PHY 110 Engineering Physics

Lecture 2

UNIT 1 – Electromagnetic theory

EMT-Topics

1. Scalar and Vector fields.
2. Concepts of gradient, divergence and curl.
3. **Gauss Theorem and Stokes theorem.**
4. Poisson and Laplace’s Equation.
5. Continuity equation.
6. Maxwell’s Electromagnetic equation.
7. Physical significance of Maxwell’s equation.
8. Ampere’s Circuital law.
9. Maxwell displacement current and correction in Amperes law.

Line Integral, Surface Integral and Volume Integral

In electrodynamics, we encounter several different kinds of integrals, among which the most important are **line** (or **path**) **integrals**, **surface integrals** (or **flux**), and **volume integrals**.

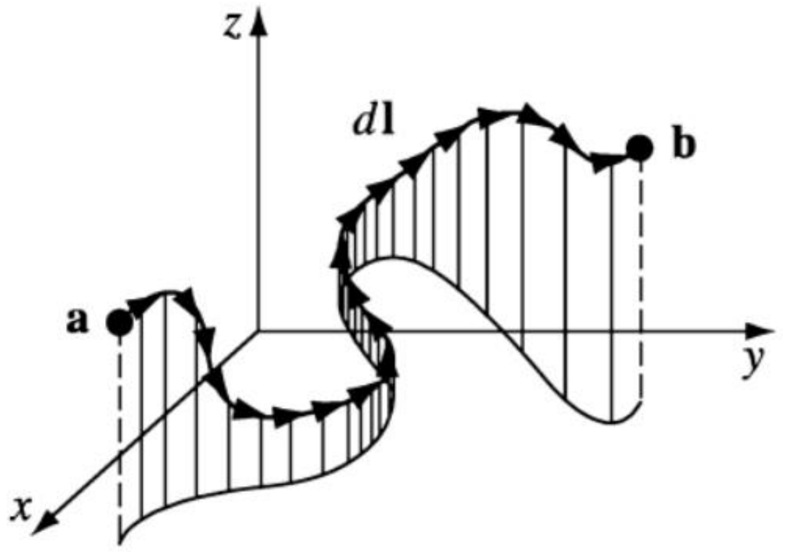
Line Integral

(a) **Line Integrals.** A line integral is an expression of the form

$$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{v} \cdot d\mathbf{l},$$

where **v** is a vector function, **dl** is the infinitesimal displacement vector, and the integral is to be carried out along a prescribed path V from point **a** to point **b**. If the path in question forms a closed loop (that is, if b = a), then we write,

$$\oint \mathbf{v} \cdot d\mathbf{l}.$$



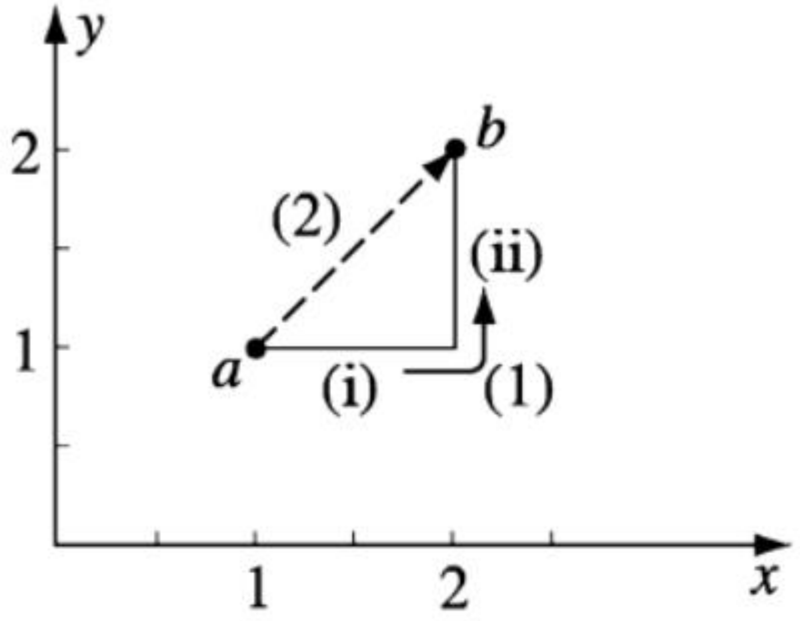
At each point on the path, we take the dot product of **v** (evaluated at that point) with the displacement $d\mathbf{l}$ to the next point on the path. To a physicist, the most familiar example of a line integral is the work done by a force **F**: $W = \int \mathbf{F} \cdot d\mathbf{l}$.

Line Integral

- Ordinarily, the value of a line integral depends critically on the path taken from **a** to **b**, but there is an important special class of vector functions for which the line integral is independent of path and is determined entirely by the end points. Such vector fields are known as conservative fields.

- In physics if a force has this property then we call that force conservative.

- Gravitational force is an example of a conservative force, while frictional force is an example of a non-conservative force.

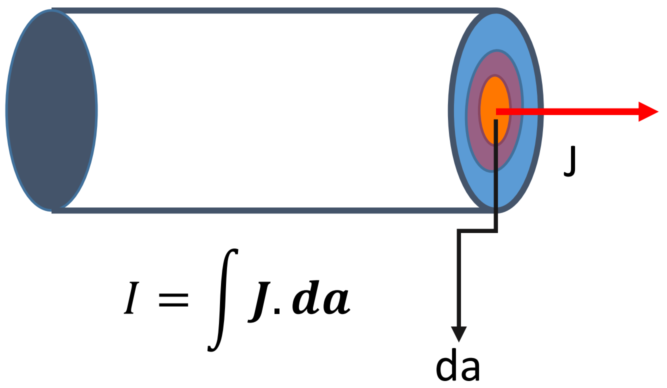


Surface Integral

(b) **Surface Integrals.** A surface integral is an expression of the form

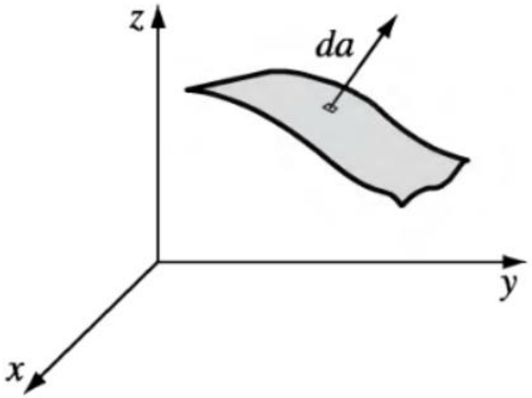
$$\int_{\mathcal{S}} \mathbf{v} \cdot d\mathbf{a},$$

Where, **v** is again some vector function, and the integral is over a specified surface S. Here **da** is an infinitesimal patch of area, with direction perpendicular to the surface as shown in figure. If the surface is closed (forming a "balloon"), then again put a circle on the integral sign.



$$I = \int J.da$$

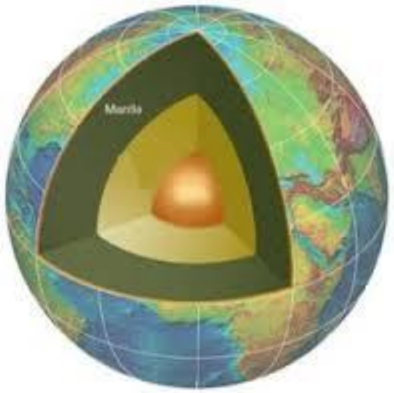
$$\oint \mathbf{v} \cdot d\mathbf{a},$$



Volume Integral

Volume integral in given by:

$$\int \mathbf{v}\, d\tau = \int (v_x\, \hat{\mathbf{x}} + v_y\, \hat{\mathbf{y}} + v_z\, \hat{\mathbf{z}}) d\tau = \hat{\mathbf{x}} \int v_x d\tau + \hat{\mathbf{y}} \int v_y d\tau + \hat{\mathbf{z}} \int v_z d\tau;$$



$$\boldsymbol{Mass} = \int \boldsymbol{\rho}\, \boldsymbol{d\tau}$$

$$\boldsymbol{\rho} = \boldsymbol{density}$$

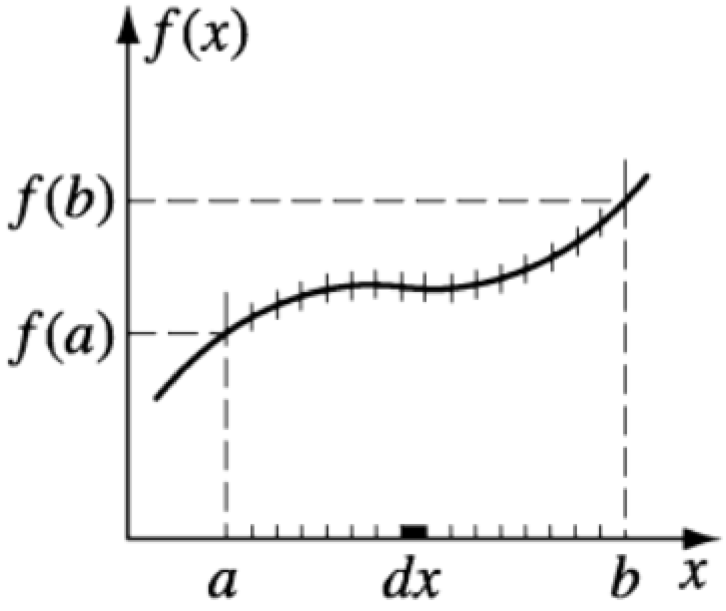
Fundamental Theorems of Calculus

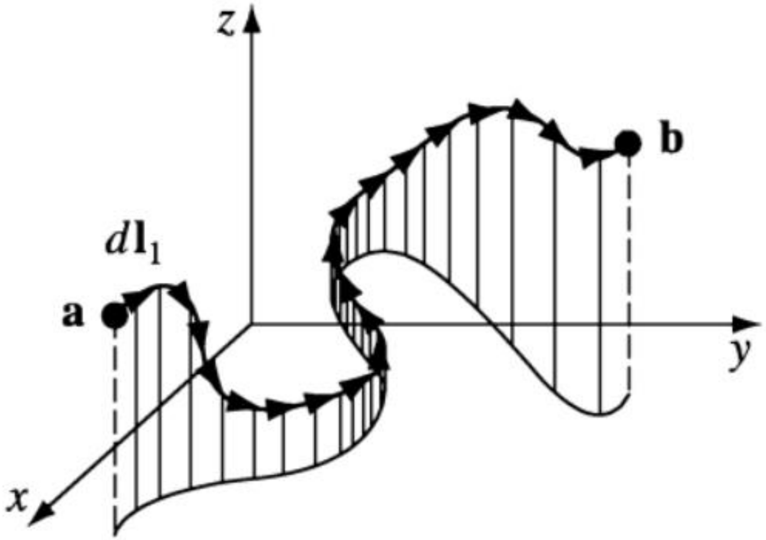
- Three fundamental theorems associated with each of the three quantities gradient, divergence and curl.
- The fundamental theorem of divergence is also known as Gauss theorem or Green's theorem.
- The fundamental theorem of gradient
- The fundamental theorem of curl is also known as Stokes Theorem.

Fundamental Theorem of Gradient

- Let us consider a function T(x, y, z).
- The function 'T' may represent any scalar quantity like temperature, potential etc.
- Starting at any point a, we move a small distance dl_1. According to the definition of gradient, the function T will change by an amount

$$dT = (\nabla T).dl_1$$

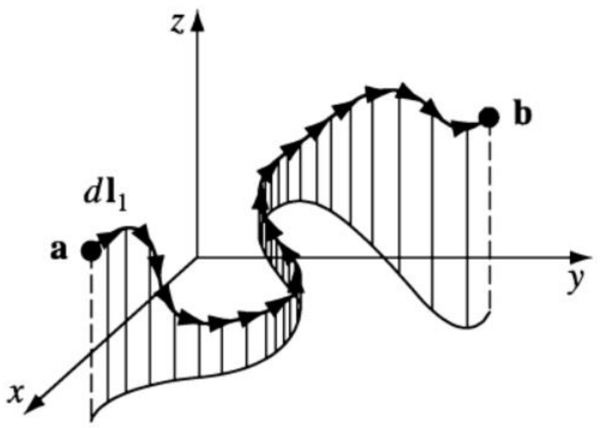




Fundamental Theorem of Gradient

Now we move a little further, by an additional small displacement $d\mathbf{l}_2$; the incremental change in T will be $(\nabla T) \cdot d\mathbf{l}_2$. In this manner, proceeding by infinitesimal steps, we make the journey to point **b**. At each step we compute the gradient of T (at that point) and dot it into the displacement $d\mathbf{l}$... this gives us the change in T. Evidently the *total* change in T in going from **a** to **b** (along the path selected) is

$$\boxed{\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T) \cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a}).}$$



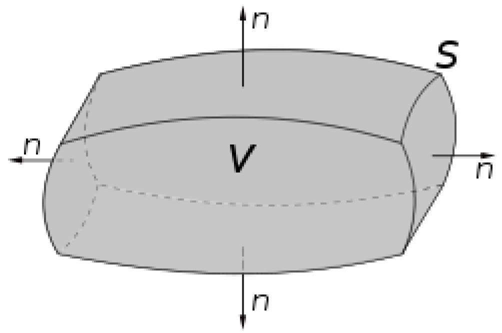
This is the **fundamental theorem for gradients**

Gauss theorem

Also called Gauss Divergence theorem; Which states volume integral of the divergence of a vector field over the volume is equal to the surface integral of that vector field enclosing the volume.

i.e.

$$\iiint_V \vec{\nabla} \cdot \vec{A}\, dV = \oiint_S \vec{A} \cdot \overrightarrow{dS}$$



Where, $\vec{A} = \hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_z$ is the vector and V is the volume bounded by the closed surface S

❑ In short by this theorem volume integral can be converted to surface integral– useful when the volume integration is difficult to achieve the result.

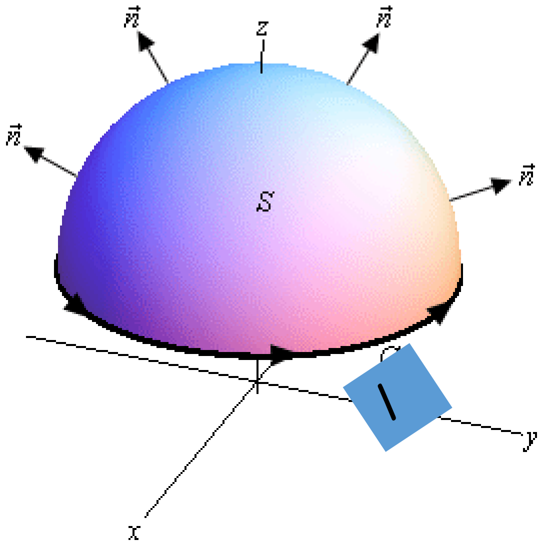
Gauss's divergence theorem relates

a) Volume integral to Surface integral
b) Line integral to surface integral
c) Surface integral to line integral

Stokes' theorem

Stokes' theorem states that surface integral (over a patch of the surface, **S**) of the curl of a vector is equal to line integral of that vector over a closed curve *(l)* defining the boundary of that surface (**S**)

$$\iint (\vec{\nabla}\times\vec{A})\cdot\vec{dS} = \oint \vec{A}\cdot\vec{dl}$$



Where, $\vec{A} = \hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_z$ is the vector and S is bounded by the closed path ***l***

- Convert surface integral into the line integral
- Curl of the vector relate to its line integration
- Right hand thumb rule to know the direction of **dS**
- Not depend on the shape of the surface
- Depends on the boundary line

- **Stokes theorem relates**

  a) Volume integral to surface integral
  b) Line integral to surface integral
  c) Surface integral to line integral