UNIT-1

①

-: probability:-

→ Basics of probability, addition theorem, conditional probability, multiplication theorem, Bayes theorem (with out proof).

Experiment :-

An experiment is an act (or) process that leads to a single outcome that can not be predicted with certainty.

→ Tossing a coin is an experiment

→ Recording the monthly sales of a business firm is an experiment

Trial :- Single performance of an experiment is called a trial.

→ A result of an experiment is called outcome.

Each experiment may yield one or more outcomes. When these are not predetermined but subject to chance, we have a random experiment. probability is concerned with the analysis of random experiment.

Random experiment :-

An experiment is conducted any number of times under essentially identical conditions, there is a set of all possible out comes associated with it. If the result is not certain and is any one of the several possible out comes, the experiment is called "random experiment"

(or)

An experiment whose outcomes can not be predicted with certainty is called a random experiment.

Eg:- 1) Reading the dialy temperature on a thermometer is a random experiment.

2) counting the number of misprints in a page is a random experiment

Event :- The possible outcomes of a random experiment is called event.

Eg:-

Tossing a coin

Then H, T are outcomes.

so, H, T are events.

Sample space :-

The set of all possible outcomes of a random experiment is called a sample space.

1. Tossing a coin — H, T are events.

$S = \{H, T\}$

2. Throwing a die — 1, 2, 3, 4, 5, 6 are events

$S = \{1, 2, 3, 4, 5, 6\}$

3. Tossing two coins —

$S = \{HH, HT, TH, TT\} - 2^2 = 4$.

4. Tossing 3 coins — $2^3 = 8$

$$S = \left\{\begin{matrix} H & H & H \\ H & H & T \\ H & T & H \\ H & T & T \\ T & H & H \\ T & H & T \\ T & T & H \\ T & T & T \end{matrix}\right\}$$

5. Tossing 4 coins — $2^4 = 16$

$$S = \left\{\begin{matrix} H & H & H & H \\ H & H & H & T \\ H & H & T & H \\ H & H & T & T \\ H & T & H & H \\ H & T & H & T \\ H & T & T & H \\ H & T & T & T \\ T & H & H & H \\ T & H & H & T \\ T & H & T & H \\ T & H & T & T \\ T & T & H & H \\ T & T & H & T \\ T & T & T & H \\ T & T & T & T \end{matrix}\right\}$$

6. Throwing 1 die $= 6^1 = 6$

$S = \{1, 2, 3, 4, 5, 6\}$

7. Throwing 2 dice $= 6^2 = 36$

$$S = \left\{\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix}\right\}$$

8. Cards 52

- Black (26)
  - spades (♣) (13)
  - clubs (♠) (13)
- Red (26)
  - diamonds (13) ◇
  - hearts ♡ (13)

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K.

Exhaustive event

All possible events in any trail are known as exhaustive events.

→ 1. tossing 1 coin
The exhaustive events are head & Tail

→ 2. throwing 1 die
The e.e are 1, or 2 or 3 or 4 or 5 or 6

→ In drawing 3 balls out of 9 balls in a box, there are $^9C_3$ exhaustive elementary events.

Mutually exclusive events

Events are said to be mutually exclusive, if the happening of any one of the event in a trial excludes the happening of any one of the others.

i.e if no two or more of the events can happen simultaneously in the same trial.

Eg:- when tossing a coin will get Head or tail are events.
when Head gets occurs at first trial
Tail does not get occur
when Tail gets occurs at another trial
Head does not get occur
then Head & Tail are mutually exclusive events.

2. for 1 die ②
$S = \{1, 2, 3, 4, 5, 6\}$
occurance of odd = $\{1, 3, 5\}$
even = $\{2, 4, 6\}$.
They are mutually exclusive events (odd, even)
are mutually exclusive events.

3. for 1 die
$S = \{1, 2, 3, 4, 5, 6\}$
even = $\{2, 4, 6\}$
prime = $\{2, 3, 5\}$

2 is either even or prime.

so, even and prime are not mutually exclusive events.

Equally likely events :-

The outcomes of a random experiment are said to be equally likely events if there is no reason for an event to occur in preference to any other event.

Eg:- If a die is thrown.
there are 1, 2, 3, 4, 5, 6 equally likely possible outcomes.

Eg:- 80% chance for rain due to weather & 20% chance that no rain here these are not equally likely outcomes since one event has more preference to the other event.

Impossible event :-

An event which is certain not to occur is called an impossible event It is denoted by ϕ.

Eg:- when two dice are thrown, getting a total score of 15 is ~~not~~ an impossible event.

Complement of an event :-

The complement of any event E is the event that E does not occur.

It is denoted by E^c (or) $\bar{E}$ (or) E'

Eg:- when die is thrown

$E = \{2, 4, 6\}$

Then $E^c = \{1, 3, 5\}$.

favourable outcomes to the event

The no. of out comes which are favour to the event are called favourable outcomes to the event.

classical definition of probability

Let 'n' be the number of all possible outcomes of a random experiment all of which are equally likely, and 'm' be the favourable to the event E, then the probability of E denoted by

$$P(E) = \frac{\text{no. of favourable outcomes to the Event } E}{\text{Total no. of out comes in the Random experiment}}$$

$$\boxed{P(E) = \frac{m}{n}}$$

Note:-

1. If E is an event, then P(E) is the probability of occurance of an event.
2. The probability value is lies b/w 0 & 1 i.e $0 \leq P(A) \leq 1$
3. The probability value can never in negative.
4. An event that occurs sure is called certain event (or) possible event.

Symbolic notations

1. E_1, E_2 & E_3 are any 3 events
The probability of an event E_1
$= P(E_1)$

2. The probability of occurance both the events
E_1 & E_2 $= P(E_1 \cap E_2)$

3. The probability of occurance of at least one of the events
E_1 & E_2 $= P(E_1 \cup E_2)$

4. The probability of occurance of neither E_1 nor E_2
$\overline{E_1}$ & $\overline{E_2}$
$= P(\overline{E_1} \cap \overline{E_2})$

5. The probability of occurance of at least one of the events
E_1, E_2, E_3
$= P(E_1 \cup E_2 \cup E_3)$

6. The probability of all the three events $= P(E_1 \cap E_2 \cap E_3)$

7. The probability of occurance of exactly one of E_1 & E_2
$= P(E_1 \cap \overline{E_2}) \cup P(E_2 \cap \overline{E_1})$

8. The probability of occurance of ~~exactly one of E_1 and E_2~~
at least two of E_1, E_2, E_3

$= P(E_1 \cap E_2) \cup P(E_2 \cap E_3) \cup P(E_3 \cap E_1)$.

9. The probability of event in which only one of E_1, E_2, E_3 occur
(or)
Exactly one of E_1, E_2 E_3
$= P(E_1 \cap \overline{E_2} \cap \overline{E_3}) \cup P(\overline{E_1} \cap E_2 \cap \overline{E_3})$
$\cup P(\overline{E_1} \cap \overline{E_2} \cap E_3)$

problems on probabilities :-

1. A coin is tossed once. Find the probability of getting Head!

Sol $S = \{H, T\} \Rightarrow n = 2$

no of favourable cases $= m = 1$
i.e $\{H\}$

$\therefore$ probability of getting a Head.

$P(E) = \dfrac{\text{no of favourable o/com to Event}}{\text{Total no of outcomes}}$

$P(E) = \dfrac{m}{n} = \dfrac{1}{2}$

2. Two coins are tossed once. Find the probability of getting (a) one head (b) at least one head.

Sol:- Two coins are tossed.
$S = \{HH, HT, TH, TT\}$
$n =$ no of total outcomes $= 4$.

(a) let E be the event of getting one head
no. of favourable cases to get one head.

$m = 2$ — $E = \{HT, TH\}$

∴ probability of getting one head.

$P(E) = \frac{m}{n} = \frac{2}{4} = \frac{1}{2}$

(b) no. of favourable cases to get at least one head. (1 head (or) two heads)

$m = 3$ — $E = \{HT, TH, HH\}$

∴ probability of getting at least one head $P(E) = \frac{m}{n} = \frac{3}{4}$.

3. A coin is tossed thrice. Find the probability of getting
(i) all tails (ii) one Tail
(iii) at least 2 Heads
(iv) all 3 Heads.

Sol) when a coin toss 3 times.

$$S = \left\{ \begin{matrix} HHH \\ HHT \\ HTH \\ HTT \\ THH \\ THT \\ TTH \\ TTT \end{matrix} \right\}$$

n = ~~total~~ no. of out comes = 8.

(a) let E be the event of getting all tails.

$E = \{TTT\}$

$m = 1$

$P(E) = \frac{m}{n} = \frac{1}{8}$.

∴ The probability of getting all tails $\boxed{P(E) = \frac{1}{8}}$.

(ii) let E be the event that getting one tail

$E = \{HHT, HTH, THH\}$.

$m = 3$

∴ $P(E) = \frac{m}{n} = \frac{3}{8}$.

(iii) let E be the event that getting at least 2 heads.

$E = \{HHH, HHT, HTH, HTT, THH, THT, TT\}$

$m = 7$

$P(E) = \frac{7}{8}$.

(iv) let E be the event that of getting all Heads.

$E = \{HHH\} \Rightarrow m = 1$

$P(E) = \frac{m}{n} = \frac{1}{8}$.

two di
prob

Two dice are thrown find the probability of

(i) Both the dice shows the same number

(ii) the total ~~of~~ the numbers on the dice is '8'

(iii) the total of the numbers on the dice is greater than 8.

(iv) the total of the numbers on the two dice is any number 2 to 12.

(v) The first die shows 6.

Sol:- The sample space for two dice

$$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$$
$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$$
$$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$$
$$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$$
$$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$$
$$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$$

Total no. of out comes $= n = 36$

(i) let E be the event of both dice shows the same number

$$E = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$$

$m = 6$

$$\therefore P(E) = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}.$$

$\therefore$ The probability of getting both the dice shows the same number

$$\boxed{P(E) = \frac{1}{6}.}$$

(ii) let E be the event of total of the numbers on the dice is 8

$$E = \{(2,6)\ (3,5)\ (4,4)\ (5,3)\ (6,2)\}$$

$m = 5$

$$\boxed{P(E) = \frac{m}{n} = \frac{5}{36}.}$$

(iii) let E be the event of the total numbers on the dice is greater than 8.

$$E = \{\cancel{(3,6)}\ (3,6)\ (4,5)\ (4,6)$$
$$(5,4)\ (5,5)\ (5,6)$$
$$(6,3)\ (6,4)\ (6,5)\ (6,6)\}$$

$m = 10$

$$\boxed{P(E) = \frac{m}{n} = \frac{10}{36}}.$$

(iv) Let E be the event of the total of the numbers on the two dice is any number 2 to 12

$$E = \{(1,1)\ (1,2)\ ----\ (6,6)\}$$

$m = 36.$

$$\boxed{P(E) = \frac{m}{n} = \frac{36}{36} = 1}$$

(v) let E be the event of first die shows 6

$$E = \{(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$$

$m = 6$ $\quad \therefore \boxed{P(E) = \frac{m}{n} = \frac{6}{36}}$

5. Among the digits 1, 2, 3, 4, 5 at first one choosen and then a second selection is made among the remaining four digits - Assumed that all twenty possible out comes have equal probabilities —

Find the probability that an odd digit will be selected.

(i) the first time

(ii) the second time

(iii) bot the times.

Sol

$$S = \left\{ \begin{array}{llll} (1,2) & (1,3) & (1,4) & (1,5) \\ (2,1) & (2,3) & (2,4) & (2,5) \\ (3,1) & (3,2) & (3,4) & (3,5) \\ (4,1) & (4,2) & (4,3) & (4,5) \\ (5,1) & (5,2) & (5,3) & (5,4) \end{array} \right\}$$

n = Total no. of out comes = 20

(i) Let E be the event of odd digit will be selected the first time.

$$E = \left\{ \begin{array}{llll} (1,2) & (1,3) & (1,4) & (1,5) \\ (3,1) & (3,2) & (3,4) & (3,5) \\ (5,1) & (5,2) & (5,3) & (5,4) \end{array} \right\}$$

m = 12

$$P(E) = \frac{m}{n} = \frac{12}{20} = \frac{3}{5}$$

$$\boxed{P(E) = \frac{3}{5}}$$

(ii) Let E be the event of odd will be selected the second time

$$E = \left\{ \begin{array}{lllll} (1,3) & (1,5) & (2,1) & (2,3) & (2,5) \\ (3,1) & (3,5) \\ (4,1) & (4,3) & (4,5) \\ (5,1) & (5,3) \end{array} \right\}$$

m = 12

The probability of an odd digit will be selected the second time

$$\boxed{P(E) = \frac{m}{n} = \frac{12}{20} = \frac{3}{5}}$$

(iii) Let E be the event of odd digit will be selected both the times

$$E = \left\{ \begin{array}{ll} (1,3) & (1,5) \\ (3,1) & (3,5) \\ (5,1) & (5,3) \end{array} \right\}$$

m = 6

$$\therefore \boxed{P(E) = \frac{m}{n} = \frac{6}{20} = \frac{3}{10}}$$

6) In a single throw with two dice, find the probability of throwing a sum (i) 10 (ii) which is a perfect square.

Sol $S = \{(1,1) \; - - - - \; (6,6)\}$

n = 36

(i) Let E be the event of getting sum is 10

$E = \{(4,6)\ (5,5)\ (6,4)\}$

m = 3

$$\boxed{P(E) = \frac{m}{n} = \frac{3}{36} = \frac{1}{12}}$$

(ii) Let E be the event of getting sum is a perfect square.

i.e the required sum is 4 & 9.

$E = \{(1,3)\ (2,2)\ (3,1)\ (3,6)\ (4,5)\ (5,4)\ (6,3)\}$

$m = 7$

$$\boxed{P(E) = \frac{m}{n} = \frac{7}{36}}$$

5

8) A box contains 6 white, 4 red and 9 black balls.

If 3 balls are drawn at random, find the probability that

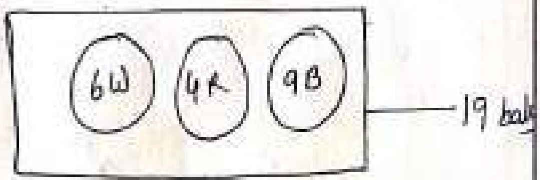
(i) Two of the balls drawn are white

(ii) one is of each color

(iii) none is red

(iv) at least one is white.

Sol:-



from

S = { Total 19 balls take 3 balls }

= $19C_3$ ways can be taken 3 out of 19

$\boxed{n = 19C_3}$

(i) Let E be the event of two balls are white.

(another one might be from red or black)

$m = 6C_2 \times 13C_1$

$$\boxed{P(E) = \frac{m}{n} = \frac{6C_2 \times 13C_1}{19C_3}}$$

(ii) Let E be the event that one is of each colour

$m = 6C_1 \times 4C_1 \times 9C_1$

$$P(E) = \frac{m}{n} = \frac{6C_1 \times 4C_1 \times 9C_1}{19C_3}$$

(iii) Let E be the event that none is red

$m = 15C_3$ (w + B = 9 + 6 = 15)

$$P(E) = \frac{m}{n} = \frac{15C_3}{19C_3}$$

(iv) Let E be the event that at least one is white

$m = (6C_1 \times 13C_2) + (6C_2 \times 13C_1) + (6C_3)$

$$P(E) = \frac{m}{n} = \frac{(6C_1 \times 13C_2) + (6C_2 \times 13C_1) + (6C_3 \times 13C_0)}{19C_3}$$

In a shuffling a pack of cards four are dropped. find the chance that the missing cards should be one from each list.

Sol S = four cards are dropped from pack of 52 cards.

$n = 52C_4$

let E be the event that missing cards from each list

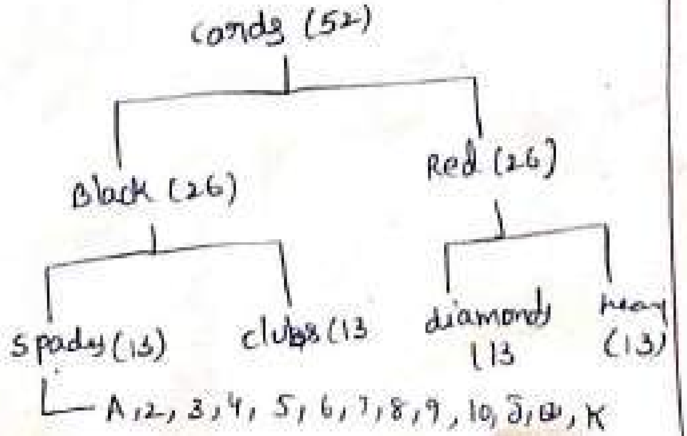
$m = 13C_1 \times 13C_1 \times 13C_1 \times 13C_1$

$\therefore$ the probability of missing cards from each list

$$P(E) = \frac{m}{n} = \frac{13C_1 \times 13C_1 \times 13C_1 \times 13C_1}{52C_4}$$

(Q). Four cards are drawn at random from a pack of 52 cards.
Find the probability that
(i) they are king, a queen, a jack and an ace
(ii) Two are kings & two are queens
(iii) Two are black and two are red.
(iv) there are two cards of hearts and two cards of diamonds.

Sol



sample space

S = four cards are drawn from out of 52 cards

$\boxed{n = 52C_4}$

(i) let E be the event that the card is king, a queen, a jack & an ace

$m = 4C_1 \times 4C_1 \times 4C_1 \times 4C_1$

$\therefore$ The probability that they are king, a queen, a jack & an ace $P(E) = \frac{m}{n}$

$$P(E) = \frac{4C_1 \times 4C_1 \times 4C_1 \times 4C_1}{52C_4}$$

(ii) let E be the event of two cards are kings & two are queens

$m = 4C_2 \times 4C_2$

$$P(E) = \frac{m}{n} = \frac{4C_2 \times 4C_2}{52C_4}$$

(iii) let E be the event of two are black & two are red

$m = 26C_2 \times 26C_2$

$$P(E) = \frac{m}{n} = \frac{26C_2 \times 26C_2}{52C_4}$$

(iv) Let E be the event of

two cards of hearts and

two cards of diamonds.

$m = 13C_2 \times 13C_2$

$$\boxed{P(E) = \frac{13C_2 \times 13C_2}{52C_4}}$$

11. A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase dpt, 2 officers of the sales dpt. and 1 chartered accountant. Find the probability of forming the committee in the following manner.

(i) There must be one from each category

(ii) It should have at least one from the purchase dpt

(iii) The chartered accountant must be in the committee.

Independent and dependent events ①

If the occurance of the event E_2 is not affected by the occurance or non occurance of the event E_1. Then the occurance of the event E_2 is said to be independent of E_1.

→ Two events are said to be independent events if

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

conditional probability :-

If E_1, E_2 are ~~any~~ two events in a sample space S, and the event E_1 is already occured, $P(E_1) \neq 0$, then the probability of E_2 after the event E_1 has occured, is called the conditional probability of the event of E_2 given E_1. and it is denoted by $P\left(\frac{E_2}{E_1}\right)$

$$\boxed{P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}}$$

similarly we define

$$\boxed{P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}}$$

1. One shot is fired from each of the three guns. E_1, E_2, E_3 denote the events that the target is hit by the first, second and third guns respectively.

If $P(E_1) = 0.5$, $P(E_2) = 0.6$, $P(E_3) = 0.8$ and E_1, E_2, E_3 are independent events, find the probability that

(a) Exactly one hit is registered

(b) at least two hits are registered.

Sol:- Given that

$P(E_1) = 0.5$	$P(\bar{E_1}) = 1 - P(E_1) = 0.5$
$P(E_2) = 0.6$	$P(\bar{E_2}) = 1 - P(E_2) = 0.4$
$P(E_3) = 0.8$	$P(\bar{E_3}) = 1 - P(E_3) = 0.2$

(a) The probability of exactly one hit is registered =

$$= P(E_1 \cap \bar{E_2} \cap \bar{E_3}) + P(\bar{E_1} \cap E_2 \cap \bar{E_3}) + P(\bar{E_1} \cap \bar{E_2} \cap E_3)$$

$$= P(E_1) \cdot P(\bar{E_2}) \cdot P(\bar{E_3}) + P(\bar{E_1}) \cdot P(E_2) P(\bar{E_3}) + P(\bar{E_1}) \cdot P(\bar{E_2}) \cdot P(E_3)$$

$$= (0.5)(0.4)(0.2) + (0.5)(0.6)(0.2) + (0.5)(0.4)(0.8)$$

$$= 0.26$$

(b) The probability to at least two hits can be registered

$$= P(E_1 \cap E_2 \cap \bar{E_3}) + P(E_1 \cap \bar{E_2} \cap E_3) + P(\bar{E_1} \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

$$= P(E_1) P(E_2) P(\bar{E_3}) + P(E_1) P(\bar{E_2}) P(E_3) + P(\bar{E_1}) P(E_2) P(E_3) + P(E_1) \cdot P(E_2) P(E_3)$$

$$= 0.70$$

Taking Probability

2. If A, B, C are independent events and $P(A)$ is $3/4$ & $P(B)$ is $4/5$ and $P(C)$ is $\frac{5}{6}$ what is the probability that exactly one is true.

Sol

$P(A) = 3/4$	$P(\bar{A}) = \frac{1}{4}$
$P(B) = 4/5$	$P(\bar{B}) = 1/5$
$P(C) = 5/6$	$P(\bar{C}) = 1/6$

The probability of exactly one is true =

$$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$$

$$= P(A) \cdot P(\bar{B}) P(\bar{C}) + P(\bar{A}) P(B) P(\bar{C}) + P(\bar{A}) P(\bar{B}) P(C)$$

$$= \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{1}{6} + \frac{1}{4} \cdot \frac{4}{5} \cdot \frac{1}{6} + \frac{1}{4} \cdot \frac{1}{5} \cdot \frac{5}{6}$$

$$= 0.1$$

…ion has 2 doctors X and Y practising independently. If the probability that doctor X is available is 0.9 and Y is 0.8. Then what is the probability that at least one doctor is available.

Sol:- $P(X)=0.9 \Rightarrow P(\bar{X})=0.1$

$P(Y)=0.8 \Rightarrow P(\bar{Y})=0.2$

$\therefore$ the probability that at least one doctor is available in the following

$= P(X\cap\bar{Y}) + P(\bar{X}\cap Y) + P(X\cap Y)$

$= P(X)\,P(\bar{Y}) + P(\bar{X})\,P(Y) + P(X)\,P(Y)$

$= 0.98$

$$\boxed{\begin{aligned} P(A\cup B) &= 1-P(\overline{A\cup B}) \\ &= 1-P(\bar{A}\cap\bar{B}) \\ &= 1-P(\bar{A})\cdot P(\bar{B}) \end{aligned}}$$

4. A problem in statistics is given to three students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$ respectively. What is the probability that the problem is solved.

Sol G.T

$P(A)=1/2$	$P(\bar{A})=1/2$
$P(B)=3/4$	$P(\bar{B})=1/4$
$P(C)=1/4$	$P(\bar{C})=3/4$

$\therefore$ The probability that the problem is solved $= P(A\cup B\cup C)$

$= \cancel{P(A)}$

$P(A\cup B\cup C) = 1-P(\overline{A\cup B\cup C})$

$= 1-P(\bar{A}\cap\bar{B}\cap\bar{C})$

$= 1-P(\bar{A})\cdot P(\bar{B})\cdot P(\bar{C})$

$= 1-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}$

$= 1-\frac{3}{32} = \frac{29}{32}$

(or)

can we find by this way

$P(\bar{A}\cap\bar{B}\cap C) + P(\bar{A}\cap B\cap\bar{C}) + P(A\cap\bar{B}\cap\bar{C})$
$+ P(\bar{A}\cap B\cap C) + P(\bar{A}\cap B\cap\bar{C}) + P(A\cap\bar{B}\cap\ldots$
$+ P(A\cap B\cap C)$.

5. The person X speaks the truths are 3:2 and that the person Y speaks the truths are 5:3. In what percentage of cases are they likely to contradict each other on an identical point

Sol X: be the event that X speaks the truth

Y: be the event that Y speaks the truth

contradict on same point means
If X says truths Y says lie on same issue
X says … Y says true on " "

G.t

$P(X) = \frac{3}{5}$ | $P(Y) = \frac{5}{8}$

$P(\bar{X}) = \frac{2}{5}$ | $P(\bar{Y}) = \frac{3}{8}$.

∴ The probability that they contradict each other $= P(X \cap \bar{Y}) + P(\bar{X} \cap Y)$

$= P(X) \cdot P(\bar{Y}) + P(\bar{X}) \cdot P(Y)$

$= \frac{19}{40}$

6. The odds that a book on statistics will be favourable reviewed by 3 independent critics are 3 to 2, 4 to 3 & 2 to 3 respectively.
what is the probability that of the three reviews.

(i) All will be favourable.
(ii) majority of the reviews will be favourable.
(iii) Exactly one review will be favourable
(iv) Exactly two reviews will be favourable.
(v) At least one of the reviews. will be favourable.

Sol) Let A, B, C denote the first, second & third critics for a book of statistics.

$P(A) = \frac{3}{5}$ | $P(\bar{A}) = \frac{2}{5}$

$P(B) = 4/7$ | $P(\bar{B}) = 3/7$

$P(C) = 2/5$ | $P(\bar{C}) = 3/5$.

(i) The probability that all will be favourable is

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$= \frac{3}{5} \cdot \frac{4}{7} \cdot \frac{2}{5} = \frac{24}{175}$

(ii) The probability that the majority of reviews will be favourable is

$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C)$

$= P(A) P(B) P(\bar{C}) + P(A) P(\bar{B}) P(C) + P(\bar{A}) P(B) P(C)$

$= \frac{94}{175}$.

(iii) The probability that exactly one review will be favourable is

$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap \bar{C})$

$= \frac{63}{175}$

(iv) The probability that exactly two reviews will be favourable

$= P(A \cap B \cap \bar{C}) + P(\bar{A} \cap B \cap C) + P(A \cap \bar{B} \cap C)$

$= \frac{63}{175}$

(v) ... one of th...

(v) The probability that at least one of the reviews will be favourable

$$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C}) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= 1 - P(\bar{A}) \cap P(\bar{B}) \cap P(\bar{C})$$
$$= 1 - \left(\tfrac{2}{3}\right)\left(\tfrac{3}{7}\right)\left(\tfrac{3}{5}\right)$$
$$= \frac{157}{175}.$$

7. It is 8:5 against the wife who is 40 years old living till she is 70. and 4:3 against her husband now 50 living till he is 80.

Find the probability that

(i) Both will be alive

(ii) None will be alive

(iii) only wife will be alive

(iv) only husband will be alive

(v) only one be alive

(vi) At least one will be alive

30 years.

Sol A : be the event of wife living.

B : be the " " husband living.

Given that 8:5 against wife ~~is~~ 40 years old living till she is 70

$P(A) = \frac{5}{13}$. | $P(\bar{A}) = \frac{8}{13}$

Given that 4:3 against husband 50 years living till he is 80.

$P(B) = \frac{3}{7}$ | $P(\bar{B}) = \frac{4}{7}$.

(i) The probability that both will be alive $= P(A \cap B)$

$$P(A \cap B) = P(A) \cdot P(B)$$
$$= \frac{5}{13} \cdot \frac{3}{7} = \frac{15}{91}$$

(ii) The probability the none will be alive $= P(\bar{A} \cap \bar{B})$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B}) = \frac{8}{13} \cdot \frac{4}{7}.$$

(iii) The probability that

conditional probability

If E_1 & E_2 are any two events of a sample space S. and the event E_1 is already occured, ~~then~~ and $P(E_1) \neq 0$, then the probability of E_2 after event E_1 has occured is called conditional probability of the event of E_2 given E_1

$$\boxed{P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}}$$

Eg:- Throwing a die

$S = \{1, 2, 3, 4, 5, 6\}$

even no: $A = \{2, 4, 6\}$

prime no: $B = \{2, 3, 5\}$

$P[\text{Even (prime)}] = P\left[\frac{A}{B}\right]$

$P[\text{prime (Even)}] = P\left[\frac{B}{A}\right]$

1. If 2 dice are thrown. Then find the probability that whose sum is '7' which contains '2' in one of the dice.

Sol $\boxed{P[\text{sum 7 (2 in one of the dice)}]}$

Sol:- $S = \{(1,1) - - - - \ - - - - (6,6)\}$

$n = 36$.

A: be the event of whose sum is 7

$= \{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (6,1)\}$

$P(A) = \frac{m}{n} = \frac{6}{36}$.

B: be the event of 2 in one of the dice.

$= \{(1,2)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (5,2)\ (2,5)\ (2,6)\ (3,2)\ (4,2)\ (6,2)\}$

$P(B) = \frac{m}{n} = \frac{11}{36}$.

$A \cap B$: be the event that ~~whose~~ sum is 7 and containing any one of the dice is 2

$= \{(2,5)\ (5,2)\}$.

$P(A \cap B) = \frac{2}{36}$

∴ The probability that whose sum is 7 which contains 2 in one of the dice

$= P[\text{sum 7 (2 is one of the dice)}]$

$= P\left[\frac{A}{B}\right]$

W.K.T $P\left[\frac{A}{B}\right] = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{11}{36}} = \frac{2}{11}$

$\boxed{\therefore P\left[\frac{A}{B}\right] = 2/11}$

2. A Bag containg 10 gold and 8 silver coins. Two successive drawing of 4 coins are made such that.

(i) coins are replaced before the second trail

(ii) the coins are not replaced before the second trail

find the probability that the first drawing will give 4 gold & second 4 silver coins.

Sol:- (i) Coins are replaced:-

$\left(S = \{ \text{selecting 4 coins out of 18} \} \quad \boxed{n = {}^{18}C_4} \right)$

E_1: be the event of 4 gold from 10 Coins out of 18 coins.

~~be the e event of depositsilver~~

$$P(E_1) = \frac{{}^{10}C_4}{{}^{18}C_4}$$

B: be the event of 4 silver.

The probability of first drawing is 4 gold $= P(A) = \frac{m}{n} = \frac{{}^{10}C_4}{{}^{18}C_4}$

after coins are replaced.

$\boxed{n = {}^{18}C_4}$

The probability of second drawing 4 silver $= P(B) = \frac{m}{n} = \frac{{}^{8}C_4}{{}^{18}C_4}$

∴ The probability of getting 4 gold & 4 silver coins with replacement is $P(A \cap B)$

$$P(A \cap B) = P(A) \cdot P(B) \quad \text{— [A \& B events are independent]}$$

$$= \frac{{}^{10}C_4}{{}^{18}C_4} \cdot \frac{{}^{8}C_4}{{}^{18}C_4}$$

(ii) coins are not replaced.

A: be the event that drawing 4 gold from 10

$$P(A) = \frac{{}^{10}C_4}{{}^{18}C_4} \quad \left(\text{when first trail the box has total 18 coins}\right)$$

B: be the event that drawing 4 silver from 8

$$P(B) = \frac{{}^{8}C_4}{{}^{14}C_4} \quad \left(\text{when second trail the box has total 14 coins}\right)$$

The probability of getting 4 gold & 4 silver coins is $P(A \cap B)$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P\left(\frac{B}{A}\right) \cdot P(A)$$

$$= \frac{{}^{8}C_4}{{}^{14}C_4} \cdot \frac{{}^{10}C_4}{{}^{18}C_4}$$

$$P(A \cap B) = \frac{{}^{8}C_4}{{}^{14}C_4} \cdot \frac{{}^{10}C_4}{{}^{18}C_4}$$

3. From a city population, the probability of selecting

(i) a male (or) a smoker is $7/10$

(ii) a male smoker is $2/5$

(iii) a male, if a smoker is already selected is $2/3$.

Find the probability of selecting

(a) a non-smoker

(b) a male

(c) a smoker if a male is first selected.

Sol:- A: be the event that a male

B: " " a smoker

Given that

$P(A \cup B) = 7/10$

$P(A \cap B) = 2/5$

$P\left(\frac{A}{B}\right) = 2/3$.

(a) The probability of selecting a non smoker $= P(\bar{B})$

$P(\bar{B}) = 1 - P(B)$

$= 1 - \frac{P(A \cap B)}{P(A/B)}$ $\left[P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \right.$

$= 1 - \frac{2/5}{2/3} = 1 - 3/5 = 2/5$

$\therefore \boxed{P(\bar{B}) = 2/5}$

(ii) The probability of selecting a male $= P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = P(A \cup B) - P(B) + P(A \cap B)$

$= \frac{7}{10} - 3/5 + 2/5 = \frac{1}{2}$

(iii) The probability of selecting a smoker if a male is first selected.

$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{2/5}{1/2}$

$\boxed{P\left(\frac{B}{A}\right) = 4/5}$

---

4. In a certain town, 40% have brown hair, 25% have brown eyes and 15% have both brown hair & brown eyes. A person is selected at random from the town.

(i) if he has brown hair, what is the probability that he has brown eyes also

(ii) if he has brown eyes, determine the probability that he does not have brown hair.

Sol:- A: be the even that do having brown hair

B: be the event that do having brown eyes.

$A \cap B$: be the even that both brown hair & eyes

$P(A) = .40\% = 0.4$

$P(B) = 25\% = 0.25$

$P(A \cap B) = 15\% = 0.15$

(i) The probability that the person the selected person has brown eyes gt he already has brown hair $= P\left(\frac{B}{A}\right)$

$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{0.15}{0.4} = 0.375$

(ii) the probability that the person has not brown hair gt he already has brown eyes $= P\left(\frac{\bar{A}}{B}\right)$

$\therefore P\left(\frac{\bar{A}}{B}\right) = \frac{P(\bar{A} \cap B)}{P(B)}$ $\left[\because P(\bar{A} \cap B) = P(B) - P(A \cap B)\right]$

$P\left(\frac{\bar{A}}{B}\right) = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{0.25 - 0.15}{0.25} = \underline{0.4}$

5. Two marbles are drawn in succession from a box containing 10 red, 30 white and 15 orange marbles, with replacement being made after each draw. Find the probability that

(i) Both are white

(ii) First is red and second is white.

Sol

$n = 75C_1$

(i) Let E_1 be the event of the first drawn marble is white.

then $P(E_1) = \frac{30C_1}{75C_1} = \frac{30}{75}$

Let E_2 be the event of the second drawn marble is also white. Then

$P(E_2) = \frac{30}{75}$.

$\therefore$ The probability that both marbles are white (with replace ment)

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$= \frac{30}{75} \cdot \frac{30}{75} = \frac{4}{25}$

(ii) Let E_1 be the event of the first drawn marble is Red

$P(E_1) = \frac{10C_1}{75C_1} = \frac{2}{15}$

let E_2 be the event that the second drawn marble is white.

$$P(E_2) = \frac{30}{75} = \frac{2}{5}$$

∴ The probability that the first marble is red and second marble is white (with replacement)

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

$$= \frac{2}{15} \cdot \frac{2}{5} = \frac{4}{75}$$

6 A, B, C are aiming to shoot a balloon. A will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3 out of 4 and that of C is 2 out of 3. If the three aim the balloon simultaneously, then find the probability that at least two of them hit the balloon.

Sol:- The probability of A hitting the target $= P(A) = 4/5$

The probability of B hitting the target $= P(B) = 3/4$

The probability of C hitting the target $= P(C) = 2/3$.

∴ The probabilities of A, B, C not hitting the target respectively are

$P(\bar{A}) = 1/5$, $P(\bar{B}) = 1/4$

$P(\bar{C}) = 1/3$

Now the probability that exactly two will hit the balloon

$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(\bar{C}) + P(A) P(\bar{B}) P(C)$
$+ P(\bar{A}) P(B) P(C)$

$= \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3}$

$= \frac{13}{30}$

The probability that all will hit the balloon $= P(A \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(C)$

$= \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{2}{5}$

∴ The probability that at least two of them will hit the target

$= \frac{13}{30} + \frac{2}{5} = \frac{13+12}{30} = \frac{25}{30} = \frac{5}{6}$

Bayes' theorem

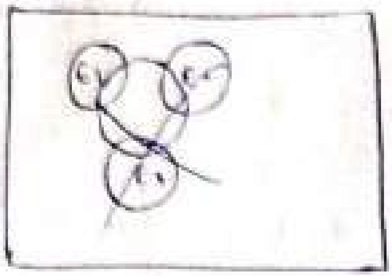
Baye's theorem (or) Rule of Inverse probability

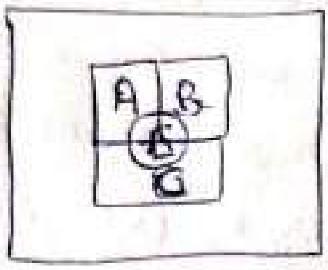
St:- If $E_1, E_2, \ldots E_n$ are 'n' mutually exclusive and exhaustive events with $P(E_i) > 0$ in a sample space 'S'. and A is any another event in S ~~inter~~ intersecting with every E_i (i.e A can only occur in combination with any one of events $E_1, E_2, \ldots E_n$) that $P(A) > 0$. then we have

$$\boxed{P\left(\frac{E_i}{A}\right) = \frac{P(E_i) \cdot P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)}}$$

1. Three machines A, B, C produces 20%, 25%, 55% of the total number of items of a factory. the percentage of defective items of these machines are 7%, 3% & 3% respectively. An item is selected at random and found to be defective. Find the probability that it is from (i) machine - A (ii) machine - B (iii) machine - C

301





The probability that machine A produces items

$P(A) = 20\% = 0.2$

The probability that machine B produces items

$P(B) = 25\% = 0.25$

the probability that the items produced by machine C

$P(C) = 55\% = 0.55$

E : be the event that the defective items in machine A, B, C

$P\left(\frac{E}{A}\right) = 7\% = \frac{7}{100} = 0.07$

$P\left(\frac{E}{B}\right) =$ the defective items produced in machine B

$= 3\% = 0.03$

$P\left(\frac{E}{C}\right) =$ the defective items produced in machine C

$= 3\% = 0.03$

(i) If the selected item is defective then the probability that it is from machine A

$$P\left(\frac{A}{E}\right) = \frac{P(A)\, P\left(\frac{E}{A}\right)}{P(A)\, P\left(\frac{E}{A}\right) + P(B)\, P\left(\frac{E}{B}\right) + P(C)\cdot P\left(\frac{E}{C}\right)}$$

$= \cancel{\{0.2\}(0.07)\} + 0.25\,(0.03)\}}$

$$= \frac{(0.2)\ (0.07)}{(0.2)(0.07) + (0.25)\ (0.03) + (0.55)(0.03)}$$

$= 0.36$

(ii) If the selected item is defective then the probability that it is from machine B

$$P\left(\frac{B}{E}\right) = \frac{P(B)\cdot P\left(\frac{E}{B}\right)}{P(A)\ P\left(\frac{E}{A}\right) + P(B)\cdot P\left(\frac{E}{B}\right) + P(C)\, P\left(\frac{E}{C}\right)}$$

$$= \frac{(0.25)\ (0.03)}{(0.2)(0.07) + (0.25)\ (0.03) + (0.55)(0.03)}$$

$= 0.197$

(iii) If the selected item is defective then the probability that it is from machin C

$$P\left(\frac{C}{E}\right) = \frac{P(C)\cdot P\left(\frac{E}{C}\right)}{P(A)\cdot P\left(\frac{E}{A}\right) + P(B)\cdot P\left(\frac{E}{B}\right) + P(C)\, P\left(\frac{E}{C}\right)}$$

$= 0.43$

2. In a certain college, 25% of the boys and 10% of girls are studying mathematics. The girls constitute 60% of the students.

(a) what is the probability that mathematics is being studied?

(b) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl.

(c) find the probability that the student is a boy if he is studying mathematics

Sol: Given that the girls constitute 60% of the students

$P(G) = 0.6$
$P(B) = 0.4$

(G: be the event of girls
B: be the event of boys)

G.t the probabily that the boys study mathematis is $P\left(\frac{M}{B}\right)$

$P\left(\frac{M}{B}\right) = 25\% = 0.25$

G.t the probability that the girl studies mathematics is

$P\left(\frac{M}{G}\right) = 10\% = 0.1$

$P\left(\frac{M}{G}\right) = 0.1$

(a) The probability that mathematics is selected.

$P(M) = P(G)\cdot P\left(\frac{M}{G}\right) + P(B)\cdot P\left(\frac{M}{B}\right)$

$= (0.6)\cdot(0.1) + (0.4)(0.25)$

$= \frac{4}{25} = 0.15$

(b) The probability that the selected student is a girl if that girl studies mathematics

$P\left(\frac{G}{M}\right) = ?$

$$P\left(\frac{G}{M}\right) = \frac{P(G)\cdot P\left(\frac{M}{G}\right)}{P(G)\cdot P\left(\frac{M}{G}\right) + P(B)\, P\left(\frac{M}{B}\right)}$$

$$= \frac{(0.6)(0.1)}{(0.6)(0.1) + (0.4)(0.25)}$$

$$= \frac{3}{8}$$

(c) The probability that the selected student is a boy if the boy studies mathe

$$P\left(\frac{B}{M}\right) = \frac{P(B)\cdot P\left(\frac{M}{B}\right)}{P(G)\cdot P\left(\frac{M}{G}\right) + P(B)\cdot P\left(\frac{M}{B}\right)}$$

$$= \frac{(0.4)\cdot(0.25)}{(0.6)(0.1) + (0.4)(0.25)}$$

$$= \frac{5}{8}$$

3. Three machines A, B, C produce 40%, 30%, 30% of the total number of items of factory. The percentages of defective items of these machines are 4%, 2%, 3%. If an item is selected at random, find the probability that the item is defective.

Sol:- Given that A, B, C are three machines.

$P(A) = 0.40$

$P(B) = 0.30$

$P(C) = 0.30$

D be the event which denotes the defective item.

The percentage of the defective items in machine A is 4%.

i.e $P\left(\frac{D}{A}\right) = \frac{4}{100} = 0.04$

Similarly $P\left(\frac{D}{B}\right) = \frac{2}{100} = 0.02$

$P\left(\frac{D}{C}\right) = \frac{3}{100} = 0.03$

$\therefore$ The probability that the selected item is defective is

$$P(D) = P(A) \cdot P\left(\frac{D}{A}\right) + P(B) \cdot P\left(\frac{D}{B}\right) + P(C) \cdot P\left(\frac{D}{C}\right).$$

$$= (0.40 \times 0.04) + (0.30)(0.02) + (0.30)(0.03)$$

$$= \frac{31}{1000}$$

4. In a group consisting of equal number men and women, 10% of the men and 45% of the women are unemployed. If a person is selected randomly from the group then find the probability that the person is an employee.

Sol ~~we have~~

M : men

W : women

$P(M) = 1/2$ & $P(W) = 1/2$

Let E_1 be the event ~~that~~ employed

E_2 " unemployed

Given that the percentage of unemployed in men is 10%.

$P\left(\frac{E_1}{M}\right) = 0.1$

Similarly $P\left(\frac{E_1}{W}\right) = 0.45$

The percentage of employed in men is 90%.

$P\left(\frac{E_2}{M}\right) = 0.9$

||ly $P\left(\frac{E_1}{W}\right) = 0.55$

$\therefore$ The probabily of a selected person is employed.

$$P(E_2) = P(M) \cdot P\left(\frac{E_2}{M}\right) + P(W) \cdot P\left(\frac{E_2}{W}\right)$$

$$= \frac{1}{2} \cdot \frac{9}{10} + \frac{1}{2} \cdot \frac{55}{100}$$

$$= \frac{1}{2} 0.9 + \frac{1}{2}(0.55)$$

$$= 0.725$$

5. A businessman goes to hotels X, Y, Z 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbings what is the probability that business man's room having faulty plumbing is assigned to hotel Z?

Sol Let the probabilities of business man going to hotels X, Y, Z be respectively $P(X)$, $P(Y)$, $P(Z)$.

Then

$$P(X) = \frac{2}{10}, \quad P(Y) = \frac{5}{10}, \quad P(Z) = \frac{3}{10}.$$

Let E be the event that the hotel has faulty pumbling.

given that

The probabilities that hotels X, Y, Z have faulty pumblings are

$$P\left(\frac{E}{X}\right) = 0.05$$

$$P\left(\frac{E}{Y}\right) = 0.04$$

$$P\left(\frac{E}{Z}\right) = 0.08.$$

$\therefore$ ~~The probability that the hotel room has faulty plumbing~~

$\therefore$ The probability that the business man's room having faulty pluming is assigned to hotel Z is

$$P\left(\frac{Z}{E}\right) = \frac{P(Z) \cdot P\left(\frac{E}{Z}\right)}{P(X)\, P\left(\frac{E}{X}\right) + P(Y) \cdot P\left(\frac{E}{Y}\right) + P(Z) \cdot P\left(\frac{E}{Z}\right)}$$

$$= \frac{\frac{3}{10}(0.08)}{\frac{2}{10}(0.05) + \frac{5}{10}(0.04) + \frac{3}{10}(0.08)}$$

$$= \frac{4}{9}.$$

6. suppose 5 men out of 100 and 25 women out of 10,000 are colour blind. A colour blind person is choosen at random what is the probability of the person being a male (Assumed male & female to be in equal numbers).

sol:-

The probability that the choosen person is male

$P(M) = 1/2$

The probability that the choosen person is female

$P(W) = 1/2$

[Given that 5 men out of 100 are colour blind.]

let B be the event that colour blind.

∴ The probability of 5 men out of 100 are colour blind.

$$P\left(\frac{B}{M}\right) = \frac{5}{100} = 0.05.$$

The probability of 25 women out of 10000 are colour bling

$$P\left(\frac{B}{W}\right) = \frac{25}{10000} = 0.0025.$$

∴ The probability of the person is a male is given by colour blind.

$$P\left(\frac{M}{B}\right) = \frac{P(M)\cdot P\left(\frac{B}{M}\right)}{P(M)\cdot P\left(\frac{B}{M}\right) + P(W)\cdot P\left(\frac{B}{W}\right)}$$

$$= \frac{0.05 \times 0.5}{(0.05)\times 0.5 + (0.5)(0.00}$$

$$= 0.95$$

Axiomatic definition of probability

If $A_1, A_2, A_3 \dots A_n$ are 'n' events in the sample space then

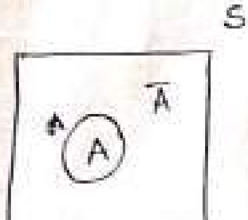
(i) $P(A_i) \geq 0 \ \forall i$

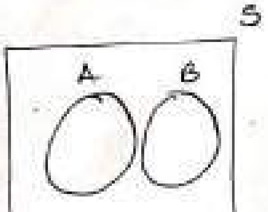
(ii) $P(S) = 1$

(iii) If $A_1, A_2, A_3 \dots A_n$ are 'n' distinct events then

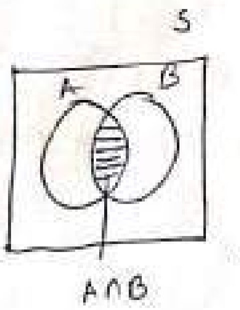
$P(A_1 \cup A_2 \cup A_3 \dots A_n)$

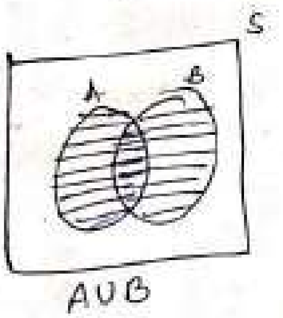
$= P(A_1) + P(A_2) + \dots P(A_n)$.

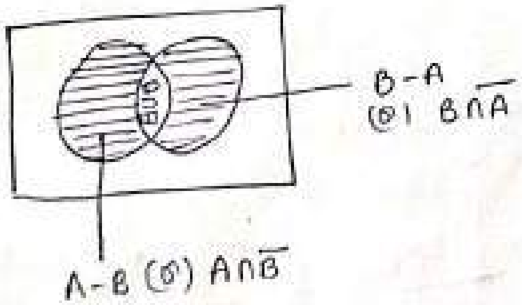




disjoint events





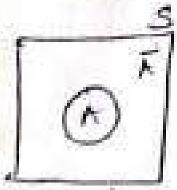


It is A but not B.

Theorem:- If A is an event in the sample space 'S' then

$P(\bar{A}) = 1 - P(A)$

proof:-



Given that 'A' is an event in the sample space 's'.

Then $A \cup \bar{A} = S$ (A & $\bar{A}$ are disjoint events).

$P(A \cup \bar{A}) = P(S)$

$P(A) + P(\bar{A}) = P(S)$ ($\because$ By the axiomatic def)

$P(A) + P(\bar{A}) = 1$ ($\because$ By the a.D)

$\boxed{P(\bar{A}) = 1 - P(A).}$

Theorem:- If ϕ is an impossible event in the sample space 'S' then $P(\phi) = 0$

proof:- since 'ϕ' is an impossible event in the sample space 's'

so that $S \cup \phi = S$

$P(S \cup \phi) = P(S)$

since 's' & ϕ are disjoint events

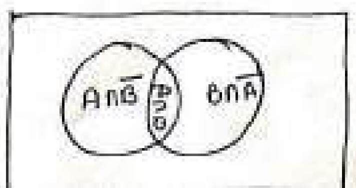
$P(S) + P(\phi) = P(S)$

$P(\phi) = 0$

Theorem:- If A, B are events in the sample space 's' then

1. $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
2. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$.

Sol



proof:- Given that A & B are even
in the sample space 's'.

(i) consider $B = (A \cap B) \cup (\bar{A} \cap B)$

$$P(B) = P\left[(A \cap B) \cup (\bar{A} \cap B)\right]$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B)$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B).$$

(ii) consider $A = (A \cap B) \cup (A \cap \bar{B})$

$$P(A) = P\left[(A \cap B) \cup (A \cap \bar{B})\right]$$

$$= P(A \cap B) + P(A \cap \bar{B})$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

Problems on Addition theorem

1. Two dice are tossed find the probability of getting an even number on the first die (or) a total of 8.

Sol $S = \{(1,1) \; - - - \; (6,6)\}$

$n = 36$.

→ Let A be the event of getting an even number on the first die.

$A = \{(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(4,1) - - - - (4,6)$
$(6,1) - - - - (6,6)\}$

$m = 18$.

$P(A) = \frac{m}{n} = \frac{18}{36} = \frac{1}{2}$

→ Let B be the even of getting total sum on the two dice is 8

$B = \{(2,6)\ (3,5)\ (4,4)\ (5,3)\ (6,2)\}$

$m = 5$

$P(B) = \frac{m}{n} = \frac{5}{36}$.

probability of getting an even number on the first die (or) a total of 8.

$= P(A \cup B)$

By the addition theorem

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$A \cap B = \{(2,6)\ (4,4)\ (6,2)\}$

$P(A \cap B) = \frac{3}{36}$.

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{20}{36}$

$$\boxed{P(A \cup B) = \frac{20}{36}}$$

2. The probability that a student passes a physics test is $\frac{2}{3}$ and the probability that he passes both physics and english test is $\frac{14}{45}$. The probability that he passes at least one test is $\frac{4}{5}$ what is the probability that he passes the English test.

Sol A: be the event that the student passes physics test

B:

$A \cap B$:

G.t $P(A) = 2/3$

$P(A \cap B) = \frac{14}{45}$

$P(A \cup B) = 4/5$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$4/5 = \frac{2}{3} + P(B) - \frac{14}{45}$

$$P(B) = \frac{4}{5} - \frac{2}{3} + \frac{14}{45}$$

$$\boxed{P(B) = 4/9}$$

3 An integer is choosen at random from two hundred numbers what is the probability that the integer is divisible by 6 or 8

Sol $S = \{1, 2, \ldots 200\}$

$n = 200$

A: be the event that the integers are divisible by 6

$m = \frac{200}{6} = 33$

$P(A) = \frac{33}{200}$

$P(B) = \frac{25}{200}$

$A \cap B$: divisible by 6 & 8

$m = \frac{200}{24} = 8$

$P(A \cap B) = \frac{8}{200}$

L.C.M of 6, 8 is 24

$\frac{200}{24} = 8$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{1}{4}$

4. A bag contains 4 green, 6 black & 7 white balls. A ball is drawn at random. what is the probability that it is either a green or a black ball?

Sol:- S = A ball is drawn out of 4 green, 6 black, 7 white

$\boxed{n = 17C_1}$

→ let A be the event of the ball is green

$m = 4C_1 \Rightarrow P(A) = \frac{4C_1}{17C_1}$

→ let B be the event of the ball is black

$m = 6C_1$

$\Rightarrow P(B) = \frac{6C_1}{17C_1}$

→ let $A \cap B$ be the event of the ball is green & black.

$m = 0$

$P(A \cap B) = 0$

∴ The probability of either green or black ball is $= P(A \cup B)$

w.k.t By the addition theorem.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{4C_1}{17C_1} + \frac{6C_1}{17C_1} - 0$

$= \frac{4}{17} + \frac{6}{17} = \frac{10}{17}$

5. 3 students A, B, C are in running race. A & B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins.

Sol

$\boxed{A \cup B \cup C = S = \text{sample space of Race}}$

Gt $P(A) = P(B)$ & $P(A) = 2P(C)$

we have $P(A) + P(B) + P(C) = 1$

$2P(C) + 2P(C) + P(C) = 1$

$5P(C) = 1$

$\boxed{P(C) = 1/5}$

$P(A) = 2 \cdot P(C)$

$= 2 \cdot \frac{1}{5} = \frac{2}{5}$

$\boxed{P(A) = 2/5 = P(B)}$

∴ the probability that B or C wins

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= \frac{2}{5} + 1/5 - 0$

$\boxed{P(B \cup C) = \frac{3}{5}}$

6. From a city 3 news papers A, B, C are being published. A is read by 20%, B is read by 16%, C is read by 14%, both A & B are read by 8%, both A & C are read by 5%, both B & C are read by 4% and all three A, B, C are read by 2%. What is the percentage of the population that read at least one paper.

Sol :- given $P(A) = \frac{20}{100} = 0.2$

$P(B) = \frac{16}{100} = 0.16$

$P(C) = \frac{14}{100} = 0.14$

$P(A \cap B) = \frac{8}{100} = 0.08$

$P(B \cap C) = \frac{4}{100} = 0.04$

$P(A \cap C) = \frac{5}{100} = 0.05$

$P(A \cap B \cap C) = \frac{2}{100} = 0.02$

$P(A \cup B \cup C)$ = probability of atleast one paper.

By the addition theorem

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= 0.2 + 0.16 + 0.14 - 0.08 - 0.04 - 0.05 + 0.02$

$= \underline{\underline{0.35}}$.

7. An MBA applies for a job in two companies X & Y. The probability of he is being selected in company in X is 0.7, and being ~~selected~~ rejected in company in Y is 0.5. The probability of at least one company ~~being~~ rejected is 0.6. What is the probability that he will be selected in one of the company?

Sol → The probability that MBA is selected in company X is

$P(X) = 0.7$

~~The " MBA " company Y is~~

$\cancel{P(Y) = 0.}$

→ The probability that MBA is rejected in company Y is

$P(\overline{Y}) = 0.5$

$P(Y) = 1 - P(\overline{Y}) = 1 - 0.5 = 0.5$

→ The probability of at least one company rejected is

$P(\overline{X} \cup \overline{Y}) = 0.6$.

∴ The probability that he will be selected in one of the company X, Y is

$P(X \cup Y) = ?$

W.K.T by the addition theorem

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$ → ①

~~since $P(\overline{X} \cup \overline{Y}) = 0.6$~~

~~$P(\overline{X} \cup \overline{Y}) = P(\overline{X}) + P(\overline{Y}) - P(\overline{X} \cap \overline{Y})$~~

~~$0.6 = 0.3 + 0.5 - P(\overline{X} \cap \overline{Y})$~~

~~$P(\overline{X} \cap \overline{Y}) = 0.2$~~

~~$P(\overline{X \cup Y}) = P(\overline{X} \cap \overline{Y}) = 0.2$~~

~~$P(\overline{X \cup Y}) = 0.2$~~

~~$P(X \cup Y) = 0.7 + 0.5 - P(\overline{X} \cap \overline{Y})$~~

∴ $P(X \cap Y) = 1 - P(\overline{X \cap Y})$

$= 1 - P(\overline{X} \cup \overline{Y})$

$= 1 - 0.6$

$= 0.4$

$P(X \cap Y) = 0.4$

∴ from ①

$P(X \cup Y) = 0.7 + 0.5 - 0.4$

$= 0.8$

$\boxed{P(X \cup Y) = 0.8}$

8. A card is drawn from pack of 52 cards find the probability of getting a King (or) a heart (or) a red card.

Sol S = 1 card drawn from pack of 52 cards

$n = 52C_1$.

A : be the event of a King.

$P(A) = \frac{4C_1}{52C_1}$

B : be the event of a heart

$P(B) = \frac{13C_1}{52C_1}$

C : be the event of red card.

$P(C) = \frac{26C_1}{52C_1}$

∴ the probability of getting a king (or) a heart (or) a red card.

$= P(A \cup B \cup C)$.

w.k.t by the addition theorem

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \longrightarrow (1).$$

$A \cap B$: be the event of a king and a heart.

$$P(A \cap B) = \frac{1C_1}{52C_1}$$

$B \cap C$: be the event of a heart in Red.

$$P(B \cap C) = \frac{13C_1}{52C_1}$$

$A \cap C$: be the event of a king in Red.

$$P(A \cap C) = \frac{2C_1}{52C_1}.$$

$A \cap B \cap C$: be the event of a king in a heart in a red.

$$P(A \cap B \cap C) = \frac{1C_1}{52C_1}.$$

from (1),

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{\cancel{1}}{52} - \frac{13}{52} - \frac{2}{52} + \frac{\cancel{1}}{52}$$

$$= \frac{28}{46} = \frac{14}{23}$$

1.15 ADDITION THEOREM ON PROBABILITY

Theorem : If S is a sample space, and E_1, E_2 are any events in S then

$$P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

[JNTU 2007 (Set No. 1), 2009 S (Set No. 2); (A) 2009, June 2012, (H) Dec

Proof : Case (*i*) : $E_1 \cap E_2 \neq \phi$. Let E_1, E_2 contain the sample points

$a_1, a_2, ..., a_k, a_{k+1}, ..., a_{k+l}$ and $a_{k+1}, ..., a_{k+l}, a_{k+l+1}, ..., a_{k+l+m}$ respectively

$\therefore \quad E_1 = \{a_1, a_2, ..., a_k, a_{k+1}, ..., a_{k+l}\}$

and $\quad E_2 = \{a_{k+1}, a_{k+2}, ... a_{k+l}, a_{k+l+1}, ..., a_{k+l+m}\}$

$\therefore \quad E_1 \cup E_2 = \{a_1, a_2, ..., a_k, a_{k+1}, ..., a_{k+l}, a_{k+l+1}, ..., a_{k+l+m}\}$

and $E_1 \cap E_2 = \{a_{k+1}, a_{k+2}, ..., a_{k+l}\}$

Hence $P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= P(a_1) + P(a_2) + ... + P(a_k) + P(a_{k+1}) + ... + P(a_{k+l})$$
$$+ P(a_{k+1}) + ...+ P(a_{k+l}) + P(a_{k+l+1}) + ... + P(a_{k+l+m})$$
$$- [P(a_{k+1}) + P(a_{k+2}) + ... + P(a_{k+l})]$$
$$= P(a_1) + ...+ P(a_k) + P(a_{k+1}) + ... + P(a_{k+l}) + P(a_{k+l+1}) + ...+ P(a_{k+l+m})$$
$$= P(E_1 \cup E_2)$$

Case (*ii*) : $E_1 \cap E_2 = \phi$

$$\therefore \; P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= P(E_1) + P(E_2) - 0$$
$$= P(E_1) + P(E_2) - P(\phi)$$
$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

for 3 varn: $P(E_1 \cup E_2 \cup E_3) = P(E_1 \cup$
$= P(E_1) + P(E_2 \cup$
$= P(E_1) + P(E_2) + P($
$P(E_1) + P(E_2) +$

Cor 1. If E_1, E_2 are two mutually exclusive events, then

1.19 MULTIPLICATION THEOREM OF PROBABILITY

Statement : In a random experiment if E_1, E_2 are two events such that $P(E_1) \neq 0$ and $P(E_2) \neq 0$, then

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)\ ,\ P(E_2 \cap E_1) = P(E_2) \cdot P(E_1/E_2)$$

Proof : Let S be the sample space associated with the random experiment. Let E_1, E_2 be two events of S such that $P(E_1) \neq 0$, $P(E_2) \neq 0$. Since $P(E_1) \neq 0$, by the definition of conditional probability of E_2 given E_1,

$$P(E_2 \mid E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$\therefore\ P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$$

Since $P(E_2) \neq 0$, we have $P(E_1 \mid E_2) = \dfrac{P(E_1 \cap E_2)}{P(E_2)}$.

Hence $P(E_2 \cap E_1) = P(E_2) . P(E_1 \mid E_2)$

Random variable:-

A random variable X whose value is determined by the outcome of a random experiment is called random variable.

→ always random variables are denoted by capital letters.

Eg:- If a dice is rolled and If 'x' denotes the number obtained then x is called random variable.

∴ x takes any one of the particular values as 1, 2, 3, 4, 5, 6 each with probability 1/6.

These values can be tabulated as

X	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Eg:- The sample space corresponding to tossing of two coins.

→ when two coins are tossed.

S = {HH, HT, TH, TT}.

after the performance of an experiment we count the number of ~~heads~~ tails and denoted by X.

The first out come is HH. that ~~has~~ 0 tail ie $X=0$

Similarly $X=1$

& $X=2$

∴ Thus X takes the values

$X = 0, 1, 2$

X = 0	1	2
P(x) = 1/4	2/4	1/4

Types of Random variables

Random variables are 2 types.

1. Discrete Random variable
2. continuous Random variable.

1. Discrete Random variable

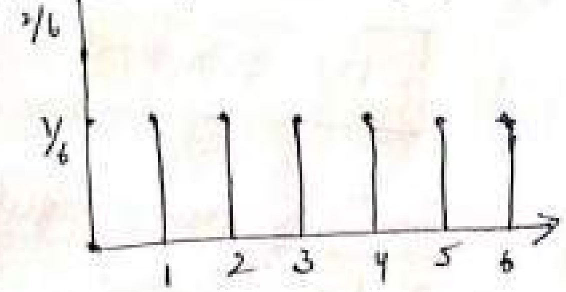
A random variable X which can take only a finite number of discrete values in an interval of a domain is called discrete random variable.

In other words, If the random variable takes the values only on the set {0, 1, 2, --- n} is called a Discrete R.V.

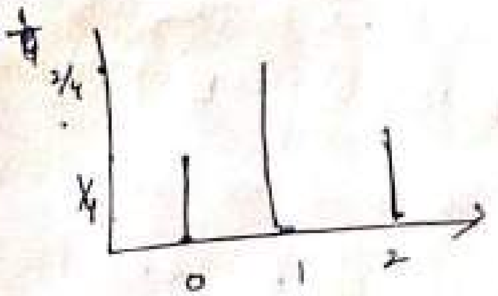
Eg:- Tossing a coin, throwing a die, the number of defective items in a sample of electric bulbs, The number of Telephonic calls received by the operator

(probab)ility function

When a single die is thrown, the probability associated with each number on the on the face of the dice can take the values with probability being $1/6$ for all the numbers 1 to 6. This can be shown with the help of the graph



→ when two coins are thrown. the probability over number of tails $\frac{1}{4}, \frac{1}{2}$ & $\frac{1}{4}$.



These are referred to graphs of probability distributions.

→ In many cases, these graphs are described by a mathematical function which is referred to as a probability functions (or) statistical function.

probability function of a discrete Random variable

Let x be the discrete random variable. The discrete probability function $f(x)$ or $P(x)$ for x is given by $f(x) = P(X=x)$ for all real x.

We can state that discrete probability distribution (or) probability mass function of a discrete random variable X as the function $P(x)$ satisfying conditions.

(i) $P(x) \geq 0$ for all x.

(ii) $\Sigma P(X=x) = 1$

(iii) $P(x)$ can not be negative for all value of x.

Then the ~~function~~ $P(x)$ is called the probability mass function of the random variable X.

Eg:- Tossing a coin 2 times.

$S = \{HH, HT, TH, TT\}$

i.e $n=4$.

X is a random variable that occurance of Heads.

0 – heads – $\{TT\} = P(X=0) = 1/4$

1 – head – $\{HT, TH\} = P(X=1) = \frac{2}{4}$

2 heads – $\{HH\} = P(X=2) = 1/4$

$X=x_i$	0	1	2
$P(X=x_i)$	$1/4$	$2/4$	$1/4$

checking that $P(x)$ is probability mass function (or) not

1. $P(X=x_i) \geq 0 \ \forall i$ clearly
2. $\Sigma P(X=x_i) = \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$
3. $P(X=x_i)$ is not in negative.

$\therefore$ It is a 'probability mass function.

Cumulative distribution function of a Discrete Random variable

Suppose X is a discrete random variable. Then the discrete distribution function (or) cumulative distribution function $F(x)$ is defined by

$$F(x) = P(X \leq x_i) = \sum_{i=1}^{x} P(x_i)$$

Expectation of a Discrete variable

X is a r.v with the values $x_1, x_2, \ldots x_n$ with respective probabilities $P_1, P_2, \ldots P_n$ is

Discrete random variable problems:-

1.

defined as the sum of products of different values of x and the corresponding probabilities.

$$E(X) = x_1 P_1 + x_2 P_2 + \ldots + x_n P_n$$

i.e $\boxed{E(X) = \Sigma x_i P_i}$

Note:- some important results on Expectations.

1. $E(X+K) = E(X) + K$ where K is a constant
2. If X is a random variable and a, b are constants then $E(aX \pm b) = aE(X) \pm b$.
3. If X, Y are two r.v $E(X+Y) = E(X) + E(Y)$
4. If X, Y are two independent r.v then $E(XY) = E(X)E(Y)$
5. $E(aX+bY) = aE(X) + bE(Y)$
6. $E(X - \bar{X}) = 0$ . .
7. $E(X_1 + X_2 + \ldots + X_n) = E(X_1) + E(X_2) + \ldots + E(X_n)$

then

from the definition

If $E(X) = \Sigma x_i p_i$ & $E(Y) = \Sigma y_j p_j$

then $E(XY) = \Sigma \Sigma x_i y_j p_{ij}$

1. $E(XYZ) = E(X \cdot YZ) = E(X) \cdot E(YZ)$
$= E(X) E(Y) E(Z)$.

Mean

The mean value μ of the discrete distribution function is given by

$$\mu = \frac{\Sigma p_i x_i}{\Sigma p_i} = \Sigma p_i x_i = E(X).$$

i.e $\boxed{\mu = E(X)} = \Sigma x_i P(X=x_i)$

Variance :-

variance of the probability distribution of a random variable X is denoted by σ^2 (or) $V(X)$

$$Var(X) = \sum_{i=1}^{n} x_i^2 P(X=x_i) - (\text{mean})^2$$

$$\boxed{V(X) = \sum_{i=1}^{n} x^2 P(X=x_i) - (\mu)^2}$$

(or)

$$\boxed{V(X) = E[X^2] - [E(X)]^2}$$

Standard deviation :-

It is the positive square root of the variance

$$\therefore S.D = \sigma = \sqrt{\Sigma x^2 p_i - \mu^2}$$
$$= \sqrt{E(x^2) - \mu^2}$$
$$= \sqrt{E(x^2) - [E(x)]^2}$$

Some important results on Variance

(3)

1. variance of constant is zero i.e $V(K) = 0$
2. If K is constant then $V(KX) = K^2 V(X)$
3. If X is a r.v and K is a constant then $V(X+K) = V(X)$ $(\because V(K)=0)$
4. $V(aX+b) = a^2 V(X)$
5. $V(aX) = a^2 V(X)$.
6. $V(X+b) = V(X)$.
7. $V(X \pm Y) = V(X) \pm V(Y)$.

Discrete random variable problems

1. A random variable X has the following probability function

$X=x$	0	1	2	3	4	5	6
$P(X=x)$	K	3K	5K	7K	9K	11K	13K

find (i) K
(ii) $P(X<4)$
(iii) $P(X \geq 5)$
(iv) $P(3 < X \leq 7)$

(i) To find K.

We know that $\Sigma P(X=x_i)=1$

i.e $P(X=0)+P(X=1)+P(X=2)+P(X=3)$
$+P(X=4)+P(X=5)+P(X=6)=1$

$\Rightarrow K+3K+5K+7K+9K+11K+13K=1$

$\Rightarrow 49K=1$

$\Rightarrow \boxed{K=1/49}$

(ii) $P(X<4)$

$P(X<4)=P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$=K+3K+5K+7K$

$P(X<4)=16K=16\left(1/49\right)=\frac{16}{49}$

$\boxed{P(X<4)=0.32}$

(iii) $P(X\geq 5)$

$P(X\geq 5)=P(X=5)+P(X=6)$

$=11K+13K=24K$

$=\frac{24}{49}=0.82$

$\therefore \boxed{P(X\geq 5)=0.82}$

(iv) $P(3<X\leq 7)$

$P(3<X\leq 7)=P(X=4)+P(X=5)+P(X=6)+P(X=7)$

$=9K+11K+13K$

$=33K=\frac{33}{49}=0.67$

$\boxed{P(3<X\leq 7)=0.67}$

2. A random variable (iii) the following probability &

x	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

(i) find K

(ii) Evaluate $P(X<6)$, $P(X\geq 6)$, $P(0<X<5)$ and $P(0\leq X\leq 4)$

(iii) find the minimum value of K such that $P(X\leq K)>1/2$

(iv) Determine the distribution function of X.

(v) Mean

(vi) variance.

Sol

(i) To find K.

w.k.t $\Sigma P(X=x_i)=1$

i.e $P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)+P(X=6)+P(X=7)=1$

$\Rightarrow 0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$

$10K^2+9K-1=0$

$(10K-1)(K+1)=0$

$K=-1,\ K=1/10$

$\therefore \boxed{K=1/10}$ $[\because P(X)\geq 0)$ so $K\neq -1)$

(ii) $P(X<6)$

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0+K+2K+2K+3K+K^2$

$= 8K+K^2$

$= 8\left(\frac{1}{10}\right)+\frac{1}{100} = 0.8+0.01$

$= 0.81$

$\boxed{P(X<6)=0.81}$

$P(X\geq 6) = P(X=6)+P(X=7)$

$= 2K^2+7K^2+K$

$= 9K^2+K$

$= 9\left(\frac{1}{100}\right)+\frac{1}{10}$

$= 0.09+0.1 = 0.19$

$\therefore \boxed{P(X\geq 6)=0.19}$

$P(0\leq X\leq 4) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 0+K+2K+2K+3K$

$= 8K = 8\left(\frac{1}{10}\right) = 0.8$

(iii) The minimum value of K

~~at K=0 ⇒ P(X ≤ K) = P(X ≤ 0) = P(X=0) = 0~~

at $K=1 \Rightarrow P(X\leq K)$

$= P(X\leq 1)$

$= P(X=0)+P(X=1)$

$= 0+K = 1/10 = 0.1 \not> \frac{1}{2}$

at $K=2 \Rightarrow P(X\leq 2)$

$= P(X=0)+P(X=1)+P(X=2)$

$= 0+K+2K = 3K = 0.3 \not> 1/2$

at $K=3$, $P(X\leq 3)$

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$= 5K = \frac{5}{10} = 0.5 \not> 1/2$

at $K=4$, $P(X\leq 4)$

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 8K = \frac{8}{10} = 0.8 > 1/2$

$\therefore$ The minimum value of K for which $P(X\leq K) > 1/2$ is $\boxed{K=4}$.

(v) The distribution function of X is given by

X	$P(X=x)$	$F(x)=P(X\leq x)$
0	0	0
1	$K=\frac{1}{10}$	$0+\frac{1}{10}=1/10$
2	$2K=\frac{2}{10}$	$\frac{1}{10}+\frac{2}{10}=\frac{3}{10}$
3	$2K=\frac{2}{10}$	$\frac{3}{10}+\frac{2}{10}=\frac{5}{10}$
4	$3K=\frac{3}{10}$	$\frac{5}{10}+\frac{3}{10}=\frac{8}{10}$
5	$K^2=\frac{1}{100}$	$\frac{8}{10}+\frac{1}{100}=\frac{81}{100}$
6	$2K^2=\frac{2}{100}$	$\frac{81}{100}+\frac{2}{100}=\frac{83}{100}$
7	$7K^2+K=\frac{7}{100}+\frac{1}{10}=\frac{17}{100}$	$\frac{83}{100}+\frac{17}{100}=1$

(v) mean $= \mu = \Sigma x_i p_i$

$= \Sigma x_i p_i$

$= 0\cdot(0) + 1(K) + 2(2K) + 3(2K)$
$+ 4(3K) + 5(K^2) + 6(2K^2)$
$+ 7(7K^2 + K)$

$= 66K^2 + 30K = \frac{66}{100} + \frac{30}{10} = 3.66$

(vi) variance $= \sigma^2 = \Sigma x^2 p_i - (\mu)^2$

$= 0(0) + 1(K) + 4(2K) + 9(2K)$
$+ 16(3K) + 25K^2 + 36(2K^2)$
$+ 49(7K^2 + K) - (3.66)^2$

$= K + 8K + 18K + 48K + 25K^2$
$+ 72K^2 + 343K^2 + 49K - (3.66)^2$

$= 440K^2 + 124K - (3.66)^2$

$= \frac{440}{100} + \frac{124}{10} - (3.66)^2$

$= 3.4044$

3. A random variable x has the following probability distribution

x	1	2	3	4	5	6	7	8
P(x)	K	2K	3K	4K	5K	6K	7K	8K

find the value of

(i) K (ii) $P(X \le 2)$ (iii) $P(2 \le X \le 5)$

Sol

Sol (i) To find K

w.k.t $\Sigma P(X = x_i) = 1$

$K + 2K + 3K + 4K + 5K + 6K$
$+ 7K + 8K = 1$

$\Rightarrow 36K = 1$

$\Rightarrow \boxed{K = 1/36}$

(ii) $P(X \le 2)$

$P(X \le 2) = P(X=0) + P(X=1)$
$+ P(X=2)$

$= \cancel{K} 2K + 3K = 6K$

$= \frac{3}{36} = \frac{1}{12}$

(iii) $P(2 \le X \le 5)$

$P(2 \le X \le 5) = P(X=2) + P(X=3)$
$+ P(X=4) + P(X=5)$

$= 2K + 3K + 4K + 5K$

$= 14K = 14 \cdot \frac{1}{36} = \frac{7}{18}$.

4. A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) find a

(ii) $P(X<3)$, $P(X \ge 3)$ & $P(0 < X < 5)$

(iii) Find the distribution function $F(x)$.

Sol:- (i) To find a

w.k.t $\Sigma P(X = x_i) = 1$

$\therefore a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$\Rightarrow 81a = 1$

$\Rightarrow a = \frac{1}{81}$

(ii) $P(X < 3)$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= a + 3a + 5a = 9a = 9 \cdot \frac{1}{81} = \frac{1}{9}$

$P(X \geq 3)$

$P(X \geq 3) = P(X=3) + P(X=4) + \cdots + P(X=8)$

(or)

$= 1 - P(X<3)$

$= 1 - \frac{1}{9} = \frac{8}{9}$

$P(0 < X < 5)$

$P(0<X<5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 3a + 5a + 7a + 9a$

$= 24a = \frac{24}{81}$

(iii) The distribution function $F(x)$ of the discrete variate X is defined by

$F(x) = P(X \leq x) = \Sigma P(x_i)$

4.25 ...... are 10/- when he gets

X	$P(X=x)$	$F(x) = P(X \leq x)$
0	$a = \frac{1}{81}$	$\frac{1}{18}$
1	$3a = \frac{3}{81}$	$\frac{4}{81}$
2	$5a = \frac{5}{81}$	$\frac{9}{81}$
3	$7a = \frac{7}{81}$	$\frac{16}{81}$
4	$9a = \frac{9}{81}$	$\frac{25}{81}$
5	$11a = \frac{11}{81}$	$\frac{36}{81}$
6	$13a = \frac{13}{81}$	$\frac{49}{81}$
7	$15a = \frac{15}{81}$	$\frac{64}{81}$
8	$17a = \frac{17}{81}$	$\frac{81}{81} = 1$

5. The probability density function of a variate X is

X	0	1	2	3	4	5	6
$P(X)$	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$

(i) find K

(ii) find $(X<4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$

(iii) what will be the minimum value of K so that $P(X \leq 2) > 0.3$

Sol (i) find K

w.k.t $\Sigma P(X=x) = 1$

i.e $K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$\Rightarrow 49K = 1$

$\boxed{K = 1/49}$

(ii) $P(X<4) = P(X=0)+P(X=1)+P(X=2)$
$+P(X=3)$

$= K+3K+5K+7K = 16K = \frac{16}{49}$

$P(X \geq 5) = P(X=5)+P(X=6)$

$= 11K+13K = 24K = \frac{24}{49}$

$P(3<X\leq 6) = P(X=4)+P(X=5)+P(X=6)$

$= 9K+11K+13K = 33K = \frac{33}{49}$

(iii) $P(X\leq 2) = P(X=0)+P(X=1)+P(X=2)$

$= K+3K+5K = 9K$

$P(X\leq 2) > 0.3$

$\Rightarrow 9K > 0.3$

$K > \frac{0.3}{9}$

$K > \frac{1}{30}$

$\therefore$ The minimum value of K is $\frac{1}{30}$

6. A random variable X has the following probability function

X	-3	-2	-1	0	1	2	3
$P(x_i)$	K	0.1	K	0.2	$2K$	0.4	$2K$

find (i) K (ii) Mean (iii) variance.

Sol (i) To find K

W.K.T $\Sigma P_i = 1$

$\Rightarrow K+0.1+K+0.2+2K+0.4+2K=1$

$6K+0.7=1$

$K = \frac{0.3}{6} = 0.05$

$\boxed{K=0.05}$

7.

$X=x$	0	1	2	3	4	5	6	7	[illegible]
$P(X=x)$	$\frac{K}{45}$	$\frac{K}{15}$	$\frac{K}{9}$	$\frac{K}{5}$	$\frac{2K}{45}$	$\frac{6K}{45}$	$\frac{7K}{45}$	$\frac{6K}{45}$	[illegible]

find (i) K (ii) mean
(iii) variance.

Theoritical problems on D R V

1. Two dice are thrown.

Let X assign to each point in S that the maximum of its numbers

i.e $X(a,b) = \max\{a,b\}$ then find

(i) probability distribution

(ii) mean

(iii) variance.

Sol) Two dice are thrown

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

$n = 36$.

$S = \{ 1, 2, 3, 4, 5, 6$
$2, 2, 3, 4, 5, 6$
$3, 3, 3, 4, 5, 6$
$4, 4, 4, 4, 5, 6 \}$

X assign to each point in S that the maximum of its numbers

i.e $X = 1, 2, 3, 4, 5, 6$.

when $X = 1$

i.e 1 is max $= \{(1,1)\} = 1 = m$

$P(1) = \frac{1}{36}$.

when $X = 2$

i.e 2 is max $= \{\max (2,1)\ (2,2)\ (1,2)\} = 3$

$P(2) = \frac{3}{36}$.

when $X = 3$

i.e 3 is max $= \{\max (1,3)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\} = 5$

$P(3) = 5/36$

$X = 4$

i.e $\max(a,b) = 4$

when $\max\{ (1,4)\ (2,4)\ (3,4)\ (4,4)\ (4,1)\ (4,2)\ (4,3)\}$

$P(4) = 7/36$

$X = 5$ i.e $\max\{ (1,5)\ (2,5)\ (3,5)\ (4,5)\ (5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\} = 9$

$P(5) = 9/36$

$X = 6$ i.e $\max\{ (1,6)\ (2,6)\ (3,6)\ (4,6)\ (5,6)\ (6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\} = 11$

$P(6) = \frac{11}{36}$

(i) probability distribution.

$X=x$	1	2	3	4	5	6
$P(X=x)$	$1/36$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$
$F(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{16}{36}$	$\frac{25}{36}$	$\frac{36}{36} = 1$

(ii) mean :-

$\mu = \sum_x x . P(X=x)$

$= 1\cdot\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right).$

$= \frac{1}{36}\left[1 + 6 + 15 + 28 + 45 + 66\right]$

$= 4.47$

$\boxed{\text{Mean} = \mu = 4.47}$

(iii) Variance =

$$\text{variance} = \sigma^2 = \Sigma x^2 p(X=x) - (\text{mean})^2$$

$$= 1^2\left(\frac{1}{36}\right) + 2^2\left(\frac{3}{36}\right) + 3^2\left(\frac{5}{36}\right) + 4^2\left(\frac{7}{36}\right) + 5^2\left(\frac{9}{36}\right) + 6^2\left(\frac{11}{36}\right) - (4.47)^2$$

$$= \frac{1}{36}\left[1 + 12 + 45 + 112 + 225 + 396\right] - (4.47)^2$$

$$= 21.97 - 19.9 = \underline{1.99}.$$

2. A random variable X is defined as sum of the numbers on the faces when two dices are thrown then find

(i) probability distribution

(ii) mean

(iii) variance.

Sol) Two dice are thrown.

$S = \{(1,1)\ (1,2)\ \dots\ \dots\ (6,6)\}$

sum of the numbers on the faces of two dice

$$= \left\{\begin{matrix} 2, 3, 4, 5, 6, 7 \\ 3, 4, 5, 6, 7, 8 \\ 4, 5, 6, 7, 8, 9 \\ 5, 6, 7, 8, 9, 10 \\ 6, 7, 8, 9, 10, 11 \\ 7, 8, 9, 10, 11, 12 \end{matrix}\right\}$$

X is defined that the sum of the numbers on the two dice

$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$X=2 \Rightarrow P(2) = \frac{1}{36}$

$X=3 \Rightarrow P(3) = \frac{2}{36}$

$X=4 \Rightarrow P(4) = \frac{3}{36}$

$X=5 \Rightarrow P(5) = \frac{4}{36}$

$X=6 \Rightarrow P(6) = \frac{5}{36}$

$X=7 \Rightarrow P(7) = \frac{6}{36}$

$X=8 \Rightarrow P(8) = \frac{5}{36}$

$X=9 \Rightarrow P(9) = \frac{4}{36}$

$X=10 \Rightarrow P(10) = \frac{3}{36}$

$X=11 \Rightarrow P(11) = \frac{2}{36}$

$X=12 \Rightarrow P(12) = \frac{1}{36}$

$X=x$	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Mean :- $\mu = \Sigma x\, P(X=x_i)$

$$= 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right) = \frac{252}{36} = 7$$

$\mu = 7.$

variance

$\sigma^2 = \Sigma x^2 p(x \dots$

$= [4(\frac{1}{\dots}$

variance

$$\sigma^2 = \Sigma x^2 p(x=x_i) - (\mu)^2$$

$$= \left[4\left(\frac{1}{36}\right) + 9\left(\frac{2}{36}\right) + 16\left(\frac{3}{36}\right) + 25\left(\frac{4}{36}\right) + 36\left(\frac{5}{36}\right) + 49\left(\frac{6}{36}\right) + 64\left(\frac{5}{36}\right) + 81\left(\frac{4}{36}\right) + 100\left(\frac{3}{36}\right) + 121\left(\frac{2}{36}\right) + 144\left(\frac{1}{36}\right)\right] - 49.$$

$$= \frac{1}{36}\left[4 + 18 + 48 + 100 + 180 + 294 + 120 + 324 + 300 + 242 + 144\right] - 49$$

$$= 0.277 = 5.834$$

3. A player wins if he gets five on a single throw of a die. He loses if he gets two or four. If he wins he gets 50/- if he looses he gets 10/- otherwise he has to pay 15/- Is the game favourable to the player?

Sol) when a single die throw

$S = \{1, 2, 3, 4, 5, 6\}$.

X is assined that 50/-, 10/-, -15/-

Game Roles

player get 50/- when he gets 5 on the die.

the probability of player getting 50/- is

$$P(X=50) = \frac{1}{6}$$

player gets 10/- when he gets 2 (or) 4 on the die

$$P(X=10) = \frac{2}{6}. \quad (\because \text{single die throw})$$

player gets -15/- (or) player has to pay 15/- when he gets 1, 3, 6 on the die

$$P(X=-15) = \frac{3}{6}.$$

$X =$	50	10	-15
$P(X=x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

$$\text{Mean} = \mu = \Sigma x\, P(X=x_i)$$

$$= 50\left(\frac{1}{6}\right) + 10\left(\frac{2}{6}\right) - 15\left(\frac{3}{6}\right)$$

$$= \frac{25}{6} = 4.16.$$

$\therefore$ The game is favorable to the player.

4. A player tosses two fair coins he wins 100/- if a head appears, he wins 200/- if two heads appears on the other hand he looses 500/- i.e if no head appears. Is the game favorable to the player?

Sol:- when two coins are tossed

$S = \{HH, HT, TH, TT\}$.

$n = 4$.

X is a R.V that assigns 100/-, 200/-, -500/-

Game rules

wins 100/-, when a single head appears.

i.e $\{HT, TH\}$

probability to wins 100/- is

$P(X=100) = \frac{2}{4}$.

wins 200/- when two heads appear

i.e $\{HH\}$

$P(X=200) = \frac{1}{4}$.

looses 500/- when no head appears.

i.e $\{TT\}$

$P(X=-500) = \frac{1}{4}$.

$\therefore$

X	100	200	-500
P(X=x)	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$$\text{mean} = \mu = \sum_x x\,P(X=x)$$

$$= 100\left(\frac{2}{4}\right) + 200\left(\frac{1}{4}\right) - 500\left(\frac{1}{4}\right)$$

$$= 50 + 50 - 125 = -25$$

The game is not favorable to the player.

~~5. A player toses three co~~

~~he wins 500/- if~~

5. A player toses three coins, he wins 500/- if three heads appeared. and wins 300/- if two heads appeared, and 100/- if one head appeared. on the other hand he looses 1500/-, if tails appeared

Is the game favourable to the player?

Sol $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n = 8$.

$P(X=500) = \frac{1}{8}$ ($\because$ 3 heads occurs one and cone) i.e HHH)

$P(X=300) = \frac{3}{8}$ ($\because$ 2 heads $\{HHT, HTH, THH\}$

$P(X=100) = \frac{3}{8}$. ($\because$ 1 heads $\{HTT, THT, TTH\}$

$P(X=-1500) = \frac{1}{8}$. ($\because$ No heads $\{TTT\}$

X	500	300	100	-1500
P(X=x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
F(x)	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	$\frac{8}{8}=1$

$\text{mean} = \sum x\,P(X=x) = 25$.

$\therefore$ Game is favorable to the player

6. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denotes the number of defectives in the sample. Answer the following when the sample is drawn with replacement

(i) find the probability distribution of X.

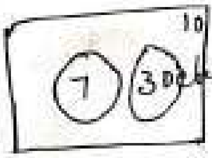
(ii) find $P(X \le 1)$ & $P(0 < X < 2)$

Sol From a lot of 10 items, 4 items are selected randomly then

$n = 10C_4$.

X denotes the number of defective items

i.e $X = 0, 1, 2, 3$



The probability of '0' defective items in a sample of 4

$$P(X=0) = \frac{7C_4 \times 3C_0}{10C_4} = \frac{1}{6}$$

The probability of '1' defective item in a sample of 4

$$P(X=1) = \frac{7C_3 \times 3C_1}{10C_4} = \frac{1}{2}$$

The probability of '2' defective items in a sample of 4

$$P(X=2) = \frac{7C_2 \times 3C_2}{10C_4} = \frac{3}{10}$$

The probability of 3 defective items in a sample of 4

$$P(X=3) = \frac{7C_1 \times 3C_3}{10C_4} = \frac{1}{30}.$$

(i) The probability distribution is

X	0	1	2	3
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$
$F(x)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{29}{30}$	1

(ii) $P(X \le 1)$ & $P(0 < X < 2)$

$$P(X \le 1) = P(X=0) + P(X=1)$$
$$= \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$$

$$P(0 < X < 2) = P(X=1) = \frac{1}{2}.$$

7. Four coins are tossed. Let X be the number of heads and Y be the number of heads minus the no. of tails. Find the probability ~~function~~ X probability function Y and $P(-2 \le Y \le 4)$.

Sol

Sol:- when 4 coins are tossed

S =		x no. of heads	y Heads − Tails
	H H H H	4	$4-0=4$
	H H H T	3	$3-1=2$
	H H T H	3	$3-1=2$
	H H T T	2	$2-2=0$
	H T H H	3	$3-1=2$
	H T H T	2	$2-2=0$
	H T T H	2	$2-2=0$
	H T T T	1	$1-3=-2$
	T H H H	3	$3-1=2$
	T H H T	2	$2-2=0$
	T H T H	2	$2-2=0$
	T H T T	1	$1-3=-2$
	T T H H	2	$2-2=0$
	T T H T	1	$1-3=-2$
	T T T H	1	$1-3=-2$
	T T T T	0	$0-4=-4$

i) X : the no. of heads .

$X = 0, 1, 2, 3, 4$

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$
$F(x)$	$\frac{1}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{15}{16}$	$\frac{16}{16}=1$

ii) Y : the no. of heads − no. of tails .

Y	−4	−2	0	2	4
$P(Y=y)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$
$F(y)$	$\frac{1}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{15}{16}$	$\frac{16}{16}=1$

(iii) $P(-2 \le Y < 4)$

$= P(Y=-2) + P(Y=0) + P(Y=2)$

$= \frac{4}{16} + \frac{6}{16} + \frac{4}{16} = \frac{14}{16} = \frac{7}{8}$

8. Let X be the R.V ∋

$P(X=-2) = P(X=-1)$

$P(X=2) = P(X=1)$

and $P(X>0) = P(X<0) = P(X=0)$

find probability distribution function. and check the function is probability mass function.

Sol we know that

$\sum_x P(X=x) = 1$

from given that

$P(X=-2) = P(X=-1)$

$P(X=2) = P(X=1)$

& $P(X>0) = P(X<0) = P(X=0)$

so

$P(X=-2) + P(X=-1) + P(X=0)$

$+ P(X=1) + P(X=2) = 1$

$P(X<0) + P(X=0) + P(X>0) = 1$

$P(X=0) + P(X=0) + P(X=0) = 1$

$\boxed{P(X=0) = 1/3}$

∴ $P(X<0) = P(X=0)$

so $P(X=-2) + P(X=-1) = 1/3$

$P(X=-2) + P(X=-2) = 1/3$

$2P(X=-2) = 1/3$

$$\boxed{P(X=-2) = 1/6}$$

so $\boxed{P(X=-1) = 1/6}$

$P(X>0) = P(X=0)$

i.e $P(X=1) + P(X=2) = 1/3$

$P(X=1) + P(X=1) = 1/3$

$\boxed{P(X=1) = 1/6} \Rightarrow \boxed{P(X=2) = 1/6}$

X	-2	-1	0	1	2
$P(X=x)$	$1/6$	$1/6$	$\frac{1}{3}$	$1/6$	$1/6$
$F(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{6}{6}=1$

(i) $\Sigma P(X=x_i) = \cancel{\frac{1}{6}+\frac{1}{6}+\frac{2}{3}+\frac{5}{6}+\frac{6}{6}}$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{3} + 1/6 + 1/6 = 1$

(ii) $\Sigma P(X=x_i) \geq 0 \quad \forall i$

$\therefore$ given function is probability mass function.

9. A random variable X assumes the values $-3, -2, -1, 0, 1, 2, 3$ such that $P(X=-3) = P(X=-2) = P(X=-1)$ and $P(X=1) = P(X=2) = P(X=3)$ & $P(X=0) = P(X>0) = P(X<0)$ obtain the probability mass function of X and distribution function. and find further the probability mass function of $Y = 2X^2 + 3X + 4$

Sol: Since w.k.t

$\Sigma P(X=x_i) = 1$

$P(X=-3) + P(X=-2) + P(X=-1) + P(X=0) + P(X=1) + P(X=2) + P(X=3) = 1$

$P(X<0) + P(X=0) + P(X>0) = 1$

$P(X=0) + P(X=0) + P(X=0) = 1$

$3P(X=0) = 1$

$$\boxed{P(X=0) = 1/3}$$

$\therefore P(X>0) = P(X=0)$

$P(X=1) + P(X=2) + P(X=3) = 1/3$

$P(X=1) + P(X=1) + P(X=1) = 1/3$

$3P(X=1) = 1/3$

$$\boxed{P(X=1) = 1/9}$$

so $\boxed{P(X=2) = 1/9}$, $\boxed{P(X=3) = 1/9}$

$\therefore P(X<0) = P(X=0)$

$P(X=-1) + P(X=-2) + P(X=-3) = P(X=0)$

$P(X=-1) + P(X=-1) + P(X=-1) = \frac{1}{3}$

$3P(X=-1) = 1/3$

$$\boxed{P(X=-1) = 1/9}$$

so, $\boxed{P(X=-2) = 1/9}$ & $\boxed{P(X=-3) = 1/9}$

The probability distribution function is

x	-3	-2	-1	0	1	2	3
P(x)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
F(x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}=1$

The probability mass function is

(i) $P(X=x_i) \geq 0 \ \forall i$ clearly

(ii) $\Sigma P(X=x_i)$

$= \frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{3}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$

$=1$

Probability mass function of Y when $Y=2x^2+3x+4$

X	-3	-2	-1	0	1	2	3
Y	13	6	3	4	9	18	31
P(y)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

(∵ Two r.v are defining on one sample space then probability of two r.v are same).

10. The probability mass function of a r.v X is zero except at the point $X=0,1,2$

At these points it has the value

$P(0)=3c^3$, $P(1)=4c-10c^2$

$P(2)=5c-1$ for some $c>0$.

(i) Determine c

(ii) find $P(X<2)$ & $P(1<X\leq 2)$

(iii) find the largest x ∋ $F(x) > 1/2$

(iv) find small x ∋ $F(x) \geq 1/3$

Sol From the given problem

X	0	1	2	3	4	5	6 - -
P(x)	$3c^3$	$4c-10c^2$	$5c-1$	0	0	0	0 - -

(i) To fin c

$\Sigma P(X=x_i)=1$

$3c^3+4c-10c^2+5c-1=1$

$3c^3-10c^2+9c=2$

$c=1, 1/3, 2$

$c=1/3 \quad (c\neq 1,2)$

(ii) $P(X<2) = P(X=1)+P(X=0)$

$= \cancel{9c-10c^2+5c-1}$

$= \cancel{-10c^2+9c-1}$

$= 3c^3+4c-10c^2$

$= 3c^3+4c-10c^2$

$= 3\left(\frac{1}{3}\right)^3+4\left(\frac{1}{3}\right)-10\left(\frac{1}{3}\right)^2$

$=\frac{1}{3}$.

$P(1<X\leq 2) = P(X=2)$

$=5c-1$

$=\frac{5}{3}-1=\frac{2}{3}$.

(iii) Find the largest x s.t.

$F(x) \geq \frac{1}{2}$

$X = x$	0	1	2	3	4
$P(X=x)$	[illegible]	[illegible]	[illegible]	0	0
$F(x)$	[illegible]	[illegible]	[illegible]	1	1

$x = 0$: $F(0) = $ [illegible] $\geq \frac{1}{2}$

$x = 1$: $F(1) = $ [illegible] $\geq \frac{1}{2}$

$x = 2$: $F(2) = 1 \geq \frac{1}{2}$

$\therefore$ largest value of $x = 2$ [illegible]

$F(x) \geq \frac{1}{2}$

(iv) [illegible]

[illegible] $\geq \frac{1}{2}$

[illegible] $\geq \frac{1}{2}$

[illegible] $\geq \frac{1}{2}$

Continuous Random variable

A r.v X is said to be continuous if its range is an interval.

eg:- Temperature of the body.

probability density function

A continuous random variable x is said to be probability density function if

(i) $f(x) \geq 0 \ \forall x$

(ii) $\int_{x=-\infty}^{\infty} f(x)\, dx = 1$

Mean:- Mean of a distribution is given by $\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

If x is defined from a to b then

$$\boxed{\mu = E(x) = \int_a^b x f(x)\, dx}$$

In general, mean or Expectation of any function $\phi(x)$ is given by

$$\boxed{E[\phi(x)] = \int_{-\infty}^{\infty} \phi(x) f(x)\, dx}$$

variance:- Variance of a distribution is given by

$$\boxed{\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2}$$

1. If a random variable has the probability density $f(x)$ as

$$f(x) = \begin{cases} 2e^{-2x}, & \text{for } x>0 \\ 0, & \text{for } x \leq 0 \end{cases}$$

find the probabilities that it will take on a value

(i) between 1 and 3

(ii) greater than 0.5

Sol

(i) between 1 & 3

The probability that takes a value b/w 1 & 3 is given by

$$P(1 \leq x \leq 3) = \int_1^3 f(x)\, dx$$

$$= \int_1^3 2e^{-2x}\, dx$$

$$= 2\left(\frac{e^{-2x}}{-2}\right)_1^3 = -(e^{-6} - e^{-2})$$

$$= e^{-2} - e^{-6}$$

(ii) The probability that takes a value greater than 0.5 is

$$P(x \geq 0.5) = \int_{0.5}^{\infty} f(x)\, dx$$

$$= \int_{0.5}^{\infty} 2e^{-2x}\, dx = \left(\frac{2e^{-2x}}{-2}\right)_{0.5}^{\infty}$$

$$-\left[e^{-\infty} - e^{-2(0.5)}\right] = e^{-2(0.5)}$$

$$= e^{-1} = \frac{1}{e}$$

If the probability density of a random variable given by

$$f(x) = \begin{cases} k(1-x^2), & \text{for } 0<x<1 \\ 0, & \text{otherwise.} \end{cases}$$

find the value of k and the probabilities that a random variable having this probability density will take on a value (i) b/w 0.1 & 0.2
(ii) greater than 0.5

Sol Given $f(x) = \begin{cases} k(1-x^2), & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$

$\therefore$ w.k.t $\int_{-\infty}^{\infty} f(x)\,dx = 1$

i.e. $\int_{-\infty}^{\infty} k(1-x^2)\,dx = 1$

$\Rightarrow \int_{0}^{1} k(1-x^2)\,dx = 1$

$k\left(x - \frac{x^3}{3}\right)_0^1 = 1$

$k\left[\left(1-\frac{1}{3}\right) - (0-0)\right] = 1$

$k\left(\frac{2}{3}\right) = 1$

$\boxed{k = 3/2}$

(i) The probability b/w 0.1 & 0.2 is

$P(0.1<X<0.2)$

$= \int_{0.1}^{0.2} f(x)\,dx = \int_{0.1}^{0.2} k(1-x^2)\,dx$

$= k\left(x - \frac{x^3}{3}\right)_{0.1}^{0.2}$

$= \frac{3}{2}\left[\left(0.2 - \frac{(0.2)^3}{3}\right) - \left(0.1 - \frac{(0.1)^3}{3}\right)\right]$

$= \frac{3}{2}\left[0.1 - \frac{0.007}{3}\right] = 0.2965$

(ii) the probability that takes the value greater than 0.5 is

$P(X>0.5)$

$= \int_{0.5}^{\infty} f(x)\,dx$

$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$

$= \int_{0.5}^{1} k(1-x^2)\,dx + 0$

$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$

$= \frac{3}{2}\left[\left(1-\frac{1}{3}\right) - \left(0.5 - \frac{(0.5)^3}{3}\right)\right]$

$= \frac{3}{2}\left(\frac{2}{3} - 0.4583\right) = 0.3125$

$= \frac{}{5} - \frac{}{64}$

note: If upper limit is given then take lower limit from the question.

If lower limit is given then take upper limit from the question.

$] - (\mu)^2$

2. If $f(x) = \begin{cases} kx^3, & 0 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$

find (i) K (ii) prob b/w $\frac{1}{2}$ & $\frac{3}{2}$

sol:-

(i) To find K

w.k.t $\int_{x=-\infty}^{\infty} f(x)\, dx = 1$.

$\Rightarrow \int_{x=-\infty}^{0} f(x)\, dx + \int_{x=0}^{3} f(x)\, dx + \int_{x=3}^{\infty} f(x)\, dx = 1$

$\Rightarrow 0 + \int_{x=0}^{3} f(x)\, dx + 0 = 1$

$\Rightarrow \int_{x=0}^{3} K x^3\, dx = 1$

$\Rightarrow K \int_{x=0}^{3} x^3\, dx = 1$

$$K\left[\frac{x^4}{4}\right]_{x=0}^{3} = 1$$

$$\Rightarrow \frac{K}{4}\left[(3)^4 - 0\right] = 1$$

$$\Rightarrow \frac{K}{4}\left[81 - 0\right] = 1$$

$$\Rightarrow 81K = 4$$

$$\Rightarrow \boxed{K = \frac{4}{81}}$$

(ii) probability between $\frac{1}{2}$ & $\frac{3}{2}$

$$\left(\because 0 < \frac{1}{2} < x < \frac{3}{2} < 3\right).$$

i.e $$\int_{x=1/2}^{3/2} f(x)\,dx$$

$$= \int_{x=1/2}^{3/2} Kx^3\,dx$$

$$= K\int_{x=1/2}^{3/2} x^3\,dx$$

$$= \frac{4}{81}\left[\frac{x^4}{4}\right]_{1/2}^{3/2}$$

$$= \frac{4}{81}\times\frac{1}{4}\left[\left(\frac{3}{2}\right)^4 - \left(\frac{1}{2}\right)^4\right]$$

$$= \frac{1}{81}\left[\frac{81}{16} - \frac{1}{16}\right]$$

$$= \frac{1}{81}\left[\frac{80}{16}\right] = \frac{5}{81}$$

3. $f(x) = \begin{cases} Kx^2; & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$

then find.

(i) K (ii) prob bet 1 & 2

(iii) prob more than 1.5

(iv) prob less than 2.5

Sol (i) To find K

w.k.t $$\int_{x=-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{x=-\infty}^{0} f(x)\,dx + \int_{x=0}^{3} f(x)\,dx + \int_{x=3}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_{x=0}^{3} f(x)\,dx + 0 = 1$$

$$\Rightarrow \int_{x=0}^{3} K\cdot x^2\,dx = 1$$

$$\Rightarrow K\int_{x=0}^{3} x^2\,dx = 1$$

$$\Rightarrow K\left[\frac{x^3}{3}\right]_{x=0}^{3} = 1$$

$$\Rightarrow K\cdot\frac{1}{3}\left[(3)^3 - 0\right] = 1$$

$$\Rightarrow \frac{K}{3}\left[27\right] = 1$$

$$\Rightarrow K(9) = 1$$

$$\boxed{K = 1/9}$$

(ii) prob b/w 1 & 2

$$\int_{x=-\infty}^{\infty} f(x)\,dx = \int_{x=1}^{2} f(x)\,dx$$

$$= \int_{x=1}^{2} Kx^2\,dx$$

$$= K\int_{x=1}^{2} x^2\,dx$$

$$= K\left[\frac{x^3}{3}\right]_{x=1}^{2}$$

$$= K\cdot\frac{1}{3}\left[8-1\right] = \frac{1}{9}\cdot\frac{1}{3}\cdot(7)$$

$$= \frac{7}{27}$$

(iii) probability ~~...~~ is more than 1.5

i.e $x = 1.5$. i.e $P(X > 1.5)$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{x=1.5}^{\infty} f(x)\,dx$$

$$= \int_{x=1.5}^{3} f(x)\,dx + \int_{x=3}^{\infty} f(x)\,dx$$

$$= \int_{x=1.5}^{3} Kx^2\,dx + 0$$

$$= K\int_{x=1.5}^{3} x^2\,dx + 0$$

$$= \frac{1}{9}\int_{x=1.5}^{3} x^2\,dx$$

$$= \frac{1}{9}\left[\frac{x^3}{3}\right]_{x=1.5}^{3}$$

$$= \frac{1}{27}\left[(3)^3-(1.5)^3\right].$$

(iv) probability less than 2.5

$$P(x<2.5) = \int_{x=-\infty}^{2.5} f(x)\,dx$$

$$= \int_{x=-\infty}^{0} f(x)\,dx + \int_{x=0}^{2.5} f(x)\,dx$$

$$= 0 + \int_{x=0}^{2.5} Kx^2\,dx$$

$$= K\int_{x=0}^{2.5} x^2\,dx$$

$$= \frac{1}{9}\left[\frac{x^3}{3}\right]_{x=0}^{2.5}$$

$$= \frac{1}{27}\left[(2.5)^3-0\right]$$

$$= \frac{1}{27}(2.5)^3$$

4. $f(x) = Ke^{-3x}, \ x \geq 0$.

find (i) K

(ii) probability B/w $\frac{1}{4}$ & $\frac{3}{4}$

(iii) Greater than $\frac{2}{3}$.

Sol (i) W.K.T $\int_{x=-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{x=-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{x=-\infty}^{0} f(x)\,dx + \int_{x=0}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{x=0}^{\infty} Ke^{-3x}\,dx = 1$

$\Rightarrow K\int_{x=0}^{\infty} e^{-3x}\,dx = 1$

$\Rightarrow K \cdot \left[\frac{e^{-3x}}{-3}\right]_{x=0}^{\infty} = 1$

$\Rightarrow K\left(\frac{-1}{3}\right)\left[e^{-3x}\right]_{x=0}^{\infty} = 1$

$\Rightarrow \frac{-K}{3}\left[e^{-\infty} - e^{0}\right] = 1$ $\left(\because e^{\infty} = \infty, \ e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0\right)$

$\Rightarrow \frac{-K}{3}\left[0-1\right] = 1$ $\left(e^{0} = 1, \ e^{-\infty} = 0\right)$

$\Rightarrow \frac{K}{3} = 1 \Rightarrow \boxed{K=3}$

(ii) prob b/w $\frac{1}{4}$ & $\frac{3}{4}$ i.e $P(\frac{1}{4} < x < \frac{3}{4})$

i.e $\int_{x=-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{1/4} f(x)\,dx + \int_{1/4}^{3/4} f(x)\,dx + \int_{3/4}^{\infty} f(x)\,dx$

$= 0 + \int_{1/4}^{3/4} f(x)\,dx + 0$

$= \int_{1/4}^{3/4} Ke^{-3x}\,dx$

$= K\int_{1/4}^{3/4} e^{-3x}\,dx$

$= K\left[\frac{e^{-3x}}{-3}\right]_{x=1/4}^{3/4}$

$= K\left(\frac{-1}{3}\right)\left[e^{-3(\frac{3}{4})} - e^{-3(\frac{1}{4})}\right]$

$= 3\left(\frac{-1}{3}\right)\left[e^{-9/4} - e^{-3/4}\right]$

$= -\left[e^{-9/4} - e^{-3/4}\right]$

$= e^{-3/4} - e^{-9/4}$

$= 0.366.$

(iii) Probability of Greater than 2/3.

i.e $P(x > \frac{2}{3})$

$$\int_{x=-\infty}^{\infty} f(x)\,dx = \int_{2/3}^{\infty} f(x)\,dx$$

$$= \int_{2/3}^{\infty} f(x)\,dx$$

$$= \int_{2/3}^{\infty} K \cdot e^{-3x}\,dx.$$

$$= K \int_{2/3}^{\infty} e^{-3x}\,dx$$

$$= K \cdot \left[\frac{e^{-3x}}{-3}\right]_{2/3}^{\infty}.$$

$$= (3)\cdot\left(\frac{-1}{3}\right)\cdot\left[e^{-3x}\right]_{2/3}^{\infty}$$

$$= -1\left[e^{-\infty} - e^{-\cancel{3}\left(\frac{2}{\cancel{3}}\right)}\right]$$

$$= -1\left[0 - e^{-2}\right]$$

$$= e^{-2} = 0.135.$$

5. $f(x) = K(1-x^2)$, $0 < x < 1$

find

(i) K (ii) b/w 0.1 & 0.2

(iii) more than 0.5

Sol (i) To find K

$$\int_{x=-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_{0}^{1} f(x)\,dx + 0 = 1$$

$$\Rightarrow \int_{0}^{1} K(1-x^2)\,dx = 1$$

$$\Rightarrow K \int_{x=0}^{1} (1-x^2)\,dx = 1$$

$$\Rightarrow K\left[x - \frac{x^3}{3}\right]_{x=0}^{1} = 1$$

$$\Rightarrow K\left[(1-0) - \left(\frac{1}{3} - 0\right)\right] = 1$$

$$\Rightarrow K\left[1 - 1/3\right] = 1$$

$$\Rightarrow K\left[\frac{2}{3}\right] = 1$$

$$\Rightarrow \boxed{K = 3/2}$$

(ii) prob b/w 0.1 & 0.2

i.e $P(0.1 < X < 0.2)$

$$= \int_{0.1}^{0.2} f(x)\,dx$$

$$= \int_{0.1}^{0.2} K(1-x^2)\,dx$$

$$= K \int_{0.1}^{0.2} (1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.1}^{0.2}$$

$$= \frac{3}{2}\left[(0.2-0.1) - \frac{1}{3}\left[(0.2)^3 - (0.1)^3\right]\right]$$

$$= \frac{3}{2}\left[(0.1) - \frac{1}{3}\left((0.2)^3 - (0.1)^3\right)\right].$$

(iii) prob more than 0.5 i.e $(P(X>0.5))$

$$\int_{x=0.5}^{1} f(x)\,dx = \int_{x=0.5}^{1} K(1-x^2)\,dx$$

$$= K\int_{x=0.5}^{1} (1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{x=0.5}^{1}$$

$$= \frac{3}{2}\left[(1-0.5) - \frac{1}{3}\left(1-(0.5)^3\right)\right]$$

6. $f(x) = 2e^{-2x},\ x \geq 0.$

find (i) b/w 1 & 3

(ii) more than 0.5

Sol $f(x) = 2e^{-2x}$.

(i) prob b/w 1 & 3

i.e $P(1 < x < 3)$

$$= \int_{x=1}^{3} f(x)\,dx$$

$$= \int_{x=1}^{3} 2e^{-2x}\,dx$$

$$= 2\int_{x=1}^{3} e^{-2x}\,dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_{x=1}^{3}$$

$$= \frac{2}{-2}\left[e^{-6} - e^{-2}\right]$$

$$= -\left[e^{-6} - e^{-2}\right]$$

$$= e^{-2} - e^{-6} = 0.01$$

(ii) prob more than 0.5

$$\int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{\infty} 2e^{-2x}\,dx$$

$$= 2\int_{0.5}^{\infty} e^{-2x}\,dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_{0.5}^{\infty}$$

$$= -\left[e^{-2x}\right]_{0.5}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-(0.5)2}\right]$$

$$= e^{-(0.5)2} - 0 = e^{-1} = \frac{1}{e}$$

7. $f(x) = K(2x+3), \quad 2 \le x \le 4$.

find (i) K (ii) b/w 1 & 4

Sol) (i) <u>To find K</u>

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{2}^{4} f(x)\,dx = 1$$

$$\Rightarrow \int_{2}^{4} K(2x+3)\,dx = 1$$

$$\Rightarrow K \cdot \int_{2}^{4} 2x+3 \; dx = 1$$

$$\Rightarrow K\left[\frac{2x^2}{2} + 3x\right]_2^4 = 1$$

$$\Rightarrow K\left[x^2 + 3x\right]_2^4 = 1$$

$$\Rightarrow K\left[(16-4) + 3(4-2)\right] = 1$$

$$\Rightarrow K\left[12 + 6\right] = 1$$

$$\Rightarrow \boxed{K = 1/18}$$

(ii) <u>prob b/w 1 & 4:</u>

$$\int_{x=1}^{4} f(x)\,dx = \int_{x=1}^{2} f(x)\,dx + \int_{x=2}^{4} f(x)\,dx$$

$$= \int_{x=1}^{2} 0\,dx + \int_{x=2}^{4} K(2x+3)\,dx$$

$$= 0 + \frac{1}{18}\int_{x=2}^{4} (2x+3)\,dx$$

$$= \frac{1}{18}\left[\frac{2x^2}{2} + 3x\right]_{x=2}^{4}$$

$$= \frac{1}{18}\left[(16-4) + 3(4-2)\right]$$

$$= \frac{1}{18}\left[12+6\right] = 1.$$

8. $f(x) = cx(2-x), \quad 0 \le x \le 2$

find (i) c (ii) mean (iii) variance

Sol) (i) <u>To find 'c'</u>

$$\text{w.k.t} \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{2} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{2} cx(2-x)\,dx = 1$$

$$\Rightarrow c\int_{0}^{2} (2x - x^2)\,dx = 1$$

$$\Rightarrow c\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow c\left[x^2 - \frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow c\left[(4-0) - \frac{1}{3}(8-0)\right] = 1$$

$$\Rightarrow c\left[4 - \frac{8}{3}\right] = 1$$

$$\Rightarrow c\left[\frac{12-8}{3}\right] = 1$$

$$\Rightarrow c\left[\frac{4}{3}\right] = 1$$

$$\Rightarrow \boxed{c = \frac{3}{4}}$$

(ii) mean

$$\text{mean } \mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^2 x\, f(x)\, dx$$

$$= \int_0^2 x \cdot cx(2-x)\, dx$$

$$= c\int_0^2 x^2(2-x)\, dx$$

$$= c\int_0^2 (2x^2 - x^3)\, dx$$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{2}{3}(8-0) - \frac{1}{4}(16-0)\right]$$

$$= \frac{3}{4}\left[\frac{16}{3} - \frac{16}{4}\right].$$

$$= \frac{3}{4}\left[\frac{64-48}{12}\right]$$

$$= \frac{3}{4}\left[\frac{16}{12}\right] = 1$$

$$\boxed{\mu = 1}$$

(iii) variance

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - (\mu)^2$$

$$= \int_0^2 x^2 f(x)\, dx - (\mu)^2$$

$$= \int_0^2 x^2 \cdot cx(2-x)\, dx - (1)^2$$

$$= c\int_0^2 x^3(2-x)\, dx - 1$$

$$= c\int_0^2 (2x^3 - x^4)\, dx - 1$$

$$= \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_{x=0}^2 - 1$$

$$= \frac{3}{4}\left[\frac{1}{2}(16-0) - \frac{1}{5}(32-0)\right] - 1$$

$$= \frac{3}{4}\left[8 - \frac{32}{5}\right] - 1$$

$$= \frac{3}{4}\left[\frac{40-32}{5}\right] - 1$$

$$= \frac{3}{4}\left[\frac{8}{5}\right] - 1$$

$$= \frac{6}{5} - 1 = \frac{1}{5}$$

$$\Rightarrow \boxed{\sigma^2 = 1/5}$$

9. $f(x) = k(3x^2-1), \quad -1 \le x \le 2$

find (i) k.

(ii) mean

(iii) variance.

Sol: (i) To find k

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-1}^{2} f(x)\,dx = 1$$

$$\Rightarrow \int_{-1}^{2} k(3x^2-1)\,dx = 1$$

$$\Rightarrow k\int_{-1}^{2} (3x^2-1)\,dx = 1$$

$$\Rightarrow k\left[\frac{3x^3}{3} - x\right]_{-1}^{2} = 1$$

$$\Rightarrow k\left[x^3 - x\right]_{-1}^{2} = 1$$

$$\Rightarrow k\left[(8+1) - (2+1)\right] = 1$$

$$\Rightarrow k[9-3] = 1$$

$$\Rightarrow k(6) = 1 \Rightarrow \boxed{k = 1/6}$$

(ii) Mean (μ)

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-1}^{2} x f(x)\,dx$$

$$= \int_{-1}^{2} x \cdot k(3x^2-1)\,dx$$

$$= k\int_{-1}^{2} x(3x^2-1)\,dx$$

$$= k\int_{-1}^{2} 3x^3 - x\,dx.$$

$$= k\left[\frac{3x^4}{4} - \frac{x^2}{2}\right]_{-1}^{2}$$

$$= \frac{1}{6}\left[\frac{3}{4}(16-1) - \frac{1}{2}(4-1)\right]$$

$$= \frac{1}{6}\left[\frac{45}{4} - \frac{3}{2}\right] = \frac{13}{8}.$$

(iii) variance (σ^2)

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - (\mu)^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left(\frac{13}{8}\right)^2$$

$$= \int_{-1}^{2} x^2\, k(3x^2-1)\,dx - \frac{169}{64}$$

$$= k\int_{-1}^{2} 3x^4 - x^2\,dx - \frac{169}{64}$$

$$= \frac{1}{6}\left[\frac{3x^5}{5} - \frac{x^3}{3}\right]_{-1}^{2} - \frac{169}{64}$$

$$= \frac{1}{6}\left[\frac{3}{5}(32+1) - \frac{1}{3}(8+1)\right] - \frac{169}{64}$$

$$= \frac{1}{6}\left[\frac{3}{5}(33) - \frac{9}{3}\right] - \frac{169}{64} =$$