PHY 110 Engineering Physics

Lecture 4

UNIT 1 – Electromagnetic theory

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What we learned so far!

1. **Scalar and Vector quantities**
   - It is *enough* to have a magnitude for **scalar** physical quantities where as it is *essential* to have both magnitude and direction for the **vector** physical quantities.
2. **Scalar and vector field**
   - **Region of space/domain** in which a function, f(x,y,z), signifies a physical quantity (Temperature, Velocity) is the **field**.
   - **Scalar field**: Each point in space is associated with a **scalar point function** (Temperature, potential) having magnitude.
   - **Vector field**: Each point space is associated with a **vector point function** (Electric field, Gravitational field) having magnitude and direction, both of which changes from point to point.
3. **Del operator (∇)**
   - It is a differential operator
   - It is not a vector by itself
   - It operate on scalar and vector functions and the resulting function may be a vector or scalar function depending on the type of operation.

Rectangular (x,y,z), cylindrical (s,ϕ,z) and spherical polar(r, ϕ,θ) coordinate systems

- **Curvilinear coordinate system**
- **Coordinate transformation**
- **Partial differential calculus**

→ Advanced Engineering Mathematics
By ERWIN KREYSZIG

What we learned so far?

4. Operation with del (∇) operator:

- Gradient of <u>*scalar function F*</u> – Directional derivative..maximum change of the scalar function is along the direction of vector ∇F, which nothing but the direction of outward surface normal vector; Advantage: A vector can be obtained from a scalar function which can be handled more easily than a vector.

- Divergence of a <u>*Vector function*</u> **A** - Gives the measure of the vector function's spread out at a point- is solenoidal or divergenceless when divergence of the vector is zero which means that flux of the such vector field entering into a region is equal to that leaving the region, a condition known as incompressibility; also gives an idea about source ($\nabla.\mathbf{A}>0$) means vector diverge and sink($\nabla.\mathbf{A}<0$) means vector converge.

- Curl of a <u>*Vector function*</u> **A**– regarding the rotation of the vector and the vector function is irrotational when curl of the vector is zero, such fields are known as conservative fields.

What we learned so far

5 Gauss's law in Electrostatic

$$\oiint \vec{\mathrm{E}} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

- Where E is the electric field vector, q is the charge and Φ is the electric flux
- Important tool since it permits the assessment of the amount of enclosed charge by mapping the **electric field normal** to the surface outside the charge distribution or vice versa

6 Poisson & Laplace Equations

$$\iiint \rho\, dV = q \qquad \vec{\nabla} \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0} \qquad \vec{\mathrm{E}} = -grad\, V = -\vec{\nabla} V$$

Where ρ is the electric charge density in the closed volume. And V is the electric potential

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

Poisson Equations

(a region with charge)

$$\nabla^2 V = 0$$

Laplace Equations

(a region free of charge)

What we learned so far

7. Continuity Equation

$$\oiiint \vec{\nabla} \cdot \vec{J}\, dV = - \oiiint \frac{d\rho}{dt}\, dV$$

The above equations is true for any volume. So we can put the integrands to be equal

$$\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$$ **Continuity Equation**

Current density flowing out of the closed volume is equal to the rate of decrease of charge within that volume.

Gauss's divergence theorem relates

a) Surface integral to volume integral
b) Volume integral to surface integral
c) Line integral to surface integral
d) Surface integral to line integral

Answer: A and B

Stokes theorem relates

a) Surface integral to volume integral
b) Volume integral to surface integral
c) Line integral to surface integral
d) Surface integral to line integral

Answer: C and D

Gauss theorem uses which of the following operations?

a) Gradient

b) Curl

c) Divergence

d) Laplacian

Answer: C

Stokes theorem uses which of the following operations?

a) Gradient

b) Curl

c) Divergence

d) Laplacian

Answer: B

Concept of magnetic flux (ϕ_B)

We can calculate the amount of electric field (**E**) that passes through a surface by a quantity called electric flux (Φ_E), which we have seen in the last lecture as

$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS}$$

Similarly, we can calculate the amount of magnetic field (**B**) that passes through a surface by a quantity called magnetic flux (Φ_B),

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS}$$

Gauss's laws of magnetostatics and Electrostatics

Gauss law of magnetostatic (Gauss's 2nd law) asserts that the net magnetic flux through any closed Gaussian surface is zero.

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS} = 0$$

Gauss's 2nd law for magnetic field (**1813**)

$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

Gauss's 1st law for electric field (**1813**)

Magnetic monopoles do not exist, where as electric monopoles do exist

- ✓ **Forms the basis of Maxwell's first and second equations of Electromagnetic theory**

Gauss law of magnetostatic (Gauss's 2nd law) asserts that the net magnetic flux through any closed Gaussian surface is

a) Infinity
b) Zero
c) Constant
d) None of the above

Maxwell’s equations of electromagnetism

- All relationships between electric & magnetic fields & their sources summarized by four equations.

- James Clerk Maxwell 13 June 1831 – 5 November 1879

Scottish scientist in the field of mathematical physics

ELECTROMAGNETISM-MAXWELL'S EQUATIONS

Now we have the necessary background for deriving the Maxwell's equations ☺

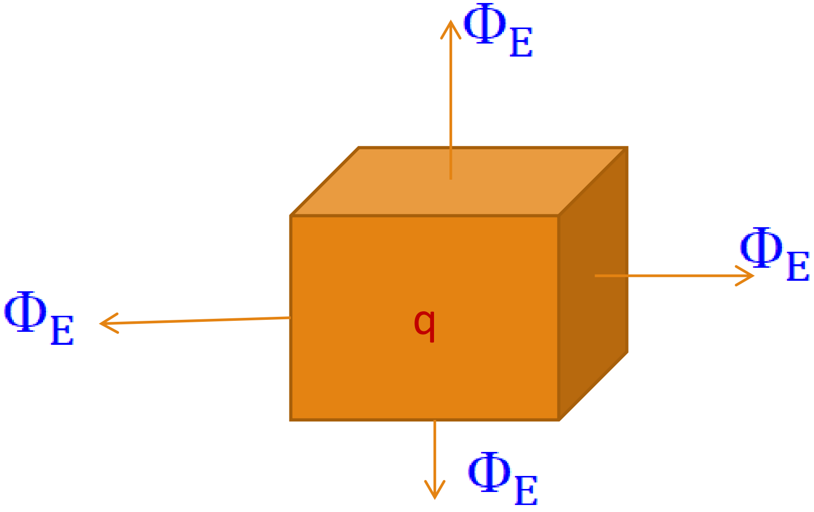
1. Derivation of Maxwell's First Equation

Let us consider the Gauss's law for Electrostatics, which relate the net electric flux Φ_E through a Gaussian surface to net enclosed electric charge;

$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

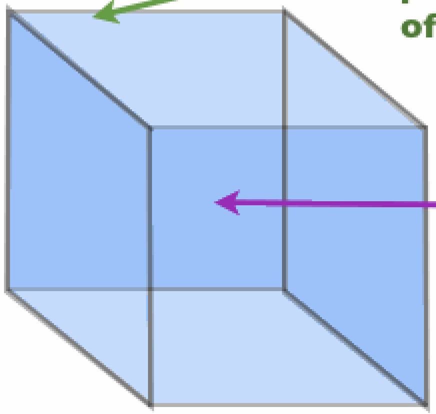
$$\oiint \varepsilon_0 \vec{E} \cdot \overrightarrow{dS} = q$$

But and $\vec{D} = \varepsilon \vec{E}$ and $q = \iiint \rho \, dV$



ELECTROMAGNETISM-MAXWELL'S EQUATIONS

$$\oiint \vec{D} \cdot \vec{dS} = \iiint \rho \, dV$$



Now apply Gauss's divergence theorem on the LHS..

$$\iiint \vec{\nabla} \cdot \vec{D} \, dV = \iiint \rho \, dV$$

This equation hold true for any arbitrary volume and for that, the integrands must be same. So we have now

$$\vec{\nabla} \cdot \vec{D} = \rho$$

Or

$$\text{div} \, \vec{D} = \rho$$

This is the Maxwell's FIRST EQUATION

ELECTROMAGNETISM-MAXWELL'S EQUATIONS

2. Derivation of Maxwell's 2nd Equation

Like electric flux Φ_E, magnetic flux Φ_B is defined as

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS}$$

But Gauss's law for Magnetostatics say Φ_B =0, flux of magnetic field B across any closed **surface** is zero

So, we have
$$\oiint \vec{B} \cdot \overrightarrow{dS} = 0$$

Now apply Gauss's divergence theorem on the LHS..

$$\iiint \vec{\nabla} \cdot \vec{B} \, dV = 0$$

ELECTROMAGNETISM-MAXWELL'S EQUATIONS

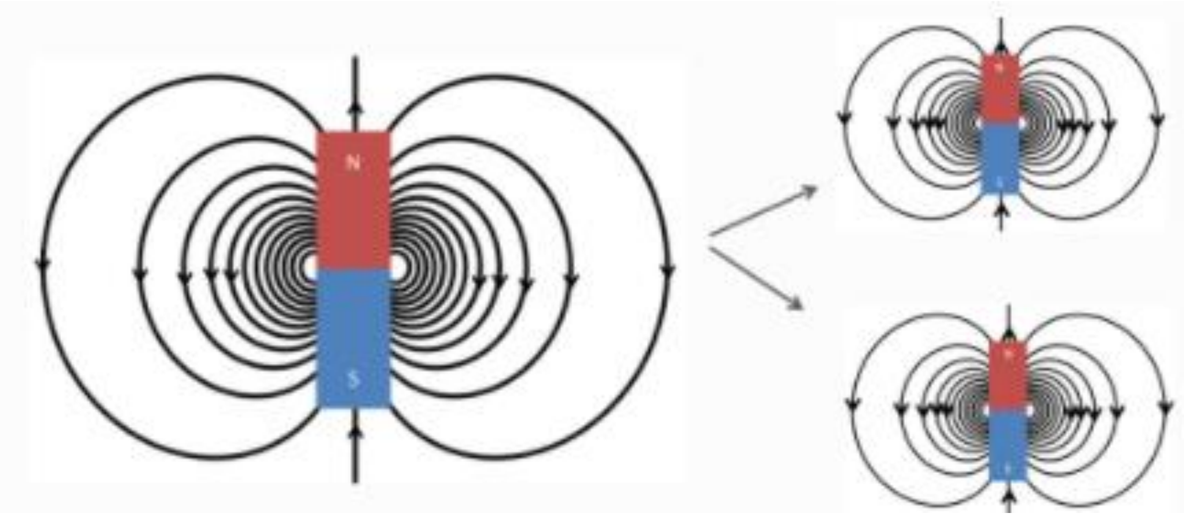
above equation hold true for any arbitrary volume and for that, the integrands must be zero. So we have now

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \textit{or} \qquad \operatorname{div} \vec{B} = 0$$

This is the Maxwell's SECOND EQUATION

The **magnetic line of force** are either closed or go off to infinity, the number of magnetic lines entering any **volume** is exactly equal to the number of lines leaving volume.

ELECTROMAGNETISM-MAXWELL'S EQUATIONS



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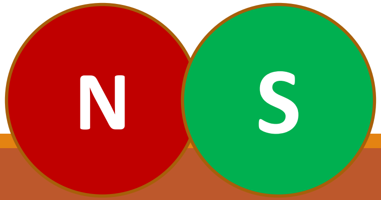
$$\vec{\nabla} \cdot \vec{B} = 0$$

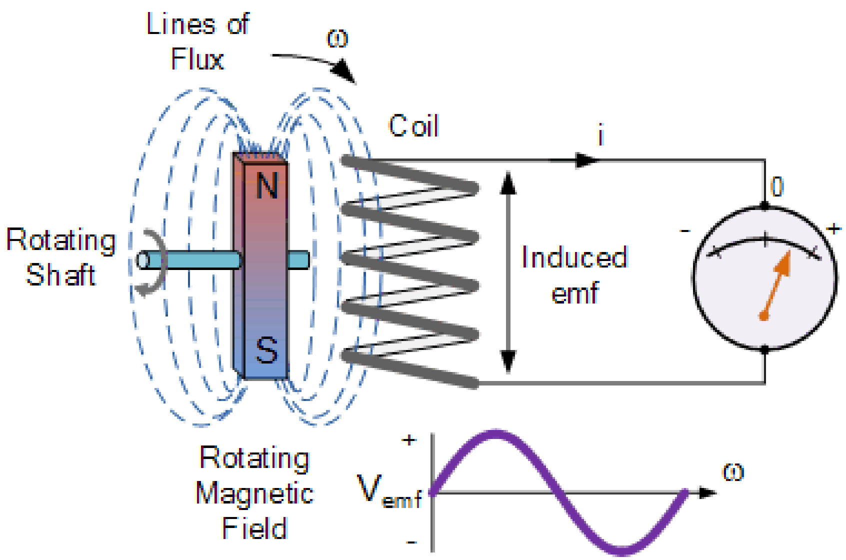
NON-EXISTANCE OF MAGETIC MONOPOLE. Lowest unit is 'dipole'. Magnetic fields have no source or sink but it is always a **solenoidal vector field**.



Magnetic dipoles exist but monopoles do not exist

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION (1831)

Faraday's first law: Whenever the magnetic flux (ϕ_B) linked with a circuit changes an emf (**E_{emf}**) is induced in the circuit.



Faraday's second law: This induced emf, **E_{emf}** is equal to the negative rate of change of magnetic flux (ϕ_B) with time linked with the circuit.

$$E_{emf} = -\frac{\partial \phi_B}{\partial t}$$

Negative sign indicating that induced emf (**E_{emf}**) always **opposes** the change in flux