PHY 110 Engineering Physics

Lecture 1

UNIT 1 – Electromagnetic theory

Physical Quantity: Any quantity that can be measured/determined and has a magnitude and unit.
Examples: Mass, weight, distance, length displacement, speed, velocity, pressure, temperature, force, acceleration, energy, current ..etc..

SCALAR

Physical quantity that has only magnitude and has no direction

VECTOR

Physical quantity that has both magnitude and direction

Ways of writing vector notation

$$\mathbf{F} = m\mathbf{a}$$

$$\vec{F} = m\vec{a}$$

$$\underline{F} = m\underline{a}$$

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Representation of a vector, Unit Vector and Rectangular Resolution of vectors

➢ A vector $\vec{r}$ may be represented as

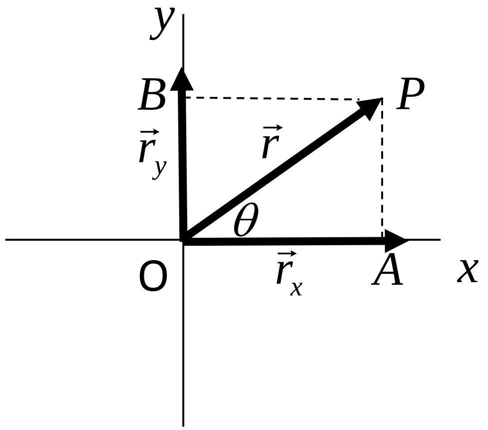
$$\vec{\boldsymbol{r}} = r\,\hat{r}$$

Here, $\vec{\boldsymbol{r}}$ is the vector, r is the magnitude of the vector and $\hat{r}$ is the unit vector.

➢ Thus, r gives the magnitude and $\hat{r}$ gives the direction of the vector.

We may then write $\hat{r} = \frac{\vec{r}}{|r|}$.

➢ Unit vectors have unit magnitude but a definite direction.



From the right angled triangle OAP we may write the magnitude of the given vector,

$$OP^2 = OA^2 + AP^2$$

$$r^2 = r_x^2 + r_y^2$$

$$r = \sqrt{r_x^2 + r_y^2}$$

- Let OX and OY be the two rectangular axes. Let $\hat{\imath}$ and $\hat{\jmath}$ are the two unit vectors along x and y respectively.
- If $\overrightarrow{OP} = \hat{r}$ and the coordinates of P be (x,y), then,

$\overrightarrow{OA} = r_x\hat{\imath}$ and $\overrightarrow{OB} = r_y\hat{\jmath}$.

- Again by the law of vector addition (For convenience bold face will be considered as vector notation)

$$\boldsymbol{OP} = \boldsymbol{OA} + \boldsymbol{AP}$$

Or,

$$\boldsymbol{r} = r_x\hat{\imath} + r_y\hat{\jmath}$$

This is known as rectangular resolution of vectors.

From the same triangle OAP

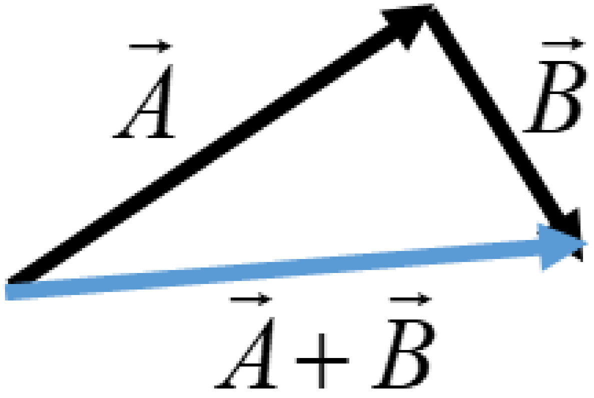
$$\tan(\theta) = \frac{AP}{OA}$$

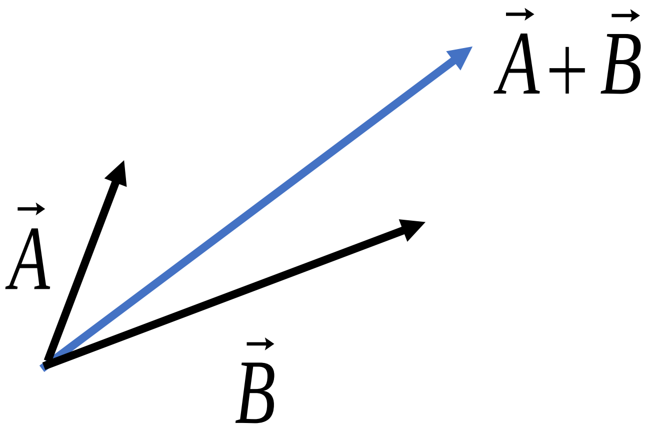
$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

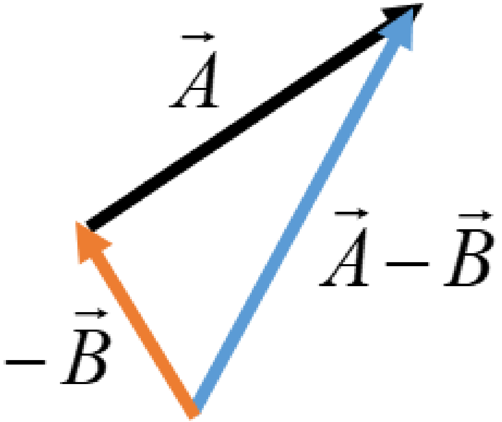
Vector algebra:

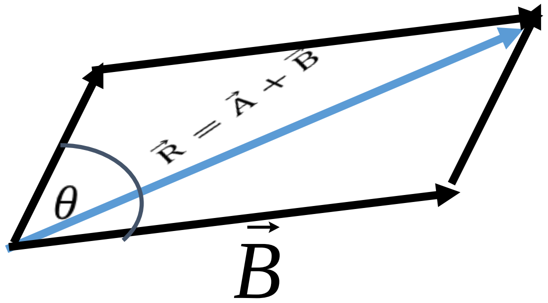
Algebraic operation on Vectors

1. **Addition**
2. **Subtraction**
3. **Products**
    1. **Dot product**
    2. **Cross product**









Magnitude of the null vector is

a) 1
b) 0
c) -1
d) ∞

Magnitude of the unit vector is

a) 1
b) 0
c) -1
d) ∞

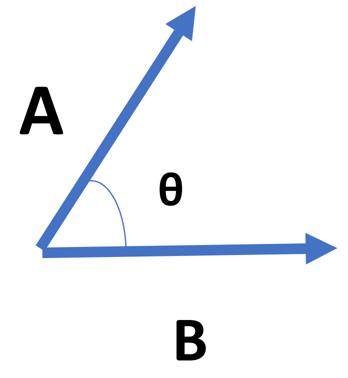
Dot Product:

- The dot product (scalar product) of two vectors can be thought of as the projection of one onto the direction of the other.

A.B=|A||B|cosθ

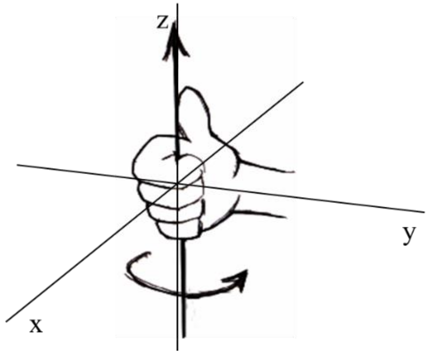
In the component form

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$



- Dot product of two vectors is a scalar

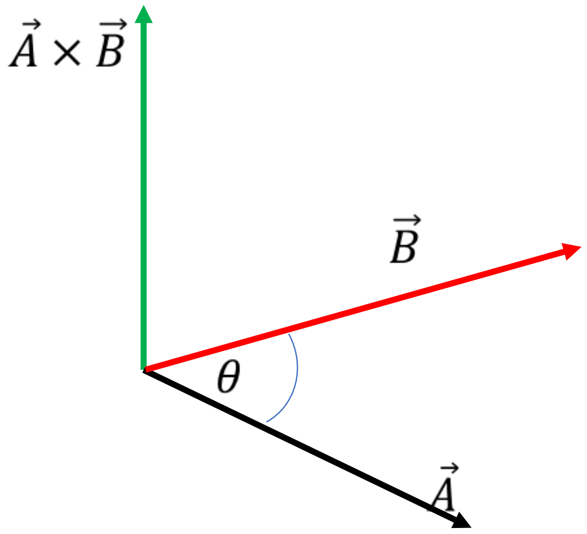
Vector Product:



- The cross product of two vectors says something about how perpendicular they are. You will find it in the context of rotation, or twist.

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$



- ➢Cross product of two vectors is vector
- ➢Lies perpendicular to the both **A** and **B**

- Cross product of two vectors is a
    a) Vector quantity
    b) Scalar quantity

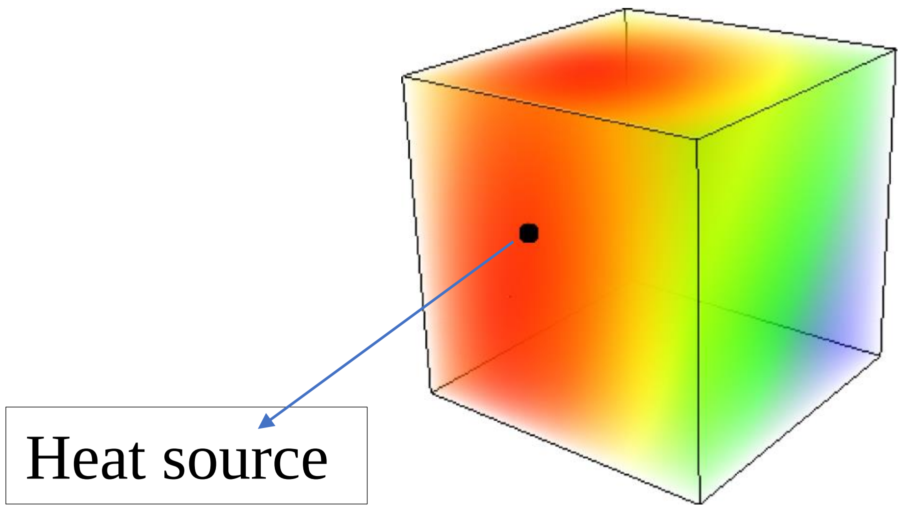
- Dot product of two vectors is a
    a) Vector quantity
    b) Scalar quantity

Scalar and Vector Field

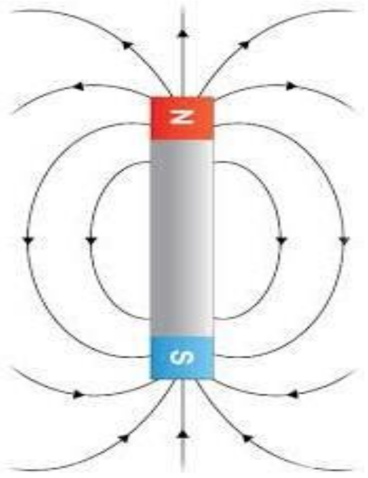
A continuous function of position of a point in a region is called a point function. The region of space in which It specifies a physical quantity is known as a field. These fields are classified into two groups:

Scalar Field: It is defined as that region of space where each point is associated with a scalar point function. That is a continuous function which gives the value of a physical quantity such as, temperature, potential etc. In a scalar field, all the points having the same scalar physical quantity are connected by the means of surfaces called equal or the level surfaces.

Vector Field: A vector field is specified by a continuous vector point function having magnitude and direction and both of which may change from point to point in a given region of field. The field is represented by vector lines or lines of surfaces. The tangent at a vector line gives the direction of the vector at that point.



Temperature field or a scalar field



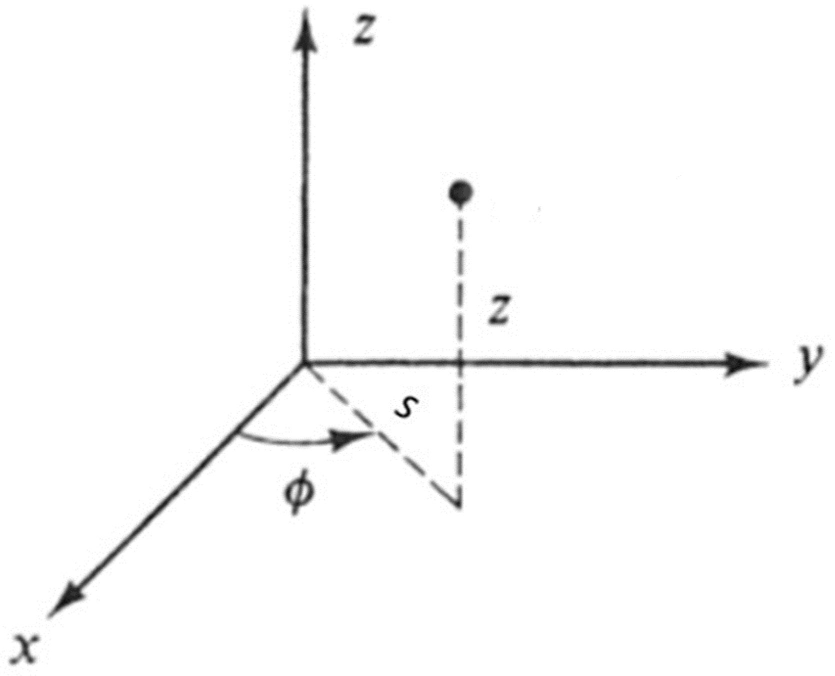
Magnetic field of bar magnet

Cylindrical Coordinates:

$x = s\cos\phi = f\,(\mathbf{s},\boldsymbol{\phi})$

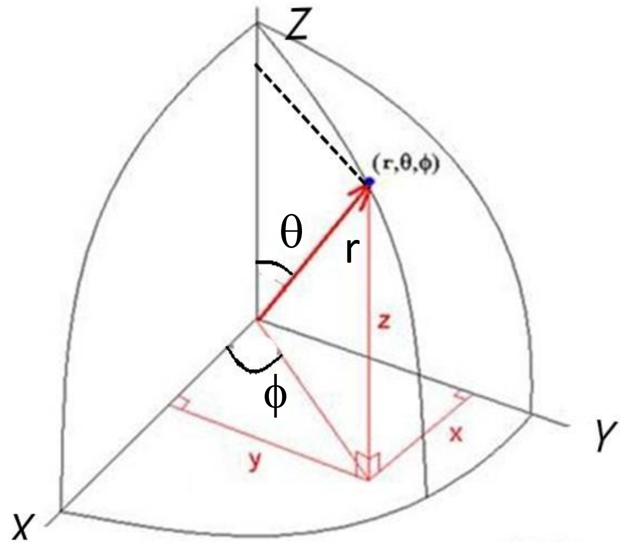
$y = s\sin\phi = f\,(\mathbf{s},\boldsymbol{\phi})$

$z = z$



Spherical Polar coordinates:

- r-projection = r Sinθ= f **(r,θ)**
- x=r-projection. Cosϕ = r Sinθ Cosϕ= f **(r,θ,ϕ)**
- y=r-projection.Sinϕ = r Sinθ Sinϕ= f **(r,θ,ϕ)**
- z= r Cosθ= f **(r,θ)**



Del operator:

Study the rate of change of scalar and vector fields.

In Cartesian coordinate:

$$\vec{\nabla} = \hat{\boldsymbol{i}}\frac{\partial}{\partial x} + \hat{\boldsymbol{j}}\frac{\partial}{\partial y} + \widehat{\boldsymbol{k}}\frac{\partial}{\partial z}$$

In Cylindrical coordinate:

$$\vec{\nabla} = \hat{s}\frac{\partial}{\partial s} + \hat{\boldsymbol{\phi}}\frac{1}{\mathrm{s}}\frac{\partial}{\partial \phi} + \hat{\boldsymbol{z}}\frac{\partial}{\partial z}$$

In Spherical polar coordinate:

$$\vec{\nabla} = \hat{\boldsymbol{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{\mathrm{r}}\frac{\partial}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{\mathrm{r}\sin\theta}\frac{\partial}{\partial \phi}$$

Gradient:

- In a Scalar field, the maximum rate of change of scalar function in space is known as gradient of scalar field.
- Gradient is a vector quantity.
- Its direction is in which rate of change is maximum.
- Gradient can be used to find the **directional derivative.**
- Gradient is a differential operator by means of which we can associate vector field with a scalar field

Example: **E**=-grad V,

where **E** (Vector quantity) denotes intensity of Electric Field

V (Scalar quantity) denotes Potential

Gradient:

In the rectangular coordinate, the Gradient of a Scalar function F(x,y,z)

$$\vec{\nabla}F = \hat{\boldsymbol{\imath}}\frac{\partial F}{\partial x} + \hat{\boldsymbol{\jmath}}\frac{\partial F}{\partial y} + \widehat{\boldsymbol{k}}\frac{\partial F}{\partial z}$$

In Cylindrical coordinate system

$$\vec{\nabla}F = \frac{\partial F}{\partial s}\hat{s} + \frac{1}{\mathrm{s}}\frac{\partial F}{\partial \phi}\widehat{\boldsymbol{\phi}} + \frac{\partial F}{\partial z}\hat{\boldsymbol{z}}$$

In Spherical polar coordinate system

$$\vec{\nabla}F = \frac{\partial F}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{\mathrm{r}}\frac{\partial F}{\partial \theta}\widehat{\boldsymbol{\theta}} + \frac{1}{\mathrm{rsin\theta}}\frac{\partial F}{\partial \phi}\widehat{\boldsymbol{\phi}}$$

If electric potential in a region of space is given by $V=5x-7x^2y+8y^2+16yz-5z$ volt, where distances are measured in meter. what will be the intensity of electric field intensity E. Calculate the y-component of the field at point (2,4,-3)

Gradient-Directional derivative

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

dF is variation in F for a small change x, y and z

And is nothing but the dot product of ∇F with $\vec{dl} = \hat{\imath}dx + \hat{\jmath}dy + \hat{k}dz$

i.e.….dF= $\vec{\nabla}F \bullet \vec{dl} = |\nabla F||dl|\cos\theta$

So maximum when θ=0; i.e when spatial change is in the direction of the vector ∇F

so…its gives an idea about the direction along which maximum change in the scalar function (F) occurs- and that will be in the direction of the vector

Which is/are correct statement(s)regarding the gradient of a scalar function **(F)**,

a) Maximum change in the scalar function (F) is along
b) It is a vector quantity
c) Both a and b
d) None of the above

Divergence:

- The divergence of a vector field at any point is defined as the flux per unit volume diverging from the point.
- Example: if V represent the velocity of a fluid (liquid or gas) then div V gives the rate of flow of the fluid at that point per unit volume.
- Divergence is a scalar.
- Divergence can be Positive, Negative and Zero
- **Positive** means fluid is undergoing expansion (source is present)
- **Negative** means fluid is undergoing contraction (density is rising) (Sink is present)
- **Zero** means fluid entering and leaving the element is the same (there is no change in the density or fluid is incompressible) (Solenoidal)

In the Rectangular coordinate

$$\vec{\nabla} \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

In Cylindrical coordinate system

$$\vec{\nabla} . \vec{A} = \frac{1}{s} \frac{\partial (s A_s)}{\partial s} + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

In Spherical polar coordinate system

$$\vec{\nabla} . \vec{A} = \frac{1}{r^2} \frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{\mathrm{rsin}\theta} \frac{\partial (sin\theta A_\theta)}{\partial \theta} + \frac{1}{rsin\theta} \frac{\partial A_\phi}{\partial \phi}$$

- Find the divergence of vector **A** at a point (1,-1,1) when **A**= $x^2z\mathbf{i}-2y^3z^2\mathbf{j}+xy^2z\mathbf{k}$
- Find the value of constant 'c' for which the vector A is Solenoidal. **A**= $\mathbf{i}(x+3y)+\mathbf{j}(y-2z)+\mathbf{k}(x+cz)$

Curl:

- Curl of a vector is a vector quantity--- as it has both magnitude and direction, and is a rotational vector
- Its magnitude is the maximum circulation per unit area
- It is not possible to have the curl of a scalar quantity
- General meaning of curl is rotation
- Curl A is Zero means that no rotation is attached with vector **A** and if curl **A** is non-zero, it means that rotation is attached with vector.
- If the curl of a vector field is zero then it is called **IRROTATIONAL** and **CONSERVATIVE**

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

In the rectangular coordinate

$$\vec{\nabla} \times \vec{A} = \hat{\imath}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

In Cylindrical coordinate system

$$\vec{\nabla} \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (sA_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$$

In Spherical Polar coordinate system

$$\vec{\nabla} \times \vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial(rA_\phi}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial(rA_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{\phi}$$

If $\mathbf{A}=2x\mathbf{i}-2y\mathbf{j}+3z\mathbf{k}$ then find curl $\mathbf{A}$

Which is the correct statement for the 'Curl of a vector' $\vec{\nabla} \times \vec{A}$?

a) Curl of a vector is a vector quantity.
b) Curl of a vector is a rotational vector
c) Curl of a vector is normal to the area that make circulation maximum.
d) It is not possible to have the curl of a scalar quantity.
e) All of the above
f) None of the above