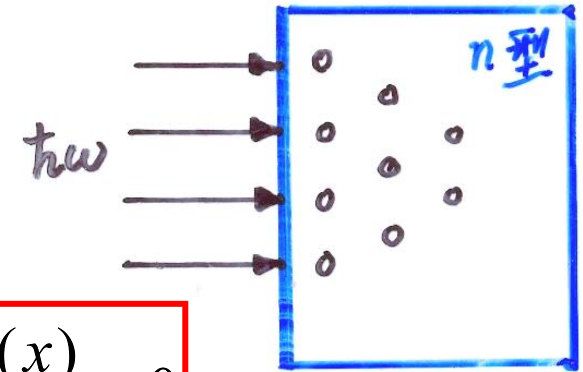
7.5 载流子的扩散运动2

7.5.2 一维扩散方程的稳态解

$$\boxed{\frac{\partial \Delta p(x,t)}{\partial t} = D_p \frac{\partial^2 \Delta p(x,t)}{\partial x^2} - \frac{\Delta p(x,t)}{\tau}}$$



稳态 $\frac{\partial \Delta p(x,t)}{\partial t} = 0$

$$\boxed{D_p \frac{d^2 \Delta p(x)}{dx^2} - \frac{\Delta p(x)}{\tau} = 0}$$

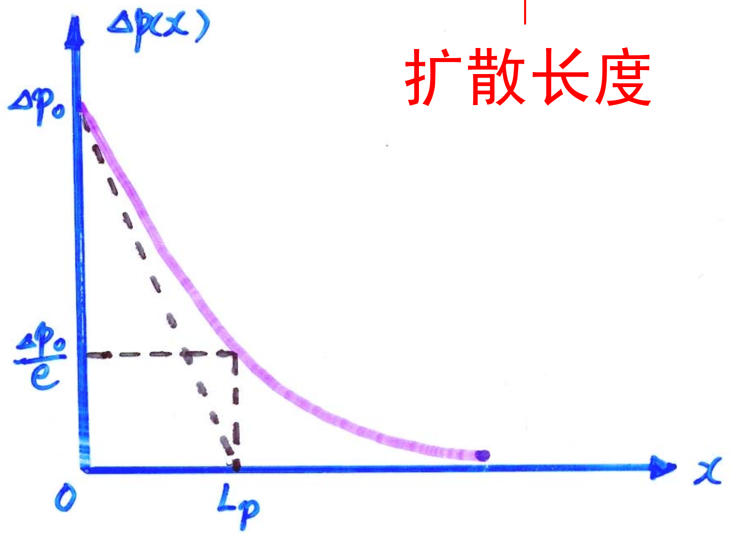
通解 $\Delta p(x) = A\exp(-x/L_p) + B\exp(x/L_p)$ 其中 $\boxed{L_p = \sqrt{D_p \tau}}$

扩散长度

(1) 样品厚度足够厚

边界条件 $\begin{cases} \Delta p(0) = \Delta p_0 \\ \Delta p(+\infty)\text{有限} \end{cases}$

$$\boxed{\Delta p(x) = \Delta p_0 \exp(-x/L_p)}$$



7.5 载流子的扩散运动3

7.5.2 一维扩散方程的稳态解

$$\boxed{\Delta p(x) = \Delta p_0 \exp\left(-x/L_p\right)}$$

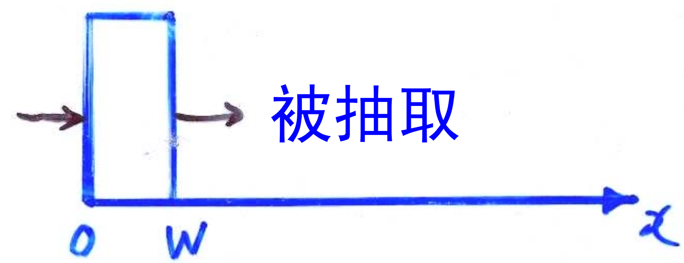
平均扩散距离 $\bar{x} = \dfrac{\int_0^\infty x\Delta p(x)dx}{\int_0^\infty \Delta p(x)dx} = L_p$

流密度 $s_p(x) = \dfrac{D_p}{L_p}\Delta p_0 \exp\left(-x/L_p\right) = \left(\dfrac{D_p}{L_p}\right)\Delta p(x)$

扩散速度 [cm/s]

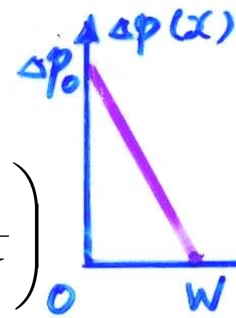
通解 $\Delta p(x) = A\exp\left(-x/L_p\right) + B\exp\left(x/L_p\right)$

(2) 样品厚度足够薄，且在另一端被抽出



边界条件 $\begin{cases} \Delta p(0) = \Delta p_0 \\ \Delta p(W) = 0 \end{cases}$

$$\Delta p(x) = \Delta p_0 \frac{\sinh\left[(W-x)/L_p\right]}{\sinh\left(W/L_p\right)} \overset{W \ll L_p}{=} \Delta p_0\left(1 - \frac{x}{W}\right)$$



$$\frac{d\Delta p(x)}{dx} = 常数$$

复合 ＝ 0

7.5 载流子的扩散运动4

7.5.3 扩散电流

扩散流

$$s_p = -D_p \frac{d\Delta p(x)}{dx}$$

$$s_n = -D_n \frac{d\Delta n(x)}{dx}$$

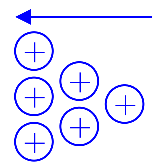
扩散电流

$$J_p = -qD_p \frac{d\Delta p(x)}{dx}$$

$$J_n = qD_n \frac{d\Delta n(x)}{dx}$$

扩散电流

梯度方向

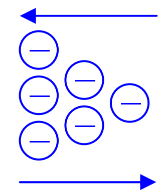


扩散方向

电流方向

电流方向

梯度方向



扩散方向

漂移电流

电子、空穴电流都与电场方向一致

总电流

扩散电流+漂移电流

$$\boxed{J_p = -qD_p \frac{d\Delta p(x)}{dx} + qp\mu_p E}$$

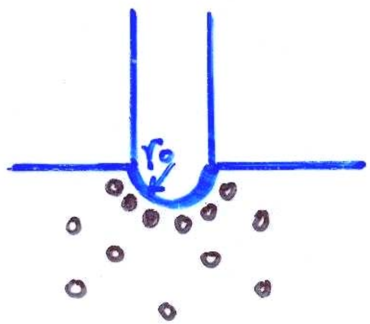
空穴电流

$$\boxed{J_n = qD_n \frac{d\Delta n(x)}{dx} + qn\mu_n E}$$

电子电流

7.5 载流子的扩散运动5

7.5.4 例子：三维探针注入



三维 $$\frac{\partial \Delta p(\mathbf{r},t)}{\partial t} = D_p \nabla^2 \Delta p(\mathbf{r},t) - \frac{\Delta p(\mathbf{r},t)}{\tau}$$

稳态 $$\frac{\partial \Delta p(\mathbf{r},t)}{\partial t} = 0 \longrightarrow \boxed{D_p \nabla^2 \Delta p(\mathbf{r},t) = \frac{\Delta p(\mathbf{r},t)}{\tau}}$$

球坐标 $$D_p \frac{1}{r^2} \frac{d}{dr}\left[r^2 \frac{d\Delta p}{dr} \right] = \frac{\Delta p(\mathbf{r},t)}{\tau}$$

$$\longrightarrow \boxed{\Delta p(r) = \Delta p_0 \frac{r_0}{r} \exp\left(-\frac{r-r_0}{L_p} \right)}$$

$$\mathbf{s}_p\Big|_{r=r_0} = -D_p \nabla \Delta p(r)\Big|_{r=r_0} = \left(\boxed{\frac{D_p}{r_0}} + \frac{D_p}{L_p} \right) \Delta p_0$$

几何形状引起的扩散速度

第七章 非平衡载流子

7.6 载流子的漂移运动、双极扩散1

7.6.1 浓度梯度引起的自建电场

—热平衡状态

—n型半导体，掺杂不均匀

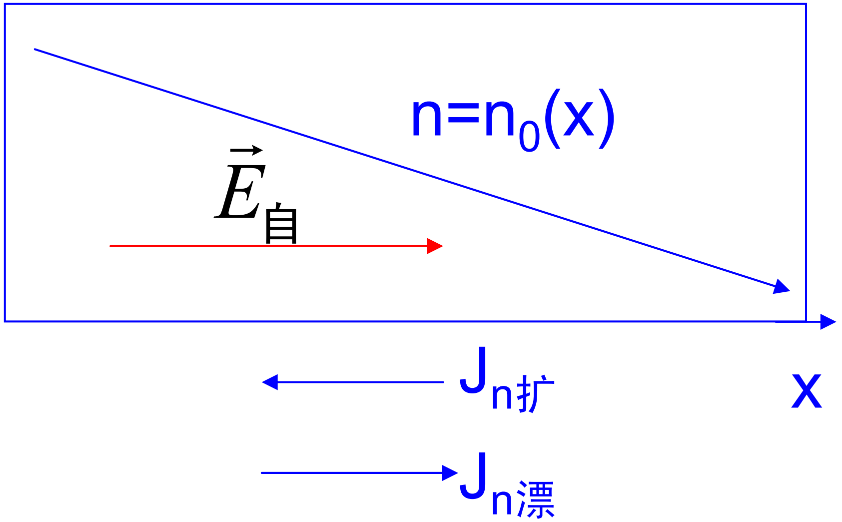
—$n_0(x)$梯度引起扩散电流

—电中性条件破坏，引起自建电场

—考虑漂移电流

$$J_{n扩} = qD_n \frac{dn_0(x)}{dx}$$

$$J_{n漂} = qn_0\mu_n E_{自}$$



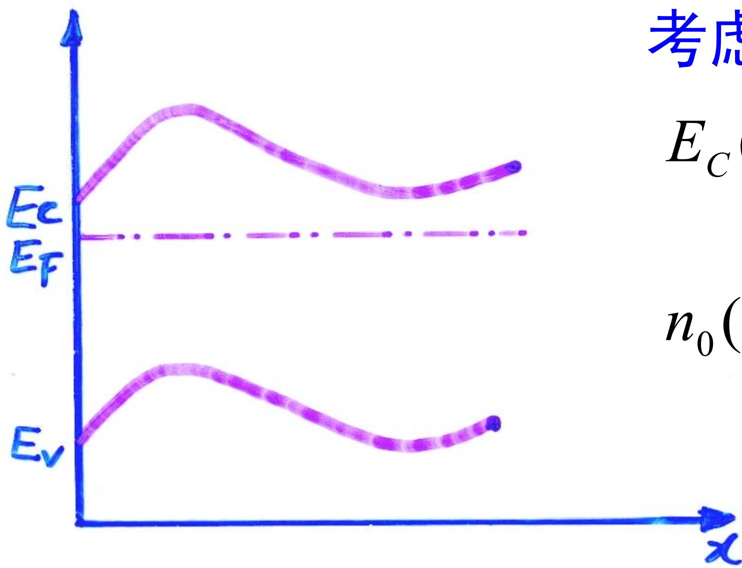
热平衡状态

$$J_{n扩} + J_{n漂} = 0$$

$$\boxed{E_{自} = -\frac{D_n}{\mu_n n_0(x)} \frac{dn_0(x)}{dx}} \neq 0$$

7.6 载流子的漂移运动、双极扩散2

7.6.2 爱因斯坦关系



考虑一平衡的不均匀半导体，静电势 $V(x)$

$$E_C(x) = E_C + (-q)V(x) \qquad E_{自} = -\frac{dV(x)}{dx}$$

$$n_0(x) = N_C \exp\left[-\frac{E_C - qV(x) - E_F}{kT}\right]$$

$$\boxed{J_{总} = qn_0(x)\mu_n E_{自}(x) + qD_n\frac{dn_0(x)}{dx} = 0}$$

将 $n_0(x)$ 代入，得

$$\left.\begin{array}{l}\boxed{D_n = \dfrac{kT}{q}\mu_n} \\ \boxed{D_p = \dfrac{kT}{q}\mu_p}\end{array}\right\} \text{爱因斯坦关系}$$

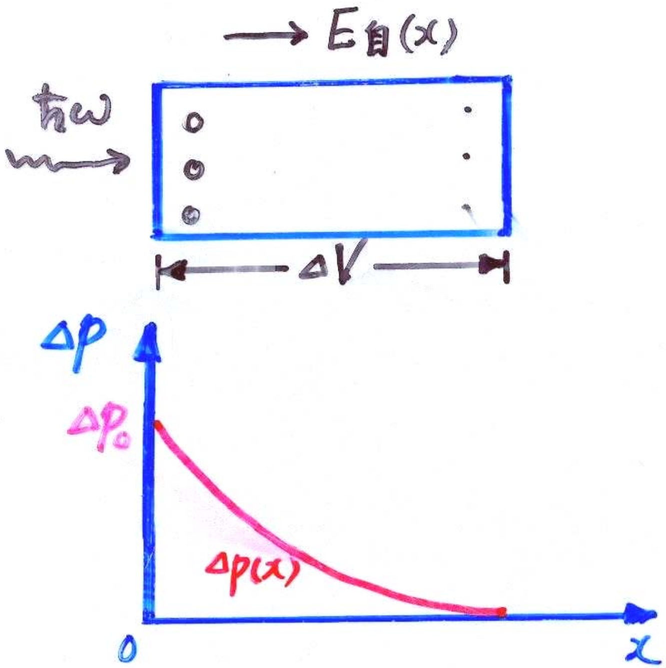
也适合非平衡载流子

通常 $\mu_n > \mu_p \longrightarrow D_n > D_p$ —— 电子与空穴的扩散不同步，电子快，空穴慢

7.6 载流子的漂移运动、双极扩散3

7.6.3 丹倍效应

自建场（丹倍电场）



$$J_{扩}+J_{漂}=-qD_p\frac{d\Delta p(x)}{dx}+qD_n\frac{d\Delta n(x)}{dx}$$
$$+qp\mu_pE_{自}+qn\mu_nE_{自}=0$$

近似关系 $\frac{d\Delta p}{dx}=\frac{d\Delta n}{dx}$

$$E_{自}=\frac{D_p-D_n}{n\mu_n+p\mu_p}\frac{d\Delta p}{dx}$$

$$\Delta V=-\int E_{自}(x)dx=\int\frac{D_n-D_p}{n\mu_n+p\mu_p}\frac{d\Delta p}{dx}dx$$

对于 $W>>L_p$ 情形

$$\Delta p=\Delta p_0\exp\left(-x/L_p\right)$$

$$\Delta V=\frac{D_n-D_p}{n\mu_n+p\mu_p}\Delta p_0=\frac{kT}{q}\frac{\mu_n-\mu_p}{n\mu_n+p\mu_p}\Delta p_0=\begin{cases}\frac{kT}{q}\frac{\mu_n-\mu_p}{\mu_n}\frac{\Delta p_0}{n_0} & \text{n 型}\\ \frac{kT}{q}\frac{\mu_n-\mu_p}{\mu_p}\frac{\Delta p_0}{p_0} & \text{p 型}\end{cases}$$

7.6 载流子的漂移运动、双极扩散4

7.6.4 双极扩散

空穴电流

$$J_p = -qD_p \frac{d\Delta p(x)}{dx} + qp\mu_p E$$

$E_{自} + E_{外}$

$$= \underbrace{-qD_p \frac{d\Delta p(x)}{dx} + qp\mu_p E_{自}}_{J'_{p扩}} + qp\mu_p E_{外}$$

$$J'_{p扩} = -qD_p \frac{d\Delta p(x)}{dx} + \underbrace{qp\mu_p \frac{D_p - D_n}{p\mu_p + n\mu_n}}_{qpD_p \frac{D_p - D_n}{pD_p + nD_n}} \frac{d\Delta p}{dx}$$

$$\boxed{D_n = \frac{kT}{q}\mu_n}$$

$$\boxed{D_p = \frac{kT}{q}\mu_p}$$

$$\boxed{J'_{p扩} = -qD\frac{d\Delta p(x)}{dx}}$$

其中

$$\boxed{D = \frac{D_p D_n (p+n)}{pD_p + nD_n}}$$

双极扩散

7.6 载流子的漂移运动、双极扩散5

7.6.4 双极扩散

双极扩散系数 $D=\dfrac{D_pD_n(p+n)}{pD_p+nD_n}$ $\begin{cases} D_p & \text{n 型，} n>>p \\ D_n & \text{p 型，} p>>n \end{cases}$

电子电流 $J_n=qD_n\dfrac{d\Delta n}{dx}+qn\mu_nE$

$E_{自}+E_{外}$

$$=\boxed{qD_n\frac{d\Delta n}{dx}+qn\mu_nE_{自}}+qn\mu_nE_{外}$$

$$J'_{n扩}=qD\frac{d\Delta n}{dx}$$

丹倍电场、双极扩散的物理意义

1º 丹倍电场的来源——电子与空穴扩散不同步，电子比空穴快；

2º 丹倍电场的作用——降低电子扩散，加速空穴扩散，努力使它们同步；

7.6 载流子的漂移运动、双极扩散6

7.6.4 双极扩散

丹倍电场、双极扩散的物理意义

3° 双极扩散系数 D —— 概括了丹倍电场对电子、空穴扩散的影响.

$$D_n > D_{双极} > D_p$$

$$\boxed{D = \frac{D_p D_n (p+n)}{pD_p + nD_n}}$$

对于 n 型半导体 $D_n > D_{双极} \sim D_p$ 即丹倍电场对空穴扩散的影响小
对电子扩散的影响大

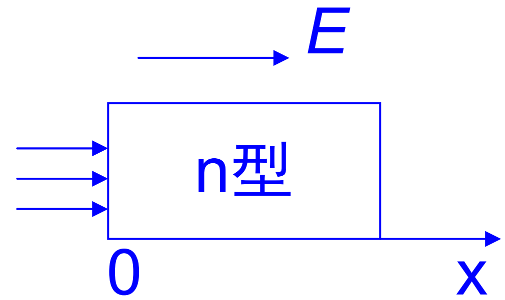
对于 p 型半导体 $D_n \sim D_{双极} > D_p$ 即丹倍电场对电子扩散的影响小
对空穴扩散的影响大

结论：丹倍电场对少子扩散影响小
对多子扩散影响大

第七章 非平衡载流子

7.7 连续性方程1

7.7.1 连续性方程



一维， n 型，外电场 *E*

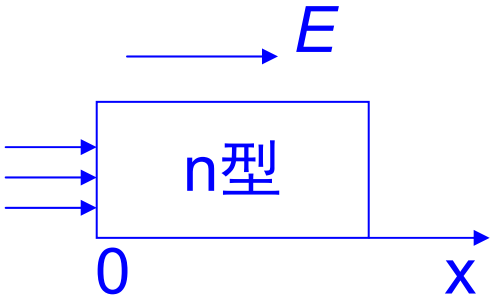
少子 $p(x,t)$ $$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - \mu_p p \frac{\partial E}{\partial x} - \frac{\Delta p}{\tau} + g_p$$

扩散 漂移 复合 产生

7.7 连续性方程2

7.7.2 连续性方程的特例情况

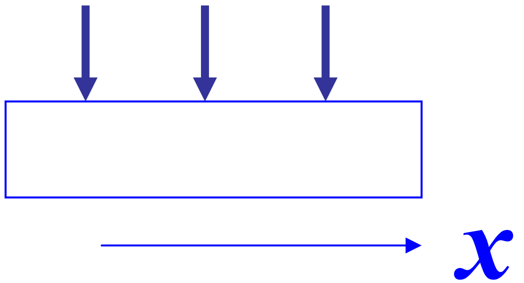
$$\boxed{\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - \mu_p p \frac{\partial E}{\partial x} - \frac{\Delta p}{\tau} + g_p}$$



1.光激发的载流子衰减

$$E = 0 \quad \frac{\partial E}{\partial x} = 0 \quad \frac{\partial \Delta p}{\partial x} = 0 \quad g_p = 0$$

（t = 0 时撤去光照）



均匀掺杂薄样品

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau} \Longrightarrow \Delta p = \Delta p_0 \exp(-t/\tau)$$

7.7 连续性方程3

7.7.2 连续性方程的特例情况

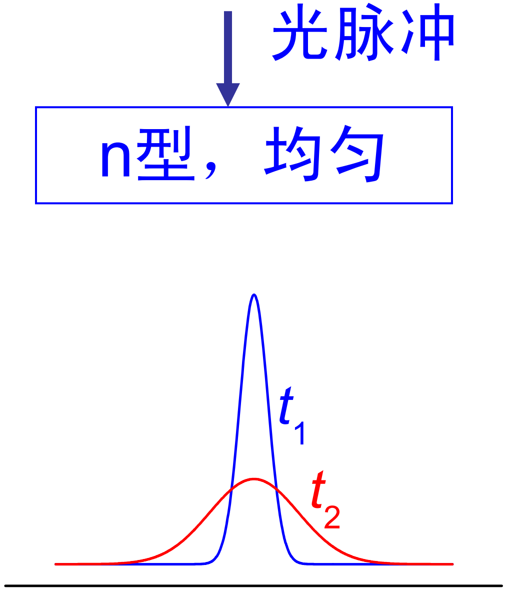
$$\boxed{\frac{\partial p}{\partial t}=D_p\frac{\partial^2 p}{\partial x^2}-\mu_p E\frac{\partial p}{\partial x}-\mu_p p\frac{\partial E}{\partial x}-\frac{\Delta p}{\tau}+g_p}$$

2. 瞬时光脉冲 $E=0\quad \frac{\partial E}{\partial x}=0\quad g_p=0$

$$\frac{\partial(\Delta p)}{\partial t}=D_p\frac{\partial^2(\Delta p)}{\partial x^2}-\frac{\Delta p}{\tau}$$

$$\Delta p(x,t)=\frac{N_p}{\sqrt{4\pi D_p t}}\exp\left(-\frac{t}{\tau}\right)\exp\left(-\frac{x^2}{4D_p t}\right)$$

$$\int_{-\infty}^{+\infty}\Delta p(x,t)dx=N_p\exp(-t/\tau)$$

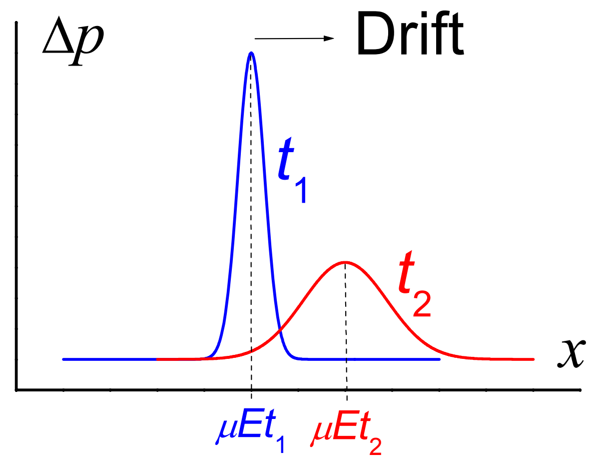


高斯分布

7.7 连续性方程4

7.7.2 连续性方程的特例情况

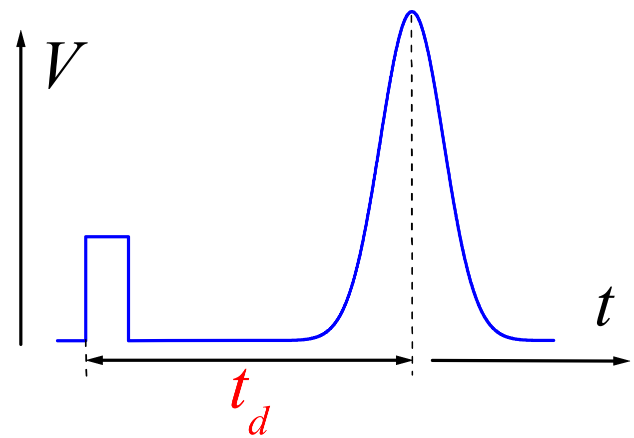
$$\boxed{\frac{\partial p}{\partial t}=D_p\frac{\partial^2 p}{\partial x^2}-\mu_p E\frac{\partial p}{\partial x}-\mu_p p\frac{\partial E}{\partial x}-\frac{\Delta p}{\tau}+g_p}$$

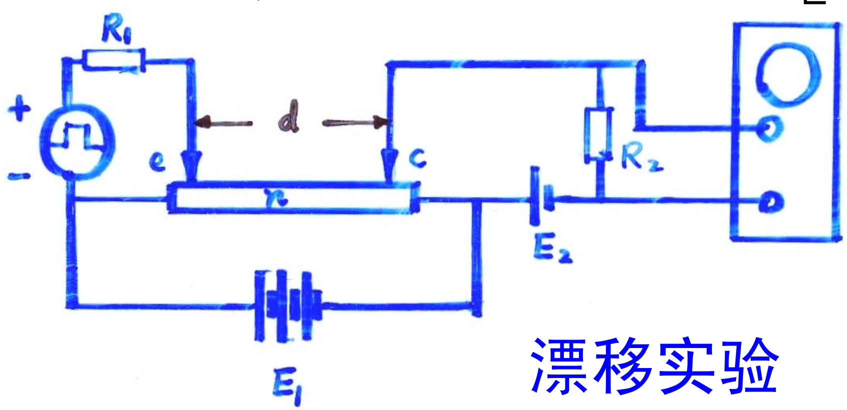


3. 瞬时电脉冲 $E\neq 0 \quad \frac{\partial E}{\partial x}=0 \quad g_p=0$

$$\frac{\partial(\Delta p)}{\partial t}=D_p\frac{\partial^2(\Delta p)}{\partial x^2}-\mu_p E\frac{\partial(\Delta p)}{\partial x}-\frac{\Delta p}{\tau}$$

$$\Delta p(x,t)=\frac{N_p}{\sqrt{4\pi D_p t}}\exp\left(-\frac{t}{\tau}\right)\exp\left[-\frac{(x-\mu_p E t)^2}{4D_p t}\right]$$





漂移实验

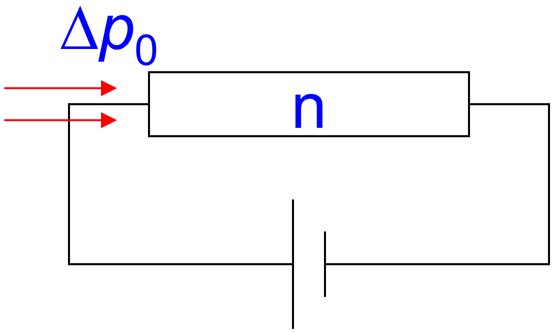
$$\boxed{\mu_{drift}=\frac{d}{Et_d}}$$

7.7 连续性方程5

7.7.2 连续性方程的特例情况

4. 光照恒定，稳态

$$\boxed{\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - \mu_p p \frac{\partial E}{\partial x} - \frac{\Delta p}{\tau} + g_p}$$



$$0 = D_p \frac{\partial^2 (\Delta p)}{\partial x^2} - \mu_p E \frac{\partial (\Delta p)}{\partial x} - \frac{\Delta p}{\tau}$$

通解 $\Delta p(x) = A\exp(\lambda_1 x) + B\exp(\lambda_2 x)$

$$\lambda_{1,2} = \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}$$

这里 $L_p(E) = \mu_p E \tau$ 牵引长度

只能是衰减解，正根舍去

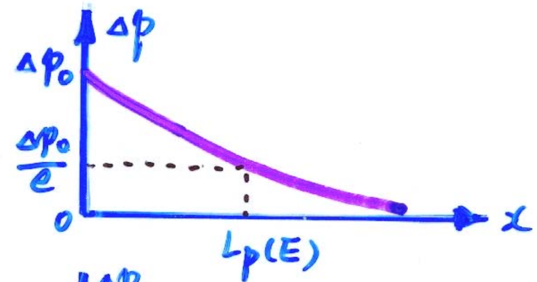
$$\Delta p(x) = \Delta p_0 \exp\left[\frac{L_p(E) - \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x\right]$$

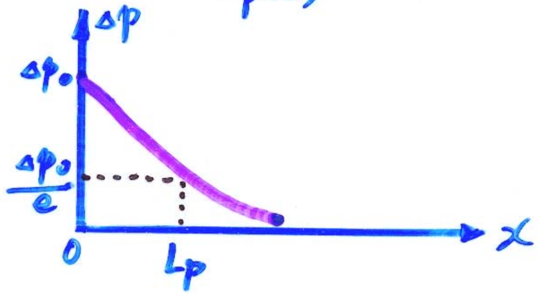
$$= \Delta p_0 \exp\left[-\frac{2}{\sqrt{L_p^2(E) + 4L_p^2} + L_p(E)} x\right]$$

7.7 连续性方程6

7.7.2 连续性方程的特例情况

4．光照恒定，稳态





$$\Delta p(x) = \Delta p_0 \exp\left[-\frac{2}{\sqrt{L_p^2(E)+4L_p^2}+L_p(E)}x\right]$$

1° 当 E 很大时，$L_p(E) >> L_p$，$\lambda_2 \to -1/L_p(E)$

$$\Delta p(x) = \Delta p_0 \exp\left[-x/L_p(E)\right]$$

$$L_p(E) = \mu_p E\tau$$

2° 当 E 很小时，$L_p(E) << L_p$，$\lambda_2 \to -1/L_p$

$$\Delta p(x) = \Delta p_0 \exp\left(-x/L_p\right)$$

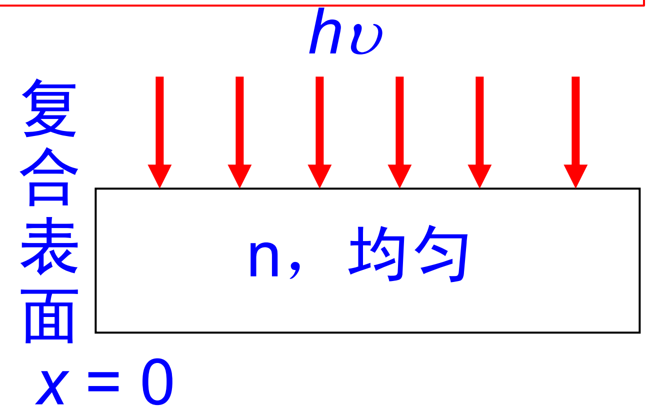
7.7 连续性方程7

7.7.2 连续性方程的特例情况

$$\boxed{\frac{\partial p}{\partial t}=D_p\frac{\partial^2 p}{\partial x^2}-\mu_p E\frac{\partial p}{\partial x}-\mu_p p\frac{\partial E}{\partial x}-\frac{\Delta p}{\tau}+g_p}$$

5. 稳态下的表面复合

$$D_p\frac{\partial^2(\Delta p)}{\partial x^2}-\frac{\Delta p}{\tau}+g_p=0$$

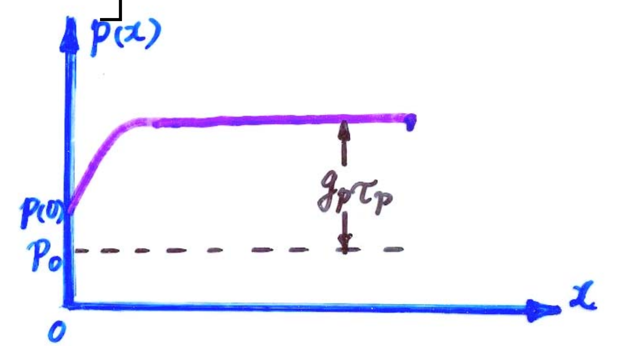


边界条件 $\begin{cases}\Delta p(+\infty)=g_p\tau_p \\ D_p\left.\dfrac{\partial(\Delta p)}{\partial x}\right|_{x=0}=s_p\left[p(0)-p_0\right]\end{cases}$

解为 $p(x)=p_0+g_p\tau_p\left[1-\dfrac{s_p\tau_p}{L_p+s_p\tau_p}\exp\left(-x/L_p\right)\right]$

当 $s_p\to 0$，$p(x)\to p_0+g_p\tau_p$ 均匀分布

当 $s_p\to\infty$，$p(0)\to p_0$



7.7 连续性方程8

7.7.3 连续性方程的一般情形

一般情形 $n \sim p$（近本征情形）

$$\begin{cases} \dfrac{\partial \Delta p}{\partial t} = D_p \dfrac{\partial^2 \Delta p}{\partial x^2} - \mu_p E \dfrac{\partial \Delta p}{\partial x} - \mu_p p \dfrac{\partial E}{\partial x} - \dfrac{\Delta p}{\tau} & ① \\ \dfrac{\partial \Delta n}{\partial t} = D_n \dfrac{\partial^2 \Delta n}{\partial x^2} + \mu_n E \dfrac{\partial \Delta n}{\partial x} + \mu_n n \dfrac{\partial E}{\partial x} - \dfrac{\Delta n}{\tau} & ② \end{cases}$$

假设是均匀半导体，外加均匀电场

$$E = E_{外} + E_{自}$$

来源于Δp, Δn 扩散的不同步

均匀

一般情况下 $E_{自} << E_{外}$ $\longrightarrow$ $E \approx E_{外}$

而 $$\frac{\partial E}{\partial x} = \frac{\partial E_{自}}{\partial x} = \frac{q(\Delta p - \Delta n)}{\varepsilon_0 \varepsilon_r}$$ —— 泊松方程 $\boxed{\nabla \cdot \vec{D} = \rho}$

7.7 连续性方程9

7.7.3 连续性方程的一般情形

$$\begin{cases} \dfrac{\partial \Delta p}{\partial t} = D_p \dfrac{\partial^2 \Delta p}{\partial x^2} - \mu_p E \dfrac{\partial \Delta p}{\partial x} - \mu_p p \dfrac{\partial E}{\partial x} - \dfrac{\Delta p}{\tau} & ① \\ \dfrac{\partial \Delta n}{\partial t} = D_n \dfrac{\partial^2 \Delta n}{\partial x^2} + \mu_n E \dfrac{\partial \Delta n}{\partial x} + \mu_n n \dfrac{\partial E}{\partial x} - \dfrac{\Delta n}{\tau} & ② \end{cases}$$

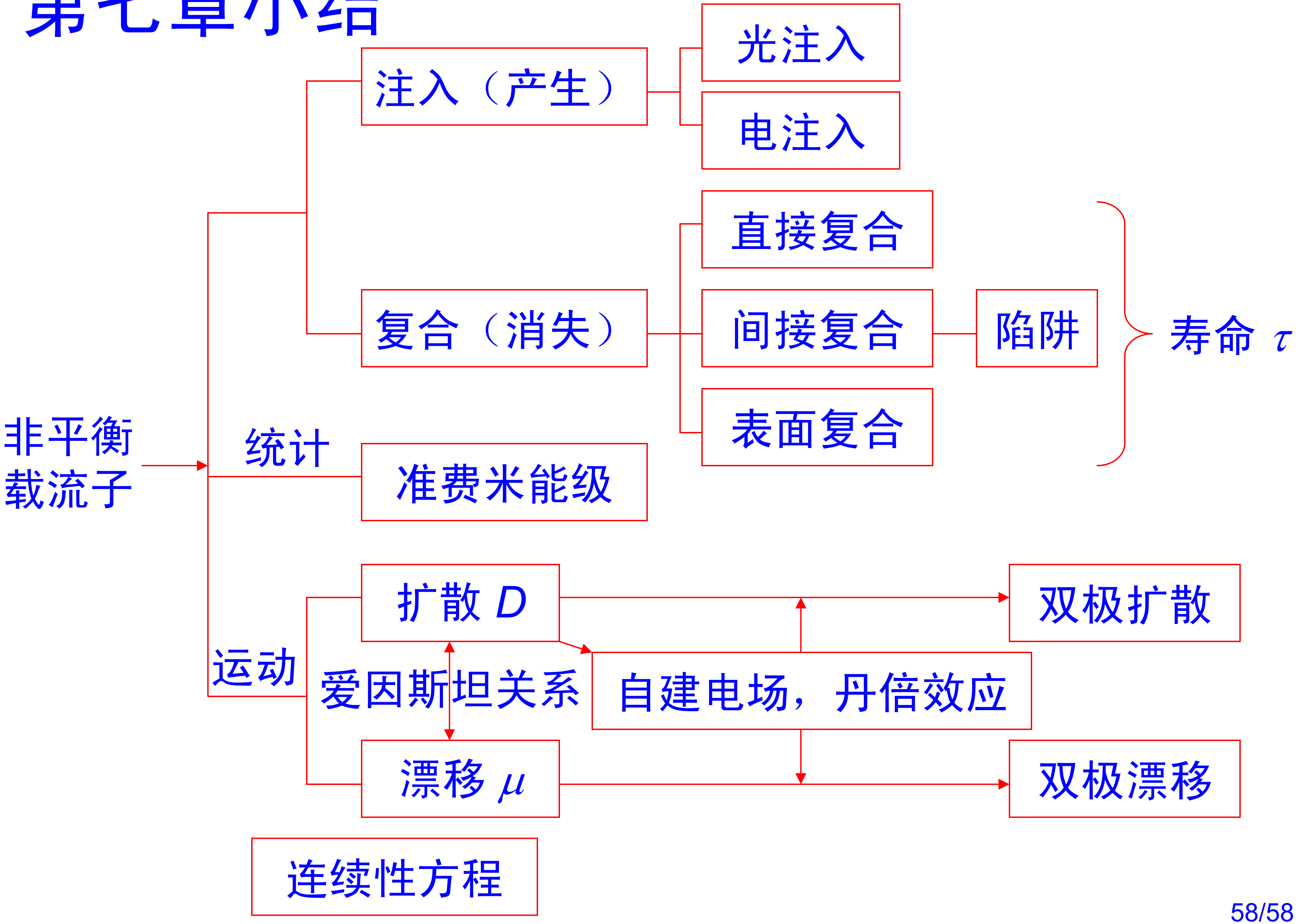
①×$\mu_n n$+②×$\mu_p p$ （假设 $\Delta n \approx \Delta p \quad \dfrac{\partial \Delta n}{\partial x} \approx \dfrac{\partial \Delta p}{\partial x}$ ）

$$(\mu_n n + \mu_p p)\frac{\partial \Delta p}{\partial t} = (\mu_n n D_p + \mu_p p D_n)\frac{\partial^2 \Delta p}{\partial x^2} - (n-p)\mu_n \mu_p E \frac{\partial \Delta p}{\partial x} - (\mu_n n + \mu_p p)\frac{\Delta p}{\tau}$$

$$\longrightarrow \boxed{\frac{\partial \Delta p}{\partial t} = D \frac{\partial^2 \Delta p}{\partial x^2} - \mu E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau}} \qquad \mu = \frac{(n-p)\mu_n \mu_p}{n\mu_n + p\mu_p}$$ —— 双极迁移率

$$D = \frac{n\mu_n D_p + p\mu_p D_n}{n\mu_n + p\mu_p} = \frac{(n+p)D_n D_p}{nD_n + pD_p}$$ —— 双极扩散系数

第七章小结



非平衡载流子
注入（产生）
光注入
电注入
复合（消失）
直接复合
间接复合
陷阱
表面复合
寿命 τ
统计
准费米能级
运动
扩散 D
漂移 μ
爱因斯坦关系
自建电场，丹倍效应
双极扩散
双极漂移
连续性方程