

$$\text{Error} = \sum (y - \hat{y})^2$$

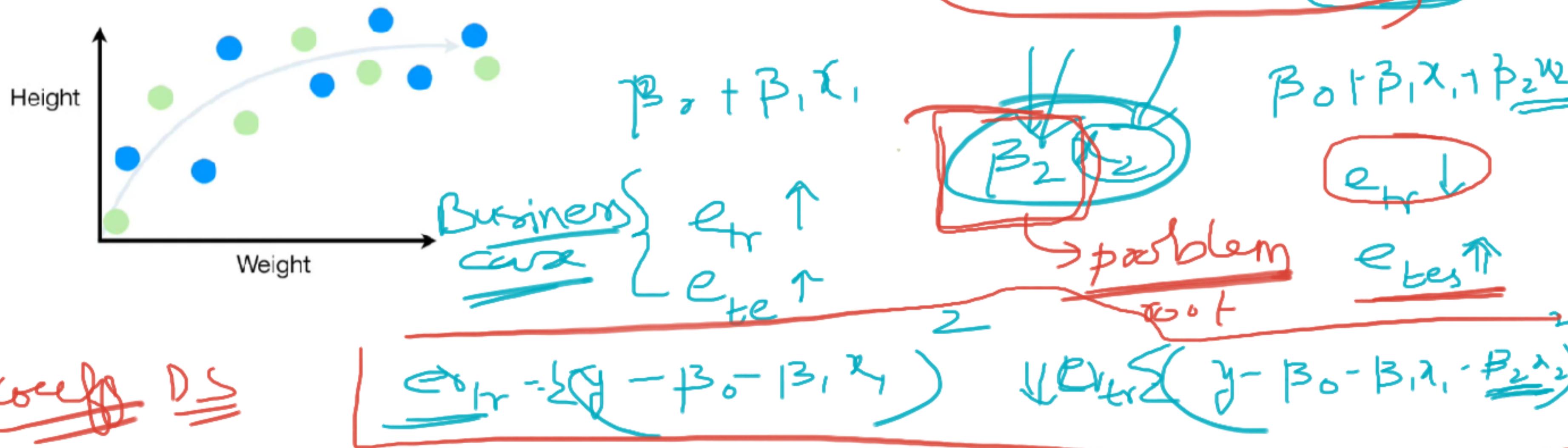
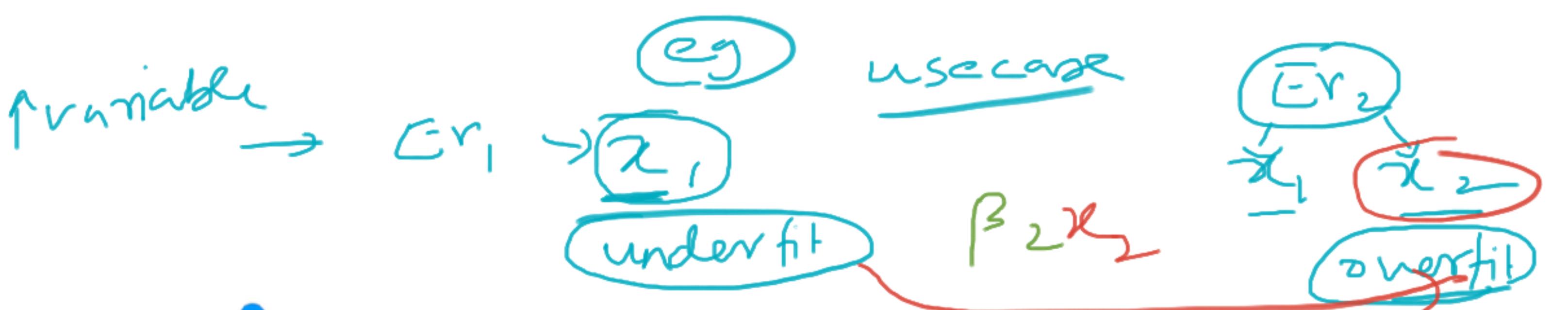
$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$\text{Error}_1 = \sum (y - \beta_0 - \beta_1 x_1)^2$$

$$\downarrow \underline{\text{Error}_2 < \text{Error}_1}$$

$$\text{Error}_2 = \sum (y - (\beta_0 + \beta_1 x_1 + \beta_2 x_2))^2$$

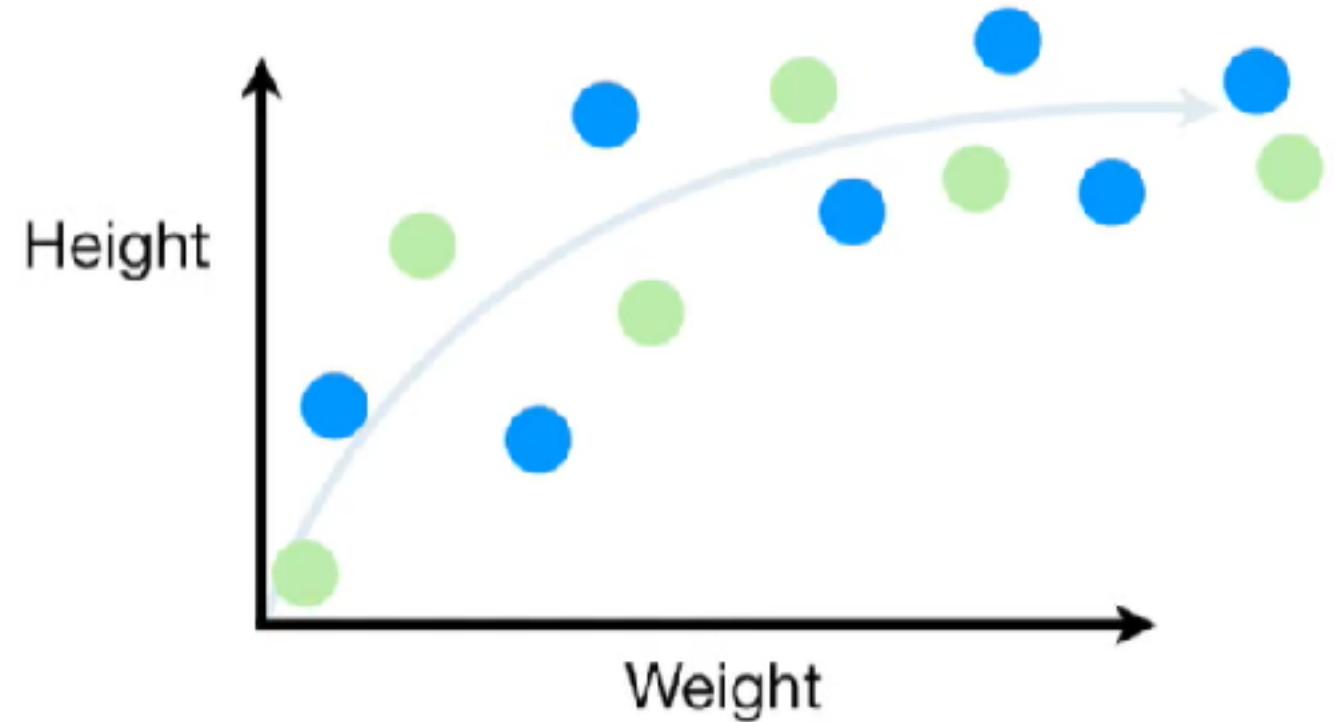
equ<sup>n</sup>



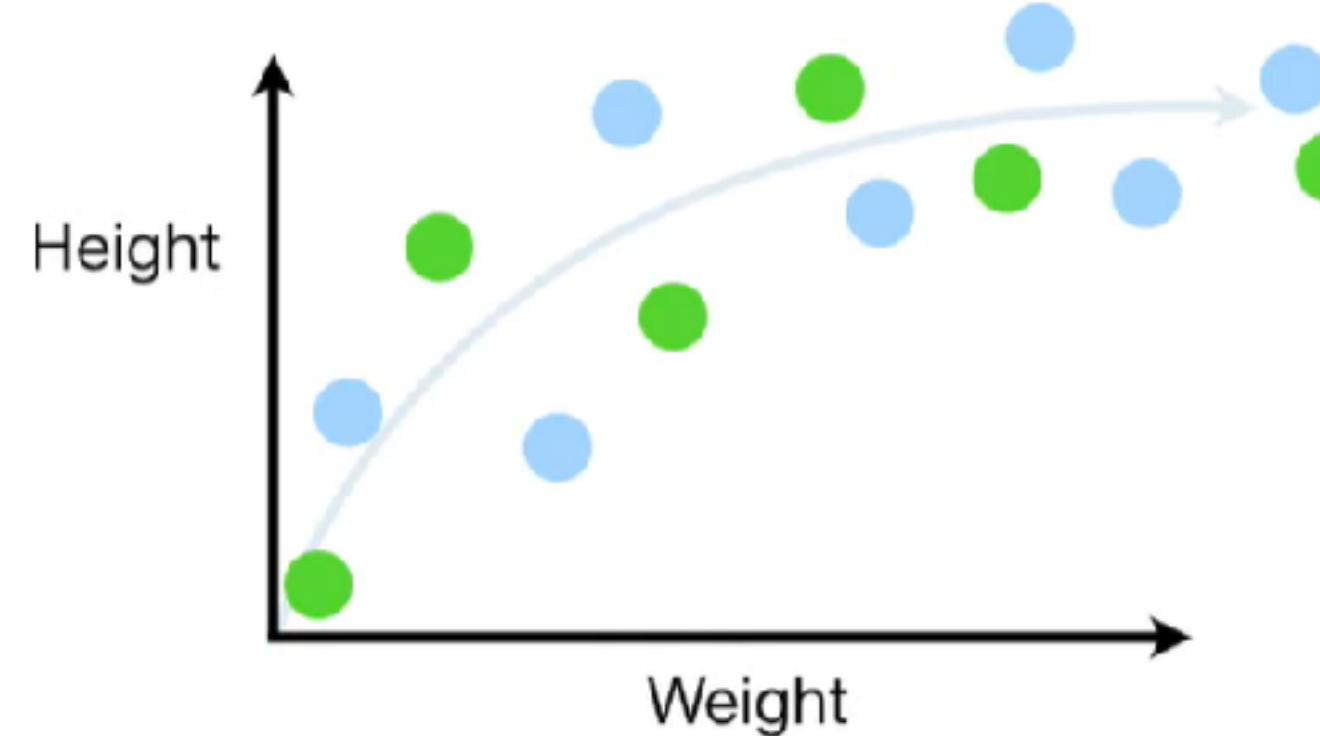
dataset  $\beta_0 + \beta_1 x_1 + \beta_2 x_2$

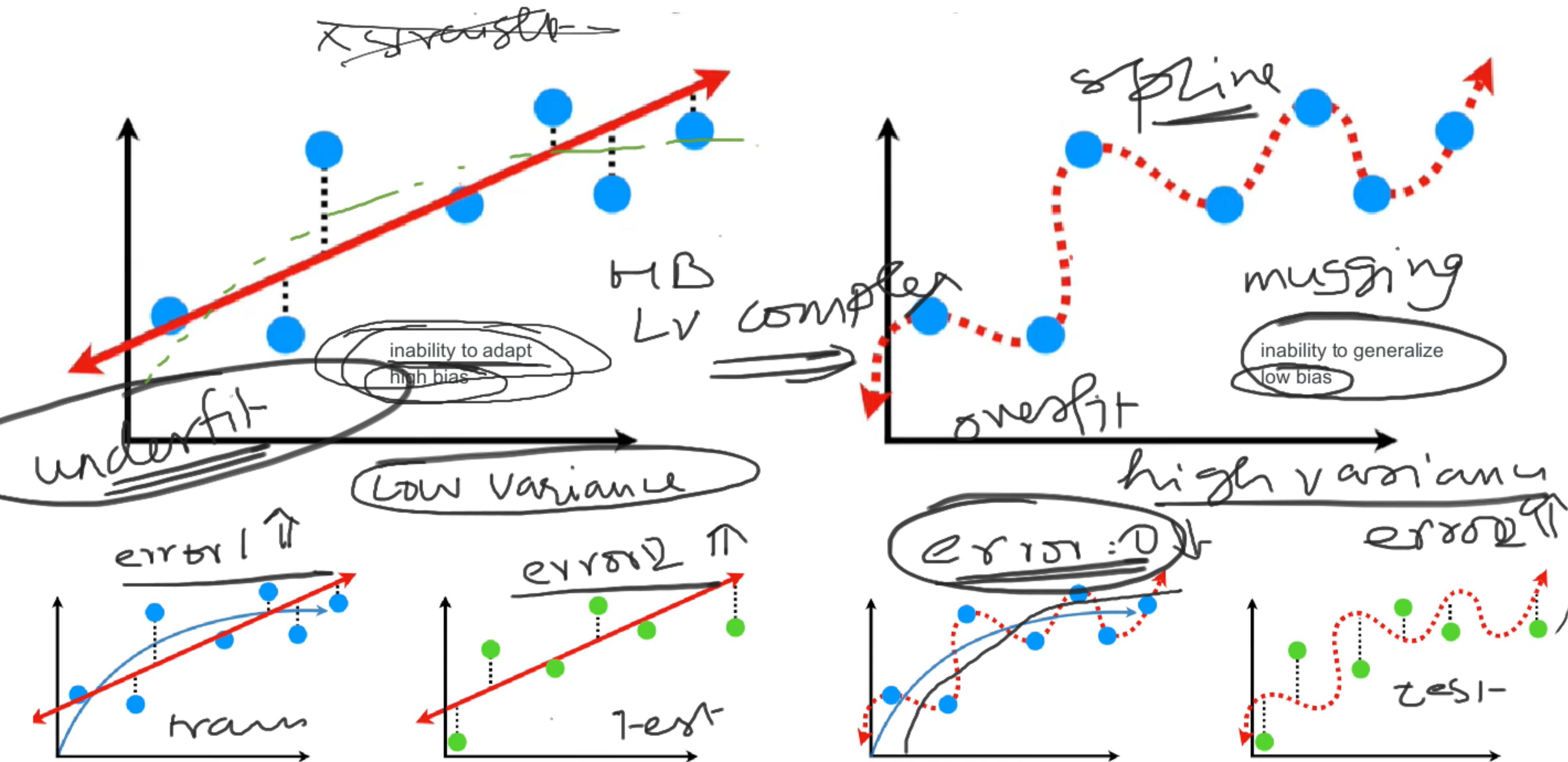
mean  $\leftarrow 0.5 x_2 \rightarrow 0.5$

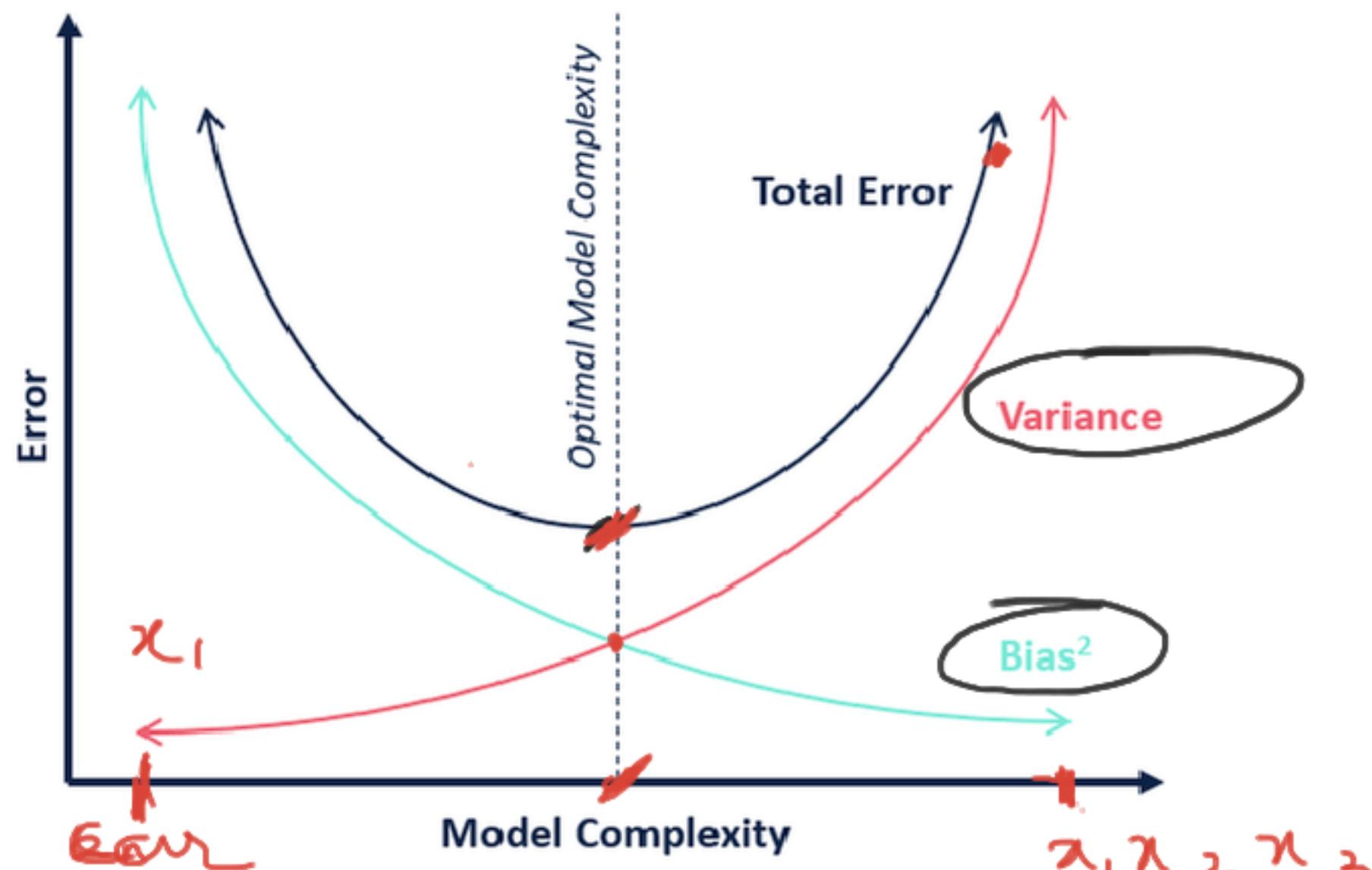
The **Blue Dots** are the **training set**...



...and the **Green Dots** are the **testing set**.







DLS  $\rightarrow$

GD  $\rightarrow$

$$\begin{pmatrix} \beta_i \\ \beta_0 \end{pmatrix}$$

$$\begin{pmatrix} \beta_1 \\ \beta_0 \end{pmatrix}$$

$$\begin{pmatrix} \beta_2 \\ \beta_1 \\ \beta_0 \end{pmatrix}$$

$$TE = \text{Bias}^2 + \text{Var} + \text{Irreducible error}$$

$$\hat{y} = \underbrace{\beta_0 + \beta_1 x_1 + \beta_2 x_2}_{\text{Bias}}$$

Bias Variance

Remedy  $\rightarrow$  Penalties

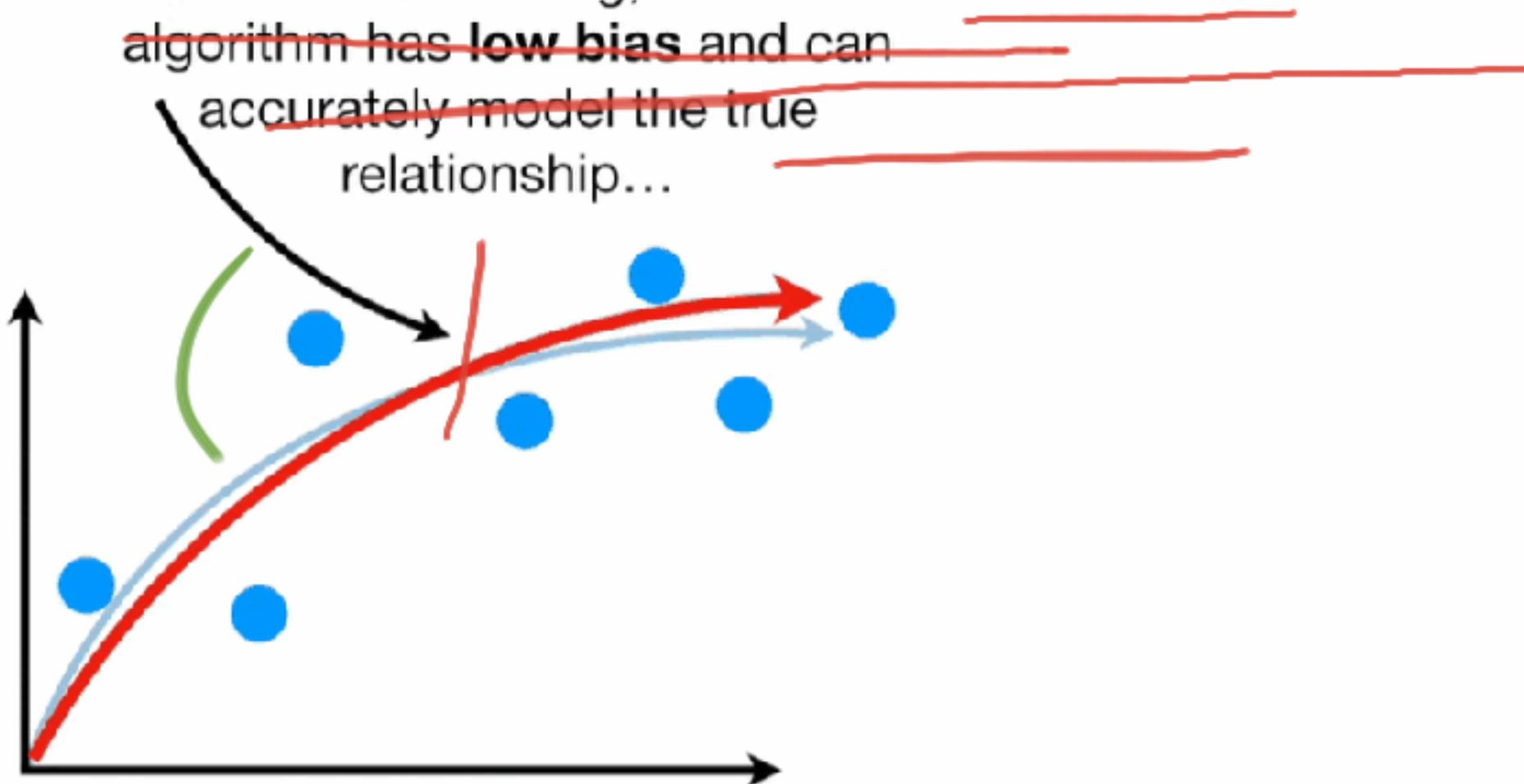
Penalties

$$\beta_0 \rightarrow \text{hiccups}$$

overfit

## ~~b~~ Penalize

In machine learning, the ideal algorithm has **low bias** and can accurately model the true relationship...



Three commonly used methods for finding the sweet spot between simple and complicated models are:

**regularization, boosting and bagging.**

class  
Decision

$$RSS = \sum (y - \hat{y})^2 + \lambda f(\beta_0)$$

$$\text{err} = \sum (y - \beta_0 - \beta_1 x_1 - \beta_2 x_2)^2 + \lambda (\beta_1 + \beta_2)$$

$$\beta_i^* = \beta_i + \alpha \frac{\partial e}{\partial \beta_i}$$

quick  
 $\beta_0^2$   
 $\frac{\partial e}{\partial \beta_0}$   
 $\beta_1^2$   
 $\frac{\partial e}{\partial \beta_1}$   
 $\beta_2^2$   
 $\frac{\partial e}{\partial \beta_2}$   
 $\frac{\partial e}{\partial \beta_1} + \frac{\partial e}{\partial \beta_2}$   
 $\frac{2\lambda \beta_0}{\beta_0}$

train  
 $\text{err} = \sum (y - \beta_0 - \beta_1 x_1 - \beta_2 x_2)^2 + \lambda (\beta_1 + \beta_2)$

$\downarrow \beta_2$   
 $5x_2$   
 $D.S. x_2$   
 $\rightarrow \text{underfit} \rightarrow \text{overfit}$

$n$   
↓ error

$\beta_0$

$$\sum (y - \beta_0 - \beta_1 x_1 - \beta_2 x_2)^2$$

$f(\beta_0)$

underfit

$\beta_2$

overfit

$\beta_2$   
term variation

$x''$

confidence

error

intro

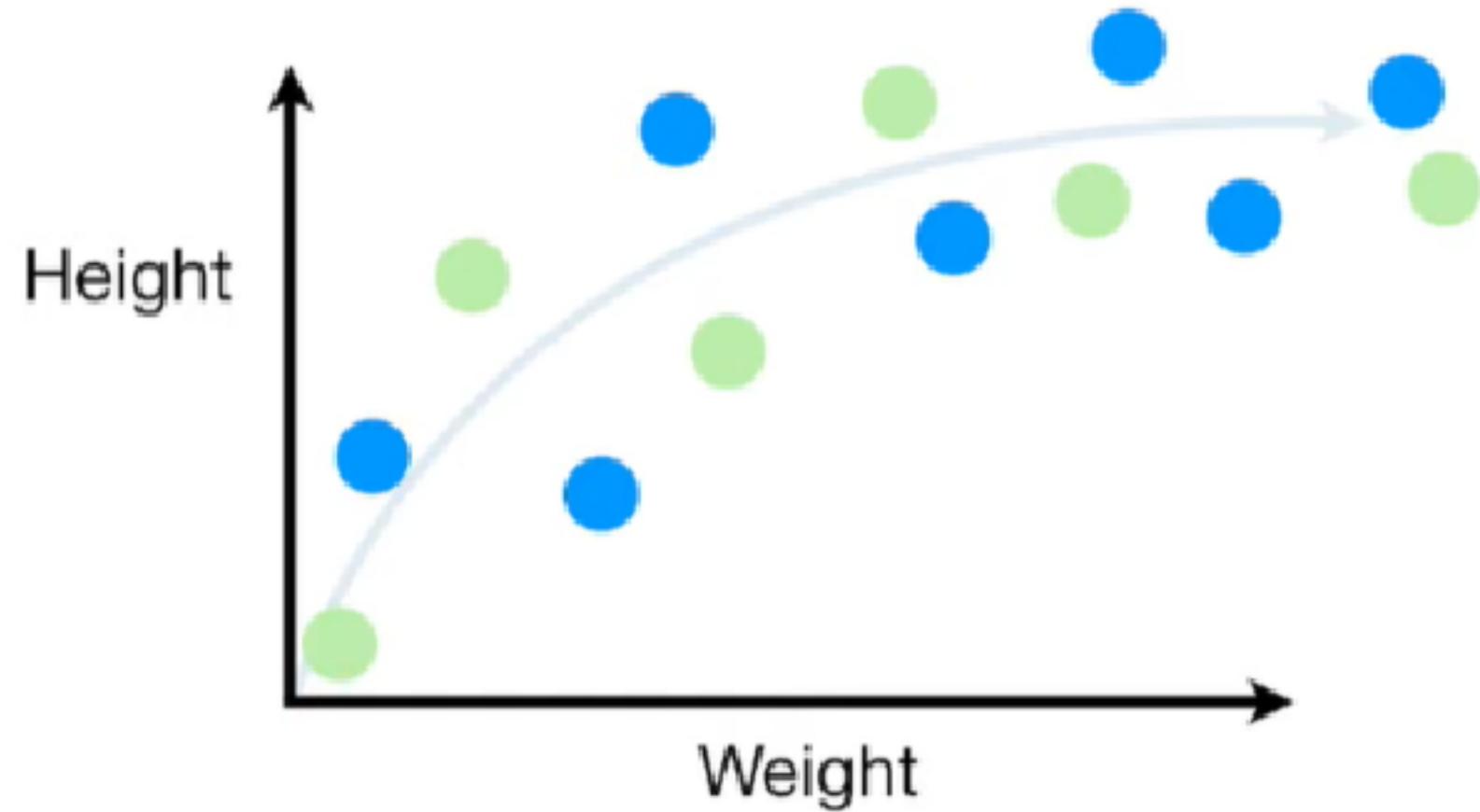
miniz

A

D<sup>-1</sup>D<sup>T</sup>

=





$\gamma$

$\epsilon_{test}$

model

lowest

$\gamma$

0.01 0.1 1 10 100

train

test

How ?  $\rightarrow$  grid search

$\gamma \uparrow$

~~$f(\beta_i) \uparrow$~~

$f(\beta_i) \downarrow$



$$f(\beta_i) = |\beta_i| / \beta_i^2$$

(1)      (2)

Ridge    LASSO

Bitwise  
diff





$\text{err}_{\text{old}}$

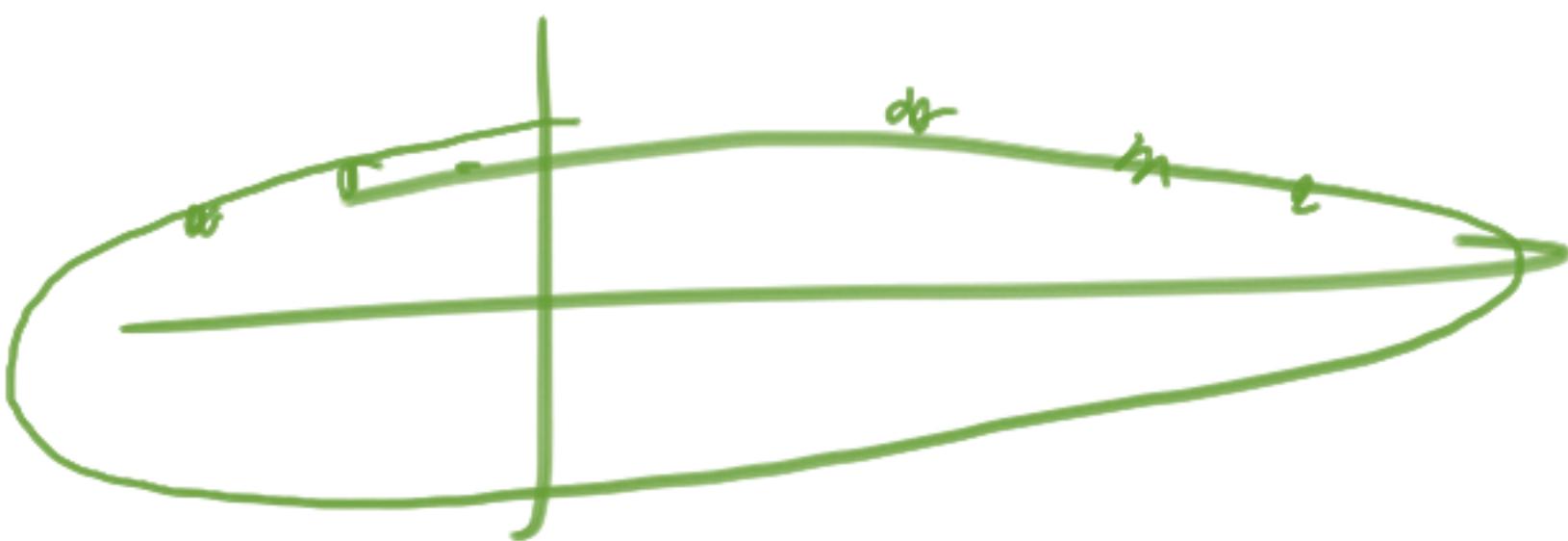
$$y - \beta_0 - \beta_1 x - \beta_2 u_2$$

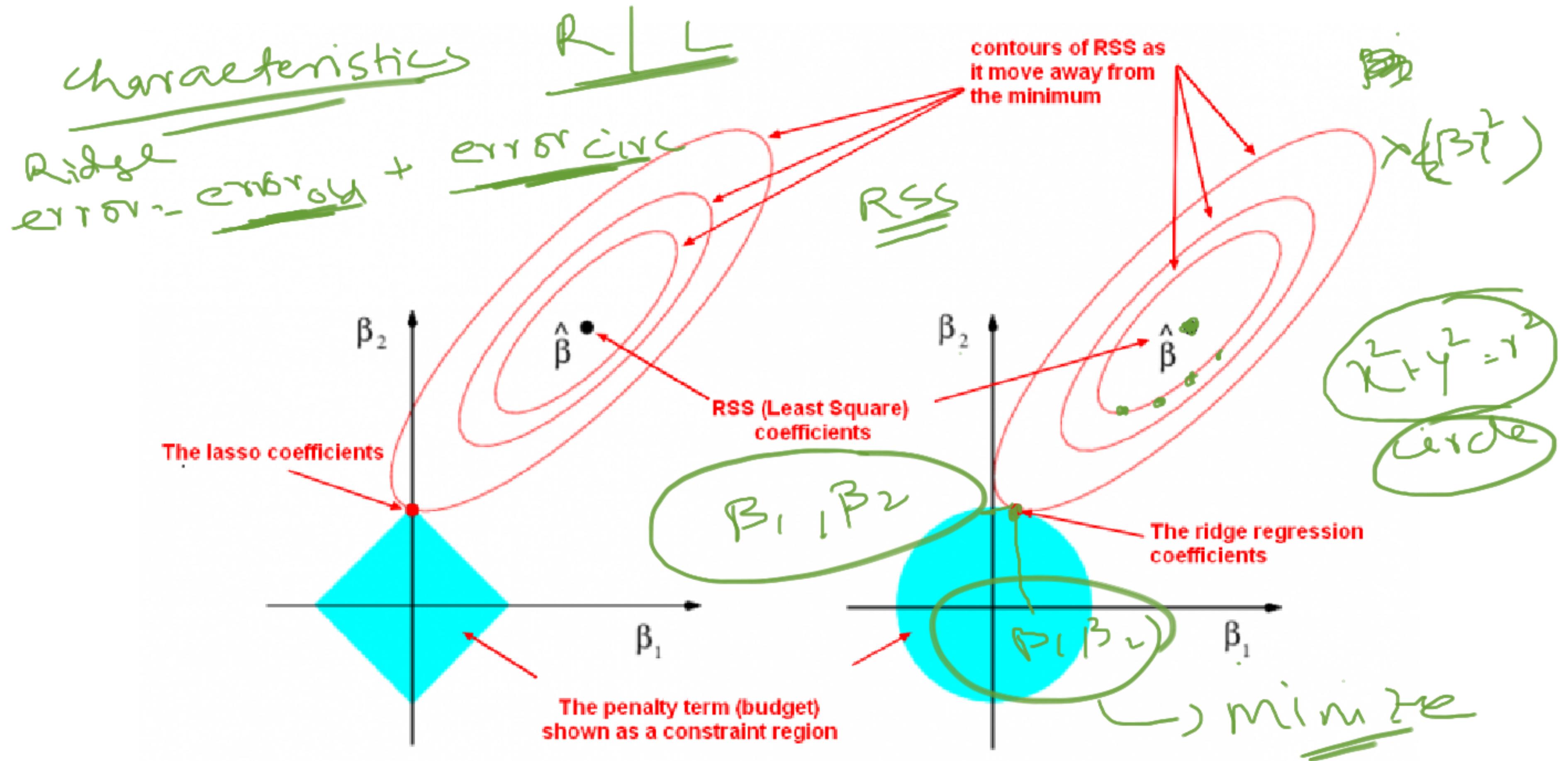
elliptical

$(a+b+\tilde{g})$

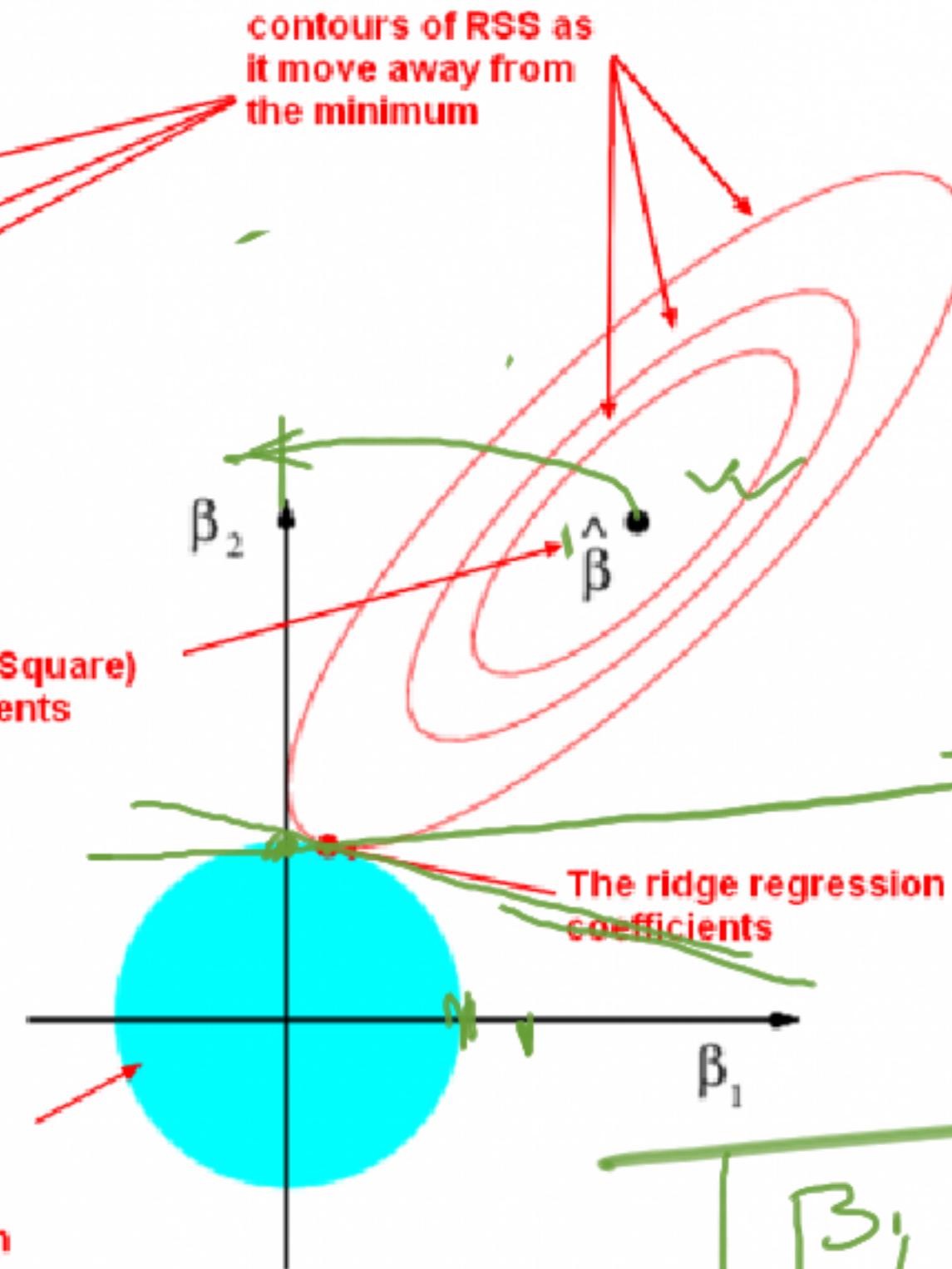
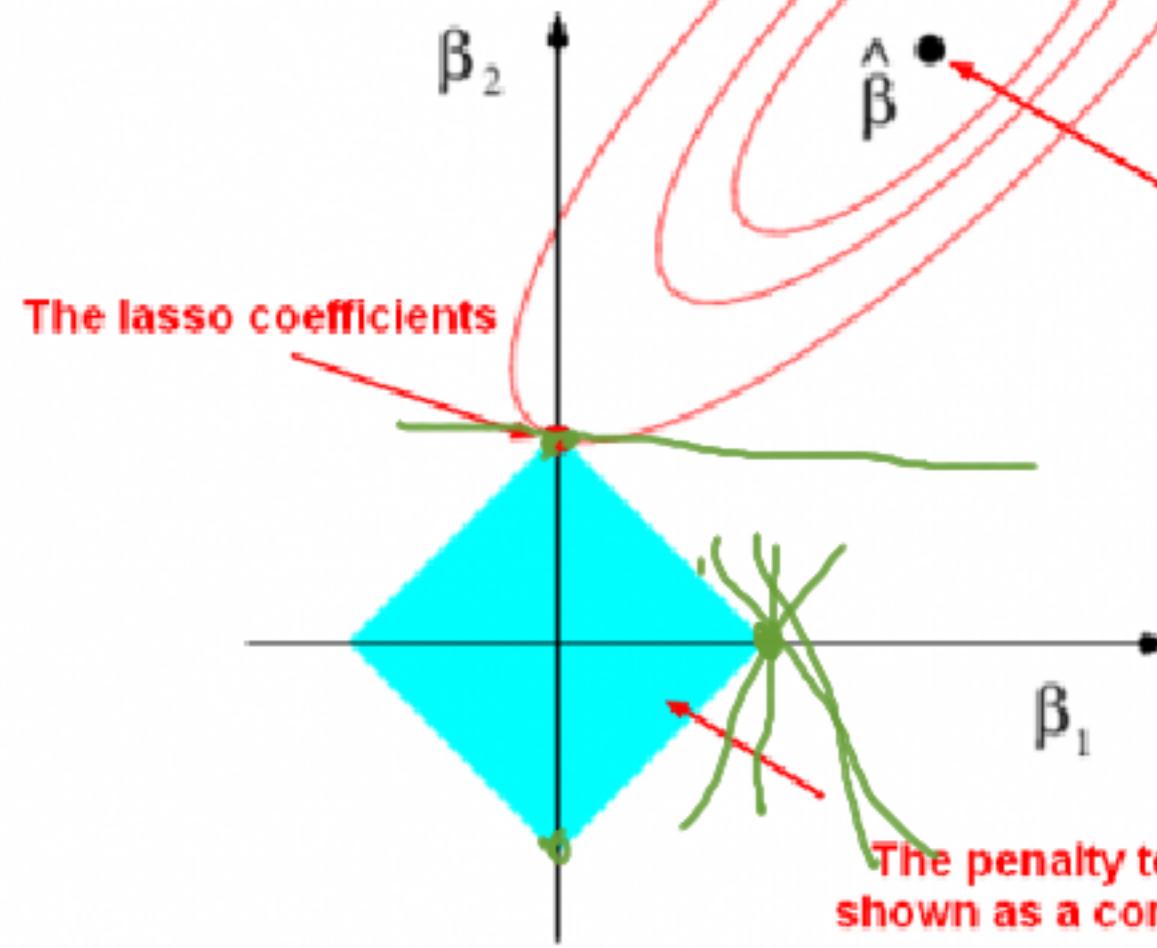
$$\frac{a^2}{r} + \frac{b^2}{r} + \frac{c^2}{r}$$
$$+ 2ab + 2bc$$

$$+ 2ca$$





$$y = \sqrt{a}$$



Lasso ADimin sing

$\beta_0$

curves

ellipse  
circle

$$\beta_l \downarrow = 0$$

RIDGE REGRESSION

LASSO

$$\beta_l = 0$$

$$y = |x|$$

$\oplus$

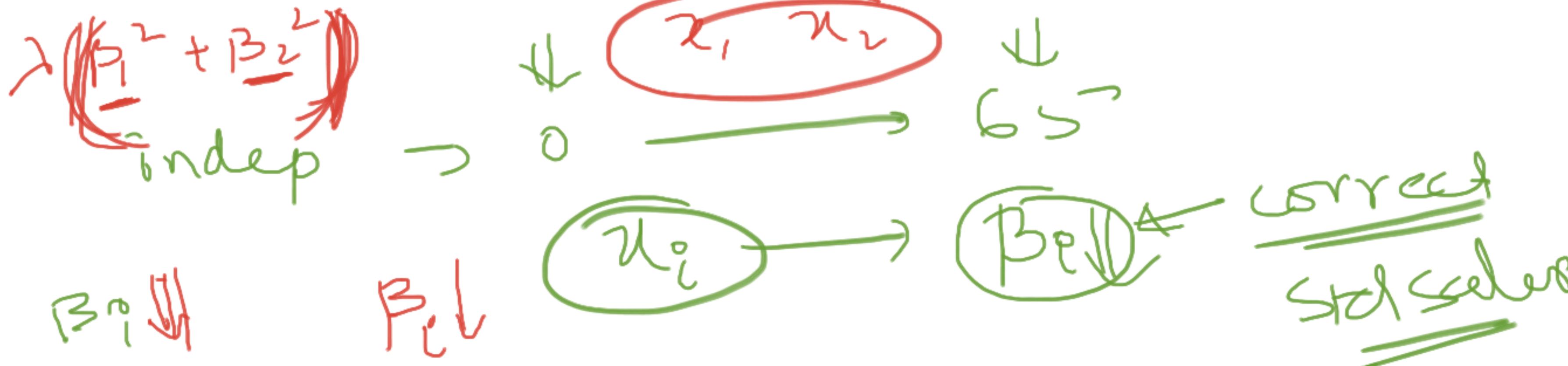
$$\begin{cases} x = 4 \Rightarrow y = 4 \\ x = -4 \Rightarrow y = 4 \end{cases}$$

$$\begin{aligned} &P_2 \\ &x+y=1 \\ &y=1-x \\ &P_1 \\ &x-y=-1 \\ &y=-1-x \end{aligned}$$

$$y = \begin{cases} 1 & \text{or } x > 0 \\ -1 & \text{or } x < 0 \end{cases}$$

" $x$ "  $\rightarrow$   $\vee$

Bitwise shift



```

[array([-1.49246448, 0.37088936, -0.70836731, 1.08568161, -0.80970633,
        4.4075122, -0.80450999]),
 array([-1.30486468, 0.5170331, -0.85951603, 0.96594376, -0.81006847,
        3.54696735, -0.74796938]),
 array([-0.6486645, 0.4993468, -0.67576213, 0.39800779, -0.61503278,
        1.33852138, -0.56022566]),
 array([-0.11842591, 0.11262874, -0.1443168, 0.05431575, -0.13553009,
        0.20563081, -0.12574102])]
```

