UNIT 4 QUANTUM MECHANICS

LECTURE 4

What we learned so far about Quantum mechanics?

1. We had a short walk down the memory lane (1900-1927)

   - ✓ Classical mechanics, relativistic mechanics, quantum mechanics and quantum field theory
   - ✓ Hertz, Planck, Einstein, Bohr, Crompton, Raman, de Broglie, Heisenberg, Schrödinger, Dirac, Pauli, Born
   - ✓ Development of quantum mechanics

2. Classical mechanics Explained

Newton's corpuscular concepts of light-particles
Maxwell’s wave concepts of light from EM theory
Reflection, refraction –explained through particle concept-ray optic
Interference, diffraction, polarization– wave nature

It was all about light!

2. How QM concept helped in overcoming classical limitation?

Black body radiation ,
Wien and Rayliegh-Jean formula,
UV catastrophe
Planck's quantum oscillator,

$$I_\nu d\nu = \frac{8\pi \nu^2}{c^3} kT \, d\nu$$

$$I_\nu d\nu = \frac{A \nu^3}{c^4} e^{-B\nu/T} \, d\nu$$

$$I_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

Photoelectric effect,
Hertz's discovery
Einstein's photoelectric equation,
The name photon

$$\mathrm{E_k = h\nu - h\nu_0}$$

ϕ-Work function

Compton effect-scattering of light by electron
Raman effect-vibration spectra of molecules upon photon irradiation

All these phenomenon were successfully explained by QM

3. Characteristic properties of a wave : **ν and λ**

4. Characteristic properties of a particle: **p and E**

5. Radiation (wave)-particle duel nature

$$p = mc = \frac{h}{\lambda}$$

6. Matter –wave duel nature

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

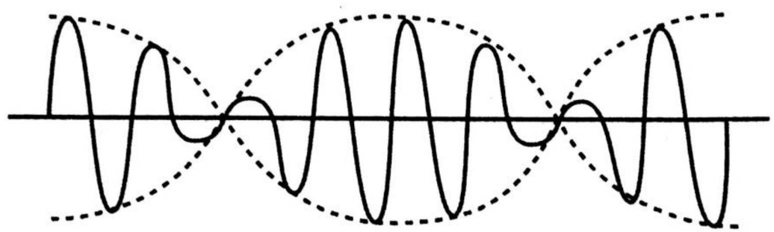
De Broglie's hypothesis; connecting the wave nature with particle nature through the Planck's constant..

Used Einstein's famous mass-energy relation $E=mc^2$

Which of the following phenomena can not be explained by the classical theory?

a) Photoelectric effect
b) Compton effect
c) Raman effect
d) All of the above

Ans: D



7. Characteristics of matter wave

8. Wave velocity, group velocity and particle velocity

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{\Delta\omega}{\Delta k} \qquad \therefore v_g = -\frac{\lambda^2}{2\pi}\frac{\partial\omega}{\partial\lambda}$$

v particle velocity

non-dispersive- $v_p = v_g$

normal-dispersive- $v_p > v_g$

anomalous dispersive mediums- $v_p < v_g$

9. Relationship between v_g and v_p & v_g and v

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

dispersion

$$v_g = v$$

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

For a non - dispersive medium derivative of the phase or wave velocity (V_p) with wavelength (λ) i.e $\frac{dV_p}{d\lambda}$ is

(a) Negative
(b) Positive
(c) Zero
(d) None of the above

Ans: C

For non-dispersive medium phase velocity (v_p) is independent on the wavelength of the wave and hence group velocity v_g is

a) $v_g > v_p$
b) $v_g < v_p$
c) $v_g = v_p$
d) none of the above

Ans: C

For dispersive medium phase velocity (v_p) is dependent on the wavelength of the wave and hence group velocity v_g is

a) $v_g > v_p$
b) $v_g < v_p$
c) $v_g = v_p$
d) none of the above

Ans: B

Matter-wave is associated with moving particle. In that case, the particle velocity is equal to the group velocity. True or False?

(a) True
(b) False

Ans: A

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

For a normal dispersive medium derivative of the phase or wave velocity (V_p) with wavelength (λ) i.e $\frac{dV_p}{d\lambda}$ is

(a) Negative
(b) Positive
(c) Zero
(d) None of the above

Ans: B

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

For an anomalous dispersive medium derivative of the phase or wave velocity (V_p) with wavelength (λ) i.e $\frac{dV_p}{d\lambda}$ is

(a) Negative
(b) Positive
(c) Zero
(d) None of the above

Ans: A

Heisenberg uncertainty principle

The wave-particle duality lead naturally to an uncertainty relation: In physics it is the Heisenberg uncertainty principle

Particle nature $\dashrightarrow$ $$p = \frac{h}{\lambda}$$ $\dashleftarrow$ Wave nature

Wave-particle duality introduce the concept of uncertainty. This concept suggest that if the particle nature of the matter becomes certain, the wave nature becomes uncertain and vice versa.

There are three **conjugate variables** of great importance in **quantum mechanics**: position and momentum, angular orientation and angular momentum, and energy and time.

Heisenberg uncertainty principle

Concept of wave or wave packet associated with a moving atoms/subatomic particle introduce uncertainty. According to Heisenberg uncertainty principle it is impossible to measure the exact position and momentum(or velocity) of very small particles like, molecules, atoms or subatomic species like electron, proton, neutron etc…

1. $\Delta p \, \Delta x \geq \hbar$ Original statement of Heisenberg uncertainty principle

2. $\Delta E \, \Delta t \geq \hbar$ Time –Energy uncertainty principle

3. $\Delta L_{\theta} \, \Delta \theta \geq \hbar$ Angular momentum -Angular orientation uncertainty principle

Heisenberg uncertainty principle

1. Momentum (p) and position (x)

$$p = \frac{h}{\lambda}$$

We know the propagation constant, **k** related to the wavelength λ by

$$k = \frac{2\pi}{\lambda}$$

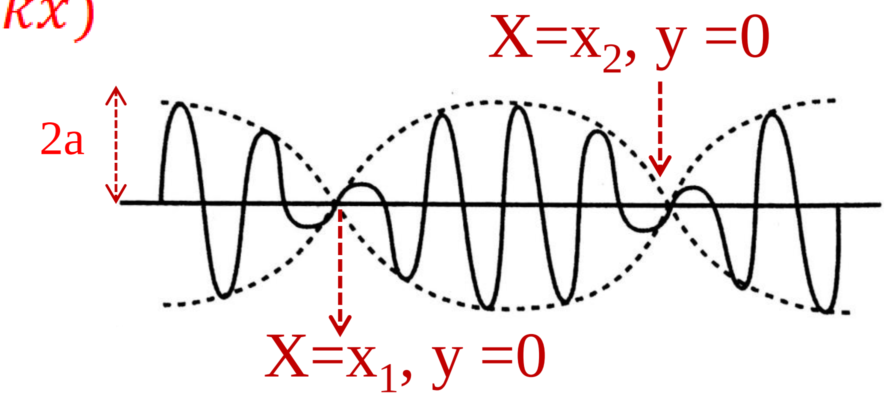
$$p = \frac{h}{\lambda} = \frac{h}{2\pi/k} = \hbar k$$

Where $\hbar = \dfrac{h}{2\pi}$

$$\Delta p = \hbar\,\Delta k \quad \text{--------------}\!\!\to \quad (1)$$

Now we have to find a relation connecting Δk to Δx !! For that we have to go back to what we learned about **wave packet and group velocity** in the last class

$$y = 2a\ cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta kx}{2}\right]\sin(\omega t - kx)$$



Group velocity of the de Broglie wave $v_g = v$, particle velocity. The position of the particle can be anywhere in the loop (dotted curve)

In the above equation y=0, corresponds to the node, in that case

$$cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta kx}{2}\right] = 0 \quad \text{or} \quad \frac{\Delta\omega t}{2} - \frac{\Delta kx}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots . \frac{(2n+1)\pi}{2}$$

For the two consecutive nodes at ($y=0$, $x=x_1$) and ($x=x_2$. $y=0$) the above equation can be written as

$$\frac{\Delta\omega t}{2} - \frac{\Delta k x_1}{2} = \frac{(2n+1)\pi}{2} \dashrightarrow (2)$$

$$\frac{\Delta\omega t}{2} - \frac{\Delta k x_2}{2} = \frac{(2n+3)\pi}{2} \dashrightarrow (3)$$

Upon subtracting Eq.(3)- Eq.2(2) we get $\frac{\Delta k(x_1 - x_2)}{2} = \pi$

but $x_1 - x_2 = \Delta x$ $\quad \Delta k\, \Delta x = 2\pi \dashrightarrow (4)$

Substitute Eq. 4 in Eq.1 $\Delta p = \hbar \frac{2\pi}{\Delta x}$ $\quad \Delta p\, \Delta x = 2\pi\hbar$

However, more accurate measurements show that the product of the uncertainties in momentum (Δp) and position (Δx) can not be less than $\hbar$

$$\Delta p\, \Delta x \geq \hbar \dashrightarrow (5)$$

This is Heisenberg uncertainty principle

If I know the position of a subatomic particle precisely, then

a) I know nothing about the particle’s momentum.
b) I know a little about the particle’s momentum
c) The particle must be at rest.
d) None of the above.

Ans: B

Heisenberg uncertainty principle

2. Energy (E) and time (t)

Momentum, force, Work/Energy

$$F = ma = m\frac{dv}{dt} = \frac{d(mv)}{dt} = \frac{dp}{dt}$$

Force and momentum are also conjugate quantities

So the uncertainties in the measurement of force is related to the errors in p and t as follows

$$\Delta F = \frac{\Delta \mathrm{p}}{\Delta t}$$ or $$\Delta p = \Delta F \times \Delta t$$

But just now we proved $\Delta \mathrm{p}\, \Delta \mathrm{x} \geq \hbar$ substitute in the above eqn.

$$\frac{\hbar}{\Delta x} = \Delta F \times \Delta t$$

And upon re-arranging

$$\frac{\hbar}{\Delta x} = \Delta F \times \Delta t \quad \Longrightarrow \quad (\Delta x \Delta F) \times \Delta t = \hbar$$

But Force x distance is work, nothing but the energy $\Delta x \, \Delta F = \Delta E$

$$\Delta E \, \Delta t = \hbar$$

OR

$$\Delta E \, \Delta t \geq \hbar$$

Uncertainty principle represented in terms of energy and time

Heisenberg uncertainty principle:

3. Angular momentum (L_θ) and angular orientation (θ)

A particle of mass m and velocity v making circular motion with radius r. Its angular momentum is given by

$$L_\theta = mv\,r = pr$$

In moving a distance x along the circle, the particle sweep an angle θ given by

$$\theta = \frac{x}{r} \qquad \Delta L_\theta = \Delta p\, r \qquad \Delta\theta = \frac{\Delta x}{r}$$

$$\Delta L_\theta\, \Delta\theta = \Delta p\, r\, \frac{\Delta x}{r} = \Delta p \Delta x$$

$$\Delta L_\theta\, \Delta\theta = \hbar$$

$$\Delta L_\theta\, \Delta\theta \geq \hbar$$

Uncertainty principle represented in terms of angular momentum and orientation

Applications of Heisenberg uncertainty principle

Applications of Heisenberg uncertainty principle are

1. Non existence of electron in the nucleus
2. Existence of proton, neutrons and α-particles in the nucleus
3. Binding energy of an electron in an atom
4. Radius of Bohr's first orbit
5. Energy of a particle in a box
6. Ground state energy of the linear harmonic oscillator
7. Radiation of light from an excited atom

1. Non existence of electron in the nucleus

The radius of the nucleus of an atom ~ 10^{-14} m. To be in the nucleus the position of the electron must be less than 10^{-14} m. In in terms of the uncertainty principle the highest limit the electron can have is the diameter of the nucleus i.e $\Delta x \sim 2 \times 10^{-14}$ m

$$\Delta x \Delta p = \frac{h}{2\pi} \qquad \Delta p = \frac{h}{2\pi\, \Delta x}$$

$$\Delta p = \frac{6.625 \ \times 10^{-34}\, J - s}{2 \times 3.14 \ \times 10^{-14}\, m} = 5.27 \times 10^{-27}\, kg - m/s$$

This is the uncertainty in the momentum of the electron. It means momentum of the electron 'p' would not be less than Δp.

$$p = \Delta p = 5.27 \times 10^{-27}\, kg - m/s$$

Kinetic energy of the electron (E_k)

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(5.27 \times 10^{-27})^2}{2 \times 9.1 \times 10^{-31}} \times \frac{1}{1.6 \times 10^{-91}}$$

$= 95.55 \times 10^6$ eV ~ 96 MeV

So electron can stay inside the nucleus only when it is having an energy about 96 MeV.

However the maximum energy an electron can have is 4 MeV as it is emitted from the radioactive decay of the nucleus. Hence electrons cannot stay at the nucleus.

UNIT 4-Quantum Mechanics

Text Book: ENGINEERING PHYSICS by HITENDRA K MALIK AND A K SINGH, MCGRAW HILL EDUCATION, 1st Edition, (2009)

References:

- ENGINEERING PHYSICS by B K PANDEY AND S CHATURVEDI, CENGAGE LEARNING, 1st Edition, (2009).

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