PHY 110 Engineering Physics

Lecture 3

UNIT 1 – Electromagnetic theory

What we learned so far!

1. **Scalar and Vector quantities**
   - It is *enough* to have a magnitude for **scalar** physical quantities where as it is *essential* to have both magnitude and direction for the **vector** physical quantities.
2. **Scalar and vector field**
   - **Region of space/domain** in which a function, f(x,y,z), signifies a physical quantity (Temperature, Velocity) is the **field**.
   - **Scalar field**: Each point in space is associated with a **scalar point function** (Temperature, potential) having magnitude.
   - **Vector field**: Each point space is associated with a **vector point function** (Electric field, Gravitational field) having magnitude and direction, both of which changes from point to point.
3. **Del operator (∇)**
   - It is a differential operator
   - It is not a vector by itself
   - It operate on scalar and vector functions and the resulting function may be a vector or scalar function depending on the type of operation.

Rectangular (x,y,z), cylindrical (s,ϕ,z) and spherical polar(r, ϕ,θ) coordinate systems

- **Curvilinear coordinate system**
- **Coordinate transformation**
- **Partial differential calculus**

→ Advanced Engineering Mathematics
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What we learned so far?

4. Operation with del (∇) operator

- Gradient of *scalar function F* – Directional derivative..maximum change of the scalar function F is along the direction of vector ∇F, which nothing but the direction of outward surface normal vector

- Divergence of a *Vector function* **A** - Gives the measure of the vector function's spread out at a point- is solenoidal or divergenceless when divergence of the vector is zero which means that flux of the such vector field entering into a region is equal to that leaving the region, a condition known as incompressibility; also gives an idea about source ($\nabla.\mathbf{A}>0$) means vector diverge and sink($\nabla.\mathbf{A}<0$) means vector converge.

- Curl of a *Vector function* **A**– regarding the rotation of the vector and the vector function is irrotational when curl of the vector is zero, such fields are known as conservative fields.

Which is/are correct statement(s)regarding the gradient of a scalar function (**F**), $\vec{\nabla}F$

a) Maximum change in the scalar function (F) is along $\vec{\nabla}F$
b) It is a vector quantity
c) Both a and b
d) None of the above

Answer: **C**

If the divergence of the vector is zero i.e $\vec{\nabla} \cdot \vec{A} = 0$. Then that vector $\vec{A}$ is called

a) Solenoidal vector
b) Rotational Vector
c) Null vector
d) Unit vector

Answer: **A**

Which is the correct statement for the 'Curl of a vector' $\vec{\nabla} \times \vec{A}$?

a) Curl of a vector is a vector quantity.
b) Curl of a vector is a rotational vector
c) Curl of a vector is normal to the area that make circulation maximum.
d) It is not possible to have the curl of a scalar quantity.
e) All of the above
f) None of the above

Answer: **E**

‘Curl of a vector’ $\vec{\nabla} \times \vec{A}$ is Zero, then the vector is

a) Solenoidal.
b) Irrotational
c) Null vector
d) Unit vecor

Stokes theorem relates

a) Volume integral to surface integral
b) Line integral to surface integral
c) Surface integral to line integral

Gauss's law in Electrostatic (First law)

Electric flux (Φ_E) : The area integral of the Electric field (**E**) over any closed surface is the Φ_E or electric field is the flux per unit area

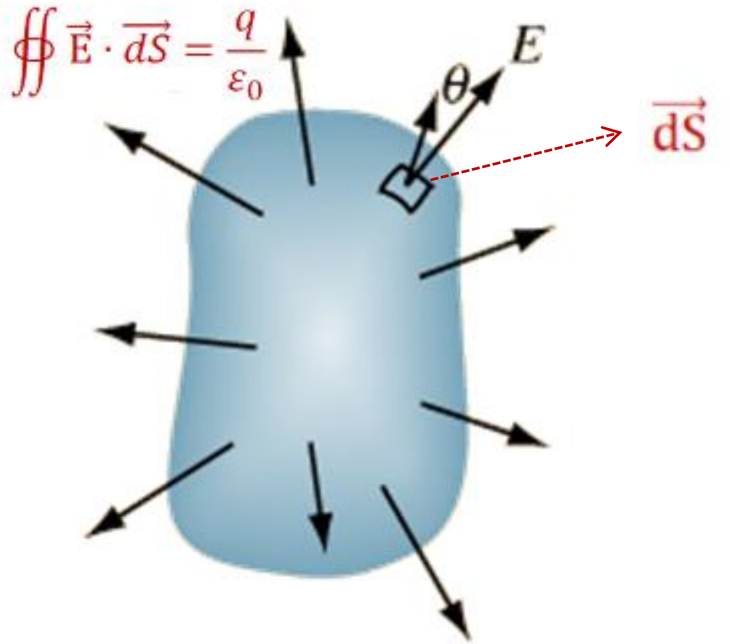
$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS} \longrightarrow \text{Eq..1}$$

Gauss's law: Electric flux (Φ_E) from a closed surface (**Gaussian surface**) is equal to $1/\varepsilon_o$ times the charge (q) enclosed by the surface

$$\Phi_E = \frac{q}{\varepsilon_0} \longrightarrow \text{Eq..2}$$

ε_0 is the permittivity of free space. We know absolute permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$
It is the ability of a material to store electric energy under influence of an electric field in its dielectric medium.

From Eq.1 and 2 $$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$



•Gauss's Law is a general law applying to any closed surface.

• It is an important tool since it permits the assessment of the amount of enclosed charge by mapping the **electric field normal** to the surface outside the charge distribution

•Or can be used to calculate electric field

Poisson's Equations: is a simple second order differential equation that come up in most of the engineering and physics fields.

$$\nabla^2 X = -\text{constant}$$

Siméon Denis Poisson (1781-1840), French mathematician, engineer, and physicist who made many scientific advances

For example, the solution to **Poisson's equation** is the potential field caused by a given electric charge or mass density distribution; with the potential field known, one can then calculate electrostatic or gravitational (force) field

It applies to electrostatics

Eg. In Electrostatics

Poisson's equation states that the Laplacian (**∇²**) of electric potential at a point is equal to the ratio of the volume charge density (**ρ**) to the absolute permittivity of the medium (**$\varepsilon=\varepsilon_o\varepsilon_r$**). Laplace's equation tells us that the laplacian of electric potential at a point is equal to zero.

Eq.1 $$\oiint \vec{\mathrm{E}} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$ Gauss's first law

Charge distributed over a volume with ρ is the volume charge density

Eq.2 $$\iiint \rho \, dV = q$$

Eq.3 $$\varepsilon_0 \oiint \vec{\mathrm{E}} \cdot \overrightarrow{dS} = q$$

Applying divergence theorem to eq.3

$$\varepsilon_0 \iiint \vec{\nabla} \cdot \vec{E} \, d\mathrm{V} = \iiint \rho \, dV \dashrightarrow \vec{\nabla} \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0}$$ **Eq.4**

Integrands must be equal for LHS and RHS

Poisson Equation in Electrostatics

$$\vec{\nabla} \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0}$$ **Eq.4**

Electric field (**E**) and potential (V) are related as

But $\vec{\mathrm{E}} = -grad\, V = -\vec{\nabla} V$ **Eq.5**

V is the Energy required for moving a unit +ve charge from a reference point to a specific point in an electric field

$$\vec{\nabla} \cdot (-\vec{\nabla} V) = \frac{\rho}{\varepsilon_0}$$ **Eq. 5 in Eq.4**

By using the vector identity for $\vec{\mathrm{A}} \cdot \vec{\mathrm{A}} = \mathrm{A}^2$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$ **Eq.6 is the Poisson's Equation in electrostatics**

Laplace Equations: is also simple second order differential equation that come up in most of the engineering and physics fields. $\nabla^2 X = 0$

Like Poisson equation, it also applies to electrostatics.

Eg. In Electrostatics

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$ Poisson's Equation

For a charge free region i.e $\rho=0$, then the Poisson's Equation changes to

Eq.7 $$\nabla^2 V = 0$$

This, Eq.7 is known as Laplace's equation and ∇^2 is the Laplacian operator.

Laplace Equation

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Cartesian coordinate

$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} = 0$$

Cylindrical coordinate

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{\mathrm{r}^2\sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{\mathrm{r}^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

Spherical coordinate

Laplace’s equation is named for Pierre-Simon Laplace, a French mathematician

Continuity Equation

We know current, **I** is the rate of change of charge, **q** **i.e.** $I = \frac{dq}{dt}$

If **ρ** is the charge density, then the charge, **q**, enclosing the volume is given by

$$q = \iiint \rho \, dV$$

Also if J is the current density $I = \iint \vec{j} \cdot \overrightarrow{dS}$

$$\iint \vec{j} \cdot \overrightarrow{dS} = -\frac{d}{dt} \iiint \rho \, dV$$

-ve sign means decreasing ρ as current flows out of the volume

By using the Gauss divergence theorem to LHS, we get

Continuity Equation

$$\oiiint \vec{\nabla} \cdot \vec{J}\, dV = -\oiiint \frac{d\rho}{dt}\, dV$$

The above equations is true for any volume. So we can put the integrands to be equal

$$\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$$ **Continuity Equation**

Current density flowing out of the closed volume is equal to the rate of decrease of charge within that volume.

Gauss theorem uses which of the following operations?

a) Gradient
b) Curl
c) Divergence
d) Laplacian

Stokes theorem uses which of the following operations?

a) Gradient

b) Curl

c) Divergence

d) Laplacian