Esercitazione 8 -

Exercise 1 -

ideal axial turbine $\rightarrow \Delta s = 0$

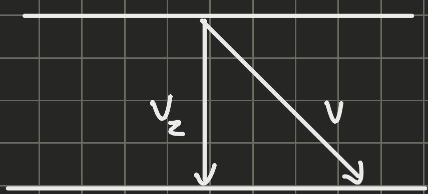
$M_m = 18 \ \frac{kg}{kmole}$ $\quad \gamma = 1{,}33$

$T_{T0} = 800 K$ $\quad D_m = 1.25 m$

$p_{T0} = 50 \ bar$ $\quad n = 3000 \ rpm$

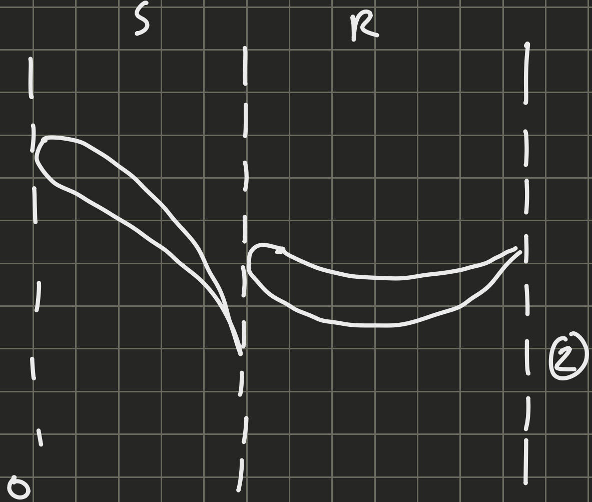
$\dot{m} = 100 \ \frac{kg}{s}$ $\quad$ optimized turbine

$v_x = \text{const}$ $\quad \hookrightarrow v_{2t} = 0$



$\alpha_1 = 73^\circ$

1) impulse stage



$$\chi = \frac{\Delta h_R}{l} = \frac{h_2 - h_1}{l} = 0$$

$\Rightarrow h_2 = h_1$

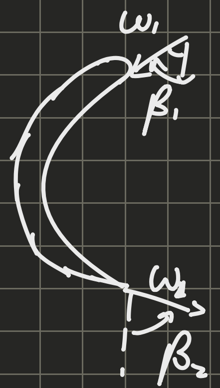
$$I_2 = I_1 \rightarrow h_2 + \frac{w_2^2}{2} = h_1 + \frac{w_1^2}{2}$$

$\Rightarrow w_2^2 = w_1^2$

$\Rightarrow w_{2t}^2 = w_{1t}^2 \Rightarrow \tan^2\beta_2 = \tan^2\beta_1 \rightarrow |\beta_2| = |\beta_1|$

$\rightarrow w_{2t} = w_{1t} \quad \rightarrow \beta_2 = -\beta_1$

If we want to exchange work



1) velocity triangles?

$$V_{2t} = 0$$

$$u_m = \frac{2\pi n}{60} \cdot \frac{D_m}{2} = 196{,}3 \ \frac{m}{s}$$

$$w_{2t} = \cancel{V_{2t}} - u_m = -u_m = -196{,}3 \ \frac{m}{s}$$

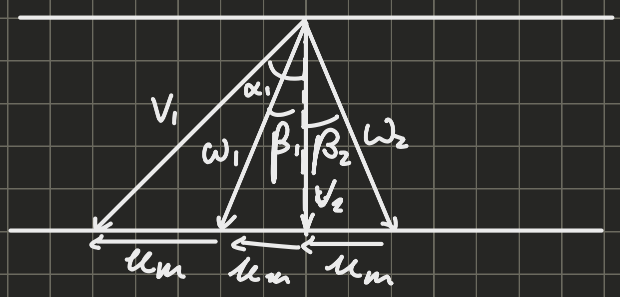
$$w_{1t} = -w_{2t} = 196{,}3 \ \frac{m}{s} \longrightarrow V_{1t} = w_{1t} + u_m = 392.6 \ m/s$$

$$V_x = \frac{V_{1t}}{\tan \alpha_1} = 120 \ \frac{m}{s} = V_{1x} = w_{1x}$$

$$V_1 = 410{,}5 \ m/s \longrightarrow \beta_1 = \tan^{-1} \frac{w_{1t}}{V_x} = 58.6^\circ$$

$$w_1 = 230.1 \ m/s$$

We can now draw the triangles:



$w_{2t} = -196{,}3 \ \frac{m}{s}$ $\qquad\qquad$ $\beta_2 = \tan^{-1}\left(\frac{w_{2t}}{V_x}\right) = -58{,}6$

$V_{2t} = 0$

$V_{2x} = w_{2x} = V_x = 120 \ m/s$

$\alpha_2 = 0 \qquad V_2 = 120 \ m/s$

$$k_p = \frac{u}{V_1} = \frac{u}{V_{1t}/\sin\alpha_1} = \frac{\sin\alpha_1}{2}$$

2) $l_{eulerian}$? $\qquad \eta_{TS}$? $\qquad \dot{L} = ?$

$$l_{eul} = u_m(\cancel{V_{2t}} - \underbrace{V_{1t}}_{2u_m}) = -2u_m^2 = -77 \ \frac{kJ}{kg}$$

$$\dot{L} = \dot{m} \ l_{eul} = -7.7 \ MW$$

$$\eta_{TS} = \frac{|l_{eul}|}{h_{T0} - h_{2S}} = \frac{|l_{eul}|}{\underbrace{h_{T0} - (h_{T2,s}}_{|l_s|} - \frac{V_{2S}^2}{2})}$$

$h_{T1} - h_{T2,1S}$

$V_{2S} \simeq V_2 \qquad \Longrightarrow \eta_{TS} \simeq \frac{|l_{eul}|}{|l_s| + \frac{V_2^2}{2}} = 91.4\%$

$\hookrightarrow$ we don't have β to determine it

$= |l_{eul}|$ since $\Delta s = 0$

$$\eta_{TT} = \frac{|l_{eul}|}{\underbrace{h_{T0} - h_{T2,s}}_{|l_{eul}|}} = 1 \text{ since } \Delta s = 0$$

If had a non-optimized stage to increase $|l_{eul}|$ we decrease η_{TS}

2) Find thermodynamic (static) quantities

$P_1 = ?$ $\quad P_2 = ?$

$T_1 = ?$ $\quad T_2 = ?$

$$\left.\begin{array}{l} T_{T1} = T_{T0} \\ P_{T1} = P_{T0} \end{array}\right\} \text{since } l_{stator} = 0$$

$$T_{T1} = T_1 + \frac{V_1^2}{2c_p} \longrightarrow T_1 = T_{T1} - \frac{V_1^2}{2c_p} = T_{T0} - \frac{V_1^2}{2c_p}$$

since we are at the same point $= 754.7\ K$

$$\left(\frac{P_{T1}}{P_1}\right) = \left(\frac{T_{T1}}{T_1}\right)^{\gamma/\gamma-1} = 39.5\ bar$$

$$T_{T2} = T_1 + \frac{l_{eul}}{c_p} = 758.6\ K \longrightarrow T_2 = T_{T2} - \frac{V_2^2}{2c_p} = 754.7\ K$$

$P_{T2} =$

Since it's an impulse stage, $\Delta h_R = 0 \Rightarrow T_2 = T_1$

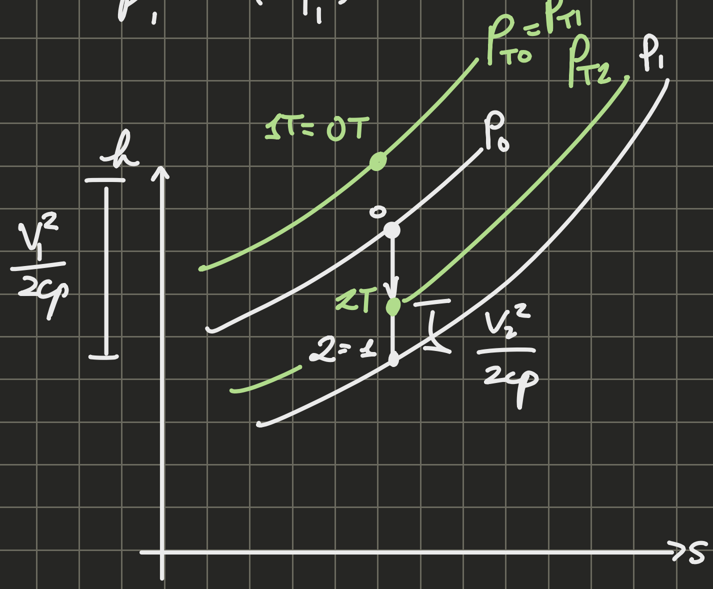
Also since $\Delta s = 0 \Rightarrow q = 0$

$\Rightarrow P_2 = P_1$

since $\cancel{dh} = \cancel{TdS} + vdP$

since the machine is ideal we can write isentropic equations along the machine:

$$\frac{P_2}{P_1} = \left(\frac{T_{T2}}{T_1}\right)^{\gamma/\gamma-1} \longrightarrow P_2$$



While $1 = 2$

$1T \neq 2T$

4) Calculate $b_1 = ?$

$$\dot{m} = \rho_1 (\pi D_m b_1) V_x$$

$$\rho_1 = \frac{P_1}{R T_1} = 11.3 \ kg/m^3 \longrightarrow b_1 = \frac{\dot{m}}{\pi D_m \rho_1 V_x} = 0.019 \, m$$

$$R = \frac{\mathcal{R}}{\mathcal{M}_m} = \frac{8314 \ J/kmol \cdot K}{18 \ kg/kmol} = 461.9 \ J/kg \cdot K$$

5) Verify the constraint on axial turbines

$$0.025 < \frac{b_1}{D_m} < 0.4$$

$$Q \propto u_m^2 \frac{b_1}{D_m}$$

↳ in the early stages $\rho \uparrow$, $Q \downarrow$ so we tend to $\frac{b_1}{D_m} \downarrow$

$$\frac{b_1}{D_m} = 0.015 < 0.025$$

What we can do is decrease the area to reduce the flow rate by blocking some of the inter-blade volumes.

D_m cannot change (unless it's really really really needed)

b_1 can be increased, increasing the cross section, but this also changes the velocity triangles and area, so to keep the area the same we can block some of the area between the blades

$$\dot{m} = \rho_1 \pi D_m b_1^* V_x \underbrace{\varepsilon}_{\frac{A_{open}}{A_{tot}}}$$

$$\frac{b_1^*}{D_m} = 0{,}025$$ → at least, so we can calculate the new b_1^*, and the ε needed to compensate

$$b_1^* = 0{,}031\,m$$

$$\varepsilon = \frac{\dot{m}}{\rho_1 \pi D_m b_1^* V_x}$$

2) symmetric, not impulse, rotor now $\chi = 0{,}5$

$$\longrightarrow \chi = \frac{\Delta h_R}{l_{eul}} = \frac{w_1^2 - w_2^2}{w_1^2 - w_2^2 + V_2^2 - V_1^2} = 0{,}5$$

$$l_{eul} \to \frac{\Delta v^2}{2} - \frac{\Delta w^2}{2}$$

$$\longrightarrow \begin{cases} \beta_1 = -\alpha_2 \\ \beta_2 = -\alpha_1 \end{cases}$$

$V_{2t} = 0 \longrightarrow \omega_{2t} = -u_m = -196.3 \ \frac{m}{s}$

$$\left.\begin{aligned} \tan\alpha_1 &= \frac{V_{1t}}{V_x} \\ \tan\beta_2 &= \frac{\omega_{2t}}{V_x} \end{aligned}\right\} \xrightarrow[\beta_2 = -\alpha_1]{} V_{1t} = -\omega_{2t}$$

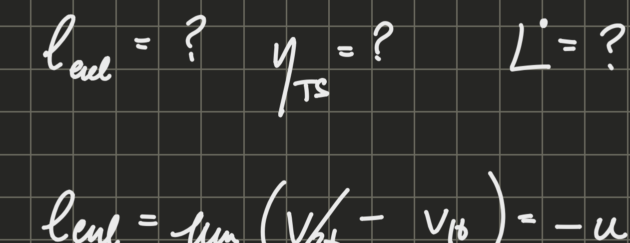
$$\left.\begin{aligned} V_{1t} &= 196,3 \ m/s \\ V_x &= \frac{V_{1t}}{\tan\alpha_1} = 60 \ \frac{m}{s} \end{aligned}\right\} \rightarrow V_1 = 205,3 \ m/s$$

$\omega_1 = V_{1t} - u_m = 0 \longrightarrow \beta_1 = 0 \Rightarrow \alpha_2 = 0 \longrightarrow \beta_1 = -\alpha_2$ ✓

$V_{2t} = 0$

$\omega_{2t} = -196,3 \longrightarrow \beta_2 = -73$

$V_{2x} = \omega_{2x} = V_x = 60 \ m/s$



$l_{eul} = ? \quad \eta_{TS} = ? \quad \dot{L} = ?$

$$l_{eul} = u_m(\cancel{V_{2t}} - \underbrace{V_{1t}}_{=u_m}) = -u_m^2 = -38.5 \ \frac{kJ}{kg}$$

$\dot{L} = \dot{m} l_{eul} = -3.85 \ MW$

$$\eta_{TS} = \frac{|l_{eul}|}{|l_s| + \frac{V_2^2}{2}} = 95.5 > 94.1 \text{ from before}$$

lower than impulse

$|l_{eul}|$

$\boxed{12:00}$

$P_1, P_2 = ?$

$T_1, T_2 = ?$

$$T_{T0} = T_{T1} = T_1 + \frac{V_1^2}{2c_p} \Rightarrow T_1 = 788.7\ K$$

$$T_{T2} = T_{T1} + \frac{l_{eul}}{c_p} = 779.3\ k \rightarrow T_2 = T_{T2} - \frac{V_2^2}{2c_p} = 778.3\ K$$

$$\frac{P_{T1}}{P_1} = \left(\frac{T_{T1}}{T_1}\right)^{\gamma/\gamma-1} \rightarrow P_1 = 47.2\ bar$$

Since the machine is isoentropic, we can write an isoentropic equation along the machine.

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} \longrightarrow P_2 = 44.7\ bar$$

$P_2 < P_1$

$b_1? \rightarrow 0.026 > 0.025$

$$\rho_1 = \frac{P_1}{R T_1} = 12.96\ \frac{kg}{m^3}$$

$$b_1 = \frac{\dot{m}}{\rho_1 \pi D_m V_m} = 0.033\ m$$

Advantage of impulse is higher work but efficiency is lower.

Exercise 2

Optimized $\longrightarrow v_{2b} = 0$

$$\text{Argon} \longrightarrow \begin{cases} M_m = 39.3 \ \text{kg/kmol} \\ \gamma = 1.6 \end{cases}$$

$$\beta_{TS} = \frac{P_{T0}}{P_2} = 1.8$$