Lezione 19 -

Conclusion of Betz theory:

We got the two expressions:

$$\begin{cases} T = (P_2 - P_1) A_D \overset{\text{BME } 0 \to 3}{=} \frac{1}{2}\rho (V_3^2 - V_0^2) A \\ T = \dot{m} (V_3 - V_0) = \rho V_D A_D (V_3 - V_0) \end{cases}$$

Setting equality:

$$\Rightarrow \frac{1}{2} \cancel{\rho} \cancel{A_D} \cancel{(V_3 - V_0)} (V_3 + V_0) = \cancel{\rho} \cancel{A_D} V_D \cancel{(V_3 - V_0)}$$

$$\Rightarrow V_D = \frac{V_3 + V_0}{2} \rightarrow \text{Froude Velocity}$$

Before we optimize we study $\dot{L}$ and introduce a few parameters:

$$a = \frac{V_0 - V_D}{V_0} \rightarrow \text{induction coefficient}$$

↳ decelleration of flow after the turbine.

The forces generated on the blades are so large that they cause stagnation downstream which causes the v to decrease and P to increase.

We want to maximise $\dot{L}$, so we optimise a, what we won't see is how we change the turbines to change a.

$$\begin{cases} v_D = (1-a)\,v_0 \\ v_3 = (1-2a)\,v_0 \end{cases}$$

So, thrust is

$$T = \rho v_0 A_D (v_3 - v_0) = \rho (1-a)\, v_0 A_D \left((\cancel{1}-2a)v_0 - \cancel{v_0}\right)$$

$$= \rho A_D v_0^2 \left(-2a(1-a)\right)$$

Power is:

$$\dot{L} = T v_D = \rho A_D v_0^3 \left(-2a(1-a)^2\right)$$

As said power is function of v_0^3

$$c_T = \frac{|T|}{\frac{1}{2}\rho A_D v_0^2} = 4a(1-a) \rightarrow \text{Thrust coefficient}$$

→ while A_D and v_0 technically can't go together, they are controllable, but it's convenient since appear together in the equation for T.

$$c_p = \frac{|\dot{L}|}{\frac{1}{2}\rho A_D v_0^3} \rightarrow \text{Power coefficient}$$

$$= 4a(1-a)^2$$

Motivation of optimization

↳ $\dot{L}$ → depends on ΔV^2, but reducing v_3, through a large exchange of work, reduces v_D, so we have to have a balance to maximize it.

↳ and so the flow rate

The $\dot{L}$ has a counter balancing effect of needing to increase ΔV^2, but this change causing change in v_D reducing the power

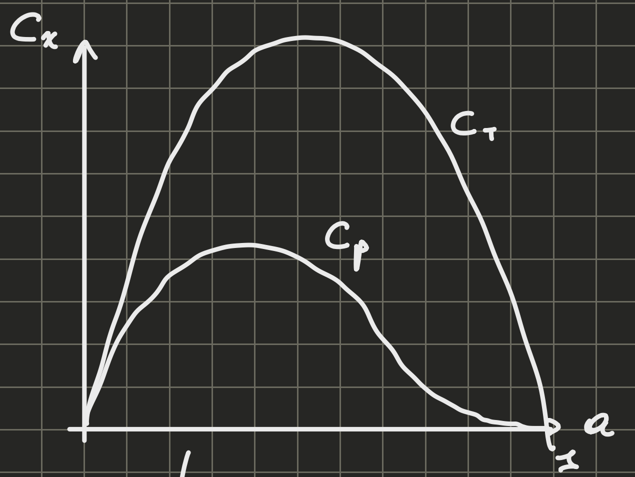
→ to increase the power.

In this balance we find an optimum, and we are lucky this lies in the valid range of the Betz theory as its assumptions are still valid to produce a technically meaningful result.

↳ End of analytical development, we will look at the theoritical result of the optimization.

Maximum c_p at $a = \frac{V_0 - V_D}{V_0} = 1/3 \Rightarrow c_p = 16/27$
↳ slightly lower than .6.

Maximum c_T at $a = 1/2 \implies V_3 = 0$
↳ not possible



Results for $a > 0,5$ are meaningless with the theory.

Above $a = 0,4$, the streamtube theory is no longer valid.

If a is too big, this is not valid, we need experimental trends.

Luckily the maximum for c_p is achieved at less than $a < 0,4$.
In reality there might be some error; so for the design this is still valid.

Design of turbines:

During operation we only know v.

What the power of the wind increases, the c_p varies, the point which c_p is higher is the point at which we can extract the most power, this is not necessarily the point at which the winds power is highest. What we have to do is design the machine to withstand a certain velocity at which to size the machine so they don't break, this is a speed we should never exceed, and if it is exceeded we pivot the blades to save the machine and make it purposely reduce L^* so it doesn't exceed the design limit. We forcefully decrease c_p doing so, but it is ok.

There exists some kind of Balje diagram, for different types of blade designs.

Compressible Fluid Machines - Thermal Machines

↳ Temperatures becomes relevant.

<u>Thermal Machines</u>

We have introduced balances already

$$\begin{cases} \text{Mass Balance: } \dot{m} = \text{const} \Rightarrow \rho v_n A = \text{const} \\ \text{Energy Balance: } l + q = \Delta h + \dfrac{\Delta v^2}{2} + g\Delta z \\ \text{Mechanical Energy Balance: } l - l_w = \underbrace{\displaystyle\int_{IN}^{OUT} v \, dP}_{\text{Technical Work}} + \dfrac{\Delta v^2}{2} + g\Delta z \end{cases}$$

We combine these with the equations of state (for a gas):

$$\begin{cases} \Delta h = c_p \Delta T \\ Pv = RT = \dfrac{P}{\rho} \end{cases} \qquad v = \frac{RT}{P} \qquad \rho = \frac{P}{RT}$$

From the mass balance, we know: (we develop the balance now)

$$d(\ln(\rho v_n A)) = d(\ln(\text{const})) = 0$$

$$d(\ln \rho + \ln v_n + \ln A) = 0$$

$$\Rightarrow \frac{d\rho}{\rho} + \frac{dv_n}{v_n} + \frac{dA}{A} = 0$$

$$\Rightarrow \frac{dv_n}{v_n} = -\frac{dA}{A} - \frac{d\rho}{\rho}$$

→ The add ρ term differentiates it from the incompressible case.

If we accelerate the fluid in a nozzle, we increase v_n, usually decreasing ρ and A. But if something weird happens to ρ it is possible to have a non-convergent (divergent) nozzle.

There will be a varying form depending on the case.

Same for the diffuser.

We will clarify during our study of gas dynamics.

Energy Balance:

→ ΔT cannot be ignored

$$\ell + q = c_p \Delta T + \frac{\Delta v^2}{2} + g \Delta z$$

Mechanical Energy Balance:

$$\ell - \ell w = \int_{IN}^{OUT} RT \frac{dP}{P} + \frac{\Delta v^2}{2} + g \Delta z$$

Temperature also effects l-lw

→ After depending we will introduce a pressure ratio, which will impact our results, it's the pressure difference ratio before and after our transformation

→ Integral is unsolvable unless we know the type of transformation, since T and P are unknown.

Types:

↳ Adiabatic (ideal but close to reality)

↳ Isothermal (more difficult to achieve in reality)

Simplification for our machine, $\Delta z = 0$, since in general in thermal plant like we study it's very negligible.

For these kinds of machine we will not look inside the machine, just at the inlet and outlet, as if it were a black box, in this approach Δv^2 is also very low, so we also consider it to be negligible.

The black box approach is what is used when studying thermal energy systems.

Thermodynamics of Gas Compression

Since there is no way to resolt the integral, we choose to look at an isothermal and reversible transformation.

Isothermal Reversible Compression

$$l + q = c_p \Delta T = c_p (\overbrace{T_{OUT} - T_{IN}}) = 0 \Rightarrow l = -q$$

compression causes heating so if $\Delta T = 0$, l needs to extract all the heat energy.

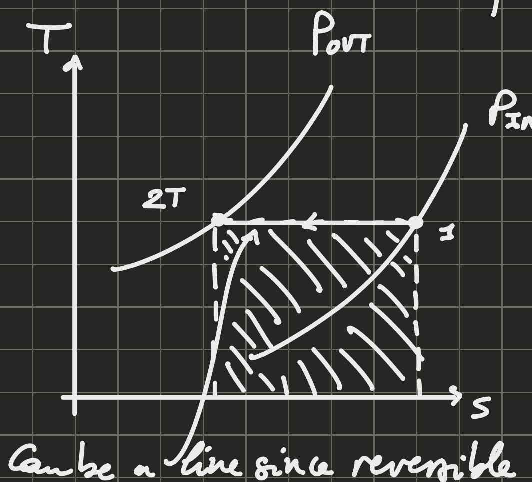
$$l - l_w = \int_{IN}^{OUT} RT \frac{dP}{P} = RT \ln \underbrace{\frac{P_{OUT}}{P_{IN}}}_{\beta}$$

To compress isothermal, we need to compress the fluid and reduce the temperature and more fast.

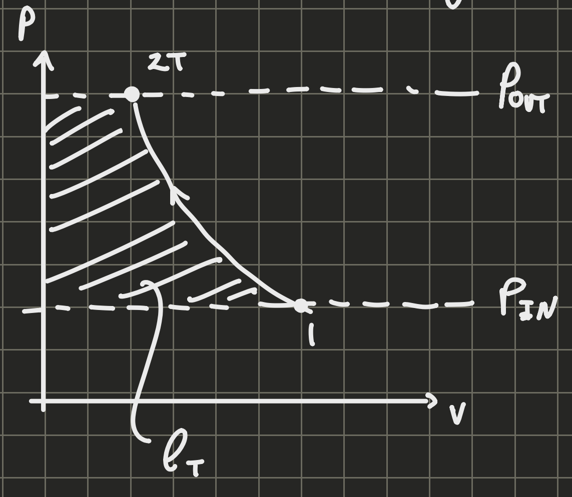
→ This is why machines like this don't exist so far, machines tend to reduce the area to reduce friction, but to remove heat we need a large surface area to exchange heat slowly

β → pressure ratio

We can represent the operation in two different ways



Can be a line since reversible



Pv = const

$$l - \underbrace{l_w}_{\to 0,\ \text{ideal}} = RT \ln \beta = l_T$$

$$q_{REST} = \int_1^2 T\,ds$$

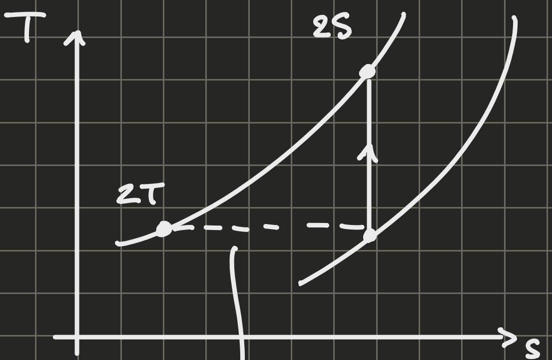
While this is great, isothermal machine are not available, so we look at adiabatic reversible machines.

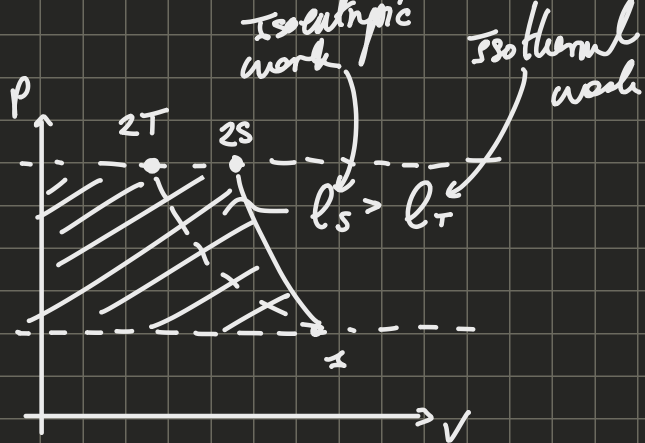
Adiabatic Reversible Machines $\rightarrow$ Isoentropic = Adiabatic + Reversible

$$l + \underbrace{\cancel{q}}_{\to 0 \to \text{adiabatic}} = c_p \Delta T \Longrightarrow l = c_p\,(T_{OUT} - T_{IN})$$

$$l - \underbrace{\cancel{l_w}}_{\to 0} = \int_{IN}^{OUT} RT \frac{dP}{P} = l = \int_{IN}^{OUT} RT \frac{dP}{P}$$

Representing the transformation:





$$P V^{\gamma} = \text{const}$$

$$\gamma = \frac{c_p}{c_v} > 1$$

since $c_p = c_v + R$

$$\Delta s = \cancel{c_p} \ln \left(\frac{T_{2S}}{T_1}\right)^{c_p} - \cancel{R} \ln \frac{P_2}{P_1} = 0$$

$\frac{P_2}{P_1} = \beta$

For perfect gas (Esercitazione)

we assume we keep P_2 and P_1 the same regardless of the process, because they will be plant dependant

$$\Longrightarrow \frac{T_{2S}}{T_1} = \beta^{R/c_p}$$

$$R/c_p = \frac{c_p - c_v}{c_p} = \frac{c_p/c_v - 1}{c_p/c_v} = \frac{\gamma - 1}{\gamma}$$

$$\Rightarrow \frac{T_{2S}}{T_1} = \beta^{\frac{\gamma-1}{\gamma}}$$ ⇒ once we chosen β, and chosen a fluid (⇒ γ defined), we have found ΔT for an adiabatic process.

since $\frac{\gamma-1}{\gamma} < 1$, the temperature ratio will be smaller than β.

$$\Rightarrow l = c_p T_1 \left(\frac{T_{2S}}{T_1} - 1\right) = c_p T_1 \left(\beta^{\frac{\gamma-1}{\gamma}} - 1\right)$$

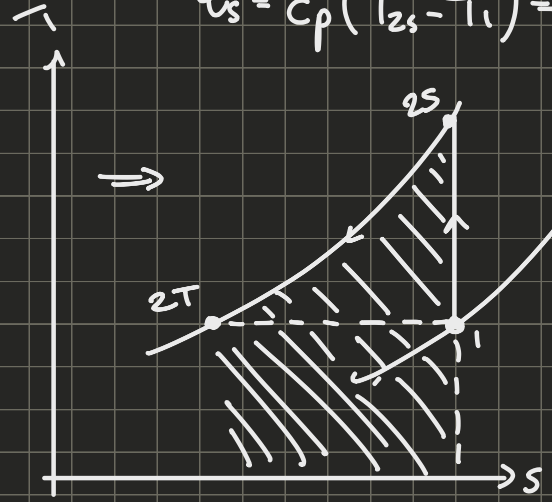
$$\boxed{l_s = \underbrace{\frac{\gamma}{\gamma-1}}_{c_p} \underbrace{\frac{R}{MM}}_{R} T_1 \left(\beta^{\frac{\gamma-1}{\gamma}} - 1\right)}$$

↳ molar mass (mcal (M))

↳ Isentropic work

While this is conceptually useful, if we have the T_{2S} and T_1, we can calculate l directly.

$$l_s = c_p (T_{2S} - T_1) = c_p (T_{2S} - T_{2T}) = q_p$$



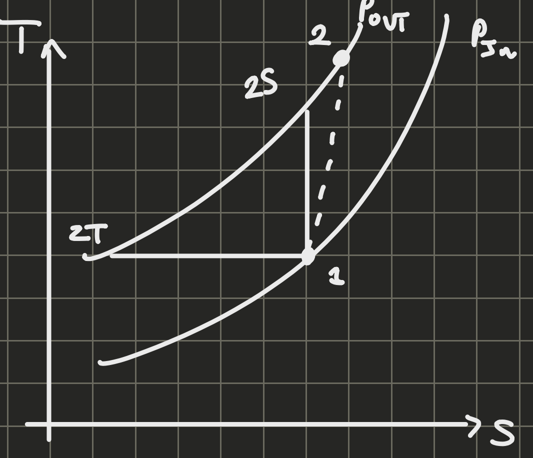
⇒ The work of the isoentropic is quantitatively the same as that of an isobaric transformation between 2S and 2T.

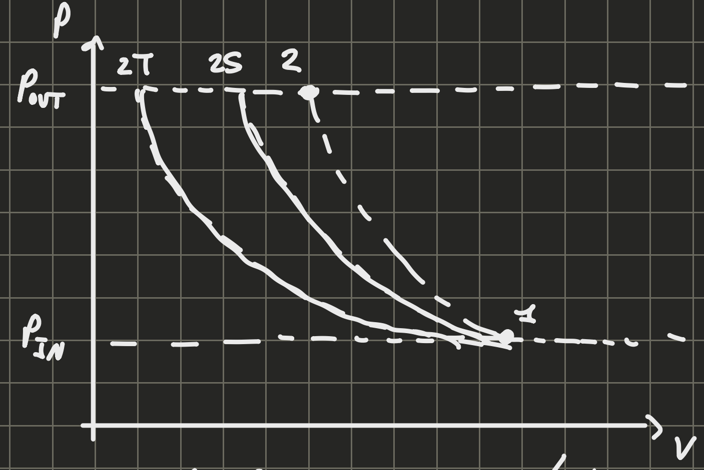
Adiabatic Irreversible Compression

$q = 0$ & $l_w \neq 0$

$$(A)_3 \begin{cases} \Rightarrow l + \cancel{q}^{\,0} = c_p (T_{OUT} - T_{IN}) = c_p T_{IN} \left(\frac{T_{OUT}}{T_{IN}} - 1\right) \\ l - l_w = \int_{IN}^{OUT} v\,dP \quad (A)_4 \end{cases}$$

↳ cannot calculate since there is no relationship to β.





Dashed line since irreversible

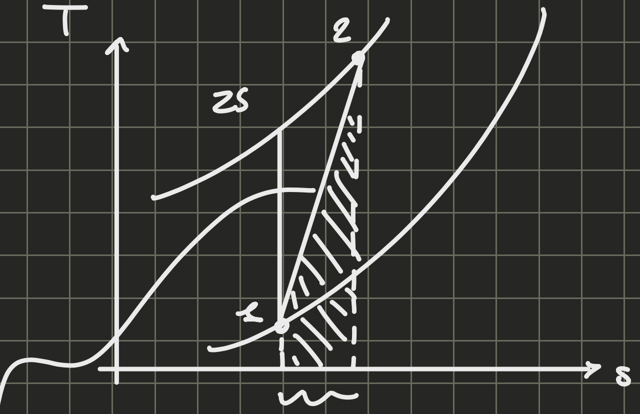
Extra heat causes p to be lower, so V higher.

The idea to solve the issue of finding 2, is to represent the global effects of the transformation

We need to use a reversible transformation/s that represent globally the energy transfer.

↳ We can do this by modelling the transformation as a general polytropic transformation to represent how work is exchanged, and model the overheating we immagine it is achieved by a reversible heating process. → due to friction

We use to model the real transformation an idealistic reversible transformation with work exchanged and reversible heating, the process will therefore be deabatic, it has positive heat exchange, so the fluid recieves a heat exchange to move 2S to 2.
In other words there is a transformation which exchanges the work and the change in S will be caused by a certain amount of reversibly exchanged heat which is represented by the areas.



$q_{REV} = l_w$

↳ since it's not truly reversi

Solid since substituted for "reversible process"

So to do our calculations: ③

Instead of solving them, which we cannot do, we use a model in which $q = l_w$

This kind of transformation is known as polytropic, in which:

$$Pv^n = \text{const}$$

$$\Rightarrow \frac{T_2}{T_1} = \beta^{\frac{n-1}{n}}$$

n is an index of the quality of the transformation

The closer n is to γ, the more efficient is the machine.

n is a property of the machine, not of the fluid.

$$\boxed{l = c_p T_1 \left(\frac{T_{OUT}}{T_1} - 1\right) = c_p T_1 \left(\beta^{\frac{n-1}{n}} - 1\right)}$$

$$\Rightarrow l = \frac{\gamma}{\gamma - 1} \frac{R}{M} T_1 \left(\beta^{\frac{n-1}{n}} - 1\right)$$

We can also calculate integral since:

$$P v^n = P_1 v_1^n \Rightarrow v = v_1 \left(\frac{P_1}{P}\right)^{1/n}$$

Plugging onto the integral we get:

$$(\circledast_4)$$

$$l - l_w = \int_1^2 v_1 \left(\frac{P_1}{P}\right)^{1/n} dP = \frac{n}{n-1} \frac{R}{M} T_1 \left(\beta^{\frac{n-1}{n}} - 1\right)$$

"Fun thing":

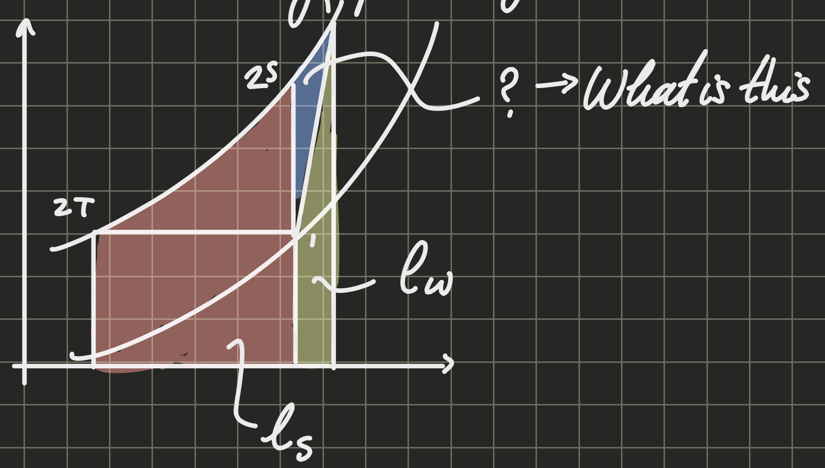
In hydraulics we said:

$l - l_w = \Delta T \rightarrow$ Work exchanged by machine if we neglect losses,

$= l_s$ $\rightarrow$ isoentropic transformations are loss less.

But $\quad l \neq l_s + l_w$

We can see this graphically because:



$l \neq l_s + l_w$

In reality there is an additional bit of extra work that isn't the isoentropic or loss work

Technical Work $\int v\,dP$

Isoentropic $\to \int_1^{2s} v(s_1, P)\,dP$

Real $\to \int_1^{2} v(s, P)\,dP$

$\wedge$ → since, in the real case s can be $> s_1$, and we don't limit ourselves to $2s$.

which we model with q_{REV}

Because of the losses we two negative effects, the direct losses (l_w) and the second is an indirect reversible effect because the heating has increased v and technical work.

We call still additional effect, reheat effect, so we can say:

$$l = \underbrace{l_s}_{\text{Isoentropic Work}} + \underbrace{l_w}_{\text{Wasted work}} + \underbrace{l_{RH}}_{\text{Re-heat}}$$

l_{RH} → Re-heat losses.