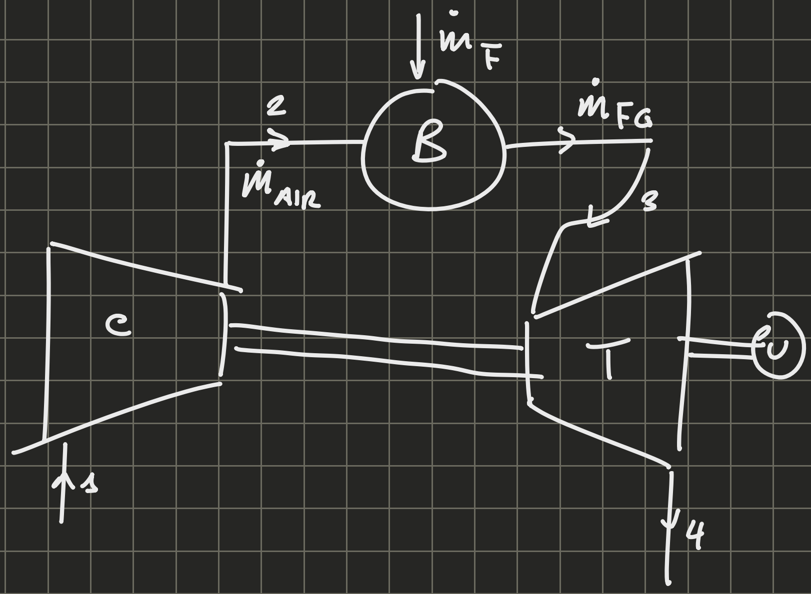
Esercitazione 9 – Turbogas / Gas-Turbine Cycles

Esercizio 1

$\dot{L}_u = \dot{L}_{el} = 10\,MW$

useful



$p_1 = 1\,bar$

$T_1 = 15°C = 288\,K$

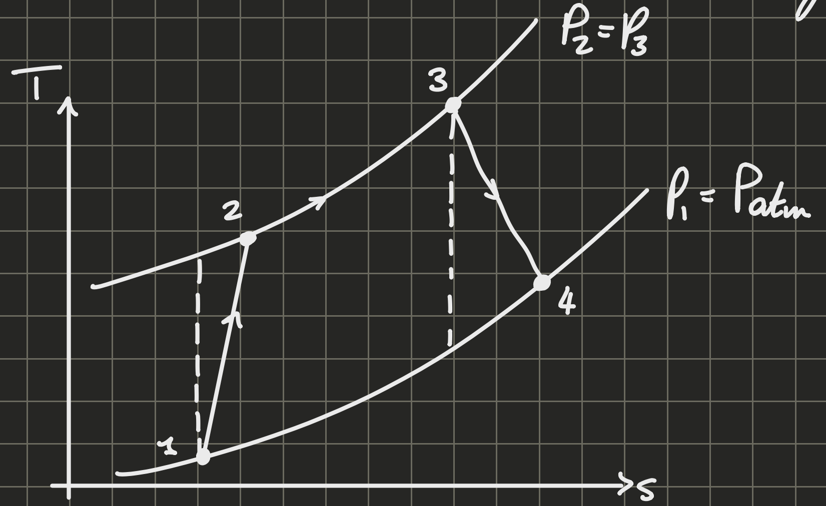
$T_{max} = 1373\,K = T_3$

$\beta = 25$

$\eta_{iso,C} = 80\%$ $\eta_{iso,T} = 85\%$

$T_3 \gg T_1$

$\dot{m}_F$ is negligible with respect to $\dot{m}_{AIR}$



For the whole cycle we can use air.

$\Rightarrow c_{p\,air} = c_{p\,fuel} = c_{p\,FG} = \overline{c_p} = 1004\ J/kgK\ (= c_{p\,air})$

$\gamma_{air} = \gamma_F = \gamma_{FG} = \gamma = 1{,}4$

$LHV = 44\ MJ/kg$

$T_{ref} = 298\,K \longrightarrow h_f(T_{ref}) + c_p(T - T_{ref})$

1) $\dot{m}_{air}, \dot{m}_F$?

unknowns:

$T_2, T_4, \dot{m}_{air}, \dot{m}_F \rightarrow$ 4 unknown $\Rightarrow$ 4 equations

(Generally $P_4 = P_1$, unless explicitly stated in text)

Compressor:

$$l_c = c_{p\,air} T_1 \left(\beta^{\frac{\gamma-1}{\gamma}} - 1\right) = 436.2 \ \frac{kJ}{kg}$$

$$\eta_{iso,c} = \frac{l_{cis}}{l_c} \longrightarrow l_c = 545.3 \ \frac{kJ}{kg}$$

$$l_c = c_{p\,air}(T_2 - T_1) \rightarrow T_2 = T_1 + \frac{l_c}{c_{p,air}} = 831.1 \ K$$

Turbine:

$$l_{iso,T} = c_{p,FG}\left(T_{4,is} - \underbrace{T_3}_{=T_{MAX}}\right)$$

$$= c_{p,FG}\, T_3 \left(\underbrace{\frac{T_{4,is}}{T_3}}_{=\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \beta^{\frac{1-\gamma}{\gamma}}} - 1\right) = c_{p,FG}\, T_3 \left(\beta^{\frac{1-\gamma_{FG}}{\gamma_{FG}}} - 1\right)$$

since T_{4iso}

because turbine β not compressor one

$$\longrightarrow l_{iso,T} = 829 \ \frac{kJ}{kg}$$

$$\eta_{iso,T} = \frac{l_T}{l_{iso,T}} = -704.65 \ \frac{kJ}{kg}$$

$$T_4 = T_3 + \frac{\ell_T}{c_{p,FG}} = 671.2\,K$$

Based on the balance on the shaft we know:

$$\dot{L}_U + \dot{L}_T + \dot{L}_c = 0$$

$$\dot{L}_U - |\dot{L}_T| + \dot{L}_c = 0$$

$$\dot{L}_U = |\dot{L}_T| - \dot{L}_c = (\dot{m}_{air} + \dot{m}_{FUEL})|\ell_T| - \dot{m}_{air}\,\ell_c$$

$$\frac{\dot{L}_{el}}{\dot{m}_f} = (\alpha + 1)|\ell_T| - \alpha\,\ell_c$$

We don't $\dot{m}_F$ or α, so we need another equation to solve.

We can use the burner to find α:

(We start from scratch in the burner, since it's good to review)

$$\cancelto{0}{\dot{L}} + \cancelto{0}{\dot{Q}} = \dot{m}_{FG}\left(h_{f,FG} + c_{p,FG}\left(T_{FG} - T_{ref}\right)\right)$$
$$- \dot{m}_{ox}\left(h_{f,ox} + c_{p,ox}\left(T_{ox} - T_{ref}\right)\right)$$
$$- \dot{m}_F\left(h_{f,F} + c_{p,F}\left(T_F - T_{ref}\right)\right)$$

Generally air is our oxidiser but we just write a general formula for now

$$\dot{m}_{ox} + c_{p,ox}\left(T_{ox} - T_{ref}\right) + \dot{m}_F\, c_{p,F}\left(T_F - T_{ref}\right) +$$
$$+ \dot{m}_{ox}\, h_{f,ox} + \dot{m}_F\, h_{f,F} - \dot{m}_{FG}\, h_{f,FG} = \dot{m}_{FG}\, c_{p,FG}\left(T_{FG} - T_{ref}\right)$$

Dividing by $\dot{m}_F$:

$$\alpha\, c_{p,ox}\left(T_{ox} - T_{ref}\right) + c_{p,F}\left(T_F - T_{ref}\right) + \overbrace{\left[h_{f,F} + \alpha\, h_{f,ox} - (1+\alpha)\, h_{f,FG}\right]}^{LHV_F}$$

$$= (1+\alpha)\, c_{p,FG} \left(T_{FG} - T_{ref}\right)$$

Approximations:

$$\begin{cases} c_{p,ox} = c_{p,F} \\ T_{ox} = T_F \rightarrow \text{more realistic} \end{cases}$$

Applying these:

$$\alpha + 1\, c_{p,ox}\left(T_{ox} - T_{ref}\right) + LVH_F = (1+\alpha)\, c_{p,FG}\left(T_{FG} - T_{ref}\right)$$

Solving for α:

$$\boxed{\alpha = \frac{LHV_F}{c_{p,FG}\left(T_{FG} - T_{ref}\right) - c_{p,ox}\left(T_{ox} - T_{ref}\right)} - 1}$$

To simplify more:

$$\bar{c}_p = \frac{\dot{m}_F c_{p,F} + \dot{m}_{ox} c_{p,ox} + \dot{m}_{FG} c_{p,FG}}{\dot{m}_F + \dot{m}_{ox} + \dot{m}_{FG}}$$

$$= \frac{\cancel{(1+\alpha)}\left(c_{p,ox} + c_{p,FG}\right)}{2\cancel{(1+\alpha)}} = \frac{c_{p,ox} + c_{p,FG}}{2}$$

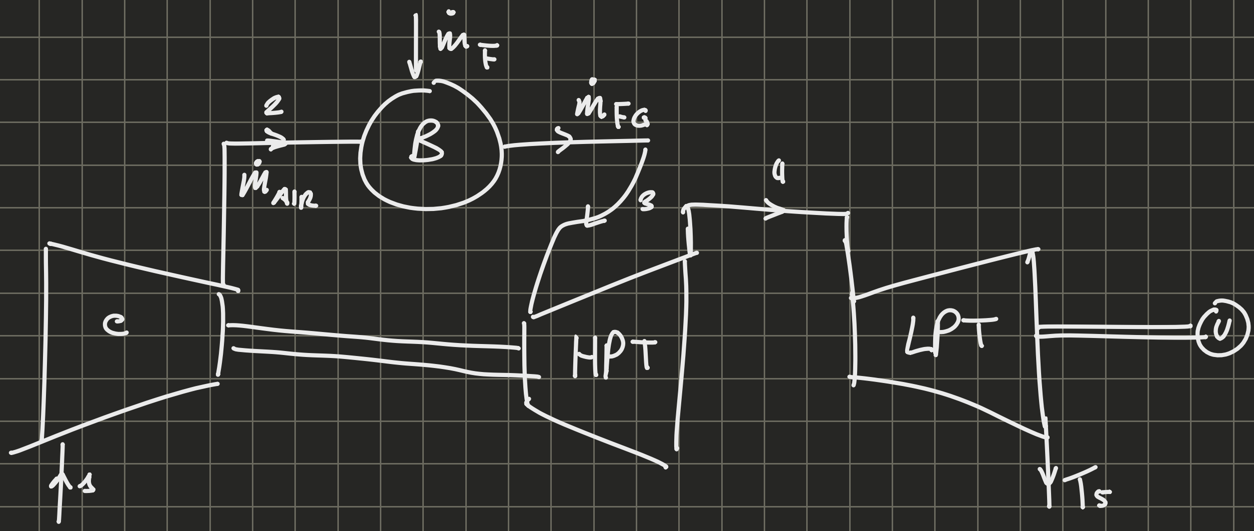
$$\Rightarrow \boxed{\alpha = \frac{LHV_F}{\bar{c}_p\left(T_{FG} - T_{ox}\right)} - 1} = 79.9 = \frac{\dot{m}_{air}}{\dot{m}_F}$$

$\Longrightarrow \dot{m}_F = 0.744\ kg/s \rightarrow$ Using in combination with other equation.

$\dot{m}_{air} = \alpha \dot{m}_F = 59.4 \ kg/s$

2) $\eta_{cycle} = \dfrac{\dot{L}_{EL}}{\dot{m}_F LHV_F} = 30.5\%$

<u>Exercise 2</u>



$\dot{m}_{air} = 15 \ kg/s$

$T_1 = 293K$

$\beta = 8$

$T_{max} = 1250K$

$LHV = 45 \ MJ/kg$

$\eta_{BURNER} = 1$

$\eta_C = 0.8 \quad \eta_{HPT} = \eta_{LPT} = 0{,}85$

$\eta_{GEN} = 0.97 = \dfrac{\dot{L}_{el}}{|\dot{L}_{LPT}|}$

$T_{ref} = 298 \ K$

$c_{p,air} = c_{p,F} = 1004 \ \dfrac{kJ}{kgK} \qquad \gamma_{air} = \gamma_F = 1.4$

$c_{p,FG} = 1.2 \ \dfrac{kJ}{kg\,K} \qquad \gamma_{FG} = 1.32$

a) $\dot{m}_F = ?$

unknowns $\cancel{T_2}$, T_4, P_4, $\cancel{\dot{m}_F}$, T_5, $\dot{L}_{el}$

(α) either is fine

Compressor:

$$l_{c,iso} = c_{p,air} T_1 \left(\beta^{\frac{\gamma-1}{\gamma}} - 1 \right) = 237.8 \frac{kJ}{kg}$$

$$l_c = \frac{l_{c,is}}{\eta_c} = 297.2 \; kg/kg$$

$$T_2 = T_1 + \frac{l_c}{c_{p,air}} = 590.2 \; K$$

$$\alpha = \frac{LHV_F}{c_{p,FG}(\underbrace{T_{FG}}_{T_3} - T_{ref}) - c_{p,ox}(\underbrace{T_{ox}}_{T_2} - T_{ref})} - 1$$

$$\longrightarrow \alpha = 51.9 = \frac{\dot{m}_{air}}{\dot{m}_F}$$

$$\dot{m}_F = \frac{\dot{m}_{air}}{\alpha} = 0.29 \; \frac{kg}{s}$$

b) $\dot{L}_{el}$, $\eta_{cycle} = ?$

$$l_{is,HPT} = c_{p,FG} T_3 \left(\beta_{HPT}^{\frac{1-\gamma_F}{\gamma_F}} - 1 \right) \rightarrow \text{Cannot use}$$

Through a balance on the $\underbrace{\text{shaft}}_{\text{high pressure}}$ we find $\dot{L}_c = \dot{L}_{HPT}$

$$\cancel{\dot{L}_u} + \dot{L}_c - |\dot{L}_{HPT}| = 0$$

$$\rightarrow \dot{L}_c = |\dot{L}_{HPT}| \longrightarrow \dot{m}_{air} \, l_c = (\dot{m}_{air} + \dot{m}_{FG}) |l_{HPT}|$$

$$\alpha \, l_c = (1 + \alpha) |l_{HPT}|$$

$$|l_{HPT}| = \frac{\alpha}{1+\alpha} l_c = -291.6 \frac{kJ}{kg}$$

$$T_4 = T_3 + \frac{l_{HPT}}{c_{p,FG}} = 1007 \ K$$

$$l_{is,HPT} = \frac{l_{HPT}}{\eta_{HPT,is}} = -343.06 \ \frac{kJ}{kg}$$

$$\longrightarrow \beta_{HPT} = 2.92 = \frac{P_3}{P_4} = \frac{\beta P_1}{P_4} \rightarrow P_4 = \frac{\beta P_1}{\beta_{HPT}} = 2.74 \text{ bar}$$

$$\beta_{LPT} = \frac{P_4}{P_5} = \frac{P_4}{P_{ATM}} = 2.74$$

$$l_{is,LPT} = c_{p,FG} T_4 \left(\beta_{LPT}^{\frac{1-\gamma}{\gamma}} - 1 \right)$$

$$\longrightarrow l_{is,LPT} = -261.97 \ \frac{kJ}{kg}$$

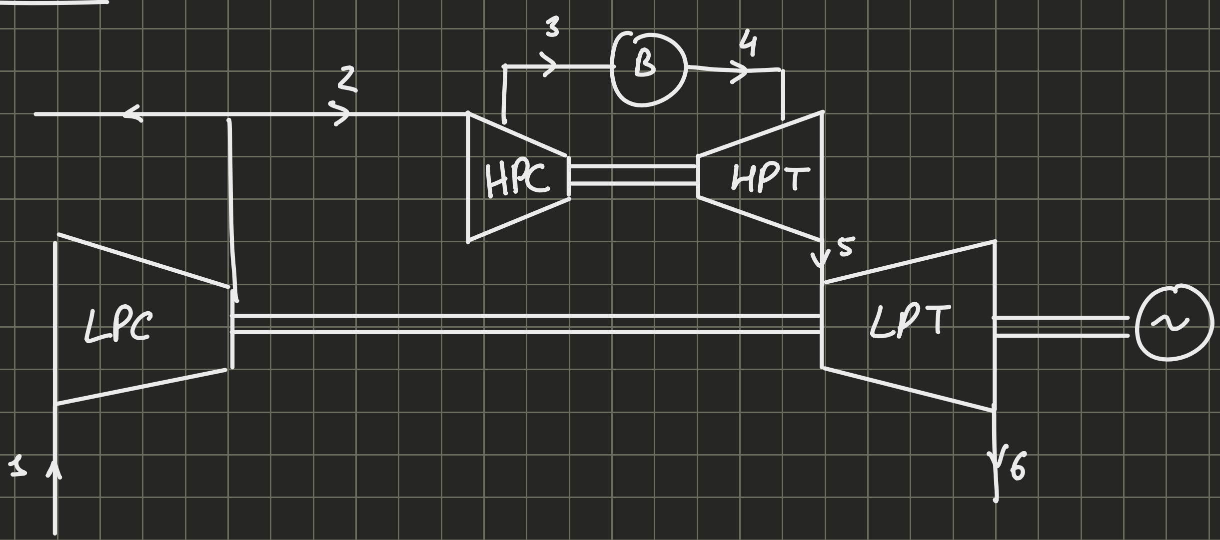
$$l_{LPT} = l_{is,LPT} \cdot \eta_{LPT} = -222.67 \ \frac{kJ}{kg}$$

$$T_5 = T_4 + \frac{l_{LPT}}{c_{p,FG}} \longrightarrow T_5 = 821.4 \ K$$

$$\eta_{GEN} = \frac{\dot{L}_{el}}{|\dot{L}_{LPT}|} = \frac{\dot{L}_{el}}{|(\dot{m}_F + \dot{m}_{air}) l_{LPT}|} \Rightarrow \dot{L}_{el} = 3302 \ kW = 3.3 MW$$

$$\eta_{cycle} = \frac{\dot{L}_{el}}{\dot{m}_F LHV_F} = 25.3\%$$

Exercise 3



$\beta = 32 \qquad \beta_{LPC} = \beta_{HPC} = \sqrt{\beta} = 5.66$

$T_{max} = 1473\ K \qquad \dot{L}_{el} = \dot{L}_u = 20 MW$

$\eta_{is} = 0.85 \rightarrow$ for all the components

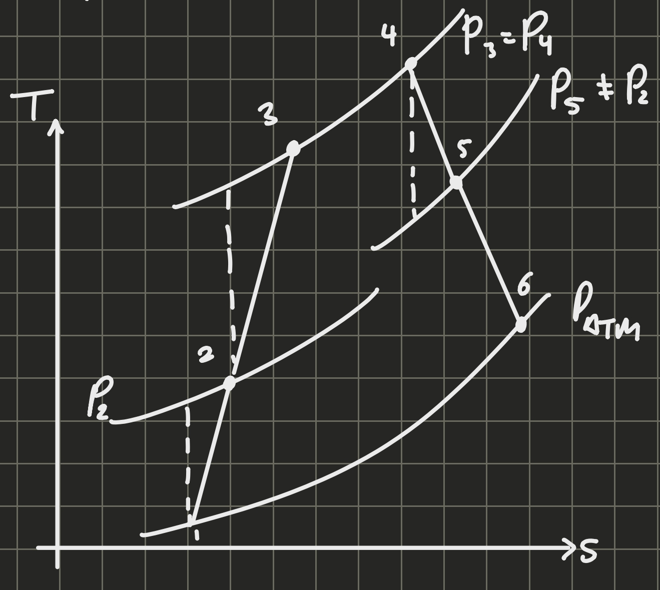
a) $\dot{m}_{air}$, $\dot{m}_F$?

$c_{p,F} = c_{p,FG} = c_{p,ox} = 1 \frac{kJ}{kg\,k}$

$\gamma_F = \gamma_{FG} = \gamma_{air} = 1.4$

$P_1 = 1\ bar \qquad T_{ref} = 298\ k$

$T_1 = 293\ k \qquad LHV_F = 40 \frac{MJ}{kg}$



unknowns:

$T_2, T_3, T_5, P_5, T_6, \dot{m}_{air}, \dot{m}_F \longrightarrow$ 7 unknowns, 7 equations

4 turbomachines, 2 shafts, 1 burner $\rightarrow$ 1 equation for each

<u>LPC</u>

$$l_{is,LPC} = c_{p,air} T_1 \left(\beta_{LPC}^{\frac{\gamma-1}{\gamma}} - 1 \right) = 188.5 \frac{kJ}{kg}$$

$$l_{LPC} = \frac{l_{LPC,is}}{\eta_{LPC}} = 221.8 \frac{kJ}{kg} \longrightarrow T_2 = T_1 + \frac{l_{LPC}}{c_{p,air}} = 514 K$$

<u>HPC</u>

$$l_{HPC,is} = c_{p,air} T_2 \left(\beta_{HPC}^{\frac{\gamma-1}{\gamma}} - 1 \right) = 330.8 \frac{kJ}{kg}$$

$$l_{HPC} = \frac{l_{HPC,is}}{\eta_{HPC}} = 389.2 \frac{kJ}{kg}$$

$$T_3 = T_2 + \frac{l_{HPC}}{c_{p,air}} = 901.6 \ K$$

<u>Combustor</u>

since 10% of air taken away.

$$\alpha^* = \frac{LHV_F}{\bar{c}_p (T_4 - T_3)} - 1 = 68.7 = \frac{0.9 \, \dot{m}_{air}}{\dot{m}_F}$$

↳ assuming $\bar{c}_p$, the T_{ref} is not considered

$$\alpha = \frac{\dot{m}_{air}}{\dot{m}_F} = \frac{\alpha^*}{0.9} = 76.3$$

HP shaft

$$|\dot{L}_{HPT}| = \dot{L}_{HPC}$$

$$(0.9\,\dot{m}_{air} + \dot{m}_F)\, l_{HPT} = 0.9\,\dot{m}_{air}\, l_{HPC}$$

$$\longrightarrow (\alpha^* + 1)\,|l_{HPT}| = \alpha^*\, l_{HPC}$$

$$|l_{HPT}| = \frac{\alpha^*}{\alpha^*+1}\, l_{HPC} = 383.6\,\frac{kJ}{kg}$$

$$l_{HPT} = -383.6\,\frac{kJ}{kg} \longrightarrow T_5 = T_4 + \frac{l_{HPT}}{c_{p,FG}} = 1091\,K$$

$$\eta_{HPT} = \frac{l_{HPT}}{l_{HPT,is}} \rightarrow l_{HPT,is} = \frac{l_{HPT}}{\eta_{HPT}} = -451.3\,\frac{kJ}{kg}$$

$$l_{HPT,is} = c_{p,FG} \cdot T_4\left(\beta_{HPT}^{\frac{1-\gamma}{\gamma}} - 1\right) \longrightarrow \beta_{HPT} = \left(\frac{l_{HPT,is}}{c_{p,FG}\,T_4} + 1\right)^{\frac{\gamma_{FG}}{1-\gamma_{FG}}} = 3.58$$

$$\frac{P_4}{P_5} = \frac{\beta P_1}{P_5} \longrightarrow P_5 = \frac{\beta P_1}{\beta_{HPT}} = 8.94\text{ bar } \underbrace{(> P_2)}_{\text{as we drew}}$$

LPT

$$l_{LPT,is} = c_{p,FG}\, T_5\left(\beta_{LPT}^{\frac{1-\gamma_{FG}}{\gamma_{FG}}} - 1\right)$$

$$\frac{P_5}{P_6 = 1} = 8.94$$

$$\longrightarrow l_{LPT,is} = -509.6\,\frac{kJ}{kg}$$

$$l_{LPT} = \eta_{LPT}\, l_{LPT,is} = -433.2\,\frac{kJ}{kg}$$

$$T_6 = T_5 + \frac{l_{LPT}}{c_{p,FG}} = 659.5\ K$$

LP shaft

$$\dot{L}_u - |\dot{L}_{shaft}| + \dot{L}_{LPC} =$$

$$\rightarrow \dot{L}_u = |\dot{L}_{LPT}| - \dot{L}_{LPC}$$

$$\dot{L}_{el} = (0.9\ \dot{m}_{air} + \dot{m}_F)|l_{LPT}| - \dot{m}_{air}\, l_{LPC}$$

$$\rightarrow \frac{\dot{L}_{el}}{\dot{m}_F} = (\alpha^* + 1)\,|l_{LPT}| - \alpha\, l_{LPC}$$

$$\rightarrow \dot{m}_F = \frac{\dot{L}_{el}}{(\alpha^* + 1)|l_{LPT}| - \alpha\, l_{LPC}}$$

$$\rightarrow \dot{m}_F = 1.51\ kg/s \rightarrow \dot{m}_{air} = \alpha\, \dot{m}_F = 115.2\ kg/s$$

$$\eta_{cycle} = \frac{\dot{L}_{el}}{\dot{m}_F \cdot LHV_F} = 33.1\%$$