PHY 110 Engineering Physics

Lecture 6

UNIT 1 – Electromagnetic theory

What we learned so far

$$F = f(x, y, z)$$

$$\vec{\nabla} = \hat{\boldsymbol{\imath}}\frac{\partial}{\partial x} + \hat{\boldsymbol{\jmath}}\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial}{\partial z}$$

$$\vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y)$$

$$\vec{\nabla}F$$

$$\vec{\nabla} \bullet \vec{A} \qquad \vec{\nabla} \times \vec{A}$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\oiiint_V \vec{\nabla} \cdot \vec{A}\, dV = \oiint_S \vec{A} \cdot \overrightarrow{dS}$$

$$\iint (\vec{\nabla} \times \vec{A}) \cdot \overrightarrow{dS} = \oint \vec{A} \cdot \overrightarrow{dl}$$

$$\nabla^2 V = 0$$

$$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0} = \Phi_E$$

$$\Delta \equiv \nabla^2$$ Laplacian operator

$$\iiint \rho\, dV = q$$

$$I = \frac{dq}{dt}$$

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS} = 0$$

$$\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$$

$$I = \iint \vec{J} \cdot \overrightarrow{dS}$$

What we learned so far

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \phi_B}{\partial t}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t}$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt} \right) = \mu_0 (I + I_d)$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

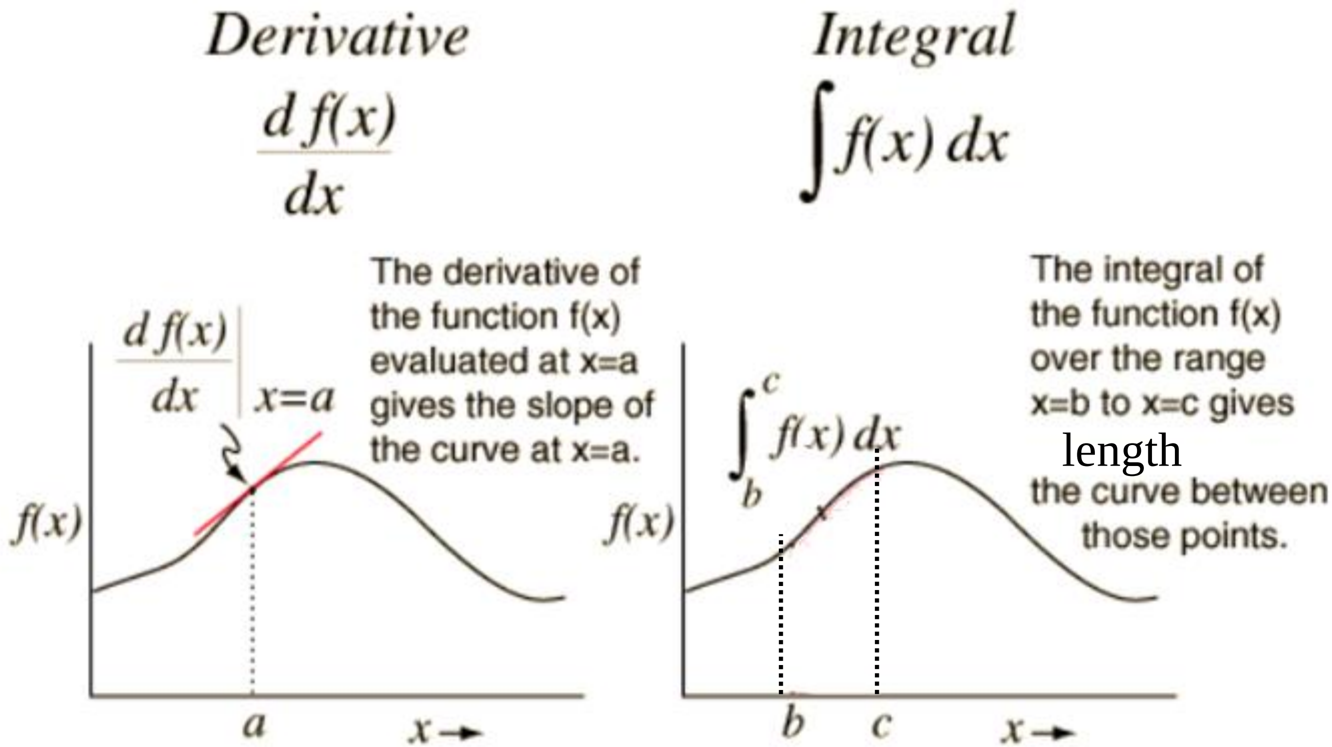
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

What we learned so far

	Differential forms		Integral forms
Eq.1	$\vec{\nabla}\cdot\vec{D}=\rho$	Gauss law of electrostatics	$\oiint \vec{D}\cdot\overrightarrow{dS}=q$
Eq.2	$\vec{\nabla}\cdot\vec{B}=0$	Gauss law of magnetostatics	$\oiint \vec{B}\cdot\overrightarrow{dS}=0$
Eq.3	$\vec{\nabla}\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}$	Faraday's law of EM induction (***emf***)	$\oint \vec{E}\cdot\overrightarrow{dl}=-\oiint\dfrac{\partial\vec{B}}{\partial t}\cdot\overrightarrow{dS}$
Eq.4	$\vec{\nabla}\times\vec{H}=\vec{J}+\dfrac{\partial\vec{D}}{\partial t}$	Ampere-Maxwell's law	$\oint \vec{H}\cdot\overrightarrow{dl}=\oiint(\vec{J}+\dfrac{d\vec{D}}{dt})\cdot\overrightarrow{dS}$

Differential and integral forms..



Slopes of tangent lines and velocities

Deals with total size or value, such as lengths, areas, and volumes

Integral of $\iint f(x, y)dxdy$ gives

a) Length
b) Area
c) Volume
d) None of these

Integral of $\iiint f(x,y,z)dx\,dy\,dz$ gives

a) Length
b) Area
c) Volume
d) None of these

In Maxwell's fourth equation, he corrected Ampere's circuital law. What is the new term he added?

a) Electric Displacement
b) Displacement current
c) Conduction current
d) None of the above

Ans: B

Maxwell's equations give

a) The variation of magnetic field only
b) The variation of electric and magnetic field in quantum domain
c) The unified approach called electromagnetic theory explaining the variation of static and time varying electric and magnetic field
d) Variation of electric field only

Ans: C

Physical significance of Maxwell Equations

1) Maxwell's first equation $\vec{\nabla} \cdot \vec{D} = \rho$ or $\iiint \vec{\nabla} \cdot \vec{D}\, dV = q$ states electric displacement flux through any closed surface is equal to the total charge enclosed by the surface.

Electric field lines originate on positive charges and terminate on negative charges

2) Maxwell's second equation $\vec{\nabla} \cdot \vec{B} = 0$ or $\oint\!\!\oint \vec{B} \cdot \vec{dS} = 0$ states net magnetic flux through any closed surface is zero. Since a magnetic monopole does not exist, any closed volume always contains equal and opposite magnetic poles (north and south poles), resulting in the zero net magnetic pole strength. It also signifies that magnetic line of flux are continuous i.e., the number of magnetic lines of flux entering into a region is equal to the lines of flux leaving it.

Magnetic field lines always form closed loops—they don't begin or end anywhere.

Physical significance of Maxwell Equations

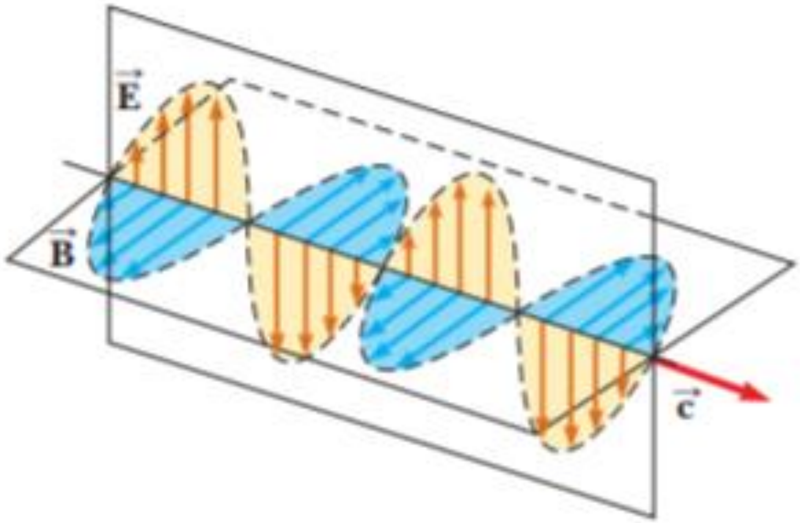
3) Maxwell's third equations $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or $\oint \vec{E} \cdot \overrightarrow{dl} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$ states that the induced electromotive force around any closed path is equal to the negative time rate of change of magnetic flux through the path enclosing the surface. This signifies that an electric field can also be produced by a changing magnetic flux.

4) Maxwell's fourth equation $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ or $\oint \vec{H} \cdot \overrightarrow{dl} = \iint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$ states that magnetomotive force around any closed path is equal to the sum of conduction current and the displacement current through the surface bounded by that path. This signifies that a conduction current or a changing electric flux produces a magnetic field.

5) In addition to unifying the formerly separate fields of electricity and magnetism, Maxwell's electromagnetic theory predicted that electric and magnetic fields can move through space as waves with the speed of light. So light is nothing but an electromagnetic wave.

Physical significance of Maxwell Equations

Maxwell's equations (1865) and the prediction of electromagnetic waves were truly one of the greatest discoveries of science, on a par with Newton's discovery of the laws of motion (1687). Like Newton's laws, it had a profound influence on later scientific developments.



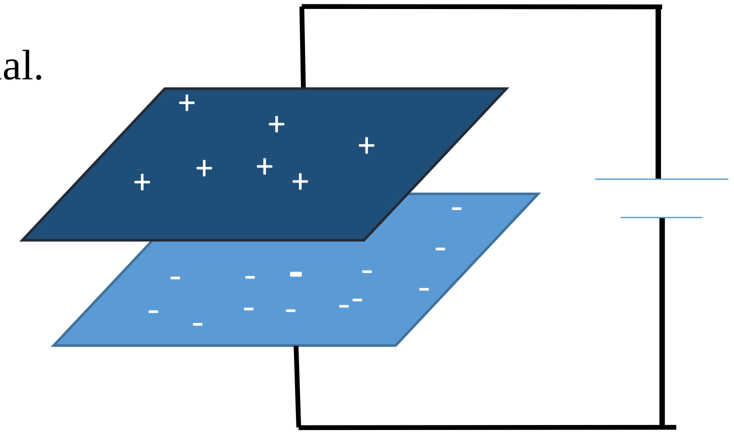
Important point of Maxwell's EM theory is that accelerated charges radiate electromagnetic waves.

Dielectrics

- Dielectrics are insulators which do not conduct electricity. However, based on the function they perform, their nomenclature is different.
- When the main function of non-conducting materials is to provide electrical insulation, they are called **insulators.**
- On the other hand, non conducting materials when placed in an electric field, modifies the electric field and themselves undergo appreciable changes. As a consequence, they act as electrical charge stores.
- Thus, when charge storage is the main function of the of the non conducting material, they are called **dielectrics**.

Dielectric Constant

- The dielectric constant characterizes a dielectric material.
- It is also called the relative permittivity.



If V is the voltage applied across parallel plate capacitor, then charges on one plate will be +Q and on the other plate will be –Q. The capacitance C is related by the quantity of charge stored on either plate. Thus,

$$C = \frac{Q}{V}$$

If the plates have area A and a vacuum in between then the capacitance is written as,

$$C_0 = \frac{\epsilon_0 A}{d}$$

Here, ϵ_0 is called the permittivity of free space or vacuum. This is a universal constant with value $8.85 \times 10^{-12}\ F/m$.

Dielectric Constant

Let us insert a piece of dielectric material inside the plates. The capacitance can be written as

$$C = \frac{\epsilon A}{d}$$

Where, ϵ is the permittivity of this dielectric medium and $\epsilon > \epsilon_0$.

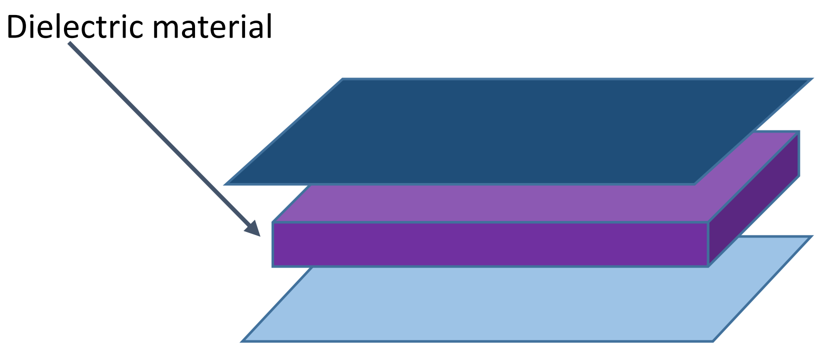
The dielectric constant of the inserted material is thus defined as

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

It is a dimensionless quantity and it is independent of the size and shape of the dielectric. This is also called the relative permittivity of the material.

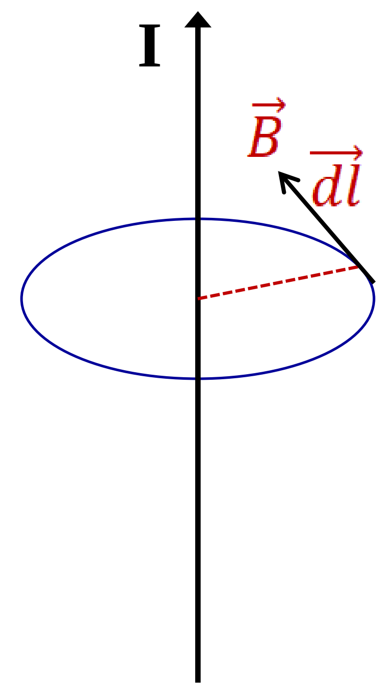
For dielectrics,

$$\epsilon_r > 1$$

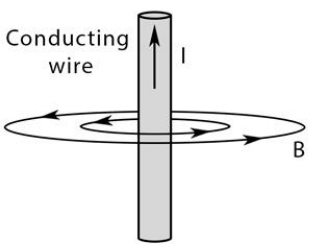


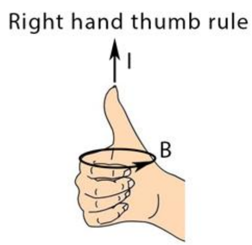
Ampere Circuital Law in Magnetostatics

Ampere circuital law (1823): The line integral of the magnetic field (**B**) around any closed loop is equal to μ_0 (***permeability of the free space***) times the net current (**I**) flowing through the area enclosed by the loop.



Mathematically, this can be expressed as





$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

Amperian loop

Gauss's Electrostatic vs. Ampere Magnetostatics

$$\oiint \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Gauss's law of Electrostatic

Gauss's law charge constant (do not change with time)

$$\oiint \vec{B} \cdot \vec{dS} = 0$$

Gauss's law of magnetostatic

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

Ampere's law of Magnetostatic

Ampere's Law all currents have to be steady (i.e. do not change with time).

Incomplete Ampere's law??

According to the continuity equation $\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$, the rate of change of charge give rise to current density

$\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}$ And will be zero only when there is no change in the charge density within a closed volume

That is $\vec{\nabla} \cdot \vec{J} = 0$ is zero only when $\frac{d\rho}{dt} = 0$

Let us consider Ampere's law and the relation between current density (**J**) and current (**I**)

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I \qquad I = \iint \vec{j} \cdot \overrightarrow{dS}$$

$$\oint \vec{B}\cdot\overrightarrow{dl} = \mu_0 \iint \vec{J}\cdot\overrightarrow{dS}$$

Now apply Stoke's theorem to the LHS

$$\iint (\vec{\nabla}\times\vec{B})\cdot\overrightarrow{dS} = \mu_0 \iint \vec{J}\cdot\overrightarrow{dS} \quad \Rightarrow \quad \iint \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})\cdot\overrightarrow{dS} = \iint \vec{J}\cdot\overrightarrow{dS}$$

Integrands of the LHS and RHS must be equal, so we have

$$\frac{1}{\mu_0}(\vec{\nabla}\times\vec{\mathrm{B}}) = \vec{\mathrm{J}}$$

Take the divergence of both LHS and RHS we will get

$$\vec{\nabla}\cdot\frac{1}{\mu_0}(\vec{\nabla}\times\vec{\mathrm{B}}) = \vec{\nabla}\cdot\vec{\mathrm{J}} \quad \Rightarrow \quad \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{\nabla}\times\vec{\mathrm{B}}) = \vec{\nabla}\cdot\vec{\mathrm{J}}$$

But the divergence of the curl of *any* vector field **A** is always zero

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$$

That means in the last equation $\frac{1}{\mu_0} \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot \vec{J}$

the term $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B})$ is ZERO and we will end up with

$$0 = \vec{\nabla} \cdot \vec{J}$$

But we know that there is current and hence there is a rate of change of charge density… $\frac{d\rho}{dt}$ *is not zero*

So Ampere's law conflicts with the continuity equation $\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}$
And hence the correction by Maxwell is justified.

In Ampere's Law all currents have to be steady (i.e. do not change with time). State true or false

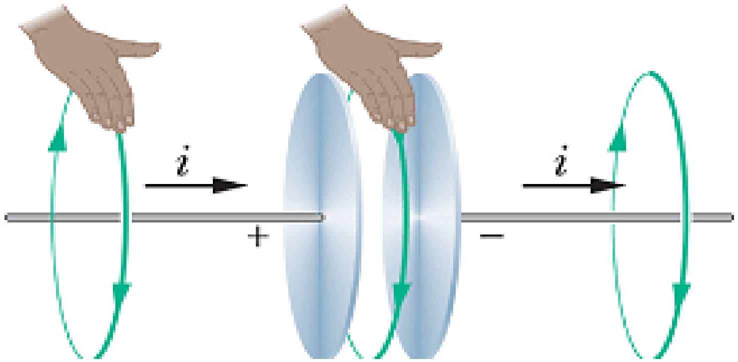
a. True
b. False

Correction in Ampere Circuital Law

Concept of 'displacement current' due to the charge/discharge of a capacitor leads to the correction/modification to the Ampere's law

$$\oint_{s_1} \vec{B} \cdot \overrightarrow{dl} = \mu_0 I \qquad \text{Eq.1}$$

$$\oint_{s_2} \vec{B} \cdot \overrightarrow{dl} = 0 \qquad \text{Eq.2}$$



Because no current is enclosed by S_2

Eq.1 and Eq.2 are contradicting and Maxwell corrected Ampere's law by putting another 'current' term in equation 1… let us see

MAXWELL's LAW OF ELECTROMAGNETIC INDUCTION

Maxwell (like Faraday's) introduced the idea of changing electric field as the source of magnetic field in the gap between the capacitor plate (during charging) and introduced the idea displacement current I_d

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

Add this I_d into

$$\oint_{s_1} \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt} \right) = \mu_0 (I + I_d)$$

Thus corrected Ampere's law to take care of the continuity equation and will be verified later..

Maxwell displacement current

- **Current** in a conductor produces magnetic field – **Ampere’s circuital law**
- However, a changing electric field produces a **magnetic field** in vacuum or in a dielectric??☺
- That means **a changing electric field** is equivalent to a **current** and is called the **DISPLACEMENT CURRENT**
- **Displacement current** produces the same effect as a conventional current in a metallic wire/conductor

Refer R-3 FUNDAMENTALS OF PHYSICS HALLIDAY. RESNICK, WALKER Chapter 8 278-281

Have you read this book ???

James Clerk Maxwell (1831-1879) considered as the father of classical electrodynamics: He corrected Ampere's law by adding another term, which he called the "displacement current”, On what does Maxwell's "displacement current" depend?

a) The derivative of the electric field with respect to time
b) The divergence of the magnetic field
c) The derivative of the magnetic field with respect to time
d) The electromagnetic force on a charged particle