned}$$

Such that

$$L(t) = \sqrt{2kt + l_0^2}$$

As specified, at $t = 1\mathrm{h}$, $L(1) = 3l_0/4$, therefore

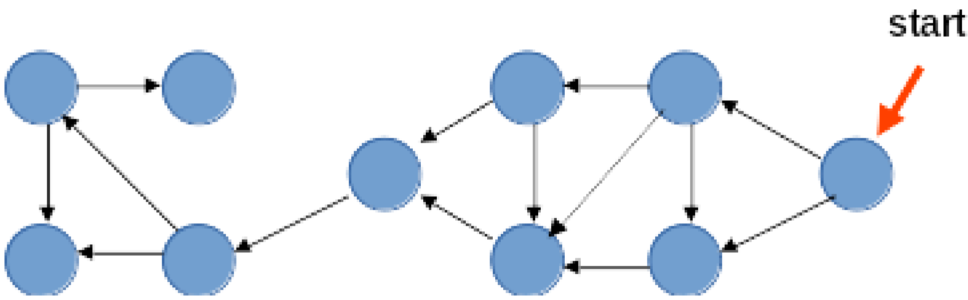
$$\begin{aligned} L(1) = \sqrt{2k\cdot 1 + l_0^2} &= \frac{3l_0}{4} \\ 2k + l_0^2 &= \frac{9l_0^2}{16} \\ 32k + 16l_0^2 &= 9l_0^2 \\ 32k + 7l_0^2 &= 0 \\ k &= -\frac{7}{32}l_0^2 \end{aligned}$$

Then

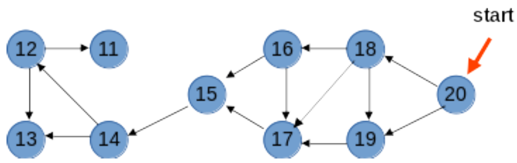
$$\begin{aligned} L(t) &= \sqrt{-\frac{2\cdot 7}{32}l_0^2 t + l_0^2} \\ &= l_0\sqrt{-\frac{7}{16}t + 1} \\ &= l_0\frac{\sqrt{-7t + 16}}{\sqrt{16}} \\ &= \frac{l_0}{4}\sqrt{16 - 7t} \end{aligned}$$

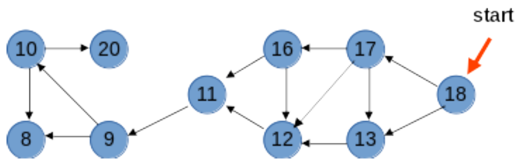
2022-119

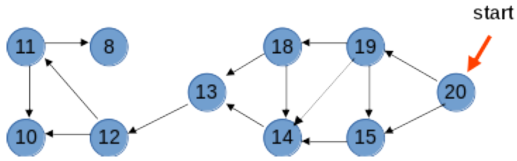
Starting from the node indicated as 'start', use DFS and label the nodes with their ending times. Which of the alternatives below corresponds to a possible answer?

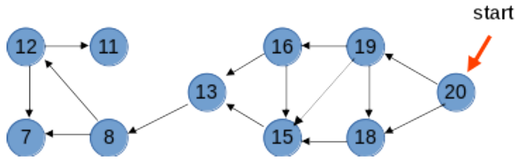


Tip: the reverse order of finishing DFS times in a directed acyclic graph is a topological order.

A. 

B. 

C. 

D. 

E. None of the above

Original ideia by: Filipe Maciel Roberto

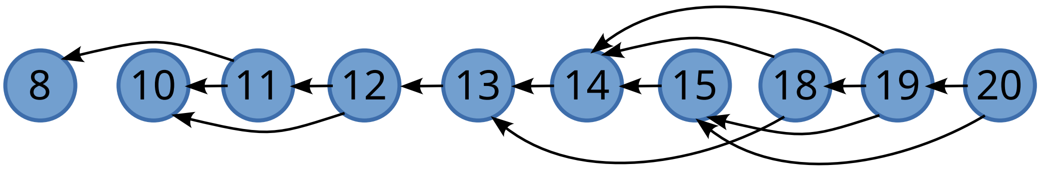
Answer: C

The given digraph is acyclic, therefore the finishing time must form a topological sorting of its vertices.

For **A.**, we have 18 as a successor to 19 in the topological order, but there's a back arc from 18 to 19, invalidating this order.

For **B.**, there's an arc from 10 to 20. In fact, the start node is the last to finish and must have the largest finishing time.

For **C.**, the topological order is valid:



For **D.**, the single invalid arc is from 8 to 12.

2022-130

Consider the following statements about degree correlation in networks:

I. In neutral networks, nodes link to each other randomly, which in turn results in a lack of degree correlation for the linking pattern.

II. A perfectly assortative network is always a complete graph.

III. The correlation exponent can help determine the type of the network. When the correlation exponent is positive, we may say the network is assortative.

IV. In assortative networks, nodes tend to link to nodes of similar degree. In other words, hubs tend to connect with hubs, and small-degree nodes tend to connect with small-degree nodes.

V. Degree correlations for directed network's are defined by two coefficients: $r_{\text{in,out}}$ and $r_{\text{out,in}}$.

Select the alternative that lists the correct statements:

A. I, II, and V are correct.

B. Only V is correct.

C. II, III, and IV are correct.

D. I, II, III, and IV are correct.

E. None of the above.

Original idea by: Heitor Mattosinho

Answer: E

I. True.

II. False. Any uniform graph is perfectly assortative, including cycles and complete bipartite graphs.

III. True.

IV. True.

V. False. There's $r_{\text{in,in}}$ and $r_{\text{out,out}}$ too.