UNIT 4 QUANTUM MECHANICS

LECTURE 2

LECTURE 1 - Revision

Need of quantum mechanics

To overcome the limitation of classical mechanics

Classical mechanics failed to explain….

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1) Stability of atom
2) Spectral distribution of black body radiation

   Planck's quantum hypothesis

3) Origin of discrete spectra of atoms
4) Photoelectric effect

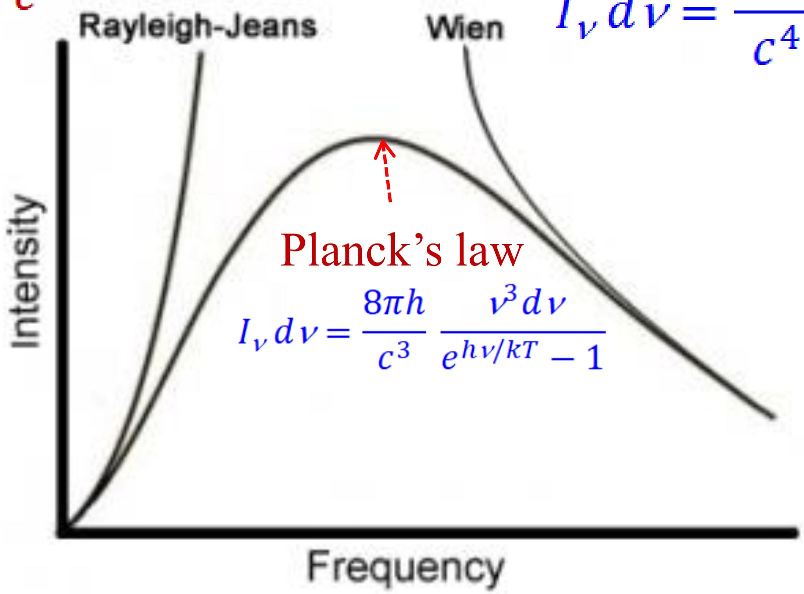
   particle nature of light by Einstein

5) Compton effect
6) Raman effect

1. Black body radiation and Planck's hypothesis

$$I_\nu \, d\nu = \frac{8\pi \nu^2}{c^3} kT \, d\nu$$

$$I_\nu \, d\nu = \frac{A \nu^3}{c^4} e^{-B\nu/T} \, d\nu$$



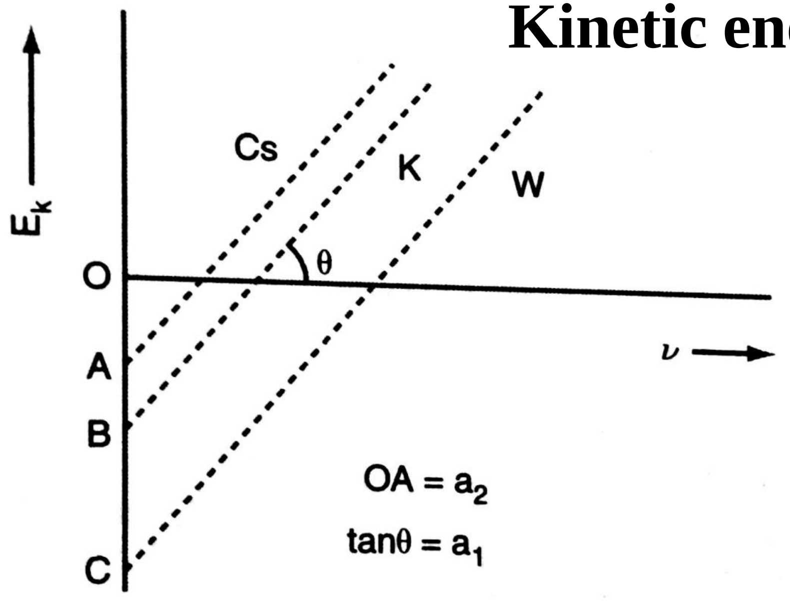
Planck's law

$$I_\nu \, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 \, d\nu}{e^{h\nu/kT} - 1}$$

- ✓ Rayleigh-Jeans can be deduced from Planck's law for low frequency (long wavelength) and high temperature
- ✓ Similarly Wien's law can be deduced from Planck's law for high frequency (low wavelength) and low temperature

2. Photoelectric effect

Discovered by Hertz but explained by Einstein



Kinetic energy of the ejected electron E_k

$$E_k = h\nu - h\nu_0$$

Work function

Incident photon energy

This effect says about the emission/ejection of electrons from the metal surface upon irradiation with light waves

- We knew the wave nature of light or electromagnetic radiation with the help of Maxwell's equation- electromagnetic theory
- Then assumed particle nature of light with the Planck's black body radiation
- With Einstein's photoelectric equation we experimentally proved particle nature of light

✓ *Importance of quantum mechanics and quantum/particle nature of light*

Rayleigh-Jeans law is deduced from Planck's radiation formula under the condition of

a) **High frequency and low temperature**
b) **Low frequency and high temperature**
c) **High frequency and high temperature**
d) **Low frequency and low temperature**

Ans: B

Wien law is deduced from Planck's radiation formula under the condition of

a) **High frequency and low temperature**
b) **Low frequency and high temperature**
c) **High frequency and high temperature**
d) **Low frequency and low temperature**

Ans: A

A black body appears black because it

a) Does not reflect light
b) Does not transmit light
c) Does absorb light
d) All of the above

D

If we heat a black body, it does

a) radiate electromagnetic radiation in the visible region only
b) radiate electromagnetic radiation in the infrared region only
c) radiate electromagnetic radiation in the ultra violet region only
d) radiate electromagnetic radiation in the entire EM spectrum

D

According to Planck's hypothesis, the frequency of radiation from a black body is not continuous, but only in the multiples of a small unit called

a) Phonon
b) Proton
c) Photon
d) Polaron

C

Who gave the correct theoretical explanation for photoelectric effect by considering the particle nature of light?

a) Einstein
b) Planck
c) Hertz
d) Maxwell

A

Who gave the name ‘photon’ for the quantum of light?

a) Planck
b) Rayleigh
c) Wien
d) Einstein

D

Which of the following phenomena show the particle nature of light?

a) Photoelectric effect
b) Interference
c) Diffraction
d) Polarization

A

Einstein's photoelectric equation relate the kinetic energy of the ejected electron to the incident photon energy and --------??

a) Work function
b) Wave function
c) Both a and b
d) None of these

Ans: A

Saturated photoelectric current depends

a) on the energy of the incident radiation
b) on the intensity of the incident radiation
c) Both a and b
d) None of the above

Ans: B

What we know about particle and wave

Concept of particle

i. Mass: has definite mass
ii. Position; located to a place or point
iii. Velocity: move with certain velocity
iv. Momentum (p): mass and velocity so it has momentum
v. Energy (E): it has PE, KE

Concept of wave

i. Phase of wave: Phase gives instantaneous position and direction
ii. Frequency and wavelength: main characteristics of a wave
iii. Amplitude: Gives the intensity of the disturbance in the medium

- E and p are the characteristics of the particle
- ν and λ are the characteristics of the wave

Can we find a mathematical formulation to connect these two?

Is it Possible? Can we put it in a mathematical relation

If we think about wave and particle in the above description, it is very difficult accept the wave-particle duality.

Superposition of wave? Yes

Superposition of particle? No..two particles can not have same position at the same time

If we can assign these,

i. Phase of wave:
ii. Frequency and wavelength:

Then matter can be considered as wave as well

If we can assign these

i. Mass: difficult to think mass being associated with a wave..
ii. Energy
iii. Momentum
iv. Velocity

Then, a wave can be considered as particle as well

We will see that now for the radiation first!

Dual nature of radiation

In the case of radiation (Plank's theory), we know Energy, $E = h\nu$

Now will go to Einstein special theory of relativity and that famous equation $E = mc^2$

De Broglie hypothesized that the two energies would be equal

$$mc^2 = h\nu = \frac{hc}{\lambda} \longrightarrow mc = \frac{h}{\lambda}$$

But *mc* is nothing but the momentum of photon, $p = \frac{h}{\lambda}$

.. by mixing Einstein's famous matter-energy relation with Planck's famous quantum oscillator theory.. Wavelength of the wave is related to the momentum of its particle through the Planck's constant ..

Now we see the relation for the mass of photon

Dual nature of radiation

From the same equation of energy, we have $mc^2 = h\nu$ we can have the relation for the mass of photon,

$$m = \frac{h\nu}{c^2}$$

the photon has no rest mass but does have relativistic mass

So photons has mass ($m = \frac{h\nu}{c^2}$), momentum ($p = \frac{h}{\lambda}$), energy ($E = mc^2$) wavelength (λ)and frequency (ν)

This was proposed by de Broglie in 1923 in his doctoral thesis

Since E, m and P are particle characteristics, and we got it for radiation, a wave having wavelength and frequency!! The dual nature has a mathematical basis now for radiation..

Dual nature of MATTER

If a wave can be so then why not a particle?

de Broglie extended matter concept of radiation and applied to particles as well..

Because real particles do not travel at the speed of light, De Broglie used velocity (v) for the speed of light (c).

$$E = mv^2 = h\nu \quad \Rightarrow \quad mv = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

λ is the de Broglie wavelength of the matter wave of the particle of mass m moving with velocity v and momentum p

The **de Broglie wavelength** is the **wavelength**, λ, associated with a massive particle and is related to its momentum, p, through the Planck constant, h: In other words, you can say that matter also behaves like waves.

Dual nature of MATTER- de Broglie wavelength

λ can be expressed in various ways, depending on the process by which the particle gain energy for the travel

1. If particle is accelerated through the kinetic energy

$$E_k = \frac{1}{2} m v^2$$

$$= \frac{1}{2m}(mv)^2 = \frac{p^2}{2m} \qquad \therefore \; p = \sqrt{2mE_k}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

Dual nature of MATTER- de Broglie wavelength

2. If a charged particle having charge (q) is accelerated through electrostatic potential V, then the kinetic energy is expressed as

$$E_k = \frac{1}{2} m v^2 = qV$$

$$E_k = \frac{1}{2m} (mv)^2 = qV$$

$$(mv)^2 = p^2 = 2mqV$$

$$p = \sqrt{2mqV}$$

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\sqrt{2mqV}}$$

Dual nature of MATTER- de Broglie wavelength

3. If the particle having mass (m) is accelerated by means of thermal energy

$$E_k = \frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$E_k = \frac{1}{2m}(mv)^2 = \frac{3}{2}kT$$

$$(mv)^2 = p^2 = 3mkT$$

$$\therefore \quad p = \sqrt{3mkT}$$

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\sqrt{3mkT}}$$

4. If the particle having rest mass (m_0) is moving with a velocity(v) comparable to the speed of light (c)

In this case mass is not constant, and relativistic equation come into play and instantaneous mass is given by

$$m = \frac{m_0}{\sqrt{1 - (v/c)^2}} \qquad p = mv = \frac{m_0 v}{\sqrt{1 - (v/c)^2}}$$

$$\lambda = \frac{h}{p} = \frac{h\sqrt{1 - (v/c)^2}}{m_0 v}$$

Although de Broglie was credited for his hypothesis, he had no actual experimental evidence for his conjecture. In 1927, Clinton J. Davisson and Lester H. Germer shot electron particles onto on to a nickel crystal. What they saw was the diffraction of the electron similar to waves diffraction against crystals (x-rays). In the same year, an English physicist, George P. Thomson fired electrons towards thin metal foil providing him with the same results as Davisson and Germer.

Properties of matter-wave

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

i. Lighter particles have large de Broglie wavelength than heavier one

ii. Smaller the velocity of the particle, the greater is the de Broglie wavelength associated with it

iii. Matter waves are generated only when particle is in motion. [$v=0$, $\lambda= \infty$]

iv. Matter waves are not electromagnetic ..i.e. independent of charge

v. Velocity of the matter-wave is not constant. It depends on the velocity of the particle, while velocity of the electromagnetic wave is constant

- Velocity of matter wave may be greater than the velocity of light. Difficult to believe and hence phase velocity and group velocity came into play..
- Wave-particle duality introduce the concept of uncertainty, This concept suggest that if the particle nature of the matter becomes certain, the wave nature becomes uncertain and vice versa.

We will see in the next two classes……

UNIT 4-Quantum Mechanics

Text Book: ENGINEERING PHYSICS by HITENDRA K MALIK AND A K SINGH, MCGRAW HILL EDUCATION, 1st Edition, (2009)

References:

- ENGINEERING PHYSICS by B K PANDEY AND S CHATURVEDI, CENGAGE LEARNING, 1st Edition, (2009).

- ENGINEERING PHYSICS by D K BHATTACHARYA, POONAM TONDON OXFORD UNIVERSITY PRESS.