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Anisotropic strange star with de Sitter spacetime

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Abstract Stars can be treated as self-gravitating fluid. Krori and Barua (J. Phys. A., Math. Gen. 8:508, 1975) gave an analytical solution to that kind of fluids. In this connection, we propose a de Sitter model for an anisotropic strange star with the Krori–Barua spacetime. We incorporate the existence of the cosmological constant on a small scale to study the structure of anisotropic strange stars and come to the conclusion that this doping is very well compatible with the well-known physical features of strange stars.

1 Introduction

Recent observational data and results in modern cosmology revealed that the dark energy which is described in majority by the cosmological constant Λ is of dominant importance in the dynamics of our Universe. Measurements conducted by Wilkinson Microwave Anisotropic Probe (WMAP) indicate that almost three fourth of total mass-energy in the Universe is Dark Energy [2, 3] and the leading theory of dark energy is based on the cosmological constant characterized by repulsive pressure which was introduced by Einstein in 1917 to obtain a static cosmological model. Later on Zel'dovich [4, 5] interpreted this quantity physically as a vacuum energy of quantum fluctuation whose size is of the order of $\sim 3\times 10^{-56}$ cm^{-2} [6, 7].

However, for viability of the present-day accelerated Universe with dark energy the erstwhile cosmological constant Λ, in general, assumed to be time-dependent in the cosmological realm [2, 3]. On the other hand, space-dependent Λ has an expected effect in the astrophysical context as argued by several authors [8–11] in relation to the nature of local massive objects like galaxies and elsewhere. In the present context of compact stars, however, we assume the dark energy in the form of Einstein's cosmological constant as a purely constant quantity as follows: either $\Lambda_{\text{eff}} = \Lambda_0 - \Lambda(r)$, where Λ_{eff} is the effective cosmological parameter [12] or $\Lambda_{\text{eff}} = \Lambda_0 + 8\pi E$, where E is the energy density of the energy state [13] so that, for the time being, variation of time and/or space-dependence of Λ is ignored. This constancy of Λ can not be ruled out for the systems of very small dimension like compact star systems or elsewhere with different physical requirements [14–17].

To study mass and radii of neutron star Egeland [18] incorporated the existence of cosmological constant proportionality depending on the density of vacuum. Egeland did it by using the Fermi equation of state together with the Tolman–Oppenheimer–Volkov (TOV) equation. Motivated by the above facts we incorporate the existence of cosmological constant in a small scale to study the structure of strange stars and arrived to a conclusion that incorporation of Λ describes the well known strange stars, for example strange stars, X-ray buster, 4U 1820-30, X-ray pulsar Her X-1, Millisecond pulsar SAX J 1808.4-3658 etc., in good manners. Dey et al. [19], Usov [20], Ruderman [21], Mak and Harko [22–24], Li et al. [25, 26], Chodos et al. [27] and many more have also studied structure of strange stars in different way. If we look at the anisotropy and TOV equation of strange stars then our model fits appropriately with the above said stars.

In the present work, we modeled a strange star which has been proposed by Alcock et al. and Haensel et al. [28, 29] through their fundamental work. However, our model of

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the strange star is associated with cosmological constant which satisfies all the energy conditions including the TOV-equation. We have checked the stability and mass–radius relation. Finally, we have calculated the surface redshift for our solutions which may be interesting to the observers for possible detection of strange stars.

2 Anisotropic de Sitter model

To describe the space-time of the strange stars stellar configuration, we take the Krori and Barua [1] metric (henceforth KB) given by

$$ds^2 = -e^{\nu(r)}\,dt^2 + e^{\lambda(r)}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \qquad (1)$$

with $\lambda(r) = Ar^2$ and $\nu(r) = Br^2 + C$ where A, B and C are arbitrary constants to be determined on physical grounds. We further assume that the energy-momentum tensor for the strange matter filling the interior of the star may be expressed in the standard form as

$$T_{ij} = \mathrm{diag}(\rho, -p_r, -p_t, -p_t),$$

where ρ, p_r and p_t correspond to the energy density, radial pressure and transverse pressure of the baryonic matter, respectively.

The Einstein field equations for the metric (1) in presence of Λ are then obtained as (with $G = c = 1$ under geometrized relativistic units)

$$8\pi\rho + \Lambda = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}, \qquad (2)$$

$$8\pi p_r - \Lambda = e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}, \qquad (3)$$

$$8\pi\ p_t - \Lambda = \frac{e^{-\lambda}}{2}\left[\frac{\nu'^2 - \lambda'\nu'}{2} + \frac{\nu' - \lambda'}{r} + \nu''\right]. \qquad (4)$$

Now, from the metric (1) and (2)–(4), we get the energy density (ρ), the radial pressure (p_r) and the tangential pressure (p_t) as

$$\rho = \frac{1}{8\pi}\left[e^{-Ar^2}\left(2A - \frac{1}{r^2}\right) + \frac{1}{r^2} - \Lambda\right], \qquad (5)$$

$$p_r = \frac{1}{8\pi}\left[e^{-Ar^2}\left(2B + \frac{1}{r^2}\right) - \frac{1}{r^2} + \Lambda\right], \qquad (6)$$

$$p_t = \frac{1}{8\pi}\left[e^{-Ar^2}\left(\left(B^2 - AB\right)r^2 + (2B - A)\right) + \Lambda\right]. \qquad (7)$$

Using (5)–(7) the equation of state (EOS) corresponding to radial and transverse directions may be written as

$$\omega_r(r) = \frac{[e^{-Ar^2}(2B + \frac{1}{r^2}) - \frac{1}{r^2} + \Lambda]}{[e^{-Ar^2}(2A - \frac{1}{r^2}) + \frac{1}{r^2} - \Lambda]}, \qquad (8)$$

$$\omega_t(r) = \frac{e^{-Ar^2}[(B^2 - AB)r^2 + (2B - A)] + \Lambda}{[e^{-Ar^2}(2A - \frac{1}{r^2}) + \frac{1}{r^2} - \Lambda]}. \qquad (9)$$

3 Physical analysis

It is known that $\Lambda > 0$ implies the space is open. To explain the present acceleration state of the universe, it is believed that energy in the vacuum is responsible for this expansion. As a consequence, vacuum energy provides some gravitational effect on the stellar structures. It is suggested that cosmological constant plays the role of energy of the vacuum. In this section we will study the following features of our model assuming the value of $\Lambda = 0.00018\ \mathrm{km}^{-2}$. We have assumed this value as required for the stability of the strange star and mathematical consistency.

3.1 Anisotropic behavior

From (5) we have

$$\frac{d\rho}{dr} = -\frac{1}{8\pi}\left[\left(4A^2 r - \frac{2A}{r} - \frac{2}{r^3}\right)e^{-Ar^2} + \frac{2}{r^3}\right] < 0,$$

and (6) leads to

$$\frac{dp_r}{dr} < 0.$$

The density and pressure are decreasing with the increase of radius of the star.

Figures 1 and 2 support the above results.

We observe that, at $r = 0$, our model yields

$$\frac{d\rho}{dr} = 0, \qquad \frac{dp_r}{dr} = 0,$$

$$\frac{d^2\rho}{dr^2} = -\frac{A^2}{4\pi} < 0,$$

and

$$\frac{d^2 p_r}{dr^2} < 0,$$

which indicate maximality of central density and central pressure. Interestingly, similar to an ordinary matter distribution, the EOS is restricted between 0 and 1 (see Fig. 3) despite the fact that star is constituted by the combination

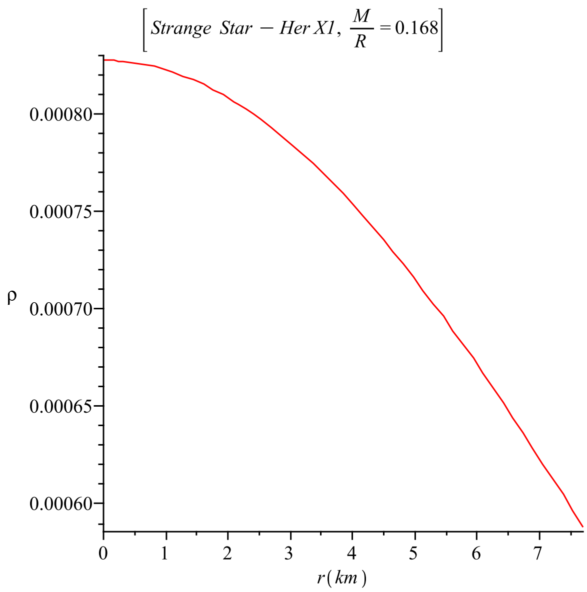

Fig. 1 Density variation at the stellar interior of Strange Star-Her X-1

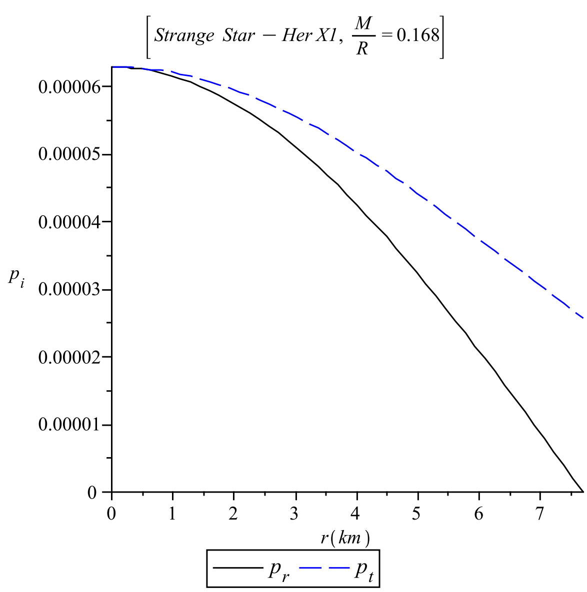

Fig. 2 Radial and Transverse Pressure variation at the stellar interior of Strange Star-Her X-1

of strange matter and effect of Λ. We call it strange matter, as the EOS depends on the radius of the star rather than behaving as a constant as in ordinary matter.

The measure of anisotropy, $\Delta = (p_t - p_r)$ in this model is obtained as

$$\Delta = \frac{1}{8\pi}\left[e^{-Ar^2}\left(\left(B^2 - AB\right)r^2 - A - \frac{1}{r^2}\right) + \frac{1}{r^2}\right]. \tag{10}$$

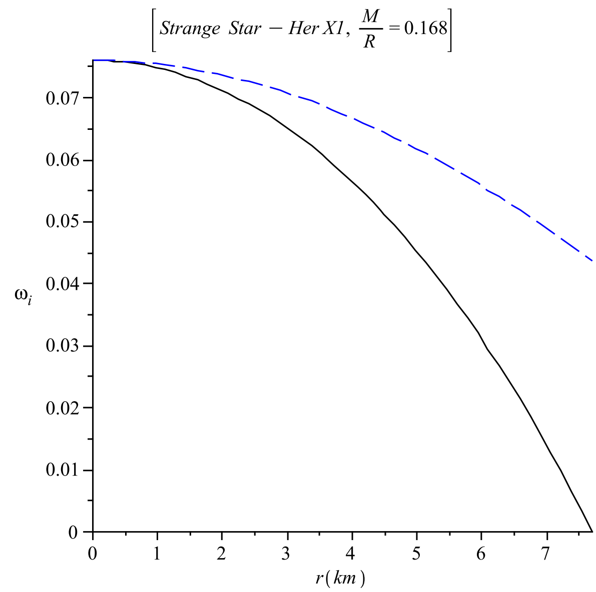

Fig. 3 Variation of equation of state parameter with distance of Strange Star-Her X-1

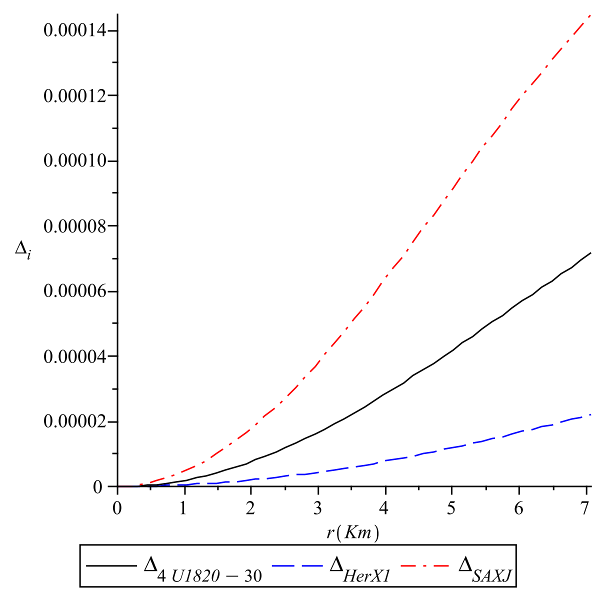

Fig. 4 Comparison of anisotropic behaviors at the stellar interior of Strange Star-4U 1820-30, Her X-1 and SAX J 1808.4-3658

We note that measure of anisotropy is independent of Λ. In other words, vacuum energy does not affect on the anisotropic force. The 'anisotropy' will be directed outward when $P_t > P_r$ i.e. $\Delta > 0$, and inward if $P_t < P_r$ i.e. $\Delta < 0$. Figure 4 of our model indicates that $\Delta > 0$ i.e. a repulsive 'anisotropic' force exists for Strange Star-4U 1820-30, Her X-1 and SAX J 1808.4-3658. The positivity of Δ allows the construction of more massive distributions.

3.2 Matching conditions

Here we match the interior metric to the Schwarzschild de Sitter exterior

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right) dt^2 + \left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{11}$$

at the boundary $r = R$. Continuity of the metric functions g_{tt}, g_{rr}, and $\frac{\partial g_{tt}}{\partial r}$ at the boundary surface S yields

$$A = -\frac{1}{R^2}\ln\left[1 - \frac{2M}{R} - \frac{1}{3}\Lambda R^2\right], \tag{12}$$

$$B = \frac{1}{R}\left[\frac{M}{R^2} - \frac{1}{3}\Lambda R\right]\left[1 - \frac{2M}{R} - \frac{1}{3}\Lambda R^2\right]^{-1}, \tag{13}$$

$$C = \ln\left[1 - \frac{2M}{R} - \frac{1}{3}\Lambda R^2\right] - \frac{R[\frac{M}{R^2} - \frac{1}{3}\Lambda R]}{[1 - \frac{2M}{R} - \frac{1}{3}\Lambda R^2]}. \tag{14}$$

Imposing the boundary conditions $p_r(r = R) = 0$ and $\rho(r = 0) = b$ (= a constant), where b is the central density, we have A and B in the following forms:

$$A = \frac{8\pi b + \Lambda}{3}, \tag{15}$$

$$B = \frac{1}{2}\left[e^{\frac{8\pi b+\Lambda}{3}R^2}\left(\frac{1}{R^2} - \Lambda\right) - \frac{1}{R^2}\right]. \tag{16}$$

Combining, (12) and (15), we get

$$A = \frac{8\pi b + \Lambda}{3} = -\frac{1}{R^2}\ln\left[1 - \frac{2M}{R} - \frac{1}{3}\Lambda R^2\right]. \tag{17}$$

At this juncture, to get an insight of our model, we have evaluated the numerical values of the parameters A, B and b for the Strange Star-4U 1820-30, Her X-1 and SAX J 1808.4-3658 (see Table 1).

We have verified for particular choices of the values of mass and radius, leading to solutions for the unknown parameters, that they satisfy the following energy conditions namely, the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC) through out the configuration:

$$\rho \geq 0, \qquad \rho + p_r \geq 0, \qquad \rho + p_t \geq 0,$$
$$\rho + p_r + 2p_t \geq 0, \qquad \rho > |p_r|, \quad \text{and} \quad \rho > |p_t|.$$

It is interesting to note here that the model satisfies the strong energy condition, which implies that the space-time does contain a black hole region.

The anisotropy, as expected, vanishes at the centre i.e., $p_t = p_r = p_0 = \frac{2B-A+\Lambda}{8\pi}$ at $r = 0$. The energy density and the two pressures are also well behaved in the interior of the stellar configuration.

3.3 TOV equation

For an anisotropic fluid distribution, the generalized TOV equation is given by

$$\frac{d}{dr}\left(p_r - \frac{\Lambda}{8\pi}\right) + \frac{1}{2}\nu'(\rho + p_r) + \frac{2}{r}(p_r - p_t) = 0. \tag{18}$$

According to Ponce de León [31], the above TOV equation can be rewritten as

$$-\frac{M_G(\rho + p_r)}{r^2}e^{\frac{\lambda-\nu}{2}} - \frac{d}{dr}\left(p_r - \frac{\Lambda}{8\pi}\right) + \frac{2}{r}(p_t - p_r) = 0, \tag{19}$$

where $M_G = M_G(r)$ is the gravitational mass inside a sphere of radius r and is given by

$$M_G(r) = \frac{1}{2}r^2 e^{\frac{\nu-\lambda}{2}}\nu', \tag{20}$$

which can easily be derived from the Tolman–Whittaker formula and the Einstein's field equations. This new form of TOV equation provides the equilibrium condition for the strange star subject to gravitational and hydrostatic plus another force due to the anisotropic nature of the stellar object. Using (5)–(7), the above equation can be written as

$$F_g + F_h + F_a = 0, \tag{21}$$

where

$$F_g = -Br(\rho + p_r), \tag{22}$$

Table 1 Values of the model parameters for different Strange stars

Strange Quark Star	M ($M_\odot$)	R (km)	$\frac{M}{R}$	A (km^{-2})	B (km^{-2})	b (km^{-2})
Her X-1	0.88	7.7	0.168	0.0069968808	0.004199502132	0.000828449
SAX J 1808.4-3658(SS1)	1.435	7.07	0.299	0.0138138300	0.01484158661	0.002188063
4U 1820-30	2.25	10.0	0.332	0.01108662625	0.009878787879	0.001316874

$$F_h = -\frac{d}{dr}\left(p_r - \frac{\Lambda}{8\pi}\right), \tag{23}$$

$$F_a = \frac{2}{r}(p_t - p_r). \tag{24}$$

The profiles of F_g, F_h, and F_a for our chosen source are shown in Fig. 5. This figure indicates that the static equilibrium can be attained due to pressure anisotropy, gravitational and hydrostatic forces.

3.4 Stability

The velocity of sound $v_s^2 = (\frac{dp}{d\rho})$ should be less than one for a realistic model [32, 33]. Now, we calculate the radial and transverse speed for our anisotropic model,

$$v_{sr}^2 = \frac{dp_r}{d\rho} = -1 + \frac{4Are^{-Ar^2}(A+B)}{e^{-Ar^2}(4A^2r - \frac{2A}{r} - \frac{2}{r^3}) + \frac{2}{r^3}}, \tag{25}$$

$$v_{st}^2 = \frac{dp_t}{d\rho} = -\frac{e^{-Ar^2}[2r(A^2+B^2-3AB) - 2A(B^2-AB)r^3]}{e^{-Ar^2}(4A^2r - \frac{2A}{r} - \frac{2}{r^3}) + \frac{2}{r^3}}. \tag{26}$$

To check whether the sound speeds lie between 0 and 1, we plot the radial and transverse sound speeds in Fig. 6 and observe that these parameters satisfy the inequalities

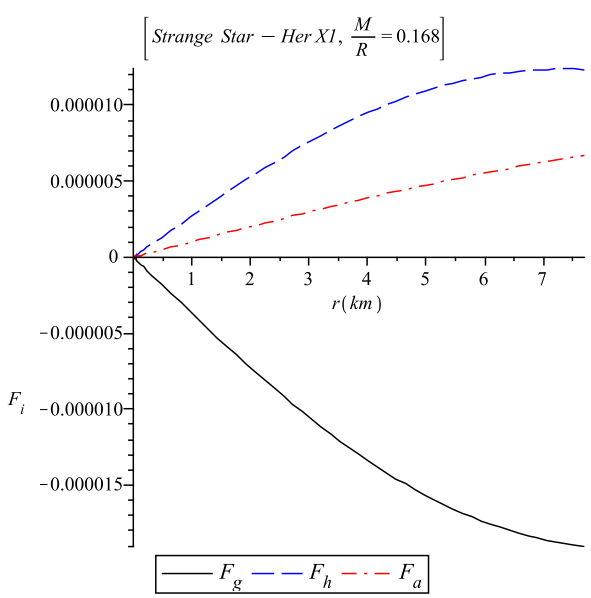

Fig. 5 Behaviors of pressure anisotropy, gravitational and hydrostatic forces at the stellar interior of Strange Star Her X-1

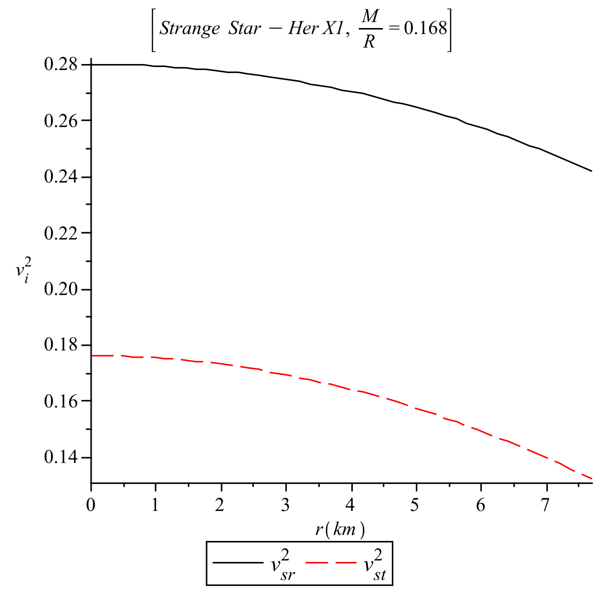

Fig. 6 Variation of radial and transverse sound speed of Strange Star-Her X-1

$0 \le v_{sr}^2 \le 1$ and $0 \le v_{st}^2 \le 1$ everywhere within the stellar object.

Equations (25) and (26) lead to

$$v_{st}^2 - v_{sr}^2 = 1 - \frac{e^{-Ar^2}[2r(3A^2+B^2-AB) + 2AB(A-B)r^3]}{e^{-Ar^2}(4A^2r - \frac{2A}{r} - \frac{2}{r^3}) + \frac{2}{r^3}}. \tag{27}$$

As sound speeds lie between 0 and 1, we have $|v_{st}^2 - v_{sr}^2| \le 1$.

A few years back, Herrera [32] proposed a technique for stability check of local anisotropic matter distribution. This technique is known as the cracking (or overturning) concept which states that the region for which radial speed of sound is greater than the transverse speed of sound is a potentially stable region. In our case, Fig. 7 indicates that there is no change of sign for the term $v_{st}^2 - v_{sr}^2$ within the specific configuration. Also, the plot for $v_{st}^2 - v_{sr}^2$ (Fig. 7) shows negativity in its nature. Therefore, we conclude that our strange star model is stable.

3.5 Surface redshift

To get observational evidence of anisotropies in the internal pressure distribution, it is necessary to study redshift of light emitted at the surface of the compact objects. At first, we try to see whether our model will follow the Buchdahl [30] maximum allowable mass radius ratio limit. We have calculated $\frac{M_{\mathrm{eff}}}{R} = \frac{4\pi\int_0^R \rho\, dr}{R}$ for Strange Star Her X-1 of our model and have found that $(\frac{M_{\mathrm{eff}}}{R})_{\max} = .336$. Thus our model satisfies Buchdahl's limit and hence is physically acceptable. It

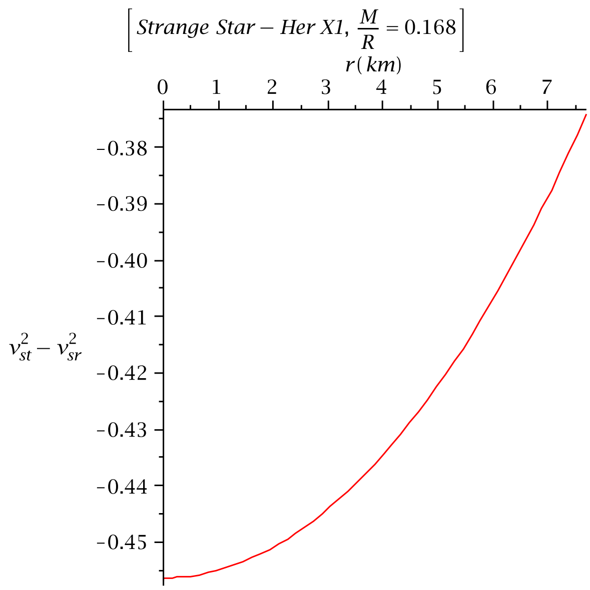

Fig. 7 Variation of the $v_{st}^2 - v_{sr}^2$ of Strange Star-Her X-1

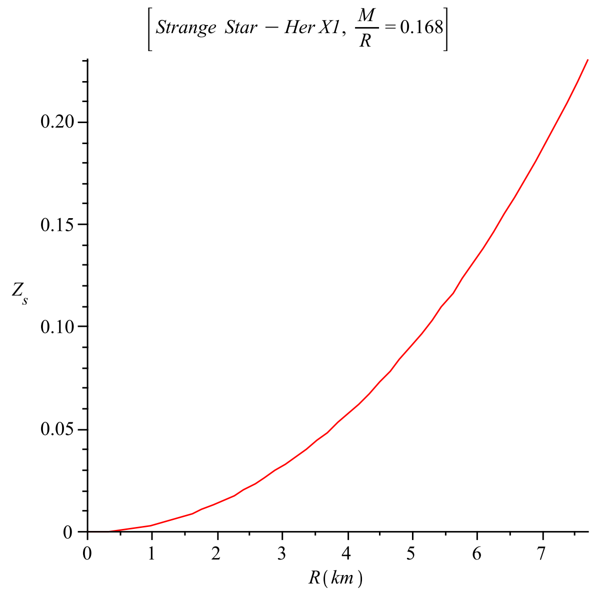

Fig. 8 Variation of surface redshift of Strange Star-Her X-1

is worthwhile to mention that our model provides the same mass–radius ratio for the observed Strange Star Her X-1.

The compactness of the star is given by

$$u = \frac{M_{\mathrm{eff}}}{R} = \frac{1}{2}\left(1 - e^{-AR^2}\right) - \frac{\Lambda R^2}{6}. \tag{28}$$

The surface redshift (Z_s) corresponding to the above compactness (u) is obtained as

$$1 + Z_s = \left[1 - \left(2u + \frac{\Lambda R^2}{3}\right)\right]^{-\frac{1}{2}}, \tag{29}$$

where

$$Z_s = e^{\frac{A}{2}R^2} - 1. \tag{30}$$

Thus, the maximum surface redshift for a strange star Her X-1 of radius 7.7 km turns out to be $Z_s = 0.022$ (see Fig. 8).

4 Discussion

We have studied in the present work a self-gravitating fluid of strange star under the metric of KB [1]. The spacetime turns out to be de Sitter type and anisotropic in nature due to the presence of tangential pressure. We have incorporated the erstwhile cosmological constant Λ in the field equation to study the structure of anisotropic strange stars. Successfully we find an analytical solution to the fluids which are quite interesting in connection to several physical features of strange stars.

In this regard it is to note that the present investigation is a sequel of the earlier work with KB models [34–37]. Varela et al. [34] considered KB model in the context of electrically charged system whereas Rahaman et al. [35] studied it under the influence of anisotropic charged fluids with Chaplygin equation of state. Also, Rahaman et al. [36] studied dark energy star constituted by two fluids, namely ordinary fluid and dark energy. Very recently Rahaman et al. [37] have uniquely considered the system of strange star with MIT Bag model under KB metric. However, in the present investigation we have considered a de Sitter model for an anisotropic strange star with the same KB [1] spacetime. It is yet unclear to what extent the strange stars can be described by the KB [1] metric. At least, at this stage of theoretical investigation on strange star, the question of 'if they exist at all' is not a serious issue or adding one more arbitrary ingredient to match with the observational data cannot seriously disprove the subject.

We would like to mention here that we have doped here Λ as a purely constant quantity and have shown that the results are very well compatible with the well-known physical features of strange stars. This at once demands a space-variable Λ [8–11] to be incorporated to see the effect of this inclusion in the astrophysical system like strange star. Another issue, a possibly very intriguing one, the assumed value of the cosmological constant here is taken as $\Lambda = 0.00018\ \mathrm{km}^{-2}$. This has certainly nothing to do with the standard "cosmology" of mainstream arena as it looks quite artificial. However, all these issues related to Λ may be considered in a future project.

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