UNIT 5 SOLID STATE PHYSICS

LECTURE 5

Effective mass

- Electrons in the solid are not completely free and it interact with ions or lattice
- Response to external force (electric or magnetic) is different compared to free electron
- Effective mass (m*) concept introduced to account the difference between free electrons and the electrons in the crystal lattice.
- m* is different from the free electron mass (m) and depends on the nature of crystal lattice
- m* varies with the direction of motion of the electron the lattice

Suppose an electron is moving along the x-axis in the crystal due to the application of an electric field **E**. So the force on the electron is given by

$$F = e\mathbf{E} \qquad \text{(i)}$$

If this electron move a distance 'dx' in a time 'dt' due to this force, then the work done by this electron is given by

$$\mathrm{dW} = \mathrm{d}E = F dx = e\mathbf{E}\,\mathrm{v}\, dt \qquad \text{(ii)} \qquad \therefore v = \frac{dx}{dt}$$

However, according to quantum mechanics the velocity of this electron (v) is nothing but the group velocity (v_g) and is given by

$$\mathrm{v} = \mathrm{v_g} = \frac{\partial\omega}{\partial k} \qquad \text{(iii)}$$

$$\mathrm{v} = \mathrm{v_g} = \frac{2\pi}{h}\frac{\partial E}{\partial k} \qquad \text{(iii a)}$$

$$\mathrm{dW} = dE = \mathrm{e}\mathbf{E}\frac{\partial \mathrm{w}}{\partial \mathrm{k}}\mathrm{dt} \qquad \text{(iv)}$$

where 'ω' is the angular frequency and 'k' is the propagation constant. Substitute (iii) into (ii)

But the energy of the particle E according to Einstein and De Broglie is

given by $E = h\nu = \frac{h}{2\pi}\omega$ (v)

Differentiate eqn (v) with respect to 'k'

$$\frac{dE}{dk} = \frac{h}{2\pi}\frac{d\omega}{dk} \quad \text{or} \quad dE = \frac{h}{2\pi}\mathrm{v_g}\, dk \qquad \text{(vi)}$$

$$\mathrm{d}E = Fdx = e\mathbf{E}\,\mathrm{v}\, dt \qquad \text{(ii)}$$

By comparing equation (vi) and (ii) we get $\frac{h}{2\pi}\mathrm{v_g}\, dk = e\mathbf{E}\,\mathrm{v}\, dt$ but $\mathrm{v} = \mathrm{v_g}$

$$\frac{h}{2\pi}dk = e\mathbf{E}\, dt \quad \text{or} \quad \frac{dk}{dt} = \frac{2\pi}{h}e\mathbf{E} \qquad \text{(vii)}$$

$$\frac{dk}{dt} = \frac{2\pi}{h} e\mathbf{E} \qquad \text{(vii)}$$

$$\mathrm{v} = \mathrm{v_g} = \frac{2\pi}{h}\frac{\partial E}{\partial k} \qquad \text{(iii a)}$$

differentiate with respect to time ‘t’

$$\frac{\mathrm{dv}}{\mathrm{dt}} = \frac{\mathrm{dv_g}}{\mathrm{dt}} = \frac{2\pi}{h}\frac{\partial^2 E}{dt\,\partial k}$$

or

$$\frac{\mathrm{dv_g}}{\mathrm{dt}} = \frac{2\pi}{h}\frac{d^2E}{dk^2}\frac{dk}{dt} \qquad \text{(viii)}$$

substitute (vii) in (viii)

$$\frac{\mathrm{dv_g}}{\mathrm{dt}} = \frac{2\pi}{h}\frac{d^2E}{dk^2}\frac{2\pi}{h} e\mathbf{E}$$

$$\frac{\mathrm{dv}}{\mathrm{dt}} = \left(\frac{4\pi^2}{h^2}\frac{d^2E}{dk^2}\right) e\mathbf{E} \quad \text{or} \quad \frac{\mathrm{dv}}{\mathrm{dt}} = \left(\frac{4\pi^2}{h^2}\frac{d^2E}{dk^2}\right) F \qquad \text{(ix).}$$

This eqn connect the acceleration to the force, through a proportionality constant, $\left(\frac{4\pi^2}{h^2}\frac{d^2E}{dk^2}\right)$

$$F = ma = m\frac{d\mathrm{v}}{dt} \quad \text{or} \quad \frac{d\mathrm{v}}{dt} = \frac{\mathrm{F}}{\mathrm{m}} \qquad \ldots\text{(x)}$$

By comparing eqns (ix) and (x), we get $\frac{1}{m*} = \left(\frac{4\pi^2}{h^2}\frac{d^2E}{dk^2}\right)$

E is the energy of the electron and **E** is the electric field applied

m* is the effective mass of the electron in the crystal lattice

Another Approach to get m*

According to de Broglie, electron moving with a velocity v is having an energy $m\text{v}^2 = h\nu$ where h is the Planck's constant , ν is the frequency (v/λ) and λ is the de Broglie wavelength of the moving electron

$$m\text{v}^2 = \frac{h\text{v}}{\lambda} \qquad m\text{v} = \frac{h}{2\pi/_k} \qquad m\text{v} = \frac{h}{2\pi/_k} \Rightarrow p = \hbar k$$

$$\text{E} = \frac{1}{2}m\text{v}^2 = \frac{1}{2m}m^2\text{v}^2 = \frac{\text{p}^2}{2m} \qquad \text{E} = \frac{\hbar^2 k^2}{2m^*}$$

For free electron theory m*= m

Differentiating the above equation *w.r.t* k twice we get

$$\frac{dE}{dk} = \frac{\hbar^2}{2m^*}2k \longrightarrow \frac{d^2E}{dk^2} = \frac{\hbar^2}{m^*}$$

$$\therefore \quad m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

or

$$\frac{1}{m*} = \frac{4\pi^2}{h^2}\frac{d^2E}{d^2k}$$

Effective mass is inversely proportional to the curvature ($\frac{d^2E}{d^2k}$) of the band

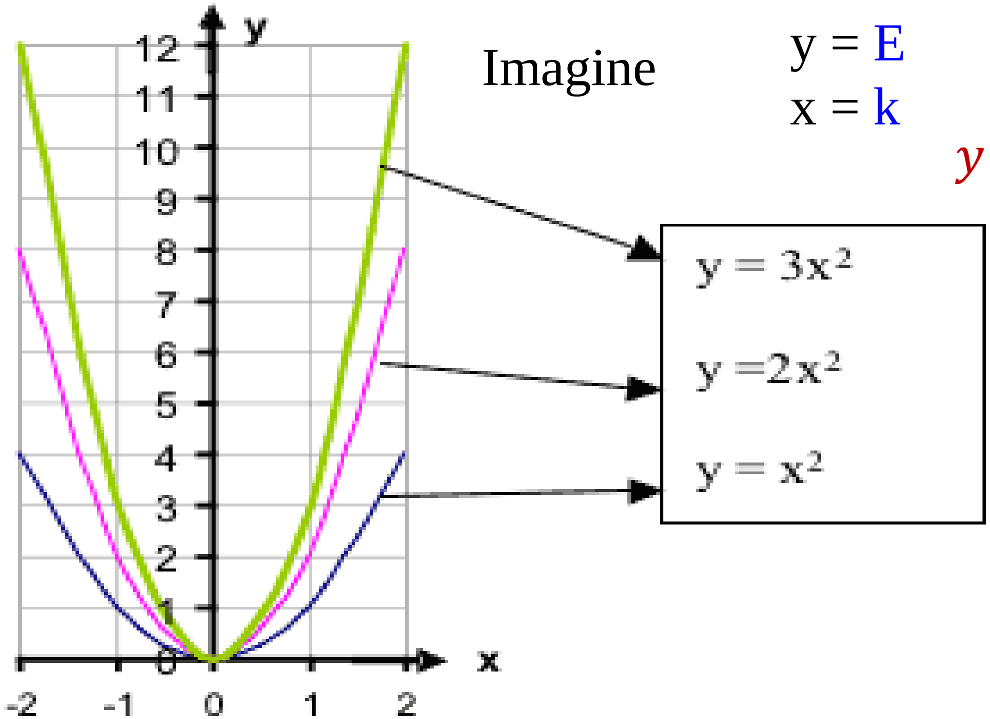
Energy of electron in the band $E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m} k^2$

Curvature $\left(\frac{d^2E}{d^2k}\right)$??

Imagine

y = E
x = k

$y = c\,x^2$, parabolic equation!

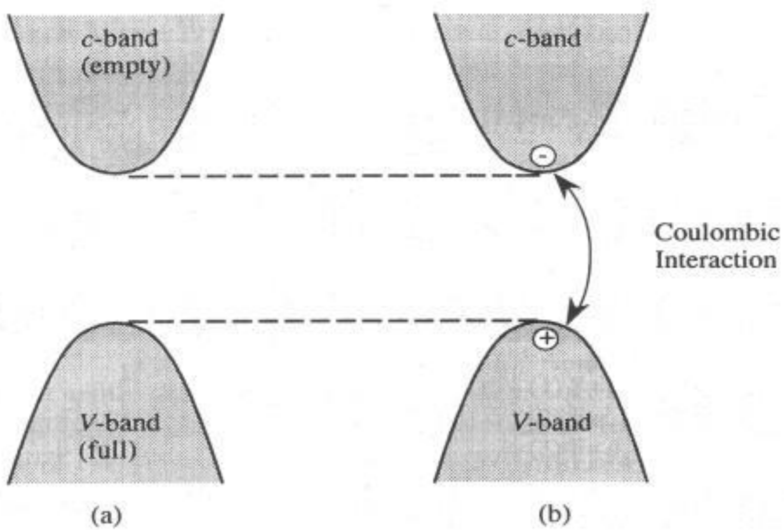


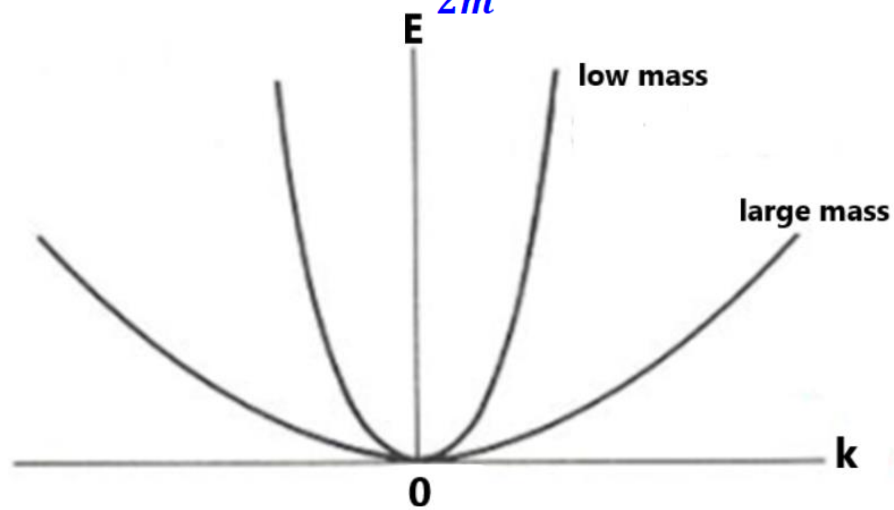
If the shape of the E-k diagram of the same electron is different in different energy bands then the mass of the electron must be different??. . **As mass of electron increases the curvature of E-k curve also increases.**

Effective mass concept formulated to explain it..

- There are crystals in which the effective mass of the carriers (electrons) is much larger or much smaller than the free electron m_0.
- The effective mass may be anisotropic (directional dependency), and it may even be negative??. Holes ??
- The important point is that the electron in a periodic potential is accelerated relative to the lattice in an applied electric or magnetic field as if its mass is equal to an effective mass.

$$E = \frac{\hbar^2}{2m^*}k^2$$





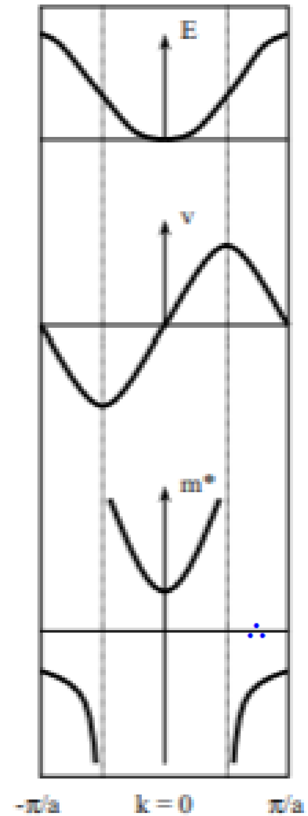
The concept of effective mass (**m***) introduced to explain the curvature of the E-k diagram -

E-k Diagram, Velocity and Effective Mass

The figure depicts the graphs for E, dE/dk, and d^2E/dk^2 for CB in the first BZ. At k=0, electron has a constant positive value and it rises rapidly as k value increases. After experiencing a singularity (infinite mass) the effective mass becomes negative up to the top of the first BZ.

Therefore:

- m^* is positive near the bottoms of all bands,
- m^* is negative near the tops of all bands.



$$E = \frac{\hbar^2}{2m^*}k^2$$

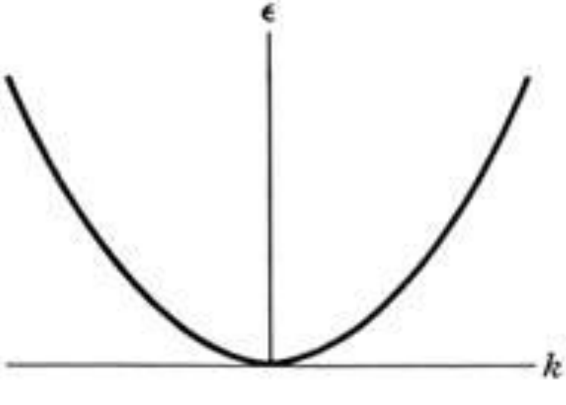
$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

$$\therefore \; m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

BZ-Brillouin zone: first -π/a to +π/a

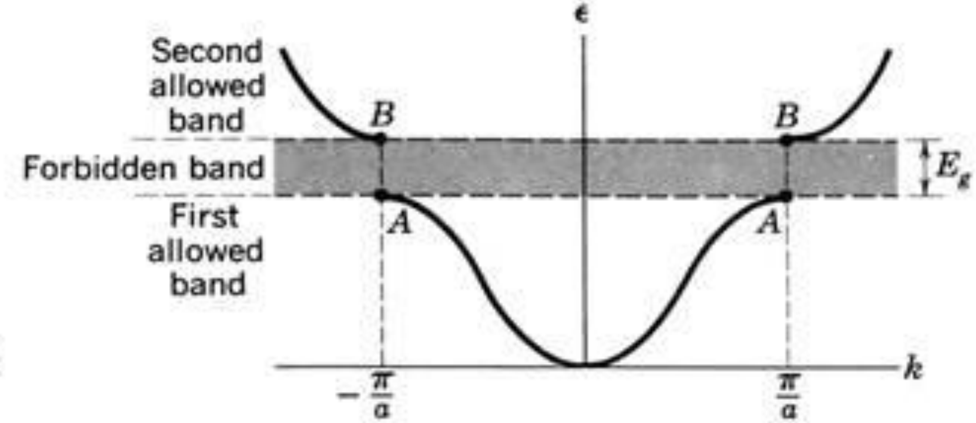
second - between -2π/a and -3π/a to +2π/a and 3π/a

$$E = \frac{\hbar^2}{2m^*}k^2$$



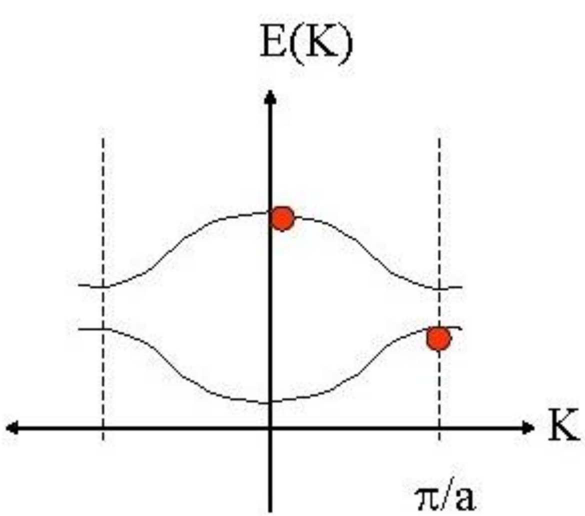
m*=m is a constant for free electron

$$E = \frac{\hbar^2}{2m^*}k^2$$



m* is changing at different energies of the electron or along the energy band in the case of ion-electron interaction?…

The Hole

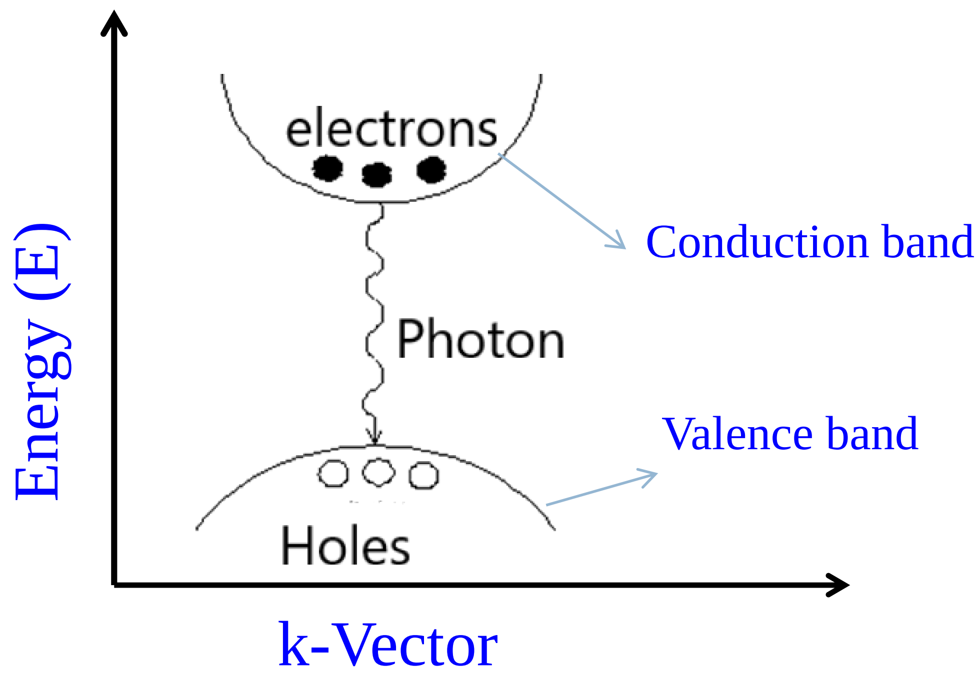


- The hole can be understood as an electron with negative effective mass
- An electron near the top of an energy band will have a negative effective mass
- A negatively charged particle with a negative mass will be accelerated like a positive particle with a positive mass (a hole!)
- Without the crystal lattice, the hole cannot exist. It is an artifact of the periodic potential created by the crystal.

Direct Band gap semiconductors

In E-k diagram, when the lowest-energy point of the conduction band lies directly above the highest-energy point of the valence band in a semiconductor, the movement of a electron across the band gap conserves momentum and the gap is classified as "direct"

Examples: GaAs, InP, CdS are direct band gap semiconductors

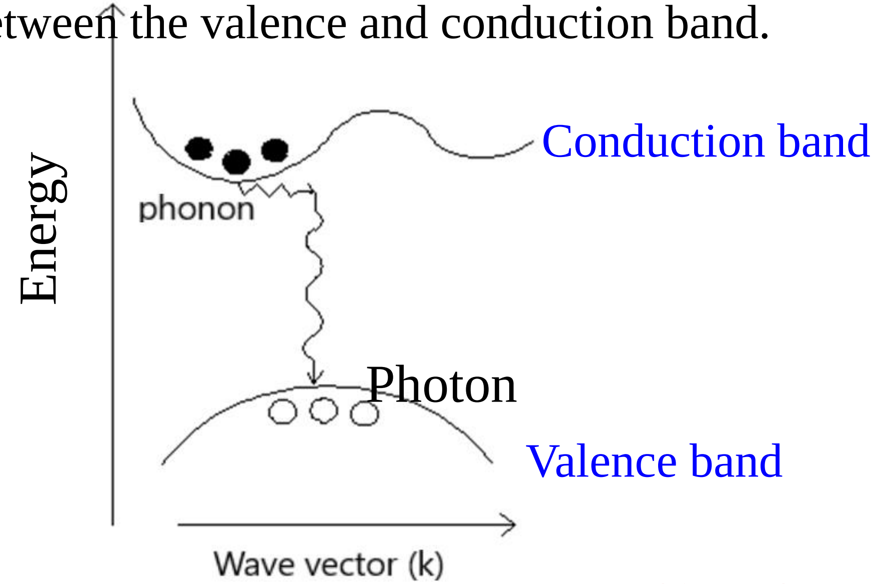


optical transitions more likely in materials with direct band gap compared to indirect bandgap.

Used in solar cell, LED, photo diode etc

Indirect Band gap semiconductors

When the highest-energy point of the valence band is not directly below the lowest-energy point in the conduction band , a phonon must carry away the momentum off-set if a transition is to occur between the valence and conduction band.



Si or Ge are indirect bandgap semiconductors

Indirect band gap: Transition assisted with phonon (quantized lattice vibrations)

- This causes optical transitions to be much less likely in materials with indirect band gap compared to direct bandgap semiconductors.

COMPARISON- Direct vs Indirect

Direct band gap

- Bottom of conduction band(CB) lies directly above top of the valance band(VB)
- Electron recombines with holes gives photon
- The photon have energy equal to the band gap
- It is radiative recombination
- It is used to build light emitting devices
- Eg) GaAs

Indirect band gap

- Bottom of CB not lies directly above the top of VB
- So for the conservation of momentum electron losses its energy by interacting with phonons
- The electron and hole recombines getting energy as form of heat
- It is Non radiative recombination
- Eg) Si,Ge

Under forward bias condition, the depletion layer thickness of p-n junction

(a) Increase
(b)Decrease
(c) remains same
(d) None of the above

Ans: B

Which is a direct band gap semiconductor?

a) Silicon (Si)
b) Germanium (Ge)
c) GaAs
d) Aluminum Arsenide (AlAs)
e) Gallium Phosphide (GaP).

Ans: C

To measure light intensity we use

(a) LED with forward bias
(b) LED with reverse bias
(c) Photodiode with reverse bias
(d) Photodiode with forward bias

Ans: C

Quantized lattice vibration is known as

(a) Photon
(b) Polaron
(c) Polariton
(d) Phonon

Ans: D

A polaron is a particle comprised of an electron surrounded by a cloud of phonons (i.e. lattice vibrations).

Polaritons are particles made up of a photon strongly coupled to an electric dipole.

Which is an in-direct band gap semiconductor?

a) Crystalline Silicon (c-Si)
b) Gallium Arsenide (GaAs)
c) Amorphous Silicon (a-Si)
d) Indium Arsenide (InAs).

Ans: A

To generate light we use

(a) LED with forward bias
(b) LED with reverse bias
(c) Photodiode with reverse bias
(d) Photodiode with forward bias

Ans: A

PHY110 – ENGINEERING PHYSICS

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