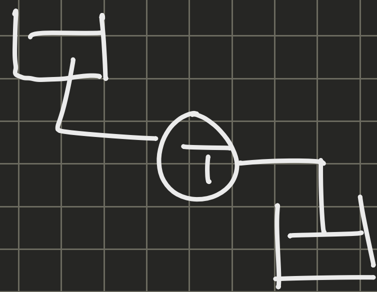
Lezione 16 -

Cavitation in Turbines

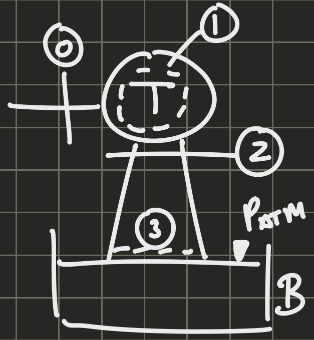


$$NPSH_R = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} - \frac{P_{MIN}}{\rho g}$$

$$NPSH_A = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} - \frac{P_{SAT} + P_{DIS}}{\rho g}$$

$$P_{MIN} > P_{SAT} + P_{DIS} \iff NPSH_A > NPSH_R$$

Turbines cost a lot so they are typically custom made.



BEM $2 \to B$

$$\cancel{e - e_w} - y_{DT} = I_B - I_2 =$$

$$= \frac{P_{ATM}}{\rho} + g z_B - \frac{P_2}{\rho} - \frac{V_2^2}{2} - g z_2$$

$$\Rightarrow \frac{P_2}{\rho} + \frac{V_2^2}{2g} = \frac{P_{ATM}}{\rho} - g\overbrace{\underbrace{(z_2 - z_B)}_{\text{Installation height of turbine}}}^{h_T} + y_{DT}$$

$$y_{DT} = \frac{(\xi_{DT} + 1) V_3^2}{2}$$

Our boundary condition is that $P_B = P_{ATM}$, so the fluid has to exist to adhere to the boundary condition.

From the perspective of efficiency, not having a draft tube, it is better for us, since we have less losses.

But reducing the losses, reduce the $NPSH_A$.

We have to compensate with h_T the negate effect of the draught tube.

$$NPSH_A = \frac{P_{ATM}}{\rho g} - h_T - \frac{P_{SAT} + P_{DIS}}{\rho g} + \left(\xi_{DT} + 1\right)\frac{V_3^2}{2g}$$

The heigher is the turbine relative to the basin the forse the cavitation risk.

Dam are good for high H and low Q, low cos.

Water-flowing plant are good for low H, high Q, high cos

Dams use Francis turbines radial

Water flowing use Kaplan turbines. axial

$\boxed{\text{Balje diagram for turbines exist.}}$

like pumps it's for one machine, combining we can achieve any H or Q.

These machinnes have to be at maximum efficiency when possible. Francis and kaplan turbines are able to change their shape to maximise η when to operating condition changes.

Draft Tube (diffuser)

↳ axial

Pelton Turbine (use case for extremely high H and low Q)
↳ used in dams.

They are easy to make approximations for so it's what we start with.

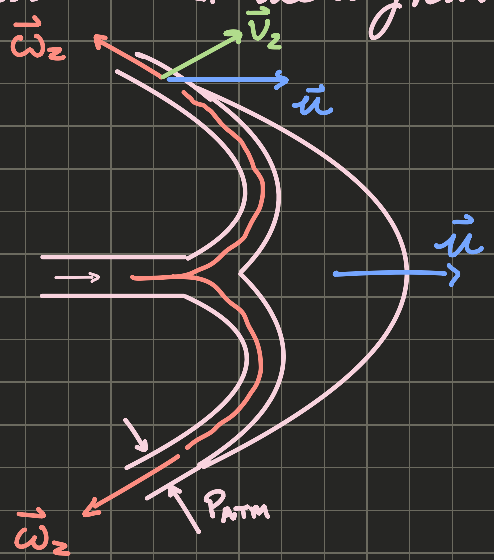
It's the most complicated to work with but after some simplifications it becomes the easiest.

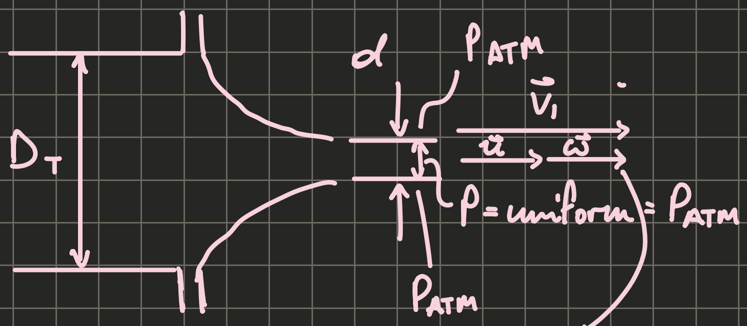
Pelton turbines spin in air and is fed a jet of water locally. This jet coming from the nozzle is made to go very quickly, as the nozzle is conical.

Conical Nozzles (1-6).

The pressure up stream and downstream from bucket is the same.

The water leaving just falls into basin due to gravity.





$\vec{u}$ cannot be too high, since that means reducing $\vec{\omega}$ more than we want.

Pelton is by definition an axial machine, since the change in radius, does not change $\vec{u}$.

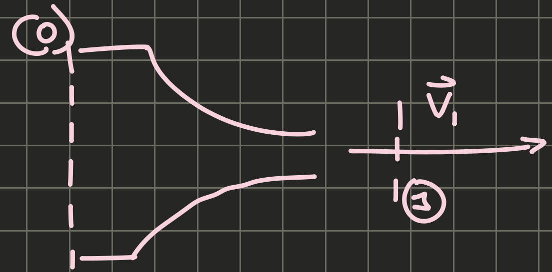
This machine does not suffer from any cavitation, since a jet hitting surface increases the pressure, so on the bucket internal surface pressure can only increase. This is why it is useful for when H_m is high.

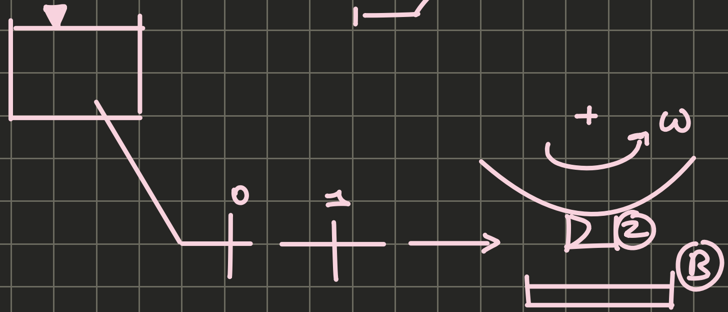
To bucket suffer from erosion not cavitation. It's possible to have cavitation if the wedge is not designed well, since we are at P_{ATM} so $\frac{P_{ATM}}{\rho} = 10\,m$

$$\chi = \frac{P_1 - P_2}{\rho g H_m} = 0 \rightarrow$$ Pelton turbines are called <u>impulse</u> turbines (by design).

Kaplan & Francis $\chi \neq 0 \Rightarrow$ they are called reaction turbines.

<u>Nozzle</u>





In general we cannot write BME $0 \rightarrow 1$, since we don't know the χ, but in this special case we do, so we can consider section 1.

$z_0 = z_1 = z_2 = z_B$

$\xi_{DT} = 0$, $V_3 = V_2 \rightarrow$ an optimization in this case reduces V_2 not V_{2m}, but this is a special case.

BME $D \to 1$

$$\cancel{\ell} - \ell_{w_N} - y_p = T_1 - T_D = \cancel{\frac{P_1}{\rho}} + \frac{V_1^2}{2} + g z_1 - \cancel{\frac{P_{ATM}}{\rho}} - g z_D$$

↳ the nozzle is a part of the machine, so we have to consider it, since it will not be part of the losses in the penstock. (before D).

$$\Rightarrow \frac{V_1^2}{2g} = \underbrace{g(z_D - z_B) - y_p}_{T_0 - T_B = g H_m} - \ell_{w_N}$$

$\underset{D \to 0}{\Delta z} \to \underset{0 \to 1}{\Delta P} \to \Delta V$ ↝ How we convert the energy at different steps.

$$\Rightarrow \frac{V_1^2}{2} = g H_m - \ell$$

$V_1' : V_1^2/2 = g H_m \Rightarrow V_1' = \sqrt{2 g H_m}$

↳ Ideal velocity with no losses over the nozzle.

$$\frac{V_1}{V_1'} = \varphi \Rightarrow V_1 = \varphi V_1' = \varphi \sqrt{2 g H_m} \to = 0{,}96 - 0{,}98$$

↳ Parameter which measures losses in V_1 due to ℓ_{w_N}

↳ like efficiency but for velocity.

We can use this to account for the water which the machine processes:

$i \to$ number of jets / nozzles
$d \to$ diameter of jets

$$\Rightarrow Q = i \frac{\pi d^2}{4} \cdot \varphi \sqrt{2 g H_m}$$

↳ Flow rate that passes into our system.

If $y_p = 0$, $V_1 = \varphi \sqrt{2 g \Delta z}$ stays the same.

Changing d, Q changes but V_1 doesn't change, since we are not changing P_{ATM} around the jet which means that v has to be the same, so eventhough it's strange changing d, means that we change Q and not v, usually it's the opposite.

Next step is to consider how the flow moves to calculate the power

$$|\ell| = v_{2t}\, u_2 - v_{1t}\, u_1 = \left| v_{1t}\, u_1 - v_{2t}\, u_2 \right| \overset{\text{since turbine}}{=} \left| v_{1t} - v_{2t} \right| u$$

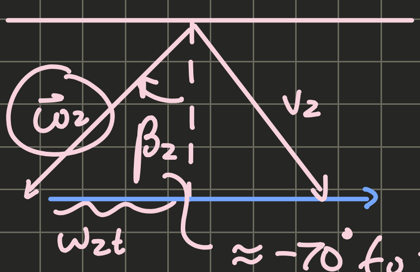
tangential velocity is the one parallel to u

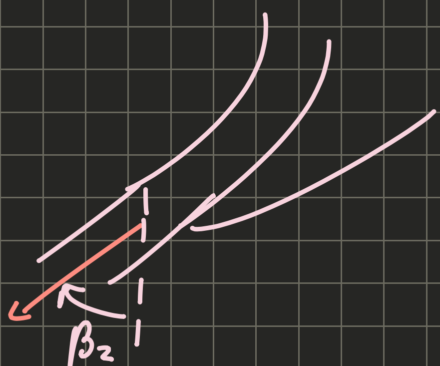
$\Rightarrow v_{1t} = v_1,\ v_{1m} = v_1$

↳ conceptually since at the inlet, it's a degenerated triangle, so $v_{1t} = v_{1m}$, this allows us to calculate Q.

$$= \left| v_1 - v_{2t} \right| u = \left| v_1 - (w_{2t} + u) \right| u \overset{(*)_1}{=}$$

$w_{2t} = w_2 \cdot \sin\beta_2$





$\approx -70°$ to $-78°$

↳ negative since opposite direction of u.

BERF $1 \to 2$:

↳ Balance of Energy in Rotating Frame

$$q_{\to 0} = \left(h_2 + \frac{w_2^2}{2} - \frac{u_2^2}{2} \right) - \left(h_1 + \frac{w_1^2}{2} - \frac{u_1^2}{2} \right)$$

Rothalpy → generally conserved

$$\frac{\omega_2^2}{2} - \frac{\omega_1^2}{2} = h_1 - h_2 = -\Delta h = -\left(\int_1^2 T\,dS + \int_1^2 v\,dP\right) = -\ell_{coR}$$

Rotor

The friction causes a change in the velocity.

This is only true for Pelton, not kaplan or Francis

$$\frac{\omega_2}{\omega_2'} = \frac{\omega_2}{\omega_1} = \psi \approx 0,9$$

↳ isoenthropic ω_2

$$= \psi \omega_1 \sin \beta_2$$

$$\textcircled{A}_1 = \left|(v_1 - u) - \omega_1 \psi \sin\beta_2\right| u$$

$\omega_1 = v_1 - u$ → Degenerate triangle

$$= \left|(v_1 - u) - (v_1 - u)\psi \sin\beta_2\right| u$$

$(v_1 - u) > 0$, otherwise the machine is too fast

$$|\ell| = (v_1 - u)\left|1 - \psi \sin\beta_2\right| u$$

↳ We have calculated the work

Now we calculate η:

Efficiency of bucket

$$\eta = \frac{|\ell|}{gH_m} = \frac{u(v_1 - u)\left|1 - \psi \sin\beta_2\right|}{v_1'^2/2g} = 2\,\frac{u}{v_1'}\left(\underbrace{\frac{v_1}{v_1'}}_{\varphi} - \frac{u}{v_1'}\right)\left|1 - \psi\sin\beta_2\right|$$

nozzle efficiency

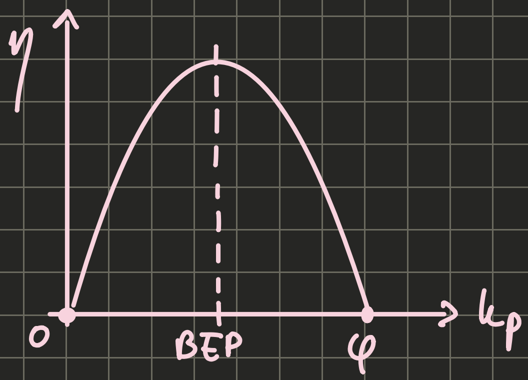
geometry of bucket

$$k_p = \frac{u}{v_1'}$$

↳ peripheral speed coefficient

$$\eta = 2k_p(\varphi - k_p)\left|1 - \psi \sin\beta_2\right|$$

It gives us the unique solution to maximize the efficiency (unique for this course)



why?

is $\eta = 0$, when $k_p = \varphi \Rightarrow V_1 = u \Rightarrow \omega = 0$

$$\xrightarrow{\quad V_1 \quad}$$
$$\xrightarrow[\quad u \quad]{}$$

when $u > v$, they

η_{BEP} : $k_p = \frac{\varphi}{2} \Rightarrow V_1 = 2u \Rightarrow u = \frac{1}{2} V_1, \Rightarrow \omega_1 = u_1$

We design the machine to operate at the BEP.

Power and Torque

$$\dot{L} = \dot{m}\, \ell = \rho\, Q\, |\ell|$$

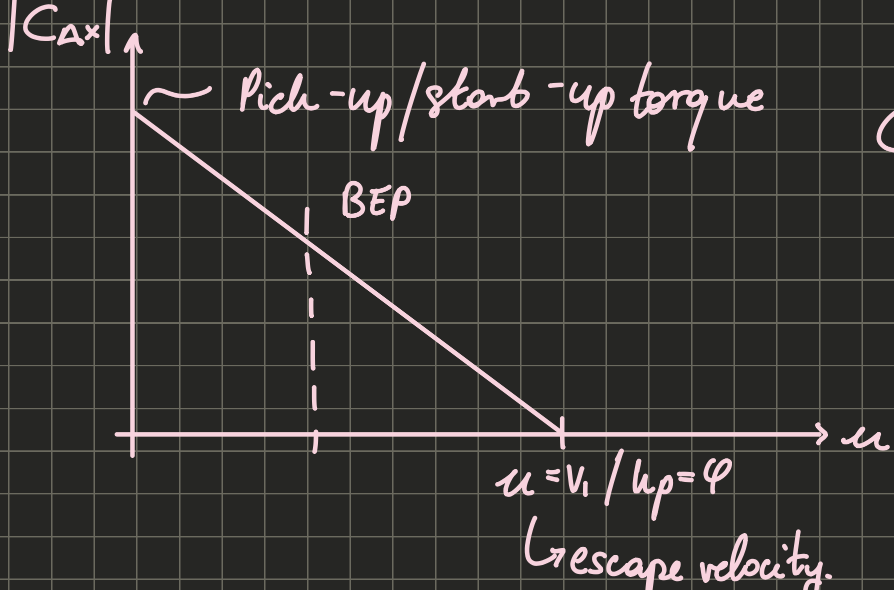
$$= \rho\, i\, \pi \frac{d^2}{4} \cdot \varphi \sqrt{2gH_m} \cdot u\left(\varphi\sqrt{2gH_m} - u\right)\left|1 - \psi \sin\beta_2\right|$$

$\dot{L}$ is a function of d^2, this makes it extremely easy to regulate (neglecting changes in H_m) since only v_1 will change, but everything remains the same, it's an engineers dream. (Assuming φ is constant, since reducing d, will cause losses to decrease, but the effect on v_1 will be minimal.)

$$C_{Ax} = \frac{|\dot{L}|}{\omega} = \frac{|\dot{L}|}{\omega} \cdot \frac{D}{2} = \rho i \frac{d^2}{4} \varphi \sqrt{2gH_m} \left(\varphi \sqrt{2gH_m} - u \right) \frac{D}{2} \left| 1 - \psi \sin \beta_2 \right|$$

$$u = \frac{\omega D}{2} \Rightarrow \omega = \frac{u D}{2}$$

when $u = 0$, $\dot{L} = 0$, but $C_{Ax} \neq 0$



$C_{Ax}(0) \neq 0$, is useful since the machine will immediately start.

v_1 doesn't change much, and Q is near constant, since from 0 to 1, the diameter is reduced by a lot already so changing d a bit more doesn't do much, this is compared with a garden hose where we are changing the d and v_1 changes by a lot but this is because comparative this d is a much greater magnitude change relative to the initial diameter.