Assignment 6
Computational Intelligence
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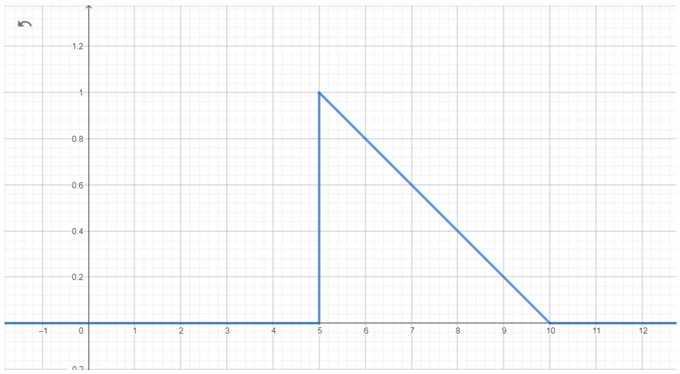
Contents

1 Question 1

One possible fuzzy function for this problem is as follows:

$$\mu(x) = \begin{cases} -0.2\,x + 2 & 5 \leq x \leq 10 \\ 0 & \text{O.W.} \end{cases} \quad , \quad [\mu]_\alpha = [5,\, 10 - 5\alpha] \quad , \quad 0 < \alpha \leq 1$$



2 Question 2

A t-norm must satisfy the following properties:

Part 1: $T(x, y)$

$(T1)$ $\quad t(\alpha, \beta) = t(\beta, \alpha) \qquad T(x, y) = T(y, x) \qquad \checkmark$

$(T2)$ $\quad t\,(t(\alpha, \beta), \gamma) = t\,(\alpha, t(\beta, \gamma))$

$$\text{if } \alpha = 0.55\ ,\ \beta = \gamma = 0.95 \Rightarrow \begin{cases} T(0.55, 0.95) = 0.5 \Rightarrow T(T(0.55, 0.95), 0.95) = T(0.5, 0.95) = 0.45 \\ T(0.95, 0.95) = 0.9 \Rightarrow T(0.55, T(0.95, 0.95)) = T(0.55, 0.9) = 0.5 \end{cases} \Rightarrow \times$$

The second constrain is not satisfied so $T(x, y)$ can not be a t-norm.

Part 2: $I(x, y)$

$(T1)$ $\quad t(\alpha, \beta) = t(\beta, \alpha) \qquad I(x, y) = I(y, x) \qquad \checkmark$

$(T2)$ $\quad t\,(t(\alpha, \beta), \gamma) = t\,(\alpha, t(\beta, \gamma)) \qquad \checkmark$

$$I(I(x, y), z) = \begin{cases} 0 & x, y \in (0, 0.5) \\ I(x, z) & x < y \\ I(y, z) & \text{O.W.} \end{cases} \quad = \quad I(x, I(y, z)) = \begin{cases} I(x, 0) = 0 & x, y \in (0, 0.5)\ ,\ z \in (0, 0.5) \\ I(x, 0) = 0 & x, y \in (0, 0.5)\ ,\ z = 0 \\ I(x, y) = 0 & x, y \in (0, 0.5)\ ,\ z \in (0, 0.5) \\ I(y, z) \quad x > y & \\ I(y, z) \quad \text{O.W.} & \end{cases}$$

$T(3)$ $\quad \beta \leq \gamma \Rightarrow t(\alpha, \beta) \leq t(\alpha, \gamma) \qquad \checkmark$

$$\text{if } x \leq y \Rightarrow \begin{cases} \text{if } x, y, z \in (0, 0.5) \Rightarrow I(x, z) = 0 = I(y, z) \\ \text{if } z = 0 \Rightarrow I(x, z) = 0 = I(y, z) \\ \text{if } z \geq 0.5\ ,\ x > z \Rightarrow \min(x, z) = z\ ,\ \min(y, z) = z \Rightarrow I(x, z) = I(y, z) \\ \text{if } z \geq 0.5\ ,\ x < z \Rightarrow \min(x, z) = x\ ,\ \min(y, z) = z or y \Rightarrow I(x, z) \geq I(y, z) \end{cases}$$

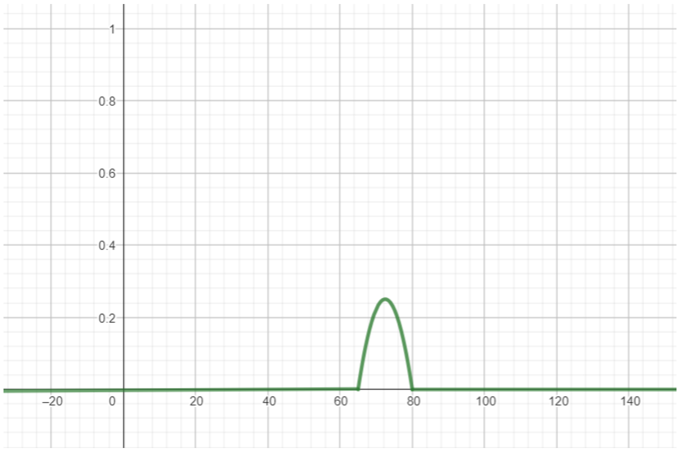
$T(4)$ $\quad I(x, 1) = \min(x, 1) = x \qquad \checkmark$

3 Question 3

$$\mu_h(x) = \begin{cases} \frac{1}{15}\,x - \frac{10}{3} & 50 \leq x \leq 65 \\ \\ \frac{-1}{15}\,x + \frac{16}{3} & 65 \leq x \leq 80 \\ \\ 0 & \text{O.W.} \end{cases} \quad , \quad \mu_{vh}(x) = \begin{cases} 0 & x < 65 \\ \\ \frac{1}{15}\,x - \frac{13}{3} & 65 \leq x \leq 80 \\ \\ 1 & x > 80 \end{cases}$$

a

$$t(\alpha,\beta) = \alpha.\beta \Rightarrow \mu_h \cap_t \mu_{vh} = \begin{cases} \left(\frac{-1}{15}\,x + \frac{16}{3}\right)\left(\frac{1}{15}\,x - \frac{13}{3}\right) = \frac{1}{225}(-x^2 + 145x - 5200) & 65 \leq x \leq 80 \\ \\ 0 & \text{O.W.} \end{cases}$$

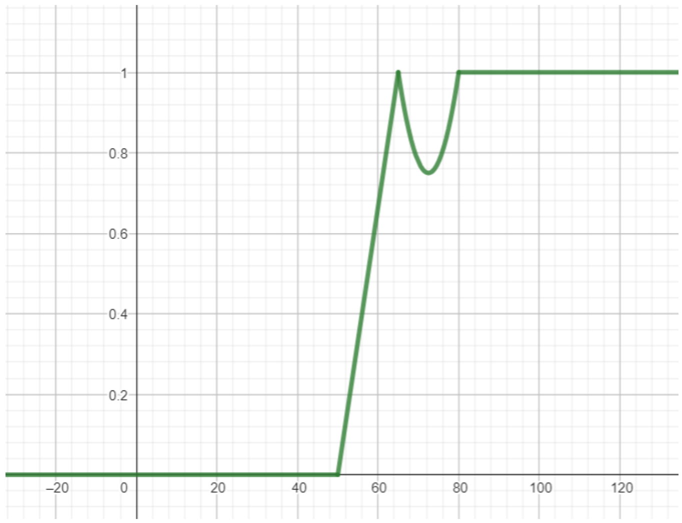


b

$$t(\alpha,\beta) = \begin{cases} 0 & \text{if } 1 \notin \{\alpha,\beta\} \\ \min(\alpha,\beta) & \text{O.W.} \end{cases} \qquad \Rightarrow \qquad \mu_h \cap_t \mu_{vh} = \begin{cases} \min(1,0) = 0 & x = 65 \\ \min(0,1) = 0 & x \geq 80 \\ 0 & \text{O.W.} \end{cases}$$

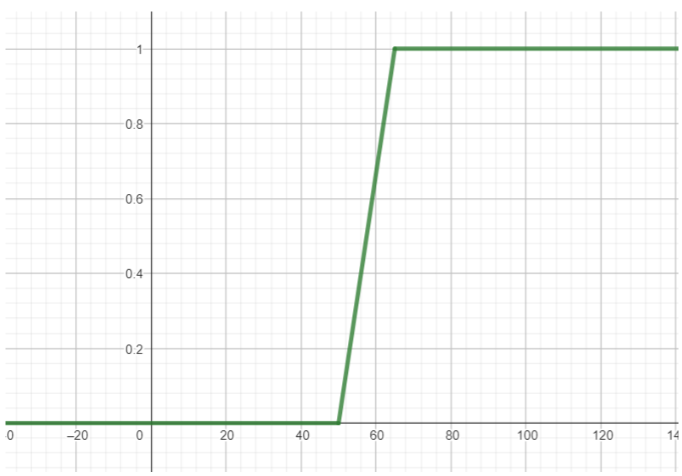
c

$$s(\alpha,\beta) = \alpha + \beta - \alpha\beta \qquad \Rightarrow \qquad \mu_h \cup_t \mu_{vh} = \begin{cases} 0 & x < 50 \\ \\ \frac{1}{15}\,x - \frac{10}{3} & 50 \leq x \leq 65 \\ \\ 1 - \frac{1}{225}(-x^2 + 145x - 5200) & 65 \leq x \leq 80 \\ \\ 1 & x > 80 \end{cases}$$



d

$$s(\alpha,\beta) = \begin{cases} 1 & 0 \notin \{\alpha,\beta\} \\ \max\{\alpha,\beta\} & \text{O.W.} \end{cases} \quad \Rightarrow \quad \mu_h \cup_t \mu_{vh} = \begin{cases} 0 & x < 50 \\ \frac{1}{15}x - \frac{10}{3} & 50 \leq x \leq 65 \\ 1 & x > 65 \end{cases}$$



4 Question 4

In order to be able to calculate the union and intersection of these fuzzy sets using the α-cuts, we have to use min and max as t-norm and t-conorm functions. Using the figure of the previous question, we have the followings:

$$[\mu_{vl} \cap \mu_l]_\alpha = [\mu_{vl}]_\alpha \cap [\mu_l]_\alpha \quad , \quad [\mu_{vl}]_\alpha = [0, 35 - 15\alpha] \quad , \quad [\mu_l]_\alpha = [15\alpha + 20, 50 - 15\alpha] \quad \Rightarrow$$

$$\text{if } \alpha > 0.5 \Rightarrow 15\alpha > 7.5 \Rightarrow \begin{cases} 35 - 15\alpha < 27.5 \\ 15\alpha + 20 > 27.5 \\ 50 - 15\alpha < 42.5 \end{cases} \Rightarrow [\mu_{vl}]_\alpha \cap [\mu_l]_\alpha = \emptyset$$

$$\text{if } \alpha \leq 0.5 \Rightarrow 15\alpha \leq 7.5 \Rightarrow \begin{cases} 35 - 15\alpha \geq 27.5 \\ 15\alpha + 20 \leq 27.5 \\ 50 - 15\alpha \geq 42.5 \end{cases} \Rightarrow [\mu_{vl}]_\alpha \cap [\mu_l]_\alpha = [15\alpha + 20,\ 35 - 15\alpha]$$

$$\Rightarrow [\mu_{vl} \cap \mu_l]_\alpha = \begin{cases} \emptyset & \alpha > 0.5 \\ [15\alpha + 20,\ 35 - 15\alpha] & \text{O.W.} \end{cases}$$

$$[\mu_{vl} \cup \mu_l]_\alpha = [\mu_{vl}]_\alpha \cup [\mu_l]_\alpha \quad , \quad [\mu_{vl}]_\alpha = [0, 35 - 15\alpha] \quad , \quad [\mu_l]_\alpha = [15\alpha + 20, 50 - 15\alpha] \quad \Rightarrow$$

$$\text{if } \alpha > 0.5 \Rightarrow 15\alpha > 7.5 \Rightarrow \begin{cases} 35 - 15\alpha < 27.5 \\ 15\alpha + 20 > 27.5 \\ 50 - 15\alpha < 42.5 \end{cases} \Rightarrow [\mu_{vl}]_\alpha \cup [\mu_l]_\alpha = [0, 35 - 15\alpha] \cup [15\alpha + 20, 50 - 15\alpha]$$

$$\text{if } \alpha \leq 0.5 \Rightarrow 15\alpha \leq 7.5 \Rightarrow \begin{cases} 35 - 15\alpha \geq 27.5 \\ 15\alpha + 20 \leq 27.5 \\ 50 - 15\alpha \geq 42.5 \end{cases} \Rightarrow [\mu_{vl}]_\alpha \cup [\mu_l]_\alpha = [0,\ 50 - 15\alpha]$$

$$\Rightarrow [\mu_{vl} \cup \mu_l]_\alpha = \begin{cases} [0, 35 - 15\alpha] \cup [15\alpha + 20, 50 - 15\alpha] & \alpha > 0.5 \\ [0,\ 50 - 15\alpha] & \text{O.W.} \end{cases}$$

5 Question 5

a

Drastic product & and drastic sum

$$s(\alpha,\beta)=\begin{cases}1 & 0\notin\{\alpha,\beta\}\\ \max\{\alpha,\beta\} & \text{O.W.}\end{cases}\quad,\quad t(\alpha,\beta)=\begin{cases}0 & \text{if } 1\notin\{\alpha,\beta\}\\ \min(\alpha,\beta) & \text{O.W.}\end{cases}$$

$\overleftrightarrow{t}(\mu_M(6),\mu_H(10))=t\left(\vec{t}(\mu_M(6),\mu_H(10))\,,\,\vec{t}(\mu_H(10),\mu_M(6))\right)$

$\vec{t}(\mu_M(6),\mu_H(10))=\sup\{\gamma\in[0,1]|t(\mu_M(6),\gamma)\le\mu_H(10)\}$

$\mu_M(6)=\dfrac{2}{3}\ ,\ \mu_H(10)=\dfrac{1}{3}\ ,\ t(\mu_M(6),\gamma)=\begin{cases}\frac{2}{3} & \gamma=1\\ 0 & \gamma\neq 1\end{cases}\quad\Rightarrow\quad\vec{t}(\mu_M(6),\mu_H(10))=1$

$\vec{t}(\mu_H(10),\mu_M(6))=\sup\left\{\gamma\in[0,1]|t(\dfrac{1}{3},\gamma)\le\dfrac{2}{3}\right\}\quad,\quad t(\dfrac{1}{3},\gamma)=\begin{cases}\frac{1}{3} & \gamma=1\\ 0 & \gamma\neq 1\end{cases}\quad\Rightarrow\quad\vec{t}(\mu_H(10),\mu_M(6))=1$

$\Rightarrow\quad\overleftrightarrow{t}(\mu_M(6),\mu_H(10))=t(1,1)=1$

Łukasiewicz

$t(\alpha,\beta)=\max\{\alpha+\beta-1,0\}\quad,\quad\vec{t}\{\alpha,\beta\}=\min\{1-\alpha+\beta,1.\}$

$\vec{t}(\mu_M(6),\mu_H(10))=\min\{1-\dfrac{2}{3}+\dfrac{1}{3}\,,\,1\}=\dfrac{2}{3}\quad,\quad\vec{t}(\mu_H(10),\mu_M(6))=\min\{1-\dfrac{1}{3}+\dfrac{2}{3}\,,\,1\}=1$

$$\overleftrightarrow{t}(\mu_M(6),\mu_H(10))=\begin{cases}\min\{1\,,\,\frac{2}{3}\}=\frac{2}{3}\\ \\ \max\{1+\frac{2}{3}-1\,,\,0\}=\frac{2}{3}\end{cases}$$

b

$[\![\forall x\ge 10\,;\,x\in\mu_H]\!]=\min\limits_{x\ge 10}[\![x\in\mu_H]\!]=\mu_H(10)=\dfrac{1}{3}$

c

Min & Max : $[\![5\in\mu_L\ ,\ 5\in\mu_H]\!]=\min\{\dfrac{1}{2}\,,\,\dfrac{3}{4}\}=\dfrac{1}{2}$

Łukasiewicz : $[\![5\in\mu_L\ ,\ 5\in\mu_H]\!]=\max\{\dfrac{1}{2}+\dfrac{3}{4}-1\,,\,0\}=\dfrac{1}{4}$

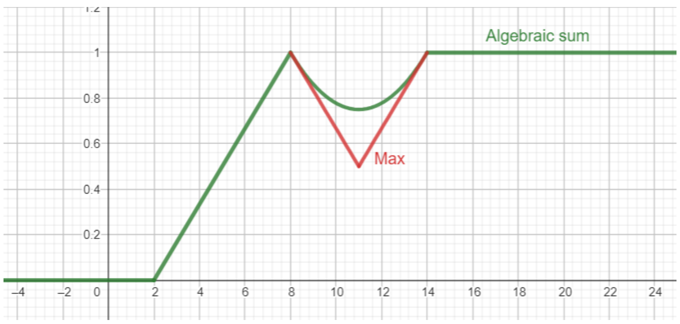
6 Question 6

a

$$\mu_2(x) = \begin{cases} \frac{1}{6}x - \frac{1}{3} & 2 \le x \le 8 \\ \frac{-1}{6}x + \frac{7}{3} & 8 \le x \le 14 \\ 0 & \text{O.W.} \end{cases} \quad , \quad \mu_3(x) = \begin{cases} \frac{1}{6}x - \frac{4}{3} & 8 \le x \le 14 \\ 1 & x \ge 14 \\ 0 & \text{O.W.} \end{cases}$$

$$\text{Max t-conorm}: \ \mu_2 \cup_t \mu_3 = \begin{cases} \frac{1}{6}x - \frac{1}{3} & 2 \le x \le 8 \\ \frac{-1}{6}x + \frac{7}{3} & 8 \le x \le 11 \\ \frac{1}{6}x - \frac{4}{3} & 11 \le x \le 14 \\ 1 & x \ge 14 \end{cases}$$

$$\text{algebraic sum t-conorm}: \ s(\alpha, \beta) = \alpha + \beta - \alpha\beta \quad \Rightarrow \quad \mu_2 \cup_t \mu_3 = \begin{cases} \frac{1}{6}x - \frac{1}{3} & 2 \le x \le 8 \\ 1 - \left(\frac{-1}{6}x + \frac{7}{3}\right)\left(\frac{1}{6}x - \frac{4}{3}\right) & 8 \le x \le 14 \\ 1 & x \ge 14 \end{cases}$$



b

$$\mu_1 \ : \ \frac{1}{2}x = \alpha \Rightarrow x = 2\alpha \quad , \quad \frac{-1}{4}x + 2 = \alpha \Rightarrow 8 - 4\alpha \quad \Rightarrow \quad [\mu_2]_\alpha = [2\alpha, 8 - 4\alpha]$$

$$\mu_2 \ : \ \frac{1}{6}x - \frac{1}{3} = \alpha \Rightarrow x = 6\alpha + 2 \quad , \quad \frac{-1}{6}x + \frac{7}{3} = \alpha \Rightarrow 14 - 6\alpha \quad \Rightarrow \quad [\mu_2]_\alpha = [6\alpha + 2, 14 - 6\alpha]$$

c

$$-\frac{1}{4}x + 2 = \frac{1}{6}x - \frac{1}{3} \Rightarrow x = 5.6 \quad \Rightarrow \quad \mu_1 \cap \mu_2 = \begin{cases} \frac{1}{6}x - \frac{1}{3} & 2 \le x \le 5.6 \\ 2 - \frac{1}{4}x & 5.6 \le x \le 8 \\ 0 & \text{O.W.} \end{cases}$$

d

$$[\mu_1 \cap \mu_2]_\alpha = [\mu_1]_\alpha \cap [\mu_2]_\alpha \xrightarrow{\alpha \in (0,1)} [\mu_1 \cap \mu_2]_\alpha = [6\alpha + 2, 8 - 4\alpha]$$

e

$$x = 6\alpha + 2 \Rightarrow \alpha = \frac{1}{6}x - \frac{1}{3} \quad , \quad x = 8 - 4\alpha \Rightarrow \alpha 2 - \frac{1}{4}x \quad , \quad 0 < \alpha \leq 0.6$$

Normal intersection of these α-cuts, is the α-cut of the fuzzy intersection of the sets. This is only true for minimum t-norm.

7 Question 7

a

Regarding the fact that the drastic sum is the biggest t-conorm and the max is the smallest one, the provided t-conorm must be included in this range.

$$\text{Drastic sum}: \ s_d(\alpha, \beta) = \begin{cases} 1 & 0 \notin \{\alpha, \beta\} \\ \max\{\alpha, \beta\} & \text{O.W.} \end{cases} \quad , \quad \text{Max}: \ s_m(\alpha, \beta) = \max \alpha, \beta$$

$$s_d(0.2, 0.3) = 1 \quad , \quad s_m(0.2, 0.3) = 0.3 \quad \Rightarrow \quad s_m < s_{new} < s_d \quad \checkmark$$

Also we note that this new t-conorm must satisfy the T_1, T_2, T_3, T_4' conditions.

b

$$s(\alpha, 0) = s(0, \alpha) = \alpha \quad , \quad \text{if } \beta \leq \gamma \Rightarrow s(\alpha, \beta) \leq s(\alpha, \gamma)$$

$s_{new}(0,0)$	$s_{new}(0.2,0)$	$s_{new}(0.3,0)$	$s_{new}(1,0)$	$s_{new}(0,0.2)$	$s_{new}(0.2,0.2)$	$s_{new}(0.3,0.2)$	$s_{new}(1,0.2)$
0	0.2	0.3	1	0.2	(0.2, 0.4)	0.4	1

$s_{new}(0,0.3)$	$s_{new}(0.2,0.3)$	$s_{new}(0.3,0.3)$	$s_{new}(1,0.3)$	$s_{new}(0,1)$	$s_{new}(0.2,1)$	$s_{new}(0.3,1)$	$s_{new}(1,1)$
0.3	0.4	(0.4, 1)	1	1	1	1	1

$$0.3 < A_1 < s_{new}(0.2, 1) \quad , \quad 0.4 < A_2 < 1 \quad , \quad s_{new}(0.3, 0.3) < A_3 < 1 \quad , \quad 0.2 < A_4 < 0.4$$
$$0.2 < A_5 < s_{new}(0.3, 0.3) \quad , \quad 0.4 < A_6 < 1 \quad , \quad 0 < A_7 < 0.4 \quad , \quad 0.2 < A_8 < 0.4 \quad , \quad 0.3 < A_9 < 1$$

c

$$t_{new}(\alpha, \beta) = 1 - s_{new}(1 - \alpha, 1 - \beta) \quad \Rightarrow \quad t_{new}(0, \alpha) = 0 \quad , \quad t_{new}(0.7, 0.8) = 1 - s_{new}(0.3, 0.2) = 1 - 0.4 = 0.6$$
$$t_{new}(0.8, 1) = 0.8 \quad , \quad t_{new}(0.7, 1) = 0.7 \quad , \quad t_{new}(\alpha, \beta) = t_{new}(\beta, \alpha)$$