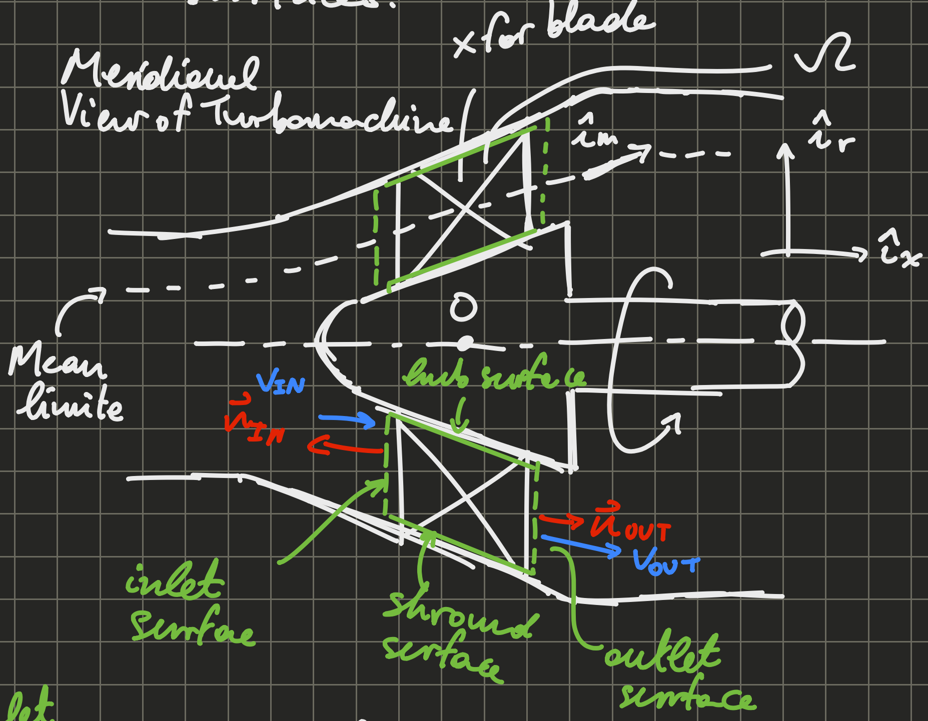
Lezione 7- Work exchanged by turbomachinery rotor

↳ Review this class

From the mixed flow machine, we get a model which we can get the axial and radial model from.

We use the Fu model for the meridional (MERIDIANA) and blade to blade surfaces.

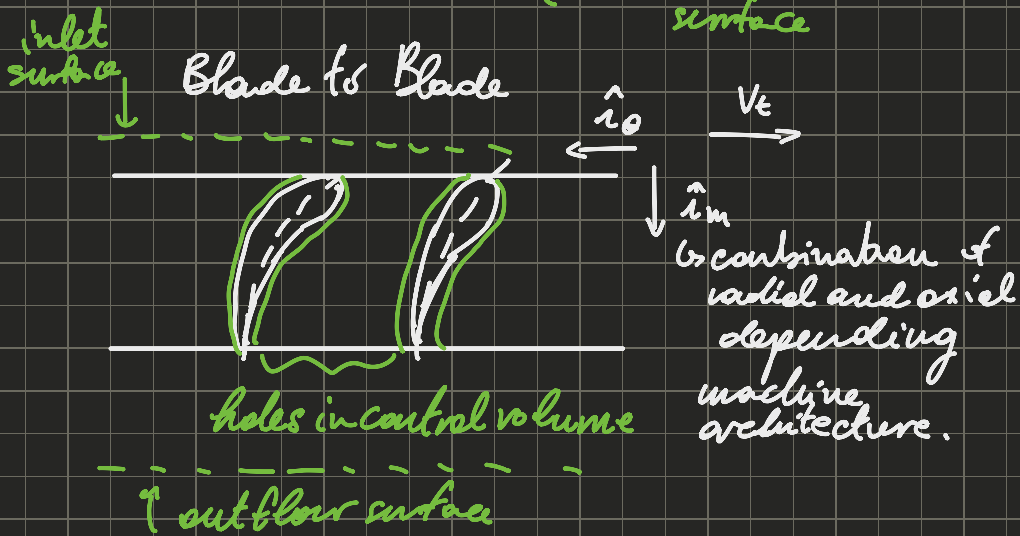


→ tangential is going in an out of board.

→ boundary of our control volume.

Shrouded/closed impeller.

Rotor → rotore
girande → impeller.



They rotors can be open/unshrouded or closer/shrouded, this makes is the top of the machine is also spinning

The deflection causes the exchange in work, we can therefore assume there will be same torque and we will have angular moment.

We write the forces exchanged by writing new balances of linear and angular momentum

It also means we won't have a gap between the blade and cover, which causes leakable flows which causes losses.

We are going to use the shrouded surface model

We can immagine a volume of revolution which bounded by the surface of the blades, the hub and shroud as well as the inlet and outlet surface.

We have to make holes in the control volume to contain the blades.

Ω = control volume

$$\partial\Omega = \underbrace{A_{in} + A_{out}}_{\text{material surfaces}} + \underbrace{A_{HUB} + A_{shroud} + A_{BL}}_{A_W \to \text{area of the walls.}}$$

solid surface

immaterial

Balance of Momentum (Linear)

linear momentum

$$\frac{d\vec{\pi}}{dt} = \underbrace{\vec{F}}_{\text{Volume and contact forces}} = \underbrace{\int_{\Omega(t)} \rho \vec{f}\, dV}_{\text{Volume force}} + \int_{\partial\Omega(t)} \vec{\sigma}\, dA \overset{t=\tau}{=} \int_{\Omega} \rho \vec{f}\, dV + \int_{\partial\Omega} \vec{\sigma}\, dA$$

we take at $t = \tau$ such that $\Omega(\tau) = \Omega$

To write for the control volume

$$\boxed{\frac{d}{dt}\int_{\Omega(t)} \rho \vec{v}\, dV} \overset{RTT}{=} \frac{d}{dt}\int_{\Omega} \rho \vec{v}\, dV + \int_{A_{IN}+A_{OUT}} \rho \vec{v}(\vec{v}\cdot\vec{n})\, dA$$

$\vec{v}$: velocity

Ω: control volume

we can do since we are not taking the derivative, and take a given moment.

since one taking the derivative with respect to t

$$\frac{d}{dt}\int_{\Omega}\rho\vec{v}\,dV+\int_{A_{OUT}}\vec{v}\rho\cdot\underbrace{(\vec{v}\cdot\vec{n})}_{V_{m,OUT}}dA-\int_{A_{IN}}\vec{v}\rho\underbrace{(\vec{v}\cdot\vec{n})}_{-V_{m,in}}dA$$

like before $V_m > 0$ always

Lumped Parameter Approach

$$\frac{d}{dt}\int_{\Omega}\rho\vec{v}\,dV+\int_{A_{OUT}}\vec{v}\rho\cdot V_m-\int_{A_{IN}}\vec{v}\rho V_m\,dA\overset{LPA}{=}\frac{d}{dt}\int_{\Omega}\rho\vec{v}\,dV+\underbrace{\vec{V}_{OUT}\dot{m}_{OUT}-\vec{V}_{IN}\dot{m}_{IN}}_{\text{Momentum flux}}$$

The meridional velocity is not constant but we will use $\dot{m}$ which is constant and the evaluation of the change remain in the fact that we need to evaluate in A_{IN} an A_{OUT}

$$\vec{F}=\int_{\Omega}\rho\vec{g}\,dV+\int_{A_{IN}+A_{OUT}}\overbrace{(-P\vec{n}+\vec{\tau})}^{\vec{\sigma}}dA+\int_{A_w}\vec{\sigma}\,dA=$$

We do the same as before, we put the boundaries far from the blades, so there is less gradient (the flow is more uniform) and so $\vec{\tau}$ is close to 0.

$$\overset{LPA}{=}\int_{\Omega}\rho\vec{g}\,dV-P_{OUT}\vec{n}_{OUT}A_{OUT}-P_{IN}\vec{n}_{IN}A_{IN}+\vec{F}_{aero}$$

usually it's the unknown

"aerodynamic forces" on solid surfaces ← is the name

Will end up negative

$$\frac{d}{dt}\int\rho\vec{v}\,dV+\underbrace{\dot{m}_{OUT}\vec{V}_{OUT}-\dot{m}_{IN}V_{IN}+P_{OUT}\vec{n}_{OUT}A_{OUT}+P_{IN}\vec{n}_{IN}A_{IN}}_{\text{simple but require more info}}-\int_{\Omega}\rho\vec{g}\,dV=\vec{F}_{aero}$$

More complicated

For steady flow and for one inlet and outlet: $(\dot m_{OUT} = \dot m_{in})$

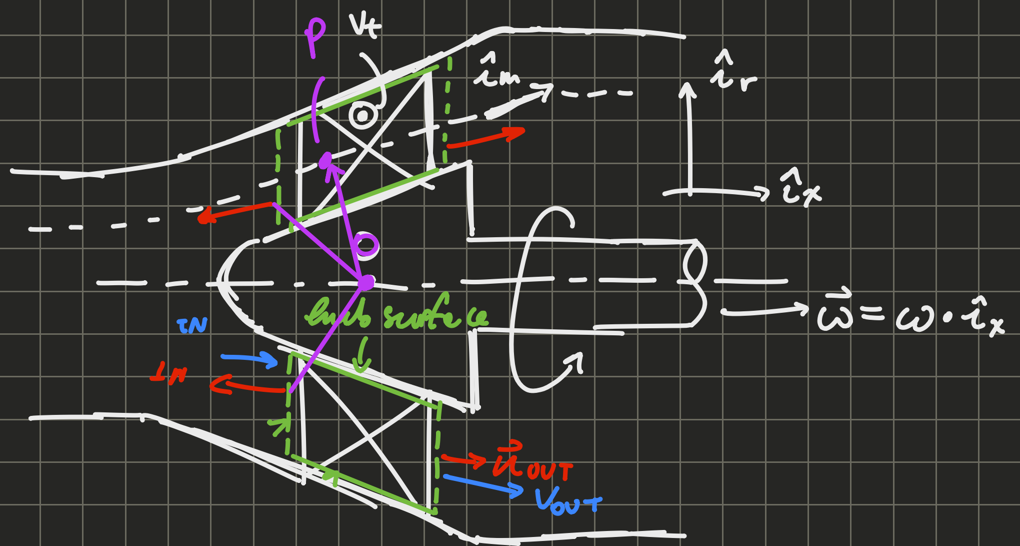
$$\underbrace{\frac{d}{dt}\int \rho \vec v \, dV}_{0} + \dot m \left(\vec V_{OUT} - \vec V_{IN}\right) + P_{OUT}\,\vec n_{OUT}\,A_{OUT} + \vec P_{IN}\,\vec n_{IN}\,A_{IN} - \int_{\Omega} \rho \vec g \, dV = \vec F_{aero}$$

This is not very useful since it's very complex.

Balance of Angular Momentum

$$\underbrace{\frac{d\vec\Gamma}{dt}}_{\text{Angular momentum}} = \vec M = \int_{\Omega(t)} \vec{OP}\times\underbrace{\rho\vec g}_{\text{should have been } f}\, dV + \int_{\partial\Omega(t)} \vec{OP}\times\vec\sigma\, dA =$$

The point O in the initial drawing is the reference point on the axis of rotation for the torque.



$$= \int_{\Omega} \vec{OP}\times\rho\vec g\, dV + \underbrace{\int_{A_{in}+A_{out}} \vec{OP}\times\left(-\overset{0}{\cancel{P\vec n}} + \underset{0}{\cancel{\vec\tau}}\right) dA}_{} + \underbrace{\int_{A_w} \vec{OP}\times\vec\sigma\, dA}_{\vec C_{aero}}$$

since there pressure is uniform, the part on top and bottom create moments that are balance, and

$\vec C_{aero}$ (Coppia) Aerodynamic Torque so the contribution of the pressure is 0. $\boxed{15:33}$

This reduces what we have to calculate.

$$\vec{\Gamma} = \int_{\Omega(t)} \vec{OP} \times \rho \vec{v}\, dV \xrightarrow{\text{RTT on } \frac{d\vec{\Gamma}}{dt}} \frac{d\vec{\Gamma}}{dt} = \frac{d}{dt}\int_{\Omega} \vec{OP} \times \rho\vec{v}\, dV + \int_{A_{IN}+A_{OUT}} \vec{OP} \times \rho \vec{v}(\vec{v}\cdot\vec{n})\, dA$$

$$= \int_{\Omega} \vec{OP} \times \rho \vec{g}\, dV + \vec{C}_{aero}$$

15:39 → Power talk

From the torque we can get the power associated

$$\dot{L} = \vec{C}_{aero} \cdot \vec{\omega} = \omega C_{AERO,x}$$

Angular Momentum Balance on x

We assume steady flow to kill one term.

$$\int_{A_{OUT}+A_{IN}} \vec{OP} \times \rho\vec{v} \cdot \hat{i}_x (\vec{v}\cdot\vec{n})\, dA = \int_{\Omega} \overbrace{\vec{OP} \times \rho\vec{g} \cdot \hat{i}_x}^{=0}\, dV + C_{aero x}$$

g is not along x and therefore

↳ We express $\vec{OP}$ and $\vec{v}$ in the cartesian system, which will always work for us, then we see if we can rewrite as cylindrical system if we think it possible.

We find that the term reduces to:

$= \rho\, r\, V_t$

$$\Rightarrow C_{AEROx} = \int_{A_{IN}+A_{OUT}} r V_t\, \rho(\vec{v}\cdot\vec{n})\, dA \overset{LPA}{=} r_{OUT} V_{t,OUT} \dot{m}_{OUT} - r_{IN} V_{t,in} \dot{m}_{IN}$$

$$\overset{!}{=} \dot{m}\left(r_{OUT} V_{t,out} - \overset{\text{average } r}{r_{in}} V_{t,in}\right)$$

$$\Rightarrow \dot{L} = \omega C_{aero} = \omega \dot{m}\left(r_{OUT} V_{t,out} - r_{in} V_{t,in}\right)$$

$$\overset{!}{=} \dot{m}\left(\omega r_{OUT} V_{t,OUT} - \omega r_{in} V_{t,in}\right)$$

~ 15:56 This equation explains why we need deflection, since we need a difference in V_t to be able to change angular momentum.

$$\vec{u} = \vec{\omega} \times \vec{r} = \omega \hat{i}_x + V_r \hat{i}_r = \omega r \hat{i}_\theta = u \hat{i}_\theta$$

↳ peripheral speed

$$= \dot{m}\left(u_{OUT} V_{t,OUT} - u_{IN} V_{t,in}\right)$$

$$l = \frac{\dot{L}}{\dot{m}} = u_{OUT} V_{t,OUT} - u_{IN} V_{t,in}$$ Euler Work Exchange.

Proprieties:

↳ r is the mean line, the mid-span of the blade

↳ This can be an issue if the height is very large since we can get gradients of velocity.

↳ in the radial we exploit the difference between u_{OUT} and u_{IN}

↳ This $\dot{L}$ that we got is the real work this exchanged to the fluid, meaning that the losses have already been taken into account, so we don't find l_w

We use the energy balance to find l_w:

$$l - l_w = \int_{IN}^{OUT} v\, dP + \frac{\Delta V^2}{2} + g\Delta z$$

w

$\underbrace{\hphantom{xxxxxxxxxxxxxxxx}}$ Mechanical energy chang

↳ what we have found

We use those to find lco.

$\boxed{16:33}$

Operating Machine

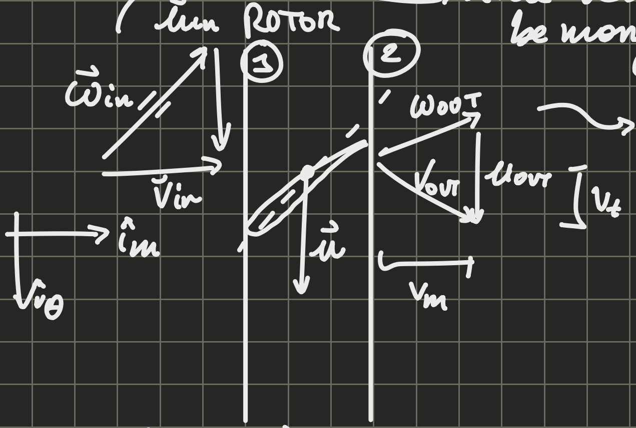
↳ $\Rightarrow l > 0 = u_{OUT} v_{\theta,OUT} - u_{IN} \underbrace{v_{\theta,in}}_{\to 0\ ?}$ $\Rightarrow$ $v_{\theta OUT} > 0$ because ($u > 0$ by def.)

We typically consider fluid as coming from a duct $\Rightarrow v_\theta = 0$

Once the blades are spinning we define the tangential direction such that u will always be positive.

v_θ can be the same or opposite sign depending if l is >0 or <0

↳ since we are going down with u, the fluid must be moving up relative to us.



$\rightarrow V_{out} = W_{OUT} + U_{OUT}$

Section ① = In
Section ② = Out } for rest of course.

$\vec{V} = \vec{\omega} + \vec{u}$

The blades at the front are inclined so it can take into account it's own rotation velocity and the initial velocity.

↳ relative velocity.

While we design based on $\vec{\omega}$, while our calculations are based on v_θ so we need to convert to:

$\vec{u} = u\ \hat{i}_\theta$

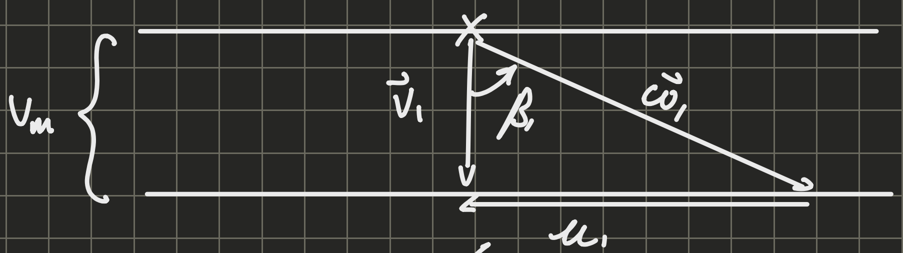
$$\begin{cases} v_x = \omega_x \\ v_r = \omega_r \end{cases} \Rightarrow v_m = \omega_m$$
$$v_\theta = \omega_t + u = \omega_t + \omega r$$

We want our airfoil to allign with w_{IN} to reduce the loss that we suffer.

We change the tangential velocity by deflecting it and changing also the $\vec{w}$. We basically force v_θ onto the fluid through rotating it.

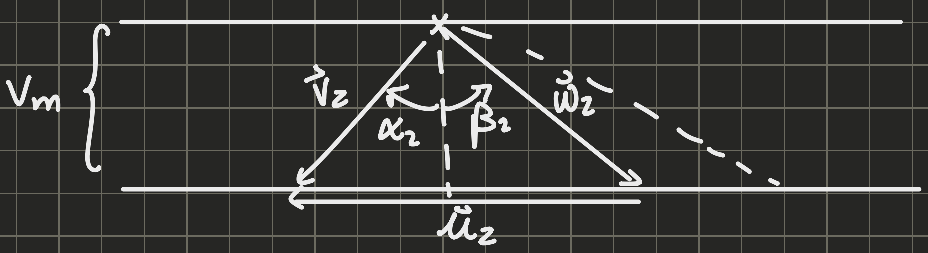
We achieved our objective of introducing a positive tangential component, since this is an operating machine so we have positive work.

Velocity Triangles



For inlet

↳ Head to head, v_1 and u_1



For outlet

Head to tail w_2 and u_2