UNIT-1
Regular Expressions(PART-2)

Regular Expressions vs. Finite Automata

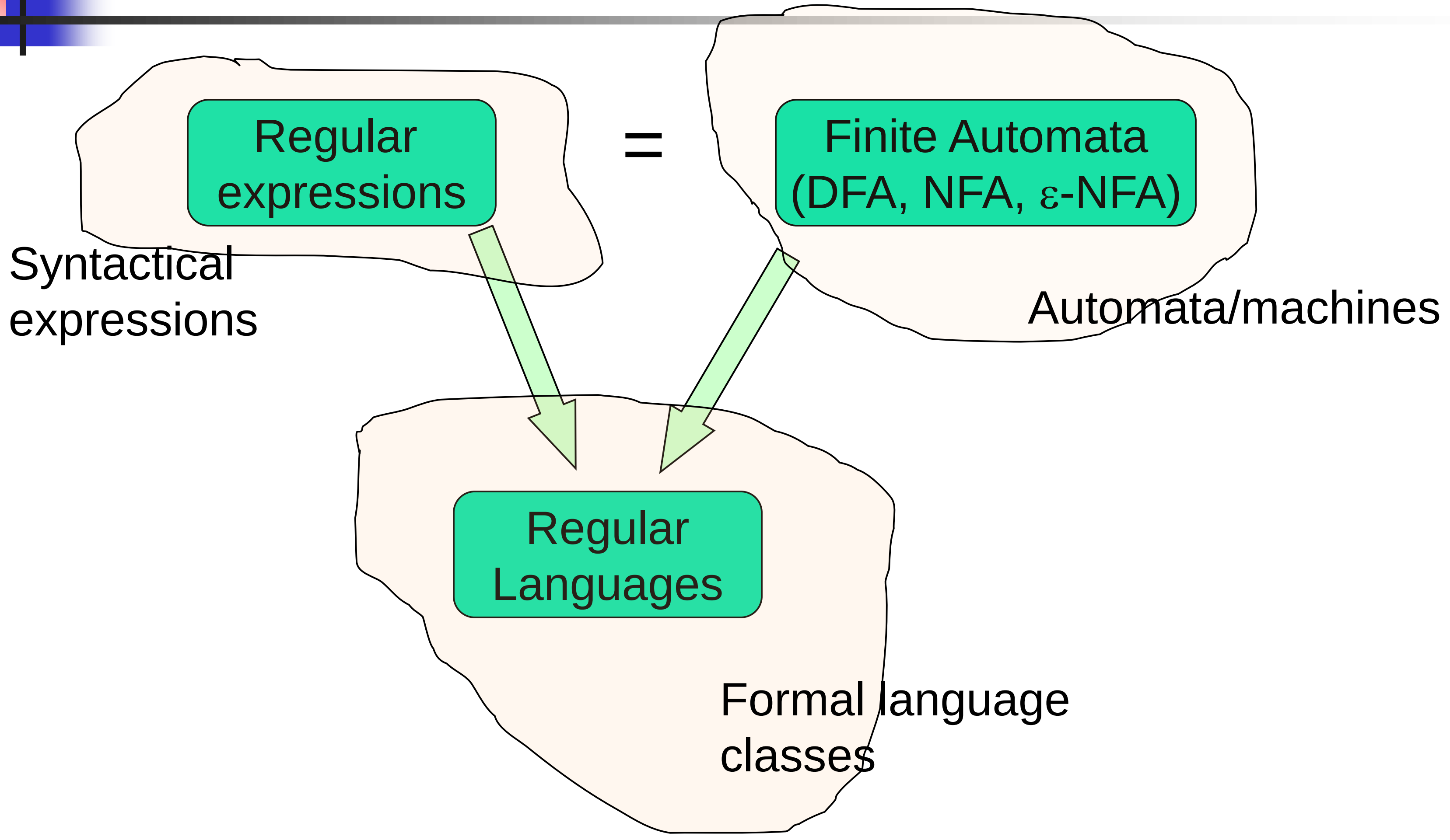
- Offers a declarative way to express the pattern of any string we want to accept
  - E.g., 01*+ 10*

- Automata => more machine-like

  < input: string , output: [accept/reject] >
- Regular expressions => more program syntax-like

- Unix environments heavily use regular expressions
  - E.g., bash shell, grep, vi & other editors, sed
- Perl scripting – good for string processing
- Lexical analyzers such as Lex or Flex

Regular Expressions



Language Operators

- Union of two languages:
  - **L U M** = all strings that are either in L or M
  - Note: A union of two languages produces a third language

- Concatenation of two languages:
  - **L . M** = all strings that are of the form *xy* s.t., $x \in L$ and $y \in M$
  - The *dot* operator is usually omitted
    - i.e., **LM** is same as L.M

“i” here refers to how many strings to concatenate from the parent language L to produce strings in the language L^i

Kleene Closure (the * operator)

- Kleene Closure of a given language L:
  - $L^0 = \{\varepsilon\}$
  - $L^1 = \{w \mid \text{for some } w \in L\}$
  - $L^2 = \{ w_1w_2 \mid w_1 \in L, w_2 \in L \text{ (duplicates allowed)}\}$
  - $L^i = \{ w_1w_2\ldots w_i \mid \text{all } w\text{'s chosen are} \in L \text{ (duplicates allowed)}\}$
  - (Note: the choice of each w_i is independent)
  - **$L^* = \bigcup_{i\geq 0} L^i$** (arbitrary number of concatenations)

Example:

- Let L = { 1, 00}
  - $L^0 = \{\varepsilon\}$
  - $L^1 = \{1,00\}$
  - $L^2 = \{11,100,001,0000\}$
  - $L^3 = \{111,1100,1001,10000,000000,00001,00100,0011\}$
  - **$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots$**

Kleene Closure (special notes)

- L^* is an infinite set iff $|L| \geq 1$ and $L \neq \{\varepsilon\}$ **Why?**
- If $L=\{\varepsilon\}$, then $L^* = \{\varepsilon\}$ **Why?**
- If $L = \Phi$, then $L^* = \{\varepsilon\}$ **Why?**

Σ^* denotes the set of all words over an alphabet Σ

- Therefore, an abbreviated way of saying there is an arbitrary language L over an alphabet Σ is:
  - $L \subseteq \Sigma^*$

Building Regular Expressions

- Let E be a regular expression and the language represented by E is L(E)
- Then:
  - (E) = E
  - L(E + F) = L(E) U L(F)
  - L(E F) = L(E) L(F)
  - L(E*) = (L(E))*

Example: how to use these regular expression properties and language operators?

- ***L = { w | w is a binary string which does not contain two consecutive 0s or two consecutive 1s anywhere)***
  - E.g., w = 01010101 is in L, while w = 10010 is not in L
- Goal: Build a regular expression for L
- Four cases for w:
  - Case A: w starts with 0 and |w| is even
  - Case B: w starts with 1 and |w| is even
  - Case C: w starts with 0 and |w| is odd
  - Case D: w starts with 1 and |w| is odd
- Regular expression for the four cases:
  - Case A: (01)*
  - Case B: (10)*
  - Case C: 0(10)*
  - Case D: 1(01)*
- Since L is the union of all 4 cases:
  - Reg Exp for L = (01)* + (10)* + 0(10)* + 1(01)*
- If we introduce ε then the regular expression can be simplified to:
  - Reg Exp for L = (ε +1)(01)*(ε +0)

Precedence of Operators

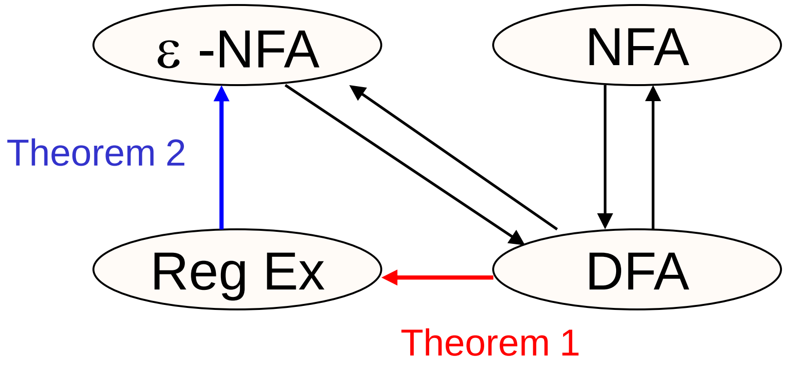
- Highest to lowest
  - * operator (star)
  - .   (concatenation)
  - + operator

- Example:
  - 01* + 1   =   (0 . ((1)*)) + 1

Finite Automata (FA) & Regular Expressions (Reg Ex)

- To show that they are interchangeable, consider the following theorems:
  - *Theorem 1:* For every DFA A there exists a regular expression R such that L(R)=L(A)
  - *Theorem 2:* For every regular expression R there exists an ε -NFA E such that L(E)=L(R)

Proofs in the book



Kleene Theorem

Algebraic Laws of Regular Expressions

- Commutative:
  - $E+F = F+E$
- Associative:
  - $(E+F)+G = E+(F+G)$
  - $(EF)G = E(FG)$
- Identity:
  - $E+\Phi = E$
  - $\varepsilon E = E \varepsilon = E$
- Annihilator:
  - $\Phi E = E\Phi = \Phi$

Algebraic Laws…

- Distributive:
  - $E(F+G) = EF + EG$
  - $(F+G)E = FE+GE$
- Idempotent: $E + E = E$
- Involving Kleene closures:
  - $(E^*)^* = E^*$
  - $\Phi^* = \varepsilon$
  - $\varepsilon^* = \varepsilon$
  - $E^+ = EE^*$
  - $E? = \varepsilon + E$

True or False?

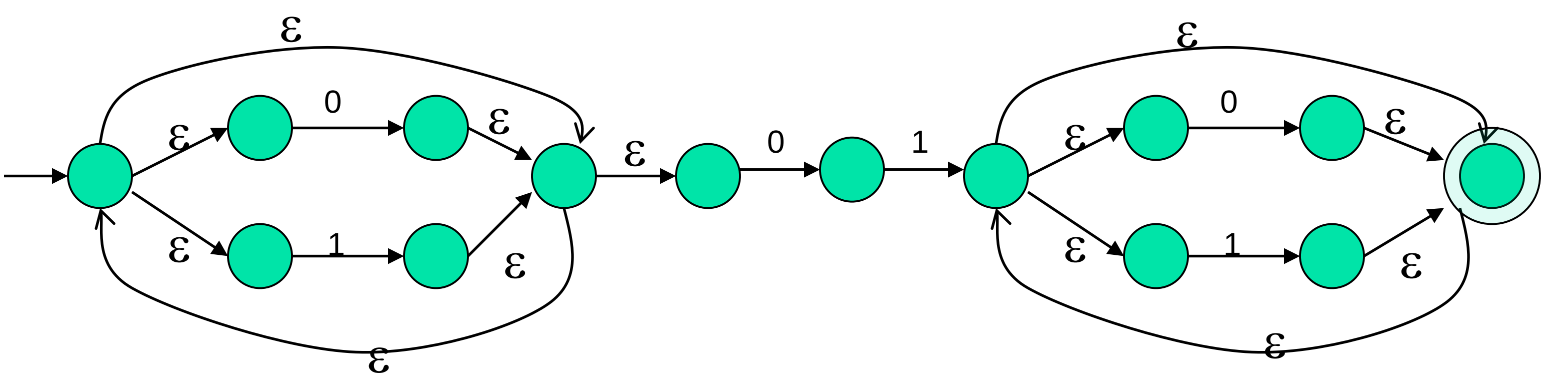
Let R and S be two regular expressions. Then:

1. $((R^*)^*)^* = R^*$?
2. $(R+S)^* = R^* + S^*$?
3. $(RS + R)^* RS = (RR^*S)^*$?

RE to ε-NFA construction

Example: (0+1)*01(0+1)*

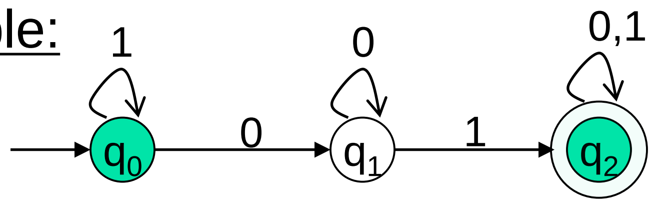
(0+1)* 01 (0+1)*



DFA to RE construction

Informally, trace all distinct paths (traversing cycles only once) from the start state to *each of the* final states and enumerate all the expressions along the way

Example:



$$\underbrace{(1^*)}_{1^*}\ \underbrace{0\ \ (0^*)}_{00^*}\ \underbrace{1}_{1}\ \underbrace{(0 + 1)^*}_{(0+1)^*}$$

$$1^*00^*1(0+1)^*$$

Q) What is the language?

Summary

- Regular expressions
- Equivalence to finite automata
- DFA to regular expression conversion
- Regular expression to ε-NFA conversion
- Algebraic laws of regular expressions
- Unix regular expressions and Lexical Analyzer

ε-clo

Regular Expressions & language :-

The language recognized by finite automata is called a regular language and these languages are described by regular expressions (RE)

→ RE are ~~must~~ used to represent certain set of strings in some algebraic manner.

→ RE are used by text editors, utilities & programming languages to search and manipulate text based on patterns.

→ They are used as i/p language for many systems like lexical analyzer generator such as lex or flex.

operators of regular expressions:-

There are three operators supported by RE

1. Union of 2 languages L_1 & L_2 is denoted by $L_1 \cup L_2$ is the set of string that are in either L_1 or L_2 or both.

2. Concatenation of 2 languages L_1 & L_2 is denoted by $L_1 \cdot L_2$

3. Closure (or) star (or) Kleen Closure of L is denoted by L^*

It represents the set of strings that can be formed by taking any number of strings from L possibly with repetations and concatenating all of them.

4. There is an additional operator called +ve closure of a language denoted by L^+ that represents the set of strings that can be formed by taking any no. of strings from L possibly with repetitions and concatenating all of them excluding the null string.

i.e., $L^+ = L^* - \epsilon$

→ If $L_1 = \{001, 10, 111\}$ $L_2 = \{\epsilon, 001\}$ find (i) $L_1 \cup L_2$

(ii) $L_1 \cdot L_2$

$L_1 \cup L_2 = \{\epsilon, 001, 10, 111\}$

$L_1 \cdot L_2 = \{001, 001001, 10, 10001, 111, 111001\}$

Definition of Regular Expression (RE):-

The class of RE over Σ is defined recursive as follows:-

1. The letter ϕ & ϵ are RE over L.
2. Every letter $a \in \Sigma$ is a RE over Σ.
3. If r_1 & r_2 are RE over Σ then so are (r_1+r_2), $r_1 \cdot r_2$, r_1^*, r_1^+, r_2^*, r_2^+ are also RE over Σ;

In General, $\boxed{R^+ = R \cdot R^*}$

Identity rules for regular expressions (RE):-

1. $\phi + R = R$.
2. $\phi R = R\phi = \phi$
3. $\epsilon R = R\epsilon = R$
4. $\epsilon^* = \epsilon$
5. $\phi^* = \epsilon$.
6. $R+R = R$.
7. $R^* \cdot R^* = R^*$
8. $RR^* = R^* \cdot R = R^+$
9. $(R^*)^* = R$.
10. $\epsilon + R \cdot R^* = R^* = \epsilon + R^* R = \cancel{\epsilon R^+}\ \epsilon + R^+$
11. $(PQ)^* P = P(QP)^*$
12. $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$.
13. $(P+Q)R = PR + QR$
14. $R(P+Q) = RP + RQ$

ARDEN'S THEOREM:-

Let P & Q be two RE's over Σ if 'P' does not contain ϵ then the following Equation $R = Q + RP$ has a unique solution given by $R = QP^*$

$\Rightarrow$ Prove that $(1+00^*1) + (1+00^*1)\cdot(0+10^*1)^*(0+10^*1)$
$= 0^*1(0+10^*1)^*$

Sol:- L.H.S:-

$(1+00^*1) + (1+00^*1)\cdot(0+10^*1)^*(0+10^*1)$

$(1+00^*1)(\epsilon + (0+10^*1)^*(0+10^*1))$ $[\epsilon + R\cdot R^* = R^*]$

$(1+00^*1)(0+10^*1)^*$

$(\epsilon+00^*)(1)(0+10^*1)^*$

$0^*1(0+10^*1)^*$

$\Rightarrow$ Write RE for following languages over $\{0,1\}^*$

(i) The set of all strings such that no. of 0's are odd is given by $\{1^*(00)^n 0 \cdot 1^* / n \geq 0\}$

Sol:- $\{1^*(00)^n 0\cdot 1^* / n \geq 0\}$

(ii) The set of all strings where 10th symbol from the right end is 1.

Sol:- $(0+1)^* 1 (0+1)^9$

(iii) The set of all strings that do not contain 1101

Sol:- $0^*10^*10^*01^*10^*$

(iv) The set of all strings of 0's & 1's not containing 101 as substring.

Sol:- RE is $0^*10^*00^*10^*$

(v) The set of strings of 0's & 1's whose no. of 0's a[illegible] divisible by 5 and no of 1's are even.

Sol:- RE is $(00000)^n (11)^n / n > 0$.

→ Describe the following set by RE

(i) The set of all strings of 0's & 1's beginning with 00

Sol:- $00(0+1)^*$

(ii) The set of all strings of 0's & 1's beginning with 1 and ending with 00.

Sol:- $1(0+1)^*00$

(iii) $L = \{w / w \in \Sigma^*, w$ has atleast one pair of consecutive 0's$\}$

Sol:- $(0+1)^*00(0+1)^*$

(iv) Find a RE for each of the following languages over $\Sigma = \{a, b\}$

(i) $L_1 = \{a^m b^n, m, n > 0\}$ = RE is $a \cdot a^* \cdot b \cdot b^* = a^+ b^+$

(ii) $L_2 = \{b^m a b^n, m, n > 0\}$ = RE is $b \cdot b^* a \, b \cdot b^* = b^+ a b^+$

(iii) $L_3 = \{(ab)^m, m > 0\}$ = RE is $(ab)(ab)^* = (ab)^+$

$$\boxed{\begin{aligned} L_1 &= \{a^m, m > 0\} = a \cdot a^* \\ L_2 &= \{b^n, n > 0\} = b \cdot b^* \end{aligned}}$$

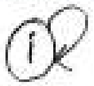
Conversion from DFA to RE:-

If $L=L(A)$ for some DFA then there is a RE 'R' said that $L=L(R)$.

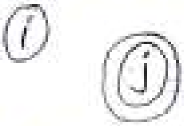
proof

(i) Basis:- If $K=0$ then there is an arc from node i to node j. The transition is represented by $R_{ij}(0)=a_1+a_2+\cdots+a_i$.

→ If there is an arc from node i itself then the transition is given by $R_{ij}(0)=a$



→ If there are no arcs from node i to node j then the transition is given by $R_{ij}=\phi$



→ If $i=j$ then $R_{ij}(0)=\epsilon+a_1+a_2+\cdots+a_i$

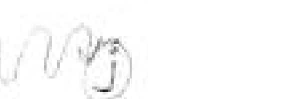
If $i\neq j$ then $R_{ij}(0)=a_1+a_2+\cdots+a_i$

Induction/hypothesis step:-

If there is a path from node i to node j then the following cases may occur

1. If the path doesn't go through state 'K' then

$R_{ij}(K)=R_{ij}(K-1)$



2. If the path goes through state atleast once then break the path into several pieces.

$R_{iK}(K-1)$ $R_{KK}(K-1)^*$ $R_{Kj}(K-1)$

→ By combining the two possible paths going through K and the one that does not go through K is given by the RE for the given automata as

$$R_{ij}(K)=R_{ij}(K-1)+R_{iK}(K-1)R_{KK}(K-1)^*R_{jK}(K-1)$$

→ The RE for the language and corresponding automata is the sum of all expressions $R_{ij}^{(n)}$ such that i is the starting state 'j' is ~~called~~ the accepting / final state, n = total no of states.

Q) convert the following DFA to RE

Sol:- $R_{ij}^{(n)}$ here $i=1$; $j=2$; $n=2$

$\therefore R_{12}^{(2)} = ?$

$\epsilon + RR^* = R^*$

$\phi + R = R$

$\phi \cdot R = \phi$

$\epsilon + R = R$

~~$RR^* = R^+$~~

$(\epsilon + R)^* = R^*$

K=0

	1	2
1	$R_{11}^{(0)} = \epsilon + 1$	$R_{12}^{(0)} = 0$
2	$R_{21}^{(0)} = \phi$	$R_{22}^{(0)} = \epsilon + 0 + 1$

K=1

	1	2
1	$R_{11}^{(1)} = 1 \cdot 1^*$	$R_{12}^{(1)} = 0 \cdot 1^*$
2	$R_{21}^{(1)} = \phi$	$R_{22}^{(1)} = \epsilon + 0 + 1$

K=2

	1	2
1	$R_{11}^{(2)} = 1 \cdot 1^*$	$R_{12}^{(2)} = 0 \cdot 1^*(0+1)^*$
2	$R_{21}^{(2)} = \phi$	$R_{22}^{(2)} = (0+1)(0+1)^*$

$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} R_{11}^{(0)*} R_{11}^{(0)}$

$= (\epsilon+1) + (\epsilon+1)(\epsilon+1)^*(\epsilon+1)$

$= (\epsilon+1)\left[\epsilon + (\epsilon+1)^*(\epsilon+1)\right]$

$= (\epsilon+1)(\epsilon+1)^*$

$= (\epsilon+1)^+ = 1\,1^*$

$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)} R_{11}^{(0)*} R_{12}^{(0)}$

$= 0 + (\epsilon+1)(\epsilon+1)^* 0$

$= 0\left[\epsilon + (\epsilon+1)(\epsilon+1)^*\right]$

$= 0(\epsilon+1)^* \Rightarrow 0 \cdot 1^*$

$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)} R_{11}^{(0)*} R_{11}^{(0)}$

$= \phi + \phi(\epsilon+1)^*(\epsilon+1)$

$= \phi\left[\epsilon + (\epsilon+1)^*(\epsilon+1)\right]$

$= \cancel{\phi \left[\epsilon + \epsilon\right.} = \phi(\epsilon+1)^*$

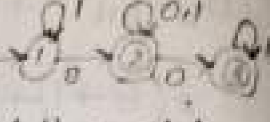
$= \phi \cdot 1^* \Rightarrow \phi$

$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} R_{11}^{(0)*} R_{12}^{(0)}$

$= (\epsilon+0+1) + \phi(\epsilon+1)^* 0$

$= (\epsilon+0+1) + \phi$

$= (\epsilon+0+1)$

g:- 

If three states are present

RE is $R_{12}^{(3)} + R_{13}^{(3)}$

$$R_{11}^{(2)} = R_{11}^{(1)} + R_{12}^{(1)} R_{22}^{(1)*} R_{21}^{(1)}$$
$$= 1.1^* + 0.1^* (\epsilon+0+1)^* \phi$$
$$= 1.1^* + \phi$$
$$= 1.1^*$$

$$R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} R_{22}^{(1)*} R_{22}^{(1)}$$
$$= 0.1^* + 0.1^* (\epsilon+0+1)^* (\epsilon+0+1)$$
$$= 0.1^* [\epsilon + (\epsilon+0+1)^* (\epsilon+0+1)]$$
$$= 0.1^* (\epsilon+0+1)^*$$
$$= 0.1^* (0+1)^*$$

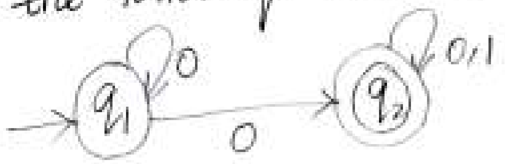
$$R_{21}^{(2)} = R_{21}^{(1)} + R_{22}^{(1)} R_{22}^{(1)*} R_{21}^{(1)}$$
$$= \phi + (\epsilon+0+1)(\epsilon+0+1)^* \phi$$
$$= \phi + \phi$$
$$= \phi$$

$$R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)} R_{22}^{(1)*} R_{22}^{(1)}$$
$$= (\epsilon+0+1) + (\epsilon+0+1)(\epsilon+0+1)^* (\epsilon+0+1)$$
$$= (\epsilon+0+1)[\epsilon + (\epsilon+0+1)(\epsilon+0+1)^*]$$
$$= (\epsilon+0+1)(\epsilon+0+1)^*$$
$$= (0+1)(0+1)^*$$

∴ The RE for the given automata is given by :

$$\boxed{R_{12}^{(2)} = 0\cdot 1^*(0+1)^*}$$

Q) Convert the following DFA to RE



Sol:- $R_{ij}^{(n)}$ here $i=1$; $j=2$; $n=2$.

∴ $R_{12}^{(2)} = ?$

$k=0$

	1	2
1	$R_{11}^{(0)} = \epsilon+0$	$R_{12}^{(0)} = 0$
2	$R_{21}^{(0)} = \phi$	$R_{22}^{(0)} = \epsilon+0+1$

$K=1$

	1	2
1	$R_{11}^{(1)} = 0\cdot 0^*$	$R_{12}^{(1)} = 0\cdot 0^*$
2	$R_{21}^{(1)} = \phi$	$R_{22}^{(1)} = \epsilon+0+1$

$K=2$

	1	2
1	$R_{11}^{(2)} = 0\cdot 0^*$	$R_{12}^{(2)} = 0\cdot 0^*(0+1)^*$
2	$R_{21}^{(2)} = \phi$	$R_{22}^{(2)} = (0+1)(0+1)^*$

$$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} R_{11}^{(0)*} R_{11}^{(0)}$$
$$= \epsilon+0 + [\epsilon+0](\epsilon+0)^* [\epsilon+0)$$
$$= \epsilon+0 \, [\epsilon + (\epsilon+0)^*(\epsilon+0)]$$
$$= \epsilon+0 \, (\epsilon+0)^*$$
$$= 0 \cdot 0^*$$

$$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)} R_{11}^{(0)*} R_{12}^{(0)}$$
$$= 0 + (\epsilon+0)(\epsilon+0)^* 0$$
$$= 0 [\epsilon + (\epsilon+0)(\epsilon+0)^*]$$
$$= 0 \cdot (\epsilon+0)^*$$
$$= 0 \cdot 0^*$$

$$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)} R_{11}^{(0)*} R_{11}^{(0)}$$
$$= \phi + \phi \, (\epsilon+0)^* (\epsilon+0)$$
$$= \phi \, [\epsilon + (\epsilon+0)^* (\epsilon+0)]$$
$$= \phi \, (\epsilon+0)^*$$
$$= \phi$$

$$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} R_{11}^{(0)*} R_{12}^{(0)}$$
$$= (\epsilon+0+1) + \phi \, (\epsilon+0)^* \cdot 0$$
$$= (\epsilon+0+1) + \phi$$
$$= (\epsilon+0+1)$$

$$R_{11}^{(2)} = R_{11}^{(1)} + R_{12}^{(1)} R_{22}^{(1)*} R_{21}^{(1)}$$

$$= 0.0^* + 0.0^* (\epsilon+0+1)^* \phi$$

$$= 0.0^* + \phi$$

$$= 0.0^*$$

$$R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} R_{22}^{(1)*} R_{22}^{(1)}$$

$$= 0.0^* + (0.0^*)(\epsilon+0+1)^* (\epsilon+0+1)$$

$$= 0.0^* [\epsilon + (\epsilon+0+1)^* (\epsilon+0+1)]$$

$$= 0.0^* (\epsilon+0+1)^*$$

$$= 0.0^* (0+1)^*.$$

$$R_{21}^{(2)} = R_{21}^{(1)} + R_{22}^{(1)} R_{22}^{(1)*} R_{21}^{(1)}$$

$$= \phi + (\epsilon+0+1)(\epsilon+0+1)^* . \phi$$

$$= \phi + \phi$$

$$= \phi$$

$$R_{22}^{(2)} = R_{22}^{(1)} + R_{22}^{(1)} R_{22}^{(1)*} R_{22}^{(1)}$$

$$= (\epsilon+0+1) + (\epsilon+0+1)(\epsilon+0+1)^* (\epsilon+0+1)$$

$$= (\epsilon+0+1) [\epsilon + (\epsilon+0+1)(\epsilon+0+1)^*]$$

$$= (\epsilon+0+1)(\epsilon+0+1)^*.$$

$$= (0+1)(0+1)^*.$$

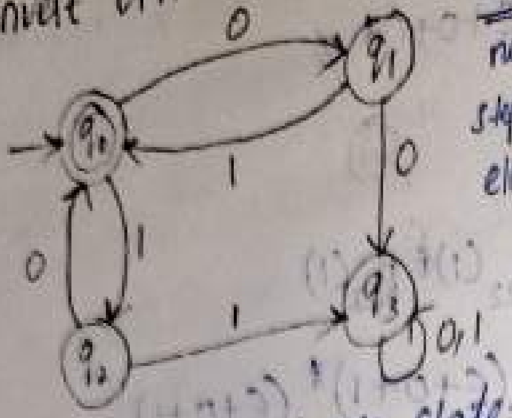
∴ The RE for the given automata is given by.

$$\boxed{R_{12}^{(2)} = 0.0^* (0+1)^*}$$

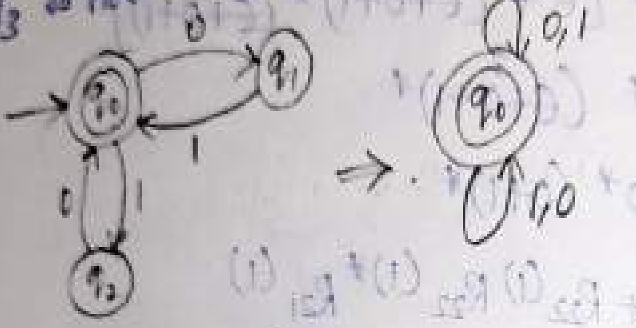
1) Convert DFA to RE by eliminating States

Step ①: Remove any state which is not reachable.

Step ②: If we simplify further by eliminating q_1 & q_2 states.



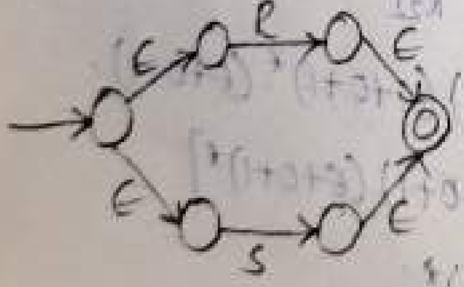
Sol:- q_3 is not reachable state so, eliminate it.



$RE = (01+10)^*$

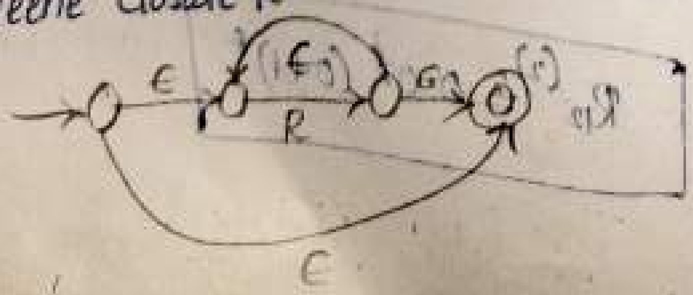
★ Convert RE to Automata:-

1. Union R+S (or) R/S

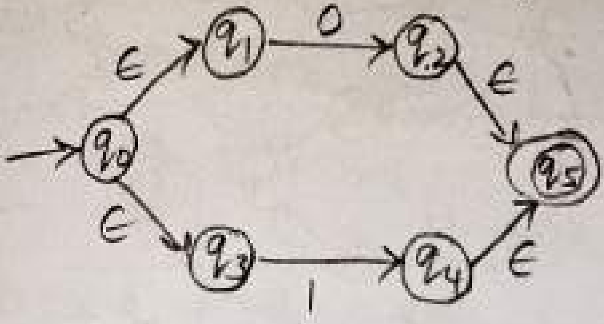


2. Concatenation R·S

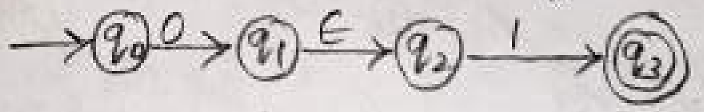
3. Kleene closure R*



Q) Draw NFA with ε for the given RE = (0+1)



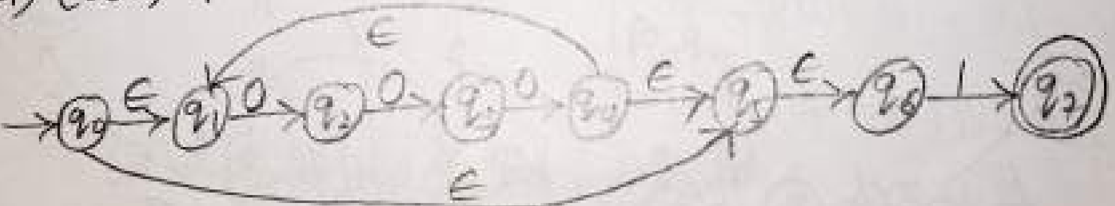
Q) Draw NFA with ε for the given RE = 01.



Q) Draw NFA with ε for the given RE = 0^*

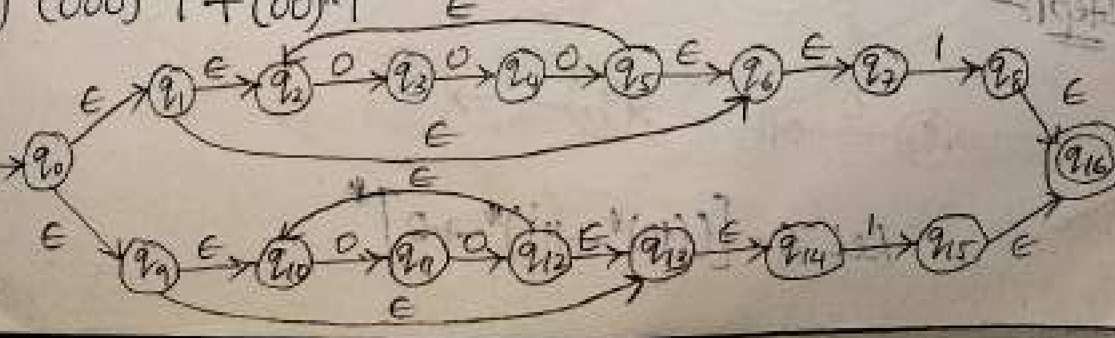
Q) Draw NFA with ε for the given RE = 1^*.

Q) $(000)^*1$

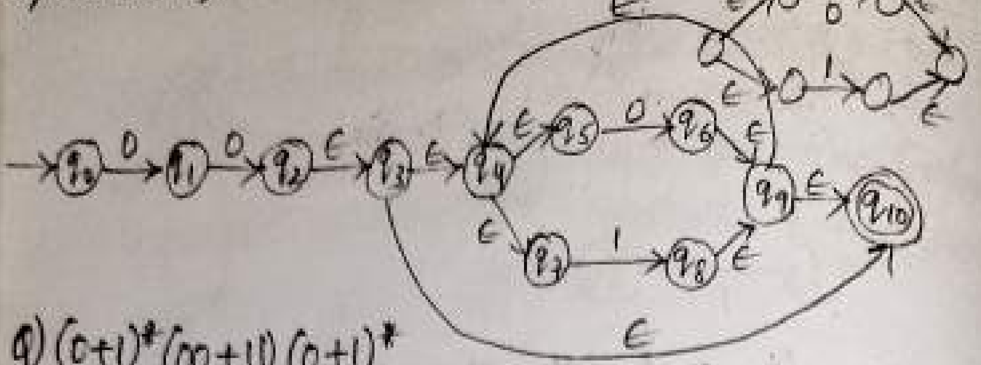


Q) $(00)^*1$

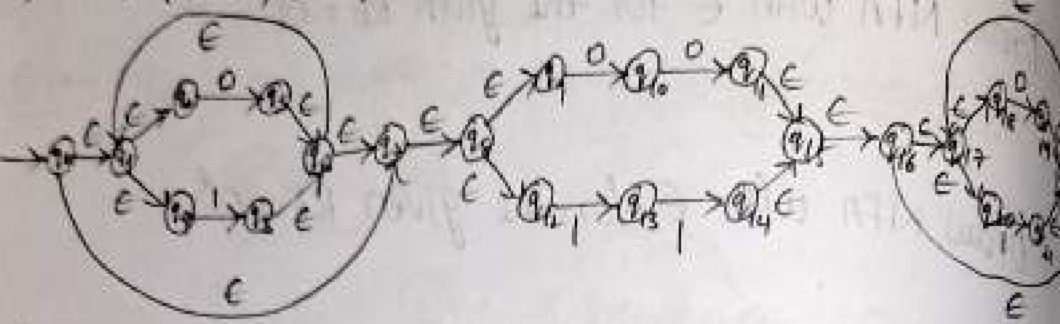
Q) $(000)^*1 + (00)^*1$



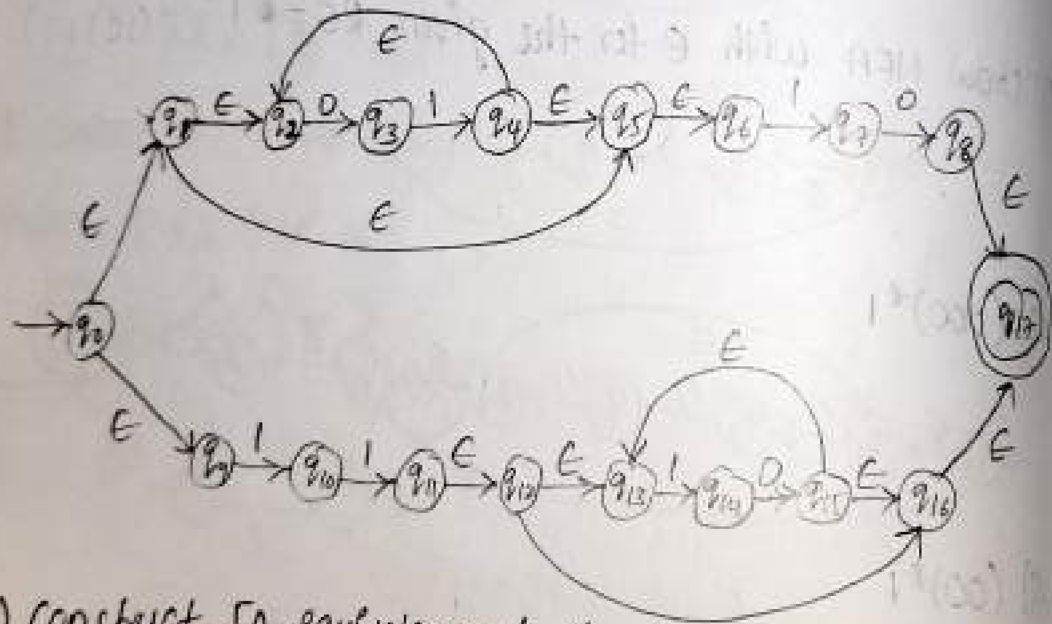
Q) $00(0+1)^*$



Q) $(0+1)^*(00+11)(0+1)^*$



Q) $(01)^*10+11(10)^*$

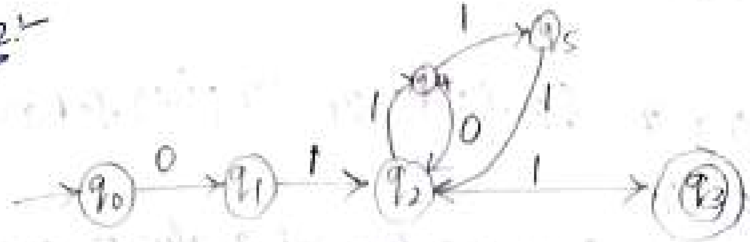


Q) Construct FA equivalence to the follõwing RE given by.
$r = 01[((10)^*+111)^*+0]^*1$

Step1:-

Let $S = [((10)^*+111)^*+0]^*$

step 2:-



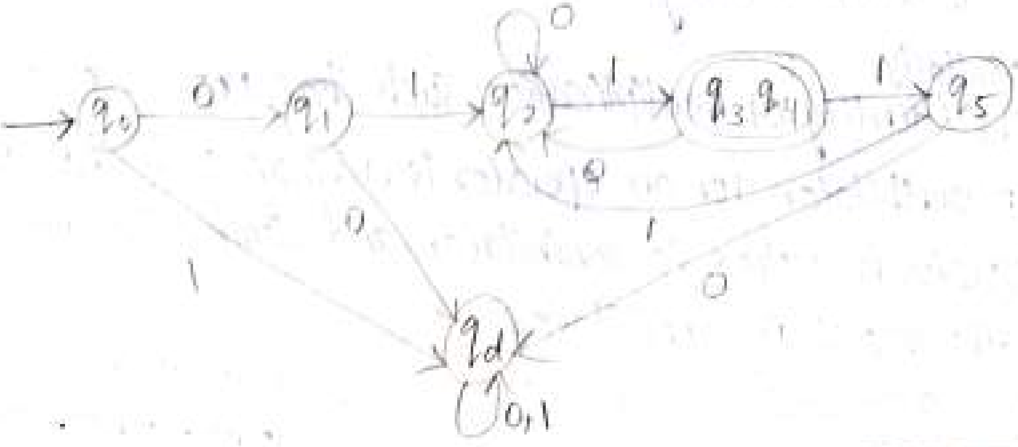
step 3:-

δ	0	1
→ q_0	q_1	ϕ
q_1	ϕ	q_2
q_2	q_2	$\{q_3, q_4\}$
* q_3	ϕ	ϕ
q_4	q_2	q_5
q_5	ϕ	q_2

This is NFA transition table

step 4:- since it is NFA convert it to DFA

δ	0	1
→ q_0	q_1	q_d
q_1	q_d	q_2
q_2	q_2	$[q_3 q_4]$
* $[q_3 q_4]$	$q_2 q_d$	$q_5 q_d$
q_5	q_d	q_2
q_d	q_d	q_d



Applications of regular Expressions:-

RE's are used in the fields of compiler & natural languages of compiler.

→ Two important applications are lexical analyzer and text search.

→ Basically RE is used in designing UNIX, lexical analyser, finding patterns in text

Algebraic laws of RE:-

1. Associativity and commutativity:-

→ Associative law of multiplication is given by

$$(x*y)*z = x*(y*z)$$

→ Commutative law of addition is given by $P+Q = Q+P$

→ Associative law of union ⇒ $(P+Q)+R = P+(Q+R)$

→ Associative law of concatenation ⇒ $(LM)N = L(MN)$

NOTE:- $LM \neq ML$ (concatenation is not commulative).

2. Identities and Annihilators:-

An identity for an operator is a value such that when the operator is applied to the identity and some other value the result is the other value.

for eg:- 0 is the identity of addition such that $0+x = x+0 = x$.

similarly,

1 is the identity of multiplication such that $1*x = x*1 = x$

→ An annihilator for an operator is a value such that when the operator is applied to annihilator and some other value then the result is annihilator.

for eg:- 0 is annihilator for multiplication such that $0*x = x*0 = 0$.

→ There is no annihilator for addition.

→ There are other 3 laws for RE.

1) $\phi + L = L + \phi = L$.

This law states that 'ϕ' is the identity for union.

2) $\epsilon L = L \cdot \epsilon = L$ where 'ϵ' is the identity for concatenation

3) $\phi \cdot L = L \cdot \phi = \phi$ where 'ϕ' is the annihilator for concatenation

3. Distributive law:-

A distributive law involves 2 operators and states one operator can be pushed down to be applied to each argument of the other operator individually.

Eg:- Distributive law of multiplication over addition is given by $x * (y+z) = x*y + x*z$

~~Distributive law of addition over multiplication is given by~~

$L(M+N) = LM + LN$ → left distributive law of concatenation over union

$(M+N)L = ML + NL$ → right distributive law of concatenation over union.

→ concatenation is not commutative.

4. Idempotent law:-

An operator is said to be idempotent if the result of applying it 2 of the same value as arguments is that value.

Eg:- $0+0=0$

$1 \cdot 1 = 1$

→ union & intersections are common examples of idemponent operators.

$L+L=L$ is the idempotent law for union.

5. <u>Laws involving closures</u>:-

(i) $(L^*)^* = L^*$

(ii) $\emptyset^* = \epsilon$

(iii) $\epsilon^* = \epsilon$

(iv) $L^+ = L \cdot L^* = L^* \cdot L$

(v) $L^* = L^+ + \epsilon$

(vi) $L? = \epsilon + L$

<u>NOTE</u>: + and ? are UNIX-style variants.

UNIT-1
Finite Automata

Finite Automaton (FA)

- Informally, a state diagram that comprehensively captures all possible states and transitions that a machine can take while responding to a stream or sequence of input symbols
- Recognizer for “Regular Languages”

- Deterministic Finite Automata (DFA)
  - The machine can exist in only one state at any given time
- Non-deterministic Finite Automata (NFA)
  - The machine can exist in multiple states at the same time

Deterministic Finite Automata - Definition

- A Deterministic Finite Automaton (DFA) consists of:
  - Q ==> a finite set of states
  - ∑ ==> a finite set of input symbols (alphabet)
  - q_0 ==> a start state
  - F ==> set of accepting states
  - δ ==> a transition function, which is a mapping between Q x ∑ ==> Q
- A DFA is defined by the 5-tuple:
  - {Q, ∑ , q_0,F, δ }

What does a DFA do on reading an input string?

- Input: a word w in Σ^*
- Question: Is w acceptable by the DFA?
- Steps:
  - Start at the "start state" q_0
  - For every input symbol in the sequence w do
    - Compute the next state from the current state, given the current input symbol in w and the transition function
  - If after all symbols in w are consumed, the current state is one of the accepting states (F) then *accept w;*
  - Otherwise, *reject w.*

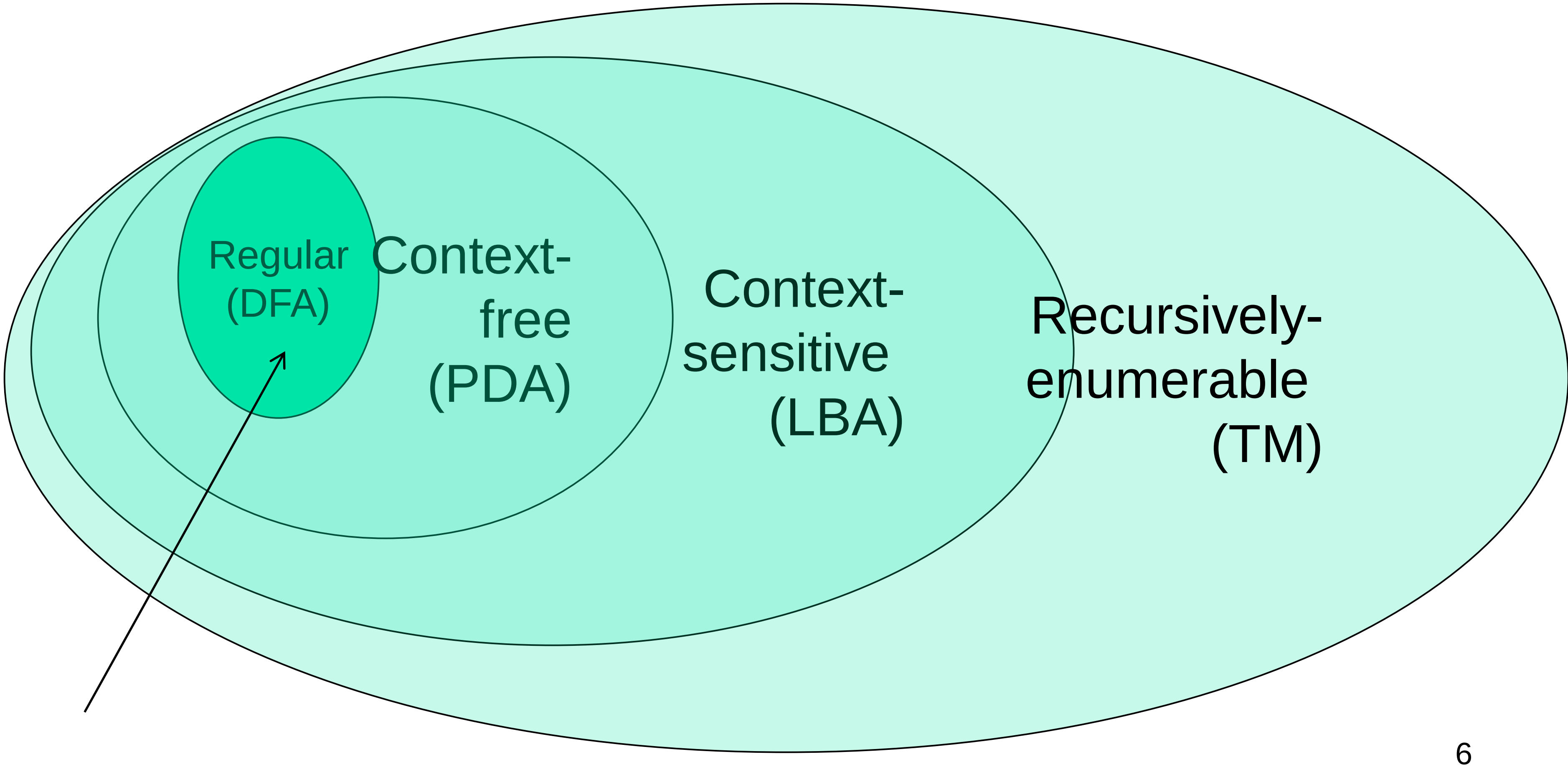
Regular Languages

- Let L(A) be a language *recognized* by a DFA A.
  - Then L(A) is called a "*Regular Language*".

- Locate regular languages in the Chomsky Hierarchy

The Chomsky Hierachy

- A containment hierarchy of classes of formal languages



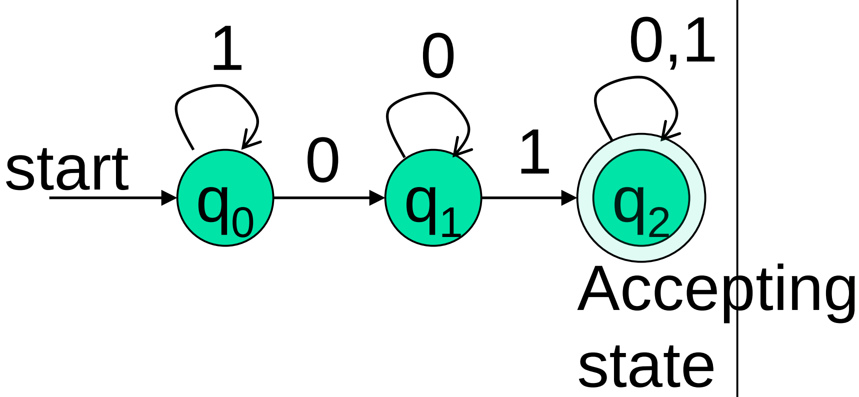
Example #1

- Build a DFA for the following language:
  - L = {w | w is a binary string that contains 01 as a substring}
- Steps for building a DFA to recognize L:
  - $\sum = \{0,1\}$
  - Decide on the states: Q
  - Designate start state and final state(s)
  - δ: Decide on the transitions:
- "Final" states == same as "accepting states"
- Other states == same as "non-accepting states"

Regular expression: $(0+1)^*01(0+1)^*$

DFA for strings containing 01

- What makes this DFA deterministic?



- $Q = \{q_0,q_1,q_2\}$
- $\Sigma = \{0,1\}$
- start state = q_0
- $F = \{q_2\}$
- Transition table

symbols

δ (states)	**0**	**1**
→ **q_0**	q_1	q_0
q_1	q_1	q_2
$*q_2$	q_2	q_2

- What if the language allows empty strings?

Example #2

Clamping Logic:

- A clamping circuit waits for a ”1” input, and turns on forever. However, to avoid clamping on spurious noise, we’ll design a DFA that waits for *two consecutive 1s* in a row before clamping on.

- Build a DFA for the following language:
  L = { w | w is a bit string which contains the substring 11}
- State Design:
  - q_0 : start state (initially off), also means the most recent input was not a 1
  - q_1: has never seen 11 but the most recent input was a 1
  - q_2: has seen 11 at least once

Example #3

- Build a DFA for the following language:
  L = { w | w is a binary string that has even number of 1s and even number of 0s}
- ?

Extension of transitions (δ) to Paths ($\hat{\delta}$)

- $\hat{\delta}$ *(q,w) = destination state* from state *q* on input string *w*

- $\hat{\delta}$ *(q,wa)* = δ ($\hat{\delta}$*(q,w), a)*

    - Work out example #3 using the input sequence w=10010, a=1:

        - $\hat{\delta}$ $(q_0, wa) = ?$

Language of a DFA

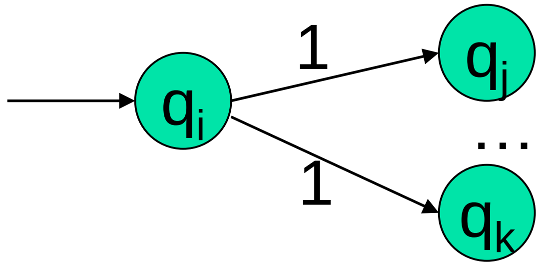
A DFA A accepts string *w* if there is a path from q_0 to an accepting (or final) state that is labeled by *w*

- *i.e.,* $L(A) = \{ w \mid \hat{\delta}(q_0,w) \in F \}$

- *I.e., L(A) = all strings that lead to an accepting state from* q_0

Non-deterministic Finite Automata (NFA)

- A Non-deterministic Finite Automaton (NFA)
  - is of course "non-deterministic"
    - Implying that the machine can exist in more than one state at the same time
    - Transitions could be non-deterministic



- Each transition function therefore maps to a <u>set</u> of states

Non-deterministic Finite Automata (NFA)

- A Non-deterministic Finite Automaton (NFA) consists of:
  - Q ==> a finite set of states
  - ∑ ==> a finite set of input symbols (alphabet)
  - q_0 ==> a start state
  - F ==> set of accepting states
  - δ ==> a transition function, which is a mapping between Q x ∑ ==> subset of Q
- An NFA is also defined by the 5-tuple:
  - $\{Q, \Sigma, q_0, F, \delta\}$

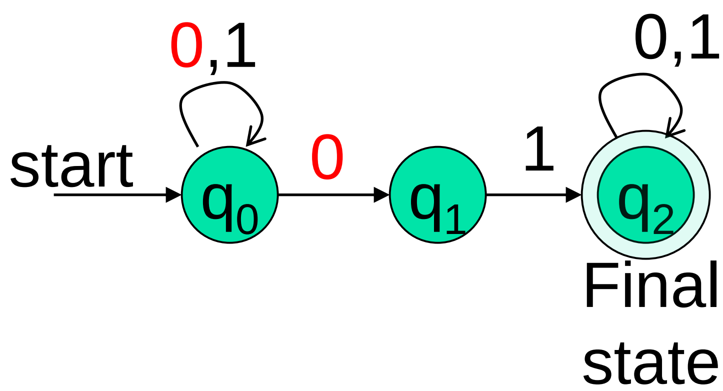
How to use an NFA?

- Input: a word w in Σ^*
- Question: Is w acceptable by the NFA?
- Steps:
  - Start at the "start state" q_0
  - For every input symbol in the sequence w do
    - Determine all possible next states from all current states, given the current input symbol in w and the transition function
  - If after all symbols in w are consumed and if at least one of the current states is a final state then *accept w;*
  - Otherwise, *reject w.*

Regular expression: (0+1)*01(0+1)*

NFA for strings containing 01

Why is this non-deterministic?



What will happen if at state q_1 an input of 0 is received?

- Q = $\{q_0,q_1,q_2\}$
- Σ = {0,1}
- start state = q_0
- F = $\{q_2\}$
- Transition table

symbols

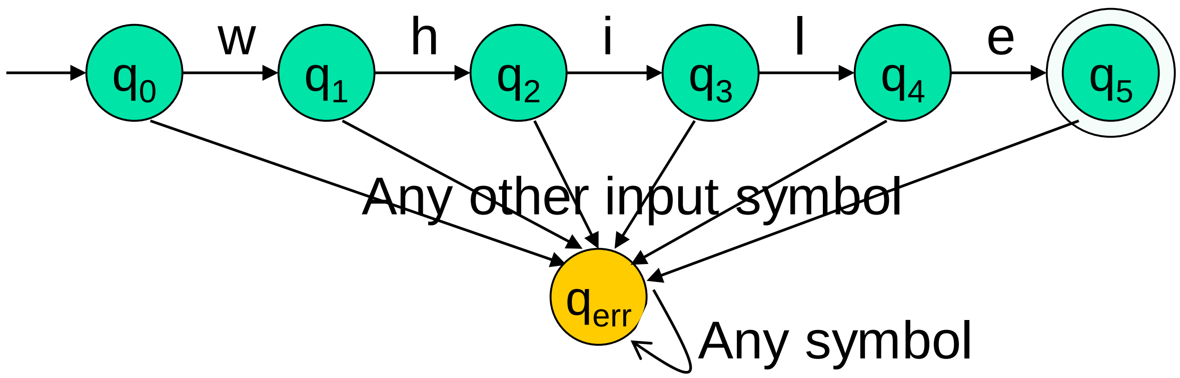
states

δ	**0**	**1**
q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Φ	$\{q_2\}$
$*q_2$	$\{q_2\}$	$\{q_2\}$

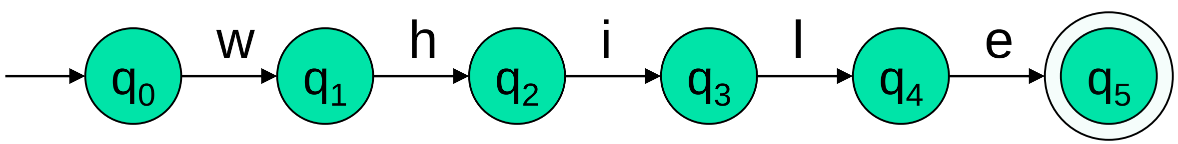
Note: Omitting to explicitly show error states is just a matter of design convenience
(one that is generally followed for NFAs), and
i.e., this feature should not be confused with the notion of non-determinism.

What is an "error state"?

- A DFA for recognizing the key word "*while*"



- An NFA for the same purpose:



Transitions into a dead state are implicit

Example #2

- Build an NFA for the following language:

  L = { w | w ends in 01}
- ?
- Other examples
  - Keyword recognizer (e.g., if, then, else, while, for, include, etc.)
  - Strings where the first symbol is present somewhere later on at least once

Extension of δ to NFA Paths

- Basis: $\hat{\delta}(q,\varepsilon) = \{q\}$

- Induction:
  - Let $\hat{\delta}(q_0,w) = \{p_1,p_2\ldots,p_k\}$
  - $\delta(p_i,a) = S_i$ for $i=1,2\ldots,k$
  - *Then,* $\hat{\delta}(q_0,wa) = S_1 \cup S_2 \cup \ldots \cup S_k$

Language of an NFA

- An NFA accepts *w* if *there exists at least one* path from the start state to an accepting (or final) state that is labeled by *w*
- $L(N) = \{ w \mid \hat{\delta}(q_0,w) \cap F \neq \Phi \}$

Advantages & Caveats for NFA

- Great for modeling regular expressions
  - String processing - e.g., grep, lexical analyzer

- Could a non-deterministic state machine be implemented in practice?
  - Probabilistic models could be viewed as extensions of non-deterministic state machines
    (e.g., toss of a coin, a roll of dice)
    - They are not the same though
  - A parallel computer could exist in multiple "states" at the same time

Technologies for NFAs

- Micron’s Automata Processor (introduced in 2013)
- 2D array of MISD (multiple instruction single data) fabric w/ thousands to millions of processing elements.
- 1 input symbol = fed to all states (i.e., cores)
- Non-determinism using circuits
- http://www.micronautomata.com/

But, DFAs and NFAs are equivalent in their power to capture langauges !!

Differences: DFA vs. NFA

- **DFA**

1. All transitions are deterministic
   - Each transition leads to exactly one state
2. For each state, transition on all possible symbols (alphabet) should be defined
3. Accepts input if the last state visited is in F
4. Sometimes harder to construct because of the number of states
5. Practical implementation is feasible

- **NFA**

1. Some transitions could be non-deterministic
   - A transition could lead to a subset of states
2. Not all symbol transitions need to be defined explicitly (if undefined will go to an error state – this is just a design convenience, not to be confused with "non-determinism")
3. Accepts input if *one of* the last states is in F
4. Generally easier than a DFA to construct
5. Practical implementations limited but emerging (e.g., Micron automata processor)

NFA to DFA by subset construction

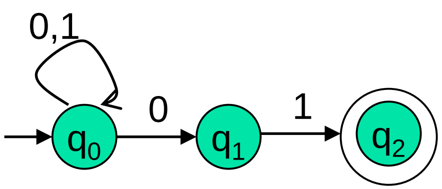
- Let $N = \{Q_N, \sum, \delta_N, q_0, F_N\}$
- <u>Goal:</u> Build $D=\{Q_D, \sum, \delta_D, \{q_0\}, F_D\}$ s.t. $L(D)=L(N)$
- <u>Construction:</u>
  1. Q_D= all subsets of Q_N (i.e., power set)
  2. F_D=set of subsets S of Q_N s.t. $S \cap F_N \neq \Phi$
  3. δ_D: for each subset S of Q_N and for each input symbol a in $\sum$:
     - $\delta_D(S,a) = \bigcup_{p \text{ in } s} \delta_N(p,a)$

Idea: To avoid enumerating all of power set, do "lazy creation of states"

NFA to DFA construction: Example

- $L = \{w \mid w \text{ ends in } 01\}$

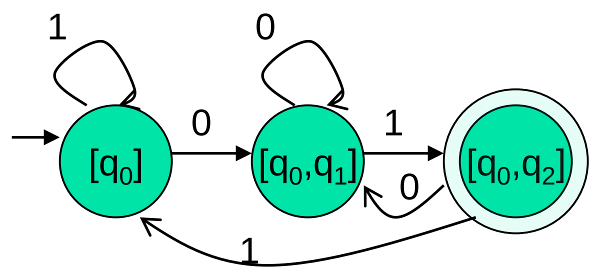
NFA:



δ_N	0	1
→q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
$^*q_2$	Ø	Ø

DFA:

δ_D
~~Ø~~
→$[q_0]$
~~$[q_1]$~~
~~$^*[q_2]$~~
$[q_0,q_1]$
$^*[q_0,q_2]$
~~$^*[q_1,q_2]$~~
~~$^*[q_0,q_1,q_2]$~~



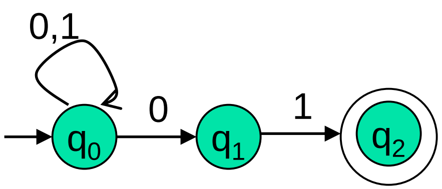
δ_D	0	1
→$[q_0]$	$[q_0,q_1]$	$[q_0]$
$[q_0,q_1]$	$[q_0,q_1]$	$[q_0,q_2]$
$^*[q_0,q_2]$	$[q_0,q_1]$	$[q_0]$

0. Enumerate all possible subsets
1. Determine transitions
2. Retain only those states reachable from $\{q_0\}$

NFA to DFA: Repeating the example using *LAZY CREATION*

- *L = {w | w ends in 01}*

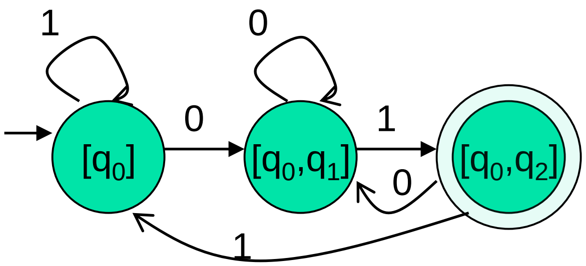
NFA:



δ_N	0	1
→q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$
$*q_2$	Ø	Ø

→

DFA:



δ_D	0	1
$[q_0]$	$[q_0,q_1]$	$[q_0]$

Main Idea:

Introduce states as you go (on a need basis)

Correctness of subset construction

Theorem: *If D is the DFA constructed from NFA N by subset construction, then L(D)=L(N)*

- Proof:
  - Show that $\hat{\delta}_D(\{q_0\},w) \equiv \hat{\delta}_N(q_0,w\}$, for all w
  - Using induction on w's length:
    - Let w = xa
    - $\hat{\delta}_D(\{q_0\},xa) \equiv \delta_D(\hat{\delta}_N(q_0,x\}, a) \equiv \hat{\delta}_N(q_0,w\}$

A bad case where #states(DFA)>>#states(NFA)

- L = {w | w is a binary string s.t., the k^{th} symbol from its end is a 1}

  - NFA has k+1 states

  - But an equivalent DFA needs to have at least 2^k states

<u>(Pigeon hole principle)</u>

  - *m* holes and >*m* pigeons
    - => at least one hole has to contain two or more pigeons

Applications

- Text indexing
  - inverted indexing
  - For each unique word in the database, store all locations that contain it using an NFA or a DFA
- Find pattern P in text T
  - Example: Google querying
- Extensions of this idea:
  - PATRICIA tree, suffix tree

A few subtle properties of DFAs and NFAs

- The machine never really terminates.
  - It is always waiting for the next input symbol or making transitions.
- The machine decides when to consume the next symbol from the input and when to ignore it.
  - (but the machine can never skip a symbol)
- => A transition can happen even *without* really consuming an input symbol (think of consuming ε as a free token) – if this happens, then it becomes an ε-NFA (see next few slides).
- A single transition *cannot* consume more than one (non-ε) symbol.

FA with ε-Transitions

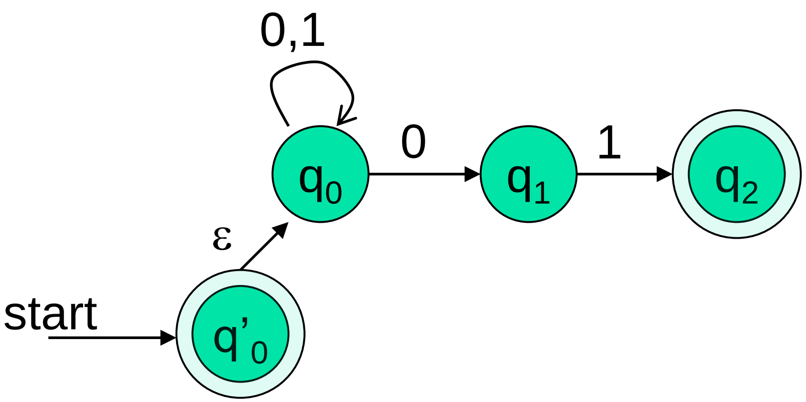
- We can allow <u>explicit</u> ε-transitions in finite automata
  - i.e., a transition from one state to another state without consuming any additional input symbol
  - Explicit ε-transitions between different states introduce non-determinism.
  - Makes it easier sometimes to construct NFAs

<u>Definition:</u> ε-NFAs are those NFAs with at least one explicit ε-transition defined.

- ε -NFAs have one more column in their transition table

Example of an ε-NFA

L = {w | w is empty, or if non-empty will end in 01}



δ_E	0	1	ε
→ $*q'_0$	Ø	Ø	$\{q'_0, q_0\}$
q_0	$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
$*q_2$	Ø	Ø	$\{q_2\}$

ECLOSE(q'_0)

ECLOSE(q_0)

ECLOSE(q_1)

ECLOSE(q_2)

- ε-closure of a state q, ***ECLOSE(q)***, is the set of all states (including itself) that can be *reached* from q by repeatedly making an arbitrary number of ε-transitions.

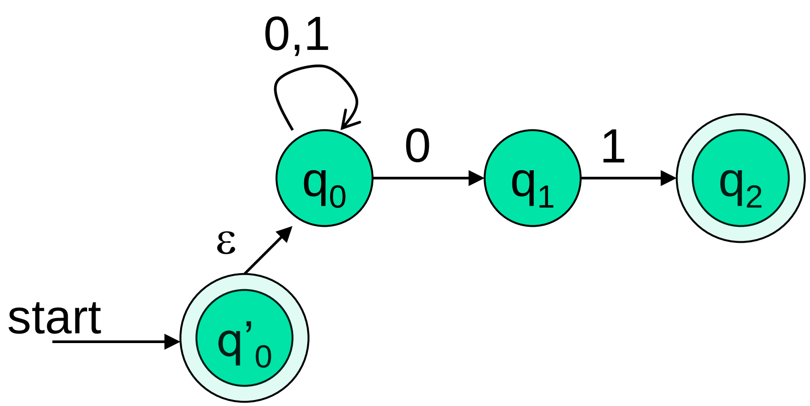
To simulate any transition:

Step 1) Go to all immediate destination states.

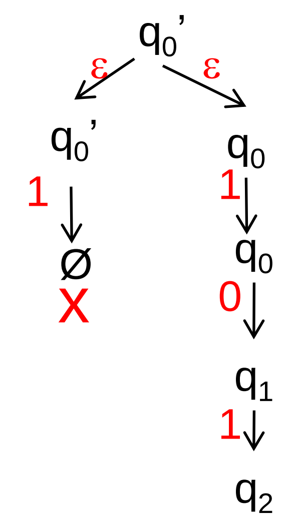
Step 2) From there go to all their ε-closure states as well.

Example of an ε-NFA

L = {w | w is empty, or if non-empty will end in 01}



Simulate for w=101:



δ_E	0	1	ε
$*q'_0$	Ø	Ø	$\{q'_0, q_0\}$
q_0	$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
$*q_2$	Ø	Ø	$\{q_2\}$

ECLOSE(q'_0)

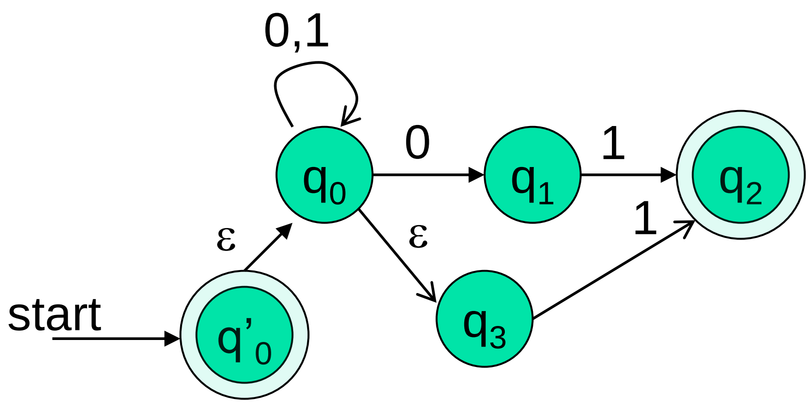
ECLOSE(q_0)

To simulate any transition:

Step 1) Go to all immediate destination states.

Step 2) From there go to all their ε-closure states as well.

Example of another ε-NFA



δ_E	0	1	ε
→*q'_0	Ø	Ø	$\{q'_0, q_0, q_3\}$
q_0	$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0, q_3\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
*q_2	Ø	Ø	$\{q_2\}$
q_3	Ø	$\{q_2\}$	$\{q_3\}$

<u>Simulate for w=101:</u>

?

Equivalency of DFA, NFA, ε-NFA

- Theorem: A language L is accepted by some ε-NFA if and only if L is accepted by some DFA

- Implication:
  - DFA ≡ NFA ≡ ε-NFA
  - (all accept Regular Languages)

Eliminating ε-transitions

Let $E = \{Q_E,\Sigma,\delta_E,q_0,F_E\}$ be an ε-NFA

Goal: To build DFA $D=\{Q_D,\Sigma,\delta_D,\{q_D\},F_D\}$ s.t. $L(D)=L(E)$

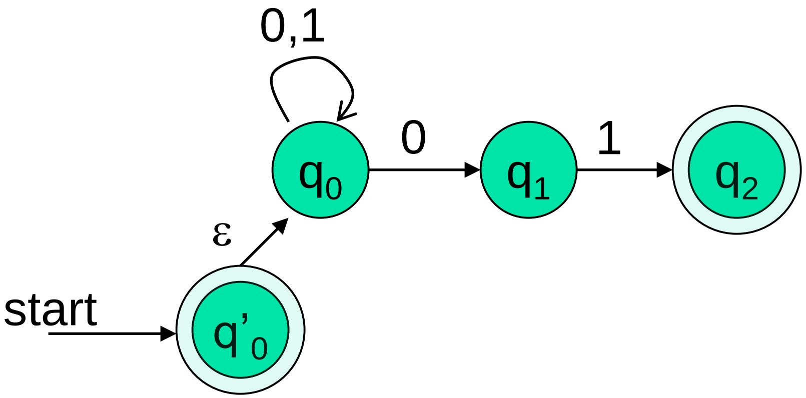
Construction:

1. Q_D= all reachable subsets of Q_E factoring in ε-closures
2. q_D = ECLOSE(q_0)
3. F_D=subsets S in Q_D s.t. $S\cap F_E \neq \Phi$
4. δ_D: for each subset S of Q_E and for each input symbol $a\in\Sigma$:
   - Let $R= \bigcup_{p \text{ in } s} \delta_E(p,a)$ // go to destination states
   - $\delta_D(S,a) = \bigcup_{r \text{ in } R} \text{ECLOSE}(r)$ // from there, take a union of all their ε-closures

Reading: Section 2.5.5 in book

Example: ε-NFA → DFA

L = {w | w is empty, or if non-empty will end in 01}

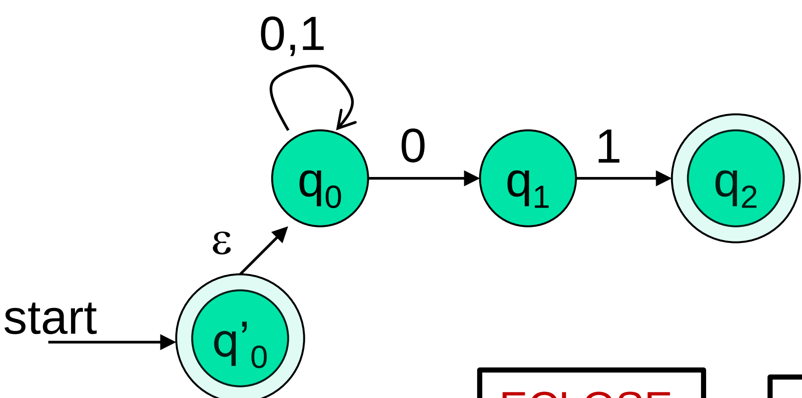


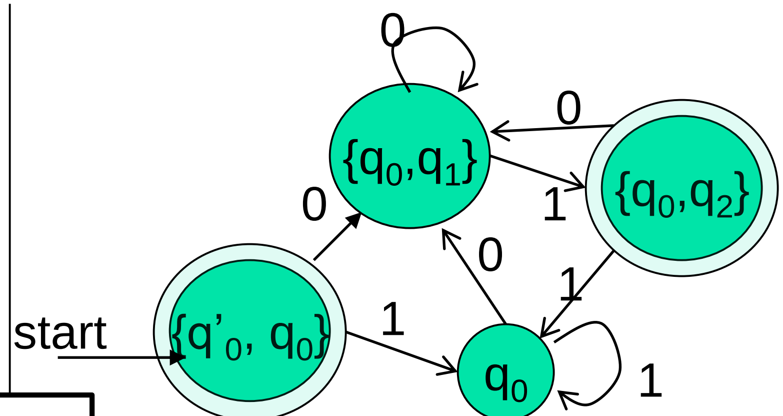
δ_E	0	1	ε
→ $*q'_0$	Ø	Ø	$\{q'_0,q_0\}$
q_0	$\{q_0,q_1\}$	$\{q_0\}$	$\{q_0\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
$*q_2$	Ø	Ø	$\{q_2\}$

δ_D	0	1
→ $*\{q'_0,q_0\}$		
…		

Example: ε-NFA → DFA

L = {w | w is empty, or if non-empty will end in 01}





ECLOSE

union

	δ_E	0	1	ε
→	$*q'_0$	Ø	Ø	$\{q'_0,q_0\}$
	q_0	$\{q_0,q_1\}$	$\{q_0\}$	$\{q_0\}$
	q_1	Ø	$\{q_2\}$	$\{q_1\}$
	$*q_2$	Ø	Ø	$\{q_2\}$

	δ_D	0	1
→	$*\{q'_0,q_0\}$	$\{q_0,q_1\}$	$\{q_0\}$
	$\{q_0,q_1\}$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
	$\{q_0\}$	$\{q_0,q_1\}$	$\{q_0\}$
	$*\{q_0,q_2\}$	$\{q_0,q_1\}$	$\{q_0\}$

Summary

- DFA
  - Definition
  - Transition diagrams & tables
- Regular language
- NFA
  - Definition
  - Transition diagrams & tables
- DFA vs. NFA
- NFA to DFA conversion using subset construction
- Equivalency of DFA & NFA
- Removal of redundant states and including dead states
- ε-transitions in NFA
- Pigeon hole principles
- Text searching applications

UNIT_1

Introduction to finite automata: *The central concepts of automata theory, Structural representation of FA, Types of FA, Conversion of NFA to DFA, NFA with epsilon to NFA without epsilon conversion.*

Regular Expression: *Introduction to Regular language, Algebraic laws for regular expressions, Conversion of FA to RE, Conversion of RE to FA, Pumping lemma for Regular language.*

==

Introduction to Finite Automata _Fundamentals – Terminology

Automation: means automatic machine / self-controlled

- Ex: computer, ATM

Why we study it:

- It allows us to think systematically about what machine do without going into Hardware details.
- Learning of languages and computational techniques.
- Designing of theoretical models for machines.
- It is core subject of computer science.

To understand a machine Language we should know

- How we are going to give input to machine
- How machine takes the input
- What are the limitations of machine?

Central concepts of automata theory

1. Symbol

- Something that has some meaning.
- Can't be further divided.

Eg: { a, b, 0, 1, +, ?, :, +, ++ }

---

2. Alphabet

- Has finite set of symbols?
- Denoted by ∑

Eg : ∑ = { 0, 1 }
∑ = { a, b }
∑ = {[#, @ }
∑ { 0. . .9 }
∑ eng = { A, B, ...Z, a, b . . .z }

∑ bin = { 0, 1}
∑ dec = { 0, 1 . . .9 }
∑ empty = Ø

3. String / Word / sentence

- Finite sequence of symbols over ∑
- Denoted by w.

  ∑ = { 0, 1 }

  W = 101,   w = 1100,   w = 111 00 11

  ∑ { a, b }

  W = aba,   w = bab,   w = aabbaa

---

4. String operations

- Empty string is denoted by ε / л / λ
- String length: * # of symbols in the word.
- Denoted by |w|

---

5. Concatenation: with Є

Є. W = w. Є = w.

Eg : w = 110110   w = Є 011100 Є

|w|= 6 |w| =6

w = Є 00 ЄЄ 011 Є 00

|w| = 7

Don't consider Є while calculating length of a string.

---

6. Prefix

- The first letter of each string is fixed. But ending may be anywhere.

Eg: w = abcd

Prefix(w) = {Є, a, ab, abc, abcd}

---

7. Suffix

- The last letter of each string is fixed. But start may be anywhere.

Eg: w = abcd

Suffix(w) = {Є, d, cd, bcd, abcd}

---

8. Subsequence

- Starting and ending is anywhere.

Eg: w = abcd

Subsequence(w)= { Є, a, b, c, d, ab, ac, ad, bc, bd, cd, abc, abd, bcd, abcd}

---

9. Substring

- Letters of consecutive symbols.

Eg: w = abcd

Substrings(w) = {Є, a, b, c, d, ab, bc, cd, abc, bcd, abcd}

---

10. Concatenation

- Concatenation of two strings w1 and w2
- W1 is followed by w2 without any space b/w them

Eg : w1 = Success w2 = fully

w1.w2 = successfully

---

11. Proper substring:

- All substrings of w except itself.

Eg : FLAT

Proper substrings: {Є, F, L, A, T, FL, LA , AT, FLA, LAT}

12.Palindrome

- Can be read w either side identically.

Eg : w = { 0, 1 }

w = 0110, w = 10001, w = 111000111

---

13. Power Alphabet

- The power alphabet, $\sum^k$
- K denotes set of all strings of length k.

Eg: let $\sum = \{0, 1\}$

$\sum^0 = \{\epsilon\}$ $\sum^1 = \{0, 1\}$ $\sum^2 = \{00, 01, 10, 11\}$

---

14. Kleene closure

- All strings of length 0 or more instances.
- Denoted by $\sum^*$

$$\sum^* = \sum^0 \cup \sum^1 \cup \sum^2 \ldots\ldots$$

$$\boxed{\bigcup_{i\geq 0}^{\infty} \Sigma^i}$$

$$\sum^* \sum^* = \sum^*$$
$$\sum^+ + \sum^0 = \sum^*$$
$$\sum^+ + \epsilon = \sum^*$$

---

15. Positive Closure

- $\sum^+$ denotes all strings of length 1 or more.

$$\sum^+ = \sum^1 \cup \sum^2 \cup \sum^3 \;\ldots\ldots$$

$$\longrightarrow \boxed{\bigcup_{i\geq 0}^{\infty} \Sigma^i}$$

$$\sum^* \cap \sum^+ = \sum^+$$
$$\sum^* \cup \sum^+ = \sum^*$$
$$\sum^* \sum^+ = \sum^+$$
$$\sum^+ \sum^+ = \sum^+$$
$$\sum^* - \epsilon = \sum^+$$
$$a.\ \epsilon = a$$

16. Set operation:

- Set is a collection of distinct objects.
- Set is denoted by S.

Eg :1. S = { 0,1, 2, 3, 4 } → finite set
2. S = { 0, 1, 2, 3, 4 . . . } → Infinite set
3. S = { } → Empty | Null set

---

17. Set size | cardinality:

- The number of objects in the set

Denoted by |S|

Eg : s = {[1, 2, 3 } → |S|= 3

---

18. Finite | countable set:

- Every element of the set is countable

Eg: S = {10, 20, 30, 40}

19. Infinite | countably Infinite set

- Basically, finite set.
- Every object is countable for sometimes.

Eg: set of natural Numbers.

---

20. Intersection

- Common elements from the given sets.

A = {1, 2, 3, 4, 5}   B = {2, 6, 7, 8}   A ∩ B = {2}

---

21. Union

- Unique elements from the given sets.

A U B = {1, 2, 3, 4, 5, 6, 7, 8}

---

22. Cartesian product

- Multiply each element of one set by another element of another set.

Eg : A = { 1, 2 } ,   B = {a, b }

A * B = { (1, a), (1, b), (2, a), (2, b) }

---

23. Language: (Finite and Infinite)

- subset of $\sum^*$
- Denoted by L.

Finite Language: Finite | countable number of objects in the set.

Eg: $\sum$ = {a, b}

$\sum^2$ = {aa, ab, ba, bb}

Infinite Language: Infinite number of elements in the set.

Eg: All strings of starts with a.

{a, ab, aa, aab, aab, aba,………}

$\sum$ = {a, b } → L_1 = { a, aa, ab, aab, abb ……….}

L_2 = { } → Φ

L_3 = { ε }

L_4 = {a, b } → { ε, ab, aabb, aaabb …….}

$L_5 = \{a^n \mid n >= 0\}$ $\{ a^0, a^1, a^2, a^3, \ldots \}$

$L_6 = \{a^n b^m \mid n, m >= 1\}$

= {ab, aab, abb, aaab,………}

23.Automata

- Mathematical model of computation.
- Abstract machine to perform computation.

Eg : Can generate primes

Can check primes.

24. Types of Automata

- Finite Automaton
- Pushdown Automaton
- Linear Bounded Automaton
- Turing Machine

---

25. Grammar

- Generates valid strings using rules.

---

26. Types of languages, Automata and grammar

Language	Automata	Grammar
Type 0[REL]	TM	UG
Type 1[CSL]	LBA	CSG
Type 2[CFL]	PDA	CFG
Type 3[Regular Language]	FA	RG

Practice Questions

1. Let $\sum$ = {0, 1} How many strings of length n are possible over $\sum$.

   A) n B) n+1 C) 2^n D) 2n

---

2. w be a string of length n. How many suffixes are possible for w.?

   A) n B) n+1 C) 2^n D) 2n

3. Let $\sum = \{0, 1, 2\}$ How many strings of length 3 or less are possible?

A) 8 B) 9 C) 18 D) None

$L^3 = 3^3$
$L^2 = 3^2$
$L^1 = 3^1$ 27+9+3+1 = 40
$L^0 = 3^0$

---

4. Let $\sum$ = {0, 1}, Let L = {01, 110, 100}

i) 0100110010 iii) 110100011110
ii) 100110100110 iv) 1010101110

Which of the combination of strings belong to L^*?
A) i & ii B) ii & iv C) iii & iv D) None (only ii)

Structural Representation of Finite Automata

- A finite automaton (FA) can be represented structurally as a directed graph, also called a state transition diagram or state machine diagram.
- The graph consists of vertices (nodes) and edges. In an FA, the vertices represent the states of the machine and the edges represent the transitions between states based on input symbols.
- The FA has one initial state, represented by an arrow pointing to the state. The initial state is where the machine begins processing input symbols.
- The machine may have one or more accepting states, indicated by a double circle around the state. An accepting state is where the machine accepts the input string.
- The edges of the graph represent transitions between states. Each edge is labeled with an input symbol, indicating the symbol that causes the machine to transition from one state to another. For a deterministic finite automaton (DFA), each state has exactly one edge leaving it for each input symbol in the alphabet.
- For a non-deterministic finite automaton (NFA), a state may have multiple edges leaving it for the same input symbol or even have epsilon (ε) transitions, which means the machine can transition to another state without reading an input symbol.

Finite Automata

- Finite automata are used to recognize patterns.
- It takes the string of symbol as input and changes its state accordingly. When the desired symbol is found, then the transition occurs.
- At the time of transition, the automata can either move to the next state or stay in the same state.

- Finite automata have two states, **Accept state** or **Reject state**. When the input string is processed successfully, and the automata reached its final state, then it will accept.

→ In other words, a FA is a 5-tuple system.

$M = (Q, \sum, \delta, q_0, F)$ where

Q: Finite set of states
$\sum$: Input alphabet
δ: Transition function mapping from $Q \times \sum \rightarrow Q$
q_0: Initial state
F: Set of final states

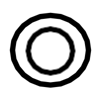
A FA is represented in two ways
1. State Diagram
2. Transition state Table

State Diagram

→ can also be called transition diagram | transition system.

→ It is a direct labelled graph in which nodes are corresponding to states and directed edges are corresponding to transition states.

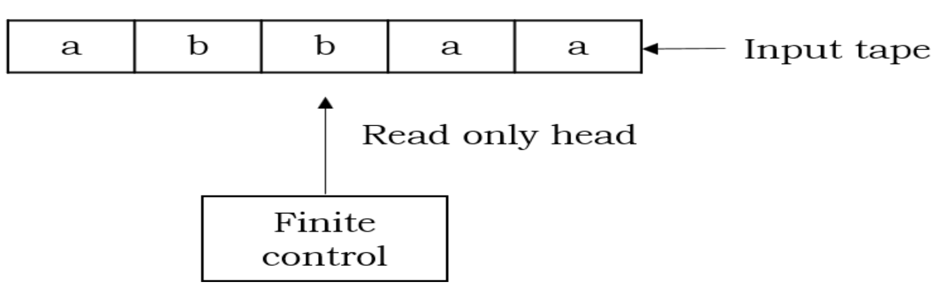
→ Initial states are denoted by → 

→ final state is denoted by 

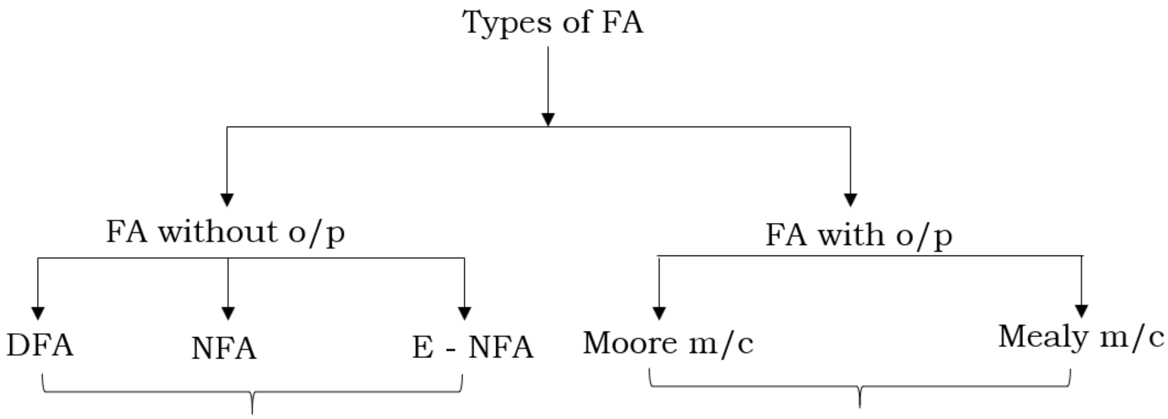
State Transition Table

- In transition table, rows are corresponding to state and columns are corresponding to i/p.
- For each combination of present state and present i/p, corresponding entry specifies next state of the system.

Configuration | Block Diagram of FA



- A FA consists of Input tape, read only head and Finite Control.
- The i/p tape consists of set of cells each hols one i/p symbol.
- The read only head moves left to right and scans one symbol at a time.
- FC determines the next state of the system using symbol under read only head and the present state of the system.



Applications

1. Used to check spellings In Editor
2. Lexical analysis in compilers
3. General Applications
4. To model the behavior of electrical ckts
5. To model the behaviour of digital ckts
6. To model the behaviour of mechanical devices
7. To solve puzzles.

Applications

1. Computing1's complement
2. Computing 2's compliment
3. Adder | Subtractor
4. Counter
5. Sequence Generator/detector

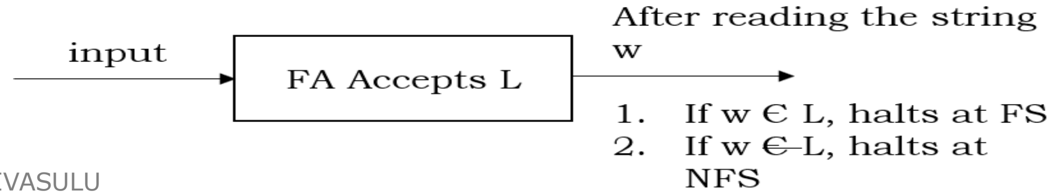
Equality

All types of FAS are equal. Equivalence means not the same but they perform the same task.

L (FA) = L (DFA) = L (NFA) = L (ε- NFA) = RL.

FA $\cong$ DFA $\cong$ NFA $\cong$ ε- NFA $\cong$ = RL.

FA without o/p

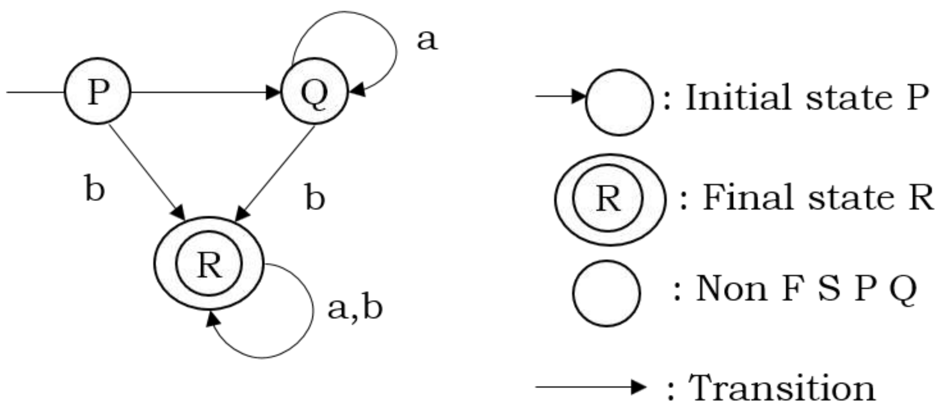


FA Representation

DFA . . . . . . δ (Q X $\sum$) $\rightarrow$ Q
NFA . . . . . . δ (Q X $\sum$) $\rightarrow$ 2^Q
Є – NFA . . . δ (Q X $\sum$ U {Є} $\rightarrow$ 2^Q

1. State Diagram
2. Transition Table
3. Transition Function

State Diagram



Transition Table

Staes	a	b
P	Q	R
Q	Q	R
R	R	R

String Acceptance by FA

- If there Exists a path from initial state to final state the string w is said to be accepted.
- In other words, w is accepted by FA
  iff δ (q_0, w) $\rightarrow$ for some P in F.

Language Accept

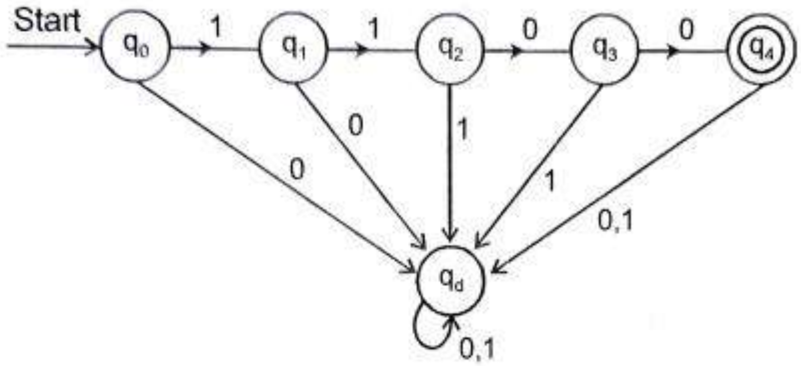
- A FA is said to be accepted a lang if all the strings in the lang are accepted.
- All the strings that are not in the lang are rejected by the FA.

DFA PROBLEMS

Problem 1:

Design DFA which accepts string 1100 only

Solution:

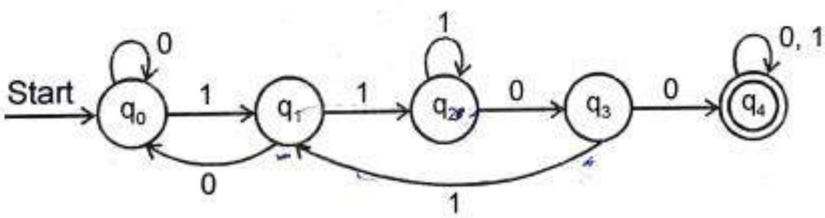


Where q_d is called Dummy state, trap state or Dead state.

Problem 2:

Design a DFA which accepts set of all strings contains 1100 as substring, where $\Sigma = \{0,1\}$

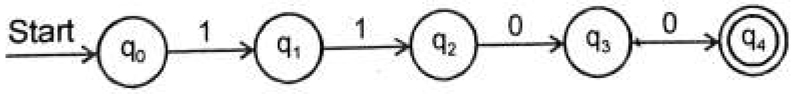
Solution:



Procedure:

Step 1:

1100 is a sub string itself so draw the finite automata for 1100



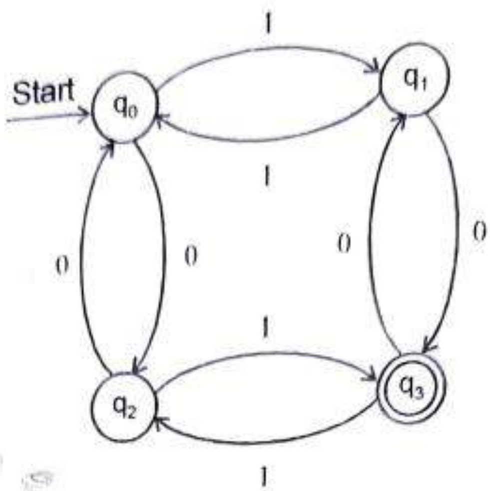
Step 2:

q_0 on 1 already path is there to q_1 and q_0 on o should stay in the q_0 because 1100 substring may have any number of 0's before it. q_1 on 0 sub string matching string is failed so it checks from the starting state q_0. q_2 on 1 string is failed so it should return back to the starting state but after two 1's any number of 1's can be there. So it returns in the same state. q_3 on 1 string matching failed so it returns back to the starting state but it already see in first alphabet of substring so it returns to q_1. After subset is matched any input can be accepted so q_4 on 0 or 1 remains in the same state.

Problem 3:

Design a DFA which accepts set of all strings containing odd number of 0's and odd number of 1's.

Solution:



Procedure:

Step 1:

Select all combinations of even and odd number of 0's and 1's as states ie, q_0 even number of 0's and even number of 1's

q_1 : even number of 0's and odd number of 1's.

q_2 : odd number of 0's and even number of 1's.

q_3 : odd number of 0's and odd number of 1's.

q_3 contains odd number of 0's and odd number of 1's so make it as a final state.

Step 2:

q_0 on 0 ie, on even number of zeros and even number of 1's if we apply another 0 it becomes odd number of 0's. So it moves to odd number of 0's and even number of is state ie, q_2 similarly for other states and inputs also,

Problem 4:

Design DFA which accepts set of all strings containing mod3 is zero where string is treated as binary numbers.

Solution:

Step 1:

Take all posible reminders as states

Step 2:

For any binary string if we add a bit at LSB the previous value gets doubled this can be genarlized as $2 \times n+a$ where n is previous number and a is the bit added.

So $(2 \times n+a)$ mod3 ie $2 \times n$ mod3 + a mod3

ie $2 \times$ (State number or Reminder)+a

Step 3:

Substitute as the above and get the values.

Solution:

For |3| the remainders are 0,1,2. So let us consider 0 at q_0, 1 at q_1 and 2 at q_2 .

$\Sigma = \{0,1\}$ and base =2.

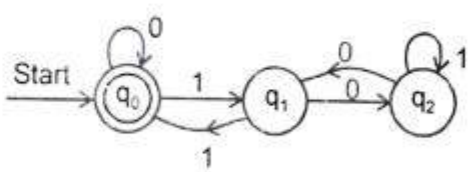
$$(q_0, 0) = 2\times 0+0 = 0 \rightarrow q_0$$
$$(q_0, 1) = 2\times 0+1 = 1 \rightarrow q_1$$
$$(q_1, 0) = 2\times 1+0 = 2 \rightarrow q_2$$
$$(q_1, 1) = 2\times 1+1 = 0 \rightarrow q_0$$
$$(q_2, 0) = 2\times 2+0 = 1 \rightarrow q_1$$
$$(q_2, 1) = 2\times 2+1 = 2 \rightarrow q_2$$

$(q_2, 1) = 2\times 2+1 = 2 \rightarrow q_2$

from the above,

$$(q_1, 1) = 2\times 1+1 = 3\%3 = 0$$

Base.

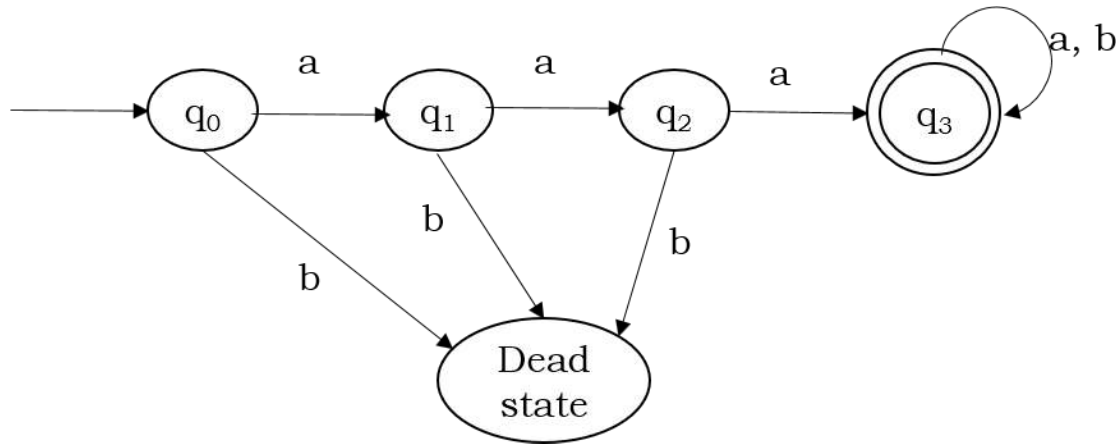


Practice QuestionsMODEL :1
(start | End | contain)

1. Dead state exists
2. # states : n+2

1. No dead state
2. # states +1

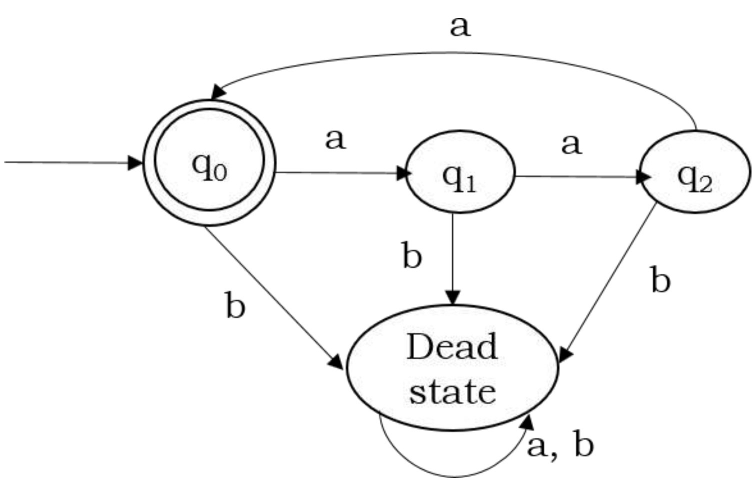
1. Design DFA for the lang RE: aaa $(a+b)^*$ → 5 states



Exercise 1: Construct DFA for the RE: $(a+b)^*abb(a+b)^*$ → contain
Exercise 2: Construct DFA for the RE: $(a+b)^*aaa$ → Ends with aaa Need 3+1 states.

Model : 2
(Language over 1 Symbol)

2. $L = (aaa)^*$ over $\sum = \{a, b\}$

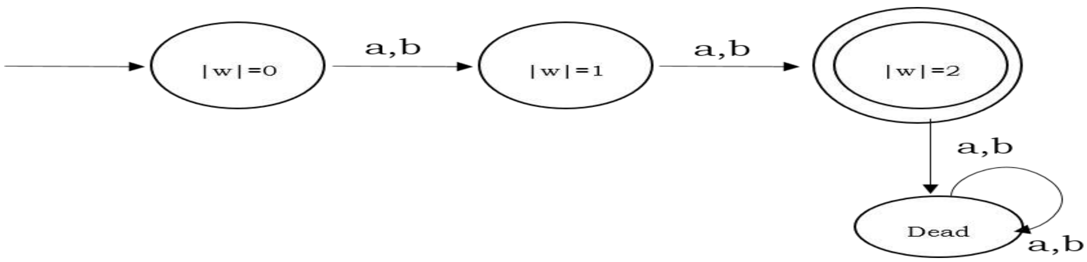


Exercise 1: $L = \{ a^m, b^n, c^L \mid m, n, L >= 1 \}$

Exercise 2: L = { a^m, b^n, c^L | m, n, L >= 0 }

Model: 3
(Length and Divisible problems)

3. L = {String length is exactly 2 } over $\sum$ = { a, b }*



Exactly n length
= n+2 states
Dead state exists

Almost = |w| <= 2
n+2 states
Dead state exists

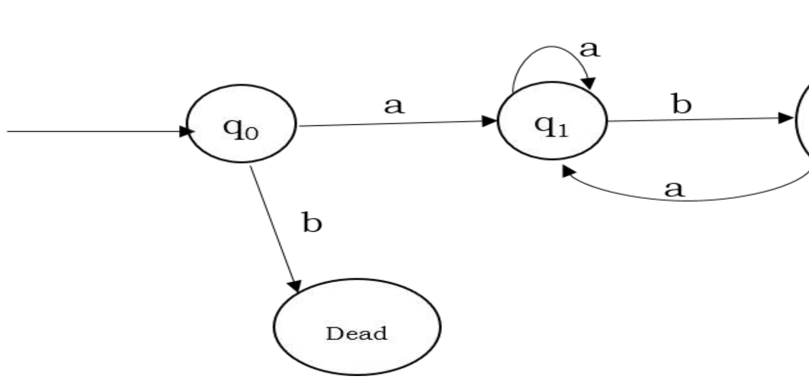
Atleast = |w| >= 2
n+1 states
No Dead state

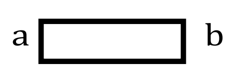
Exercise 1:L = { w|w Є {[a, b }, |w| <= 2 }

Exercise 2: L = { w|w Є { a, b }*, |w| >= 2 }

MODEL: 4
(Multiple conditions)

4. L = { starts with a and ends with b }





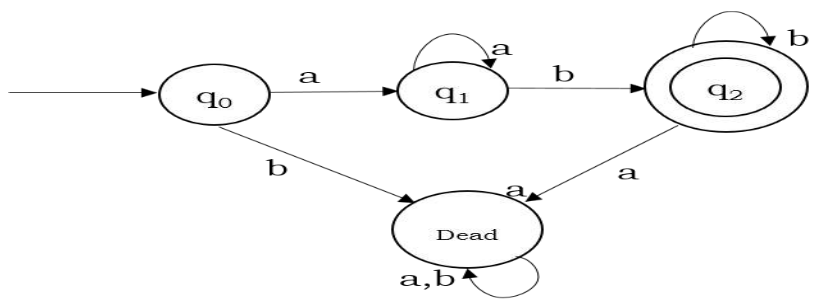
Exercise 1: L = {[strings start and end with different symbols} over ∑ = {a, b}
Exercise: 2: L = (aa+bb) (a+b)* ………… starts with aa or bb.

MODEL: 5
(Position Based problems)

Exercise 1: L = { 2^{nd} symbol from LHS is a }
Exercise 2: L = { a^3 b w a^3 / w Є { a, b }* }

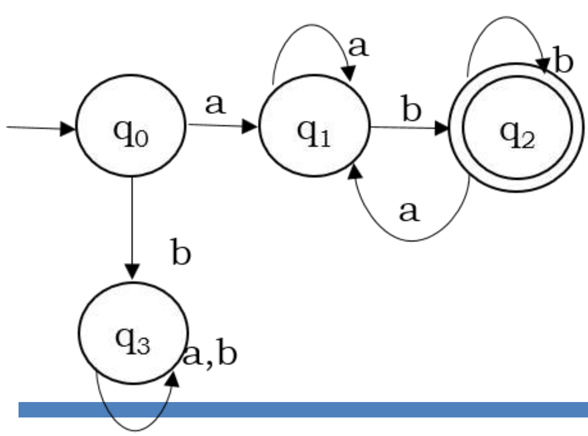
MODEL : 6
(Sequence Based problems)

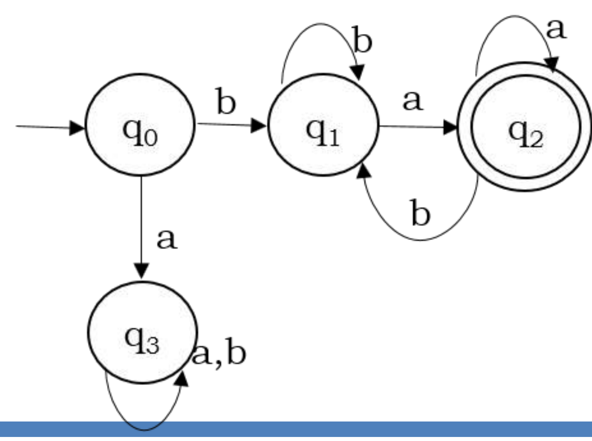
5. L = { $a^+ b^+$ }



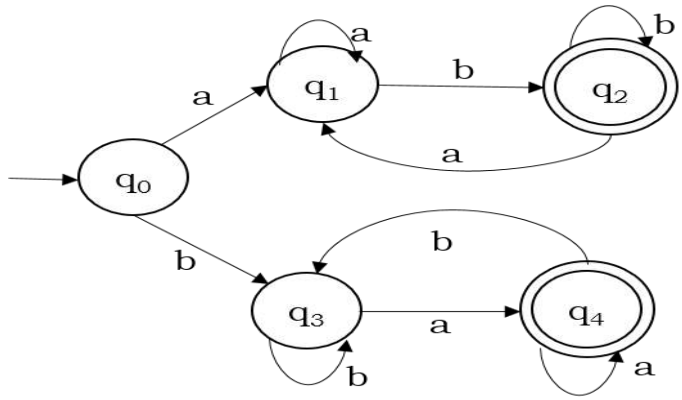
6. UNION of two FAs

L_1 = { start with a and end with b } L_2 = { start with b and end a }
= { ab, aab, abb, aaab . . . } = { ba, bba, bbba, baaa . . }





Union of the above FA (starting and ending with different Symbols) $L_1 \cup L_2$

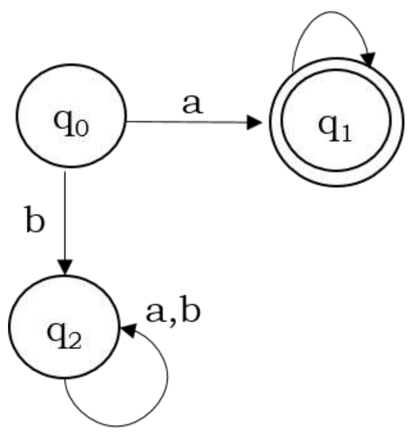


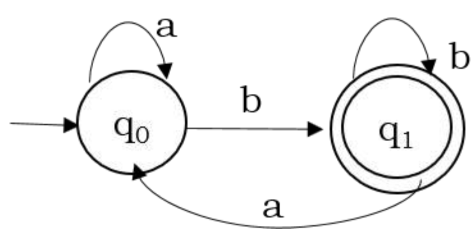
---

7. Concatenation of two FA.

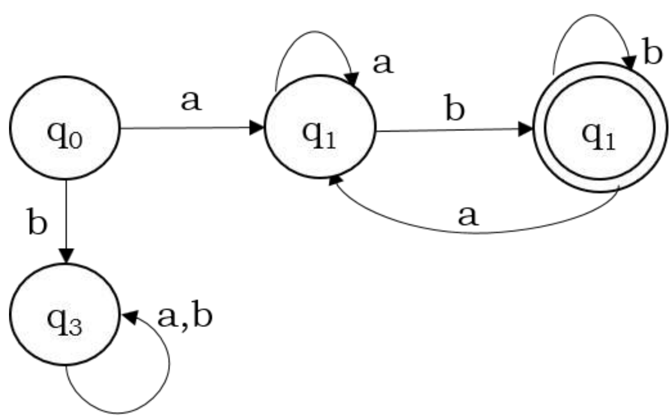
L_1 = {[start with a }
= { a, aa, ab, aaa . . . }

L_2 = {Ending with b }
+ { b, ab, aab, bbb . . . }





Concatenation of $L_1 . L_2$ (start with a and End with b)



---

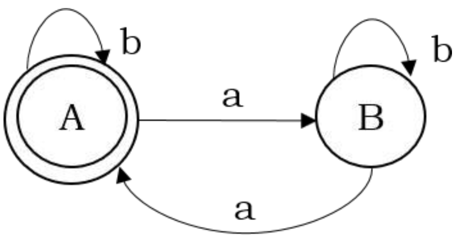
8. Cross product

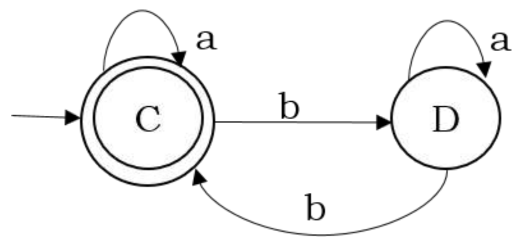
Even no of a's

L_1 = { aa, aba, aab . . . }

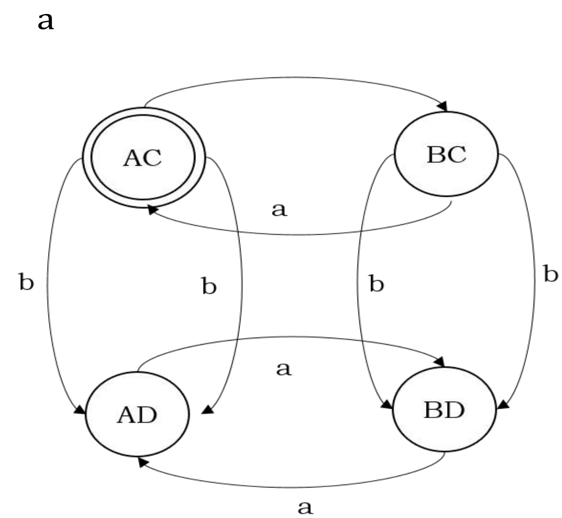
Even no of b's

L_2 = { bb, bab, bba . . . }





L_1 X L_2 = { AB X CD } = { AC, AD, BC, BD }



---

9. Compliment of language L_1

L_1 = {containing a }

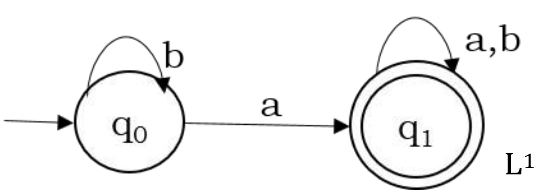
= { a, aa, ba, bab, . . . }

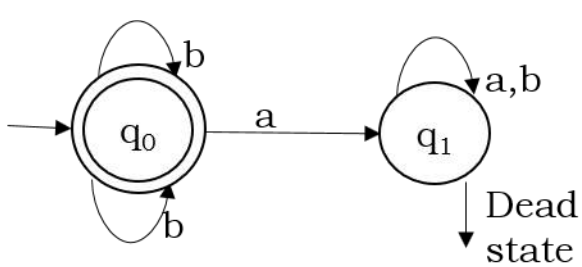
$\bar{L_1}$ = { does not contain a }

= { Є, b, bb, bbb, . . . }

i) Final to non-final

ii) Non- final to final

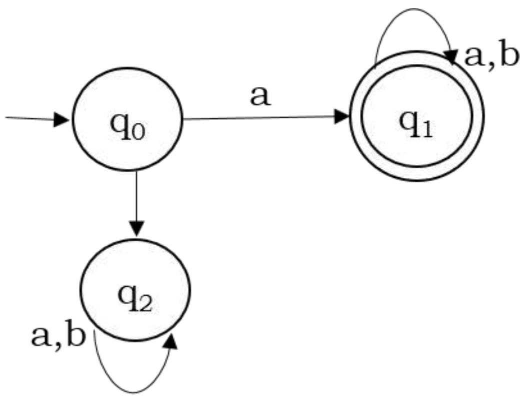




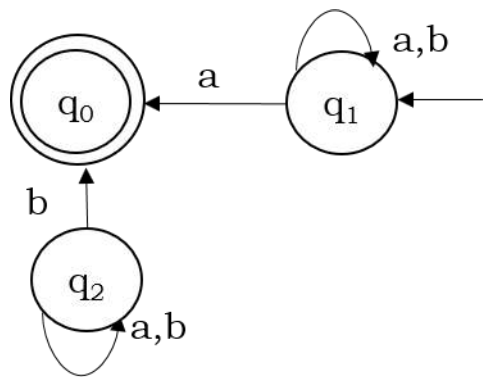
10. **Reversal:** start with a

L = { a, aa, ab, aaa . . . }

L^R = {Reverse of strings}
= { a, aaa, ba, aaa . . . }



(Ending with a . . .)



(Staring with a . . .) Its NFA

Since no transition q_0

i) Final to initial & initial to final.
ii) No change in Symbols
iii) Reverse transition.

Non-Deterministic Finite Automata

- In NDFA, for a particular input symbol, the machine can move to any combination of the states in the machine.
- In other words, the exact state to which the machine moves cannot be determined. Hence, it is called Non-deterministic Automaton.
- As it has finite number of states, the machine is called Non-deterministic Finite Machine or Non-deterministic Finite Automaton.

Formal Definition of an NDFA

An NDFA can be represented by a 5-tuple $(Q, \sum, \delta, q_0, F)$ where –

- **Q** is a finite set of states.
- **∑** is a finite set of symbols calling the alphabets.
- **δ** is the transition function where $\delta: Q \times \sum \rightarrow 2^Q$

(Here the power set of Q (2^Q) has been taken because in case of NDFA, from a state, transition can occur to any combination of Q states)

- **q_0** is the initial state from where any input is processed ($q_0 \in Q$).
- **F** is a set of final state/states of Q ($F \subseteq Q$).

Graphical Representation of an NDFA: (same as DFA)

An NDFA is represented by digraphs called state diagram.

- The vertices represent the states.
- The arcs labeled with an input alphabet show the transitions.
- The initial state is denoted by an empty single incoming arc.
- The final state is indicated by double circles.

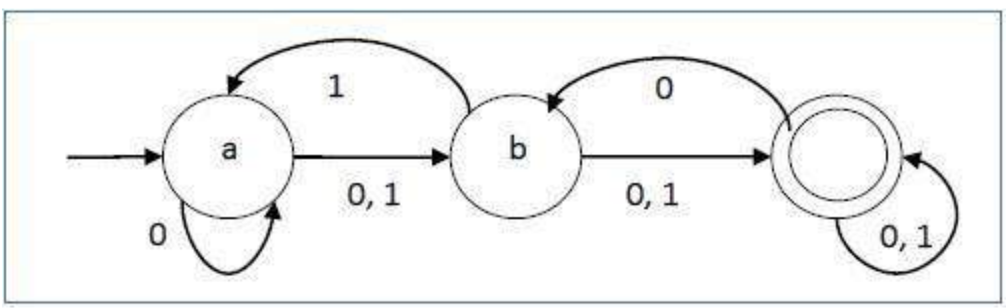
Example

Let a non-deterministic finite automaton be →

- Q = {a, b, c}
- ∑ = {0, 1}
- q_0 = {a}
- F = {c}

The transition function δ as shown below

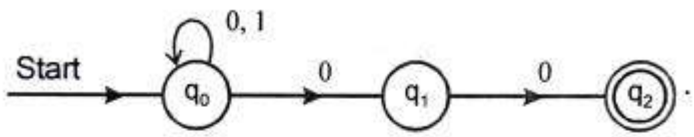
Present State	Next state for Input 0	Next state for Input 1
a	a, b	b
b	c	a, c
c	b, c	c



Problem 1:

Design an NFA to accept set of strings over alphabet set {0, 1} and ending with two consecutive O's.

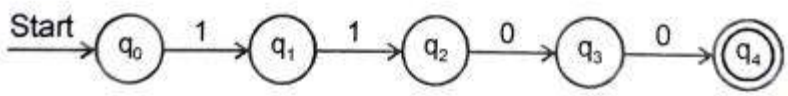
Solution:



Problem 2:

Design NFA which accept string 1100 only

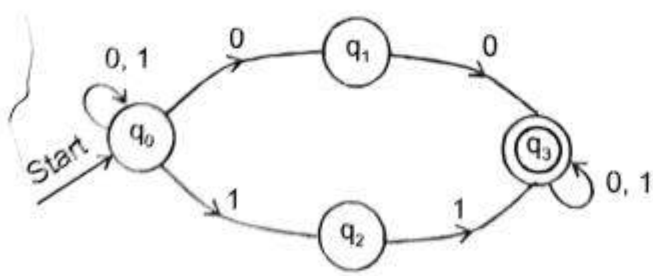
Solution:



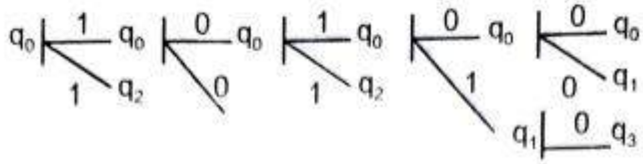
Problem 3:

Design a NFA to accept the strings with 0's and 1's such that string contains either two consecutive 0's or two consecutive 1's.

Solution:



Consider a string 10100

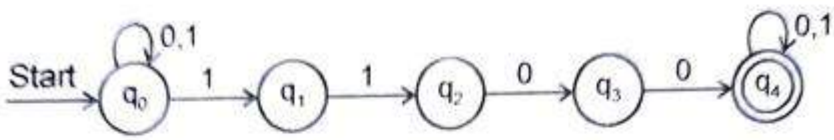


The path is $q_0 \vdash \xrightarrow{1} q_0 \vdash \xrightarrow{0} q_0 \vdash \xrightarrow{1} q_0 \vdash \xrightarrow{0} q_1 \vdash \xrightarrow{0} q_3$

Given string is accepted as transition reaches to final state, q_3.

Problem 4:

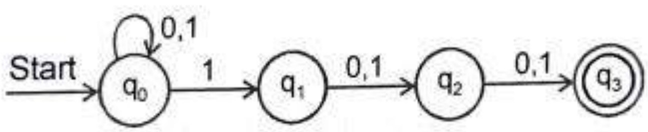
Design NFA which accepts set of all strings containing 1100 as substring.



Problem 5:

Design NFA which accepts set of all strings containing 3rd symbol from right side is 1.

Solution:



DFA vs NDFA

The following table lists the differences between DFA and NDFA.

DFA	NDFA
The transition from a state is to a single particular next state for each input symbol. Hence it is called *deterministic*.	The transition from a state can be to multiple next states for each input symbol. Hence it is called *non-deterministic*.
Empty string transitions are not seen in DFA.	NDFA permits empty string transitions.
Backtracking is allowed in DFA	In NDFA, backtracking is not always possible.
Requires more space.	Requires less space.
A string is accepted by a DFA, if it transits to a final state.	A string is accepted by a NDFA, if at least one of all possible transitions ends in a final state.

NDFA to DFA Conversion

Problem Statement

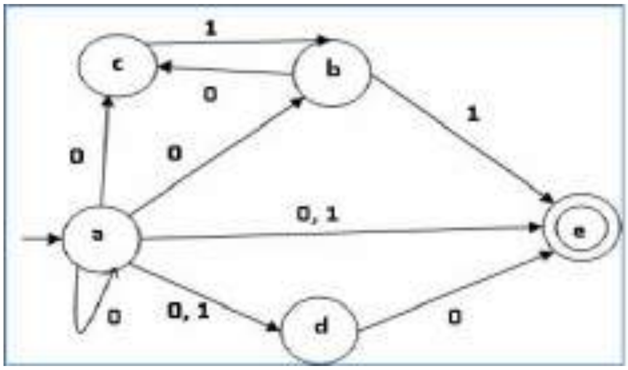
Let $X = (Qx, \Sigma, \delta x, q0, Fx)$ be an NDFA which accepts the language $L(X)$. We have to design an equivalent DFA $Y = (Qy, \Sigma, \delta y, q0, Fy)$ such that $L(Y) = L(X)$. The following procedure converts the NDFA to its equivalent DFA:

Algorithm

Input:	An NDFA
Output:	An equivalent DFA
Step 1	Create state table from the given NDFA.
Step 2	Create a blank state table under possible input alphabets for the equivalent DFA.
Step 3	Mark the start state of the DFA by q0 (Same as the NDFA).
Step 4	Find out the combination of States {Q0, Q1,... , Qn} for each possible input alphabet.
Step 5	Each time we generate a new DFA state under the input alphabet columns, we have to apply step 4 again, otherwise go to step 6.

Step 6 The states which contain any of the final states of the NDFA are the final states of the equivalent DFA.

Example
Let us consider the NDFA shown in the figure below.



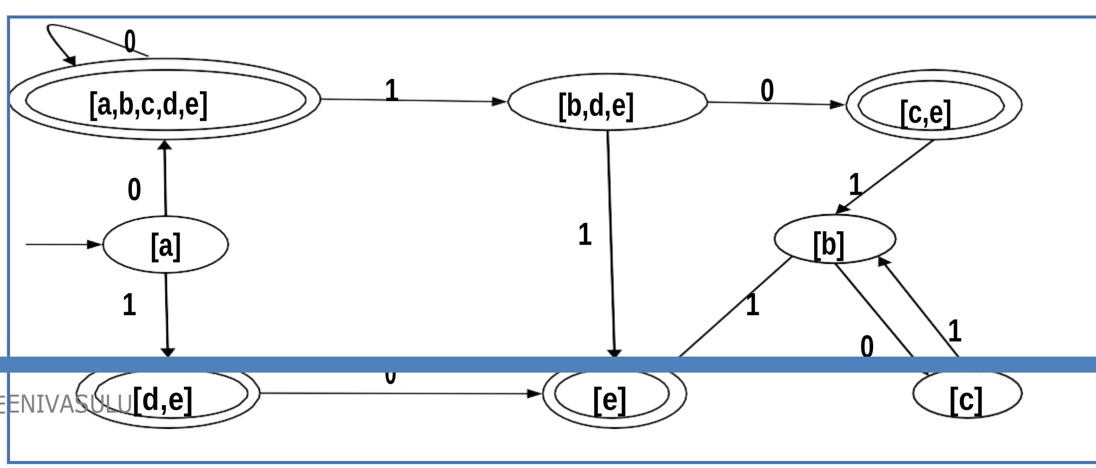
q	δ(q,0)	δ(q,1)
a	{a,b,c,d,e}	{d,e}
b	{c}	{e}
c	∅	{b}
d	{e}	∅
e	∅	∅

Using the above algorithm, we find its equivalent DFA. The state table of the DFA is shown in below.

Transition Table for DFA

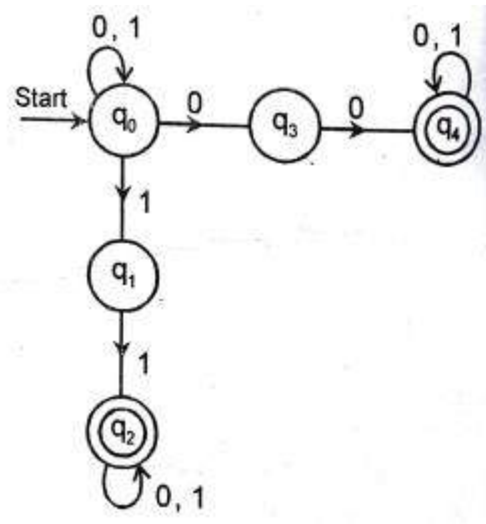
q	δ(q,0)	δ(q,1)
[a]	[a, b, c, d, e]	[d,e]
[a, b, c, d, e]	[a, b, c, d, e]	[b, d, e]
[d, e]	[e]	∅
[b, d, e]	[c, e]	[e]
[e]	∅	∅
[c, e]	∅	[b]
[b]	[c]	[e]
[c]	∅	[b]

The state diagram of the DFA is as follows:



Example 1:

Convert the following NFA to DFA



Transition table for the given NFA diagram is

State \ i/ps	0	1
q_0	q_0, q_3	q_0, q_1
q_1	ϕ	q_2
q_2	q_2	q_2
q_3	q_4	ϕ
q_4	q_4	q_4

First we make the starting state of NFA as the starting state of DFA. Apply 0 and 1 as i/p on state q_0. Now keep the o/p's in 2nd and 3rd columns. Next take the new generated states which we placed in 3rd and 4th columns keep them in first. Column and again apply 0's and 1's as i/p. Repeat the process until no new state is left.

DFA's transition table

The deterministic automation M_2 equivalent to the above M_1, defined as

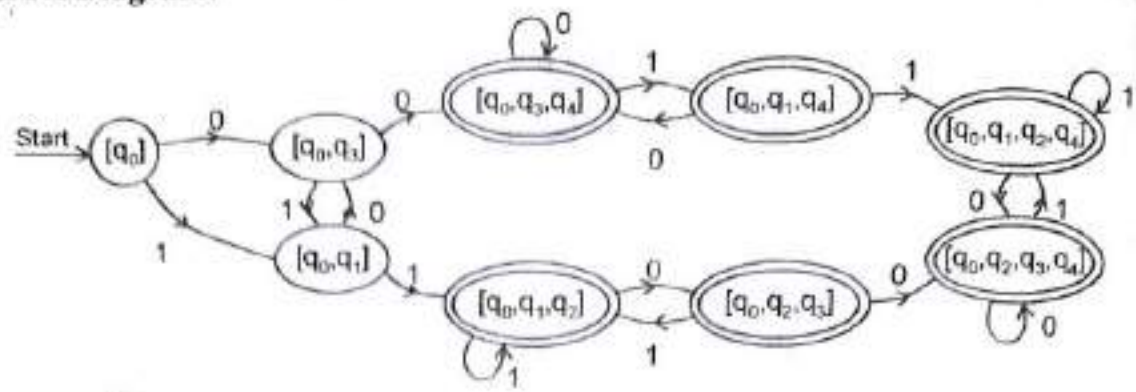
$$M_2 = (2^Q, \{0, 1\}, \delta, [q_0], F)$$

$$F = \{[q_0, q_3, q_4], [q_0, q_1, q_4], [q_0, q_1, q_2, q_4]$$
$$[q_0, q_1, q_2], [q_0, q_2, q_3], [q_0, q_2, q_3, q_4]\}$$

δ is defined as

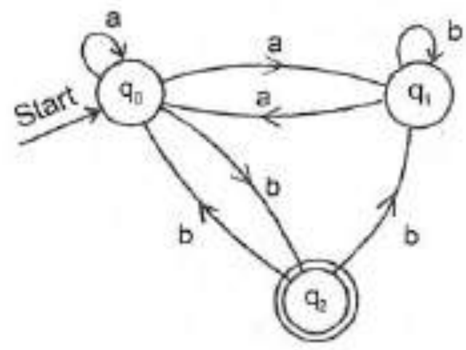
state/ip	0	1
$[q_0]$	$[q_0, q_3]$	$[q_0, q_1]$
$[q_0, q_3]$	$[q_0, q_3, q_4]$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_3]$	$[q_0, q_1, q_2]$
$[q_0, q_3, q_4]$	$[q_0, q_3, q_4]$	$[q_0, q_1, q_4]$
$[q_0, q_1, q_2]$	$[q_0, q_2, q_3]$	$[q_0, q_1, q_2]$
$[q_0, q_1, q_4]$	$[q_0, q_3, q_4]$	$[q_0, q_1, q_2, q_4]$
$[q_0, q_2, q_3]$	$[q_0, q_2, q_3, q_4]$	$[q_0, q_1, q_2]$
$[q_0, q_1, q_2, q_4]$	$[q_0, q_2, q_3, q_4]$	$[q_0, q_1, q_2, q_4]$
$[q_0, q_2, q_3, q_4]$	$[q_0, q_2, q_3, q_4]$	$[q_0, q_1, q_2, q_4]$

DFA Diagram:



Example 2:

Construct a DFA diagram for to the NFA given below



NFA

Solution:

Let equivalent DFA, $M_2 = (Q', \Sigma, \delta', q'_0, F')$

of given NFA, $M_1 = (Q, \Sigma, \delta, q_0, F)$

Q' is $\{q_0, q_2, [q_0, q_1], [q_1, q_2], q_d\}$

F' is $\{q_2, [q_2, q_1]\}$

q'_0 of DFA is q_0 of NFA

Σ of DFA Σ of NFA

δ is defined as $\delta'(q_0, a) = \{q_0, q_1\}$

$$\delta'(q_0, b) = \{q_2\}$$

$$\delta'([q_0, q_1], a) = \delta(q_0, a) \cup \delta(q_1, a)$$

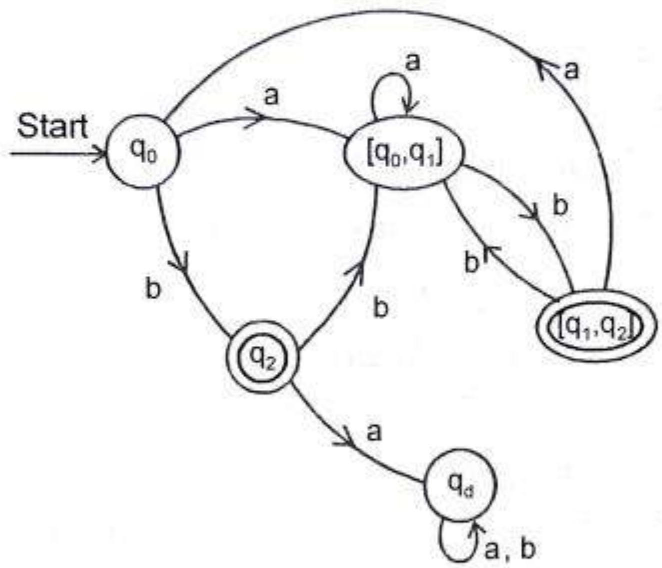
$$= \{q_0, q_1\} \cup \{q_0\}$$

$$= \{q_0, q_1\}$$

$$\delta'([q_0, q_1], b) = \delta(q_0, b) \cup \delta(q_1, b)$$

$= \{q_2\} \cup \{q_1\} = \{q_1, q_2\}$

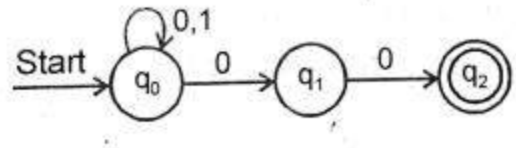
$$\delta'([q_1, q_2], a) = \delta(q_1, a) \cup \delta(q_2, a)$$
$$= \{q_0\}$$
$$\delta'([q_1, q_2], b) = \delta(q_1, b) \cup \delta(q_2, b)$$
$$= \{q_1\} \cup \{q_0, q_1\} = \{q_0, q_1\}$$



DFA

Example 3:

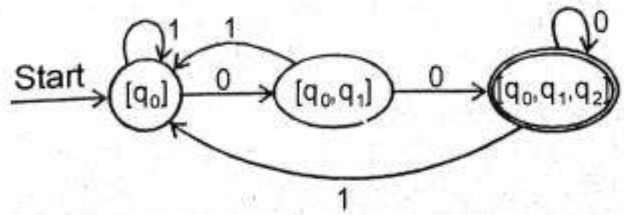
Convert NFA to DFA,



Solution:

	0	1
$[q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1, q_2]$	$[q_0]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$	$[q_0]$

The DFA is



Conversion of Epsilon-NFA to NFA

Non-deterministic Finite Automata (NFA) is a finite automata having zero, one or more than one moves from a given state on a given input symbol. Epsilon NFA is the NFA which contains epsilon move(s)/Null move(s). To remove the epsilon, move/Null move from epsilon-NFA and to convert it into NFA, we follow the steps mentioned below.

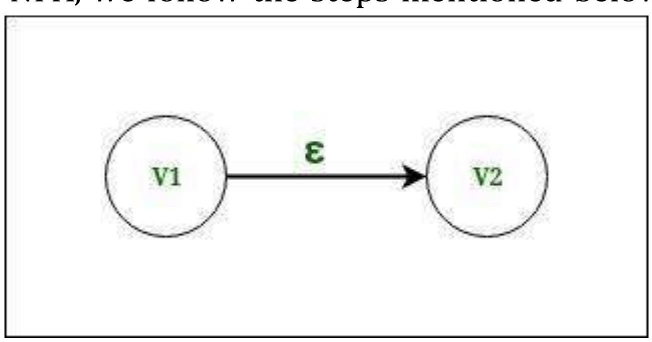


Figure – Vertex v1 and Vertex v2 having an epsilon move

Step-1:

Consider the two vertexes having the epsilon move. Here in Fig.1 we have vertex v1 and vertex v2 having epsilon move from v1 to v2.

Step-2:

Now find all the moves to any other vertex that start from vertex v2 (other than the epsilon move that is considering). After finding the moves, duplicate all the moves that start from vertex v2, with the same input to start from vertex v1 and remove the epsilon move from vertex v1 to vertex v2.

Step-3:

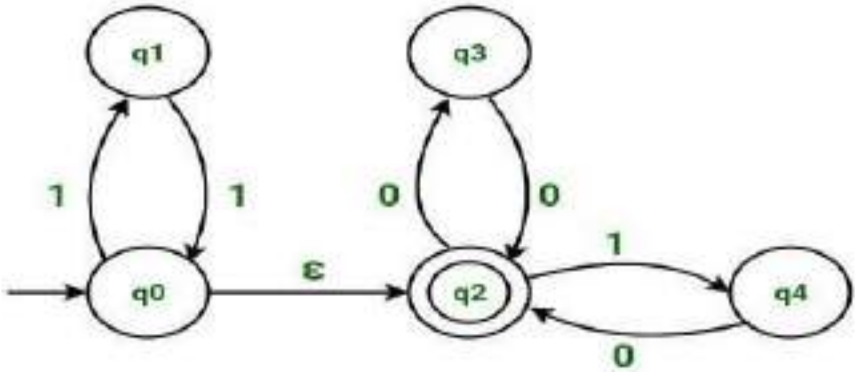
See that if the vertex v1 is a start state or not. If vertex v1 is a start state, then we will also make vertex v2 as a start state. If vertex v1 is not a start state, then there will not be any change.

Step-4:

See that if the vertex v2 is a final state or not. If vertex v2 is a final state, then we will also make vertex v1 as a final state. If vertex v2 is not a final state, then there will not be any change. Repeat the steps(from step 1 to step 4) until all the epsilon moves are removed from the NFA.

Now, to explain this conversion, let us take an example.

Example: Convert epsilon-NFA to NFA. Consider the example having states q0, q1, q2, q3, and q4.

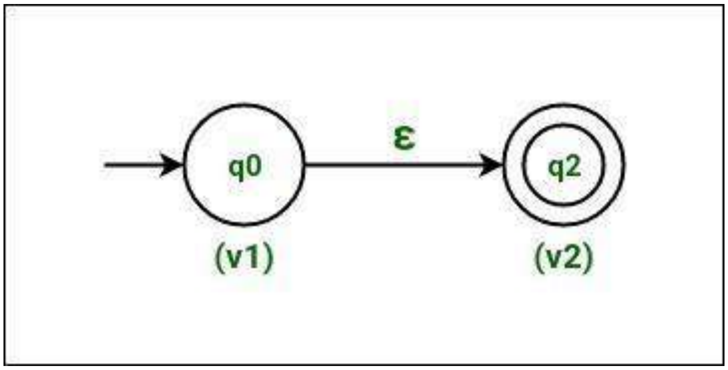


In the above example, we have 5 states named as q0, q1, q2, q3 and q4. Initially, we have q0 as start state and q2 as final state. We have q1, q3 and q4 as intermediate states.

Transition table for the above NFA is:

States/Inputs	INPUT 0	INPUT 1	INPUT EPSILON
q0	–	q1	q2
q1	–	q0	–
q2	q3	q4	–
q3	q2	–	–
q4	q2	–	–

According to the transition table above,
state q0 on getting input 1 goes to state q1.
State q0 on getting input as a null move (i.e. an epsilon move) goes to state q2.
State q1 on getting input 1 goes to state q0.
Similarly, state q2 on getting input 0 goes to state q3, state q2 on getting input 1 goes to state q4.
Similarly, state q3 on getting input 0 goes to state q2.
Similarly, state q4 on getting input 0 goes to state q2.
We can see that we have an epsilon move from state q0 to state q2, which is to be removed.
To remove epsilon move from state q0 to state q1, we will follow the steps mentioned below.
Step-1:
Considering the epsilon move from state q0 to state q2. Consider the state q0 as vertex v1 and state q2 as vertex v2.



S

Figure – State q0 as vertex v1 and state q2 as vertex v2

Step-2:

Now find all the moves that starts from vertex v2 (i.e. state q2).

After finding the moves, duplicate all the moves that start from vertex v2 (i.e state q2) with the same input to start from vertex v1 (i.e. state q0) and remove the epsilon move from vertex v1 (i.e. state q0) to vertex v2 (i.e. state q2).

Since state q2 on getting input 0 goes to state q3.

Hence on duplicating the move, we will have state q0 on getting input 0 also to go to state q3.

Similarly state q2 on getting input 1 goes to state q4.

Hence on duplicating the move, we will have state q0 on getting input 1 also to go to state q4.

So, NFA after duplicating the moves is:

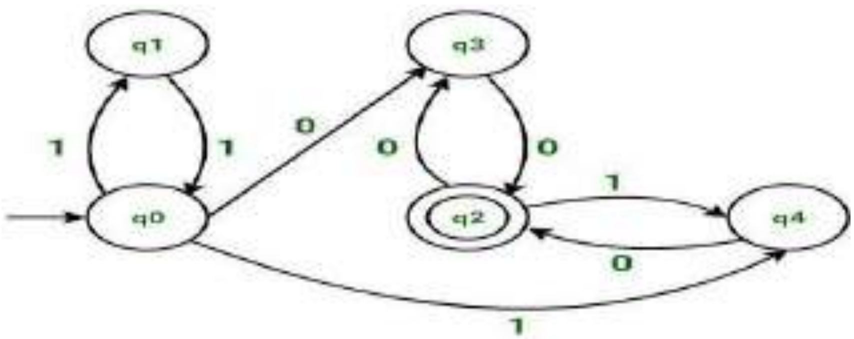


Figure – NFA on duplicating moves

Step-3:

Since vertex v1 (i.e. state q0) is a start state. Hence we will also make vertex v2 (i.e. state q2) as a start state.

Note that state q2 will also remain as a final state as we had initially.

NFA after making state q2 also as a start state is:

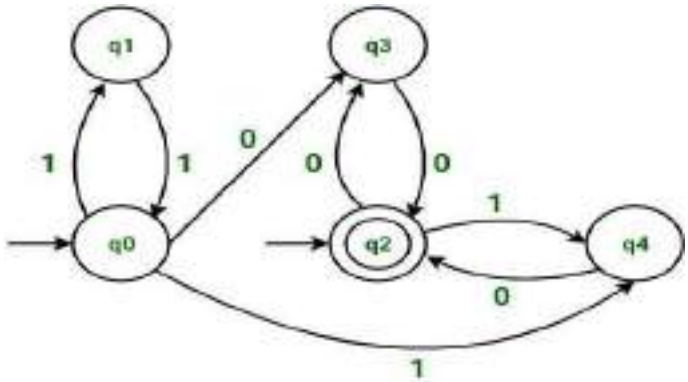


Figure – NFA after making state q2 as a start state

Step-4:

Since vertex v2 (i.e. state q2) is a final state. Hence we will also make vertex v1 (i.e. state q0) as a final state.

Note that state q0 will also remain as a start state as we had initially.

After making state q0 also as a final state, the resulting NFA is:

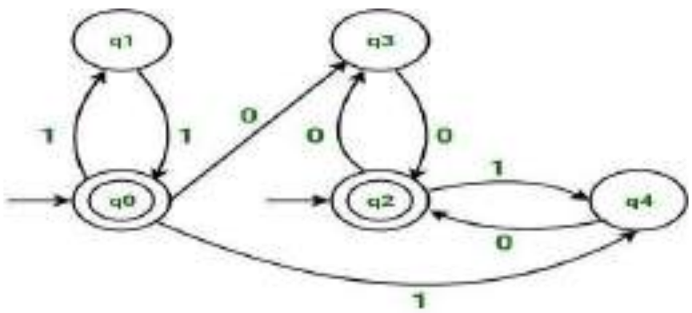


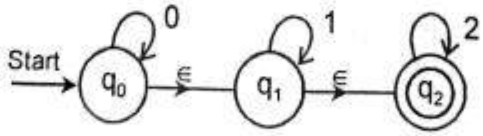
Figure – Resulting NFA (state q0 as a final state)

The transition table for the above resulting NFA is:

STATES/INPUT	INPUT 0	INPUT 1
q0	q3	q1,q4
q1	–	q0
q2	q3	q4
q3	q2	–
q4	q2	–

Example:

Convert NFA with $\in$-moves in figure given below to equivalent NFA without $\in$-moves



Solution:

From the definition of $\in$-closure,

$$\in\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$
$$\in\text{-closure}(q_1) = \{q_1, q_2\}$$
$$\in\text{-closure}(q_2) = \{q_2\}$$

There q_2 is in $\in$-closure(q_0) that means without reading any i/p symbol transition directly moves from q_0 to q_2 directly. Similarly, q_2 is the only final state for given NFA with $\in$-moves

So, F of NFA with $\in$ is $\{q_2\}$ and F′ of NFA without $\in$ is $\{q_0, q_1, q_2\}$

$$\hat{\delta}(q_0, 0) = \in\text{-closure}(\delta(\hat{\delta}(q_0, \in), 0))$$
$$= \in\text{-closure}(\delta(\{q_0, q_1, q_2\}, 0))$$
$$= \in\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2,0)) = \in\text{-closure}(\{q_0\} \cup \phi)$$
$$= \in\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$
$$\hat{\delta}(q_0, 1) = \in\text{-closure}(\delta(\hat{\delta}(q_0, \varepsilon), 1))$$
$$= \in\text{-closure}(\delta(\{q_0, q_1, q_2\}. 1))$$
$$= \in\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$$
$$= \in\text{-closure}(\phi \cup \{q_1\} \cup \phi)$$
$$= \in\text{-closure}(q_1) = \{q_1, q_2\}$$
$$\hat{\delta}(q_0, 2) = \in\text{-closure}(\delta(\hat{\delta}(q_0, \in, 2))$$
$$= \in\text{-closure}(\delta(\{q_0, q_1, q_2\}, 2))$$
$$= \in\text{-closure}(q_1) = \{q_1, q_2\}$$
$$\hat{\delta}(q_1, 0) = \in\text{-closure}(\delta(\hat{\delta}(q_1, \in), 0))$$
$$= \in\text{-closure}(\delta(\{q_1, q_2\}, 0))$$
$$= \in\text{-closure}(\phi) = \phi$$

The state q_1 is not consuming the i/p symbol '0' so, no need to perform this step.

$$\hat{\delta}(q_1, 1) = \in\text{-closure}(\delta(\hat{\delta}(q_1, \in), 1))$$
$$= \in\text{-closure}(\delta(\{q_1, q_2\}, 1))$$

$= \in\text{-closure}(q_1) = \{q_1, q_2\}$

$\hat{\delta}(q_0, 2) = \in\text{-closure}(\delta(\hat{\delta}(q_0, \in), 2))$

$= \in\text{-closure}(\delta(\{q_2\}, 1))$

$= \in\text{-closure}(\delta(q_2, 1))$

$= \in\text{-closure}(q_2) = \{q_2\}$

Similarly,

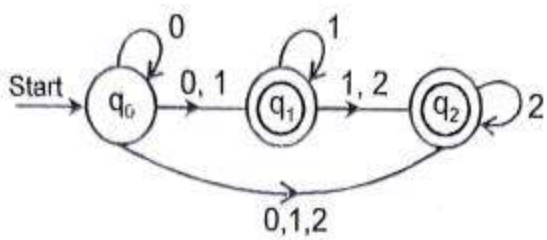
$\hat{\delta}(q_1, 2) = \{q_2\}$

$\hat{\delta}(q_2, \in) = \{\phi\}$

$\hat{\delta}(q_2, 1) = \phi$

$\hat{\delta}(q_2, 2) = \{q_2\}$

Using above information of δ' we can construct the transition diagram for NFA without $\in$.

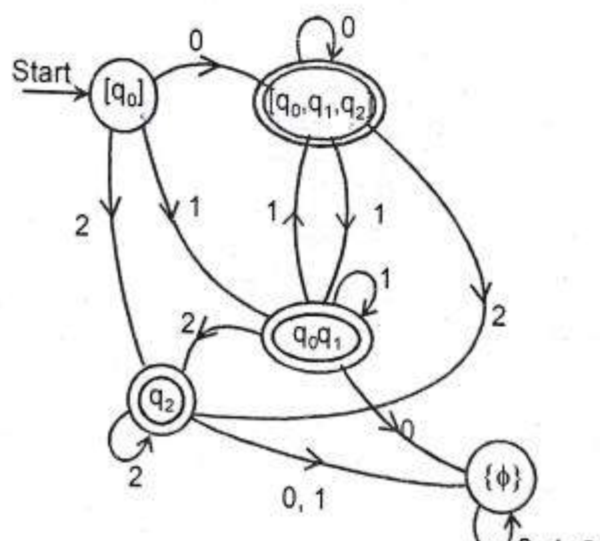


Construction of DFA from $\in$-NFA:

i) Construct first NFA from NFA with $\in$-moves

ii) NFA without $\in$ in the above figure is converted into equivalent DFA.

The DFA is



DFA Fig 2.15

Converting Regular Language to Regular Expressions

- Regular sets| Reg lang: Recognized by FA.
- Regular Expressions
  → Mathematical representations of lang accepted by FA.
  →Regular sets are expressed in simple algebraic forms.
  → Lang accepted by FA can be represented in RE.

- Applications of RE
  → pattern matching algorithm.
  → search Engines (google, yahoo)
  → Text Editors.
  → Web programming forms.

PRACTICE QUESTIONS

Reg set | Language	Reg Exp
1. L = { }	Φ
2. L = { ε }	ε
3. L = { a }	a
4. L = {ab, ba }	ab + ba or ab| ba

5. L = { ab, ba }* U { abb }	(ab + ba)* + abb
6. L = { ab, ba } ab	(ab+ba)* aab
7. L = { ε, a, b, aa, ab, baa, bb . . . }	(a+b)*
8. L = { ε, a, b, c, aa, ab, aac, baa . . .}	(aa+b+c)*
9. ∑ = { 0 } L = { ε, 0, 00, 000 . . . }	0*
10. L = { 0, 1 }* 00	(0+1)* 00

11. Set of strings begin with 1 and Ending with 0.
RE : 1 (0+1)* 0

12. Set of strings containg in 3 consecutive OS.
RE : (0+1)* 000 (0+1)*

13. Set of all strings such that 10th symbol from the right is 1.
RE : (0+1)* 1 (0+1)^9

14. Set of all strings such that 3rd symbol from LHS is 1
RE : (0+1)^2 1 (0+1)*

15. # 1's followed by exactly 2 consecutive 0s followed by # 1's.

RE : $1^* 00\ 1^*$

16. Set of all strings containing atleast two 0's.

RE : $1^*\ 01^*\ 0\ (\ 0+1)^*$ or $(\ 0+1\ )^*\ 0\ (\ 0+1)^*\ 0\ (\ 0+1)^*$

17. Set of all strings containing atmost two 0's

RE : $1^*\ (\ 0+\varepsilon\ )\ 1^*\ (\ 0+\varepsilon\ )^*\ 1$

18. Set of strings of length 2.

RE : $(\ 0+1\ )\ (\ 0+1\ ) = (\ 0+1\ )^2$

19. Strings of length 2 or more

RE : $(\ 0+1\ )\ (\ 0+1\ )\ (\ 0+1\ )^*$ or $(\ 0+1)^2\ (\ 0+1\ )^*$

20. Strings of length 3 or less

RE : $(\ 1+0+\varepsilon\ )\ (\ 1+0+\varepsilon\ )\ (\ 1+0+\varepsilon\ )$

Algebraic Laws forRegular Expressions

Given R, P, L, Q as regular expressions, the following identities hold:

1. $\emptyset^* = \varepsilon$
2. $\varepsilon^* = \varepsilon$
3. $RR^* = R^*R$
4. 4. $R^*R^* = R^*$
5. 5. $(R^*)^* = R^*$
6. $RR^* = R^*R$
7. $(PQ)^*P = P(QP)^*$

8. $(a+b)^* = (a^*b^*)^* = (a^*+b^*)^* = (a+b^*)^* = a^*(ba^*)^*$

9. $R + \emptyset = \emptyset + R = R$ (The identity for union)

10. $R\varepsilon = \varepsilon R = R$ (The identity for concatenation)

11. $\emptyset L = L\emptyset = \emptyset$ (The annihilator for concatenation)

12. $R + R = R$ (Idempotent law)

13. $L (M + N) = LM + LN$ (Left distributive law)

14. $(M + N) L = LM + LN$ (Right distributive law)

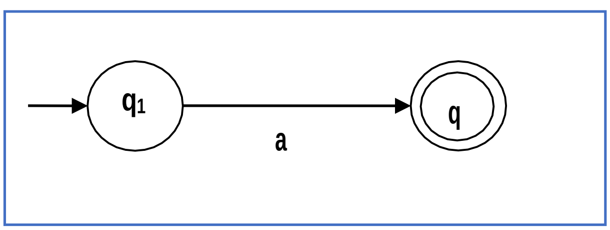
15. $\varepsilon + RR^* = \varepsilon + R^*R = R^*$

Construction of a Finite Automata from a Regular Expression

We can use Thompson's Construction to find out a Finite Automaton from a Regular Expression. We will reduce the regular expression into smallest regular expressions and converting these to NFA and finally to DFA.

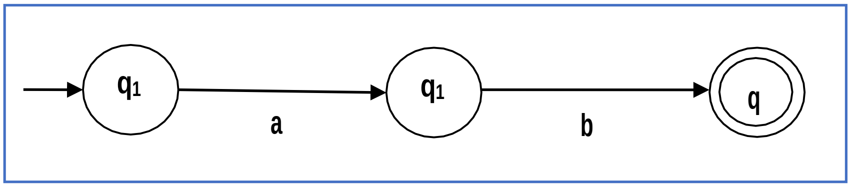
Some basic RA expressions are the following:

***Case 1*:** For a regular expression 'a', we can construct the following FA:



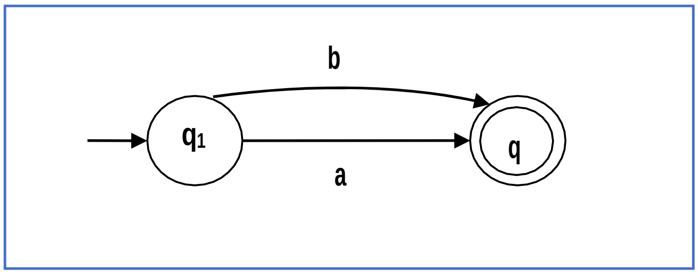
Finite automata for RE = a

***Case 2*:** For a regular expression 'ab', we can construct the following FA:



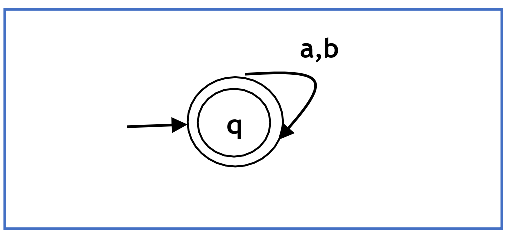
Finite automata for RE = ab

***Case 3*:** For a regular expression (a+b), we can construct the following FA:



Finite automata for RE= (a+b)

***Case 4*:** For a regular expression (a+b)*, we can construct the following FA:



Finite automata for RE= (a+b)*

Method:

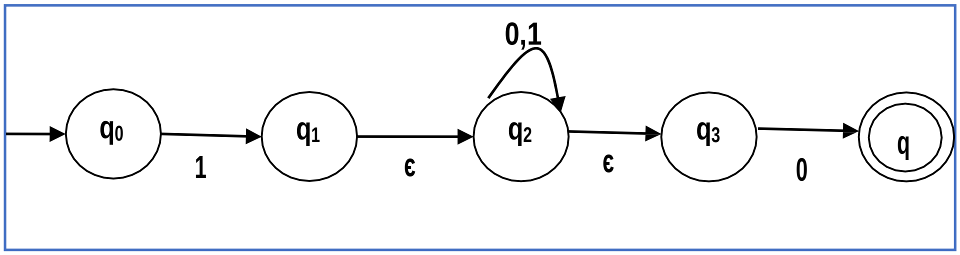
Step 1: Construct an NFA with Null moves from the given regular expression.

Step 2: Remove Null transition from the NFA and convert it into its equivalent DFA.

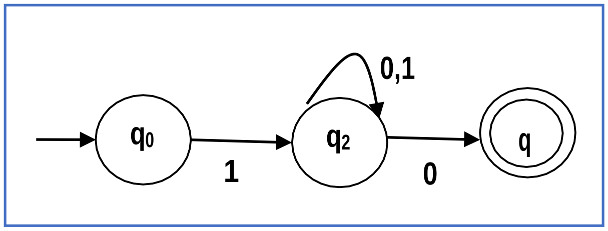
Problem **Convert the following RA into its equivalent DFA: 1 (0 + 1)* 0**

***Solution*:**

We will concatenate three expressions "1", "(0 + 1)*" and "0"



NDFA with NULL transition for RA: 1 (0 + 1)* 0



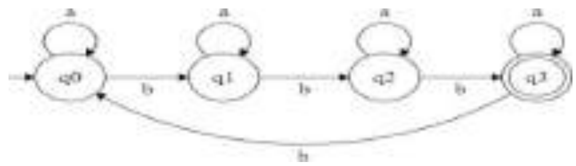
Now we will remove the **ϵ** transitions. After we remove the **ϵ** transitions from the NDFA, weget the following:NDFA without NULL transition for RA: 1 (0 + 1)* 0

It is an NDFA corresponding to the RE: 1 (0 + 1)* 0. If you want to convert it into a DFA, simplyapply the method of converting NDFA to DFA discussed in Chapter 1.

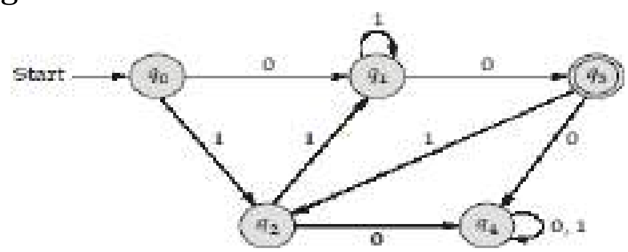
Examples:

Construct DFA for the regular expression given.

1. a (a + b) * a
2. a (a + b) * b
3. a $(a + b)^+$ a
4. a $(a + b)^+$ b
5. a*
6. a* b*
7. Construct a DFA for the regular language: ending with aab
8. Find the language accepted by the given DFA



9. Consider the following DFA.



10. Consider the following strings. Which of these below strings are accepted by the DFA?
    A. 011110 B. 101011 C. 010110 D. 1110110

==

Pumping Lemma

- Pumping lemma is used to prove that a language is Not Regular.
- But can't be used to prove that a language is regular.
-

If L is a Regular Language, L has a pumping length P such that any word W where $|w| >= p$ may be divided into 3 parts w = x y z then the following condition must be true.

1) x y z $\in$ L for every $i > 0$, the string xy^iz is also in L
2) $|y| > 0$ or $|y| \neq \varepsilon$
3) $|xy| <= P$

To prove that a language is not regular using pumping lemma follow the steps.

- Assume that L is regular.
- The pumping length is n.
- Words of length greater than p can be pumped $|w| >= n$
- Divide w into x y z.
- Show that x y $\notin$ L f for some i.
- So, none can satisfy all the above 3 conditions at a time.
- So, the assumption is wrong, proof by contradiction.

1. Prove that the language L $= \{a^n b^n \mid n >= 0\}$is not regular using pumping lemma.

Proof: Assume that L $= \{a^n b^n \mid n >= 0\}$is regular

Pumping length = K

w $= a^K b^k$ → aaaaaaabbbbbbb

Case1: The y is in the 'a' part

a a a a a a a b b b b b b b b x y z → x y^2 z

x y z a a a a a a a a a a a b b b b b b b

11 a's ≠ 7 b's

Case2: The y is in the 'b'; part

a a a a a a a b b b b b b b x y^1 z → x y^2 z

x y z a a a a a a a b b a a b b b b b b

Not even the pattern = $a^k b^k$

Language L is not regular.

==

2. Prove that the language L = {y y|y ∈{0, 1} * is not regular by using pumping lemma.
Assume that L is regular

The pumping length = n.

W = $0^n\ 1\ 0^n\ 1$

x y z k = 7

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 → x y^i z → x y^2 z

x y z 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1

110's L. $\notin$

|y| > 0

|xy| < k Not satisfied.

Contradiction by proof. So, L is not regular.

==

3.

RL or non-RL? → pumping lemma for RL.

For any RL, L ∃ an integer is dependent on L

Set ∀ Z ∈ L and |z| >= n

i) z = u v w then $\boxed{u\ v^i\ w \in L}$ ∀ i >= 0

ii) |uv| <= |z|

iii) v >= 1 or |v| # 0. u, v, w, z are words ∈

Example 1: L = {$a^m b^m$ | m >= 1} pumping means: u v' w

u v^2 w

u v^3 w . . . .

// counting, storing, comparing not possible

// finite lang. No problem.

Let assume L is a RL

Z ∈ L

Z = $a^k b^k$ → |z| >= n, we don't know the length of n or k

u v w chooses a k s.t |$a^k b^k$| = 2K >= n

1.Z → a^{k-1} a b^k → length of   v >= 1
2. | u v | = | a^{k-1} a | = | a^k | = k < = 2k
Check | v | ≠ 0, but 1
At first, it is assumed L is a RL   Hence uv^i w Є L ∀ I >= 0
Proof by contradiction   →   i = 2: u v   w
→ L is non-RL a^{k-1} a b^k = a^{k+1} b^k
Now it is m+1 not equal to b^m ∉ L
L = {a^m b^m | m >= 1} if i changes

==
Eg2: L = {a^m b^p | m > p}
Let assume L is a RL → by proof by contradiction assumption is incorrect. L is a non-RL
z ∈ L
z = a^{k+1} b^k choose k s.t |z| >= n
2k+1 >= n
a^{k+1} b b^{k-1}
1) |z| >= n satisfied

2) |uv| <= |z|
|v| ≠ 0,

3) Then u v^i w Є L ∀ I >= 0
Choose i = 2
u v   w
a^{k+1} b^2 b^{k-1} ∈ L
Because m > p
Proof by contradiction
Hence L is non regular

==
Eg 3: L = {a^{3n} | n >= 1}
1) z Є L

u   v w
a^{3n-1} a   Є → can be
but v = 1
or
v ≠ 0
2) |u v| < = | z |
3) Now check
U v w Є L ∀ I >= 0
Choose I = 2:
u v   w

a^{3n-1} a^2 Є → a^{3n+1} ∉ L

because L = a^{3n} but ↑ say a's and b's are equal.
By proof by contradiction. it is not.
Special case of pumping lemma when $\sum = \{a\}$
Singleton set.
Length of strings must follow AP for a L tube a RL.

$$u\, v^i\, w \rightarrow a\, (aa)^i\, a \rightarrow \{a\, a,\ a\, a\, a\, a,\ a\, a\, a\, a\, a\, a\}$$
2 — 4 — 6 —

==

Eg 4: L = $\{a^{3n} \mid n >= 1\}$ $\sum = \{a\}$
a^3, b^6, a^9, a^{12} .... f n = 1
RL n = 2
3 6 9 12 . . .. n = 3
Diff: 3 3 3 n = 4

Eg 6: L = $\{a^{2n+1} \mid n >= 0\}$ → RL
1, 3, 5, 7 ....AP
If n = 0
n = 1
n = 2
n = 3
1, 3, 5, 7 . . ..

==

Eg 7: L = $\{a^{n^3 + n^2 + 1} \mid n >= 0\}$ → non RL if n =0
1, 3, 13 . . . Not AP n = 1
N = 2

Eg 8: L = $\{a^{2^n} \mid n >= 0\}$ Non-RL If n = 0
n = 1
n = 2
1, 2, 4, 8, 16 . . ≠ AP n = 3

Eg 9: L = $\{a^p \mid$ p is a prime$\}$ non-RL
Prime numbers 2, 3, 5, 7, 11.........≠ AP
1 2 2 4........ ≠ AP

================================00oo00===