AGH University of Science and Technology

Selected Topics in Cryptography
Quantum cryptanalysis

Szymon Szozda

Department of Telecommunications

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Quantum crypanalysis

Agenda

1. Bra-ket notation
2. Quantum gates
3. Grover's Database Search
4. Shore's factorization algorithm
    - Fast modular exponentiation
    - Quantum Fourier Transform
5. Implementation of quantum computer
    - Cold, Confined Atomic Ions
    - Cold, Confined Atoms
    - Quantum Dots
    - Linear Optic Computers
    - Superconducting Devices
    - NMR

Bra-ket notation

Definition

Bra–ket notation: $\langle x|y\rangle$ is a standard notation for describing quantum states. It can also be used to denote abstract vectors, linear functionals and scalar product in mathematics.

The left part: $\langle x|$, called the bra, is a row vector.
The right part: $|y\rangle$, called the ket, is a column vector.

Qbit

Definition

A pure qubit state is a linear superposition of the basis states. This means that the qubit can be represented as a linear combination of $|0\rangle$ and $+|1\rangle$:
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
When we measure this qubit in the standard basis, the probability of outcome $|0\rangle$ is $|\alpha|^2$ and the probability of outcome $1\rangle$ is $|\beta|^2$. Because the absolute squares of the amplitudes equate to probabilities, it follows that α and β must be constrained by the equation

$$|\alpha|^2 + |\beta|^2 = 1$$

Gates

Definition

In quantum computing and specifically the quantum circuit model of computation, a quantum gate (or quantum logic gate) is a basic quantum circuit operating on a small number of qubits.

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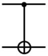
Gates

Example

Gate	Notation	Matrix
NOT (Pauli-X)	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Z	$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard	$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
CNOT (Controlled NOT)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Unitary Transformation

Definition

Unitary transformation is transformation that preserves the inner product (isometry).

It is a bijective function:

$$U : H_1 \rightarrow H_2$$

where $H_1 \quad \text{and} \quad H_2$ are Hilbert spaces, such that:

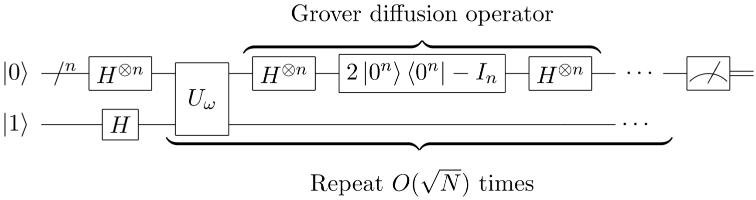
$$\langle Ux, Uy \rangle_{H_2} = \langle x, y \rangle_{H_1}$$

Grover's database search

Grover's database search uses ability of quantum computing to pararell process of qubits. The algorithm allows us to find selected element in unsorted set with complexity $\sqrt{n}$

Grover's database search

Scheme



Group Theory

Abellian Group

In abstract algebra, an abelian group, also called a commutative group, is a group in which the result of applying the group operation to two group elements does not depend on the order in which they are written.

Multiplicative group of integers modulo n is an abelian group. The set of classes relatively prime to n is closed under multiplication:

$$gcd(a, n) = \neg 1 \quad \text{and} \quad gcd(b, n) = 1 \quad => \quad gcd(ab, n) = 1$$

Shor

Overview

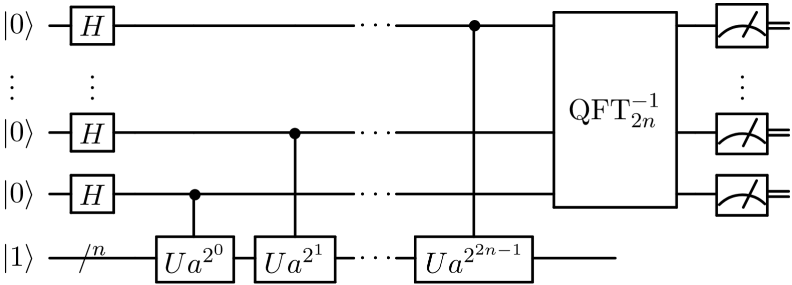
1. Pick a random number $a < N$ and compute $gcd(a, N)$.
2. If $gcd(a, N) \neq 1$, then this number is a nontrivial factor of N, so we are done.
3. Otherwise, use the period-finding subroutine (below) to find r, the period of the following function:

   $$f(x) = a^x \bmod N$$

   i.e. the order r of a in $(\mathbb{Z}_N)^\times$ $(\mathbb{Z}_N)^\times$, which is the smallest positive integer r for which $f(x + r) = f(x)$, or $f(x + r) = a^{x+r} \bmod N \equiv a^x \bmod N$
4. If r is odd, go back to step 1.
5. If $a^{\frac{r}{2}} \equiv -1 \bmod N$, go back to step 1.
6. $gcd(a^{\frac{r}{2}} + 1, N)$ and $gcd(a^{\frac{r}{2}} - 1, N)$ are nontrivial factors of N.

Shor

Scheme



$|0\rangle$
$|0\rangle$
$|0\rangle$
$|1\rangle$
H
H
H
n
Ua^{2^0}
Ua^{2^1}
$Ua^{2^{2n-1}}$
QFT_{2n}^{-1}

Fast exponentiation

We can calculate $A^B modC$ quickly, using modular multiplication rules:

$$A^2 modC = (A * A) modC = ((AmodC) * (AmodC)) modC$$

Quantum fourier transform

xyz

General Steps