Predicting German GDP Growth Rate by Using Machine Learning Algorithms

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Abstract

In this research, we work on generating a model that can predict accurately the GDP growth rate of Germany by using Machine Learning techniques. The independent variables are the time-varying correlation coefficients between the output gap of Germany and selected countries. By using Multiple Linear Regression, Best Subset Selection, and considering Mallow's Cp and R squared as the selection criteria, the best possible model is successfully generated, and it has ten independent variables. The produced model is accurate because it can forecast the sign of GDP growth rate of Germany correctly in all years of the test set and its predictions can cover an acceptable part of the actual GDP growth rate of Germany.

1 Introduction

Germany is the fourth-largest economy in the world. It is also the greatest economy in Europe. There is an idea about analyzing the German economy as the core of Europe. The idea says the story of the economic success of Germany can also be the economic success of other European countries. This idea wants to see European countries as a network that has an economic heart named Germany.

By having such a big picture, it is also crucial to analyze whether Germany's economy updates itself with European and non-European partners and counterparts such United States of America and China. To find the answer to this question, it is vital to research how the relationship between the economy of Germany and other countries can explain and forecast the economic behavior of Germany. To do such kinds of research it is needed to make meaningful data to answer the research problem because there is not any raw data which have the power to meet the needs of the research question. At the same time, the applications of Machine Learning Algorithms to predict the economic factors are highly used in economic research, and data scientists are working hard to implement different learning techniques to predict the macroeconomic factors such as inflation rate, unemployment rate, economic growth, and oil price. This research tries to build models which are generated by Machine Learning techniques to estimate the growth rate of Germany. During this process, time-varying correlation coefficients between the output gap of Germany and other countries are made and used as independent variables to estimate the GDP growth rate of Germany.

2 Literature Review

There is a large amount of research that highlights the characteristics of the German economy. This research by (Ringel, Schlomann, Krail, & Rohde, 2016) shows how Germany implemented accurate green energy policies to boost its economy.

The way that German companies encountered the digitalization trends has been discussed by (Neligan, 2018). The performance of the German economy during the financial crisis has been analyzed by (Dustmann, Fitzenberger, Schönberg, & Spitz-Oener, 2014). The research shows why the unemployment rate did not increase during the financial recession while other countries faced high unemployment rate.

The research by (Rinne & Zimmermann, 2013) analyzes the elements of labor policies in Germany and argues these policies should be seen as a model for other countries.

There are many research papers that analyze the similarity between business cycles of countries. The research by (Belke, Domnick, & Gros, 2017) provides a core and peripheral subgrouping of Business Cycle Synchronization in European countries.

The role of the European monetary union (EMU) to synchronize the business cycles of European countries has been studied by (Aguiar-Conraria, Martins, & Soares, 2013). The research about what countries in Europe have the leading role in business cycle synchronization has been done by (Soares et al., 2011).

At the same time, the application of Machine learning techniques in macroeconomic subjects has grabbed the attention of researchers. The research by (Patel, Shah, Thakkar, & Kotecha, 2015) demonstrates the application of Machine Learning Techniques such as Support Vector Regression, and Random Forest to analyze financial market indices in India. The future inflation rate in the US has been forecast by (Medeiros, Vasconcelos, Veiga, & Zilberman, 2021). In this research, Random Forest has better performance than other techniques. The role of nonlinear Machine learning techniques in predicting macroeconomic factors has been analyzed by (Coulombe, Leroux, Stevanovic, & Surprenant, 2020)

Predicting energy factors such as oil price is a field that highly using Machine learning Techniques. The research by (An, 2019) predicts the oil price by considering macroeconomic factors in the United States of America. The Support Vector Machine Algorithm in the research by (Shen, Jiang, & Zhang, 2012) is used to analyze global financial factors. Bagging and Stacked Denoising Autoencoders are used in the research by(Zhao, Li, & Yu, 2017) to predict the price of WTI oil price. The performance of the GA–NN algorithm (genetic and neural network algorithm) to predict the oil price has been researched by (Chiroma, Abdulkareem, & Herawan, 2015).

Machine learning techniques are also highly used to predict economic growth. The research by (Sokolov-Mladenović, Milovančević, Mladenović, & Alizamir, 2016) analyzing the application of using Back Propagation Learning algorithm and Extreme Learning machine to predict GDP growth rate.

3 Data Preparation

Because Machine Learning Models are extensively dependent on data, the research period must be long. According to our research, the longest period starts from 1970 since some countries such as Germany have missing values for years before 1970. At the same time, some European countries such as Poland, and Albania do not have real GDP data for some individual years from 1970 to 2018 and they have to also be removed from the model. Finally, the researcher reached to the data of GDP of 22 countries which are Germany, Austria, Belgium, China, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland United Kingdom, and the United States. The research did not limit itself to just Economic and Monetary Union, EMU, and widen its dimension by considering crucial economic partners and counterparts such as China and other European countries which are not members of EMU. In the next step of Data Preparation, instead of using the Real GDP of countries, the natural logarithm of data is used since they can lead to more accurate results.

Following this, it is important to eliminate the role of Business Cycles from the data of each country to reach more precise results. Among all the methods to detrend Macroeconomics time series data, the one which is highly used by economists is the Hodrick-Prescott filter.

3.1 Hodrick-Prescott filter

The Hodrick-Prescott filter decomposes a time series into a cyclical component and a trend component and chooses an appropriate number for a parameter which is named lambda. By doing the above-mentioned activities, the Hodrick-Prescott filter extracts the role of business cycles from the series. Because in this research the data is yearly, the Hodrick-Prescott filter chooses the number 100 for the parameter lambda to detrend the time series. As an output, the Hodrick-Prescott filter produces the Potential Gross Domestic Product for each year in each country. The Potential GDP is the amount of output that an economy can produce at a constant inflation rate. The Potential GDP is used to produce the output gap which is calculated by considering the difference between the natural log of GDP and the potential GDP for each country. The output gap for a country can be positive or negative. When the output gap is negative, it informs us that the performance of the economy is below its capacity while when it is positive it means that the economy is overperforming the expectations. The role of business cycles must be considered when the output gap is analyzed.

3.2 Time-varying correlation coefficients

At this step, the time-varying correlation coefficient between the output gap of each country and the output gap of Germany is calculated. For calculating the time-varying correlation coefficient, the researcher uses the rolling window correlation coefficient. The length of the window must be chosen as an external parameter, and it is chosen to be 10 years. The output of this step is the final independent variables that we need to run the models. Therefore, there are 21 independent variables which are the time-varying correlation coefficients between Germany and each country. During running the model, we call each independent variable with the name rollcountyname such as rolljapan.

The dependent variable is the "GDP growth rate of Germany" and it is important to consider that for each year the lag of order one of the independent variables tries to predict the dependent variable. For example, the data of independent variables in 2017 is used to predict the GDP Growth Rate of Germany in 2018.

4 Methodology

4.1 Multiple Linear Regression

The learning algorithm in this empirical research uses Multiple linear Regression to generate the model which can accurately predict the dependent variable. In the area of Machine Learning, researchers widely use Multiple Linear regression because they are easy to implement, and the results can be interpreted easily. At the same time, it must be considered that no model is always perfect. The multiple linear regression in some situations is prone to face overfitting, and it is the situation when the generated model has a good performance in the training set while its performance in the test set is not good.

$$\mathrm{Y} = \alpha + \beta_1 \mathrm{X}_1 + \beta_2 \mathrm{X}_2 + \beta_3 \mathrm{X}_3 + ... + \beta_{n-1} \mathrm{X}_{n-1} + \beta_n \mathrm{X}_n + \epsilon$$

4.2 Best Subsets Selection

The method which is used to select the best independent variables to predict the growth rate of Germany is Best Subsets Selection. This Learning Algorithm is based on huge computations and has different stages. Its process is based on selecting the following models: Firstly, the best individual feature that can accurately predict the German Growth Rate. Secondly, the best two features that can accurately predict the German Growth Rate Thirdly, the best three features that can accurately predict the German Growth Rate. This process continues until choosing all the features. Therefore, the algorithm considers 2^n combinations of features, where n is the total numebr of independent variables. According to some criteria the algorithm chooses the best combinations of features, or independent variables, among all the selected combinations. It is vital to check whether it is possible to reduce the number of independent variables when it can also explain more precisely the behavior of German GDP Growth. In this research, we choose two criteria which are R squared and Mallow's Cp criteria. According to R squared, the best model is the one that has the largest R squared.

4.2.1 Mallow's Cp Criterion

Researchers try to reach models which have small Mallows' Cp. When the figure of Mallow's Cp is low, it shows the model is approximately precise to predict accurately the coefficients of the regression model and forecast the dependent variable, when it is high it indicates that the generated model is suffering from not fitting the correct coefficients.

4.3 Dividing the Data to Test Set and Train Set

According to the methodology of Machine learning algorithms when there is an explanatory variable and the goal is to generate a model which can accurately forecast the explanatory variable, it is important to divide the whole dataset into Train Set and Test Set. This division is vital because the training set is used by the algorithm to generate an appropriate model. Then the model is tested on the test set to check how is the accuracy of the model which was produced by the algorithm. From the theoretical viewpoint, to have a model with acceptable accuracy, it is crucial to allocate a large part of the whole dataset to the training set.

4.3.1 Different Training Set Sizes

One of the goals of this research is to check the role of the size of the training set on the accuracy of the generated model. There is a long training set which is from 1980 to 2015, the models which are produced according to this training set are called long-term models, and a short training set which is from 1995-2015, the related models are called short term models. For both training sets, the partner test set is from 2015-2018.

5 Results

This part analyses the result of the models which are produced by using long-term and short-term training sets. For each of these two groups, we also choose the best selected models according to R squared and Mallow's Cp criteria.

5.1 Selected Models According to Long-term Training Set

The Learning Algorithm produces two different models which are the best combinations of independent variables According to R squared and Mallow's Cp Criteria.

5.1.1 According to Mallow's Cp Criterion

The learning Algorithm generates the best model according to Mallow's Cp criterion which has 10 independent variables. The countries are rollchina, rollfrance, rollgreece, rolliceland, rollireland, rolljapan, rollluxembourg, rollsweden, rollswitzerland, and rollunitedkingdom. From a statistical viewpoint, the p-values of the estimated coefficients are highly low and the model has R squared which is 79.96 percent, and the adjusted R squared is 71.95 percent. The time-varying correlation coefficients of the United Kingdom and Switzerland have the largest positive estimated coefficients. The statistical figures of generated model are in the appendix part.

The Multiple Linear Regression model, which is produced by the learning algorithm, according to Mallow's Cp Criterion predicts that the GDP Growth rate of Germany in 2016 will be 1.4, which can cover 57 percent of the real rate of that year (table 1). In 2017, the model predicts that the dependent variable will be 1.7 percent which can explain 73 percent of the real rate, which was 2.3 percent. In the final year, the generated model forecasts that the dependent variable will be 2.4 percent while it was 1.3 percent. When it comes to the performance of the generated model, it has no mistakes in predicting the sign of the independent variable. In the first two years, the model predictions are underestimate while in the third year it is overestimating.

Year	Real Growth Rate of GDP of Germany in percent	Estimated Growth Rate by Algorithm in percent	The Proportion of Coverage in percent
2016	2.4	1.4	57
2017	2.3	1.7	73
2018	1.3	2.4	188

Table 1: Long-term by CP criterion.

5.1.2 According to R squared Criterion:

It chooses 13 time-varying correlation coefficients as the best combination of independent variables, which can predict the German GDP growth rate. The independent variables are: rollaustria, rolldenmark, rollfrance, rolliceland, rollireland, rollitaly, rolljapan, rollluxembourg, rollnorway, rollspain, rollswitzerland, rollunited-kingdom, and rollunitedstates. The R squared of the generated model is 84.7 percent and the Adjusted R squared is 75.76 percent. The estimated coefficients for the time-varying correlation coefficients of countries such as Denmark, Japan, Switzerland have a positive relationship with the German GDP growth rate while the ones for countries such as Austria, Iceland, and Luxemburg, and Spain have a negative relationship with the Growth Rate of Germany. The estimated coefficient of the time-varying of the United Kingdom by 11.7 percent is the largest among the ones of all the other independent varaibles.

when the produced model by this criterion is used to forecast the dependent variable from 2016 to 2018, through the test set, it is seen that the generated model is successful in predicting the direction of the movement of the dependent variable(table 2). In other words, the model can predict the signs, being positive or negative, of the dependent variable successfully without any mistakes. The predicted GDP growth rate is 3.9 percent for 2016 while it was 2.4 percent in reality. Through the next two following years, the predicted German Growth rate is 4.2 percent and 3.8 percent respectively while actual rates were 2.3 and 1.3, respectively. As it seems the generated model has an overestimated performance in predicting the dependent variable.

Year	Real Growth Rate of GDP of Germany in percent	Estimated Growth Rate by Algorithm in percent	The Proportion of Coverage in percent
2016	2.4	3.9	165
2017	2.3	4.2	180
2018	1.3	3.8	280

Table 2: Long-term by R squared criterion.

5.2 Selected Models According to Short-term Training Set

5.2.1 According to Mallow's Cp Criterion:

When the learning algorithm produces the model according to Mallow's Cp Criterion by using the short-term training set, it generates the model which has just one independent variable. The independent variable is the time-varying correlation coefficient for the country Luxembourg, and its estimated coefficient is around -2.5 percent. The model is not precise because both the R-squared and Adjusted R-squared are too low which are 15.3 percent and 10.88 percent, respectively. The statistical figures of generated model are in the appendix part.

Even though the model is not accurate from the Statistical viewpoint, its performance on the test set is almost acceptable (table 3). It can predict the sign of the dependent variable correctly. In 2015, its prediction covers 68 percent of the actual figure of the dependent variable. The next year, it predicts that the Growth rate of GDP of Germany will be 1.3 percent which is around 56 percent of the actual German GDP growth rate. The coverage of the predicted dependent variable in the last year is almost 60 percent of the actual number. It is important to consider that the model for all the years of test set is suffering from underestimating.

Year	Real Growth Rate of GDP of Germany in percent	Estimated Growth Rate by Algorithm in percent	The Proportion of Coverage in percent
2016	2.4	1.6	68
2017	2.3	1.3	56
2018	1.3	0.7	61

Table 3: Short-term by CP criterion.

5.2.2 According to R squared Criterion:

The best model that is generated according to the R squared criterion by using the short-term training set, which is from 1995-2015, has 19 independent variables. The R squared of the model is 1 and the adjusted

R squared of the model is 99.98 percent. The p-values of the estimated coefficients are acceptable from a statistical viewpoint. The statistical figures of generated model are in the appendix part.

While the performance of the model in the training set is almost perfect, its performance in the test set is not acceptable (table4). It cannot even predict the sign of the growth rate of Germany. For all three years, the estimated figures for dependent variable are highly negative, which shows that the model is unrealistic.

Year	Real Growth Rate of GDP of Germany in percent	Estimated Growth Rate by Algorithm in percent
2016	2.4	-18.5
2017	2.3	-34
2018	1.3	-70

Table 4: Short-term by CP criterion.

6 Conclusion

The results of the models which are produced by long-term training set are much more accurate from the statistical and predictive viewpoints. At the same time, it seems the model, which is produced by Mallow's Cp Criterion, has better results than the one which is generated by R squared criterion. The reason is that it is using fewer independent variables and its level of coverage of the actual GDP growth is more precise.

Moreover, the research demonstrates the possibility of using time-varying correlation coefficients between a country and a network of other countries to predict macroeconomic factors such as the growth rate of GDP.

7 Discussion

Machine learning techniques are dependent on the dataset. There is a traditional viewpoint that these algorithms are just for big data, even though recent research is showing the correctness of algorithms in small datasets. This research proves the application of Machine Learning techniques on a small dataset.

8 Recommendations

There are many tasks that can be done in the future. The researchers can enlarge the dataset by using quarterly, monthly, weekly, or even daily data.

It is possible to add the data of other non-European counties that import goods and services from Germany or the countries that provide raw materials like oil or gas for the country.

Using other Machine Learning techniques such as Random Forest, Support Vector Machine, Decision Tree, and comparing their results.

It is also possible to do the same research for other European countries such as France and compare the results with the ones of Germany.

Researchers can use other macroeconomic factors such as inflation rate, unemployment rate, export to import ratio as the dependent variable and do the same research.

If the researchers have access to the larger dataset, they can also tune the ratio of training and test set to check the results, even though in all situations the proportion of the training set must be larger than the test set.

References

Aguiar-Conraria, L., Martins, M. M., & Soares, M. J. (2013). Convergence of the economic sentiment cycles in the eurozone: A time-frequency analysis. *JCMS: Journal of Common Market Studies*, *51*(3), 377–398.

An, J. (2019). Oil price predictors: Machine learning approach. *670216917*.

Belke, A., Domnick, C., & Gros, D. (2017). Business cycle synchronization in the emu: Core vs. periphery. *Open Economies Review*, *28*(5), 863–892.

Chiroma, H., Abdulkareem, S., & Herawan, T. (2015). Evolutionary neural network model for west texas intermediate crude oil price prediction. *Applied Energy*, *142*, 266–273.

Coulombe, P. G., Leroux, M., Stevanovic, D., & Surprenant, S. (2020). How is machine learning useful for macroeconomic forecasting? *arXiv preprint arXiv:2008.12477*.

Dustmann, C., Fitzenberger, B., Schönberg, U., & Spitz-Oener, A. (2014). From sick man of europe to economic superstar: Germany's resurgent economy. *Journal of Economic Perspectives*, *28*(1), 167–88.

Medeiros, M. C., Vasconcelos, G. F., Veiga, Á., & Zilberman, E. (2021). Forecasting inflation in a data-rich environment: the benefits of machine learning methods. *Journal of Business & Economic Statistics*, *39*(1), 98–119.

Neligan, A. (2018). Digitalisation as enabler towards a sustainable circular economy in germany. *Intereconomics*, *53*(2), 101–106.

Patel, J., Shah, S., Thakkar, P., & Kotecha, K. (2015). Predicting stock market index using fusion of machine learning techniques. *Expert Systems with Applications*, *42*(4), 2162–2172.

Ringel, M., Schlomann, B., Krail, M., & Rohde, C. (2016). Towards a green economy in germany? the role of energy efficiency policies. *Applied energy*, *179*, 1293–1303.

Rinne, U., & Zimmermann, K. F. (2013). Is germany the north star of labor market policy? *IMF Economic Review*, *61*(4), 702–729.

Shen, S., Jiang, H., & Zhang, T. (2012). Stock market forecasting using machine learning algorithms. *Department of Electrical Engineering, Stanford University, Stanford, CA*, 1–5.

Soares, M. J., et al. (2011). Business cycle synchronization and the euro: A wavelet analysis. *Journal of Macroeconomics*, *33*(3), 477–489.

Sokolov-Mladenović, S., Milovančević, M., Mladenović, I., & Alizamir, M. (2016). Economic growth forecasting by artificial neural network with extreme learning machine based on trade, import and export parameters. *Computers in Human Behavior*, *65*, 43–45.

Zhao, Y., Li, J., & Yu, L. (2017). A deep learning ensemble approach for crude oil price forecasting. *Energy Economics*, *66*, 9–16.

9 Appendix

Listing 1: The best subset model by using long-term data according to CP criterion

```
Call:
lm(formula = growth_rate.of.Germany ~ roll_china + roll_france +
    roll_greece + roll_iceland + roll_ireland + roll_japan +
    roll_luxembourg + roll_sweden + roll_switzerland + roll_united_kingdom,
    data = total)

Residuals:
      Min        1Q    Median        3Q       Max
-0.025683 -0.003730  0.001368  0.005899  0.013543

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)          0.053809   0.023327   2.307 0.029640 *
roll_china          -0.027929   0.014831  -1.883 0.071360 .
roll_france         -0.171232   0.035343  -4.845 5.57e-05 ***
roll_greece         -0.040250   0.010942  -3.678 0.001126 **
roll_iceland        -0.067142   0.018898  -3.553 0.001546 **
roll_ireland         0.041670   0.013295   3.134 0.004364 **
roll_japan           0.053949   0.012503   4.315 0.000220 ***
roll_luxembourg     -0.052233   0.007068  -7.390 9.66e-08 ***
roll_sweden         -0.074027   0.017604  -4.205 0.000292 ***
roll_switzerland     0.136109   0.026161   5.203 2.21e-05 ***
roll_united_kingdom  0.134660   0.022537   5.975 3.07e-06 ***
---

Residual standard error: 0.01064 on 25 degrees of freedom
Multiple R-squared:  0.7996,    Adjusted R-squared:  0.7195
F-statistic: 9.978 on 10 and 25 DF,  p-value: 1.722e-06
```

Listing 2: The best subset model by using long-term data according to R sqaured criterion

```
Call:
lm(formula = growth_rate.of.Germany ~ roll_austria + roll_denmark +
    roll_france + roll_iceland + roll_ireland + roll_italy +
    roll_japan + roll_luxembourg + roll_norway + roll_spain +
    roll_switzerland + roll_united_kingdom + roll_united_states,
    data = total)

Residuals:
      Min        1Q    Median        3Q       Max
-0.022041 -0.003760  0.001586  0.003581  0.014124

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)          0.148553   0.043988   3.377 0.002715 **
roll_austria        -0.114708   0.046757  -2.453 0.022548 *
roll_denmark         0.100465   0.022960   4.376 0.000241 ***
roll_france         -0.177358   0.048483  -3.658 0.001383 **
roll_iceland        -0.111701   0.020310  -5.500 1.58e-05 ***
roll_ireland         0.100061   0.020588   4.860 7.40e-05 ***
roll_italy           0.046374   0.031068   1.493 0.149729
roll_japan           0.046528   0.010541   4.414 0.000219 ***
roll_luxembourg     -0.050346   0.006575  -7.658 1.21e-07 ***
roll_norway         -0.055200   0.015923  -3.467 0.002192 **
roll_spain          -0.043941   0.016954  -2.592 0.016643 *
roll_switzerland     0.063742   0.013127   4.856 7.48e-05 ***
roll_united_kingdom  0.117142   0.025841   4.533 0.000164 ***
roll_united_states  -0.094403   0.032703  -2.887 0.008564 **
---
Residual standard error: 0.009889 on 22 degrees of freedom
Multiple R-squared:  0.8476,    Adjusted R-squared:  0.7576
F-statistic: 9.415 on 13 and 22 DF,  p-value: 3.354e-06
```

Listing 3: The best subset model by using short-term data according to CP criterion

```
Call:
lm(formula = growth_rate.of.Germany ~ roll_luxembourg, data = total)

Residuals:
      Min        1Q    Median        3Q       Max
-0.067973 -0.007079  0.000503  0.013294  0.026004

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      0.017126   0.004863   3.521  0.00228 **
roll_luxembourg -0.024220   0.013055  -1.855  0.07914 .
---


Residual standard error: 0.02 on 19 degrees of freedom
Multiple R-squared:  0.1534,    Adjusted R-squared:  0.1088
F-statistic: 3.442 on 1 and 19 DF,  p-value: 0.07914
```

Listing 4: The best subset model by using short-term data according to R sqaured criterion

```
Call:
lm(formula = growth_rate.of.Germany ~ roll_austria + roll_belgium +
    roll_china + roll_denmark + roll_finland + roll_france +
    roll_greece + roll_iceland + roll_ireland + roll_italy +
    roll_japan + roll_luxembourg + roll_netherlands + roll_norway +
    roll_portugal + roll_spain + roll_switzerland + roll_united_kingdom +
    roll_united_states, data = total)

Residuals:
         1          2          3          4          5          6
7          8          9
 7.597e-05 -7.537e-05 -2.245e-05  4.236e-05 -1.148e-05  5.417e-06 -3.291e-05
2.889e-05 -3.453e-06
        10         11         12         13         14         15
16         17         18
 1.444e-05  1.836e-05 -1.300e-04  1.540e-04 -8.305e-05  1.888e-05
8.176e-05 -8.856e-06 -9.767e-05
        19         20         21
 6.794e-05  2.154e-05 -6.431e-05

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)          1.785033   0.041712  42.794  0.01487 *
roll_austria        -0.196564   0.012315 -15.961  0.03983 *
roll_belgium         5.660692   0.092065  61.486  0.01035 *
roll_china          -0.639833   0.013561 -47.181  0.01349 *
roll_denmark        -6.964218   0.104754 -66.482  0.00958 **
roll_finland         1.188333   0.018935  62.760  0.01014 *
roll_france         -8.689070   0.143750 -60.446  0.01053 *
roll_greece         -2.163344   0.030215 -71.598  0.00889 **
roll_iceland        -0.842214   0.010414 -80.874  0.00787 **
roll_ireland        -0.808993   0.016740 -48.328  0.01317 *
roll_italy           0.087677   0.009352   9.376  0.06765 .
roll_japan          -0.723319   0.011843 -61.077  0.01042 *
roll_luxembourg      0.681529   0.012553  54.293  0.01172 *
roll_netherlands    -4.214651   0.075492 -55.829  0.01140 *
roll_norway          4.504972   0.072928  61.773  0.01030 *
roll_portugal        1.838243   0.033554  54.785  0.01162 *
roll_spain           3.073278   0.043237  71.080  0.00896 **
roll_switzerland     3.580043   0.051709  69.235  0.00919 **
roll_united_kingdom  0.896762   0.018377  48.798  0.01304 *
roll_united_states   2.784044   0.039037  71.317  0.00893 **
---

Residual standard error: 0.0002996 on 1 degrees of freedom
Multiple R-squared:      1,     Adjusted R-squared:  0.9998
F-statistic:  5262 on 19 and 1 DF,  p-value: 0.01085
```