

Gear Ratios and Torque

Review of Torque: Torque is a twisting “force”, like turning a screwdriver or doorknob, or turning a wrench or bike pedals. Its units are force times distance. For example, if your wrench is 10 inches long and you push on the end with a force of 48 lb, your bolt is tightened to $48 \text{ lb} * 10 \text{ inches} = 48 \text{ in}*\text{lb} = 4 \text{ ft}*\text{lb}$. Or if you push down on a bike pedal with 70 lb of body weight and the crank arm is 1 foot long, you have 70 ft*lb of torque going into your bike drivetrain.

Gears: Gears are used to change direction of rotation and/or speed of rotation.

- Two gears having the same number of teeth rotate in opposite directions at the same speed. In general, any time you have two gears meshing together the direction of rotation changes.
- Rack and pinion is used for steering systems on cars. The pinion is a small round gear; the rack is a “flat” gear. The rack and pinion converts rotating motion to linear (straight-line) motion.
- Bevel gears are used when you want to have input and output shafts non-parallel (car differential for rear-drive)
- A worm gear rotates one turn of the input shaft for each gear tooth of the output shaft. It can't be back driven, which may be useful for a robot arm lifting something. It also has higher friction than other types of gears.
- The “Input” shaft is the one you are turning with the engine, a hand crank, an electric motor, or the steering wheel. It is attached to the “drive” gear. The “Output” shaft is the one attached to the “driven” gear. The driven gear is driven by the drive gear.
- If you have an 8 tooth gear driving a 24 tooth gear, the 24 tooth (driven gear) will turn 1/3 as fast as the 8 tooth input gear since each gear moves one tooth at a time. You can calculate the speed of the output shaft as follows:

$$\underline{\text{Output speed}} = \underline{\text{Input speed}} * \underline{\text{drive teeth}} / \underline{\text{driven teeth}}$$

Example: Output speed = $30 \text{ rpm} * 8 \text{ teeth} / 24 \text{ teeth} = 10 \text{ rpm} = 10 \text{ revolutions per minute.}$

- If you have multiple gears you can multiply by each ratio of gears. Suppose the 24 tooth gear in the above example is attached to a shaft which has an 8 tooth gear. Both of these gears rotate at the same speed since they are attached to the same shaft. The 8 tooth gear drives a 40 tooth gear. So the output speed is:

$$\text{Output speed} = 30 \text{ rpm} * 8 \text{ teeth} / 24 \text{ teeth} * 8 \text{ teeth} / 40 \text{ teeth} = 2 \text{ rpm}$$

- In this case the output rotation is in the same direction as the input rotation since there are two sets of gears, reversing direction twice.
- We visited a grist mill which used four sets of wooden gears to multiply the water wheel's speed. They acted like bevel gears, changing direction four times. The grindstone's speed was 12 times the water wheel speed. This allowed the mill to grind more corn per hour than most other gristmills, which had a lower gear ratio.

Here is a basic relationship for geartrains: **Torque times Speed = Power**. Except for friction losses, power remains a constant in a gear train. In our example above, speed was 15 times smaller on the output shaft, which means torque was 15 times greater! Our robot kit's electric motors have high speed and low torque. If we want a slower speed and higher torque for drive wheels of a robot, we will have to design a geartrain that slows down the speed. This will multiply the torque.

Another example of gearing is a bicycle chain and sprocket system (each sprocket rotates one chain link at a time). Usually there are more teeth on the chainring (attached to the the crank) than the rear sprocket. So the output speed (the rear wheel) is faster than the input speed (the pedals and crank). Bikers sometimes choose their sprocket sizes by calculating the gear ratios in “gear inches.” This is done by dividing the number of teeth on the chainring (drive gear) by the number of teeth on the rear sprocket and multiplying by the wheel diameter. My touring bike has these ratios for low & high gear:

- 26 tooth “granny gear” chainring / 34 tooth rear sprocket * 27 inch wheel = 20.6 gear inches
- 48 tooth large front chainring / 13 tooth rear sprocket * 27 inch wheel = 100 gear inches.

Racing bikes have a much narrower gear range, say 60-110 gear inches. You can even calculate how fast you're going if you know your pedal “cadence” or rotating speed in revolutions per minute. 90-100 rpm is the most efficient pedal cadence for long distance riding: higher speed than the usual 60 rpm many people ride at, but easier on the knees & muscles since you don't have to push as hard (lower torque). The circumference of the wheel is 3.1416 times its diameter, so this is the distance traveled in one wheel revolution. This ratio of circumference to diameter is represented by the Greek letter Pi, written Π . So here's my bike speed when lugging up a long hill at 60 pedal rpm:

- $60 \text{ rev/min} * 26 \text{ teeth} / 34 \text{ teeth} * 27 \text{ inches} * 3.1416 = 3892 \text{ inches/minute}$. Not too useful, so we convert units:
- $3892 \text{ inches / minute} * 60 \text{ minutes / hr} * 1 \text{ ft} / 12 \text{ in} * 1 \text{ mile} / 5280 \text{ ft} = 3.7 \text{ miles per hour.}$

This isn't very fast but it sure beats pushing the bike up the hill! Coming down the other side in top gear:
 $110 \text{ rev/min} * 48 \text{ teeth} / 13 \text{ teeth} * 27 \text{ inches} * 3.1416 * 60 \text{ min/hr} * 1 \text{ ft} / 12 \text{ in} * 1 \text{ mile} / 5280 \text{ ft} = 32.6 \text{ mph.}$