Lezione 2 - Transmission Shafts and Axels (Theory)

We start with this since it's the machine with which we already have a decent basis.

(We are not doing a review of CdM, there is a recap at the end of these slides, we need to have a look on our own.)

While axels and shafts are different things, they take the same form, and fill the role of connecting rotating parts.

Axels and shaft are generally needed to transfer power between parts, but just shafts and axels are not enough, we need other elements such gears and pulley which we will be looking at.

The main difference between axels and shafts is how they transmit power.

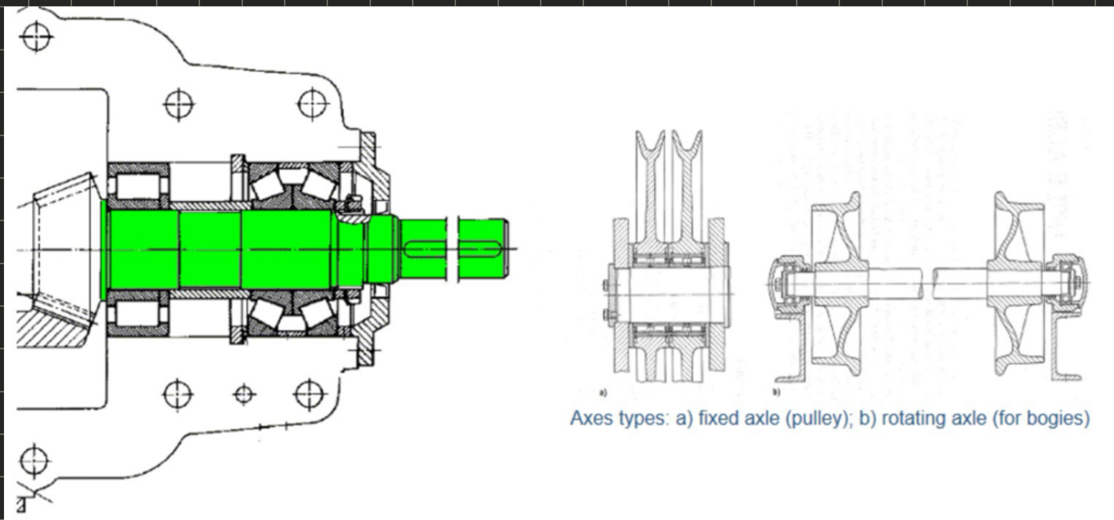
Axels
- can be fixed or rotating
- subject to just bending and not torsion.

Shafts
- will always be rotating
- subject to both bending and torsion.

If it's motorized it's a shaft

There are many possible classification with different categories:

- centerpoint path: straight, elbow, ...
- section type: hollow or solid
- section shape: circular or profiled.
- proportions: slender or non-slender
- type: single piece, flexibly joined
- constraints: ~~isostatic~~ or hyperstatic, all d.o.f other than than those of the rotations are locked.


Axes types: a) fixed axle (pulley); b) rotating axle (for bogies)

Since was see a gear, we have an applied torque so it's shaft.

The axel is fixed to the structure and the part on the axel is free to rotate

Bearings cannot apply torque, so even though there is a pulley which should transmit torque to the shaft, there is non that is applied.

Principles of Design:

↳ The main step is to size the dimension of the shaft.

Steps:

- 1. Preliminary Design (material choice & presizing):
  - ↳ based on strength and on empirical formulas.
  - ↳ we get a rough idea for the size of the object.
- 2. Preliminary Drawing
  - ↳ Define shaft geometry
- 3. Checks on the shaft
  - ↳ Static Check
  - ↳ Fatigue Check
  - ↳ Deformation Check
  - ↳ Vibration Check
  - ↳ Other possible checks.
- 4. Final drawing of the shaft.

Preliminary Drawing

↳ The first step is to choose the material, we will always be looking at steel. There is an important classification based proportions:

*

- Slender ($d/L < 0{,}1$)
  - ↳ We use Eulero - Bernoulli, in which we can

ignore shear (taglio) deformations.

→ The most important parameter is stiffness, this is related to the elastic modulus (E) and the geometry (J) ⇒ stiffness = EJ

→ The problem is not usually the strength but the stiffness because they are slender:

For steels: $E \sim 206$ GPa, this is a constant so — for Aluminium: 70 GPa

so we have to work on the geometry.

(true for all classes of materials) ⇒ in slender shafts materials don't matter, so we will always try to use carbon steels, like S235, S375, S355, unless there is a special reason

(flexible, not necessary definite

→ Non-slender ($d/L > 0{,}1$) they are usually tough so strength becomes the main issue, so choosing the steel grade is important, we will usually use high-strength steels.

→ Pre-sizing (using simplified equations) — based on most vulnerable section

→ Case: Axels (subject only to bending

$$\sigma_{MAX} = \frac{32\, M_{f,max}}{\pi d^3} \leq \sigma_{amm} \Rightarrow d \geq 2{,}17 \cdot \sqrt[3]{\frac{M_{f,max}}{\sigma_{amm}}}$$

objective

$\sigma_{amm} = \sigma_m / 6 \rightarrow$ permissible stress.
$\hookrightarrow$ defined by pre-sizing standard.

$M_{f,max} \rightarrow$ max applied bending moment.

→ Cases: Shafts

These calculations give us an idea of the true needed dimension but don't directly find it

→ Shaft subject to just torque

$$\tau_{max} = \frac{16\, M_{t,max}}{\pi d^3} \leq \underbrace{\tau_{amm}}_{\frac{R_m}{15}} \rightarrow d$$

→ Shaft subject to bending & torque

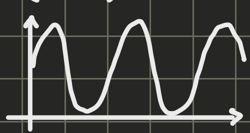
$$\sigma_{eq} = \frac{32\, M_{f,eq}}{\pi d^3} \leq \sigma_{amm} \rightarrow d$$

σ_{amm} → Same as before

$$M_{f,eq} = \sqrt{M_{f,max}^2 + (\alpha \cdot M_{t,max})^2}$$

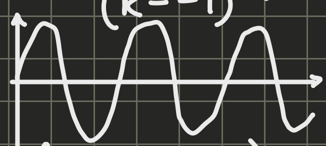
$R = \frac{\sigma_{max}}{\sigma_{min}}$ → Stress Ratio

Pulsating or constant
(R=0)



→ $\alpha^2 = 0{,}25$

Alternating
(R=−1)



→ $\alpha^2 = 0{,}75$

(Worse case)

Preliminary Drawing

→ In this step we define the shape and size, such that it fullfil the requirements of the shaft, knowing if if and where we have gears, bearings and other elements. This shape has to also define fillet ratios, tollerances and other details.

The geometry is defined by us, the designer. There are infinite solutions that fullfil the constraint, so there are no unique solutions to our problem.

Checks and Revisions of the Design

Governing Equation : $\phi_{max} \leq \dfrac{\phi_{lim}}{\eta}$ → unknown

ϕ is the quantity we are interested in checking

All machine parts are subject to checks, these checks include strength, stiffness, vibration, and other checks. Each check returns it's own safety factor, which leaves us to to determine whether we are within it or if we have to modify the geometry to bring it to an acceptable (for us) value.

We can only optimize the smallest safety factor since optimizing any other will induce the smallest to fall below the acceptable level and therefore be unsafe.

Static Check

$$\sigma^*_{VM} = \sqrt{(k_{s\sigma}\,\sigma_{nom})^2 + 3\cdot(k_{s\tau}\,\tau_{nom})^2} \leq \frac{\sigma_{su}}{\eta}$$ → Von-Mises (better)

$$\sigma^*_{GT} = \sqrt{(k_{s\sigma}\,\sigma_{nom})^2 + 4\cdot(k_{s\tau}\,\tau_{nom})^2} \leq \frac{\sigma_{su}}{\eta}$$ → Guest-Tresca

For the static checks we usually use ductile materials, one way we can check it's ductility from the data is to look at the elongation at the yield point, the higher it is the more ductile it is.

$k_{s\sigma}$ and $k_{s\tau}$ are stress concentration factors. In static checks there are typically not included but its a choice of the designer.

→ Fatigue Check

→ There is a recap for the theory of this topic at the end of the slides.

→ We need to consider different factors:

- Notch effect (k_f)
- Surface Finish (b_3)
- Dimension effect (b_2)
- effect of applied mean stress (Haigh-Diagram)

$$\sigma^*_{GP} = \sqrt{\sigma^2 + H^2\tau^2} \le \frac{\sigma_{lim}}{\eta} \longrightarrow \text{Gough-Pollard}$$

$$H = \frac{\sigma_{lim}}{\tau_{lim}}$$

	Stress σ	Stress τ	σ_{lim}	τ_{lim}
σ and τ constant	$\sigma_{max} = \sigma_{nom}$ $\sigma_{max} = K_t\sigma_{nom}$	$\tau_{max} = \tau_{nom}$ $\tau_{max} = K_t\tau_{nom}$	σ_{sn}	τ_{sn}
σ variable τ constant	σ_a	$\tau_{max} = K_t\tau_{nom}$	σ'_F	τ_{sn}
σ constant τ variable	$\sigma_{max} = K_t\sigma_{nom}$	τ_a	σ_{sn}	τ'_F
σ and τ variable	σ_a	τ_a	σ'_F	τ'_F

$\tau_{sn} = \frac{\sigma_{sn}}{\sqrt{3}}$ → According to Von-Mises

$\tau_{sn} = \frac{\sigma_{sn}}{2}$ → According to Guest-Tresca

Using these values we find that the static case is a case of the dynamic cases.

In the variable cases we cannot ignore the stress concentration factor (k_t)

How we get σ_F' and τ_F' will need to be refreshed.

→ Deformation Check (Stiffness Check)

→ In this check we calculate the maximum deflection either it position or of a rotation around a constraint, since both are effects of only the stiffness, so it's effectively a stiffness check.
We find the tables for the maximum permissible deflections, where L is the free deflection length.

If we don't have the maximum permissible in the table we just set it to 10^{-3} radians

→ Special Checks:

→ Corrosion, wear, contact fenomena, etc.

→ Flexural and torsional speed checks.

Product Drawing

↳ We modify the drawing to adhere to the changes we have made to the geometry through the checks we have done.