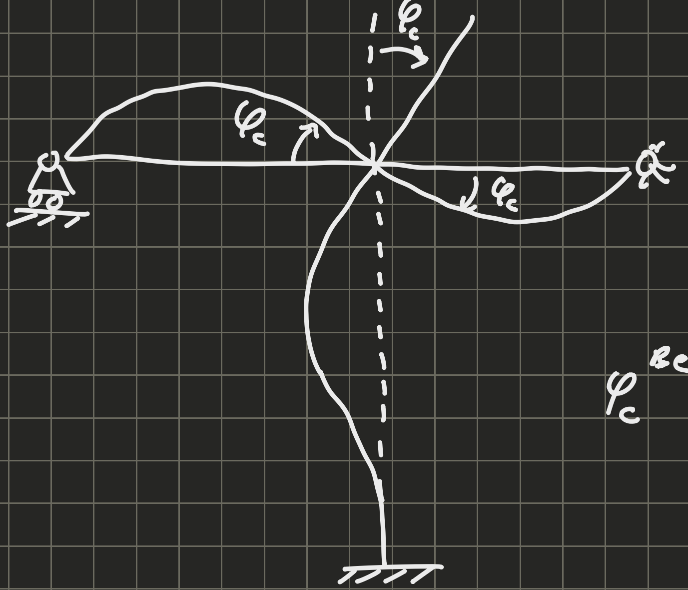
Esercitazione 4

Nelle ultime lezione, le rotazioni relative sono sempre 0, invece le rotazioni assolute esistono essendo non nulle, la deformata era:



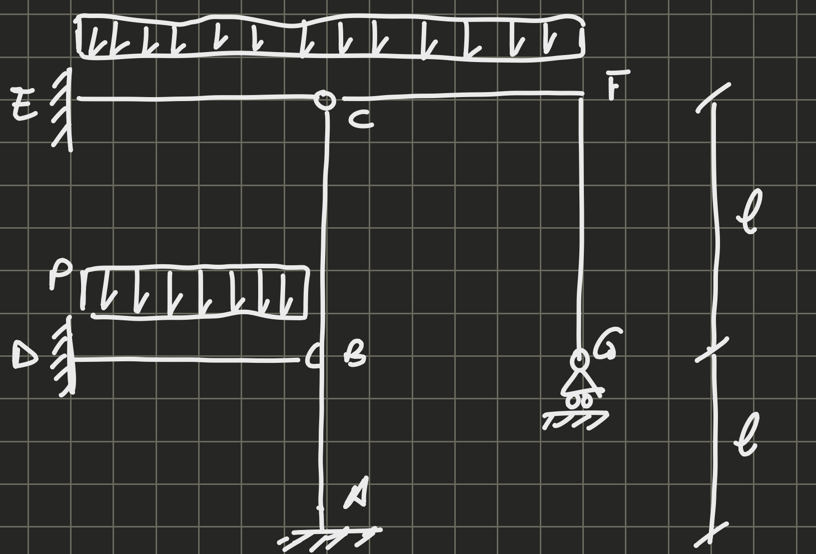
$$\varphi_C = \alpha_{CD}^{(0)} + \alpha_{CD}^{(1)} X_1 + \alpha_{CD}^{(2)} X_2 = \frac{3}{68}\frac{ql^3}{EI}$$

$$\varphi_C^{BC} = -\alpha_{CB}^{(0)} - \alpha_{CB}^{(1)} X_1 = \frac{3}{68}\frac{ql^3}{EI}$$

$\frac{f}{q}$ tale per cui $\varphi_C = 0 \rightarrow \alpha_{CB}^{(0)} = -\alpha_{CB}^{(1)} X_1$

$$\frac{fl^3}{24EI} = \frac{35}{136} ql^3 \cdot \frac{l}{3EI} \rightarrow \frac{f}{q} = \frac{35}{17}$$

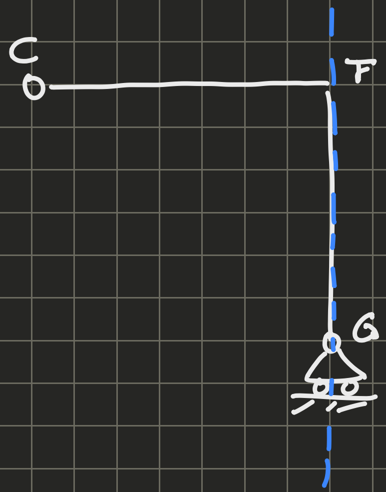
TdE 13/02/2023



$GA \rightarrow \infty$

$BA \rightarrow \infty$

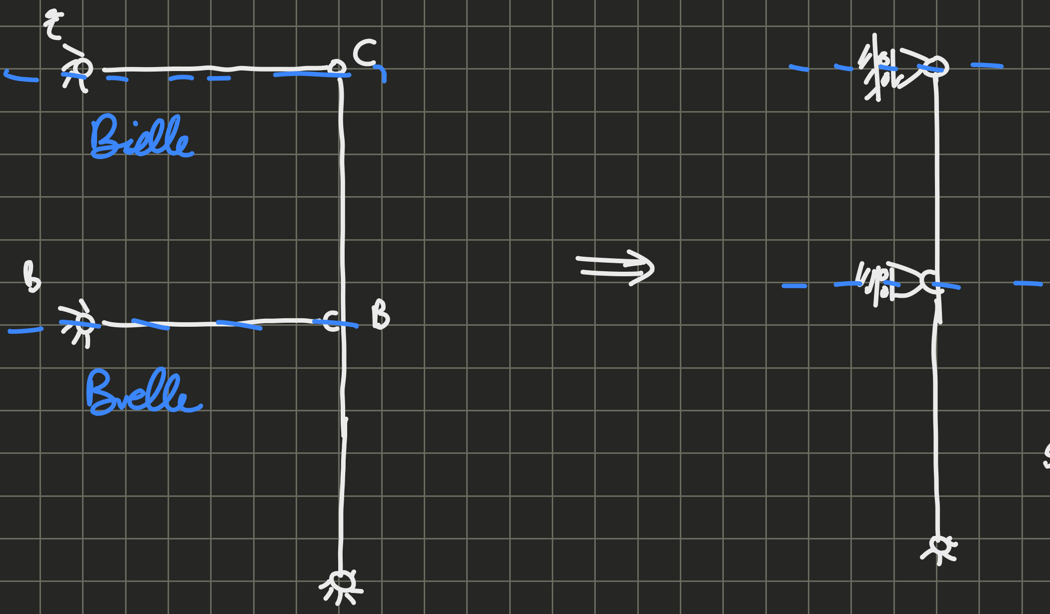
Analisi Cinematica



3gdl e 3gdv

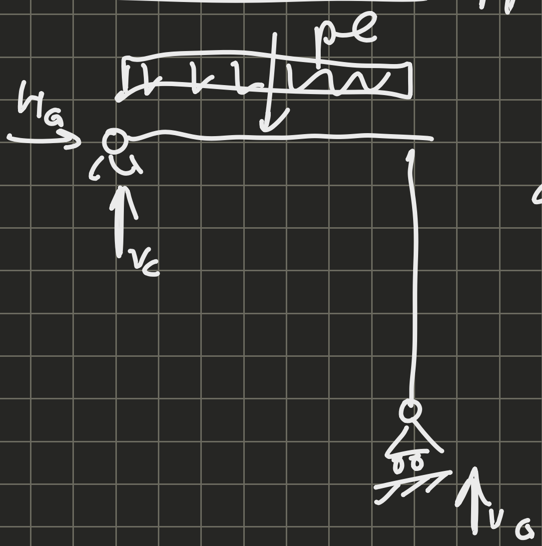
Appendice Isostatica

↳ può esser tratto a parte della struttura



CBDE è ben vincolato, non è labile ⟹ telaio di partenza è a nodi fissi.

Risultato Appendice Iso



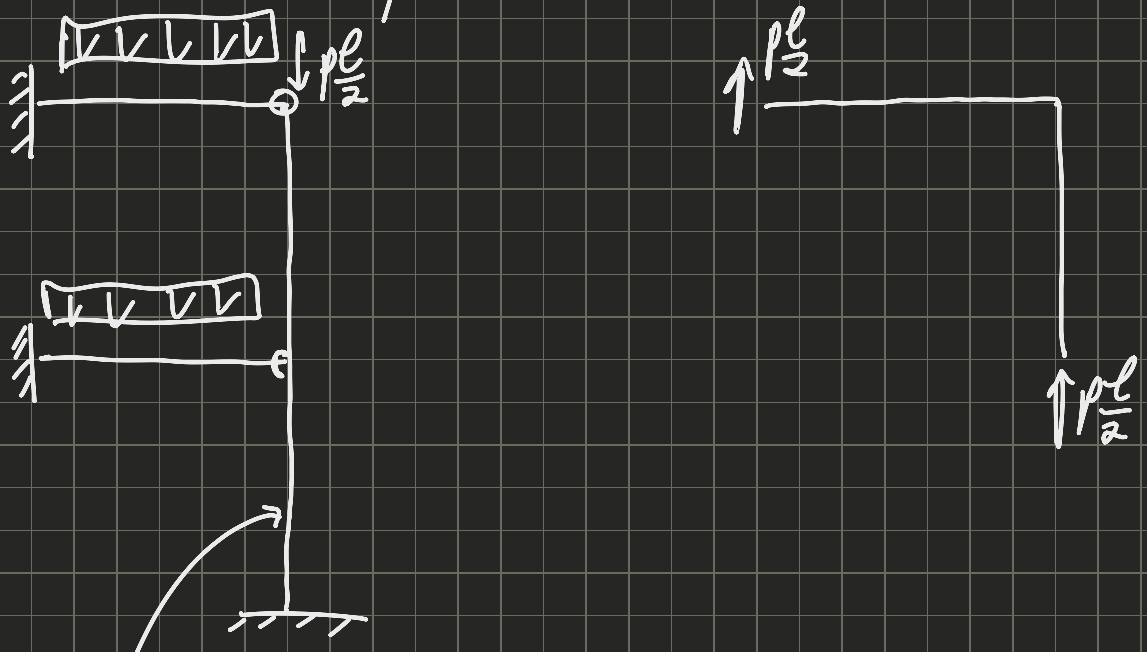
$$\sum F_x = 0 \rightarrow H_C = 0$$

$$\sum F_y = 0 \rightarrow V_C + V_G = pl$$

$$\sum M_C = 0 \rightarrow V_G l - \frac{pl^2}{2} = 0$$

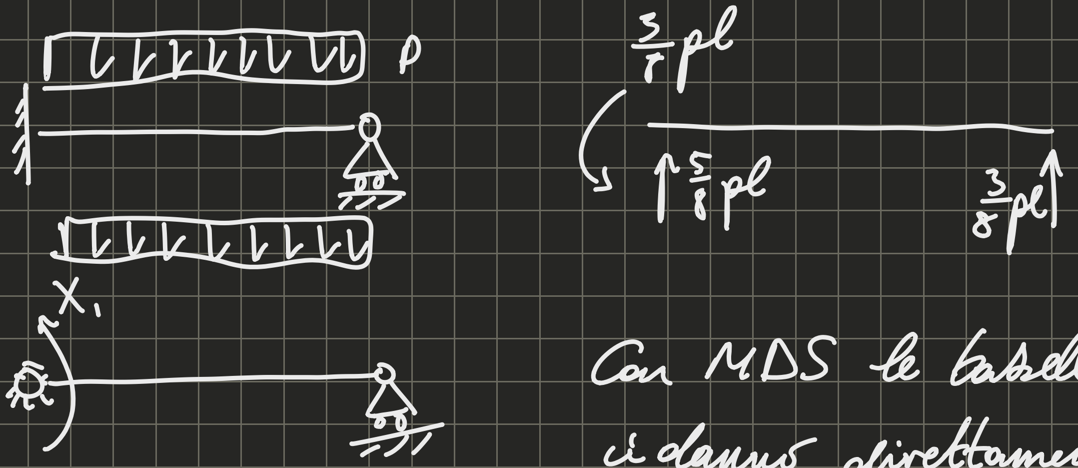
$$V_G = \frac{pl}{2} \qquad V_C = p\frac{l}{2}$$

Vogliamo portare questi carichi al telaio:



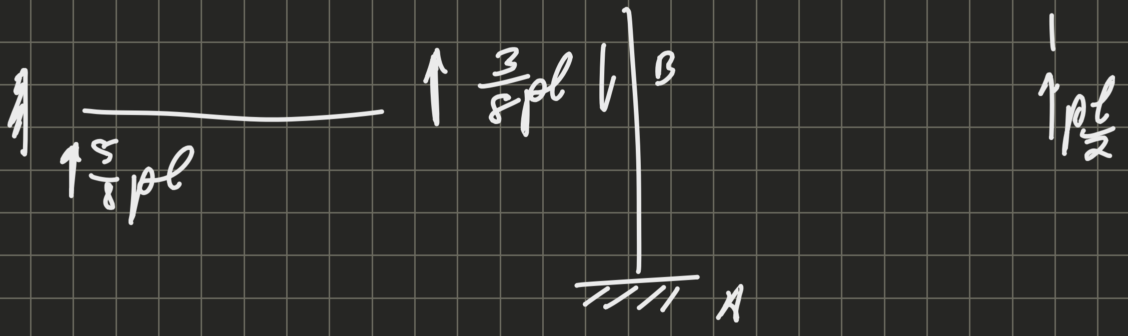
Sul pilastro non c'è momento per la configurazione dei carichi.

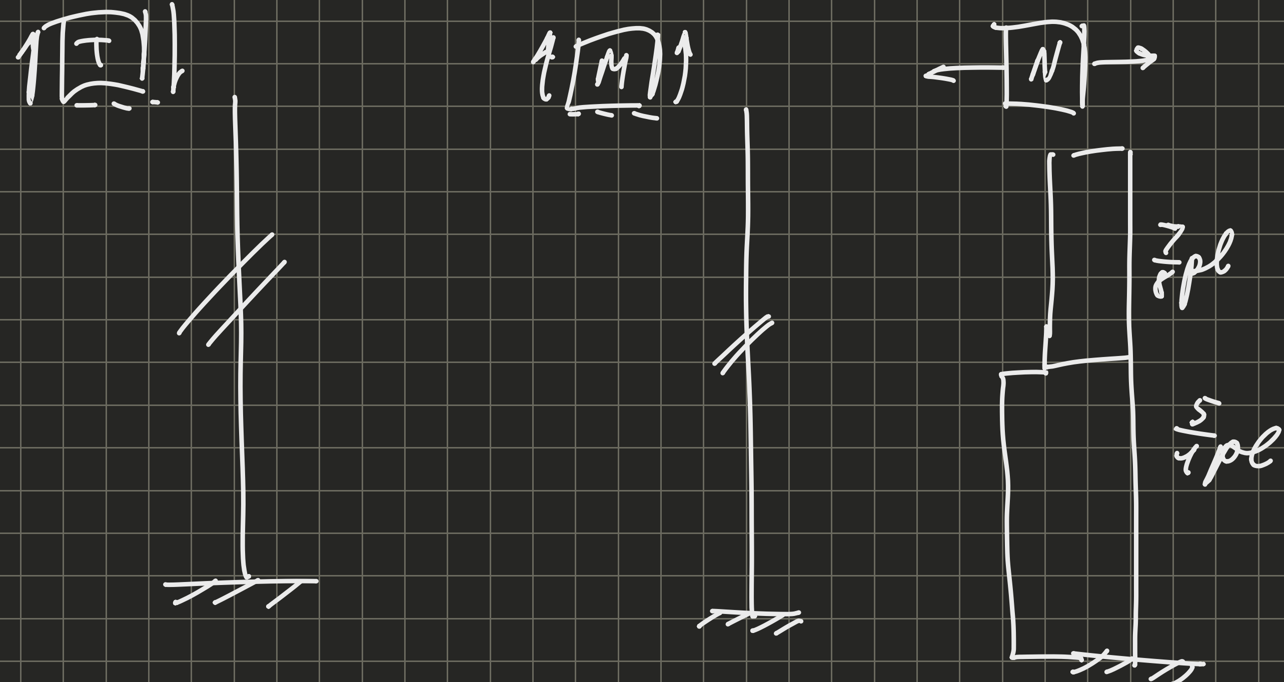
DB è uguale a BC



Con MDS le tabelle ci danno direttamente le forze sulla trave.

MdF



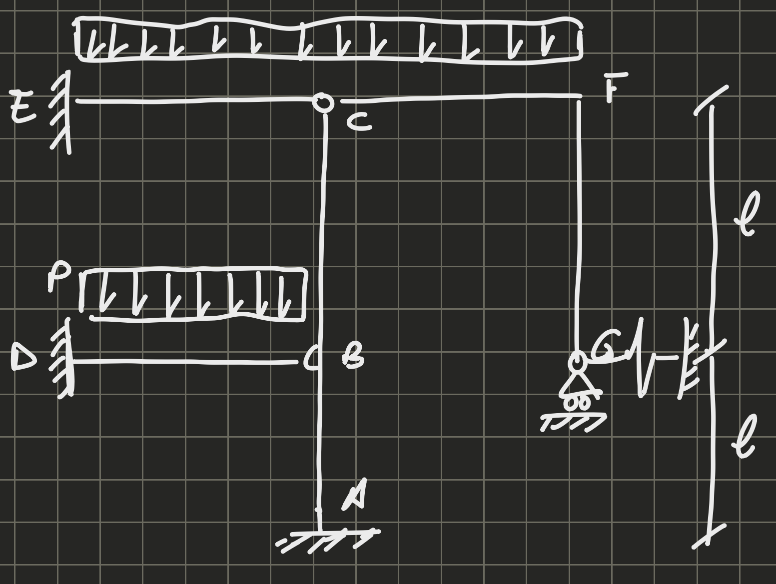


MDS

↳ Si bloccano i gdl e poi si studiano la statica

MDF è metodo della flessibilità mentre MDS è metodo della rigidezza.

Variante



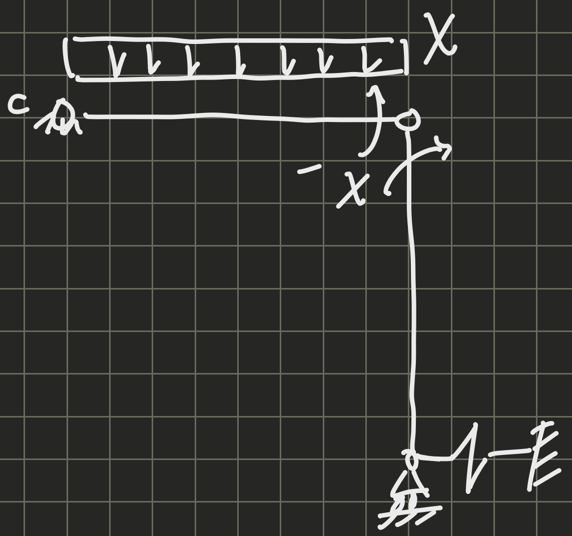
$$k_H = \frac{EJ}{l^3}$$

Cinematica è la stessa

CFG non è più appendice isostatico per il vincolo flessibile

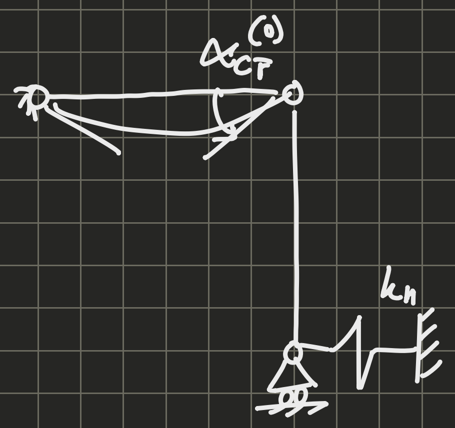
4 gdv e 3 gdl

Mettiamo una cerniera in F per avere un momento di continuità e studiare le azioni interne



$$\Delta(C\hat{F}G) = 0$$

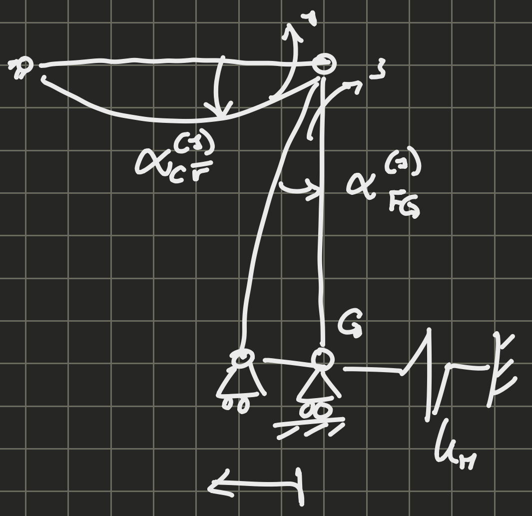
Struttura Ausiliare "0"



$$\alpha_{CF}^{(0)} = \frac{pl^3}{24EI}$$

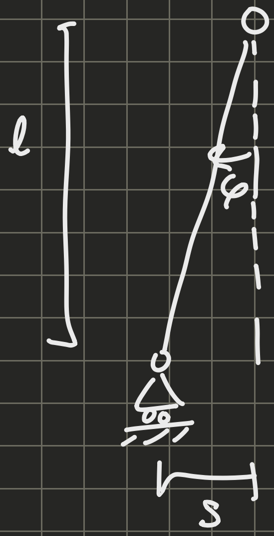
$= \eta_0$

Struttura Ausiliare "1"



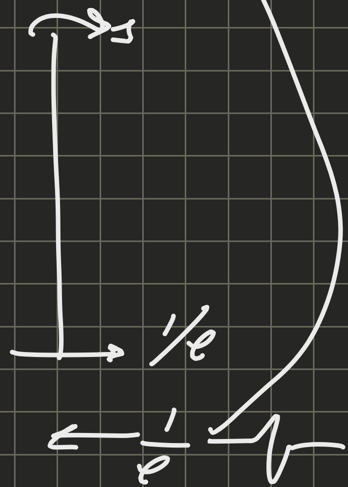
$$\alpha_{CF}^{(1)} = \frac{l}{3EI}$$

$$\alpha_{FG}^{(1)} = \frac{l}{3EI}$$



$$x = F/k$$

$$s = \frac{1}{l} \cdot \frac{1}{k_H} = \frac{l^2}{EI}$$



$$\alpha_{FG}^{*} = \frac{s}{l} = \frac{l}{EI}$$

$$\alpha_{FG}^{(1)} = \frac{4l}{3EI}$$

$$\Delta(C\hat{F}G) = 0$$

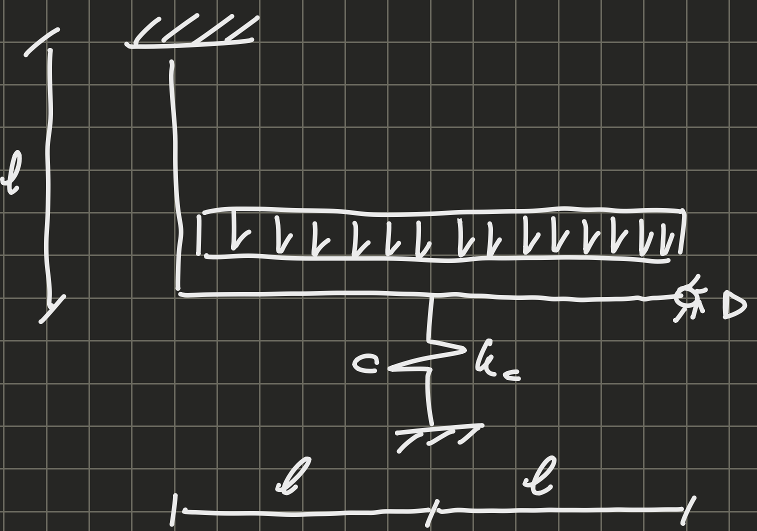
$$\eta_0 + \eta_1 X_1 = 0$$

$$\frac{pl^3}{24EI} + \left(\frac{4l}{3EI} + \frac{l}{3EI}\right) X = 0$$

$$X = -\frac{pl^3}{24E} \cdot \frac{3EI}{5l} = -\frac{pl^2}{40}$$

Primo esercizio nell'esame è più lungo di solito, il secondo di solito è più corto.

TdE Esercizio 3



$$k_c = \frac{\alpha EI}{l^3}$$

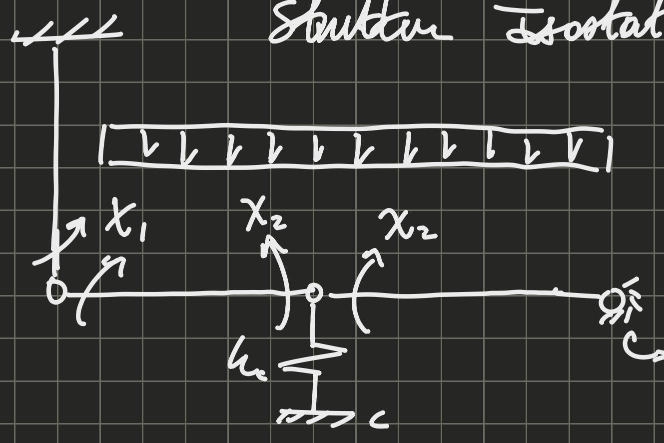
$$\alpha = 12$$

$S = 3$

↳ è un upper bound, non significa che dobbiamo svincolare tot volte.

Svincoliamo in B e C, svincolare anche in A è superfluo

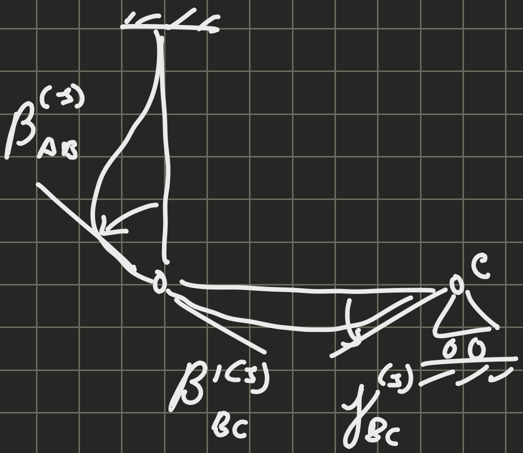
Struttura Isostatica Principale



↳ Possiamo mettere una cerniera

"1" $(X_1 = 1,\ X_2 = 0,\ p = 0)$

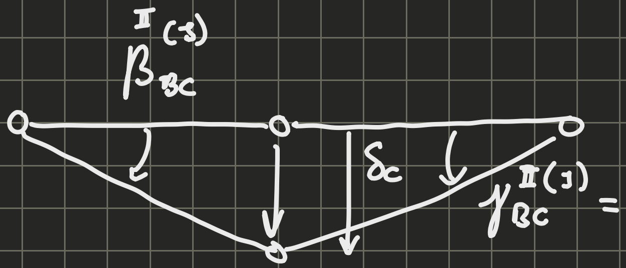
Deformabilità : EI

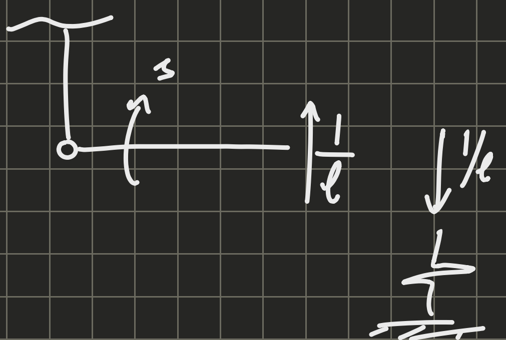


$$\beta_{AB}^{(1)} = \frac{l}{4EI}$$

$$\beta'^{(1)}_{BC} = \frac{l}{3EI}$$

$$\gamma'^{(1)}_{BC} = \frac{l}{6EI}$$





$$\delta_c = \frac{1}{l} \cdot \frac{1}{k_c} = \frac{l^3}{\alpha EI}$$

$$\beta_c^{II(1)} = \frac{\delta_c}{l} = \frac{l}{\alpha EI}$$

$$\gamma_{BC}^{II(1)} = - \frac{l}{8EI}$$

Perché è opposto a x_2

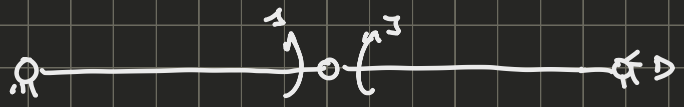
$$\beta_{BC}^{(1)} = \beta_{BC}^{I(1)} + \beta_{BC}^{II(1)}$$

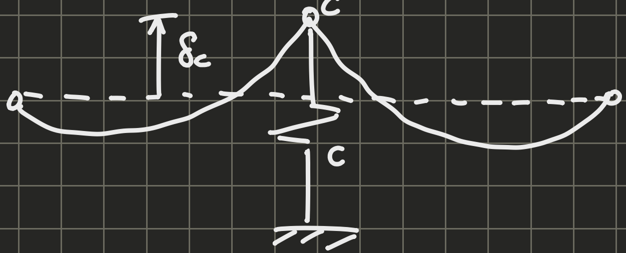
$$= \frac{l}{3EI} + \frac{l}{\alpha EI}$$

$$\gamma_{BC}^{(1)} = \gamma_{BC}^{I(1)} + \gamma_{BC}^{II(1)}$$

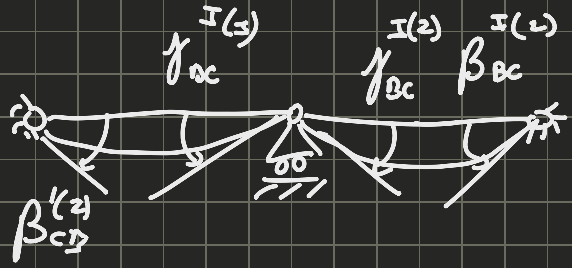
$$= \frac{l}{6EI} - \frac{l}{\alpha EI}$$

Ausiliare "2" $(x_2 = 1,\ X_1 = 0,\ p = 0)$





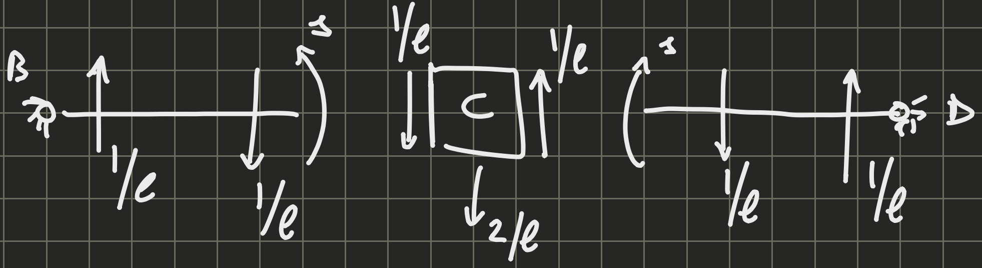
Effetto di EI



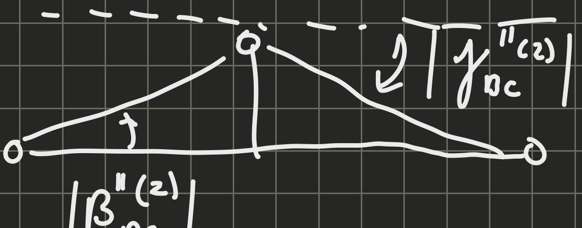
$$\gamma'^{(2)}_{BC} = \frac{\ell}{3EI}$$

$$\beta'^{(2)}_{BC} = \frac{\ell}{6EI}$$

Effetto di δ_c



$$\delta_c = \frac{2}{\ell} \cdot \frac{1}{k_c} = \frac{2\ell^3}{\alpha EI}$$



$$\gamma''^{(2)}_{BC} = \frac{\delta_c}{2\ell} = \frac{2\ell}{\alpha EI}$$

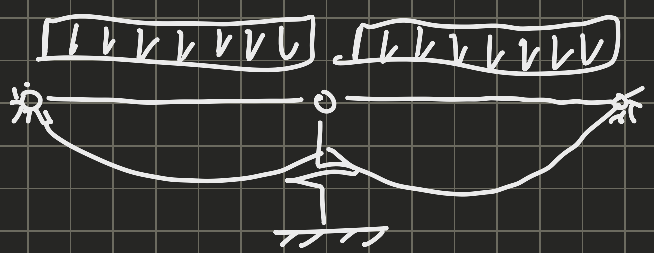
$$\beta_{BC}^{II(2)} = - \frac{\delta_C}{l} = \frac{-2l}{\alpha EI}$$

Discorde con X_1

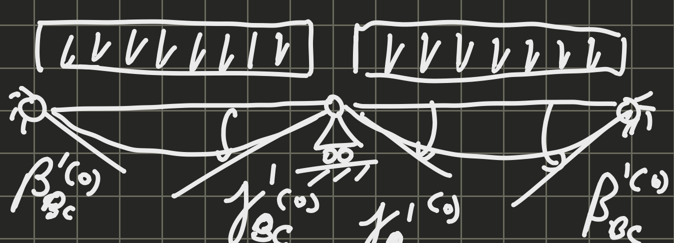
$$\beta_{BC}^{(2)} = \beta_{BC}^{I(2)} + \beta_{BC}^{II(2)}$$

$$\gamma_{BC}^{(2)} = \gamma_{BC}^{I(2)} + \gamma_{BC}^{II(2)}$$

"0" $(X_1 = 0,\ X_2 = 0,\ p \neq 0)$



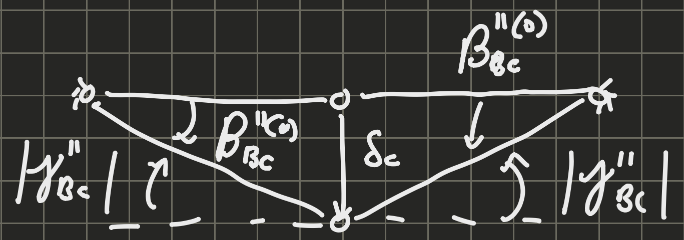
<u>Effetto di EI ?</u>



$$\beta_{BC}^{'(0)} = \frac{pl^2}{24EI}$$

$$= \gamma_{BC}^{'(0)}$$

Effetto δ_c (II)



$$\beta_{BC}^{II(0)} = \frac{\delta_c}{\ell} = \frac{p\ell^3}{\alpha EI}$$

$$\gamma_{BC}^{II(0)} = \frac{-\delta_c}{\ell} = -\frac{p\ell^3}{\alpha EI}$$

$$\beta_{BC}^{(0)} = \beta_{BC}^{I(0)} + \beta_{BC}^{II(0)}$$

$$\gamma_{BC}^{(0)} = \gamma_{BC}^{I(0)} + \gamma_{BC}^{II(0)}$$

$$\underline{\underline{\eta}}\,\underline{X} + \underline{\eta}_0 = \underline{0} \qquad \underline{X} = [X_1, X_2]^T$$

Tutti i segni saranno positivi perché sono già incorporati

$$\begin{cases} \left(\beta_{AB}^{(1)} + \beta_{BC}^{(1)}\right)X_1 + \beta_{BC}^{(2)} X_2 + \beta_{BC}^{(0)} = 0 \\ \left(\gamma_{BC}^{I(1)} + 2\gamma_{BC}^{II(1)}\right)X_1 + 2\gamma_{BC}^{(2)} X_2 + 2\gamma_{BC}^{(0)} = 0 \end{cases}$$

Per la rotazione rigida

$$\alpha = 12 \longrightarrow X_1, X_2$$

Si possono tracciare i movimenti