UNIT 4 QUANTUM MECHANICS

LECTURE 7

What we learned so far about Quantum mechanics?

1. We had a short walk down the memory lane (1900-1927)

   - ✓ Classical mechanics, relativistic mechanics, quantum mechanics and quantum field theory
   - ✓ Hertz, Planck, Einstein, Bohr, Crompton, Raman, de Broglie, Heisenberg, Schrödinger, Dirac, Pauli, Born
   - ✓ Development of quantum mechanics

2. Classical mechanics Explained

   Newton's corpuscular concepts of light-particles
   Maxwell's wave concepts of light from EM theory
   Reflection, refraction –explained through particle concept-ray optic
   Interference, diffraction, polarization– wave nature

It was all about light!

3. How QM concept helped in overcoming classical limitation?

Black body radiation ,
Wien and Rayliegh-Jean formula,
UV catastrophe
Planck's quantum oscillator,

$$I_\nu \, d\nu = \frac{8\pi \nu^2}{c^3} kT \, d\nu$$

$$I_\nu \, d\nu = \frac{A \nu^3}{c^4} e^{-B\nu/T} \, d\nu$$

$$I_\nu \, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 \, d\nu}{e^{h\nu/kT} - 1}$$

Photoelectric effect,
Hertz's discovery
Einstein's photoelectric equation,
The name photon

$$E_k = h\nu - h\nu_0$$

ϕ_m-Work function

Compton effect-scattering of light by electron
Raman effect-vibration spectra of molecules upon photon irradiation

All these phenomenon were successfully explained by QM

4. Characteristic properties of a wave : **ν and λ**

5. Characteristic properties of a particle: **p and E**

6. Radiation (wave)-particle duel nature

$$p = mc = \frac{h}{\lambda}$$

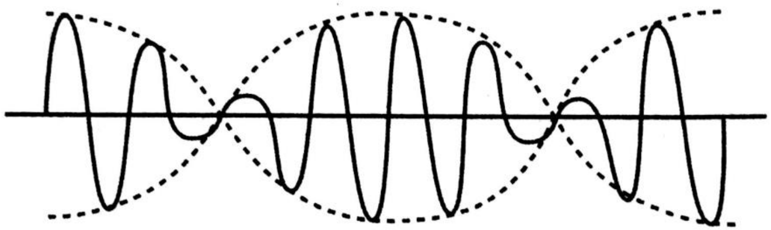
7. Matter (particle) –wave duel nature

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

De Broglie's hypothesis; connecting the wave nature with particle nature through the Planck's constant..

Used Einstein's famous mass-energy relation $E=mc^2$

8. Various relation connecting the de Broglie wavelength associated with a particle of mass m and having energy E



9. Characteristics of matter wave

10. Wave velocity, phase velocity, group velocity and particle velocity

v particle velocity

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{\Delta\omega}{\Delta k} \qquad \therefore v_g = -\frac{\lambda^2}{2\pi}\frac{\partial\omega}{\partial\lambda}$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}.$$

non-dispersive- $v_p = v_g$

normal-dispersive- $v_p > v_g$

anomalous dispersive mediums- $v_p < v_g$

11. Relationship between v_g and v_p & v_g and v

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

Dispersion

$$v_g = v$$

12. Heisenberg uncertainty principle

Wave-particle duality introduce the concept of uncertainty. This concept suggest that if the particle nature of the matter becomes certain, the wave nature becomes uncertain and vice versa.

Uncertainty in the measurements of physical quantities

There are three **conjugate variables** of great importance in **quantum mechanics**: position and momentum, angular orientation and angular momentum, and energy and time.

1. $\Delta p \, \Delta x \geq \hbar$ Original statement of Heisenberg uncertainty principle

2. $\Delta E \, \Delta t \geq \hbar$ Time –Energy uncertainty principle

3. $\Delta L_\theta \, \Delta \theta \geq \hbar$ Angular momentum -Angular orientation uncertainty principle

13. Applications of Heisenberg uncertainty principle are

1. Non existence of electron in the nucleus
2. Existence of proton, neutrons and α-particles in the nucleus
3. Binding energy of an electron in an atom
4. Radius of Bohr’s first orbit
5. Energy of a particle in a box
6. Ground state energy of the linear harmonic oscillator
7. Radiation of light from an excited atom

14. Wave Equation and function- Classical

$$\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \qquad u = a\ \sin(\omega t - kx)$$

Where 'u' is the wave function… that is the solution of the wave equation

e.g, u can be E, B or P at a position (x,y,z) at a given time t.

15. Wave function- Quantum

The characteristics of the wave functions in quantum mechanics are

- ψ must be finite, continuous and single valued everywhere
- ψ must be normalizable $\iiint_{-\infty}^{\infty} \psi^* \psi \ dV = 1$
- also $\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z}$ must be finite, continuous and single valued

18. Schrödinger wave equation

Schrödinger time- independent wave equation for free particle

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0$$

Schrödinger time- independent wave equation

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\,\psi = 0$$

Schrödinger time- dependent wave equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\mathbf{H}\psi = \mathbf{E}\psi$$

19. Operators, Eigen value and Eigen function

Classical expression for total energy

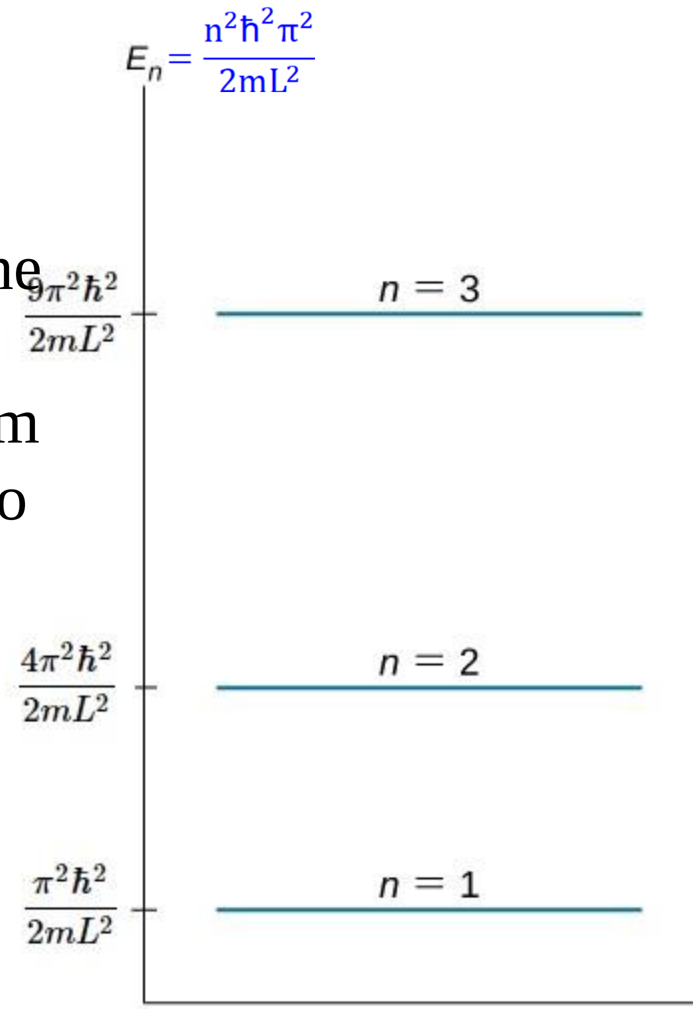
$$\frac{p^2}{2m} + V = E$$

$$E = i\hbar\frac{\partial}{\partial t}$$

$$p = -i\hbar\nabla$$

https://www.youtube.com/watch?v=LBB39u8dNw0

20. Particle in a box- Eigen value



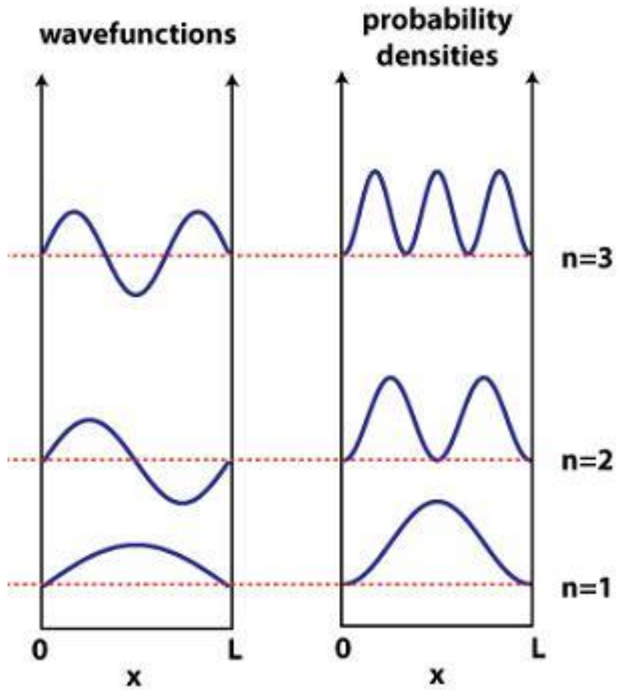
- **E** is the Eigen value of the particle in the potential well
- Constitute the energy level of the system
- ***n*** is the quantum number corresponds to the energy level $\mathbf{E_n}$

$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$$

So we found out the energy (**Eigen value**) of the particle in a box, with the help of Schrödinger equation

21. Particle in a box- Eigen Function

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$



Probability of finding the particle is

$$|\psi_n(x)|^2$$

- Classical mechanics predict the same probability to find the particle anywhere in the box
- But quantum mechanics different probability
- There are points where particle will never present
- Probability is different also with energy of the particles

The characteristics of the wave functions in quantum mechanics are

a) ψ must be finite, continuous and single valued everywhere
b) ψ must be normalizable
c) ψ must be finite, continuous and single valued
d) All of the above

Ans: D

When a Laplacian operator (∇^2) and Energy (E) operate on wave function (Ψ), we get the wave equation $\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\,\psi = 0$, What type of equation is it?

a) Time dependent Schrodinger equation
b) Time-independent Schrodinger equation for any particle
c) Time-independent Schrodinger equation for free particle
d) None of the above

Ans: B

The energy of a particle at a level n in infinite potential well is

(a) Proportional to n^2
(b) Proportional to n
(c) Inversely proportional to n^2
(d) Inversely proportional to n

Ans: A

The momentum of a particle in infinite potential well is

(a) Proportional to n^2
(b) Proportional to n
(c) Inversely proportional to n^2
(d) Inversely proportional to n

Ans: B

The momentum of a particle in infinite potential well of length L is

(a) Proportional to L^2
(b) Proportional to L
(c) Inversely proportional to L^2
(d) Inversely proportional to L

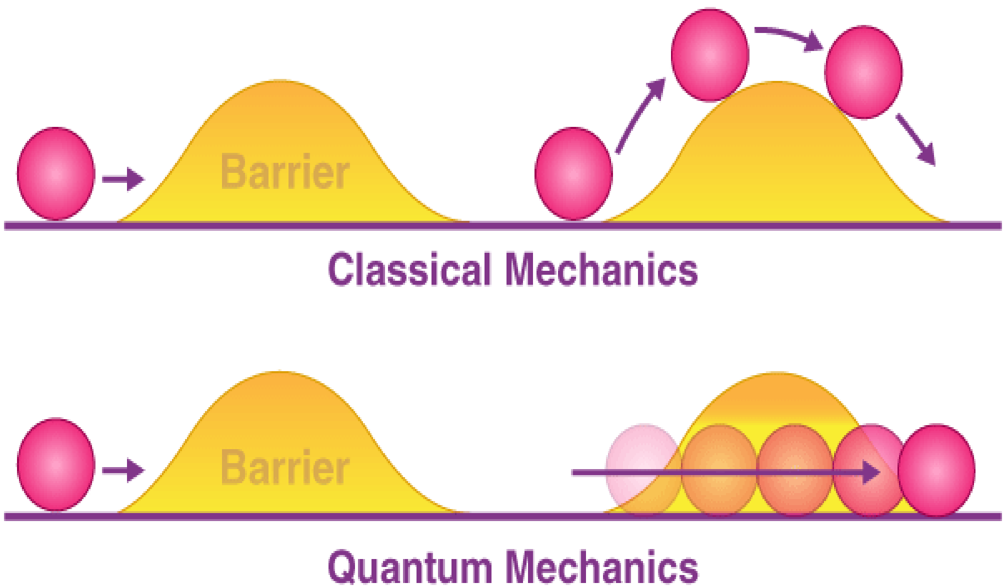
Ans: D

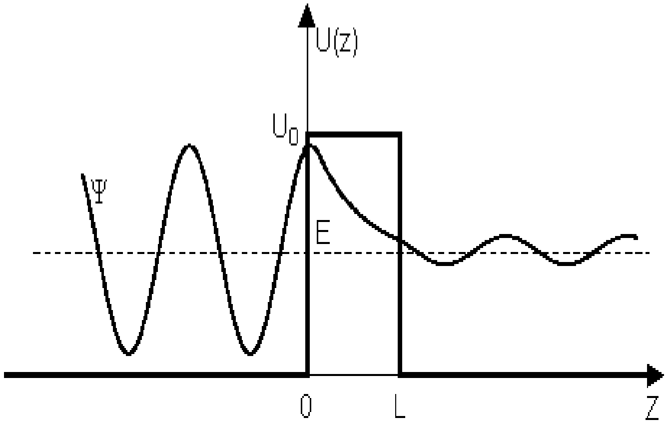
The Energy of a particle in infinite potential well of length L is

(a) Proportional to L^2
(b) Proportional to L
(c) Inversely proportional to L^2
(d) Inversely proportional to L

Ans: C

Tunneling





Found Application in

- ❖ **Quantum computing.**
- ❖ In Electronics-Limits the miniaturization of transistors.
- ❖ Nuclear fusion.
- ❖ Tunnel diodes (Esaki Diodes).
- ❖ Scanning tunneling microscopes.

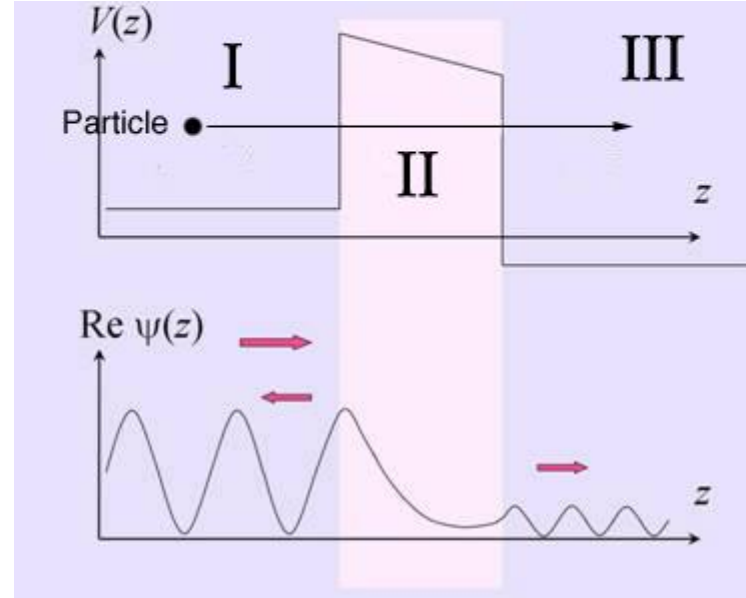
Quantum Tunneling- History

- Consequence of the wave nature of matter
- First used to explain alpha decay of heavier elements in 1928 by George Gamow
- Shown experimentally by Leo Esaki in 1958 in the tunneling diode

What is Quantum Tunneling?

- At the quantum level, matter has corpuscular and wave-like properties
- Tunneling can only be explained by the wave nature of matter as described by quantum mechanics
- Classically, when a particle is incident on a barrier of greater energy than the particle, reflection occurs
- When described as a wave, the particle has a probability of existing within the barrier region, and even on the other side of it

How Can Tunnelling Be?

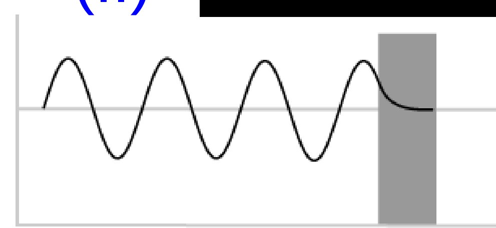


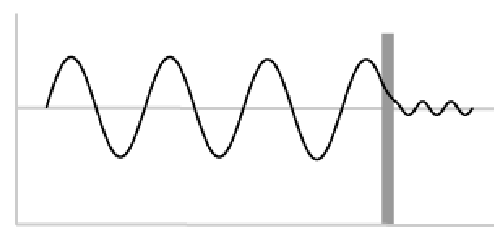
- Solutions to the wave equation have the general form (I)
- In the barrier region, the solution becomes (II)

(I) $Ae^{i\kappa x} + Be^{-i\kappa x}$

(II) $Ce^{\kappa x} + De^{-\kappa x}$

- The wave function decays exponentially in the barrier region
- If some portion of the wave function still exists on the other side of the barrier, transmission has occurred
- The width of the barrier is the most prominent factor in determining the probability of transmission





UNIT 4-Quantum Mechanics

Text Book: ENGINEERING PHYSICS by HITENDRA K MALIK AND A K SINGH, MCGRAW HILL EDUCATION, 1st Edition, (2009)

References:

- ENGINEERING PHYSICS by B K PANDEY AND S CHATURVEDI, CENGAGE LEARNING, 1st Edition, (2009).

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