OpenPulse - Open source code for numerical modelling of low-frequency acoustically induced vibration in gas pipeline systems

Theory Reference D: Weak Coupling
V2.0

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1 Acoustic-structure one-way coupling

In the previous section, the acoustic behavior of the fluid being transported throughout pipes and the structural behavior of pipeline systems were described independently (by assuming appropriate hypotheses that allow modeling each behavior independently). However, the objective of this work is to predict the dynamic response of structural systems subjected to harmonic acoustic loads.

In this research, we consider the structural pipe element as a beam with a hollow section, as can be seen in Fig. 1. The acoustic pipe element is configured in such a manner that the FETM mesh is the same as that used for FEM structural analysis. Additionally, the acoustic pipe element diameter equals to the internal structural diameter D_i considered in the respective beam element.

There are some important works on the formulation of an "elasto-acoustic" beam based on the "strong" coupling between structural and fluid fields [6, 7, 8, 3]. However, in this work it is assumed a "weak" coupling (one-way coupling) based on the following hypotheses:

- the fluid is a gas;
- the viscosity of the fluid is negligible;

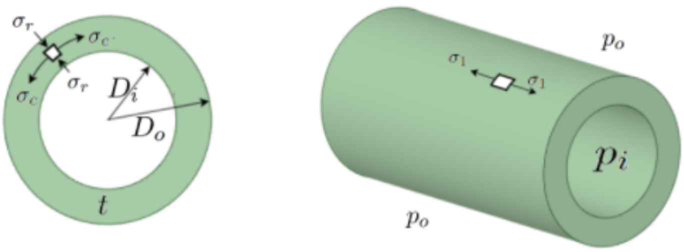

Figure 1: Section of the pipe element. Stress state.

- the acoustic plane wave propagates in axial direction (1-D acoustics);
- the pipe is thin-walled and linearly elastic;
- the radial inertia of the pipe wall is neglected if considering low frequencies ($ka << 1$) [5];
- the acoustic wave speed c_f in the gas is affected by the mechanical compliance of the pipe wall [5] and is given by:
$$c_f = \left(\frac{\rho_f}{K_f} \left(1 + \frac{D_i K_f}{Et} \right) \right)^{-1/2}, \tag{1}$$
where ρ_f is the fluid mass density, K_f is the fluid bulk modulus, E is the Young's modulus of pipe material, D_i is the internal diameter and t is the pipe wall thickness;
- resultant normal and shear stresses in the structure are derived from the internal fluid pressure under the assumptions: i) plane stress conditions in the wall; ii) cross sections remain plane to the neutral axis.

1.1 Internal pressure effect on a structural pipe: equivalent load vector

The internal pressure has a zero external resultant force acting on the pipe. However, in the approach used in this work [1, 4], considering the assumptions listed above, the load equivalent to the stress field induced by the pressure effect is applied on each element as an external axial load, considering harmonic pulsation/vibration with an excitation frequency ω.

The axial stress is resultant from the difference between internal and external pressure, and is given by [2, 1]:
$$\sigma_1(\omega) = \frac{p_i(\omega) D_i^2 - p_o D_o^2}{D_o^2 - D_i^2}, \tag{2}$$
where $p_i(\omega)$ is the internal pressure, obtained from the acoustic problem; D_i is the internal diameter; D_o is the external diameter; and p_o is the external pressure, considered constant in this work.

The radial stress can be obtained using the Lamé stress distribution [2, 1], using the internal and external pressures as a boundary condition:
$$\sigma_r(r, \omega) = \frac{p_i(\omega) D_i^2 - p_o D_o^2}{D_o^2 - D_i^2} - \frac{D_i^2 D_o^2 (p_i(\omega) - p_o)}{4r^2 (D_o^2 - D_i^2)}. \tag{3}$$

And the hoop (circumferential) stress is obtained using the same stress distribution considered in the radial case, resulting in:
$$\sigma_c(r, \omega) = \frac{p_i(\omega) D_i^2 - p_o D_o^2}{D_o^2 - D_i^2} + \frac{D_i^2 D_o^2 (p_i(\omega) - p_o)}{4r^2 (D_o^2 - D_i^2)}. \tag{4}$$

Consequently, the axial equivalent forces acting at both ends of each element can be found, and the additional element load vector $\mathbf{f}_p^e$, which must be added to the structural element load vector, is given by:

$$\mathbf{f}_p^e = \left[F_p, 0, 0, 0, 0, 0, -F_p, 0, 0, 0, 0, 0\right]', \tag{5}$$

where

$$F_p = A_e E\left[\frac{1}{E}\big(\sigma_1 - \nu(\sigma_r + \sigma_c)\big)\right]. \tag{6}$$

After this procedure, the new element load vectors $\mathbf{f}_p^e$ can be assembled and added to the global vector $\mathbf{f}$, which allows the calculation of the displacements vector $\mathbf{d}$. The axial stress σ_1 is considered only in the "capped end" condition.

1.2 Fluid mass effect

The mass of the gas is considered distributed throughout the length of the structural elements. Thus, the matrix $\mathbf{G}^{tr}$ is modified as follows

$$\mathbf{G}^{tr'} = \mathbf{G}^{tr} + \begin{bmatrix} \rho_f A_i & 0 & 0 \\ 0 & \rho_f A_i & 0 \\ 0 & 0 & \rho_f A_i \end{bmatrix}, \tag{7}$$

where A_i is the internal area of the pipe, calculated with D_i. This matrix is added to the structural element mass matrices.

References

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