Principle of Inclusion and Exclusion

$N(\bar{C_1}\bar{C_2}) = N - [N(C_1) + N(C_2)] + N(C_1C_2)$

$N(\bar{C_1}\bar{C_2}\bar{C_3}) = N - [N(C_1) + N(C_2) + N(C_3)]$
$+ [N(C_1C_2) + N(C_1C_3) + N(C_2C_3)] - N(C_1C_2C_3)$

Q. Determine the number of +ve integers between 1 and 100 (included) which are not divisible by 2, 3 and 5.

Here $S = \{1, 2, 3 \ldots 100\}$, $1 \le n \le 100$

$\therefore N = 100$

C_1 : If n is divisible by 2

C_2 : If n is divisible by 3

C_3 : If n is divisible by 5

$N(C_1) = \frac{100}{2} = 50$, $N(C_2) = \frac{100}{3} = 33$

$N(C_3) = \frac{100}{5} = 20$, $N(C_1C_2) = \frac{100}{2\times3} = 16$

$N(C_1C_3) = \frac{100}{2\times5} = 10$, $N(C_2C_3) = \frac{100}{3\times5} = 6$

$N(C_1C_2C_3) = \frac{100}{2\times3\times5} = 3$

So number of integers not divisible by 2, 3 and 5 $= N(\bar{C_1}\bar{C_2}\bar{C_3})$

$= N - [N(C_1) + N(C_2) + N(C_3)]$
$+ [N(C_1C_2) + N(C_1C_3) + N(C_2C_3)] - N(C_1C_2C_3)$
$= 100 - (50+33+20) + (16+10-6) - 3 = \underline{\underline{26}}$

Example 2.7.3

Determine the number of positive integrals $1 \leq n \leq 10000$ where n is not divisible by 5, 6, 8.

Solution:

Here $S = \{1,2,3\ldots.10000\}$ N=10000, For $n \in S$, n satisfies the conditions

C_1: if n is divisible by 5, C_2: if n is divisible by 6, C_3: if n is divisible by 8.

$N(C_1)$ =10000/5 = 2000, $N(C_2)$ =10000/6 = 1666, $N(C_3)$ =10000/8 = 1250,

$N(C_1C_2)$ =10000/30 = 333, $N(C_1C_3)$ =10000/40 = 250, $N(C_2C_3)$ =10000/24 = 416,

$N(C_1C_2C_3)$ =10000/120 =83.

$N(\overline{C}_1\overline{C}_2\overline{C}_3) = S_0 - S_1 + S_2 - S_3 + S_4 = 10000 - (2000 + 1666 + 1250) + (333 + 250 + 416) - 83 = 6000.$

Example 2.7.4

Determine the number of positive integrals $1 \leq n \leq 500$ where n is not divisible by 2, 3, 4, 6, 8, 10.

Solution:

We consider the divisors 2,3,5 since 6,8,10 are the multiples of 2,3,5.

Here $S = \{1,2,3\ldots.500\}$ N=500 , For $n \in S$, n satisfies the conditions

C_1: if n is divisible by 2, C_2: if n is divisible by 3, C_3: if n is divisible by 5.

$N(C_1)$ =500/2 = 250, $N(C_2)$ =500/3 = 166, $N(C_3)$ =500/5 = 100,

$N(C_1C_2)$ =500/6 = 83, $N(C_1C_3)$ =500/10 = 50, $N(C_2C_3)$ =500/15 = 33,

$N(C_1C_2C_3)$ =500/30 =16.

$N(\overline{C}_1\overline{C}_2\overline{C}_3) = S_0 - S_1 + S_2 - S_3 + S_4 = 500 - (250 + 166 + 1000) + (83 + 50 + 33) - 16 = 134.$