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#

Q. Symbolize "x is the father of the mother of y"

$P(x, y)$: x is the father of y

$Q(x, y)$: x is the mother of y.

$\therefore$ So $\exists z\,(Q(z, y) \wedge P(x, z))$

ie, there exists some person z, who is the mother of y and x is father of z.

Homework: Symbolize using quantifiers and connectives when

the domain consists of all students in your class.

(a) A student in your class has a cat a, dog and a ferret.

(b) All students in your class have a cat a dog or a ferret

(c) Some student in your class has a cat and a ferret but not a dog.

(d) No student in your class has a cat, a dog & a ferret.

Let $C(x)$: student x has a cat.

$D(x)$: student x has a dog.

$F(x)$: student x has a ferret.

(a) $\exists x\,(C(x) \wedge D(x) \wedge F(x))$

(b) $\forall x\,(C(x) \vee D(x) \vee F(x))$

(c) $\exists x\,(C(x) \wedge F(x) \wedge \sim D(x))$

(d) $\sim \exists(x)\,(C(x) \wedge F(x) \wedge D(x))$

#Q: All roses are beautiful.

$R(x)$: x is a rose

$B(x)$: x is beautiful

Hence $\forall x\,(R(x) \rightarrow B(x))$.

If x is a rose then it is beautiful.

Q. Translate the following statements into logical expressions using quantifiers and logical connectives. The domain is all things.

(i) Something is not correct place.

Let $K(x)$: x is in correct place

$\forall x \; \exists \lnot K(x)$.

(ii) Everything is in the correct place and in excellent condition

$E(x)$: x is in excellent condition.

$\forall x \, [K(x) \land E(x)]$.

Nothing is in the correct place and in excellent condition.

$\forall x \, [\sim K(x) \land \sim E(x)]$.

Q. If $L(x,y)$ symbolises the statement "x loves y" when the domain consisting of all people then symbolise

(i) Everybody loves z $\forall x \, L(x,z)$

(ii) Everybody loves somebody $\forall x \exists y \, L(x,y)$

Q. Express the statement "If a person is female and is a parent, then this person is someone's mother" as a logical expression.

$p(x)$: x is female

$q(x)$: x is a parent

$R(x,y)$: x is mother of y.

$\forall \, [p(x) \land q(x)] \rightarrow \exists y \, R(x,y)$

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i) $\forall x\, [p(x) \Rightarrow q(x)]$

ii) $\exists x\, [q(x) \Rightarrow p(x)]$

iii) $\forall x\, [q(x) \rightarrow p(x)]$

iv) $\exists x\, [r(x) \rightarrow p(x)]$

$x^2 - 8x + 15 = 0$

$x^2 - 3x - 5x + 15 = 0$

$x(x-3) - 5(x-3) = 0$

$(x-3)(x-5) = 0$

$x = 3, 5$

(i) $p(x) : x = 3,5$

$q(x) : x$ is odd

$\forall\, [p(x) \rightarrow q(x)]$ is true.

(ii) $q(x)$: x is odd

$p(x)$: $x = 3,5$

$\exists x\, [q(x) \rightarrow p(x)] \rightarrow$ true

as 3,5 is odd and exist.

(iii) $\forall x\, [q(x) \rightarrow p(x)] \Rightarrow$ false.

Not all odd number satisfy $p(x)$

(iv) $r(x)$: $x>0$

$r(x)$: $x = 3,5$

$\exists x\, [r(x) \rightarrow p(x)]$.

$\rightarrow$ true.

3 and 5 are true and exist.

Q. Determine the truth value of the following statement if the universe comprises all non-zero integers.

(i) $\exists x \exists y\, [xy = 2] \Rightarrow$ true.

$x = 1,\ y = 2,\ xy = 2$

at least one such value exist.

true

(ii) $\exists x \forall y\, [xy = 2] \Rightarrow$ false. there does not exist a value for x for all values of y eg: $y = 3$.

$\Rightarrow xy = 2 \Rightarrow 3x = 2$

$x = 2/3$ which is not an integer

For the universe of all integers, check the truth values of : $\forall x [p(x) \to \sim r(x)]$.

$x^2 - 8x - 8 = 0$.

$x^2 - 7x + 10 = 0$

$x^2 - 5x - 2x + 10 = 0$

$x(x-2) - 5(x-2) = 0$

$(x-2) - (x-5) = 0$

$x = 2, 5$.

$x = -1, 3$

$r(x) : x < 0$. Here the allowable

$\sim r(x) : x \geq 0$ value for x is from the set of integers.

(i) $\forall x [p(x) \to \sim r(x)] \Rightarrow$ true.

$p(x)$ means $x = 2, 5$

$\sim r(x) : x \geq 0$: x is non negative.

(ii) $\forall x [q(x) \to r(x)]$.

$q(x)$ means $x = -1, 3$

$r(x)$ means $x < 0$ but here $x = 3$ is one value which is +ve

$\forall x [q(x) \to r(x)]$ is false.

(iii) $\exists x (q(x) \to r(x)]$ $\to$ true.

At least one value exists $\Rightarrow x = -1$

its true.

Q. $p(x) : x^2 - 8x + 15 = 0$

$q(x) : x$ is odd

$r(x) : x > 0$

For the universe of all integers, determine the truth values of.

1)

ii)

(iii)

(iv)

(ii) $q(x$

$p(x$

(iii) $\forall x$

(iv) $r($

$r($

Q: Det

if

exist

For the universe of all integers, check the truth values of : $\forall x [p(x) \to \sim r(x)]$.

$x^2 - 7x + 10 = 0$ $\qquad$ $x^2 - 2x - 3 = 0$.

$x^2 - 5x - 2x + 10 = 0$

$x(x-2) - 5(x-2) = 0$

$(x-2) - (x-5) = 0$

$\underline{x = 2, 5}$ $\qquad$ $x = -1, 3$

$r(x) : x < 0$ $\qquad$ Here the allowable

$\sim r(x) : x \geq 0$ $\qquad$ value for x is from the set of integers.

(i) $\forall x [p(x) \to \sim r(x)] \Rightarrow$ true.

$p(x)$ means $x = 2, 5$

$\sim r(x) : x \geq 0$: x is non negative.

(ii) $\forall x [q(x) \to r(x)]$

$q(x)$ means $x = -1, 3$

$r(x)$ means $x < 0$ but here $x = 3$ is one value which is true

$\forall x [q(x) \to r(x)]$ is false.

(iii) $\exists x (q(x) \to r(x)] \to$ true.

At least one value exists $\Rightarrow x = 7$

It true.

Q: $p(x) : x^2 - 8x + 15 = 0$

$q(x) : x$ is odd

$r(x) : x > 0$ $\qquad$ For the universe of all integers, determine the truth values of.

i)

ii)

(iii)

(iv)

(ii) $q($

$p($

(iii) $\forall x$

(iv) $r($

$r($

Q: Dete

if

exist

$q(x)$: $x+2$ is odd
$r(x)$: $x \geq 0$.

write the truth value for

1) $q(1)$: $1+2$ is odd $\Rightarrow$ True.

2) $\sim p(3)$
$p(3)$: $3 \leq 4 \rightarrow$ True.
$\sim p(3)$ is false.

3) $p(7) \vee q(8)$
$7 \leq 4$ or $8+2$ is odd
false or false
$\Rightarrow$ false.

4) $p(7) \vee \sim q(8)$
false $\vee$ true $\Rightarrow$ true.

5) Consider the statements.
$p(x)$: $x > 0$
$q(x)$: $x^2 - 3x - 4 = 0$.
Test whether $\exists x\, [p(x) \wedge q(x)$ is true or false]
$(x-4)(x+1) = 0$
$x = 4, -1$
when $x = 4$, $p(x) \wedge q(x)$ is true.
ie $\exists x\, [p\, x \wedge q(x)]$ is true.

6) Let $p(x)$ and $r(x)$ be following statements
$p(x)$: $x^2 - 7x + 10 = 0$.
$q(x)$: $x^2 - 2x - 3 = 0$
$r(x)$: $x < 0$.

predicate calculus

Quantifiers.

open statements — the truth value to the statement changes according to the value of the variable x such statements are called open statements.

'$x+1$ is odd'
$P(2)$ is True.
$P(1)$ is false

Thus a declarative statement is an open statement if.

i) It contains one of more variables.
(ii) Becomes a statement, when the variable in the statement are replaced by certain allowable choices.

eg: Let p: '$x+1$ is odd'
$P(2)$ true.

Predicate calculus.

UNIVERSAL QUANTIFIERS [for all x: $\forall x$]
The phrases used for universal quantifiers are: 'for all x' 'for any x', 'for any x' 'for each x'

EXISTENTIAL QUANTIFIERS [$\exists x$].
The phrases used are 'for some x' there exists x'

Q: Let ($p(x)$) and $q(x)$ denote the following open statement.
$p(x)$: $x \leq 4$.

Q. If milk is white, then cow is not black

p : milk is white.

q : cow is black.

$p \to q$ $\quad \sim(p \to q) \Leftrightarrow p \land \sim q$

Milk is white and cow is not black.

Q. Paris is in France and London is in england

p : paris is in France.

q : London is in England

$(p \land q)$

$\sim(p \land q) \Leftrightarrow \sim p \lor \sim q$.

⇒ paris is not in France or London is not in england.

⇒ Paris is not in France or London is not in England

Q. Norma is doing her homework and Karan is practicing his piano.

p : Norma is doing her homework

q : Karan is practicing his

$(p \land q)$

$\sim(p \land q) = \sim p \lor \sim q$.

Norma is not doing her homework or Karan is not practicing his piano.

Q. He swims if and only if the water is warm

p : He swims.

q : The water is warm.

$p \leftrightarrow q$ $\quad \sim(p \leftrightarrow q) \Leftrightarrow \sim p \to q \Leftrightarrow p \leftrightarrow \sim q$

If he swims, then water is not warm.

Inverse. $\sim p \to \sim q$
If Its not a sunny summer day, I dont go to the beach.

Contrapositive.
$\sim q \to \sim p$.
If I havent gone to the beach its not a sunny summer day.

Q. If it snows today, I will ski tommorow
p: It snows today.
q: I will ski tommorow.
$p \to q$

converse
$q \to p$ If I will ski tomorrow then it will snow today.

Inverse.
$\sim p \to \sim q$
If it does not snow today, then, I will not ski tomorrow.
contrapositive $\sim p \to \sim q$.
If I havent gone to the beach it is not a sunny summer day.

negation of statement.
$\sim(\sim p) \Leftrightarrow p$
$\sim(p \to q) \Leftrightarrow p \wedge \sim q$
$\sim(p \leftrightarrow q) \Leftrightarrow \sim p q \Leftrightarrow p \to \sim q$

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Maths Miss

- If "$p \to q$", then "$q \to p$".
- $\sim p \to \sim q$ is the inverse
- $\sim q \to \sim p$ is the Contrapositive

Q. If p, q and r write converse, Inverse and contrapositive of $p \to (q \cap r)$.

Converse
$(q \cap r) \to p$

Contrapositive
$\sim q \vee \sim r \to \sim p$.

Inverse
$\sim p \to \sim (q \cap r)$

Q. If $a < b$ then $1 > 2$
Let $p: a < 6$ and $q: 1 > 2$
Given $p \to q$
Converse $q \to p$
i.e.; If $1 > 2$ then $a < 6$.
Inverse is $\sim p \to \sim q$
i.e. If $a \geq 6$ then $1 \leq 2$

Q. "I go to the Beach whenever it's a sunny summer day".
p: It's a sunny summer day
q: I go the beach
$p \to q$

Converse $q \to p$
If I go to the ~~Be~~ beach then it's a sunny summer day.