AGH University of Science and Technology

Selected Topics in Cryptography

Quantum cryptanalysis

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04.12.2017

Quantum crypanalysis

Agenda

1. Bra-ket notation
2. Quantum gates
3. Grover's Database Search
4. Shore's factorization algorithm
   - Fast modular exponentiation
   - Quantum Fourier Transform
5. NMR uantum Computing

Bra-ket notation

Definition

Bra–ket notation: $\langle x|y\rangle$ is a standard notation for describing quantum states. It can also be used to denote abstract vectors, linear functionals and scalar product in mathematics.

The left part: $\langle x|$, called the bra, is a row vector.
The right part: $|y\rangle$, called the ket, is a column vector.

Qbit

Definition

A pure qubit state is a linear superposition of the basis states. This means that the qubit can be represented as a linear combination of $|0\rangle$ and $+|1\rangle$:
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
When we measure this qubit in the standard basis, the probability of outcome $|0\rangle$ is $|\alpha|^2$ and the probability of outcome $1\rangle$ is $|\beta|^2$. Because the absolute squares of the amplitudes equate to probabilities, it follows that α and β must be constrained by the equation

$$|\alpha|^2 + |\beta|^2 = 1$$

Gates

Definition

In quantum computing and specifically the quantum circuit model of computation, a quantum gate (or quantum logic gate) is a basic quantum circuit operating on a small number of qubits.

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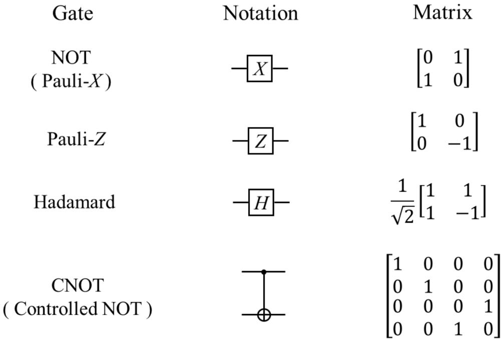
Gates

Example



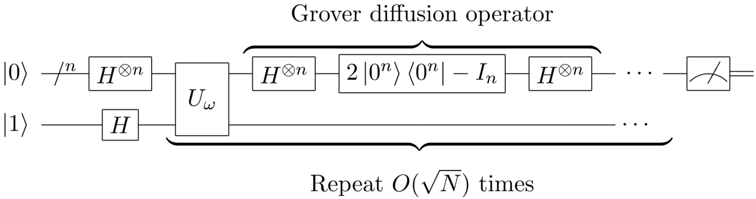
Gate	Notation	Matrix
NOT (Pauli-X)	$\boxed{X}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Z	$\boxed{Z}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard	$\boxed{H}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
CNOT (Controlled NOT)	$\bullet$ $\oplus$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Grover's database search

Grover's database search uses ability of quantum computing to pararell process of qubits. The algorithm allows us to find selected element in unsorted set with complexity $\sqrt{n}$

Grover's database search

Scheme



Group Theory

Abellian Group

In abstract algebra, an abelian group, also called a commutative group, is a group in which the result of applying the group operation to two group elements does not depend on the order in which they are written.

Group Theory

Multiplicative group of integers modulo n

Multiplicative group of integers modulo n is an abelian group. The set of classes relatively prime to n is closed under multiplication:

$$gcd(a,n)=1 \quad \text{and} \quad gcd(b,n)=1 \quad => \quad gcd(ab,n)=1$$

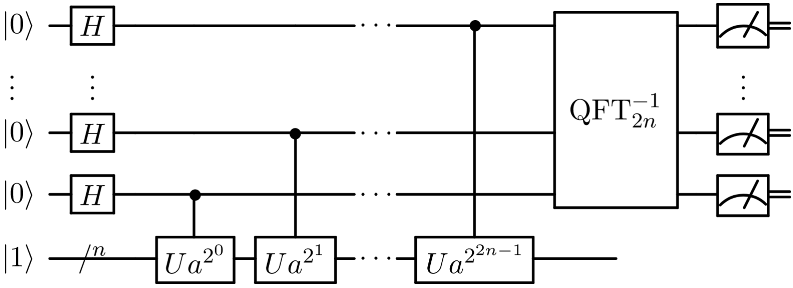
Shor

Overview

General Steps

Shor

Scheme



$|0\rangle$
$|0\rangle$
$|0\rangle$
$|1\rangle$
H
H
H
n
Ua^{2^0}
Ua^{2^1}
$Ua^{2^{2n-1}}$
QFT_{2n}^{-1}

Fast exponentiation

We can calculate $A^B modC$ quickly, using modular multiplication rules:

$$A^2 modC = (A * A)modC = ((AmodC) * (AmodC))modC$$

Quantum fourier transform

xyz

NNR

Overview

General Steps