

```

public class Tree extends BTTreePrinter {
    Node root;
    public Tree(Node root) { this.root = root; }
    public Tree() {}
    public void printTree() {
        if (root == null) printEmpty
        else super.printTree(root);
    }
    public static void printNode(Node node) {
        if (node == null) print not found
        else print note.key;
    }
    public Node find(int search_key) { // recursive
        return find(root, search_key);
    }
    public static Node find(Node node, int search_key) {
        if (node == null) return null;
        if (node.key == search_key) return node;
        else if (node.key < search_key) return find(node.right, search_key);
        else if (node.key > search_key && node.left != null)
            return find(node.left, search_key);
        else return null;
    }
    public Node findMin() { // recursive
        return findMin(root);
    }
    public static Node findMin(Node node) {
        if (node.left == null) return node;
        return findMin(node.left);
    }
    public Node findMax() { // recursive
        return findMax(root);
    }
    public static Node findMax(Node node) {
        if (node.right == null) return node;
        return findMax(node.right);
    }
    public Node findClosestLeaf(int search_key) { // recursive
        return findClosestLeaf(root, search_key);
    }
    public static Node findClosestLeaf(Node node, int search_key)
        if (node == null) return null;
        if (node.key > search_key) {
            if (node.left != null)
                return findClosestLeaf(node.left, search_key);
            else return node;
        } else if (node.key < search_key) {
            if (node.right != null)
                return findClosestLeaf(node.right, search_key);
            else return node;
        } return null;
    }
    public Node findClosest(int search_key) {
        Node current, closest;
        closest = current = root;
        int min_diff = Integer.MAX_VALUE;
        while (current != null) {
            if (Math.abs(current.key - search_key) < min_diff,
                min_diff = Math.abs(current.key - search_key));
            closest = current;
            if (current.key < search_key)
                current = current.right;
            else if (current.key > search_key)
                current = current.left;
            else break; // return closest;
        }
        public void insert(int key) {
            Node Parent = findClosestLeaf(root, key);
            Node Child = new Node(key);
            if (Parent == null) root = Child;
            else {
                Child.parent = Parent;
                if (Parent.key > key) Parent.left = Child;
                else Parent.right = Child;
            }
        }
        public void printPreOrderDFT() {
            System.out.print("PreOrder DFT node sequence [ ");
            printPreOrderDFT(root); System.out.println("]");
        }
        public static void printPreOrderDFT(Node node) {
            if (node == null) return;
            System.out.print(node.key + " ");
            printInOrderDFT(node.left);printPreOrderDFT(node.right);
        }
        public void printInOrderDFT() {
            System.out.print("InOrder DFT node sequence [ ");
            printInOrderDFT(root); System.out.println("]");
        }
        public static void printInOrderDFT(Node node) {
            if (node == null) return;
            printInOrderDFT(node.left);System.out.print(node.key + " ");
            printInOrderDFT(node.right);
        }
        public void printPostDFT() {
            System.out.print("PostOrder DFT node sequence [ ");
            printPostOrderDFT(root);System.out.println("]");
        }
        public static void printPostOrderDFT(Node node) {
            if (node == null) return;
            printPostOrderDFT(node.left); printPostOrderDFT(node.right);
            System.out.print(node.key + " ");
        }
        public static int height(Node node) {
            if (node == null) return -1;
            return Math.max(height(node.left), height(node.right)) + 1;
        }
        public static int size(Node node) {
            if (node == null) return 0;
            return size(node.left) + size(node.right) + 1;
        }
        public class Node {
            Node left; Node right; Node parent; int key;
            public Node(int key) {this.key = key; }
        }
        public class List {
            Node[] arr; int capacity; int size;
            public List(int cap) {
                arr = new Node[cap]; capacity = cap;
            }
            public void appendNode() {
                arr[size] = new Node(); arr[size].key = key;
                arr[size].size = node; size++;
            }
        }
        public void printList() {
            System.out.print("[Head] ");
            for (int i = 0; i < size; i++) {
                System.out.print(arr[i].key + " ");
            }
            System.out.println("[Tail] ");
        }
    }
}

public static int depth(Node root, Node node) {
    if (node == null) return -1;
    if (node == root) return 0;
    return depth(root, node.parent) + 1;
}
public int height() { return height(root); }
public int size() { return size(root); }
public int depth() { return depth(root); }
public Node findKthSmallest(int k) { return findKthSmallest(root, k); }
public static Node findKthSmallest(Node node, int k) {
    int l = size(node.left);
    if (k == l + 1) return node;
    else if (k < l + 1) return findKthSmallest(node.left, k);
    else return findKthSmallest(node.right, k - l - 1);
}
public static Node findNext(Node node) {
    if (node.right == null) return leftDescendant(node.right);
    else return rightAncestor(node);
}
public static Node leftDescendant(Node node) {
    if (node.left == null) return node;
    else return leftDescendant(node.left);
}
public static Node rightAncestor(Node node) {
    if (node.parent == null) return null;
    if (node.key < node.parent.key) return node.parent;
    else return rightAncestor(node.parent);
}
public List rangeSearch(int x, int y) {
    List l = new List(100); Node N = findClosest(x);
    while (N != null && N.key < y) {
        if (N.key >= x) L.append(N);
        N = findNext(N); } return l;
}
public void delete(int key) {
    Node del = find(key);
    if (root == null) System.out.println("Empty Tree!!!");
    else if (del == null) System.out.println("Key not found!!!!");
    else if (del == root) {
        if (root.left == null && root.right == null) root = null;
        else if (root.left != null && root.right == null) {
            Node newRoot = root.left; newRoot.parent = null;
            root.left = null; root = newRoot;
        } else if (root.left == null && root.right != null) {
            Node newRoot = root.right; newRoot.parent = null;
            root.right = null; root = newRoot;
        } else {
            Node min = findMin(root.right); int min_value = min.key;
            delete(min); root.key = min_value; }
        else { delete(del); }
    }
    public static void delete(Node node) {
        if (node.left == null && node.right == null) {
            Node par = node.parent; node.parent = null;
            if (par.left == node) par.left = null;
            else par.right = null;
        } else if (node.left == null && node.right != null) {
            Node par = node.parent; Node child = node.right;
            node.parent = null; node.right = null;
            if (par.key < node.key) par.right = child;
            else par.left = child;
            child.parent = par;
        } else if (node.right == null && node.left != null) {
            Node par = node.parent; Node child = node.left;
            node.parent = null; node.left = null;
            if (par.key < node.key) par.left = child;
            else par.right = child;
            child.parent = par;
        } else {
            Node min = findMin(node.right); int min_value = min.key;
            delete(min); root.key = min_value; }
        else { delete(del); }
    }
    public void doubleRotatedFromLeft(Node y) {
        Node x = y.left; Node z = x.right; Node w = y.parent;
        Node b = z.left; Node c = z.right; x.right = b;
        if (b != null) b.parent = x;
        y.left = c;
        if (c != null) c.parent = y;
        z.left = x; z.right = y; x.parent = z; y.parent = z;
        if (w != null) { w.parent = z;
            if (z.key < w.key) w.left = z;
            else w.right = z; }
        else { root = z; root.parent = null; }
    }
    public void doubleRotatedFromRight(Node y) {
        Node x = y.right; Node z = x.left; Node w = y.parent;
        Node b = z.left; Node c = z.right; x.left = b;
        if (b != null) b.parent = y;
        y.parent = x;
        if (c != null) c.parent = x;
        z.left = y; z.right = x; x.parent = z; y.parent = z;
        if (w != null) { w.parent = x;
            if (x.key < w.key) w.left = x;
            else w.right = x; }
        else { root = y; root.parent = null; }
    }
    public void singleRotateFromLeft(Node y) {
        Node x = y.left; Node w = y.parent; Node b = x.right; y.left = b;
        if (b != null) b.parent = y;
        y.parent = x; x.right = y;
        if (w != null) { x.parent = w;
            if (w.key < x.key) w.right = x;
            else w.left = x; }
        else { root = x; root.parent = null; }
    }
    public void singleRotateFromRight(Node y) {
        Node x = y.right; Node w = y.parent; Node b = x.left; y.right = b;
        if (b != null) b.parent = y;
        y.parent = x; x.left = y;
        if (w != null) { x.parent = w;
            if (w.key < x.key) w.left = x;
            else w.right = x; }
        else { root = y; root.parent = null; }
    }
    public void delete(int key) {
        Node del = find(key);
        if (root == null) System.out.println("Empty Tree!!!");
        else if (del == null) System.out.println("Key not found!!!!");
        else if (del == root) {
            if (root.left == null && root.right == null) root = null;
            else if (root.left != null && root.right == null) {
                Node newRoot = root.left; newRoot.parent = null;
                root.left = null; root = newRoot;
            } else if (root.left == null && root.right != null) {
                Node newRoot = root.right; newRoot.parent = null;
                root.right = null; root = newRoot;
            } else {
                Node min = findMin(root.right); int min_value = min.key;
                root.key = min_value; delete(min); }
            else { delete(del); }
        }
        public static void delete(AVLTree tree, Node node) {
            Node current = node.parent;
            if (node.left == null && node.right == null) {
                Node par = node.parent; node.parent = null;
                if (par.right != null && par.right.key == node.key) par.right = null;
                else par.left = null; node.parent = null; }
            else if (node.left == null && node.right != null) {
                Node par = node.parent; Node child = node.right;
                node.parent = null; node.right = null;
                if (par.right != null && par.right.key == node.key) par.right = child;
                else par.left = child; child.parent = par; }
            else if (node.left != null && node.right == null) {
                Node par = node.parent; Node child = node.left;
                node.parent = null; node.left = null;
                if (par.right != null && par.right.key == node.key) par.right = child;
                else par.left = child; child.parent = par; }
            else {
                Node min = findMin(node.right); int min_value = min.key;
                node.key = min_value; delete(tree, min);
                while (current != null) { rebalance(tree, current); current = current.parent; }
            }
            public Node findIn(int search_key) { return find(root, search_key); }
            public static Node find(Node node, int search_key) { //copy hw6
                public static Node findMin(Node node) { //copy hw6 }
                public static Node findMax(Node node) { //copy hw6 }
                public void insert(int key) { //copy hw6 }
                public static void insert(Node node, int key) { //copy hw6 }
                public void delete(int key) { //copy hw6 }
                public static void delete(Node node) { //copy hw6 }
                public static boolean isMergeable(Node r1, Node r2) {
                    Node maxR1 = findMax(r1); Node minR2 = findMin(r2);
                    return maxR1.key < minR2.key; }
                public static Node mergeWithRoot(Node r1, Node r2, Node t) {
                    if (isMergeable(r1, r2)) { t.left = r1; t.right = r2;
                        if (r1.parent != null) r1.parent = t;
                        if (r2.parent != null) r2.parent = t; return t; }
                    else {
                        System.out.println("All nodes in T1 must be smaller than all nodes from T2");
                        return r1; }
                }
                public void merge(BSTree tree2) {
                    if (isMergeable(this.root, tree2.root)) {
                        Node maxR1 = findMax(this.root); Node t = new Node(maxR1.key);
                        delete(maxR1); this.root = mergeWithRoot(this.root, tree2.root, t);
                    } else {
                        System.out.println("All nodes in T1 must be smaller than all nodes from T2"); }
                }
                public void printTree() {
                    if (root == null) { System.out.println("Empty tree!!!"); }
                    else { super.printTree(root); }
                }
            } // End BSTree.java
        }
    }

    public static Node mergeWithRoot(Node r1, Node r2, Node t) {
        if (isMergeable(r1, r2)) {
            if (Math.abs(height(r1) - height(r2)) <= 1) {
                t.left = r1;
                if (r1 != null) r1.parent = t;
                if (r2 != null) r2.parent = t; return t;
            } else if (height(r1) > height(r2)) {
                Node sub_root = mergeWithRoot(r1.right, r2, t); r1.right = sub_root;
                if (sub_root != null) sub_root.parent = r1;
                r1.parent = null; AVLTree newRoot = new AVLTree(r1);
                rebalance(newRoot, newRoot.root); return newRoot.root;
            } else if (height(r1) < height(r2)) {
                Node sub_root = mergeWithRoot(r1, r2.left, t); r2.left = sub_root;
                if (sub_root != null) sub_root.parent = r2;
                r2.parent = null; AVLTree newRoot = new AVLTree(r2);
                rebalance(newRoot, newRoot.root); return newRoot.root; }
            else { System.out.println("All nodes in T1 must be smaller than all nodes from T2");
                return r1; }
        }
        public static boolean isMergeable(Node r1, Node r2) {
            if (r1 == null || r2 == null) return true;
            int max_left_tree = findMax(r1).key;
            int min_right_tree = findMin(r2).key;
            return max_left_tree < min_right_tree; }
        public void merge(AVLTree tree2) {
            if (isMergeable(this.root, tree2.root)) { //merge
                Node maxR1 = findMax(this.root); Node t = new Node(maxR1.key);
                delete(this, maxR1); root = mergeWithRoot(this.root, tree2.root, t);
            } else {
                System.out.println("All nodes in T1 must be smaller than all nodes from T2"); }
            public Node[] split(int key) { return split(root, key); }
            public Node[] split(Node r, int key) {
                Node[] roots = new Node[2];
                if (r == null) { roots[0] = roots[1] = null; }
                else if (r.key < key) { roots[0] = split(r.right, key);
                    roots[0] = mergeWithRoot(r.left, roots[0], r); }
                else {
                    roots[0] = split(r.left, key); roots[1] = mergeWithRoot(roots[1], r.right, r);
                }
                return roots;
            }
            public static int height(Node node) {
                if (node == null) return -1;
                else return 1 + Math.max(height(node.left), height(node.right)); }
            public void printTree() {
                if (root == null) System.out.println("Empty tree!!!"); }
            public AVLTree() {} // Dummy Constructor, n need to edit
        } End AVLTree.java
    }
}
```