Nuclear Physics A493 (1989) 521-548
North-Holland, Amsterdam

VACUUM POLARIZATION EFFECTS ON NUCLEAR MATTER AND NEUTRON STARS*

Norman K. GLENDENNING

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA

Received 8 April 1988
(Revised 2 September 1988)

Abstract: Vacuum renormalization of relativistic nuclear field theory is studied for nuclear and neutron star matter in general equilibrium, and neutron stars. It is found that when the coupling constants are tightly constrained by the *five* saturation properties of nuclear matter, the binding, density, compression modulus, symmetry energy and effective nucleon mass, the theory *with or without* vacuum renormalization predicts an equation of state that differs in the two cases by only several percent over the entire density range of interest. If the effective mass and compression are not controlled, as in some works, the high density behavior is markedly different. The mass of a neutron star, even at the limiting mass, is not dominated by the dense matter at its center, half the mass being contributed by matter at densities less than about $3\rho_0$. The hyperon fraction of the limiting mass star is about 20%.

1. Introduction

So far, the only known effective relativistic field theory that can describe nuclear matter *and* finite nuclear properties is the scalar-vector-isovector (σ, ω, ρ) theory. That it provides a good description of numerous properties of nuclear matter and finite nuclei lends support to its use in deriving the properties of matter at high energy density, but below the expected transition to a quark-gluon plasma. Although it is known how to incorporate vacuum renormalization [1,2]), and this has been done in several recent works [3,4]), it so far has not been studied systematically in a way that preserves the five important properties of nuclear matter at saturation, the binding, density, compression modulus, effective mass and symmetry energy. Therefore, it has not been possible to disentangle the renormalization effects from those produced by shifting nuclear matter properties. Moreover, vacuum polarization in neutron star matter that is in generalized beta equilibrium has not been investigated previously, except in the chiral-sigma model [5]), which seems incapable of describing the normal ground state of finite nuclei, producing instead a bubble configuration [6]). In this paper we undertake such a systematic investigation of nuclear and neutron star matter for the σ, ω, ρ theory. This requires the form of the theory in which

* This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the US Department of Energy under Contract DE-AC03-76SF00098.

0375-9474/89/$03.50

cubic and quartic self-interactions of the scalar field are included [7]), and renormalized, for they, together with the nucleon interaction with the scalar, vector and vector-isovector mesons, permit the five saturation properties to be controlled.

In the following, we shall write down the equations that define the theory, derive the coupling constants that produce the five saturation properties of nuclear matter for both situations in which vacuum polarization is included or not. We compute the equation of state in both cases, for nuclear and for charge neutral matter in general beta equilibrium (neutron star matter). The constituent particles of neutron star matter are discussed. We compute the masses of neutron stars in both cases, and discuss the connection between the limiting star mass and the saturation properties of the underlying theory. Additional star properties are calculated, including the redshift, hyperon fraction, gravitational binding and radii.

2. Theory

The lagrangian of the σ, ω, ρ theory is,

$$\begin{aligned}\mathscr{L}=&\sum_{\mathrm{B}} \bar{\psi}_{\mathrm{B}}(i\gamma_\mu\partial^\mu - m_{\mathrm{B}} + g_{\sigma\mathrm{B}}\sigma - g_{\omega\mathrm{B}}\gamma_\mu\omega^\mu - \tfrac{1}{2}g_{\rho\mathrm{B}}\gamma_\mu\tau_3\rho_3^\mu)\psi_{\mathrm{B}}\\ &+\tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu\\ &-\tfrac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \tfrac{1}{3}bm_{\mathrm{n}}(g_\sigma\sigma)^3 - \tfrac{1}{4}c(g_\sigma\sigma)^4\,. \end{aligned}\tag{1}$$

The scalar meson is Yukawa coupled to the baryon scalar density and the vector and vector-isovector mesons to the baryon current and the isospin current, respectively. In preparation for application to dense matter we have included, in addition to the nucleons, other baryon species, denoted by B, where the sum is over all *charge* states of N, Λ, Σ, Ξ, Δ, etc. [8-11]). When the corresponding Euler-Lagrange equations are solved by replacing the meson fields by their mean values, and the nucleon currents by those generated in the presence of the mean meson fields, one obtains the so-called mean field approximation (MFA). It is in this approximation that nuclear field theory has been typically solved and applied. However, as is well known, the presence of matter alters the vacuum, by altering the masses of antiparticles. The energy of the filled sea therefore shifts with density. Some authors take the view that since a theory of nuclear matter based on hadron interactions is, in view of their substructure, an effective one, such an effect is not worth incorporating. There are precedents, nonetheless, for studying such effects in high density matter, both in the σ, ω, ρ theory [4]) and in the chiral-sigma theory [5,12-14]). There are well-known procedures for renormalizing the theory with respect to nucleon, and scalar and vector mesons [2]). So far, it is not known how to renormalize the vector-isovector meson, and we shall regard as phenomenological the energy contributed to asymmetric matter by the coupling of this meson to the isospin current, with this coupling chosen to reproduce the empirical symmetry energy coefficient.

The Dirac equation that follows from eq. (1), when the meson fields are replaced by their mean values, can be solved immediately to yield the eigenvalues for momentum p,

$$\varepsilon_B(p) = g_{\omega B}\omega_0 + g_{\rho B}\rho_{03} I_{3B} + \sqrt{p^2 + (m_B - g_{\sigma B}\sigma)^2}\,. \tag{2}$$

Only the time-like components of the vector fields, and the isospin 3-component of the ρ field have non-vanishing values, on account of the isotropy of nuclear matter and electric charge conservation, respectively. The energy density can be obtained now from the stress-energy tensor (cf. ref. [10]). For uniform nuclear or neutron matter in the mean field approximation it is given by

$$\begin{aligned}\varepsilon_{\mathrm{MFA}} &= \tfrac{1}{3}bm_n(g_\sigma\sigma)^3 + \tfrac{1}{4}c(g_\sigma\sigma)^4 + \tfrac{1}{2}m_\sigma^2\sigma^2 - \tfrac{1}{2}m_\omega^2\omega_0^2 - \tfrac{1}{2}m_\rho^2\rho_{03}^2 \\ &\quad + \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B}[g_{\omega B}\omega_0 + g_{\rho B}\rho_{03}I_{3B} + \sqrt{p^2+(m_B - g_{\sigma B}\sigma)^2}]p^2\,\mathrm{d}p \\ &\quad + \sum_\lambda \frac{1}{\pi^2}\int_0^{k_\lambda}\sqrt{p^2+m_\lambda^2}\,p^2\,\mathrm{d}p\,,\end{aligned} \tag{3}$$

where σ, ω_0, and ρ_{03} are the mean meson fields, k_B and J_B are the Fermi momentum and spin of the charge state of the baryon type B, and k_λ are the lepton Fermi momenta (electron and muon). For neutron star matter we need to include the contributions to the energy of the leptons, the last term in eq. (3), and incorporate in the solutions, the conditions of charge neutrality and chemical equilibrium. These complications have been discussed elsewhere [10].

With the inclusion of vacuum renormalization energies, the energy density is given by

$$\varepsilon_{\mathrm{RHA}} = \varepsilon_{\mathrm{MFA}} + V_N + V_\sigma\,. \tag{4}$$

The last two terms represent the contributions from renormalization of the nucleon scalar meson [2]), and are given by,

$$\begin{aligned}V_\sigma = \frac{m_\sigma^4}{(8\pi)^2}[(1+\phi_1+\phi_2)^2 \ln(1+\phi_1+\phi_2) - (\phi_1+\phi_2) \\ -\tfrac{3}{2}(\phi_1+\phi_2)^2 - \tfrac{1}{3}\phi_1^2(\phi_1+3\phi_2) + \tfrac{1}{12}\phi_1^4]\,,\end{aligned} \tag{5}$$

$$V_N = -\frac{m_n^4}{4\pi^2}[(1-\chi)^2\ln(1-\chi) + \chi - \tfrac{7}{2}\chi^2 + \tfrac{13}{3}\chi^3 - \tfrac{25}{12}\chi^4]\,, \tag{6}$$

where

$$\phi_1 = \frac{2bm_n g_\sigma^3\sigma}{m_\sigma^2}\,, \qquad \phi_2 = \frac{3cg_\sigma^4\sigma^2}{m_\sigma^2}\,, \qquad \chi = \frac{g_\sigma\sigma}{m_n} \tag{7}$$

and m_n and m_σ are the nucleon and σ mass. The approximation which includes the vacuum renormalization is known as the relativistic Hartree approximation (RHA) [2]).

The field equations can be found either as the solutions of the Euler–Lagrange equations, or equivalently, as the conditions that minimize the energy density at fixed baryon density. They are,

$$\omega_0 = \sum_{\mathrm{B}} \frac{g_{\omega\mathrm{B}}}{m_\omega^2} n_{\mathrm{B}}\,, \tag{8}$$

$$\rho_{03} = \sum_{\mathrm{B}} \frac{g_{\rho\mathrm{B}}}{m_\rho^2} I_{3\mathrm{B}} n_{\mathrm{B}}\,, \tag{9}$$

$$\begin{aligned} m_\sigma^2 \sigma = & -b m_{\mathrm{n}} g_\sigma (g_\sigma \sigma)^2 - c g_\sigma (g_\sigma \sigma)^3 - \frac{\partial}{\partial \sigma}[V_{\mathrm{N}} + V_\sigma] \\ & + \sum_{\mathrm{B}} \frac{2J_{\mathrm{B}}+1}{2\pi^2} g_{\sigma\mathrm{B}} \int_0^{k_{\mathrm{B}}} \frac{m_{\mathrm{B}} - g_{\sigma\mathrm{B}}\sigma}{\sqrt{p^2 + (m_{\mathrm{B}} - g_{\sigma\mathrm{B}}\sigma)^2}} p^2\, \mathrm{d}p\,, \end{aligned} \tag{10}$$

where $I_{3\mathrm{B}}$ is the isospin projection of baryon charge state B, and

$$n_{\mathrm{B}} = (2J_{\mathrm{B}} + 1) k_{\mathrm{B}}^3 / (6\pi^2) \tag{11}$$

is its density. Employing the field equations in eq. (3), we can write the energy density as,

$$\begin{aligned} \varepsilon_{\mathrm{MFA}} = & \tfrac{1}{3} b m_{\mathrm{n}} (g_\sigma \sigma)^3 + \tfrac{1}{4} c (g_\sigma \sigma)^4 + \tfrac{1}{2} m_\sigma^2 \sigma^2 + \tfrac{1}{2} m_\omega^2 \omega_0^2 + \tfrac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \sum_{\mathrm{B}} \frac{2J_{\mathrm{B}}+1}{2\pi^2} \int_0^{k_{\mathrm{B}}} \sqrt{p^2 + (m_{\mathrm{B}} - g_{\sigma\mathrm{B}}\sigma)^2}\, p^2\, \mathrm{d}p + \sum_{\lambda} \frac{1}{\pi^2} \int_0^{k_\lambda} \sqrt{p^2 + m_\lambda^2}\, p^2\, \mathrm{d}p\,. \end{aligned} \tag{12}$$

The pressure is given in RHA by,

$$p_{\mathrm{RHA}} = p_{\mathrm{MFA}} - V_{\mathrm{N}} - V_\sigma\,, \tag{13}$$

where

$$\begin{aligned} p_{\mathrm{MFA}} = & -\tfrac{1}{3} b m_{\mathrm{n}} (g_\sigma \sigma)^3 - \tfrac{1}{4} c (g_\sigma \sigma)^4 - \tfrac{1}{2} m_\sigma^2 \sigma^2 + \tfrac{1}{2} m_\omega^2 \omega_0^2 + \tfrac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \frac{1}{3} \sum_{\mathrm{B}} \frac{2J_{\mathrm{B}}+1}{2\pi^2} \int_0^{k_{\mathrm{B}}} p^4\, \mathrm{d}p / \sqrt{p^2 + (m_{\mathrm{B}} - g_{\sigma\mathrm{B}}\sigma)^2} \\ & + \frac{1}{3} \sum_{\lambda} \frac{1}{\pi^2} \int_0^{k_\lambda} p^4\, \mathrm{d}p / \sqrt{p^2 + m_\lambda^2}\,. \end{aligned} \tag{14}$$

When the field equations are solved subject to the subsidiary constraint of zero isospin and strangeness, one obtains the solution corresponding to uniform nuclear matter. When they are solved subject to the constraints of charge neutrality and general equilibrium, one obtains the solution corresponding to neutron star matter. We shall characterize both solutions by the corresponding properties of symmetric matter.

3. Nuclear and neutron matter

The five important properties of nuclear matter, mentioned in the introduction, can be used to fix the coupling constants g_σ/m_σ, g_ω/m_ω, g_ρ/m_ρ, and the parameters of the scalar self-interactions, b and c. In uniform matter, it is only the ratio of coupling constant to mass on which the theory depends, except for the scalar mass, which appears independently in the vacuum renormalization energy. For that mass we take $m_\sigma = 600$ MeV. The binding, saturation density and symmetry energy coefficient are relatively well known [15]. The compression modulus has been the subject of considerable debate in the last several years. Early work on the giant monopole resonance suggested a value $K \approx 220$ MeV. However recent new work on this topic including a much larger and more accurate data base finds $K = 300 \pm 25$ MeV [ref. [16]]. As well, a broad body of other data, both nuclear and astrophysical, favors a value of this magnitude [17]. The multifragmentation and fragment flow analyses of high energy nuclear collisions possibly favors a larger value although this work is still very much in a state of flux [18]. We shall nominally choose $K = 300$ MeV for most of our illustrations. The Landau effective nucleon mass, $m_L^*/m = 0.83$, has recently been obtained through a careful study of the mean field of heavy nuclei, and we fix this property in accord with those findings [19]. The scalar effective mass of this theory, $m^* = m - g_\sigma\sigma$, is related at saturation by

$$m_L^* = \left(\frac{k}{\partial\varepsilon(k)/\partial k}\right)_{k_F} = (m_{\rm sat}^{*2} + k_F^2)^{1/2}, \tag{15}$$

which yields $m_{\rm sat}^*/m = 0.78$. The nuclear properties are listed in table 1.

We first assess the effect of the vacuum polarization on the binding energy of normal matter, by adjusting the coupling constants so that the saturation properties listed in table 1 are reproduced in both the mean field (MFA) and the relativistic

TABLE 1

Nuclear matter properties at saturation

	ρ_0 (fm^{-3})	B/A (MeV)	$a_{\rm sym}$ (MeV)	K (MeV)	m^*/m
exp.	0.153	−16.3	32.5	≈300	≈0.78
this work					
MFA and RHA	0.153	−16.3	32.5	300	0.78
Serot–Uechi					
MFA	0.193	−15.75	22.1	540	0.557
RHA	0.193	−15.75	17.9	471	0.718

Hartree approximation (RHA). The corresponding coupling constants are given in table 2, and the comparison of the two approximations can be seen in fig. 1, for both nuclear matter and pure neutron matter. The equation of state in both approximations are surprisingly alike, differing at most by about 3% even at ten times nuclear density. This is a very encouraging result, since in the many applications of the theory to finite nuclei and neutron stars, the MFA has been employed up till now. The result could not have been anticipated on general grounds, since in the two approximations the equation of state is a *different* functional of the coupling

TABLE 2

Nucleon–meson coupling constants

	$(g_\sigma/m_\sigma)^2$ (fm^2)	$(g_\omega/m_\omega)^2$ (fm^2)	$(g_\rho/m_\rho)^2$ (fm^2)	b	c
this work ($m_\sigma = 600$ MeV)					
MFA	9.031	4.733	4.825	0.003305	0.01529
RHA	9.249	4.732	4.823	0.005723	0.000601
Serot–Uechi ($m_\sigma = 550$ MeV)					
MFA	11.805	8.6359	0	0	0
RHA	8.094	5.067	0	0	0
$K = 471$ MeV, $m^*/m = 0.718$, $m_\sigma = 550$ MeV					
MFA	7.697	5.0589	0	−0.00427	0.01922

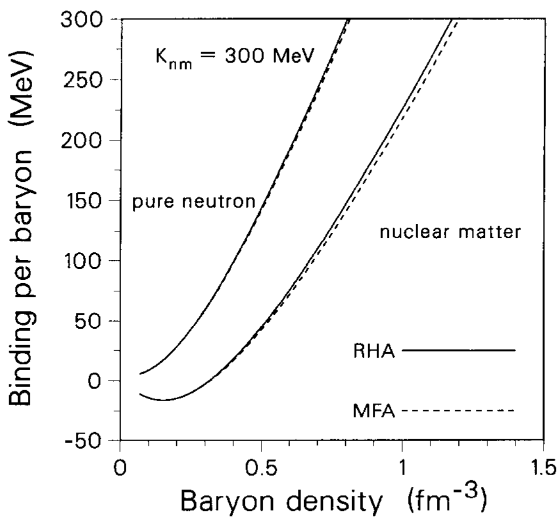

Fig. 1. Binding energy, $\varepsilon/\rho - m$, of nuclear and pure neutron matter as a function of density, computed with and without vacuum renormalization, denoted as RHA and MFA, respectively. The corresponding nuclear matter properties are listed in table 1.

constants. These are fixed in both approximations by the properties at one density, ρ_0, which assures that the functionals are coincident there, but *not* that they are nearly coincident elsewhere and most especially over such a wide range of density. Next we show in fig. 2 the separate contributions to the equation of state arising from the two-body, as well as the three and four-body terms in the energy and the two contributions V_N and V_σ of the vacuum renormalization. Aside from the region near saturation, the three and four-body terms, and the vacuum renormalization energies are all rather independent of density. They become relatively unimportant compared to the two-body energy at higher density. The scalar renormalization energy, V_σ, is particularly small. It is noteworthy that in the chiral-sigma model all of the corresponding terms are much larger, by a factor of five or so, vary more drastically with density and play a decisive role in that theory [5,14]. Indeed, without them, that theory does not possess a normal saturation curve [20]. Instead the normal state is bifurcated by an abnormal one. It is only with the addition of vacuum renormalization effects that this pathology disappears. Even so the chiral-sigma model does not have a normal ground state for finite nuclei. Instead the ground state is a bubble configuration [6]. It can be regarded as an advantage of the σ, ω, ρ model that the vacuum polarization energies are relatively constant, and have so little effect when the coupling constants are renormalized so that the five saturation properties are reproduced. The implication of the above result is that the neglect of vacuum renormalization, which in principle could produce drastic changes in the nuclear properties, is unlikely to be very important in many applications to finite nuclei and to neutron star structure.

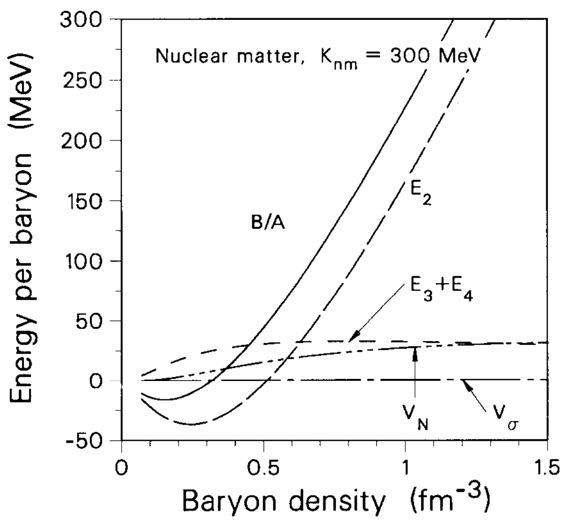

Fig. 2. For nuclear matter, the separate contributions of the two-, three- and four-body terms and the vacuum polarization energies.

In a recent paper Serot and Uechi [4]) also investigated the effects of vacuum renormalization, but they fixed only the saturation energy and density, the compression and effective mass being different in the two cases (see table 1). As a consequence of this, the two approximations yield very different results for the equation of state at higher density, as shown in fig. 3. On the other hand, when we make the comparison in the case that all five nuclear properties are identical, the two approximations again yield equations of state that are insignificantly different. The coupling constants in these three cases are shown as the last three entries respectively in table 2. The two MFA calculations shown in fig. 3 are so different from each other because both K and $m^*_{\rm sat}$ are different. The latter quantity, for given binding and saturation density, uniquely specifies the vector coupling constant through,

$$\frac{\varepsilon_0}{\rho_0}=\frac{B}{A}+m_{\rm n}=\left(\frac{g_\omega}{m_\omega}\right)^2\rho_0+\sqrt{k_0^2+m^{*2}_{\rm sat}}\,, \tag{16}$$

where the Fermi momentum at saturation, k_0, is related to the density in the usual way, $\rho_0=2k_0^3/(3\pi^2)$. (The above relation follows from eqs. (8), (12) and the saturation condition $\partial(\varepsilon/\rho)/\partial k=0$ evaluated at $k=k_0$.) For fixed K, the equation of state becomes stiffer at high density as $m^*_{\rm sat}$ decreases, as can be seen from the above relation. For fixed $m^*_{\rm sat}$, it becomes stiffer as K increases. These are the reasons why it is important to bring both of these parameters under control, through the freedom afforded by the scalar self-interaction terms in eq. (1). Without this control, the application of the theory to neutron star properties or other high density phenomena can be misleading.

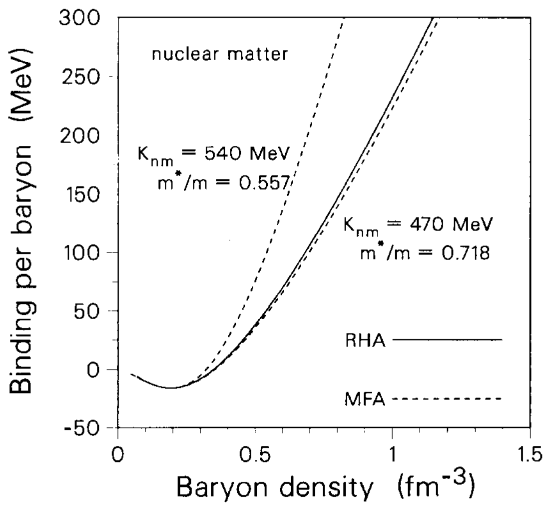

Fig. 3. Binding energy in RHA and MFA for the two sets of nuclear matter properties used by Serot and Uechi (see table 1). Also shown is the MFA in the case that the five saturation properties are the *same* as for RHA. (The dashed curve close to solid one.)

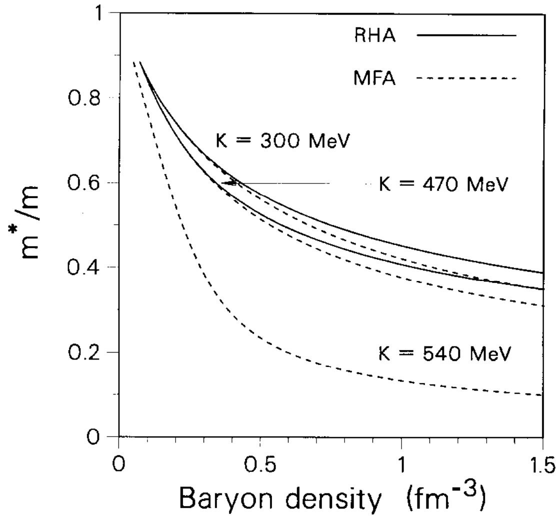

Fig. 4. Effective nucleon mass as a function of density with and without vacuum polarization (RHA and MFA respectively), for the three sets of nuclear properties of table 1 which can be identified through K.

The above conclusion is all the more reinforced by an examination of fig. 4, where the effective mass as a function of density is shown for the three sets of nuclear properties shown in table 1.

4. Neutron star matter

Neutron stars are not pure in neutron. Such a star would be beta unstable in the general sense, namely that the higher momentum neutrons have energies above the threshold for conversion to protons and leptons, and beyond a certain threshold density, they are also above the threshold for conversion to hyperons and associated kaons, and possibly to other baryon resonances. Unless there is a phase transition to a state of mixed strangeness for baryons, which we have argued is unlikely [10]), the kaons will decay, and the energy carried by any photons or neutrinos produced in this or any of the other reactions will leak out of the star eventually, in its evolution to its ground state. The ground state is therefore one of general equilibrium amongst baryons and leptons. It possesses two conserved quantities, baryon number and electric charge, the latter being vanishingly small, as required so that the Coulomb repulsion will not disrupt the star. The equations that describe such matter in this theory, are the three field equations for the mean values of the σ, ω, ρ mesons, the conditions for charge neutrality and chemical equilibrium for the electron and baryon chemical potentials, and the threshold equations for the Fermi momenta of the various baryon and lepton species. These comprise a set of self-consistent nonlinear equations in the variables just mentioned. The equations have been described elsewhere, and we refer to that reference for details [10]).

Because the time scale of star collapse is long compared to the weak interaction time, which is the scale that governs the development of the hyperon populations, the general equilibrium effects are important also for supernovae and the development of the proto-neutron star [21]).

The solution to the above system of equations can be presented as the value of the three field variables and two chemical potentials. (We represent the scalar field by the effective nucleon mass $m^* = m - g_\sigma \sigma$.) We show such solutions for the full case of general equilibrium in neutron star matter, which contains nucleons, hyperons and leptons in fig. 5, and in the case that hyperons are absent, and beta equilibrium exists between neutrons, protons and leptons, in fig. 6. The Fermi momenta of the baryons and leptons can be reconstructed from these quantities through the equations of ref. [10]). There are two points of special interest. In the case that hyperons are absent, the field strength of the (time-like component) of the neutral rho-meson, ρ_{03}, is a monotonic increasing function of density. Recall that the isospin symmetry energy density arising from this meson is

$$\varepsilon_\rho = \tfrac{1}{2} m_\rho^2 \rho_{03}^2 , \tag{17}$$

where the ρ field strength, ρ_{03} is given in terms of the isospin projection of the various constituents and their densities by eq. (9). In contrast to the above behavior of the ρ meson field strength in neutron–proton matter, or pure neutron matter, the hyperons cause it to saturate, as revealed in fig. 5. This happens for two reasons:

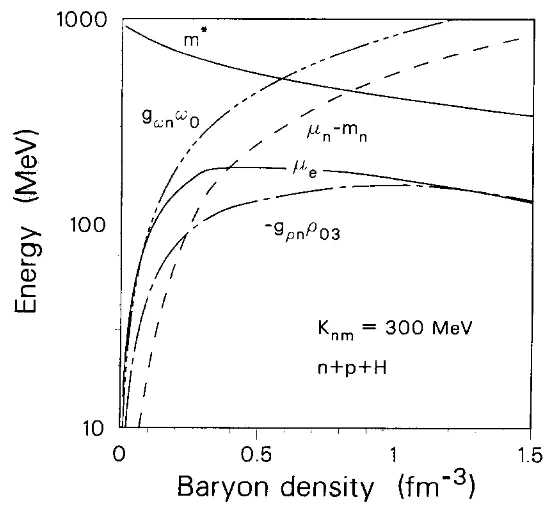

Fig. 5. Solution for neutron star matter for the general case of equilibrium among n, p, hyperons and leptons. The quantities plotted are the $m^* = m - g_\sigma \sigma$, vector and rho fields, ω_0, ρ_{03} and the two chemical potentials for charge, and baryon number, μ_e, μ_n, respectively.

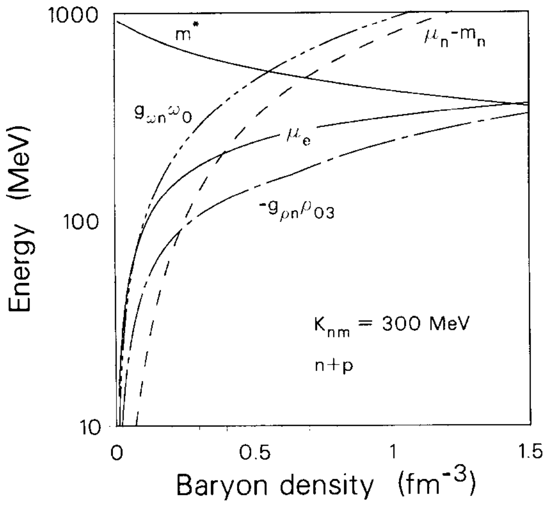

Fig. 6. Similar to fig. 5, for beta equilibrium among n, p and leptons.

As hyperons become prevalent in the star, they enter in such isospin states that tend to reduce the large negative isospin of the dominant population, the neutrons, in accord with the above symmetry energy, always consistent with the absolute constraint of charge neutrality imposed by the long-range Coulomb force. Second, as we discuss later, the hyperons are more weakly coupled to the meson fields than are the nucleons. For both reasons, hyperons tend to cause the contribution to the symmetry energy of the rho meson to saturate.

A second feature of interest in figs. 5 and 6 is the saturation of the electron chemical potential, μ_e by the hyperons. This occurs because charge neutrality can be achieved more economically among hyperons and nucleons when the electron chemical potential becomes of the order of their mass difference, than among nucleons and additional relativistic leptons. The saturation of μ_e by hyperons in neutron star matter has special significance for the possible condensation of negative pions. When μ_e exceeds the effective mass of the pion in mater*, then the negative pion is energetically more favorable for maintaining charge neutrality than additional relativistic electrons. This is because pions are bosons, and they can all condense in the lowest energy state. In this event, pions saturate μ_e. However, from fig. 5, the hyperons saturate the electron chemical potential at $\mu_e < 190$ MeV. Therefore pions cannot condense if their effective mass in matter exceeds this value. On the other hand, pions experience a repulsive s-wave and attractive p-wave interaction with nucleons. The attraction in the latter case has to be bought at the expense of finite momentum. In neutron–proton beta stable neutral matter, pions cause μ_e to saturate

* We shall refer to the dispersion relation energy, $k_0 = \sqrt{m_\pi^2 + \boldsymbol{k}^2 + \Pi(k_0, \boldsymbol{k})}$ evaluated at $k_0 = \mu_e$, as the effective pion mass.

at 177 MeV [ref. [22]]. The smallest plausible value of the pion effective mass is its vacuum value. So pions may condense in neutron star matter over a certain finite interval of density for which μ_e would otherwise exceed the pion effective mass. The interval is of finite extension because as we see in fig. 5, μ_e reaches a maximum and then decreases as hyperon populations increase. Assuming, as a maximum estimate of the effects of pion condensation, that pions condense at their vacuum mass, the solution of the equations for neutron star matter are shown in fig. 7. The plateau region in μ_e is caused by the pion condensation which arrests the growth of μ_e. Of course the use of a single-valued pion effective mass is an approximation, and the use of the vacuum mass provides an estimate of the maximum effect of pions in neutron star matter, according to the above discussion.

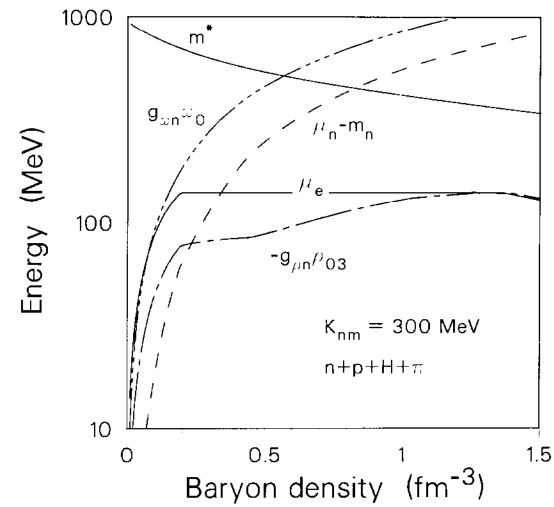

Fig. 7. Similar to fig. 5, for general equilibrium among n, p, hyperons, pions and leptons.

The equation of state of stable, charge-neutral, neutron star matter is shown in fig. 8, with and without vacuum renormalization. We also show by way of contrast, the equation of state for pure neutron matter, with vacuum renormalization. The equation of state for stable neutron star matter lies considerably below that for pure neutron matter. This softening is a result of the conversion of energetic nucleons to hyperons and the relaxation into an equilibrium population of many baryon species, in contrast to the non-equilibrium population of only the neutron in pure neutron matter. A study of these results reveals how inappropriate is the idealization of pure neutron matter for neutron stars. This is reinforced by an examination of the composition of neutron star matter, including the renormalization of the vacuum, which is shown in fig. 9. The threshold for the first hyperon lies little above two times nuclear density. In principle, other baryon resonances like the delta could

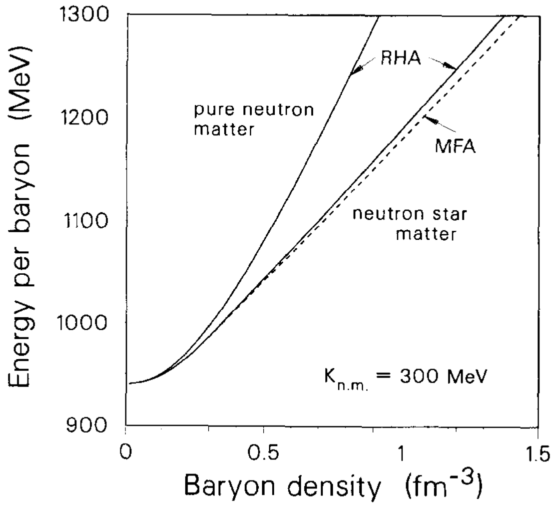

Fig. 8. Energy per nucleon for neutron star matter with and without vacuum polarization (RHA and MFA). Also shown is pure neutron matter in RHA. Properties of the corresponding nuclear matter are given in table 1, and the coupling constants in table 2.

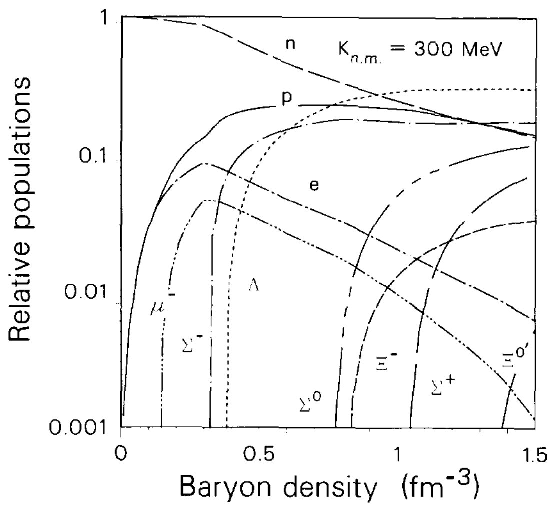

Fig. 9. Composition of stable, charge-neutral (neutron star) matter, represented as a fraction of baryon density for the various components. This calculation includes vacuum polarization (RHA), whose equation of state is shown in fig. 8.

also be present, but under the assumption of equal coupling of nucleons and deltas*, they do not appear in the density domain of neutron stars. For very low densities, neutron star matter is almost pure in neutron. However the electron and proton populations rise rapidly, even below nuclear density. These populations are initially equal, as a result of the constraint of charge-neutrality. (Excess charge would be blown off a star by the Coulomb force, which is so much stronger than the gravitational.) At densities still below nuclear density, the increasing chemical potential of the electrons makes it favorable for muons to replace electrons at the top of the Fermi distribution. At a little more than twice nuclear density, the increasing nucleon Fermi energy makes it favorable for the Σ^- to replace a neutron and lepton at the top of their respective Fermi distributions. Thereafter other thresholds are reached, the Λ, Σ^0, Ξ^-, and so on. Under the assumption of equal coupling of nucleon and delta to the meson fields, the latter does not appear in the density domain of neutron stars. This is because the most favored charge state is the Δ^-, because it can replace a higher momentum neutron *and* electron, but it has isospin projection $-\frac{3}{2}$, the same sign as that of the dominant species, the neutron, but three times the magnitude. It is therefore highly isospin unfavored. The lepton populations decrease at densities above the hyperon thresholds, as these populations increase. Eventually charge neutrality is achieved, mainly among the baryons themselves. As we have mentioned before [10]), this could effect the electrical conductivity of a neutron star, and hence the lifetime of its magnetic field and active life as a pulsar.

In the case that the effective pion mass is assumed to equal its vacuum mass, they condense in neutron star matter. The populations in this case are shown in fig. 10. The principle difference caused by pion condensation is that because they are bosons, they quench the lepton populations, and by altering the manner in which charge neutrality can be achieved, rearrange the hyperon populations. At sufficiently high density, the pions themselves are quenched by the hyperons. At moderate density above nuclear density, the pions are almost as populous as protons, being the principle agent of charge neutralization in that domain.

Such large hyperon populations in dense neutron star matter are supported by the non-relativistic calculations of Panharipande [23]) but not by Bethe and Johnson [24]). However since these early works, it has been realized that when such calculations based on two-body interactions are carried to convergence, nuclear matter saturates at twice the empirical density [25]). Moreover, even though neutron stars have dense interiors, and are the most isospin asymmetric objects known, the compression modulus and symmetry energy were not listed among the seven constraints on the early work [24]). As we show later, about half the mass of the heaviest neutron stars is composed of matter in the lower density domain below $3\rho_0$, so that such uncertainties as those mentioned are quite important for neutron star structure. Moreover, such uncertainties propagate, by continuity, into the high density domain.

* Current available evidence on the effective mass of nucleon and delta in matter is that the difference is close to that in vacuum, and that the coupling to the meson fields is therefore similar.

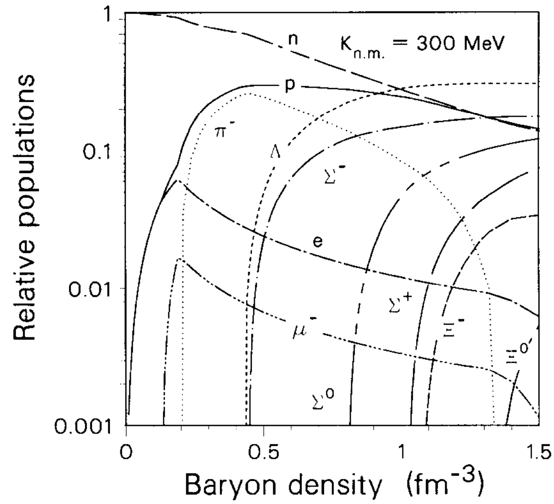

Fig. 10. Composition of stable, charge-neutral (neutron star) matter, represented as a fraction of baryon density for the various components, under the assumption that the effective mass of pions is their vacuum mass. Computed with vacuum polarization.

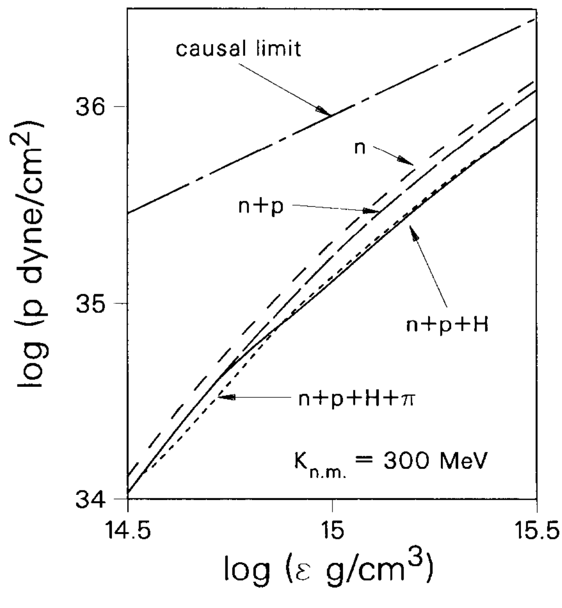

Fig. 11. Equation of state for neutron star matter in general equilibrium, (n+p+H), beta stable matter in which hyperons are ignored, (n+p), and pure neutron matter, (n). Calculations include vacuum polarization. Nuclear density is log (ε_0 g/cm^3) $\approx$ 14.4.

TABLE 3

Equation of state of neutron star matter in general equilibrium corresponding to nuclear matter properties listed in table 1 for RHA, and the corresponding coupling constants of table 2 (a number 3.5+14 means 3.5×10^{14})

ρ fm^{-3}	ϵ g/cm^3	p dynes/cm^2	ρ fm^{-3}	ϵ g/cm^3	p dynes/cm^2
0.010	1.6763 +13	1.3116 +31	0.600	1.1473 +15	1.6501 +35
0.015	2.5153 +13	1.9342 +31	0.650	1.2595 +15	1.9514 +35
0.020	3.3561 +13	2.5705 +31	0.700	1.3744 +15	2.2829 +35
0.025	4.1940 +13	3.3924 +31	0.750	1.4922 +15	2.6447 +35
0.030	5.0339 +13	4.6656 +31	0.800	1.6127 +15	3.0294 +35
0.035	5.8737 +13	6.5215 +31	0.850	1.7359 +15	3.4294 +35
0.040	6.7140 +13	9.1837 +31	0.900	1.8617 +15	3.8520 +35
0.045	7.5549 +13	1.2813 +32	0.950	1.9903 +15	4.3002 +35
0.050	8.3961 +13	1.7653 +32	1.000	2.1216 +15	4.7750 +35
0.100	1.6858 +14	1.6817 +33	1.050	2.2556 +15	5.2723 +35
0.150	2.5468 +14	5.9007 +33	1.100	2.3923 +15	5.7736 +35
0.200	3.4292 +14	1.3489 +34	1.150	2.5315 +15	6.2929 +35
0.250	4.3379 +14	2.5102 +34	1.200	2.6733 +15	6.8344 +35
0.300	5.2773 +14	4.1074 +34	1.250	2.8176 +15	7.3990 +35
0.350	6.2465 +14	5.6734 +34	1.300	2.9646 +15	7.9875 +35
0.400	7.2422 +14	7.3746 +34	1.350	3.1140 +15	8.6004 +35
0.450	8.2621 +14	9.2393 +34	1.400	3.2661 +15	9.2361 +35
0.500	9.3067 +14	1.1370 +35	1.450	3.4207 +15	9.8958 +35
0.550	1.0377 +15	1.3788 +35	1.500	3.5778 +15	1.0581 +36

TABLE 4

Equation of state of neutron star matter in general equilibrium corresponding to nuclear matter properties listed in table 1 for RHA, in the case that pions condense at their vacuum mass

ρ fm^{-3}	ϵ g/cm^3	p dynes/cm^2	ρ fm^{-3}	ϵ g/cm^3	p dynes/cm^2
0.210	3.6081 +14	1.4416 +34	0.760	1.5115 +15	2.8083 +35
0.300	5.2583 +14	3.3592 +34	0.800	1.6084 +15	3.1269 +35
0.350	6.2110 +14	5.0146 +34	0.850	1.7321 +15	3.5362 +35
0.400	7.1933 +14	7.1138 +34	0.900	1.8585 +15	3.9639 +35
0.450	8.2076 +14	9.5057 +34	0.960	2.0137 +15	4.5073 +35
0.500	9.2508 +14	1.1842 +35	1.000	2.1194 +15	4.8885 +35
0.550	1.0321 +15	1.4388 +35	1.050	2.2539 +15	5.3784 +35
0.600	1.1418 +15	1.7197 +35	1.100	2.3910 +15	5.8719 +35
0.650	1.2543 +15	2.0284 +35	1.150	2.5306 +15	6.3777 +35
0.700	1.3696 +15	2.3652 +35	1.200	2.6727 +15	6.9030 +35

The ordering of thresholds for the higher baryon states in both of the above works [23,24]) suggests that the symmetry energy at higher density becomes small in comparison with that expected from the coupling of baryons to the rho-meson, as was discussed elsewhere [10]).

The equation of state of neutron star matter in general equilibrium, represented as pressure as a function of energy density, is compared in fig. 11 with that of beta stable matter involving only neutrons, protons and leptons, and with pure neutron matter. Also the case where pions condense is shown. In all cases the vacuum renormalization is included. We note that the causal limit, $p=\varepsilon$, is respected by these relativistic theories, in contrast to theories of matter described in the Schroedinger approach. The equation of state is tabulated in table 3 for the case of general equilibrium among nucleons, hyperons and leptons and in table 4, when pions additionally condense. In the latter case, we need provide the equation of state only in the actual range of densities for which the pions appear (see fig. 10), because it is identical to table 3 below and above this range.

The equations of star structure need to be integrated to $p=0$. Therefore we supplement the high density equation of state of this work by the appropriate equation of state of the lower density domains of matter [26,27]), as in our previous work [10,11]).

5. Neutron star structure

Given an equation of state, such as those discussed in the previous section, a single parameter family of neutron stars is implied by Einstein's general theory of relativity. For a static, spherically symmetric star these equations take on the special form known as the Oppenheimer–Volkoff equations. The central energy density is a suitable parameter for characterizing the family of a particular equation of state. A general feature of any such family, is that, for stable structures, the mass is a monotonic increasing function of central density, until a maximum mass is attained. The maximum mass, known as the limiting mass, is interesting because it must exceed that of the most massive neutron star observed. So far, there are few mass measurements, because they can be performed on binary systems, and accurately only under special circumstances. The most massive measurement is for 4U0900–40, with $M=1.85^{+0.35}_{-0.30}M_\odot$, and the most accurate is for PSR1913+16, with $M=1.451\pm0.007M_\odot$ [ref. [28])].

We have calculated the families both with and without vacuum polarization. Again the coupling constants are those of table 2, which give identical nuclear matter properties in both approximations. The effect on neutron star masses is shown in fig. 12 where the MFA and the RHA are compared; the effect is found to be negligible, not because the vacuum polarization energies are negligible, but because the coupling constants in both cases give the same five saturation properties of nuclear matter. Consequently all of our earlier investigations of neutron star

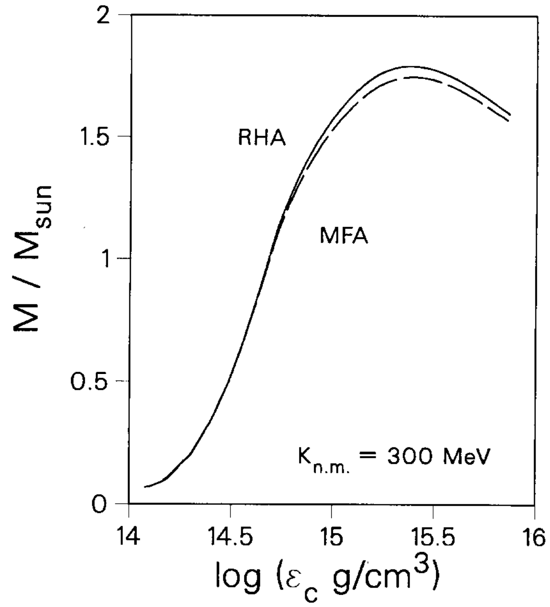

Fig. 12. Neutron star mass as function of central density computed with and without vacuum polarization (RHA and MFA) for neutron star matter in generalized equilibrium (neutrons, protons, hyperons and leptons).

structure, and in particular the limits that we found to be imposed on the equation of state by neutron star masses, stand unchanged. In the remainder of the paper we shall show the results only for the RHA (vacuum polarization included).

The properties of symmetric matter at saturation do not, of course, yield any information about the hyperon-meson couplings. We investigate first the uncertainty associaated with this. Moszkowski [29]), using quark counting arguments, suggests that these couplings should be reduced over that of nucleons by

$$g_H/g_N = x = \sqrt{\tfrac{2}{3}}. \tag{18}$$

On the other hand from evidence on hypernuclei, Walker [30]), suggests that x should be smaller, around 0.4. No matter what the coupling strength, even if free, hyperons are expected to appear in neutron star matter [31]). We compare three cases in fig. 13. A modest reduction in limiting mass results from the reduction of x from universal coupling ($x = 1$) to the value suggested by Moszkowski. The value suggested by Walker would lead to an even greater participation of hyperons in dense matter, with the first threshold occurring at little over twice nuclear density. This is because the vector repulsion is more important than the scalar attraction, and the weakening of the coupling constants is therefore favorable to hyperons in the dense region. We suspect that this last coupling is probably too drastic an estimate of the relative hyperon to nucleon couplings to mesons, and shall use Moszkowski's coupling in

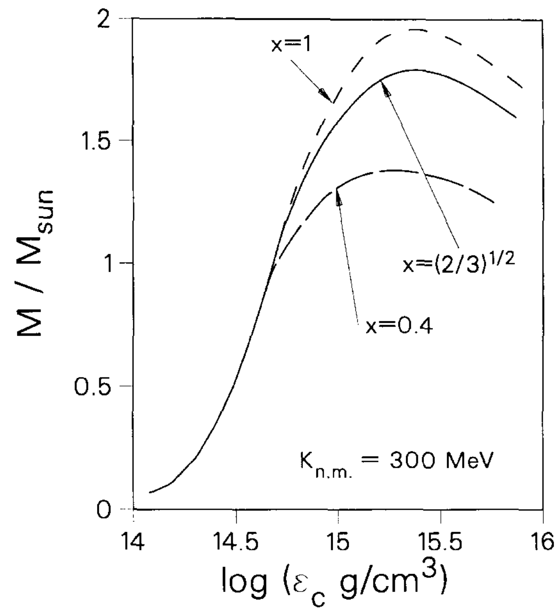

Fig. 13. Neutron star mass as a function of central density for two choices of hyperon coupling (see eq. (18)). Computed in RHA.

the remainder of the paper. This is also the coupling employed for the calculation of the equations of state shown in figs. 8–11.

Next, for the three equations of state shown in fig. 11, we show the corresponding star masses in fig. 14. Protons make their appearance in neutron star matter below the central density of the lightest neutron stars, so the pure neutron stars and the beta stable stars are shifted in mass with respect to each other, with the latter being lighter because of the softer equation of state. The first hyperon threshold occurs a little above $2\rho_0$ and is plainly visible both in the equation of state (fig. 11) and in the star mass as a function of central density. At the limiting mass, the effect of beta equilibrium is a reduction of mass of about $\frac{1}{3}M_{\odot}$ and hyperons cause a further reduction of about the same amount. Here we use the hyperon coupling suggested by Moszkowski. As shown above, the effect would be even larger if the coupling suggested by Walker was used. In either case, the effects are larger the smaller the compression modulus of symmetric matter [21]).

It is sometimes claimed that neutron star masses are not sensitive to the density domain of normal nuclei, and are therefore insensitive to nuclear matter properties. This is manifestly untrue for the lighter stars, since their central densities are not high. It is also not true for stars at the limiting mass. Although the cores of neutron stars at the limiting mass are dense, the mass of a star is not dominated by the central density. The reasons are two: the star is three dimensional *and* relativistic. The consequence is illustrated in fig. 15. There we show the fraction of mass $M(\rho)/M$

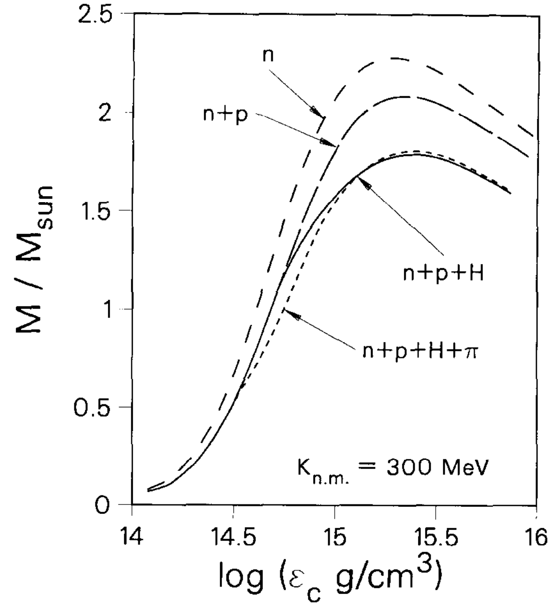

Fig. 14. Neutron star mass computed for the three equations of state shown in fig. 11. This shows how pure neutron matter overestimates the star mass.

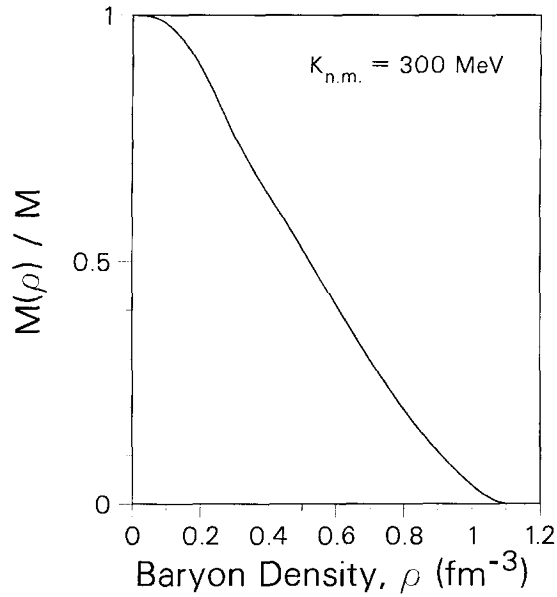

Fig. 15. Fraction of mass, $M(\rho)/M$ of the limiting mass star that is contained in matter at density greater than ρ.

that is composed of matter at densities greater than ρ. What we find is that about 50% of the mass is composed of matter at densities less than $3\rho_0$ while the central density is $\rho_c = 7.2\rho_0$, so that the limiting mass star is dominated neither by low nor high density. Besides, we have explicitly shown elsewhere how the limiting star mass depends on such saturation properties as K and m^*_{sat} [refs. [32,17])]. This is so, not only because an appreciable portion of the star's mass is contributed by matter near saturation density, as shown in the figure, but also because the equation of state is everywhere specified by its coupling constants, which determine alike the saturation properties as well as the high density behavior of the equation of state. One may argue with the model of nuclear matter but not with the inextricable connection of all domains of the equation of state through the coupling constants of theory. A disadvantage of equations of state based purely on a parameterization is that such a connection is absent.

We remark at this point on calculations reported by Serot and Uechi [4]), who find substantial differences in neutron star masses in the two approximations, RHA and MFA, in apparent contradiction to our finding. We have reproduced their calculations. The differences that they find are not attributable to the vacuum renormalization, but to the fact that the compression modulus *and* the nucleon effective mass at saturation are uncontrolled in their calculations. In particular, K has the values 470 and 540 MeV in the RHA and MFA, respectively, while m^*/m at saturation is respectively 0.718 and 0.557. Both of these differences in saturation properties effect the equation of state at high density in the same sense, as discussed earlier. When the MFA calculation is redone with parameters that give the five saturation properties corresponding to their RHA, the two compare very closely for the nuclear mattter equation of state, as was shown in fig. 3, and also for neutron stars, as shown in fig. 16.

The calculations in fig. 16 were carried out for matter in general equilibrium (i.e. nucleons, deltas, hyperons and leptons), unlike the calculations of Serot and Uechi, who restricted their calculation to pure neutron matter. This is why the limiting mass is lower than found by them. A comparison with their pure neutron matter is shown in fig. 17. The effect of equilibrium is smaller in this case as compared to fig. 14, in accord with our previous finding [21]), that the importance of equilibrium effects are inverse to the compression modulus.

Serot and Uechi also find that the effect of the rho meson, which introduces an explicit symmetry energy, has negligible effect on neutron star masses. This at first seems paradoxical. In their approximation of pure neutron matter, the contribution of the ρ meson to the energy density is quadratic in the baryon density. It is not the case, therefore, that the ρ meson is unimportant to the energy at high density, and the gravitational binding will certainly reflect its contribution. The explanation of the paradox is that in their calculation in MFA, the nucleon effective mass is very small at saturation ($0.557m$), and becomes rapidly smaller at higher density (see fig. 4). Therefore their equation of state pases quickly to one that is dominated by the vector mesons (ω and ρ), in which case, *although the baryon number content*

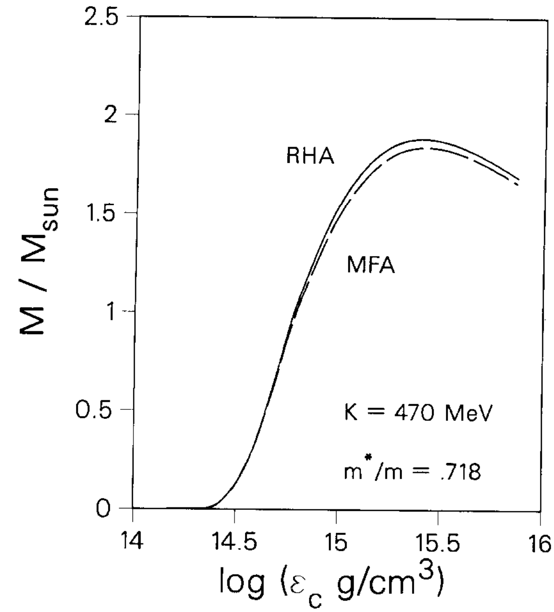

Fig. 16. Neutron star masses in RHA and MFA. In *both* cases the coupling constants produce a nuclear matter equation of state with the identical saturation properties, namely those for Serot and Uechi (table 1). Both calculations are carried out for stable neutron star matter.

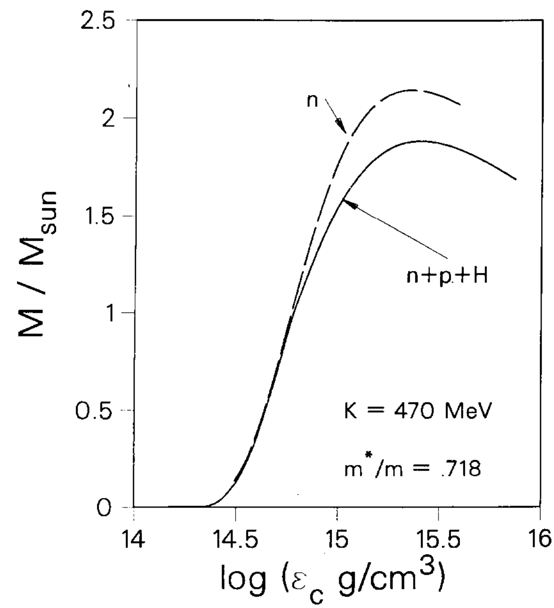

Fig. 17. Effect in RHA of generalized stability for the Serot–Uechi nuclear properties (too small a symmetry energy and too large compression modulus).

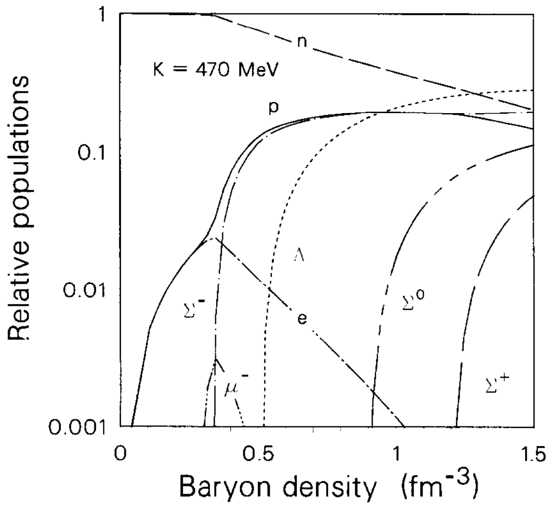

Fig. 18. Populations relative to total baryon density for the nuclear matter properties of Serot and Uechi, computed in RHA. The principal difference compared to fig. 9 is the weak symmetry energy here.

at given energy is different, the equation of state with and without the ρ meson is passing to the limit

$$p \approx \frac{1}{2}\left[\left(\frac{g_\omega}{m_\omega}\right)^2 + \frac{1}{4}\left(\frac{g_\rho}{m_\rho}\right)^2\right]\rho^2 \approx \varepsilon\,. \tag{19}$$

This is near the limit, $p = \varepsilon$ independent of the presence or absence of the ρ meson. Therefore numerically we do not disagree with their calculations. We simply point out that their conclusion is arrived at as the artifact of an effective mass that is too small compared to the empirical value cited in table 1. With the higher effective mass shown in table 1, we find an effect on the limiting star mass about four times larger than theirs. Even so, this is not a large effect, because after all the theory is dominated asymptotically by the vector mesons. Moreover, the effect of the rho meson is diluted by the hyperons. The Λ carries no isospin, and therefore does not drive the rho meson. Therefore, we agree that the rho meson does not much effect the limiting mass, though the reasons are different. It does however effect the relative baryon populations in the star, and also the gravitational binding of a star of given baryon number. The populations of neutron star matter having the saturation properties employed by Serot and Uechi, which have too small a symmetry energy, is shown in fig. 18. The large differences between this case and that of fig. 9, is not due to the difference in K but to the weaker symmetry energy of Serot and Uechi.

6. Other neutron star properties

It is possible as more observational data is gathered, that the gravitational redshift and the mass of a gamma ray burster (neutron star) will become available for the same star. If the spectral line between 300 and 500 KeV [refs. [33,34])] can be unambiguously associated with the gravitational redshift of electron-positron annihilation at the star's surface, an interesting additional constraint on the equation of state will be imposed. We show the gravitational redshift as a function of neutron star mass, and the corresponding star radii in figs. 19 and 20. The surface redshift is defined as the fractional shift in the wavelength of light that is emitted from the

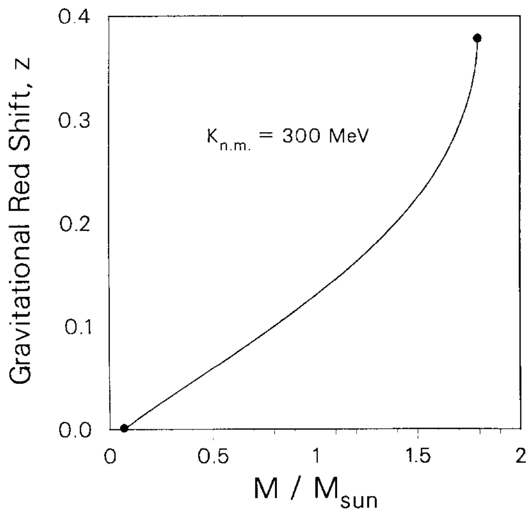

Fig. 19. Gravitational redshift as a function of neutron star mass.

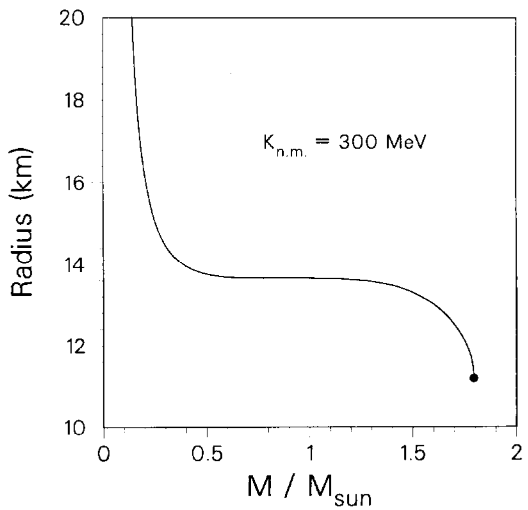

Fig. 20. Neutron star radius as a function of mass.

star surface,

$$z=\frac{\Delta\lambda}{\lambda}=e^{\lambda(R)/2}-1\,, \tag{20}$$

where the radial metric function, $\lambda(r)$, is given by,

$$\begin{aligned} e^{-\lambda(r)} &= 1-\frac{8\pi}{r}\int_0^r \varepsilon(r)r^2\,dr \\ &= 1-\frac{2M}{r}, \qquad \text{for } r>R, \end{aligned} \tag{21}$$

where M is the star mass, R its radius and $\varepsilon(r)$ is the radial distribution of energy density in the star, all of which are obtained as solutions to the Oppenheimer-Volkoff equations.

In figs. 21 and 22 we show the gravitational binding and baryon number of neutron stars as a function of their mass, and in fig. 23, the fraction of baryons that are strange is shown. This attains a value of 20% for the star at the limiting mass. This star, with central density $\rho_c \approx 7.2\rho_0$, is dominated by hyperons in the central core, as can be inferred from fig. 9. The loss of binding at the lower limit of the range of neutron stars, which occurs for $M \approx 0.069 M_\odot$ and a central density of $\rho \approx 0.46\rho_0$, and the rapid growth in radius as this limit is approached from above, correspond to each other. At the upper range of masses, the rapidly declining radius, and increasing redshift, binding and strangeness fraction are all precursors of the gravitational collapse to a block hole, as the mass approaches the limiting mass. This occurs for a central density $\rho_c \approx 7.2\rho_0$, and the corresponding mass is $M \approx 1.8 M_\odot$.

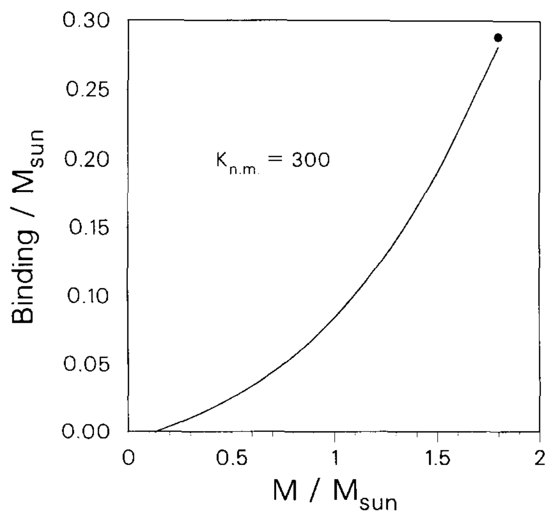

Fig. 21. Gravitational binding energy as a function of neutron star mass.

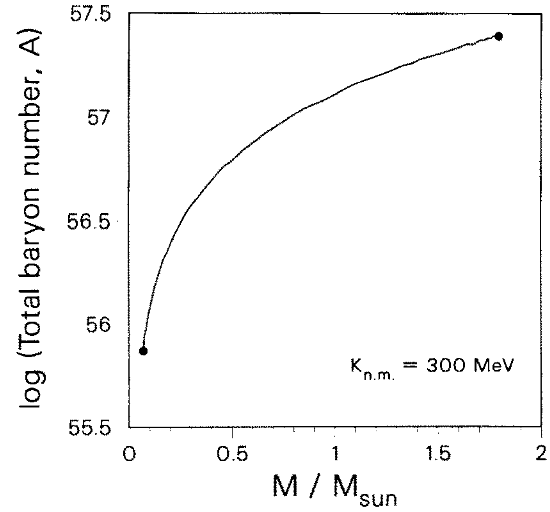

Fig. 22. Baryon number of neutron stars as a function of mass.

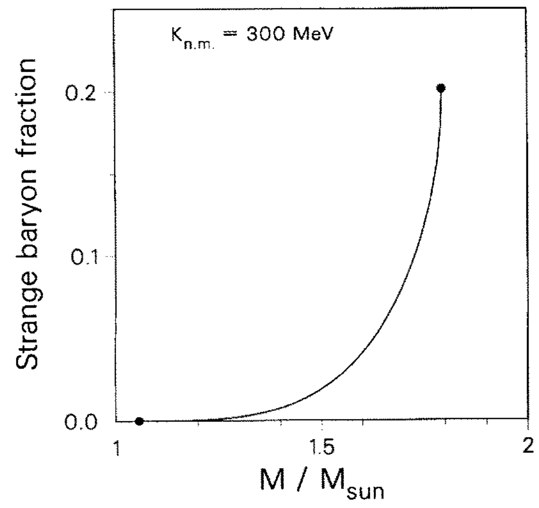

Fig. 23. Fraction of baryons that are strange as a function of neutron star mass.

7. Summary

We evaluated the vacuum polarization effects on the equation of state and neutron stars in the σ, ω, ρ theory. These effects are not negligible, although they are considerably smaller than found for the chiral sigma model [5,13]). However, it was found that when the coupling constants are renormalized so as to reproduce the five saturation properties of nuclear matter in each case, whether or not renormalization is carried out, the equation of state and neutron star properties are virtually identical in the two approximations. On the other hand, when only the saturation density and binding are controlled as in ref. [4]), the equation of state and neutron star properties, computed with and without vacuum polarization diverge at higher density. Failure to adequately constrain the equation of state at saturation can therefore lead to spurious conclusions in applications to dense matter as in neutron stars, as well perhaps in applications to nuclear structure, especially for properties that depend on K or m^*.

The question of the sensitivity of neutron star masses to the saturation properties of the corresponding nuclear matter was studied. First, as discussed above, whether the σ, ω, ρ theory yields the same or different equation of state at high density when vacuum polarization effects are incorporated, depends on how tightly the saturation properties are controlled. So within this theory, which is the only known relativistically covariant field theory of matter that can account for both nuclear matter and finite nuclei, the saturation properties and the higher density behavior are intimately connected. In fact this has to be true of any comprehensive theory of matter, since the coupling constants everywhere specify the equation of state. It need not be true of parameterizations of the equation of state, for which there is no underlying theory. Second, we explicitly demonstrated, that although the density of matter at the center of a neutron star is fairly high, $\rho_c \approx 7\rho_0$, the mass of a star, even at the limiting mass, is not dominated by dense matter. Instead, fully one half is contributed by matter at densities less than three times nuclear density. This also establishes a dependence of the limiting star mass on the equation of state near saturation.

We calculated a number of additional neutron star properties, that may become tests of the theory as more data on neutron stars becomes available. We reemphasized the role of equilibrium in neutron star structure and the equation of state. The fraction of baryons that are hyperons in the limiting mass star is about 20%, and hyperons are the dominant baryons in the central core.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the US Department of Energy under Contract DE-AC03-76SF00098.

References

1) S.A. Chin, Ann. of Phys. **108** (1977) 301
2) B.D. Serot and J.D. Walecka, The relativistic nuclear many-body problem, *in* Adv. in Nucl. Phys., ed. J.W. Negele and E. Vogt (Plenum, New York, 1986)
3) C.J. Horowitz and B.D. Serot, Nucl. Phys. **A464** (1987) 613
4) B.D. Serot and H. Uechi, Ann. of Phys. **179** (1987) 272
5) N.K. Glendenning, Nucl. Phys. **A480** (1988) 597
6) J. Kunz, D. Masak and U. Post, Phys. Lett. **B186** (1987) 124
7) J. Boguta and A.R. Bodmer, Nucl. Phys. **A292** (1977) 413
8) S.I.A. Garpman, N.K. Glendenning and Y.J. Karant, Nuc. Phys. **A322** (1979) 382
9) N.K. Glendenning, Phys. Lett. **B114** (1982) 392

10) N.K. Glendenning, Astrophys. J. **293** (1985) 470
11) N.K. Glendenning, Z. Phys. **A326** (1987) 57
12) T.D. Lee and M. Margulies, Phys. Rev. **D11** (1975) 1591;
M. Margulies, Phys. Rev. **D13** (1976) 1642
13) T.L. Ainsworth, E. Baron, G.E. Brown, J. Cooperstein and M. Prakash, Nucl. Phys. **A464** (1987) 740
14) M. Prakash and T.L. Ainsworth, Phys. Rev. **C36** (1987) 346
15) P. Möller, W.D. Myers, W.J. Swiatecki and J. Treiner, At. Data Nucl. Data Tables, **39** (1988) 225
16) M.M. Sharma, W.T.A. Borghols, S. Brandenberg, S. Crona, A. van der Woude and M.N. Harakeh Phys. Rev. **C38** (1988) 2562
17) N.K. Glendenning, Phys. Rev. **C37** (1988) 2733
18) G. Peilert, H. Stocker, W. Greiner, A. Rosenhauer, A. Bohnet and J. Aichelin, (UFTP preprint 214/1988), submitted to Phys. Rev. C
19) C.H. Johnson, D.J. Horen and C. Mahaux, Phys. Rev. **C36** (1987) 2252
20) A.K. Kerman and L.D. Miller, *in* Second high energy heavy ion summer study (1974) LBL-3675
21) N.K. Glendenning, Z. Phys. **A327** (1987) 295
22) N.K. Glendenning, P. Hecking and V. Ruck, Ann. of Phys. **149** (1983) 22
23) V.R. Pandharipande, Nucl. Phys. **A178** (1971) 123
24) H.A. Bethe and M. Johnson, Nucl. Phys. **A230** (1974) 1974
25) B.D. Day and R.B. Wiringa, Phys. Rev. **C32** (1985) 1057
26) J.W. Negele and D. Vautherin, Nucl. Phys. **A178** (1973) 123
27) B.K. Harrison and J.A. Wheeler, cited in B.K. Harrison *et al.* Gravitation theory and gravitational collapse (Univ. of Chicago Press, Chicago, 1965)
28) J.M. Weisenberg and J.H. Taylor, Phys. Rev. Lett. **52** (1984) 1348;
J.H. Taylor (private communication, April 1987)
29) S.A. Moszkowski, Phys. Rev. **D9** (1974) 1613
30) G.E. Walker, Nucl. Phys. **A450** (1986) 287c
31) V.A. Ambartsumyan and G.S. Saakyan, Astron. Zh. **37** (1963) 193 [Soviet Ast. - AJ, **4** (1960) 187]
32) N.K. Glendenning, Phys. Rev. Lett. **57** (1986) 1120
33) E.P. Mazets, S.V. Golenetski, R.L. Aptekar, Y.A. Guran and V.N. Illyinkskii, Nature **290** (1981) 378
34) E.P. Liang, Astrophys. J. **304** (1986) 682