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Central density dependent anisotropic compact stars

Mehedi Kalam[1,a], **Farook Rahaman**[2,b], **Sk. Monowar Hossein**[3,c], **Saibal Ray**[4,d]

[1]Department of Physics, Aliah University, Sector V, Salt Lake, Kolkata 700091, West Bengal, India
[2]Department of Mathematics, Jadavpur University, Kolkata 700032, West Bengal, India
[3]Department of Mathematics, Aliah University, Sector V, Salt Lake, Kolkata 700091, West Bengal, India
[4]Department of Physics, Government College of Engineering & Ceramic Technology, Kolkata 700010, West Bengal, India

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Abstract Stars can be treated as a self-gravitating fluid. In this connection, we propose a model for an anisotropic star under the relativistic framework of Krori and Barua (J. Phys. A, Math. Gen. 8:508, 1975) spacetime. It is shown that the solutions are regular and singularity free. The uniqueness of the model is that interior physical properties of the star solely depend on the central density of the matter distribution.

1 Introduction

Late time evolution of stars due to strong gravity has been an interesting investigating field in astrophysics which lets us know, via several physical processes, features of diverse characters of gravitating objects. So we actually come across a paradigm shift from the stage of normal stars to compact stars, ranging from dwarf stars to neutron stars, quark stars, dark stars, gravastars, and eventually black holes. However, in the present study we are specifically interested for compact strange stars category.

In connection to compact stars it have been shown by several workers [2–6] that the Krori and Barua [1] (henceforth KB) spacetime provides an effective platform for modeling quark and strange type compact stars. Some of the interesting works to be mentioned with KB metric have been carried out by Rahaman et al. [2] for singularity-free dark energy stars which represents an anisotropic compact stellar configuration with 8 km radius whereas the same KB spacetime with MIT Bag model was considered by Rahaman et al. [3] to describe ultra-compact object like strange star with radius of 8.26 km. Kalam et al. [6] also proposed a model for strange quark stars within the framework of MIT Bag model. This model clearly indicates that the Bag constant need not necessarily lie within the range of 60–80 $\mathrm{MeV/fm^3}$ as claimed in the literature.

In the present work, we propose a model for an anisotropic compact star where it is assumed that the radial pressure exerted on the system is proportional to the matter density. The stellar configuration, therefore, comprises two fluids—an ordinary baryonic matter together with an yet unknown form of matter (i.e. dark energy of Einstein-type varying cosmological constant) which is repulsive in nature. These two fluids are assumed to be non-interacting amongst themselves with the radial and transverse directional anisotropic property such that $p_r \neq p_t$ [2]. In favor of anisotropy we would like to point out that for fluid configuration this is not only natural but also obvious general option for describing relativistic compact stellar objects [7] (also see [8] for a review). Recent observational evidences on highly compact astrophysical objects like X-ray pulsar HerX-1, X-ray buster 4U1820-30, millisecond pulsar SAXJ1808.4-3658, X-ray sources 4U1728-34 etc. strongly favor an anisotropic pressure distributions in the case of an ultra-compact object. The mechanism at the microscopic level, though not yet well understood, may be related to variety of reasons such as the existence of type IIIA superfluid, mixture of two fluids, phase transition, pion condensation, bosonic composition, rotation, magnetic field etc.

As mentioned above basically we have considered here a two fluids model for compact star with Λ-dark energy as one of the ingredients. This consideration has been motivated by the recent observations on the accelerated expansion of the universe. It is argued that about 73 % of the energy content of the universe is of gravitationally repulsive in nature (*dark energy*) [9, 10]. As a result, cosmological models based on dark energy received much attention in the recent past either in the form of a non-zero cosmological constant (present

[a] e-mail: kalam@iucaa.ernet.in
[b] e-mail: rahaman@iucaa.ernet.in
[c] e-mail: sami_milu@yahoo.co.uk
[d] e-mail: saibal@iucaa.ernet.in

value, $\Lambda \sim 1.1 \times 10^{-56}$ cm^{-2} [11]) or in some other exotic forms of matter. However, in the astrophysical realm dark energy may have implications not only with a pure cosmological constant [12–15] but also a varying cosmological constant [16–19]. It has essentially been speculated that within a very massive dense star and/or galaxy a constant Λ may have substantial effect and in the case of a space varying Λ vary strength wise from center to boundary and decreases to extremely small non-zero value ($\Lambda \sim 10^{-56}$ cm^{-2}) outside the system. In this connection we quote from Bambi [20] as he argues that "The cosmological constant problem represents an evident tension between our present description of gravity and particle physics. Many solutions have been proposed, but experimental tests are always difficult or impossible to perform and the present phenomenological investigations focus only on possible relations with the dark energy, that is, with the accelerating expansion rate of the contemporary universe. ... strange stars, if they exist, could represent an interesting laboratory to investigate this puzzle, since their equilibrium configuration is partially determined by the QCD vacuum energy density".

We would like to mention here the very recent work of Hossein et al. [5] where the authors have assumed cosmological constant with radial dependence i.e. $\Lambda = \Lambda(r)$ to describe non-singular model for anisotropic compact stars under the KB spacetime. In the present work we are essentially following the same root, however, with a quite different motivation. Our sole aim here is to find a set of exact solution which is completely free from singularity (at $r = 0$) and all the parameters involved in the solutions depend only on the central density (ρ_0) of the stellar interior. This then facilitates one to explore different physical parameters from a single parameter ρ_0.

The plan of the present paper is as follows: In Sect. 2 we have provided the basic equations in connection to the proposed model for strange star. Section 3 is dealt for finding out a class of exact solution for the stellar interior of KB metric. Stability of the model is shown in Sect. 4 whereas mass-radius relation and hence redshift calculation are done in Sect. 5. Some concluding remarks are made in Sect. 6.

2 Basic formulations

To describe the spacetime of a compact stellar configuration, we consider the metric of KB [1] as given by

$$ds^2 = -e^{\nu(r)}\,dt^2 + e^{\lambda(r)}\,dr^2 + r^2\big(d\theta^2 + \sin^2\theta\,d\phi^2\big), \tag{1}$$

with $\lambda(r) = Ar^2$ and $\nu(r) = Br^2 + C$. Here A, B and C are arbitrary constants to be determined on physical grounds.

We further assume that the energy-momentum tensor for the strange matter filling the interior of the star may be expressed in the following standard form:

$$T_{ij} = \mathrm{diag}(\rho, -p_r, -p_t, -p_t), \tag{2}$$

where ρ, p_r and p_t correspond to, respectively, the energy density, radial pressure and transverse pressure of the baryonic matter.

The Einstein field equations in the presence of Λ can as usual be written as

$$G_{ij} + \Lambda g_{ij} = -8\pi G T_{ij}, \tag{3}$$

where the erstwhile cosmological constant Λ is assumed to be space-dependent so that $\Lambda(r) = \Lambda_r(say)$. Here $G = c = 1$ under geometrized relativistic units.

3 Interior structure

In this model we propose that the radial pressure of a star is proportional to the matter density in the following form:

$$p_r = \left(\frac{x}{1+x}\right)\rho, \tag{4}$$

where x is any '+ve' real with the equation of state parameter $\omega(r) = x/(1+x)$. Here the choice is to ensure that $0 < \omega(r) < 1$.

By plugging the above expression for p_r of Eq. (4) into Eq. (3), we get

$$\rho = \frac{(1+x)(A+B)}{4\pi(1+2x)}e^{-Ar^2}, \tag{5}$$

$$p_r = \frac{x(A+B)}{4\pi(1+2x)}e^{-Ar^2}, \tag{6}$$

$$p_t = \frac{1}{8\pi(1+2x)}\left[e^{-Ar^2}(1+2x)\big(B^2 - AB\big)r^2 + (2Bx - A) - \frac{(1+2x)}{r^2}\right] + \frac{1}{8\pi r^2}, \tag{7}$$

$$\Lambda_r = \left[2(A\omega_r - B) - \frac{1+\omega_r}{r^2}\right]\frac{e^{-Ar^2}}{1+\omega_r} + \frac{1}{r^2}. \tag{8}$$

Therefore, the equation of state parameters corresponding to the radial and transverse directions can be provided as

$$\omega_r(r) = \frac{x}{1+x}, \tag{9}$$

$$\omega_t(r) = \frac{1}{2(1+x)(A+B)}\left[\big(B^2 - AB\big)r^2(1+2x) + (2Bx - A) + A - \frac{1+2x}{r^2}) + \frac{e^{Ar^2}(1+2x)}{r^2}\right]. \tag{10}$$

The measure of anisotropy, $\Delta = (p_t - p_r)$, in this model is obtained from Eqs. (6) and (7) as follows:

$$\Delta = \frac{e^{-Ar^2}}{8\pi}\left[\left((B^2 - AB)r^2 - A - \frac{1}{r^2}\right) + \frac{e^{Ar^2}}{r^2}\right]. \tag{11}$$

To find the expressions for the constants A and B of the KB model we match the interior metric to the exterior of the Schwarzschild solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{12}$$

Then assuming the boundary conditions smoothly on the surface of the stellar configuration at $r = R =$ radius of the star, we get

$$A = \frac{8\pi\rho_0}{3}, \tag{13}$$

$$B = \frac{1}{2R^2}\left[e^{\frac{8\pi\rho_0}{3}R^2} - 1\right]. \tag{14}$$

At the center $r = 0$, the density (5) becomes

$$\rho_0 = \frac{(1 + x)(A + B)}{4\pi(1 + 2x)}. \tag{15}$$

Therefore, from Eq. (15), after substituting $\omega_r(r)$, A and B from Eqs. (9), (13), and (14), we get

$$\omega_r(r) = \frac{1}{8\pi\rho_0 R^2}\left[e^{\frac{8\pi\rho_0}{3}R^2} - 1\right] - \frac{1}{3}. \tag{16}$$

This result immediately implies that the radial equation of state parameter of a particular star completely depends on the central density only. It will then be convenient to rewrite the whole set of physical parameters from Eqs. (5)–(7) in terms of this central density as follows:

$$\rho = \rho_0 e^{-\frac{8\pi\rho_0}{3}r^2}, \tag{17}$$

$$p_r = \frac{1}{8\pi R^2}\left[e^{\frac{8\pi\rho_0}{3}(R^2 - r^2)} - e^{-\frac{8\pi\rho_0}{3}r^2}\right] - \frac{\rho_0}{3}e^{-\frac{8\pi\rho_0}{3}r^2}, \tag{18}$$

$$p_t = \frac{e^{-Ar^2}}{8\pi}\left[(B^2 - AB)r^2 + \frac{2B(A + B - 4\pi\rho_0)}{A + B} - \frac{A(8\pi\rho_0 - A - B)}{2(A + B)} - \frac{1}{r^2}\right] + \frac{1}{8\pi r^2}, \tag{19}$$

where A and B are already shown to be dependent on the central density as obvious from Eqs. (13) and (14).

From the above set of solution we observe the following two important features:

(1) Since the central density, ρ_0, being a non-zero constant quantity, all the physical parameters, viz. the matter density, radial pressure and transverse pressure of the star under consideration is entirely free from any singularity at $r = 0$.
(2) At any distance from the center all the physical parameters completely depend only on the central density. In a similar way this also implies that the measure of anisotropy (Δ) and equation of state parameters (ω_r and ω_t) are central density dependent. Therefore, we can conclude that at any distance from the center of the star, these are exactly measurable once we know the central density alone.

These results are shown graphically in Figs. 1 and 2. One can see that at $r = 0$, the density becomes the central density ρ_0 itself. We see here that the central density of the star is low for comparatively bigger stars. Figures 2 and 3 specially indicates that the present model is of compact strange star with radius about 10 km. On the other hand the measure of anisotropy are plotted in Figs. 3 and 4 to show the variation with respect to the central density. From these figures

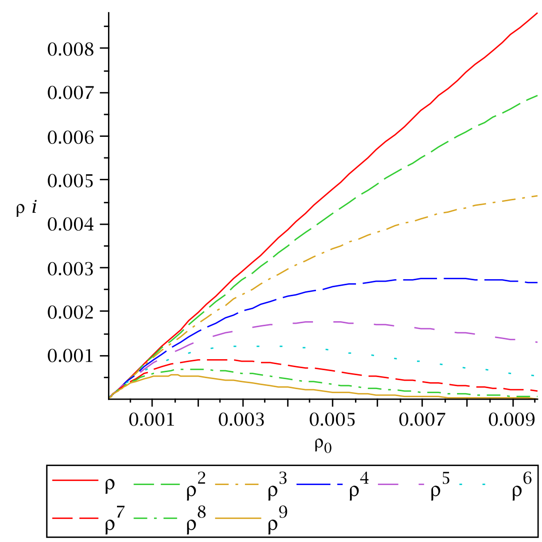

Fig. 1 Density comparison of different star radii where ρ^i indicate the density of the star of 'i' km radius

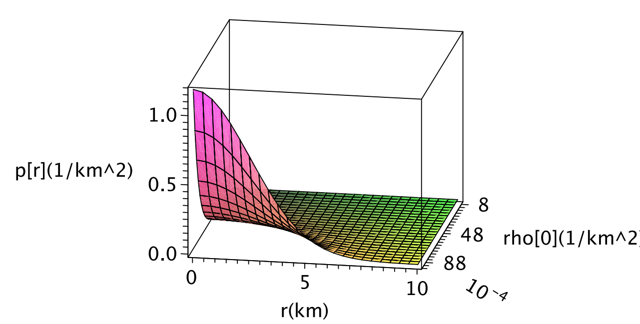

Fig. 2 Pressure variation at the stellar interior

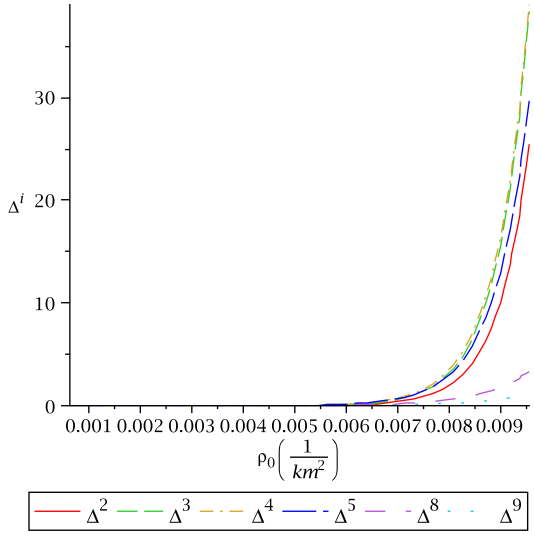

Fig. 3 Δ^i variation with respect to central density at the stellar interior for different star radii where Δ^i indicate the anisotropy of the star of 'i' km radius

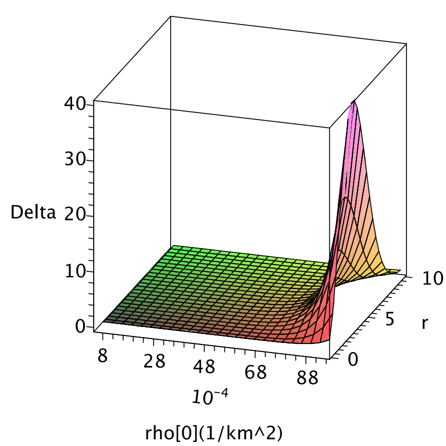

Fig. 4 Δ variation with respect to central density at the stellar interior

we can say that if the central density of the star becomes $<0.0068\ \text{km}^{-2}$ then there should be no anisotropic behavior within the stellar structure.

4 Stability

For a physically acceptable model, one expects that the velocity of sound should be within the range $0 \le v_s^2 = (\frac{dp}{d\rho}) \le 1$ [8, 21]. In our anisotropic model, we define sound speeds as

$$v_{\mathrm{sr}}^2 = \omega_r(r) = \frac{x}{1+x}, \tag{20}$$

$$v_{\mathrm{st}}^2 = \frac{dp_t}{d\rho}. \tag{21}$$

We plot the radial and transverse sound speeds in Fig. 5 and observe that these parameters satisfy the inequalities

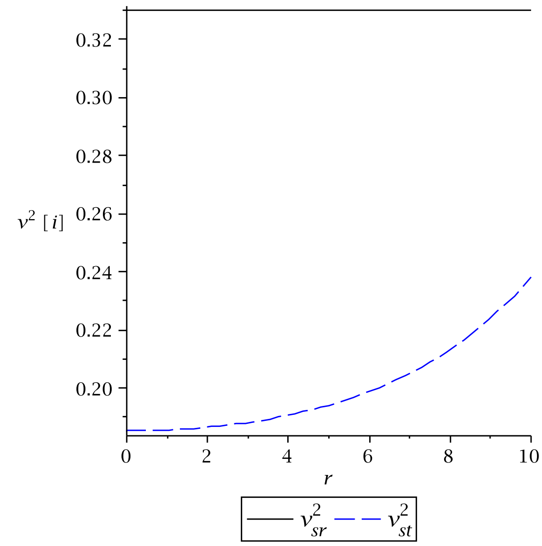

Fig. 5 Sound speed variation of the stars

$0 \le v_{\mathrm{sr}}^2 \le 1$ and $0 \le v_{\mathrm{st}}^2 \le 1$ everywhere within the stellar object.

Equations (20) and (21) lead to

$$\begin{aligned} & v_{\mathrm{st}}^2 - v_{\mathrm{sr}}^2 \\ &\quad = \frac{(\omega_r+1)}{2A(A+B)}\left[\frac{e^{Ar^2}}{r^4} + A\big(B^2 - AB\big)r^2\right. \\ &\qquad \left. - \left(\big(B^2 - AB\big) + \frac{A^2 - A\omega_r(2B+A)}{(\omega_r+1)}\right) - \frac{A}{r^2} - \frac{1}{r^4}\right] \\ &\qquad - \omega_r. \end{aligned} \tag{22}$$

From Eq. (22) we note that $|v_{\mathrm{st}}^2 - v_{\mathrm{sr}}^2| \le 1$. Since, $0 \le v_{\mathrm{sr}}^2 \le 1$ and $0 \le v_{\mathrm{st}}^2 \le 1$, therefore, $|v_{\mathrm{st}}^2 - v_{\mathrm{sr}}^2| \le 1$.

Now, to examine the stability of local anisotropic matter distribution, we use Herrera's [8] *cracking* (also known as *overturning*) concept which states that the region for which radial speed of sound is greater than the transverse speed of sound is a potentially stable region. In our case, Fig. 6 indicates that there is no change of sign for the term $v_{\mathrm{st}}^2 - v_{\mathrm{sr}}^2$ within the specific region of the configuration. This implies that the KB-type strange star model is a stable one if the radius of the star is >5 km. From Fig. 7 it is evident that if the radius of a star is <5 km then the central density should be $>68 \times 10^{-4}\ \text{km}^{-2}$ [approx] to be stable according to our proposed model.

5 Surface redshift

In this section, we study the maximum allowable mass-radius ratio in our model. For a static spherically symmetric perfect fluid star, Buchdahl [22] showed that the maximally

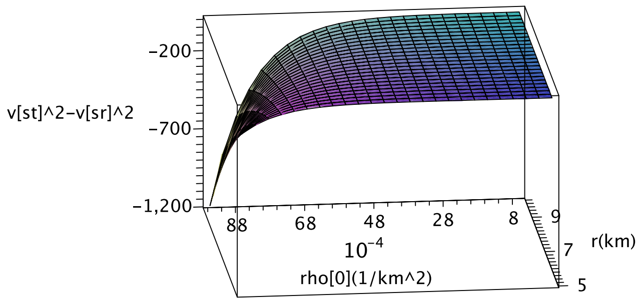

Fig. 6 Sound speed variation of the stars with radii >5 km

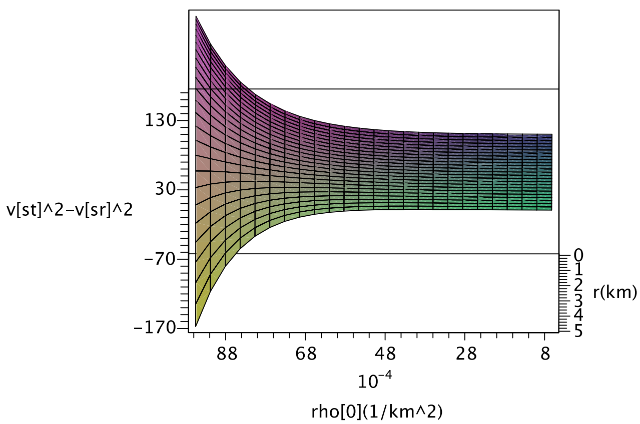

Fig. 7 Sound speed variation of the stars with radii ≤5 km

allowable mass-radius ratio is given by $\frac{2M}{R} < \frac{8}{9}$ (for a more generalized expression for the same see Ref. [23]).

Now we write the compactness of the star in the following form:

$$u = \frac{M}{R}$$
$$= -\frac{(A+B)}{4R(\omega_r+1)A^{\frac{5}{2}}}\left[2\,\mathrm{R}e^{-AR^2}A^{\frac{3}{2}} + \sqrt{\pi}\,\mathrm{erf}(\sqrt{A}R)A\right]. \tag{23}$$

The surface redshift (Z_s) corresponding to the above compactness factor (u) is obtained as

$$Z_s = [1-2u]^{-\frac{1}{2}} - 1$$
$$= \left[1 + \frac{(A+B)}{2R(\omega_r+1)A^{\frac{5}{2}}}\left(2\,\mathrm{R}e^{-AR^2}A^{\frac{3}{2}} + \sqrt{\pi}\,\mathrm{erf}(\sqrt{A}R)A\right)\right]^{-\frac{1}{2}} - 1. \tag{24}$$

It is clear from the above expression that redshift is dependent on ρ_0 and also it is well behaved (Fig. 8).

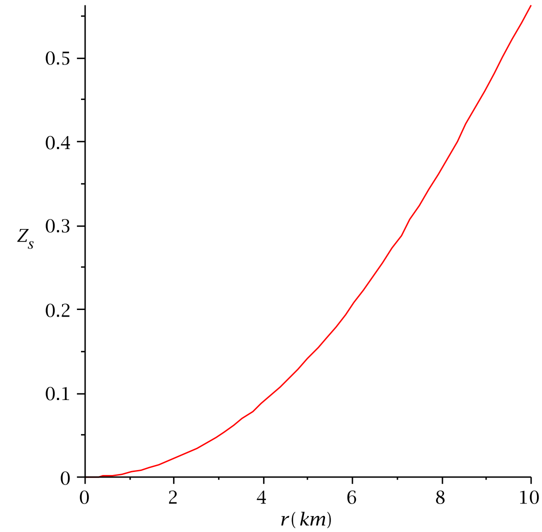

Fig. 8 Redshift variation at the stellar interior

6 Conclusion

Keeping in mind that stars can be treated as a self-gravitating fluid we have proposed a very simple and unique model for an anisotropic star. The spacetime here is of KB-type [1] which is supposed to present compact strange stars solutions [2–6]. It is shown through the solution set that (1) the central density ρ_0 being a non-zero constant quantity the star is non-singular at $r = 0$, and (2) interior physical properties of a compact star solely depend on the central density of the stable stellar configuration. Thus if we know the *mass-radius ratio* then all the interior features of a star can be exactly evaluated at any position of the interior of that star.

Further it may be observed that if we know the central density and radius of a star then assuming the spherical structure we can say that matter distribution follow a regular behavioral pattern whatever the size of a star may be. We also show that the central density is proportionately low for bigger star.

As a final comment, following the introductory discussion regarding inclusion of cosmological constant in the source term of Einstein field equations as one of the candidates of dark energy, we argue that "Strange stars, if they exist, can play an important role in the solution to the cosmological constant problem" [20].

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