PHY 110 Engineering Physics

Lecture 5

UNIT 1 – Electromagnetic theory

In Last Class We learnt

1. Maxwell's law of induction

Concept of **displacement current** due to the change/discharge of a capacitor leads to the correction/modification to the Ampere's law.

$$\oint \vec{\mathrm{B}} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{\partial \phi_{\mathrm{E}}}{\partial t}$$

2. Correction to Ampere circuital law

Concept of **displacement current** due to the charge/discharge of a capacitor leads to the correction/modification to the Ampere's law.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt} \right) = \mu_0 (I + I_d)$$

Ampere-Maxwell law

Refer R-3 FUNDAMENTALS OF PHYSICS HALLIDAY. RESNICK, WALKER Chapter 8 278-281

In electromagnetic theory, continuity equation relates ________

a) Volume conservation
b) Mass conservation
c) Charge conservation
d) Energy conservation

Answer: C

Gauss law of magnetostatic (Gauss's 2nd law) asserts that the net magnetic flux through any closed Gaussian surface is

a) Infinity
b) Zero
c) Constant
d) None of the above

Answer: B

According to Faraday’s law, the negative rate of change of the magnetic field is equal to the curl of the electric field. This forms the basis of Maxwell’s third law of electrodynamics. What's another way to describe this behavior?

a) A changing magnetic field induces an electric field
b) A nonzero electrical current creates magnetic charge
c) A changing electric field induces an magnetic field
d) As magnetic field strength increases, electric field strength decreases

Answer: A

In Ampere's Law all currents have to be steady (i.e. do not change with time). State true or false

a. True
b. False

Answer: A

James Clerk Maxwell (1831-1879) considered as the father of classical electrodynamics: He corrected Ampere's law by adding another term, which he called the "displacement current”, On what does Maxwell's "displacement current" depend?

a) The derivative of the electric field with respect to time
b) The divergence of the magnetic field
c) The derivative of the magnetic field with respect to time
d) The electromagnetic force on a charged particle

Answer: A

Maxwell's equations of electromagnetism

- All relationships between electric & magnetic fields & their sources summarized by four equations.

- James Clerk Maxwell

13 June 1831 – 5 November 1879

Scottish scientist in the field of mathematical physics

Basic laws of ELECTROMAGNETISM-MAXWELL'S EQUATIONS

Four Maxwell's equations in differential form in S.I. units are

$$\vec{\nabla} \cdot \vec{D} = \rho \quad \text{------------------------>} \quad \textbf{Eq.1}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{------------------------>} \quad \textbf{Eq.2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{------------------------>} \quad \textbf{Eq.3}$$

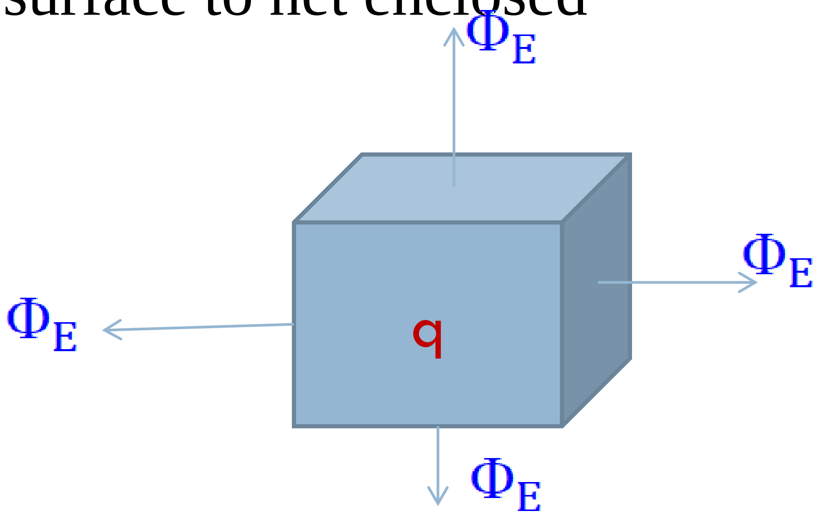
$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{------------------------>} \quad \textbf{Eq.4}$$

We knew the meaning of **E** and **B**, which are related to the Electric flux (Φ_E) and magnetic flux (Φ_B) , respectively. And now we will know how **D** and **H** are respectively, related to **E** and **B**.

Now we have the necessary background for deriving the Maxwell's equations ☺

1. Derivation of Maxwell's First Equation

Let us consider the Gauss's law for Electrostatics, which relate the net electric flux Φ_E through a Gaussian surface to net enclosed electric charge;



$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

$$\oiint \varepsilon_0 \vec{E} \cdot \overrightarrow{dS} = q$$

But and $\vec{D} = \varepsilon \vec{E}$ and $q = \iiint \rho \, dV$

$$\oiint \vec{D} \cdot \overrightarrow{dS} = \iiint \rho \, dV$$

Now apply Gauss's divergence theorem on the LHS..

$$\iiint \vec{\nabla} \cdot \vec{D} \, dV = \iiint \rho \, dV$$

This equation hold true for any arbitrary volume and for that, the integrands must be same. So we have now

$$\vec{\nabla} \cdot \vec{D} = \rho$$

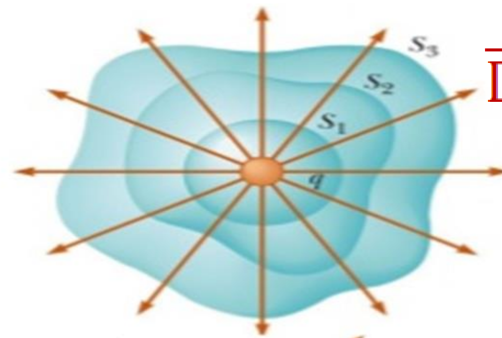
This is the Maxwell's FIRST EQUATION

Or

$$\text{div} \, \vec{D} = \rho$$

ELECTROMAGNETISM–MAXWELL'S EQUATIONS

Charge enclosed by a region



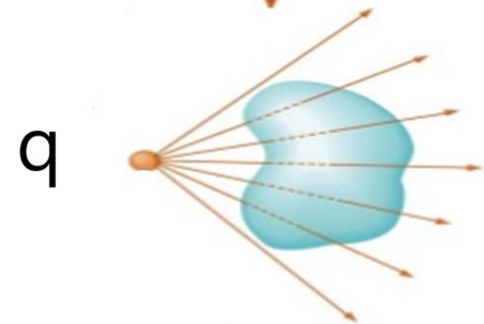
$\vec{D}$

$\vec{\nabla} \cdot \vec{D} = \rho$

ρ Positive means? **D** diverge (source)
ρ Negative means? **D** Converge (sink)
ρ=0, **D** called solenoidal

1 $\vec{\nabla} \cdot \vec{D} = \rho$

Electric field have a source or sink and can also be solenoidal vector field.



$\vec{D}$

$\vec{\nabla} \cdot \vec{D} = 0$

Charge free region

\+ −

Electric monopoles exist

2. Derivation of Maxwell's 2nd Equation

Like electric flux Φ_E, magnetic flux Φ_B is defined as

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS}$$

But Gauss's law for Magnetostatics say Φ_B =0, flux of magnetic field B across any closed **surface** is zero

So, we have $$\oiint \vec{B} \cdot \overrightarrow{dS} = 0$$

Now apply Gauss's divergence theorem on the LHS..

$$\iiint \vec{\nabla} \cdot \vec{B}\, dV = 0$$

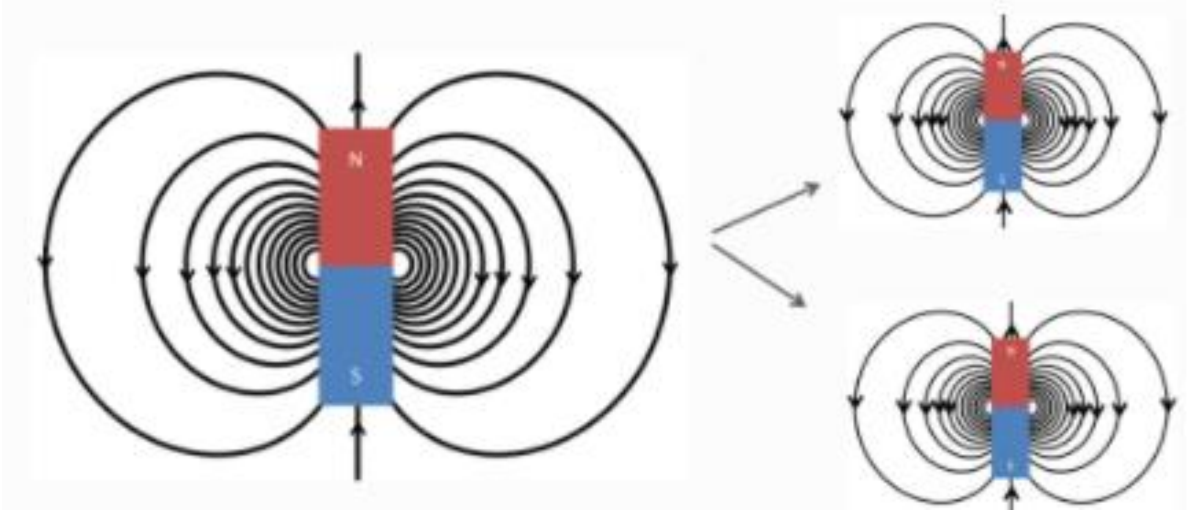
above equation hold true for any arbitrary volume and for that, the integrands must be zero. So we have now

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \textit{or} \qquad \operatorname{div} \vec{B} = 0$$

This is the Maxwell's SECOND EQUATION

The **magnetic line of force** are either closed or go off to infinity, the number of magnetic lines entering any **volume** is exactly equal to the number of lines leaving volume.

ELECTROMAGNETISM-MAXWELL'S EQUATIONS



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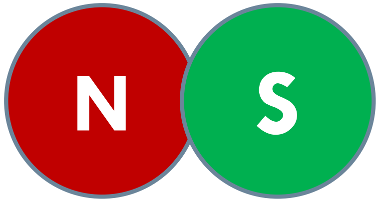
$$\vec{\nabla} \cdot \vec{B} = 0$$

NON-EXISTANCE OF MAGETIC MONOPOLE. Lowest unit is 'dipole'. Magnetic fields have no source or sink but it is always a **solenoidal vector field**.



Magnetic dipoles exist but monopoles do not exist

The divergence of the Electric displacement **D**, according to 1st Maxwell's equation is equal to charge enclosed in the volume. What does this mean?

a) An electric field has no effect on the motion of a charged particle.
b) Electric charges (i.e. monopoles) do exist.
c) Magnetic charges (i.e. monopoles) do exist.
d) Changing Electric field does induce magnetic field.

The divergence of the magnetic field, according to 2nd Maxwell's equation is equal to zero. What does this mean?

a) An external magnetic field has no effect on the motion of a charged particle.
b) Electric charges (i.e. monopoles) do not exist.
c) Magnetic charges (i.e. monopoles) do not exist.
d) changing magnetic field does not induce an electric field.

3. Derivation of Maxwell's 3rd Equation

Let us consider the Faraday's law for induction, which relates the induced electric field to changing magnetic flux;

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \phi_B}{\partial t}$$

But

$$\phi_B = \oiint \vec{B} \cdot \vec{dS}$$

then RHS can be written as

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \oiint \vec{B} \cdot \vec{dS}$$

Upon re-arranging

$$\oint \vec{E} \cdot \vec{dl} = -\oiint \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

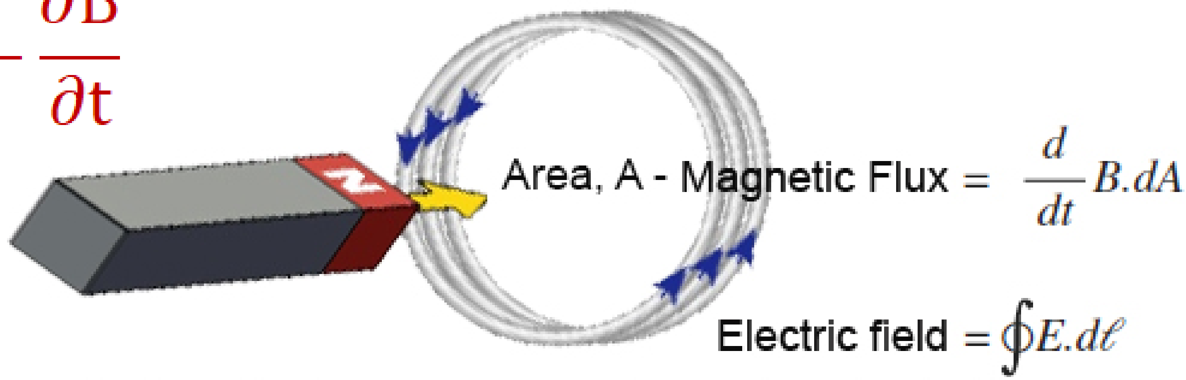
Apply Stoke's theorem to the LHS

$$\oiint (\vec{\nabla} \times \vec{E}) \cdot \vec{ds} = -\oiint \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

ELECTROMAGNETISM-MAXWELL'S EQUATIONS

3

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$



This relation may look odd. But as many people point out that - this is the formula that runs the entire economy. Because this is the basic formula that describe how power station works. When steam drive a turbine, the turbine that has effectively coil of wire will spin in a magnetic field. It doesnt matter whether the magnet is moving or the coil wire is moving.

A moving coil in the presence of stationary magnetic field or the other way around will cause a current to flow. And that is how the electricity is generated & the formula that we just derived here is the one that governs the generating of electricity. ----- That is Maxwell 3rd Law!

4. Derivation of Maxwell's 4th Equation

Let us consider the Ampere-Maxwell law, which relates the induced magnetic field to changing electric flux and current

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\left(I + \epsilon_0\frac{d\Phi_\mathrm{E}}{dt}\right)$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_\mathrm{E}}{dt}$$

But…

$$I = \oiint \vec{J}\cdot\vec{dS}$$

$$\Phi_\mathrm{E} = \oiint \vec{\mathrm{E}}\cdot\vec{dS}$$

Substituting for I and Φ_E

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\oiint \vec{J}\cdot\vec{dS} + \mu_0\epsilon_0\frac{d}{dt}\oiint \vec{\mathrm{E}}\cdot\vec{dS}$$

ELECTROMAGNETISM-MAXWELL'S EQUATIONS

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \oiint \vec{J} \cdot \vec{dS} + \mu_0 \frac{d}{dt} \oiint \varepsilon_0 \vec{E} \cdot \vec{dS}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \oiint \vec{J} \cdot \vec{dS} + \mu_0 \frac{d}{dt} \oiint \vec{D} \cdot \vec{dS}$$

$$\oint \frac{\vec{B}}{\mu_0} \cdot \vec{dl} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \vec{dS}$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\oint \vec{H} \cdot \vec{dl} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \vec{dS}$$

Now we can apply Stokes' theorem to LHS, and then line integral changes to surface integral as

$$\oiint (\nabla \times \vec{H}) \cdot \overrightarrow{dS} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

or

$$\text{Curl}\,\vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

This is the Maxwell's **FOURTH EQUATION**

Second term on the RHS is called Maxwell's correction and is known as displacement current density, this along with Ampere's law is responsible for the EM fields..

A moving coil in the presence of stationary magnetic field or vice versa generate electricity. Which of the Maxwell's equations governs electricity generation?

a) 1st Eqn.
b) 2nd Eqn.
c) 3rd Eqn.
d) 4th Eqn.

Which Maxwell's equation contains Equation of continuity ?

a) 1st Eqn.
b) 2nd Eqn.
c) 3rd Eqn.
d) 4th Eqn.

From which Maxwell's equation we can derive Coulomb's law of electrostatics?

a) 1st Eqn.
b) 2nd Eqn.
c) 3rd Eqn.
d) 4th Eqn.

In Maxwell’s fourth equation, he corrected Ampere’s circuital law. What is the new term he added?

a) Electric Displacement
b) Displacement current
c) Conduction current
d) None of the above

MAXWELL'S EQUATIONS : In integral forms from differential Equations

	Differential forms	Integral forms
Eq.1	$\vec{\nabla} \cdot \vec{D} = \rho$	$\oiint \vec{D} \cdot \vec{dS} = q$
Eq.2	$\vec{\nabla} \cdot \vec{B} = 0$	$\oiint \vec{B} \cdot \vec{dS} = 0$
Eq.3	$\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dl} = -\oiint \dfrac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$
Eq.4	$\vec{\nabla} \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$	$\oint \vec{H} \cdot \vec{dl} = \oiint (\vec{J} + \dfrac{d\vec{D}}{dt}) \cdot \vec{dS}$

Maxwell's equations in their integral form are formulated in terms of integrals, so you need to have a curve/surface/volume to integrate over.

Then we have it as

1. Guess law of electrostatic
2. Guess law magnetostatics
3. Faraday’s law of electromagnetic induction
4. Ampere-Maxwell’s law

Mathematically, the differential and integral forms are equivalent, which you can prove by applying the divergence theorem or the Stokes theorem.

MAXWELL'S 1st INTEGRAL EQUATION from differential Equation

Let us start with the first Maxwell's differential Equation $\vec{\nabla} \cdot \vec{D} = \rho$

Now integrate this equation over the volume V,

$$\iiint \vec{\nabla} \cdot \vec{D}\, dV = \iiint \rho\, dV$$

Now use divergence theorem on LHS and use the relation between charge density **ρ** and charge **q** on RHS $\iiint \rho\, dV = q$

1 $$\oiint \vec{D} \cdot \vec{dS} = q$$

Total electric displacement, **D** through the surface, **S** which define the volume, **V** is equal to the total charge contained in the volume

MAXWELL'S 2nd INTEGRAL EQUATION from differential Equation

Differential form of Maxwell's 2nd equation is $\vec{\nabla} \cdot \vec{B} = 0$

Now integrate this equation over the volume V, like before and we get

$$\iiint \vec{\nabla} \cdot \vec{B} \, dV = 0$$

Now use divergence theorem on LHS and the above equation becomes

2

$$\oint\!\!\!\oint \vec{B} \cdot \overrightarrow{dS} = 0$$

Total magnetic field, **B** through the surface, **S** which define the volume, **V** is equal to zero

MAXWELL'S 3rd INTEGRAL EQUATION from differential Equation

Differential form of Maxwell's 3rd equation is $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Here integrate over the surface, **S** bounded by the closed path

$$\oint\!\!\!\oint (\vec{\nabla} \times \vec{E}) \cdot \overrightarrow{dS} = -\oint\!\!\!\oint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$$

Now use Stoke's theorem on LHS and the above equation becomes

3

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\oint\!\!\!\oint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$$

This means the **electromotive force** around a closed path is equal to the time derivative of the magnetic field through any closed surface bounded by that path.

MAXWELL'S 4th INTEGRAL EQUATION from differential Equation

Differential form of Maxwell's 4th equation is $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Here integrate over the surface, **S** bounded by the closed path

$$\oiint (\nabla \times \vec{H}) \cdot \overrightarrow{dS} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$$

Now use Stoke's theorem on LHS and the above equation becomes

4

$$\oint \vec{H} \cdot \overrightarrow{dl} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$$

Which says that **magnetomotive force** around a closed path is equal to the conventional conduction current plus displacement current through any surface bounded by that path.

Maxwell's equations give

a) The variation of magnetic field only
b) The variation of electric and magnetic field in quantum domain
c) The unified approach called electromagnetic theory explaining the variation of static and time varying electric and magnetic field
d) Variation of electric field only