# SYSTEMATICS OF NUCLEAR MATTER PROPERTIES IN A NON-LINEAR RELATIVISTIC FIELD THEORY

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The properties of symmetric nuclear matter are investigated in a phenomenological non-linear relativistic field theory of nuclear matter. A mean field approximation is made. We find that the equation of state over a considerable density range is determined by the nuclear matter compressibility modulus. A family of equations of state is considered that fit all known bulk properties of nuclear matter, including the energy dependence of the optical potential. The importance of non-Yukawa type nuclear interactions is discussed.

One of the central aims of nuclear physics is to determine the equation of state of nuclear matter at all physically interesting densities. Thus far the equation of state is known at only one density value. This can be inferred from the structure of finite nuclei. It reveals that nuclear matter saturates at a density of about $\rho_0 = 0.145/\mathrm{fm}^3$ with a binding energy per particle of $B_0 \approx -16$ MeV [1]. Analysis of the nuclear monopole vibration allows one to infer the compressibility modulus to be $K = 210 \pm 30$ MeV [2]. Another known property of nuclear matter in its ground state is the energy dependence of the nuclear optical potential. Any reasonable theory of nuclear matter must either predict these four established properties of nuclear matter from first principles or else incorporate them in a self-consistent phenomenological approach [3]. It is these four quantities that determine the essential properties of nuclear matter and nuclear structure in the one particle sector. An efficient parametrization of these quantities in a relativistic field theoretic framework is therefore desirable, because then one can study the properties of nuclear matter as a function of any of the above quantities. This parametrization can be done in a theory that is renormalizable, though renormalizability and other field theoretic constraints will not be explicitly considered in this work.

A large number of theoretical attempts have been made to calculate the equation of state at low and high densities using two-body potentials adjusted to fit the experimental nucleon–nucleon scattering data[‡1]. These calculations are very difficult, involving a certain amount of uncertainty as to the type and origin of interaction to be used. For example, the importance of the three-body interaction is receiving new attention [5]. These calculations make it difficult to see possible simple interrelations between physically interesting quantities. For example, one of the major results of the present work is the discovery that in self-consistent relativistic mean field models, the low density equation of state is almost completely determined by the compressibility modulus $K$. The aim of the present work is to explore these interconnections in a self-consistent relativistic field theory proposed by Boguta and Bodmer [6]. This theory allows for nuclear interactions that are not strictly two-body Yukawa type.

The nucleon $\psi$ is assumed to interact with a scalar field $\sigma$ and a vector field $\omega_\mu = (\boldsymbol{\omega}, \mathrm{i}\omega_0)$ through the following lagrangian

[‡1] For a discussion of this subject, see ref. [4].

$$\mathcal{L} = -\bar{\psi}(\gamma_\mu \partial/\partial x_\mu + m_N)\psi - \tfrac{1}{2}(\partial\sigma/\partial x_\mu)^2 - U(\sigma) - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}m_v^2\omega_\mu\omega_\mu + \mathrm{i}g_v\bar{\psi}\gamma_\mu\psi\omega_\mu - g_s\bar{\psi}\psi\sigma\,, \tag{1}$$

where

$$F_{\mu\nu} = (\partial/\partial x_\nu)\omega_\mu - (\partial/\partial x_\mu)\omega_\nu\,. \tag{2}$$

The potential functional $U(\sigma)$ is taken to be quartic polynomial in the field $\sigma$ [6]. The theory is perturbatively renormalizable.

$$U(\sigma) = \tfrac{1}{2}m_s^2\sigma^2 + \tfrac{1}{3}b\sigma^3 + \tfrac{1}{4}c\sigma^4\,. \tag{3}$$

In the above expression, the coefficient $c$, strictly speaking, should be positive, to assume the existence of a lower bound for the energy. We will allow $c$ to be a free parameter and determine its value from a phenomenological fit. As will be shown, the best fits are obtained for $c < 0$. We will comment on this point later.

For translationally and rotationally invariant infinite nuclear matter, the field equations in the mean field approximation, $\sigma \to \langle\sigma\rangle = \sigma_0$, $\omega_\mu \to \langle\omega_\mu\rangle = \mathrm{i}\delta_{\mu 0}\omega_0$, are

$$m_s^2\sigma_0 + b\sigma_0^2 + c\sigma_0^3 = -g_s\rho_s\,, \tag{4a}$$

$$m_v^2\omega_0 = g_v\rho_v\,, \qquad \boldsymbol{\omega} = 0\,, \tag{4b,c}$$

where

$$\rho_v = \frac{2}{3\pi^2}k_F^3\,, \quad \rho_s = \frac{4}{(2\pi)^3}\int^{k_F} d^3k\,\frac{m^*}{(k^2+m^{*2})^{1/2}}\,, \tag{5a,b}$$

$$m^* = m_N + g_0\sigma_0\,. \tag{5c}$$

The energy density $\epsilon$, pressure $P$, and compressibility modulus $K$ at zero absolute temperature are

$$\epsilon = \tfrac{1}{2}(g_v/m_v)^2\rho_v^2 + \frac{4}{(2\pi)^3}\int^{k_F} d^3k\,(k^2+m^{*2})^{1/2} + U(\sigma)\,, \tag{6a}$$

$$P = \rho_v^2(d/d\rho_v)(\epsilon/\rho_v)\,, \quad K = \rho_v^2(d^2/dp_v^2)(\epsilon/\rho_v)\,, \tag{6b,c}$$

The energy of a particle moving through matter with momentum $k$ is given by

$$E = g_v\omega_0 + (k^2 + m^{*2})^{1/2} \tag{7a}$$

$$= (k^2 + m_N^2)^{1/2} + U_{\mathrm{eff}}\,, \tag{7b}$$

where $U_{\mathrm{eff}}$ is the nuclear optical potential. It is given by

$$U_{\mathrm{eff}} = E - [(E - g_v\omega_0)^2 + m_N^2 - m^{*2}]^{1/2}\,. \tag{7c}$$

From the field equation eq. (4a) one sees that the density dependence of the equation of state will be determined not only by the Lorentz scalar source term $\rho_s = \bar{\psi}\psi$, which leads to a Yukawa type two-body interaction, but also by the non-linear terms $b\sigma^2$ and $c\sigma^3$, which can be interpreted as many-body interactions [7].

We have taken the non-linear terms in $U(\sigma)$ of eq. (3) to be phenomenological in nature [3]. In fact, they represent a number of effects. If we start with a reasonable field theory in which the quartic coefficient were positive, vacuum fluctuations will yield a new effective potential $U(\sigma)$, which modifies the initial $U(\sigma)$. The inclusion of quantum collections is important on theoretical and practical grounds. It invariably improves the mean field results. In the case of the linear Yukawa model ($b = c = 0$), it will reduce the large compressibility ($K \approx 550$ MeV) predicted in the mean field approximation. In the non-linear model quantum corrections allow us to have $c > 0$ in the basic lagrangian and carry through our phenomenological analysis. It was shown by Serr and Walecka [8] that quantum corrections can be absorbed into the mean field expression by redefining the effective couplings; thus we need not take them into account explicitly and be concerned that the quartic coefficient $c$ is negative in our phenomenological analysis.

In the non-linear relativistic field theory discussed here, the properties of infinite nuclear matter will depend on two dimensionless constants $C_s = (g_s/m_s)m_N$, $C_v = (g_v/m_v)m_N$ and on $b = b/g_s^3$ and $c = c/g_s^4$. For every choice of $C_s$ and $C_v$, the values of $c$ and $b$ can be determined so that symmetric nuclear matter saturates at a density of $\rho_0 = 0.145/\mathrm{fm}^3$ with the binding energy of $-15.95$ MeV/particle. An important quantity to constrain is the compressibility modulus $K$. As mentioned before, the Yukawa type field theory predicts $K = 550$ MeV. In fig. 1a we show constant compressibility contours of $K = 180$ MeV, 210 MeV, 240 MeV in the $C_s$ and $C_v$ plane. This covers the limits $K = 210 \pm 30$ MeV of the compressibility modulus deduced from nuclear monopole vibration. For this range of compressibility we find the coefficient $c$ to be always negative. An interesting question

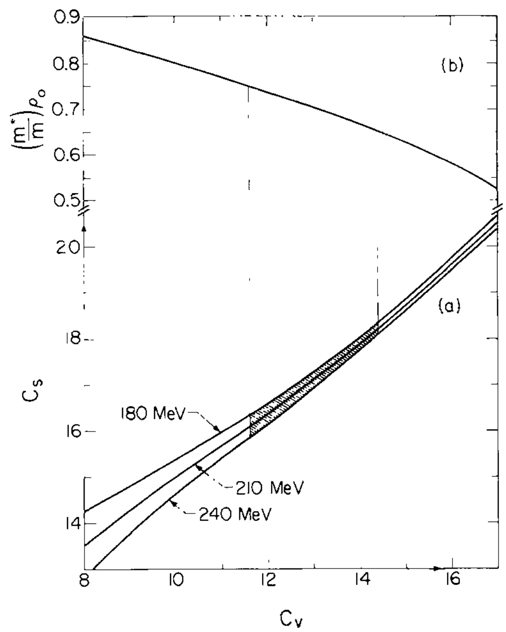


Fig. 1. (a) shows the contours of constant compressibility for $K = 180, 210$ and $240$ MeV in the $C_s$, $C_V$ plane for nuclear matter saturating at a density $\rho_0 = 0.145\ \mathrm{fm}^{-3}$ with a binding energy $B_0 = -15.96$ MeV. The shaded area indicates the narrow range of $C_s$, $C_V$ values consistent with the known bulk properties of nuclear matter and with an effective mass $m^*/m = 0.7 \pm 0.05$. (b) shows the effective mass $m^*$ at saturation versus $C_V$. The effective mass at saturation is independent of $C_s$ for fixed saturation density and binding energy.

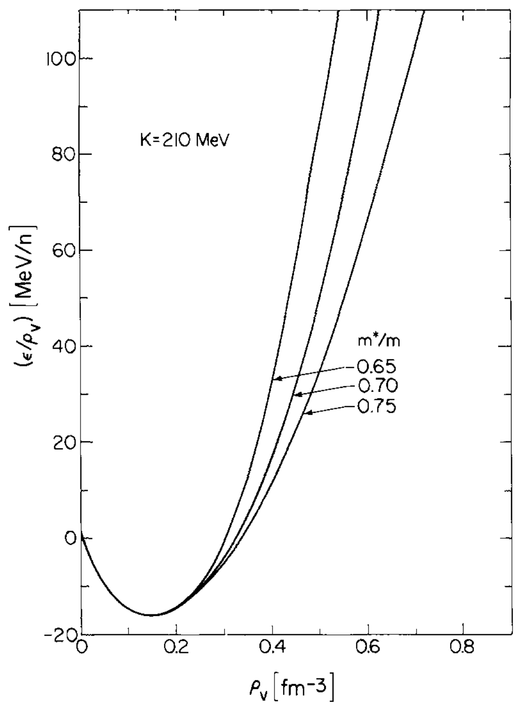


Fig. 2. The energy per particle is shown as a function of the baryon density $\rho_V$ for a fixed compressibility modulus $K = 210$ MeV, but for three different values of $C_s$, $C_V$ pair corresponding to effective masses $m^*/m_N = 0.65, 0.70, 0.75$ respectively. For densities below $\rho_0$ the three different equations of state differ by less than one percent, while the high density behavior is not uniquely determined by $K$.

is how does the equation of state vary as a function of $C_s$ and $C_v$ but constrained to give a fixed compressibility, say $K = 210$ MeV. In fig. 2 we show these equations of state for quite different values for the ($C_s$, $C_v$) pair and for physically interesting values of $m^*$. We varied the effective mass $m^*/m$ from 0.85 to 0.6 and the equations of state are almost identical for densities below the saturating density and slightly above it. The high density behavior is not determined by the compressibility modulus.

Another important quantity to determine is the energy dependence of the optical potential. In the relativistic mean field theory the effective mass $m^*$ completely determines the energy dependence of the optical potential for nuclear matter at fixed saturation density and binding energy. This can be simply seen from eq. (7a). The vector field $g_v\omega_0$ is completely determined by the density and the value of $C_v$ due to the field equation eq. (4b). The effective mass $m^*$ at saturation is completely determined by the value of $C_v$ for fixed binding energy. This is due to the validity of the Hugenholtz–Van Hove theorem in our models [3]. At saturation the Fermi energy must equal the binding energy per particle. That is

$$g_v\omega_0 + [k_F^2 + m^{*2}(k_F)]^{1/2} = \epsilon/\rho_v \tag{8a}$$

$$= -15.96\ \mathrm{MeV}\,, \tag{8b}$$

Thus we see that $m^*(k_F)$ is only a function of $C_v$ and independent of $C_s$. This means that the energy dependence of the optical potential is not uniquely related to the equation of state. A theoretically reasonable range of the effective mass is $0.65 \leqslant m^*/m_N \leqslant 0.75$, a value of 0.7 or slightly less being most consistent with the known energy dependence of the optical potential [3,9]. In fig. 1a we show the value of $m^*$ as a function of $C_v$ and in fig. 1b we have the constant compressibility contours. We are now in the position to choose judiciously the values of $C_s$ and $C_v$ that are

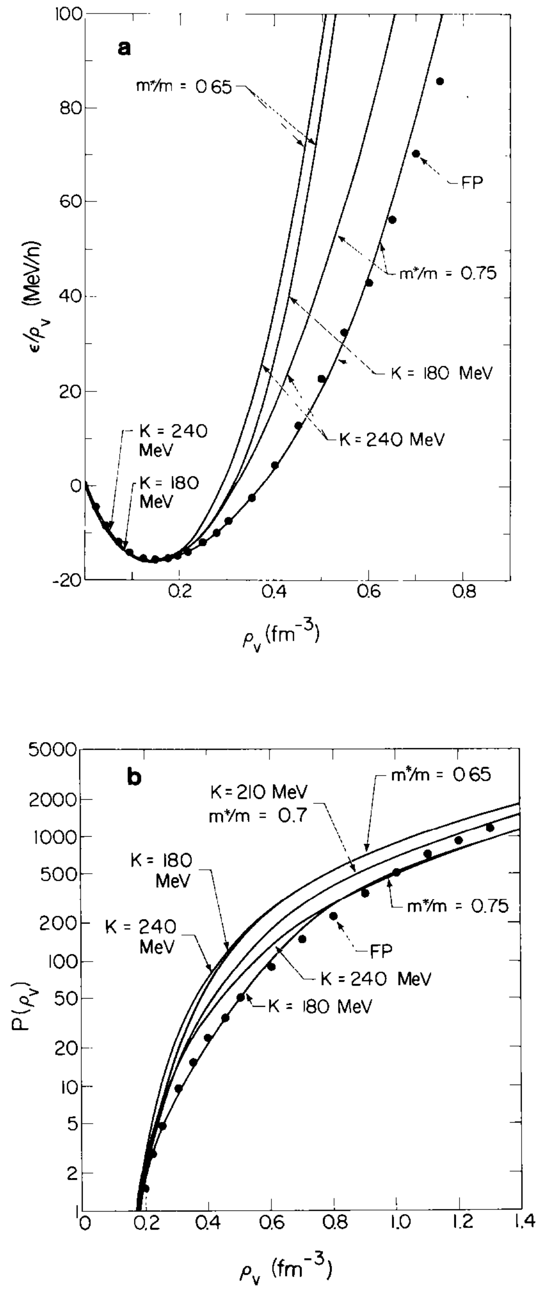


Fig. 3. (a) The energy per particle is shown as a function of baryon density for the corner points of the shaded area of fig. 1a. The dots represent the results of Friedman and Pandharipande's calculation using the variational method. (b) Shows the density dependence of the pressure for the same models as in fig. 3a. Also the pressure for the best choice of parameters, $K = 210$ MeV, $m^*/m_N = 0.7$ is shown. The dots indicate the results of Friedman and Pandharipande.

consistent with the known bulk properties of matter. This range we show in the shaded area of fig. 1b.

There are two interesting questions to answer. First, how does the equation of state at low densities. vary as one varies the interaction parameters inside the shaded region of fig. 1b. The second question is how does the equation of state vary at high densities. The answer is shown in fig. 3a, 3b. The equation of state at low density is quite insensitive to the variation of compressibility, while the equation of state at high densities can vary substantially. We can conclude that the extrapolation of the equation of state from known nuclear properties at normal densities cannot be reliable. The dots in fig. 3a, 3b represent the results of many-body variational calculation of Friedman and Pandharipande [10]. At low densities the agreement is very good. At high densities the best agreement is obtained for $m^*/m_N = 0.75$. This effective mass, we believe, is too large for the mean field theory. As already noted, a value of $m^*/m_N \approx 0.7$ is in best agreement with the energy dependence of the nuclear optical potential. There is no reason to believe that the nonrelativistic variational calculations at high nuclear density should be reliable. In fig. 3b we show the corresponding pressures as a function of density.

The above results demonstrate that even a precise determination of the nuclear equation of state at densities $\rho < 1.2\,\rho_0$ does not enable us to predict the high density behavior with reasonable accuracy. A theoretical determination is also very difficult in view of the fact that many-body forces can play an essential role [5,10]. Experiments must reinvigorate the quest for the high density equation of state.

We have shown that the equation of state at low densities in non-linear relativistic mean field theory is determined by the compressibility modulus for a fixed saturation density and binding energy. The precise mechanism (i.e. the magnitude of $b$ and $c$) for saturation is unimportant. Furthermore, the value of the effective mass is not uniquely related to the equation of state but is an additional quantity that has to be determined. Its value can be extracted from the energy dependence of the optical potential. A significant shortcoming of the Yukawa type field theory proposed by Walecka [8] is its incorrect energy dependence of the optical potential at low energies.

One would have expected that a theory of nuclear matter that does not contain many well-accepted aspects of many-body theory cannot be a reasonable starting point in the study of nuclear properties. By explicit calculation and comparison we showed that this expectation is indeed false [3]. In fact, the agreement between the non-linear relativistic mean field theory and many-body calculations is easily understood. We have shown that once one has predicted, or fitted, a few known nuclear matter properties, the equation of state at low densities as well as the energy dependence of the optical potential are determined. As far as the equation of state is concerned, its low density behavior is determined by the value of nuclear compressibility modulus at saturation and independent of the precise way nuclear saturation is achieved. This suggests that nucleon–nucleon itneraction in matter cannot be uniquely extracted from the knowlegde of the equation of state at low ($\rho < 1.2\,\rho_0$) densities. The converse is also true, because of the freedom to choose the strength of the non-linearities. This, we believe, is a strong motivation for an experimental effort to study high density phenomena achievable in intermediate and high energy heavy ion collisions. In this way, much more will be known about the equation of state and will shed light on the essential properties a many-body theory must have.

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