Week\_05

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### Z - test

# Problem 1: Z-Test for Mean (Two-Tailed Test)  
  
# A sample of 100 students has an average score of 75. The population mean score is 72, and the population standard deviation is 10. Test if the sample mean is different from the population mean at the 5% significance level.  
  
# Given values  
sample\_mean <- 75  
population\_mean <- 72  
sigma <- 10  
n <- 100  
alpha <- 0.05  
  
z\_stat <- (sample\_mean - population\_mean) / (sigma / sqrt(n))  
  
# Critical Z value for two-tailed test at alpha = 0.05  
z\_critical <- qnorm(1 - alpha / 2)  
  
# Output results  
z\_stat

## [1] 3

z\_critical

## [1] 1.959964

if(abs(z\_stat) > z\_critical) {  
 print("Reject the null hypothesis")  
} else {  
 print("Fail to reject the null hypothesis")  
}

## [1] "Reject the null hypothesis"

# Problem 2: Z-Test for Mean (Right-Tailed Test)  
  
# A factory claims the average weight of a product is 500 grams. A sample of 30 products has a mean weight of 505 grams, and the population standard deviation is 15 grams. Test if the sample weight is significantly greater than 500 grams at α = 0.01.  
  
# Given values  
sample\_mean <- 505  
population\_mean <- 500  
sigma <- 15  
n <- 30  
alpha <- 0.01  
  
  
z\_stat <- (sample\_mean - population\_mean) / (sigma / sqrt(n))  
  
# Critical Z value for right-tailed test at alpha = 0.01  
z\_critical <- qnorm(1 - alpha)  
  
# Output results  
z\_stat

## [1] 1.825742

z\_critical

## [1] 2.326348

if(z\_stat > z\_critical) {  
 print("Reject the null hypothesis")  
} else {  
 print("Fail to reject the null hypothesis")  
}

## [1] "Fail to reject the null hypothesis"

# Problem 3: Z-Test for Mean (Left-Tailed Test)  
  
# A machine is designed to fill bottles with exactly 1 liter of liquid. A quality control inspector believes the machine is underfilling the bottles. A sample of 40 bottles shows an average fill of 0.98 liters, with a population standard deviation of 0.05 liters. Test at a 1% significance level whether the machine is underfilling the bottles.  
  
# Given values  
sample\_mean <- 0.98  
population\_mean <- 1  
sigma <- 0.05  
n <- 40  
alpha <- 0.01  
  
z\_stat <- (sample\_mean - population\_mean) / (sigma / sqrt(n))  
  
# Critical Z value for left-tailed test at alpha = 0.01  
z\_critical <- qnorm(alpha)  
  
# Output results  
z\_stat

## [1] -2.529822

z\_critical

## [1] -2.326348

if(z\_stat < z\_critical) {  
 print("Reject the null hypothesis")  
} else {  
 print("Fail to reject the null hypothesis")  
}

## [1] "Reject the null hypothesis"

# Problem 4: Z-Test for Single Mean using z.test()  
  
# A company claims that the average performance score of its employees is 24. A sample of 10 employees gave the following scores: 26, 25, 10, 34, 30, 23, 28, 29, 25, 27. Assuming the population standard deviation is 10, test the company’s claim at a 0.05 significance level.  
  
# Load necessary library  
library(BSDA)

# Sample data  
sample\_data <- c(26, 25, 10, 34, 30, 23, 28, 29, 25, 27)  
  
# One-sample z-test  
z\_test <- z.test(sample\_data, mu = 24, sigma.x = 10)  
print(z\_test)

##   
## One-sample z-Test  
##   
## data: sample\_data  
## z = 0.53759, p-value = 0.5909  
## alternative hypothesis: true mean is not equal to 24  
## 95 percent confidence interval:  
## 19.50205 31.89795  
## sample estimates:  
## mean of x   
## 25.7

alpha <- 0.05 # Significance level  
  
# Decision condition  
if (z\_test$p.value <= alpha) {  
 print("Reject the null hypothesis: The average performance score is significantly different from 24.")  
} else {  
 print("Fail to reject the null hypothesis: There is no significant evidence that the average performance score is different from 24.")  
}

## [1] "Fail to reject the null hypothesis: There is no significant evidence that the average performance score is different from 24."

# Problem 5: Z-Test for Difference of Means using z.test()  
  
# Researchers want to compare the IQ levels of individuals from two different cities to see if there is a significant difference in their means. The IQ levels from 20 individuals in each city are recorded as follows:  
  
# City A: 82, 84, 85, 89, 91, 91, 92, 94, 99, 99, 105, 109, 109, 109, 110, 112, 112, 113, 114, 114  
# City B: 90, 91, 91, 91, 95, 95, 99, 99, 108, 109, 109, 114, 115, 115, 116, 117, 128, 129, 130, 133  
  
# Assume the population standard deviation for both cities is 15, and difference in means of populations is 2, test for a significant difference between the two cities' mean IQ at a 0.05 significance level.  
  
  
# Load necessary library  
library(BSDA)  
  
# Enter IQ levels for 20 individuals from each city  
cityA <- c(82, 84, 85, 89, 91, 91, 92, 94, 99, 99,   
 105, 109, 109, 109, 110, 112, 112, 113, 114, 114)  
  
cityB <- c(90, 91, 91, 91, 95, 95, 99, 99, 108, 109,   
 109, 114, 115, 115, 116, 117, 128, 129, 130, 133)  
  
# Assume different population standard deviations and difference of means  
sigma\_A <- 15  
sigma\_B <- 20  
mu\_diff <- 2  
  
# Two-sample z-test  
z\_test\_city <- z.test(cityA, cityB, mu = mu\_diff, sigma.x = sigma\_A, sigma.y = sigma\_B)  
print(z\_test\_city)

##   
## Two-sample z-Test  
##   
## data: cityA and cityB  
## z = -1.7978, p-value = 0.07221  
## alternative hypothesis: true difference in means is not equal to 2  
## 95 percent confidence interval:  
## -19.006532 2.906532  
## sample estimates:  
## mean of x mean of y   
## 100.65 108.70

alpha <- 0.05 # Significance level  
  
# Decision condition  
if (z\_test\_city$p.value <= alpha) {  
 print("Reject the null hypothesis: There is a significant difference in IQ levels between City A and City B.")  
} else {  
 print("Fail to reject the null hypothesis: There is no significant evidence of a difference in IQ levels between City A and City B.")  
}

## [1] "Fail to reject the null hypothesis: There is no significant evidence of a difference in IQ levels between City A and City B."