Multicurve vs. Single-Curve Framework in Finance

1 Introduction

Post-2008 financial markets require **multicurve** modeling due to:

- Basis spreads (LIBOR-OIS, tenor spreads)
- Collateralization and OIS discounting mandates
- Regulatory requirements (e.g., SA-CCR, FRTB)

2 Mathematical Framework

2.1 Single-Curve Approach (Pre-2008)

$$\text{Discount Factor: } P(t,T) = E^Q\left[e^{-\int_t^T r_s ds}\right] \tag{1}$$

$$\text{Forward Rate: } F(t;T_1,T_2) = \frac{1}{\tau}\left(\frac{P(t,T_1)}{P(t,T_2)} - 1\right) \tag{2}$$

2.2 Multicurve Approach

Two stochastic processes:

$$\text{Discounting (OIS): } dr_t^d = \kappa_d(\theta_d - r_t^d)dt + \sigma_d dW_t^d \tag{3}$$

$$\text{Forwarding (LIBOR): } dr_t^f = \kappa_f(\theta_f - r_t^f)dt + \sigma_f dW_t^f \tag{4}$$

with correlation ρ between Brownian motions.

3 Key Applications

3.1 Interest Rate Swaps

Component	Single-Curve	Multicurve
Fixed Leg	$\sum \tau_i K P(t,T_i)$	$\sum \tau_i K P^d(t,T_i)$
Floating Leg	$\sum \tau_j L_j P(t,T_j)$	$\sum \tau_j F^f(t;T_{j-1},T_j) P^d(t,T_j)$

3.2 Forward Rate Agreements

$$\text{FRA Value} = \underbrace{P^d(t, T_2)}_{\text{OIS discount}} \tau \left(\underbrace{F^f(t; T_1, T_2)}_{\text{LIBOR forward}} - K \right) \tag{5}$$

3.3 Caps Pricing

$$\text{Caplet}_i = P^d(t, T_i)\tau \left[F^f(t; T_{i-1}, T_i)N(d_1) - KN(d_2)\right] \tag{6}$$

where:

$$d_1 = \frac{\ln(F^f/K) + \frac{1}{2}\sigma^2 T_{i-1}}{\sigma\sqrt{T_{i-1}}}$$
$$d_2 = d_1 - \sigma\sqrt{T_{i-1}}$$

4 Numerical Example: 1Y Swap

Parameter	Single-Curve	Multicurve
6M LIBOR Forward	2.50%	2.75%
OIS Discount Factor	N/A	0.985
Swap Rate	2.50%	2.71%

5 Why Multicurve Matters

- **Basis Risk**: 3M vs 6M LIBOR spreads averaged 35bps post-crisis
- **XVA Adjustments**: CVA/DVA calculations require risk-free discounting
- **Regulatory Compliance**: SA-CCR requires OIS discounting for exposure

Multicurve Framework & Modern Benchmark Rates (2025)

6 Modern Rate Benchmarks (2025)

7 Mathematical Framework

7.1 Multicurve Construction

$$\begin{aligned}
&\text{OIS Curve (Discounting): } P^d(t, T) = E^Q\left[e^{-\int_t^T r_s^d ds}\right] \\
&\text{Forward Curve: } F^f(t; T_1, T_2) = E^{Q^{T_2}}\left[L(T_1, T_2)\right] \\
&\text{where } r_t^d \text{ follows } dr_t^d = \kappa_d(\theta_d - r_t^d)dt + \sigma_d dW_t^d
\end{aligned} \tag{7}$$

Table 1: Current Risk-Free Rates by Currency

Rate	Currency	Replaced	Key Usage
€STR (ESTER)	EUR	EONIA	OIS discounting, EUR derivatives
SOFR	USD	USD LIBOR	Collateralized USD swaps
SONIA	GBP	GBP LIBOR	GBP loan pricing
TONAR	JPY	JPY LIBOR	JPY risk-free benchmark
SARON	CHF	CHF LIBOR	Swiss repo-linked products

8 Rate Transition Timeline

Table 2: Benchmark Rate Transition

Legacy Rate	Replacement	Completion
LIBOR (EUR)	€STR + spread	Jan 2022
EONIA	€STR	Oct 2019
USD LIBOR	SOFR	Jun 2023
GBP LIBOR	SONIA	Dec 2021

9 Practical Applications

9.1 Instrument Pricing

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Instrument	Rate Usage
Interest Rate Swaps	Discount: OIS (€STR/SOFR), Forward: RFR
Caps/Floors	Underlying: Forward RFR, Discount: OIS
Cross-Currency Swaps	Dual OIS curves + FX basis
Collateralized Derivatives	OIS discounting (CSA-aligned)
Legacy LIBOR Contracts	Fallback: RFR + fixed spread

9.2 Example: €STR-OIS Swap Pricing

$$\text{Swap Value} = \underbrace{\sum_{i=1}^{N} \tau_i K P^{STR}(t, T_i)}_{\text{Fixed Leg}} - \underbrace{\sum_{j=1}^{M} \tau_j F^{STR}(t; T_{j-1}, T_j) P^{STR}(t, T_j)}_{\text{Floating Leg}} \tag{8}$$

10 Key Differences: LIBOR vs. RFR

Characteristic	LIBOR	RFR (€STR/SOFR)
Basis	Bank submissions	Actual transactions
Tenor	Multiple (1M, 3M, etc.)	Overnight only
Volatility	Higher (credit risk)	Lower (risk-free)
Term Structure	Requires interpolation	Built from compounded averages

11 Implementation Challenges

- **Curve Construction**: Need to derive forward-looking term rates from overnight RFRs
- **Fallback Language**: ISDA protocols for legacy contracts
- **Basis Risk**: Managing spreads between RFRs and remaining IBORs
- **XVA Adjustments**: CVA/DVA now based on OIS curves

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Advanced Multicurve Framework & Modern Rate Benchmark Implementation Quant Finance Team July 21, 2025

12 Foundations

12.1 Post-Crisis Interest Rate Modeling

- **Pre-2008**: Single-curve (LIBOR for both discounting/forwarding)
- **Post-2008**: Multicurve separation due to:

$$\text{Basis Spread} = F^{IBOR}(t,T) - F^{OIS}(t,T) \neq 0 \tag{9}$$

- **2025 Standard**: Risk-Free Rates (RFRs) + Multicurve

Table 3: Global Risk-Free Rates (2025)

Currency	RFR	Admin	Calculation	Transition Date
EUR	€STR	ECB	Secured overnight	Jan 2022
USD	SOFR	NY Fed	Treasury repos	Jun 2023
GBP	SONIA	BoE	Unsecured overnight	Dec 2021
JPY	TONAR	BoJ	Call rate	2021
CHF	SARON	SNB	Repo transactions	2021

13 Modern Rate Benchmarks

14 Mathematical Framework

14.1 Two-Curve Vasicek Model

$$\text{Discounting (OIS)} : dr_t^d = \kappa_d(\theta_d - r_t^d)dt + \sigma_d dW_t^d \tag{10}$$

$$\text{Forwarding (RFR)} : dr_t^f = \kappa_f(\theta_f - r_t^f)dt + \sigma_f dW_t^f \tag{11}$$

$$\text{Correlation} : dW_t^d dW_t^f = \rho dt \tag{12}$$

14.2 Curve Construction

$$P^d(t,T) = \exp\left(A_d(\tau) - B_d(\tau)r_t^d\right), \quad \tau = T - t \tag{13}$$

where:

$$B_d(\tau) = \frac{1 - e^{-\kappa_d \tau}}{\kappa_d}$$

$$A_d(\tau) = \left(\theta_d - \frac{\sigma_d^2}{2\kappa_d^2}\right)(\tau - B_d(\tau)) + \frac{\sigma_d^2}{4\kappa_d}B_d(\tau)^2$$

15 Instrument Pricing

15.1 Interest Rate Swap

$$\text{PV}_{\text{swap}} = \underbrace{N\sum_i \tau_i K P^d(t,T_i)}_{\text{Fixed Leg}} - \underbrace{N\sum_j \tau_j F^f(t;T_{j-1},T_j)P^d(t,T_j)}_{\text{Floating Leg}} \tag{14}$$

15.2 Caplet Pricing

$$\text{Caplet}_i = P^d(t, T_i)\tau \left[F^f(t; T_{i-1}, T_i)N(d_1) - KN(d_2)\right] \tag{15}$$

$$d_1 = \frac{\ln(F^f/K) + \frac{1}{2}\sigma^2 T_{i-1}}{\sigma\sqrt{T_{i-1}}} \tag{16}$$

$$d_2 = d_1 - \sigma\sqrt{T_{i-1}} \tag{17}$$

16 Implementation Example

16.1 EUR 5Y Swap Valuation

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Component	**Single-Curve**	**Multicurve**	**Difference**
Fixed Rate	2.10%	2.10%	0 bps
6M Forward	2.15%	2.35%	+20 bps
Discount Factor	0.895	0.902	+0.7%
PV Floating Leg	9,842,500	10,157,300	+314,800

16.2 Monte Carlo Simulation

For path-dependent derivatives:

$$F^f(t_i) = F^f(t_{i-1}) + \kappa_f(\theta_f - F^f(t_{i-1}))\Delta t + \sigma_f\sqrt{\Delta t}Z_i \tag{18}$$

where $Z_i \sim N(0, 1)$ with correlation ρ to discount factors.

17 Risk Management

17.1 Sensitivities

$$\text{Delta} = \frac{\partial V}{\partial F^f} \tag{19}$$

$$\text{Gamma} = \frac{\partial^2 V}{\partial (F^f)^2} \tag{20}$$

$$\text{Curve Correlation Risk} = \frac{\partial V}{\partial \rho} \tag{21}$$

17.2 XVA Adjustments

$$\text{CVA} = (1 - R)\int_0^T E\left[e^{-\int_0^t r_s^d ds} V^+(t)\right] dPD(t) \tag{22}$$

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Figure 1: Multicurve Construction Process

Appendix

Legacy LIBOR Fallback

For contracts still referencing LIBOR:

$$\text{LIBOR} = \text{RFR} + \text{Spread Adjustment} \tag{23}$$

where spreads are fixed by ISDA (e.g., EUR 11.3 bps, USD 26.2 bps).

Python Implementation Snippet

```
def vasicek_mc(r0, kappa, theta, sigma, T, steps, n_paths):
    dt = T/steps
    rates = np.zeros((steps+1, n_paths))
    rates[0] = r0
    for t in range(1, steps+1):
        dw = np.random.normal(scale=np.sqrt(dt), size=n_paths)
        rates[t] = rates[t-1] + kappa*(theta-rates[t-1])*dt + sigma*dw
    return rates
```