```
> with(tensor) :
> coords := [t, r, theta, phi] :
> g := array(symmetric, sparse, 1 ..4, 1 ..4) :
> g[1, 1] := -(A(r)) : g[2, 2] := B(r) :
> g[3, 3] := r^2 : g[4, 4] := r^2·sin(theta)^2 :
> metric := create([-1, -1], eval(g));
```

$$metric := table\left(\left[\left[index\_char = [-1, -1],\ compts = \begin{bmatrix} -A(r) & 0 & 0 & 0 \\ 0 & B(r) & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin(\theta)^2 \end{bmatrix}\right]\right]\right) \tag{1}$$

```
> tensorsGR(coords, metric, contra_metric, det_met, C1, C2, Rm, Rc, R, G, C);
> displayGR(Christoffel2, C2);
```

The Christoffel Symbols of the Second Kind

non-zero components :

$$\{1,12\} = \frac{1}{2}\ \frac{\frac{d}{dr}A(r)}{A(r)}$$

$$\{2,11\} = \frac{1}{2}\ \frac{\frac{d}{dr}A(r)}{B(r)}$$

$$\{2,22\} = \frac{1}{2}\ \frac{\frac{d}{dr}B(r)}{B(r)}$$

$$\{2,33\} = -\frac{r}{B(r)}$$

$$\{2,44\} = -\frac{r\sin(\theta)^2}{B(r)}$$

$$\{3,23\} = \frac{1}{r}$$

$$\{3,44\} = -\sin(\theta)\cos(\theta)$$

$$\{4,24\} = \frac{1}{r}$$

$$\{4,34\} = \frac{\cos(\theta)}{\sin(\theta)} \tag{2}$$

```
> displayGR(Ricci, Rc);
```

The Ricci tensor

non-zero components :

$$R11 = \frac{1}{4}\ \frac{1}{B(r)^2 A(r)\ r}\left(\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)A(r)\ r - 2\left(\frac{d^2}{dr^2}A(r)\right)B(r)\ A(r)\ r + \left(\frac{d}{dr}A(r)\right)^2 B(r)\ r - 4\left(\frac{d}{dr}A(r)\right)B(r)\ A(r)\right)$$

$$R22 = -\frac{1}{4}\ \frac{1}{A(r)^2 B(r)\ r}\left(\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)A(r)\ r - 2\left(\frac{d^2}{dr^2}A(r)\right)B(r)\ A(r)\ r + \left(\frac{d}{dr}A(r)\right)^2 B(r)\ r + 4\left(\frac{d}{dr}B(r)\right)A(r)^2\right)$$

$$R33 = -\frac{1}{2}\ \frac{\left(\frac{d}{dr}B(r)\right)r\,A(r) - \left(\frac{d}{dr}A(r)\right)r\,B(r) + 2\,B(r)^2 A(r) - 2\,A(r)\,B(r)}{B(r)^2 A(r)}$$

$$R44 = \frac{1}{2}\ \frac{1}{A(r)\ B(r)^2}\left(\left(\frac{d}{dr}B(r)\right)\cos(\theta)^2 A(r)\ r - \cos(\theta)^2\left(\frac{d}{dr}A(r)\right)B(r)\ r + 2\cos(\theta)^2 B(r)^2 A(r) - 2\cos(\theta)^2 B(r)\ A(r) - \left(\frac{d}{dr}B(r)\right)r\,A(r) + \left(\frac{d}{dr}A(r)\right)r\,B(r) - 2\,B(r)^2 A(r) + 2\,A(r)\,B(r)\right)$$

character : [-1, -1] **(3)**

> displayGR(Ricciscalar, R);

The Ricci Scalar

$$R = -\frac{1}{2}\ \frac{1}{A(r)^2 B(r)^2 r^2}\left(\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)A(r)\ r^2 - 2\left(\frac{d^2}{dr^2}A(r)\right)B(r)\ A(r)\ r^2 + \left(\frac{d}{dr}A(r)\right)^2 B(r)\ r^2 + 4\left(\frac{d}{dr}B(r)\right)A(r)^2 r - 4\left(\frac{d}{dr}A(r)\right)B(r)\ A(r)\ r + 4\,B(r)^2 A(r)^2 - 4\,A(r)^2 B(r)\right) \quad \textbf{(4)}$$

> displayGR(Einstein, G);

The Einstein Tensor
non-zero components :

$$G11 = -\frac{A(r)\left(\left(\frac{d}{dr}B(r)\right)r + B(r)^2 - B(r)\right)}{r^2 B(r)^2}$$

$$G22 = -\frac{\left(\frac{d}{dr}A(r)\right)r - A(r)\ B(r) + A(r)}{A(r)\ r^2}$$

$$G33 = \frac{1}{4}\ \frac{1}{B(r)^2 A(r)^2}\left(r\left(\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)A(r)\ r - 2\left(\frac{d^2}{dr^2}A(r)\right)B(r)\ A(r)\ r\right.\right.$$

$$+\left(\frac{d}{dr}A(r)\right)^2 B(r)\,r+2\left(\frac{d}{dr}B(r)\right)A(r)^2-2\left(\frac{d}{dr}A(r)\right)B(r)\,A(r)\Bigg)\Bigg)$$

$$G44=-\frac{1}{4}\,\frac{1}{B(r)^2A(r)^2}\Bigg(r\Bigg(\left(\frac{d}{dr}B(r)\right)\cos(\theta)^2\left(\frac{d}{dr}A(r)\right)A(r)\,r$$

$$-2\left(\frac{d^2}{dr^2}A(r)\right)\cos(\theta)^2 B(r)\,A(r)\,r+\cos(\theta)^2\left(\frac{d}{dr}A(r)\right)^2 B(r)\,r$$

$$+2\left(\frac{d}{dr}B(r)\right)\cos(\theta)^2 A(r)^2-2\cos(\theta)^2\left(\frac{d}{dr}A(r)\right)B(r)\,A(r)$$

$$-\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)A(r)\,r+2\left(\frac{d^2}{dr^2}A(r)\right)B(r)\,A(r)\,r-\left(\frac{d}{dr}A(r)\right)^2 B(r)\,r$$

$$-2\left(\frac{d}{dr}B(r)\right)A(r)^2+2\left(\frac{d}{dr}A(r)\right)B(r)\,A(r)\Bigg)\Bigg)$$

character : [-1, -1] **(5)**

```
> mixed := raise(contra_metric, G, 2);
```

$$mixed := table\left(\left[index\_char=[-1,\,1],\ compts=\left[\left[\frac{\left(\frac{d}{dr}B(r)\right)r+B(r)^2-B(r)}{r^2B(r)^2},\,0,\,0,\,0\right],\right.\right. \quad \textbf{(6)}$$

$$\left[0,\,-\frac{\left(\frac{d}{dr}A(r)\right)r-A(r)\,B(r)+A(r)}{B(r)\,A(r)\,r^2},\,0,\,0\right],$$

$$\left[0,\,0,\,\frac{1}{4}\,\frac{1}{r\,B(r)^2A(r)^2}\left(\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)A(r)\,r\right.\right.$$

$$-2\left(\frac{d^2}{dr^2}A(r)\right)B(r)\,A(r)\,r+\left(\frac{d}{dr}A(r)\right)^2 B(r)\,r+2\left(\frac{d}{dr}B(r)\right)A(r)^2$$

$$\left.-2\left(\frac{d}{dr}A(r)\right)B(r)\,A(r)\right),\,0\Bigg],$$

$$\left[0,\,0,\,0,\,\frac{1}{4}\,\frac{1}{r\,B(r)^2A(r)^2}\left(\left(\frac{d}{dr}B(r)\right)\left(\frac{d}{dr}A(r)\right)A(r)\,r\right.\right.$$

$$-2\left(\frac{d^2}{dr^2}A(r)\right)B(r)\,A(r)\,r+\left(\frac{d}{dr}A(r)\right)^2 B(r)\,r+2\left(\frac{d}{dr}B(r)\right)A(r)^2$$

$$\left.\left.\left.\left.-2\left(\frac{d}{dr}A(r)\right)B(r)\,A(r)\right)\right]\right]\right]\Bigg)$$

```
>
```