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Mathematical logic.

Mathematical logic provides a set of rules and techniques for determining whether a given argument or conclusion is valid or not.

⇒ Logic is used in computer science to verify the correctness of the program.

Statement:

A declarative ~~sta~~ sentence which is either True or false but not both simultaneously is called as a statement.

We denote statements with 'P', 'Q', 'R' ...

⇒ True, False are called Truth values denoted with 'T' & 'F' respectively

⇒ They are also denoted by 1 & 0.

⇒ There are two kinds of statements

> primitive statement

> Compound statement.

• **primitive statement**

A statement which cannot further be broken down into smaller statements is said to be primitive statement.

• **Compound statement**

Two or more primitive statements are joined together to form a compound statement using certain words called as connectives.

Connectives

The five main types of connectives that are commonly used are:

i. NOT
ii. AND
iii. OR
iv. If ... Then
v. If and only If

Truth tables

i) NOT

If P is a statement then it's Negation is NOT P. it is not the case P, it is not True that P & it is denoted by ~P or ¬P

P	~P or ¬P
T	F
F	T

ii. AND/conjunction

If P, Q are any two statements then the conjunction of P, Q is denoted by P∧Q read as P AND Q.

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

iii. Disjunction

If P, Q are any two statements the disjunction of it is denoted by P∨Q and read as P OR Q.

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

iv. Conditional statement

If P, Q are any two statements then If P then Q is denoted by P→Q and read as If P THEN Q

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

v. Biconditional statements.

If P, Q are any two statements then the biconditional statement of it is P↔Q or P⇄Q and read as P if and only if Q

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

27/10/22 statement formulas:

If P & Q are any two simple statements then the compound statements derived from P & Q like $\sim P$. $P \wedge Q$, $P \vee Q$, $\sim P \vee Q$, $P \vee \sim Q$ are called statement formulas. P, Q are called statement variables.

If there are 'n' distinct statement variables, then we get 2^n possible combinations of truth values in order to construct truth table

Well formed formulae (WFF)

Well formed formulae can be generated by following rules,

i. A statement variable alone is a WFF.

ii. If A & B are wff then the combination of these two $\sim A$, $\sim AB$. $A \wedge B$, $A \vee B$, $A \rightarrow B$, $A \leftrightarrow B$ are also wff.

iii. A string of symbols containing statement variables connectives and parenthesis are wff if and only if it can be obtained by finitly many applicatio-s of i, ii.

Tautology:

A statement formulae which is always true regardless of truth values of the statements is called a tautology

Ex:-

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

Contradiction: A statement formaulae is which is always false regardless of truth values of the statements is called a contradiction.

Ex:

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

problems

01. Write the following statement in symbolic form.

P: x is rich
Q: y is happy

a) ~~Sam~~ x is rich and y is not happy $P \wedge \neg Q$
b) x is not rich and y is happy $\neg P \wedge Q$
c) If x is rich then y is happy $P \to Q$

02. P: x is rich
Q: x is happy

a) x is rich and unhappy $P \wedge \neg Q$
b) x is neither rich nor happy $\sim P \wedge \sim Q$ / $\neg P \wedge \neg Q$

03. Construct truth table for $\neg(\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

04. $P \vee (Q \wedge R)$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$
T	T	T	T	T
F	T	T	T	T
T	F	T	F	T
F	F	T	F	F
T	T	F	F	T
F	T	F	F	F
T	F	F	F	T
F	F	F	F	F

05. Verify whether $(P \vee Q) \to P$ is a tautology

P	Q	$P \vee Q$	$(P \vee Q) \to P$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

$\therefore$ not a tautology

06. verify whether $(P \wedge (P \leftrightarrow Q)) \to Q$ is a tautology

P	Q	$P \leftrightarrow Q$	$P \wedge (P \leftrightarrow Q)$	$(P \wedge (P \leftrightarrow Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	T

$\therefore$ is a tautology

07. $(P \wedge Q) \to (P \vee Q)$

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \to (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$\therefore$ is a tautology

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Equivalence formulae:

Two formulae A & B are said to be equivalent to each other if and only if $A \Leftrightarrow B$ $(A \leftrightarrow B)$ is a tautology

NOTE: $A \Leftrightarrow B$

- $A \leftrightarrow B$ is a tautology
- Truth table for A
Truth table for B are same

Equivalent formulaes:

(i) Idempotent laws:

$P \vee P \Leftrightarrow P$

$P \wedge P \Leftrightarrow P$

(ii) Associative laws

$P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$

$P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

(iii) Commitative laws:

$P \vee Q \Leftrightarrow Q \vee P$

$P \wedge Q \Leftrightarrow Q \wedge P$

(iv) Distributive laws

$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

(v) Absorption laws:

$P \vee (P \wedge Q) \Leftrightarrow P$

$P \wedge (P \vee Q) \Leftrightarrow P$

(vi) Demorgan's laws

$\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$

$\neg(P \wedge Q) \Leftrightarrow (\neg P) \vee (\neg Q)$

(vii) $P \vee F \Leftrightarrow P$

$P \wedge T \Leftrightarrow P$

$P \vee \neg P \Leftrightarrow T$

$P \wedge \neg P \Leftrightarrow F$

$P \vee T \Leftrightarrow T$

$P \wedge F \Leftrightarrow F$

problems:

01. prove that $P \vee Q \Leftrightarrow \neg(\neg P \wedge \neg Q)$

P	Q	$P \vee Q$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$
T	T	T	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	F	T	T	T	F

$\Leftrightarrow$ ∴ Equivalent

02. $\boxed{P \to Q \Leftrightarrow \neg P \vee Q}$

P	Q	$P \to Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\Leftrightarrow$ ∴ Equivalent

Replacement process

01. prove that $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$

$P \to (Q \to R)$

$\Leftrightarrow P \to (\neg Q \vee R)$ ∵ Equivalent law

$\Leftrightarrow \neg P \vee (\neg Q \vee R)$ ∵ Equivalent law

$\Leftrightarrow (\neg P \vee \neg Q) \vee R$ ∵ Associative law

$\Leftrightarrow \neg (P \wedge Q) \vee R$ ∵ Demorgan's law

$\Leftrightarrow (P \wedge Q) \to R$ ∵ Equivalent law

Hence proved//

02. $(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg R \vee Q)$ ∵ Equivalent law

$\Leftrightarrow (\neg P \wedge \neg R) \vee Q$ ∵ Distributive law

$\Leftrightarrow \neg (P \vee R) \vee Q$ ∵ Demorgan's law

$\Leftrightarrow (P \vee R) \to Q$ ∵ Equivalent law

Hence proved//

03. $P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$

LHS $P \to (Q \to P)$

$\Leftrightarrow P \to (\neg Q \vee P)$ ∵ Equivalent law

$\Leftrightarrow \neg P \vee (\neg Q \vee P)$ ∵ Equivalent law

$\Leftrightarrow \neg P \vee (P \vee \neg Q)$ ∵ Commitative law

$\Leftrightarrow (\neg P \vee P) \vee \neg Q$ ∵ Associative law

$\Leftrightarrow T \vee \neg Q$

$\Leftrightarrow T$

RHS $\neg P \to (P \to Q)$

$\Leftrightarrow \neg P \to (\neg P \vee Q)$ ∵ Equivalent law

$\Leftrightarrow \neg(\neg P) \vee (\neg P \vee Q)$ ∵ Equivalent law

$\Leftrightarrow P \vee (\neg P \vee Q)$ ∵

$\Leftrightarrow (P \vee \neg P) \vee Q$ ∵ Associative law

$\Leftrightarrow T \vee Q$

$\Leftrightarrow T$

∴ LHS = RHS

04. $(P \to Q) \land (R \to Q) \Leftrightarrow (P \lor R) \to Q$

$\Leftrightarrow (\neg P \lor Q) \land (\neg R \lor Q)$ ∵ Equivalent law

$\Leftrightarrow (\neg P \land \neg R) \lor Q$ ∵ Distributive law

$\Leftrightarrow \neg(P \lor R) \lor Q$ ∵ Demorgan's law

$\Leftrightarrow P \lor R \to Q$ ∵ Equivalent law

Hence proved.//

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Toutological implication:

A statement 'A' is said to be toutologically implies 'B' if and only if $A \to B$ is a toutology and we write it as $A \Rightarrow B$

Implication formulae:

i. $P \land Q \Rightarrow P$

ii. $P \land Q \Rightarrow Q$

iii. $P \Rightarrow P \lor Q$

iv. $Q \Rightarrow P \lor Q$

v. $\sim P \Rightarrow P \to Q$

vi. $Q \Rightarrow P \to Q$

NOTE: We use the word product for conjunction ($\land$) and we use the word sum for disjunction ($\lor$)

Elementary product

The product of statement variables and it's negation are called elementary products.

Ex:- $P \land Q$, $P \land Q \land \sim P$, $P \land Q \land P \land \sim Q$ ... etc

Elementary sum

The sum of statement variables and it's negati are called elementary sums.

Ex:- P, Q, $\sim P$, $P \lor Q \lor \sim P$, $P \lor Q \lor P \lor \sim Q$ .... etc

Disjuctive Normal Form (DNF):

A statement formula which is equivalent to the given formula and which contains sum of elementary products is called DNF

Ex:- $P \lor (P \land Q) \lor (P \land Q \land \sim P) \lor (P \land Q \land P \land \sim Q \land P)$

sum product $\Rightarrow$ sum of elementary products

problems

01 Obtain DNF of $P \wedge (P \rightarrow Q)$

$\Leftrightarrow P \wedge (\sim P \vee Q)$ $\because$ Equivalence law

$\Leftrightarrow (P \wedge \sim P) \vee (P \wedge Q)$ $\because$ Distributive law

product sum product

Hence in DNF

02. Obtain DNF of $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \wedge \sim P))$

$\Leftrightarrow P \rightarrow ((\sim P \vee Q) \wedge (Q \vee P))$ $\because$ Demorgan's & Equivalence

$\Leftrightarrow P \rightarrow (((\sim P \vee Q) \wedge Q) \vee ((\sim P \vee Q) \wedge P))$ $\because$ Distributive

$\Leftrightarrow P \rightarrow ((\sim P \wedge Q) \vee (Q \wedge Q) \vee (\sim P \wedge P) \vee (Q \wedge P))$ $\because$ "

$\Leftrightarrow P \rightarrow ((\sim P \wedge Q) \vee Q \vee F \vee (Q \wedge P))$ $\because$ Equivalence

$\Leftrightarrow P \rightarrow (\sim P \wedge Q) \vee Q \vee (Q \wedge P)$

$\Leftrightarrow \sim P \vee ((\sim P \wedge Q) \vee Q \vee (Q \wedge P))$

$\Leftrightarrow \sim P \vee (\sim P \wedge Q) \vee Q \vee (Q \wedge P)$

$\therefore$ Hence in DNF

$\boxed{\text{NOTE: } P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)}$

03. Obtain DNF of $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$ Equivalence

$\Leftrightarrow (\sim(P \vee Q) \rightarrow (P \wedge Q)) \wedge ((P \wedge Q) \rightarrow \sim(P \vee Q))$

$\Leftrightarrow ((P \vee Q) \vee (P \wedge Q)) \wedge (\sim(P \wedge Q) \vee \sim(P \vee Q))$ $\because$ Equivalence

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Q4. find DNF of $(\sim P \to R) \land (P \leftrightarrow Q)$ using truth table

P	Q	R	$\sim P$	$\sim P \to R$	$P \leftrightarrow Q$	$(\sim P \to R) \land (P \leftrightarrow Q)$
T	T	T	F	T	T	T
F	T	T	T	T	F	F
T	F	T	F	T	F	F
F	F	T	T	T	T	T
T	T	F	F	T	T	T
F	T	F	T	F	F	F
T	F	F	F	T	F	F
F	F	F	T	F	T	F

T → same
F → negation

select the rows with 'T' in the last coloumn

$(P \land Q \land R) \lor (\sim P \land \sim Q \land R) \lor (P \land Q \land \sim R)$

is in DNF.

Conjuctive Normal Form (CNF)

A statement formula which is equivalent to the given formula and consisting of "product of sums" is called CNF of the given statement formula.

Ex:- $\underbrace{(P \lor Q \lor \sim Q)}_{\text{sums}} \underset{\text{product}}{\land} (P \lor Q \lor \sim P) \land (P \lor Q) \land P$

Q1. Obtain CNF of $P \land (P \to Q)$

$\Leftrightarrow P \land (\sim P \lor Q)$ ∵ Equivalence law

is the required CNF.

Q2. Obtain the CNF of $\sim(P \lor Q) \leftrightarrow (P \land Q)$

$\Leftrightarrow [(\sim(P \lor Q)) \to (P \land Q)] \land [(P \land Q) \to \sim(P \lor Q)]$ ∵ Equivalence

$\Leftrightarrow [(P \lor Q) \lor (P \land Q)] \land [\sim(P \land Q) \lor (\sim(P \lor Q))]$ ∵ Demorgan's

$\Leftrightarrow [(P \lor Q \lor P) \land (P \lor Q \lor Q)] \land [(\sim P \lor \sim Q) \lor (\sim P \land \sim Q)]$ ∵ Dist law

$\Leftrightarrow (P \lor Q \lor P) \land (P \lor Q \lor Q) \land [(\sim P \lor \sim Q \lor \sim P) \land (\sim P \lor \sim Q \lor \sim Q)]$

Hence in CNF.

03. Obtain CNF of $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\Leftrightarrow Q \vee (P \wedge \sim Q) \vee \sim(P \vee Q)$ ∵ Demorgan's law

$\Leftrightarrow (Q \vee P) \wedge (Q \vee \sim Q) \vee \sim(P \vee Q)$ ∵ Distributive law

$\Leftrightarrow (Q \vee P) \vee \sim(P \vee Q) \wedge (Q \vee \sim Q)$ ∵ Association

$\Leftrightarrow (Q \vee P) \vee \sim(P \vee Q) \wedge T$ ∵ $Q \vee \sim Q = T$

$\Leftrightarrow (Q \vee P) \vee \sim(P \vee Q)$ ∵ $\sim(P \vee Q) \wedge T = \sim(P \vee Q)$

$\Leftrightarrow (Q \vee P) \vee (\sim P \wedge \sim Q)$ ∵ Demorgan's

$\Leftrightarrow (Q \vee P \vee \sim P) \wedge (Q \vee P \vee \sim Q)$ ∵ Distribution

04. Obtain CNF of $(\sim P \rightarrow R) \wedge (P \rightarrow Q)$ using T-T.

P	Q	R	$\sim P$	$\sim P \rightarrow R$	$P \rightarrow Q$	$(\sim P \rightarrow R) \wedge (P \rightarrow Q)$
T	T	T	F	T	T	T
F	T	T	T	T	T	T
T	F	T	F	T	F	F
F	F	T	T	T	T	T
T	T	F	F	T	T	T
F	T	F	T	F	T	F
T	F	F	F	T	F	F
F	F	F	~~F~~T	F	T	F

F → same
T → negation

select combinations with 'F' in the last column

$(\sim P \vee Q \vee \sim R) \wedge (P \vee \sim Q \vee R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee Q \vee R)$

is in CNF.

NOTE:

In CNF we select combinations with False in the last coloumn and hence, we write the variable if it is false, and negation of the variable if it is true.

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Minterms

The conjuction of statement variables or it's negation but not both appearing only once are called minterms.

⇒ If there are 'n' variables then we get "2^n" minterms

⇒ Ex: i. n=2, P, Q are statement variables

No of minterms $= 2^2 = 4$

$P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$, $\sim P \wedge \sim Q$

ii. n=3, P, Q, R are statement variables

$P \wedge Q \wedge R$, $P \wedge Q \wedge \sim R$, $P \wedge \sim Q \wedge R$, $P \wedge \sim Q \wedge \sim R$, $\sim P \wedge Q \wedge R$, $\sim P \wedge Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge R$, $\sim P \wedge \sim Q \wedge \sim R$

Maxterms

The disjuction of statement variables or it's negation but not both appearing only once are called maxterms

→ If there are "n" variables then we get "2^n" maxterms

Ex 1:- n=2, P, Q are statement variables

$P \vee Q$, $P \vee \sim Q$, $\sim P \vee Q$, $\sim P \vee \sim Q$

2:- n=3, P, Q, R are statement variables

$P \vee Q \vee R$, $P \vee Q \vee \sim R$, $P \vee \sim Q \vee R$, $P \vee \sim Q \vee \sim R$, $\sim P \vee Q \vee R$, $\sim P \vee Q \vee \sim R$, $\sim P \vee \sim Q \vee R$, $\sim P \vee \sim Q \vee \sim R$

princial disjunctive normal form (PDNF)

An equivalence formula consisting of disjunction of minterms is known as PDNF

problems

01. obtain PDNF of $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

P	Q	R	$P \wedge Q$	$\sim P$	$\sim P \wedge R$	$Q \wedge R$	Result	Minterms
T	T	T	T	F	F	T	T	$P \wedge Q \wedge R$
F	T	T	F	T	T	T	T	$\sim P \wedge Q \wedge R$
T	F	T	F	F	F	F	F	$P \wedge \sim Q \wedge R$
F	F	T	F	T	T	F	T	$\sim P \wedge \sim Q \wedge R$
T	T	F	T	F	F	F	T	$P \wedge Q \wedge \sim R$
F	T	F	F	T	F	F	F	$\sim P \wedge Q \wedge \sim R$
T	F	F	F	F	F	F	F	$P \wedge \sim Q \wedge \sim R$
F	F	F	F	T	F	F	F	$\sim P \wedge \sim Q \wedge \sim R$

The PDNF of the given formula is,
$(P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge \sim R)$

NOTE:- PDNF & PCNF are unique

02. obtain PDNF of $P \vee (\sim P \wedge \sim Q \wedge \sim R)$

P	Q	R	$\sim P$	$\sim Q$	$\sim R$	$\sim P \wedge \sim Q \wedge \sim R$	$P \vee (\quad)$
T	T	T	F	F	F	F	T
F	T	T	T	F	F	F	F
T	F	T	F	T	F	F	T
F	F	T	T	T	F	F	F
T	T	F	F	F	T	F	T
F	T	F	T	F	T	F	F
T	F	F	F	T	T	F	T
F	F	F	T	T	T	T	T

$(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge \sim R) \vee$
$(\sim P \wedge \sim Q \wedge \sim R)$

principal conjunctive normal form (PCNF)

An equivalent formula consisting of conjunction of maxterms is called PCNF.

Q1. Obtain PCNF of $(\sim P \to R) \wedge (Q \leftrightarrow P)$

P	Q	R	$\sim P$	$\sim P \to R$	$Q \leftrightarrow P$	$(\ ) \wedge (\ )$	Maxterms
T	T	T	F	T	T	T	$\sim P \vee \sim Q \vee \sim R$
F	T	T	T	T	F	F	$P \vee \sim Q \vee \sim R$
T	F	T	F	T	F	F	$\sim P \vee Q \vee \sim R$
F	F	T	T	T	T	T	$P \vee Q \vee \sim R$
T	T	F	F	T	T	T	$\sim P \vee \sim Q \vee R$
F	T	F	T	F	F	F	$P \vee \sim Q \vee R$
T	F	F	F	T	F	F	$\sim P \vee Q \vee R$
F	F	F	T	F	T	F	$P \vee Q \vee R$

The PCNF of the given statement formula is
$(\sim P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee \sim R) \wedge (P \vee \sim Q \vee R) \wedge (\sim P \vee Q \vee R) \wedge$
$(P \vee Q \vee R)$

Q2. Obtain PCNF of $(P \wedge Q) \vee (\sim P \wedge Q) \vee (Q \wedge R)$

P	Q	R	$\sim P$	$P \wedge Q$	$\sim P \wedge Q$	$Q \wedge R$	$(P \wedge Q) \vee (\sim P \wedge Q) \vee (Q \wedge R)$
T	T	T	F	T	F	T	T
F	T	T	T	F	T	T	T
T	F	T	F	F	F	F	F
F	F	T	T	F	F	F	F
T	T	F	F	T	T	F	T
F	T	F	T	F	~~F~~ T	F	~~F~~ T
T	F	F	F	F	F	F	F
F	F	F	T	F	F	F	F

$(\sim P \vee Q \vee \sim R) \wedge (P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee Q \vee R)$

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Method to obtain PCNF from PDNF

If A is the formula representing given PDNF. $\sim A$ is also PDNF containing disjunction of remaining minterms (which are not in A). $\sim(\sim A)$ is a PCNF of A.

problems

01. Obtain PCNF from the given PDNF

$(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge\sim Q\wedge R)$

$A: (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge\sim Q\wedge R)$

$\sim A$ contains disjunction of minterms which are not in A.

$\sim A: (\sim P\wedge\sim Q\wedge\sim R)\vee(\sim P\wedge Q\wedge\sim R)\vee(P\wedge\sim Q\wedge R)\vee(P\wedge\sim Q\wedge\sim R)$

$\sim(\sim A): \sim(\sim P\wedge\sim Q\wedge\sim R)\wedge\sim(\sim P\wedge Q\wedge\sim R)\wedge\sim(P\wedge\sim Q\wedge\sim R)\wedge$
$\sim(P\wedge\sim Q\wedge R)$

$= (P\vee Q\vee R)\wedge(P\vee Q\vee\sim R)\wedge(\sim P\vee Q\vee R)\vee(\sim P\vee\sim Q\vee R)$.

which is in PDNF PCNF

02. PDNF from PCNF

$S: P\vee(\sim P\rightarrow(Q\vee(\sim Q\rightarrow R)))$

$P\vee(\sim P\rightarrow(Q\vee(Q\vee R)))$

$P\vee(\sim P\vee(Q\vee(Q\vee R)))$

$P\vee(P\vee Q)\vee(P\vee Q\vee R))$

$(P\vee P\vee Q)\vee(P\vee Q\vee R) = P\vee Q\vee R$

$\sim S=(P\vee Q\vee\sim R)\wedge(P\vee\sim Q\vee R)\wedge(P\vee\sim Q\vee\sim R)\wedge(\sim P\vee Q\vee R)\wedge$
$(\sim P\vee Q\vee\sim R)\wedge(\sim P\vee\sim Q\vee R)\wedge(\sim P\vee\sim Q\vee\sim R)$

$\sim(\sim S)=\sim(P\vee Q\vee\sim R)\vee\sim(P\vee\sim Q\vee R)\vee\sim(P\vee\sim Q\vee\sim R)\vee$
$\sim(\sim P\vee Q\vee R)\vee\sim(\sim P\vee Q\vee\sim R)\vee\sim(\sim P\vee\sim Q\vee R)\vee$
$\sim(\sim P\vee\sim Q\vee\sim R)$

$=(\sim P\wedge\sim Q\wedge R)\vee(\sim P\wedge Q\wedge\sim R)\vee(\sim P\wedge Q\wedge R)\vee(P\wedge\sim Q\wedge\sim R)$
$\vee(P\wedge\sim Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(P\wedge Q\wedge R)$

Theory of inference

The main aim of logic is to provide rules of inference to derive conclusion from certain primeses. If a conclusion is derived from a set of premises by using accepted rules of reasoning then such a process of derivation is known as deduction or formal proof and the conclusion is called valid argument.

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Formula	Name
1. $\dfrac{P \to Q \quad P}{Q}$ premise/Hypothesis, $Q \Rightarrow$ Conclusion	Modus ponens $(P \to Q) \wedge P \Rightarrow Q$
2. $\dfrac{P \to Q \quad \sim Q}{\sim P}$	Modus tollens.
3. $\dfrac{P \to Q \quad Q \to R}{P \to R}$	Hypothetical syllogism.
4. $\dfrac{P \vee Q \quad \sim P}{Q}$	Disjunctive syllogism
5. $\dfrac{P \to R \quad Q \to R \quad P \vee Q}{R}$	Dilemma
6. $\dfrac{P \vee R \quad Q \vee \sim R}{P \vee Q}$	Resolution
7. $\dfrac{P \wedge Q}{P}, \dfrac{P \wedge Q}{Q}$	simplification/Deduction
8. $\dfrac{P}{P \vee Q}, \dfrac{Q}{P \vee Q}$	Addition.

NOTE: $P \to Q \Leftrightarrow \sim Q \to \sim P$ (contra positive)

problems

Q1. Demonstrate that S is a valid inference from the premise $P \to \sim Q$, $Q \vee R$, $\sim S \to P$ and $\sim R$

S.No	Derivation	Name
1.	$Q \vee R$	premise
2.	$\sim R$	premise
3.	Q	disjunctive syllogism of 1,2
4.	$P \to \sim Q$	premise
5.	$\sim(\sim Q)$	double negation of 3
6.	$\sim P$	Modus tollens of 4,5
7.	$\sim S \to P$	premise
8.	$\sim(\sim S)$	Modus tollens of 6,7
9.	S	double negation of 8.

Q2. Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$, $(A \wedge \sim B) \to (R \vee S)$

S.No.	Derivation	Name
1.	$C \vee D$	premise
2.	$(C \vee D) \to \sim H$	premise
3.	$\sim H$	Modus ponens of 1,2
4.	$\sim H \to (A \wedge \sim B)$	premise
5.	$A \wedge \sim B$	Modus ponens of 3,4.
6.	$(A \wedge \sim B) \to (R \vee S)$	premise
7.	$R \vee S$	Modus ponens of 5,6.

Q3. show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$ and $\sim M$.

S.No.	Derivation	Name
1.	$P \to M$	premise
2.	$\sim M$	premise
3.	$\sim P$	Modus tollens of 1,2
4.	$P \vee Q$	premise
5.	Q	Disjunctive syllogism of 3,4.

6.	$Q \to R$	premise
7.	R	modus ponens of 6,5
8.	$P \vee Q$	premise
9.	$R \wedge (P \vee Q)$	Addition / Equivalence 4,7,8

Q4. Show that $P \to Q$, $R \to S$, $Q \to T$, $S \to U$, $\sim(T \wedge U)$, $P \to R$ gives $\sim P$.

S.No.	Derivation	Name
1.	$P \to Q$	premise
2.	$Q \to T$	premise
3.	$P \to T$	Hypothetical syllogism of 1,2
4.	$R \to S$	

11/11/22 Conditional proof (CP):

If we can derive S from R and a set of premises then we can derive $R \to S$ from the premises alone this is called CP rule or deduction theorem.

Q1. Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\sim R \vee P$, Q

$P \to (Q \to S)$
$\sim R \vee P$
Q

$R \to S$ ⇒ taking R as additional premise & derivative till we get S

S.No.	Derivation	Name
1.	$\sim R \vee P$	premise
2.	R	additional premise
3.	$R \rightarrow P$	Equivalence of 1
4.	P	modus ponens of 2,3
5.	$P \rightarrow (Q \rightarrow S)$	premise
6.	$Q \rightarrow S$	modus ponens of 4,5
7.	Q	premise
8.	S	modus ponens of 6,7

02 $P \rightarrow S$ can be derived from the premises $\sim P \vee Q$, $\sim Q \vee R$, $R \rightarrow S$

S.No.	Derivation	Name
1.	$\sim P \vee Q$	premise
2.	$P \rightarrow Q$	Equivalence of 1
3.	P	additional premise
4.	Q	modus ponens of 2,3
5.	$\sim Q \vee R$	premise
6.	$Q \rightarrow R$	equivalence of 5
7.	R	modus ponens ~~Equivalence~~ of 4,6
8.	$R \rightarrow S$	premise
9.	S	modus ponens of 7,8.

03. Derive $P \rightarrow (Q \rightarrow S)$ using the rule CP if neccessary from $P \rightarrow (Q \rightarrow R)$, $Q \rightarrow (R \rightarrow S)$

S.No.	Derivation	Name
1.	$P \rightarrow (Q \rightarrow R)$	premise
2	P	additional premise
3.	$Q \rightarrow R$	modus ponens of 1,2
4.	$\sim Q \vee R$	Equivalence of 3
5.	$Q \rightarrow (R \rightarrow S)$	premise
6.	$\sim Q \vee (R \rightarrow S)$	Equivalence of 5.
7	$(\sim Q \vee R) \wedge (\sim Q \vee (R \rightarrow S))$	distribution of 4,6.
8	$\sim Q \vee (R \wedge (R \rightarrow S))$	distribution of 7
9.	$\sim Q \vee (R \wedge (\sim R \vee S))$	distribution of 8

10.	$\sim Q \vee ((R \wedge \sim R) \vee (R \wedge S))$	distribution of 9
11.	$\sim Q \vee (F \vee (R \wedge S))$	distribution of 10
12.	$\sim Q \vee (R \wedge S)$	distribution of 11
13.	$(\sim Q \vee R) \wedge (\sim Q \vee S)$	distribution of 12
14.	$\sim Q \vee S$	Deduction of 13
15.	$Q \to S$	Equivalence of 14.

04. Deduce $P \to Q$ from the set of premises $R \to (S \to Q)$, $\sim P \vee R$, S.

S.No.	Derivation	Name
1.	$\sim P \vee R$	premise
2.	P	additional premise
3.	R	Disjunctive syllogism of 1, 2
4.	$R \to (S \to Q)$	premise
5.	$S \to Q$	modus ponens of 3, 4
6.	S	premise
7.	Q	modus ponens of 5, 6

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Indirect method of proof :=

In this method we introduce negation of desired conclusion as new premise from the additional premise and with the given set of premises together derive contradiction (false). Then the new set of premises is inconsistent. This implies that conclusion is true from the given set of premises alone.

01. Use indirect method of proof and derive R from the premises $\sim Q$, $P \to Q$, $P \vee R$

S.No.	Derivation	Name
1.	$\sim Q$	premise
2.	$P \to Q$	premise
3.	$\sim P$	modus tollens of 1, 2

4.	$P \vee R$	premise
5.	R	Disjunctive syllogism of 3,4
6.	$\sim R$	Additional premise
7.	$R \wedge \sim R$	Addition of 5,6
8.	F	

Q2. Derive $P \to \sim S$ from $P \to (Q \vee R), Q \to \sim P, S \to \sim R, P$.

S.No	Derivation	Name
1.	$P \to (Q \vee R)$	premise
2.	P	premise
3.	$Q \vee R$	modus ponens of 1,2
4.	$Q \to \sim R$	premise
5.	$\sim Q \vee \sim P$	Equivalence of 4.
6.	$R \vee \sim P$	Resolution of 3,5
7.	$S \to \sim R$	premise
8.	$\sim S \vee \sim R$	Equivalence of 7
9.	$\sim P \vee \sim S$	Resolution of 6,8.
10.	$P \to \sim S$	Equivalence of 9.

Q3. Solve by method of indirect proof to derive $P \to \sim S$ from $P \to (Q \vee R), Q \to \sim P, S \to \sim R, P$.

$P \to \sim S \Leftrightarrow \sim P \vee \sim S \Leftrightarrow \sim(P \wedge S)$ result. Additional premise $(P \wedge S)$

S.No	Derivation	Name
1.	$P \to (Q \vee R)$	premise
2.	P	premise
3.	$Q \vee R$	modus ponens of 1,2
4.	$P \wedge S$	Additional premise
5.	S	simplification of 4
6.	$S \to \sim R$	premise
7.	$\sim R$	modus ponens of 5,6
8.	Q	Disjunctive syllogism Resolution of 3,7
9.	$Q \to \sim P$	premise
10.	$\sim P$	modus ponens of 9.
11.	P	Simplication of 4
12	$\sim P \wedge P$	Addition of 10,11
13.	F	

Q4. By indirect proof show that $P \to Q, Q \to R, \sim(P \wedge R), P \vee R \Rightarrow R$

S.No.	Derivation	Name
1.	$Q \to R$	premise
2.	$\sim R$	additional premise
3.	$\sim Q$	modus tollens of 1,2
4.	$P \to Q$	premise
5.	$\sim P$	modus tollens of 3,4.
6.	$P \vee R$	premise
7.	R	Disjunction of 5,6
8.	$\sim(P \wedge R)$	premise
	$\sim P \vee \sim R$	
9.	$\sim P$	Disjunctive of 7,9
10.	$P \wedge \sim P$	Addition of 7,9
11.	F	

S.No.	Derivation	Name
1.	$P \vee R$	premise
2.	$\sim R$	Additional premise
3.	P	Disjunctive of 1,2
4.	$P \to Q$	premise
5.	Q	modus ponens of 3,4.
6.	$Q \to R$	premise
7.	R	modus ponens of 5,6
8.	$\sim(P \wedge R)$	premise
9.	$\sim P \vee \sim R$	Demorgan's of 8.
10.	$\sim P$	Disjunction of 7,9
11.	$P \wedge \sim P$	Addition of 7,10.
12.	F	

Q5. Show by indirect proof $P \to Q, R \to S, P \vee R \Rightarrow Q \vee S$.

S.No.	Derivation	Name
	$R \to S$	premise
1.	$\sim R \vee S$	Equivalence of 1
2.	$P \vee R$	premise
3.	$P \vee S$	Resolution of 2,3.
4.	$\sim(Q \vee S)$	additional premise
5.	$\sim Q \wedge \sim S$	Equivalence of 5
6.	$\sim Q$	Simplification of 6
7.	$P \to Q$	premise
8.	$\sim P$	modus tollens of 7,8
9.		

06. $R \to (S \to Q)$, $\sim P \vee R$ and $S \Rightarrow P \to Q$

S.No	Derivation	Name
1.	$\sim P \vee R$	premise
2.	P	additional premise
3.	R	Disjunctive of 1.2
4.	$R \to (S \to Q)$	premise
5.	$S \to Q$	modus ponens of 3.4.
6.	S	premise
7.	Q	modus ponens of 5.6

07. solve by indirect method $E \to S$, $S \to H$, $A \to \sim H \Rightarrow \sim(E \wedge A)$

S.No.	Derivation	Name
1.	$E \to S$	premise
2.	$S \to H$	premise
3.	$E \to H$	Hypothetical syllogism of 1.2
4.	$\sim E \vee H$	Equivalence of 3
5.	$A \to \sim H$	premise
6.	$\sim A \vee \sim H$	Equivalence of 5
7.	$\sim E \vee \sim A$	Resolution of 4.6
8.	$\sim(E \wedge A)$	demorgan Equivalence of 7
9.	$E \wedge A$	Additional premise
10.	F	Addition of 8.9

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01. Test the validity of the following arguments

(i) Siri is watching TV

(ii) If siri is watching TV then she is not studying.

(iii) If she is not studying then her father will not buy her scooty

(iv) Therefore siri's father will not buy a scooty

P: siri is watching TV.

Q: siri is studying

R: siri's father will buy her a scooty

$$\begin{array}{l} P \\ P \to \sim Q \\ \sim Q \to \sim R \\ \hline \sim R \end{array}$$

S.No	Derivation	Name
1.	P	premise
2.	$P \to \sim Q$	premise
3.	$\sim Q$	modus ponens of 1,2
4.	$\sim Q \to \sim R$	premise
5.	$\sim R$	modus ponens of 3,4.

Q2. Verify the validity of the following argument

(i) If there was a ball game, then travelling was difficult.

(ii). If they arrived on time then travelling was not difficult

(iii) They arrived on time

(iv) Therefore no ball game

$\boxed{P \to \sim Q \Leftrightarrow Q \to \sim P \quad \text{contra +ve}}$

P: There was a ball game

Q: The travelling was easy

R: They arrived on time

$$\begin{array}{c} P \to \sim Q \\ R \to Q \\ R \\ \hline \sim P \end{array}$$

S.No	Derivation	Name
1.	$R \to Q$	premise
2.	R	premise
3.	Q	modus ponens of 1,2
4.	$P \to \sim Q$	premise
5.	$\sim(\sim Q)$	double negation of 3
6.	$\sim P$	modus tollens of 4,5

Q2. By using method of derivation show that the following statements constitutes a valid argument

(i). If A works hard then either B or C will enjoy

(ii) If B enjoys then A will not work hard

(iii) If D enjoys then C will not.

(iv) Therefore if A works hard, D will not enjoy

P: A works hard

Q: B enjoys

R: C enjoys

S: D enjoys

$$\begin{array}{c} P \to (Q \vee R) \\ Q \to \sim P \\ S \to \sim R \\ \hline P \to \sim S \end{array}$$

S No	Derivation	Name
1	$P \to (Q \vee R)$	premise
2.	P	cp rule additional premise
3.	$Q \vee R$	modus ponens of 1,2
4.	$Q \to \sim P$	premise
5.	$\sim Q \vee \sim P$	Equivalence of 4
6	$R \vee \sim P$	Resolution of 3,5
7.	$S \to \sim R$	premise
8.	$\sim R \to \sim P$	Equivalence of 6
9	$S \to \sim P$	Hypothetical syllogism of 7,8
10	$\sim(\sim P)$	double negation of 2
11.	$\sim S$	modus tollens of 9,10

Predicate calculas

A statement vari function of one variable consisting of an individual variable and a predicate

→ A statement function becomes a statement if they replace the variable with a name

→ statement function is also called as an open statement

Ex(01): $S(x) = x$ is a student

subject property of subject

(02): $T(x,y) = x$ is taller than y

In general 'n' place predicate requires 'n' names of objects to be inserted in a fixed position in order to obtain a statement.

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Quantifiers: Certain statements involve words that indicate quantity such as all, some, none, one such words are called quantifiers.

All is called universal quantifier and we denote by symbol $(\forall x)$

Some is called existencial quantifier and is deenote by $(\exists x)$

NOTE: If a statement has a word All then it can be represented using $\rightarrow$ symbol (conditional)

If the statement has the word sum then it can be represented using $\wedge$ symbol (conjunction)

Eg: 1) All Birds can fly.

$B(x)$: x is a bird

$F(x)$: x can fly

$\forall x\, [B(x) \rightarrow F(x)]$

2) Some birds can fly

$\exists x\, [B(x) \wedge F(x)]$

3) All monkeys have tails.

$M(x)$: x is a monkey

$T(x)$: x has tail.

$\forall x\, [M(x) \rightarrow T(x)]$

Some monkeys have no tails.

$\exists x\, [M(x) \wedge \overline{T}(x)]$

4) Some people who trust others are rewarded

$P(x)$: x is a person

$T(x)$: x trusts others

$R(x)$: x is rewarded.

$\exists x\, [P(x) \wedge T(x) \wedge R(x)]$

5) someone is teasing

$Q(x)$: x is ~~teasing~~ person

$T(x)$: x is teasing

$\exists x\, [Q(x) \wedge T(x)]$.

Rules of Inference

$$\frac{(\forall x)\, P(x)}{P(t) \text{ for all } t}$$ Universal specification (US)

$$\frac{P(t) \text{ for all } t}{(\forall x)\, P(x)}$$ Universal Generalization (UG)

$$\frac{\exists x \; P(x)}{P(t) \text{ for some } t}$$ Existential specification (ES)

$$\frac{P(t) \text{ for some } t}{\exists x \; P(x)}$$ Existential Generalization (EG)

Q1. Verify the validity of the following argument
Every living thing is a plant or animal.
John's gold fish is alive and it is not a plant.
All animals have hearts.
Therefore Jhon's gold fish has a heart.

g: John's gold fish
$P(x)$: x is a plant
$A(x)$: x is a animal
$H(x)$: x have heart
$x \to$ living thing

$$\forall x\,[P(x) \vee A(x)]$$
$$\sim P(g)$$
$$\frac{\forall x\,[A(x) \to H(x)]}{H(g)}$$

S.No.	Derivation	Reason.
1.	$\forall x\,[P(x) \vee A(x)]$	premise
2.	$P(g) \vee A(g)$	US
3.	$\sim P(g)$	premise
4.	$A(g)$	Disjunctive syllogism of 2,3
5.	$\forall x\,[A(x) \to H(x)]$	premise
6.	$A(g) \to H(g)$	US
7.	$H(g)$	Hypothetical syllogism of 4,6.

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Q2 Verify the validity of the arguments.

Tigers are dangerous animals.

There are Tigers.

Therefore there are dangerous animals.

$T(x)$: x is a tiger

$D(x)$: x is a dangerous animal.

$\forall x\,[T(x) \to D(x)]$

$\exists x\; T(x)$

$\therefore \exists x\; D(x)$

S.No.	Derivation	Reason
1.	$\forall x\,[T(x) \to D(x)]$	premise
2.	$\exists x\; T(x)$	premise
3.	$T(a)$	ES
4.	$T(a) \to D(a)$	US
5.	$D(a)$	modus ponens of 3,4
6.	$\exists x\; D(a)$	EG.

Q3. Given an arguments with which will establish the validity of the following inference

All integers are rational numbers

Some integers are power's of 3

Therefore some rational number are powers of 3.

$I(x)$: x is an integer.

$R(x)$: x is a rational number

$P(x)$: x is a power of 3.

$\forall x\,[I(x) \to R(x)]$

$\exists x\,[I(x) \land P(x)]$ There exists some numbers which are integers and powers of 3

$\therefore \exists x\,[R(x) \land P(x)]$

S.No.	Derivation	Reason
1.	$\exists x[I(x) \land P(x)]$	premise
2.	$I(a) \land P(a)$	ES
3.	$I(a)$	simplification of 2
4.	$P(a)$	simplification of 2
5.	$\forall x[I(x) \to R(x)]$	premise

6.	$I(a) \to R(a)$	US
7.	$R(a)$	modus ponens of 3,6
8.	$P(a) \wedge R(a)$	Conjunction of 4,7
9.	$\exists x[P(x) \wedge R(x)]$	EG.

04 Verify the validity of the following arguments.

All men are mortal.

socrates is a men.

Therefore socrates is mortal.

$M(x)$: x is a men. socrates is a object not a predicate

$O(x)$: x is mortal

s : socrates

$$\begin{array}{l} \forall x[M(x) \to O(x)] \\ M(s) \\ \hline \therefore O(s) \end{array}$$

S.No.	Derivation	Reason
1.	$\forall x[M(x) \to O(x)]$	premise
2.	$M(s) \to O(s)$	US
3.	$M(s)$	premise
4.	$O(s)$	modus ponens of 2,3.

05. If it rains chandu will be sick

It doesnot rain.

Therefore chandu is not sick

P : It is raining

C : chandu is sick

$$\begin{array}{l} p \to c \\ \sim p \\ \hline \therefore \sim c \end{array}$$

S.No.	Derivation	Reason
1.	$p \to c$	premise
2.	$\sim p$	premise
3.	$\sim c$	modus tollens of 1,2

following premise is inconsistent (fol...)
If Gill misses many classes through illness he will fails highschool
If Gill fails highschool then he is uneducated.
If Gill reads lots of books then he is not uneducated.
Therefore Gill misses many classes through illness and reads lots of books

C : Gill misses many classes through illness
F : Gill fails his highschool.
U : Gill is uneducated.
B : Gill reads lots of books

$$\begin{array}{l} C \to F \\ F \to U \\ B \to \sim U \\ \hline C \wedge B \end{array}$$

S.No	Derivation	Reason
1.	$C \to F$	premise
2.	$F \to U$	premise
3.	$C \to U$	Hypothetical syllogism of 1,2
4.	$B \to \sim U$	premise
5.	$\sim C \to \sim U$ $\sim U \to \sim C$	Contrapositive of 3.
6.	$B \to \sim C$	Hypothetical syllogism of 4,5.
7.	$\sim B \vee \sim C$	Equivalence of 6
8.	$\sim (B \wedge C)$	Demorgan's of 7.

which is inconsistent