

Esercitazione 1 - Balance Equations

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Mass Balance:

$$\rho_{in} V_{m,in} S_{in} \xrightarrow{\text{Surface area}} \rho_{out} V_{m,out} S_{out}$$

Energy Balance:

$$\dot{m} \left[h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in}) \right] = \dot{L} + \dot{Q}$$

Assumptions we need to get to these equations: $u + \frac{V^2}{2} + gz$ Specific energy

$$\hookrightarrow 1) \text{ Steady State Flow} \Rightarrow \frac{d}{dt} \int_{R_f} \rho e dV = 0$$

2) Single inlet - single outlet

$$3) \text{ Uniform flow} \int_{S_{in} + S_{out}} \vec{v} \cdot d\vec{s} = 0$$

4) Dumper Parameter Approach:

$$u = T ds - P dv \rightarrow u = u(s, v)$$

We can change this to be a function T & P , since we know that:

$$P = P(s, v) \text{ and } T(s, v)$$

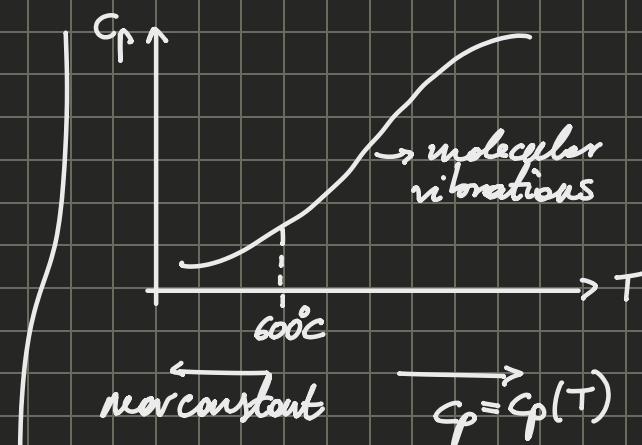
So we can write equations of state based on these two:

For ideal gases the equations of state are:

$$\begin{cases} u = u(T) \\ Pv = RT \rightarrow \text{volumetric equation of state} \end{cases}$$

$$R = 8314 \frac{J}{\text{mol} \cdot \text{K}} \rightarrow R = \frac{R}{MM} \rightarrow \text{molar mass}$$

Perfect Gases



Since $h = h(C_p(T), T)$ we cannot find it without

$$= h_f + \int_{T_{\text{ref}}}^{T_{\text{out}}} C_p(T) dT \rightarrow C_p(T) = c_p \text{ we can solve the integral}$$

and so we take it as constant,
the gases for which this is true

For perfect gases we can write: are called perfect gases.

$$\hookrightarrow dh = c_v dT \rightarrow h = h_{\text{ref}} + \int_{T_{\text{ref}}}^T C_v dT = h_{\text{ref}} + C_v (T - T_{\text{ref}})$$

$$\frac{d(h + Pv)}{dT} = \frac{dh}{dT} + R \rightarrow C_p = c_v + R$$

$$\frac{dh}{dT} = \frac{d(Pv)}{dT} = \frac{d(RT)}{dT} = R$$

In perfect gases since C_p is
constant c_v will also be constant.

$$\hookrightarrow dh = C_p dT \rightarrow \boxed{h = h_f + C_p (T - T_{\text{ref}})}$$

We can also do this for the entropy

$$dh = Tds + v dP$$

$$\rightarrow ds = \frac{dh}{T} - \frac{v}{T} dP$$

$\frac{dh}{C_p dT}$ $\frac{v}{R/P}$

$$\text{Integration by: } S = S_{\text{ref}} + C_p \ln \frac{T}{T_{\text{ref}}} - R \ln \frac{P}{P_{\text{ref}}}$$

Ideal liquids \rightarrow second type of fluid we want to consider

$$\begin{cases} u = u(T) \\ v = \text{const} \end{cases} \rightarrow C_L$$

$$du = T ds - P dv \xrightarrow{v=\text{const}} ds = \frac{du}{T} \xrightarrow{u=u(T)} \Rightarrow s = s(T)$$

$$\left. \text{The definition of } C_p = T \left(\frac{\partial s}{\partial T} \right)_p = T \frac{ds}{dT} \right\}$$

$$\left. \text{On the other hand } C_V = T \left(\frac{\partial s}{\partial T} \right)_V = T \frac{ds}{dT} \right\} \xrightarrow{\text{since } v = \text{const},} C_p = C_V = C_L$$

Perfect liquid C_L not a function of temperature

they end up being
the same

$$du = C_L dT = u = u_{\text{ref}} + C_L (T - T_{\text{ref}})$$

$$dh = d(u + Pv) = \underbrace{du}_{C_L dT} + v dP + P dv \xrightarrow{v=\text{const}}$$

$$\rightarrow h = h_f + C_L (T - T_{\text{ref}}) + v \underbrace{(P - P_{\text{ref}})}_{\text{Important term}} \Rightarrow h = h(T, P) \text{ which is perfect gas it was only of } T$$

$$ds = \frac{du}{T} = \frac{C_L dT}{T} \Rightarrow s = s_{\text{ref}} + C_L \ln \frac{T}{T_{\text{ref}}} \Rightarrow s = s(T) \text{ while in a perfect gas it was function of } P \text{ and } T$$

Exercise 1: $1 \text{ bar} = 10^5 \text{ Pa}$

$$\begin{cases} P_{\text{in}} = 10 \text{ bar} = 1000000 \text{ Pa} = 10^6 \text{ Pa} \\ T_{\text{in}} = 1000 \text{ K} \\ \rightarrow P_{\text{out}} = 6 \text{ bar} = 6 \times 10^5 \text{ Pa} \end{cases}$$

Ideal + Polytropic $\Rightarrow C_p, C_V = \text{const}$

Since it's a perfect gas h is independent on P so we can use C_p even though it's isobaric.

$$C_p = 1004 \frac{J}{kg \cdot K}$$

$$R_{air} = 287 \frac{J}{kg \cdot K}$$

$$\dot{L} = 450 \text{ kW}$$

↳ since entropy

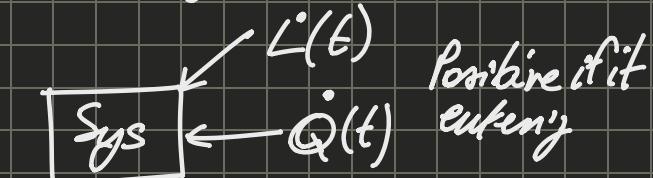
$$\dot{Q} = -50 \text{ kW}$$

$$\dot{m} = 5 \frac{kg}{s}$$

? T_{out}

Other assumptions

$$\left. \begin{array}{l} \frac{\Delta V^2}{2} \approx 0 \text{ Negligible} \\ g \Delta z \approx 0 \text{ Negligible} \end{array} \right\}$$

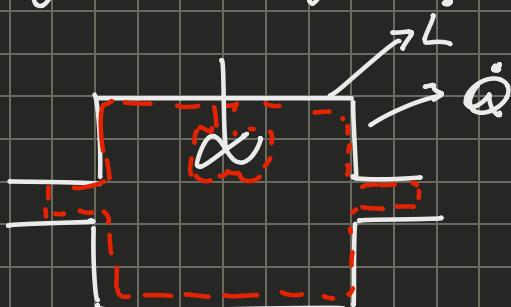


Positive if it
enters'

→ We don't need mass balance
since we know that it's single
inlet-style outlet and since \dot{m}_{cv}
is 0.

$$\dot{m}_{in} = \dot{m}_{out} = 5 \frac{kg}{s}$$

We cannot consider so
unless explicitly told.



The first two equations are the balances

$$\dot{m} \left[h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in}) \right] = \dot{L} + \dot{Q}$$

or

$$\dot{m} \left[h_f + C_p (T_{out} - T_{ref}) - h_f + C_p (T_{in} - T_{ref}) \right] = \dot{m} C_p (T_{out} - T_{in})$$

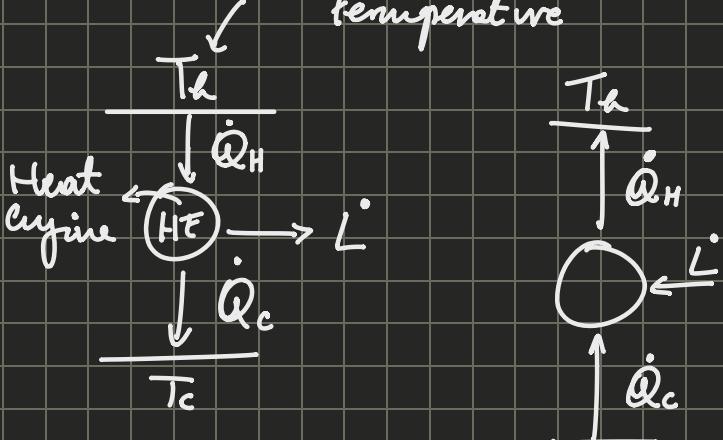
$$= \dot{L} + \dot{Q}$$

$$T_{out} = \frac{\dot{L} + \dot{Q}}{\dot{m} C_p} + T_{in} = 900.45 \text{ K}$$

Since T_{out} not very different from T_{out} we can assume C_p as
constant.

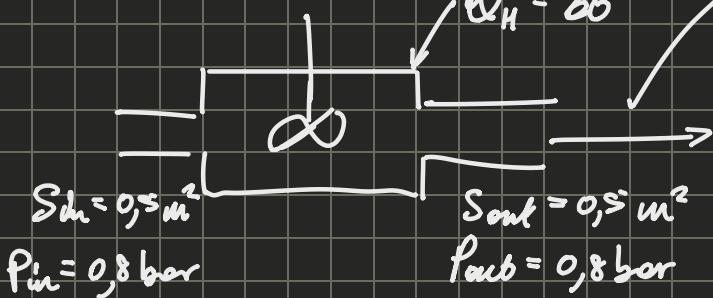
Exercise 2

Heat source at fixed temperature



Direct
Thermodynamic
Cycle

Inverse
Thermodynamic
Cycle



$$\dot{m}_{in} = 200 \frac{\text{kg}}{\text{s}}$$

$$T_{in} = 253,15 \text{ K}$$

$$T_{out} = 573,15 \text{ K}$$

$$\dot{m}_{in,inv} = \dot{m}_{out,inv} = \dot{m}_i = ?$$

we don't consider the \dot{Q}_c since we consider a heat engine.

$$R_{air} = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$C_p = 1004 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$\Delta z = 0$$

$$\frac{\Delta v^2}{2} \neq 0$$

Kinetic energy change is not negligible.

$$\dot{L} = ? \left(< 0 \text{ since heat engine} \right)$$

$$\dot{m}_{in,inv} = f_{in} V_{in,inv} S_{in}$$

$$\sim$$

$$P_v = RT$$

$$\Rightarrow \rho = \frac{P}{RT} = \frac{8 \times 10^5}{287 \cdot 253} = 1,1 \frac{\text{kg}}{\text{m}^3}$$

$$\dot{m}_{in,inv} = 110 \frac{\text{kg}}{\text{s}}$$

$$\dot{m}_{out,inv} = P_{out} \cdot V_{in,inv} S_{out} \rightarrow V_{out} = \frac{\dot{m}}{P_{out} S_{out}} = 452 \frac{\text{m}}{\text{s}}$$

$$\frac{P_{out}}{T_{out} R} = 0,486 \frac{\text{kg}}{\text{m}^3}$$

$$\dot{m} \left[\underbrace{h_{out} - h_{in}}_{C_p(T_{out} - T_{in})} + \frac{V_{out}^2 - V_{in}^2}{2} \right] = \dot{L} + \dot{Q}$$

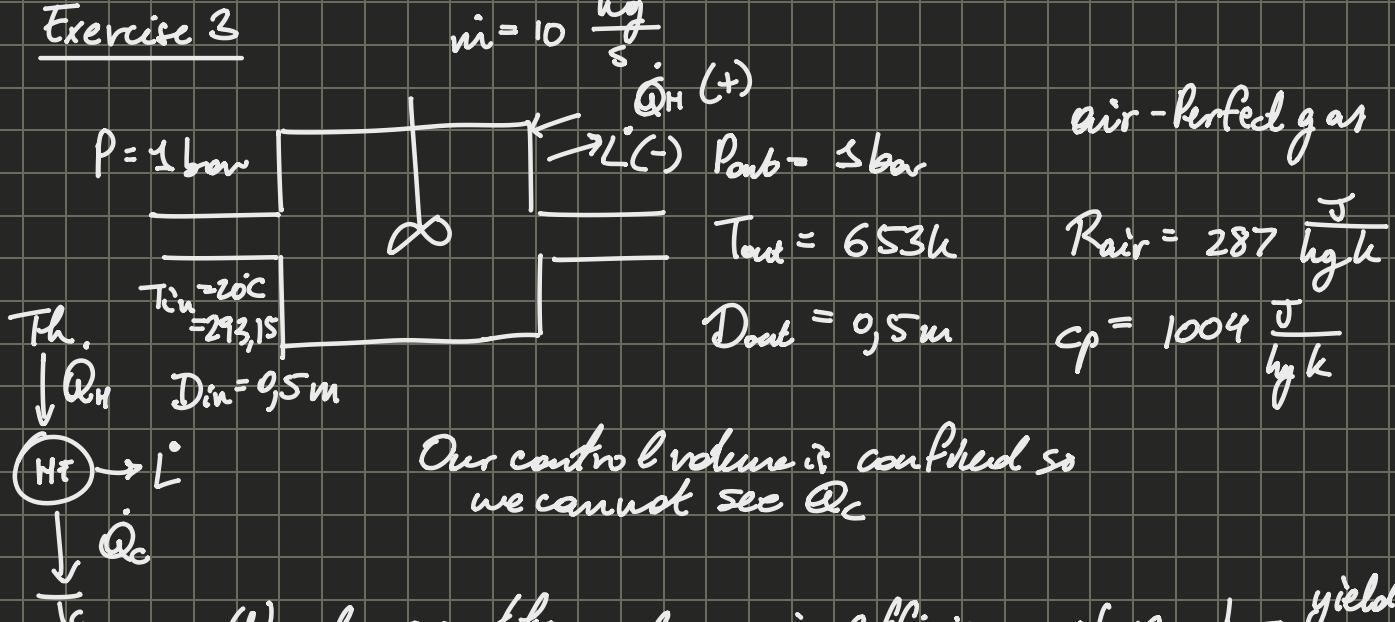
$C_p(T_{out} - T_{in})$
since perfect gas

$$\rightarrow \dot{L} = \dot{m} \left[C_p(T_{out} - T_{in}) + \frac{V_{out}^2 - V_{in}^2}{2} \right] - \dot{Q}$$

$\underline{\dot{L}} = -15,6 \text{ MW}$

It's easier to write everything in watts
and then convert.

Exercise 3



We have a thermodynamic efficiency of $\eta = \frac{1}{3} = \frac{\text{yield}}{\text{cost}}$

$$\Delta z = 0$$

$$\dot{L} = ? \quad \Delta s = ?$$

$$\dot{m}_{in} = \rho_{in} V_{in, in} S_{in} \rightarrow V_{in, in}$$

$$\rho_{in} = \frac{P_{in}}{R T_{in}} = 1,19 \frac{\text{kg}}{\text{m}^3} \rightarrow V_{in} = 42,02 \frac{\text{m}^3}{\text{s}}$$

$$S_{in} = \frac{\pi D_{in}^2}{4} = 0,2 \text{ m}^2$$

$$\dot{m} = \rho_{out} V_{m, out} S_{out} \rightarrow V_{m, out}$$

$$S_{out} = 0,2 \text{ m}^2 \quad \rho_{out} = 0,52 \frac{\text{kg}}{\text{m}^3} \rightarrow V_{out} = 96,18 \text{ m/s}$$

$$\begin{aligned} \dot{L} &= \frac{\dot{L}}{\dot{Q}_H} \\ \Rightarrow 3 |\dot{L}| &= \dot{Q}_H \\ \Rightarrow \dot{Q}_H &= -3 \dot{L} \end{aligned}$$

(small $\dot{L} < 0$)

$$\dot{m} \left[\underbrace{h_{out} - h_{in}}_{C_p(T_{out} - T_{in})} + \frac{V_{out}^2 - V_{in}^2}{2} + g(\vec{x} \vec{z})^\circ \right] = \dot{L} + \dot{Q}_H$$

$$= \dot{L} - 3\dot{L} = -2\dot{L}$$

$$\Rightarrow \dot{L} = \frac{\dot{m} \left[C_p(T_{out} - T_{in}) + \frac{V_{out}^2 - V_{in}^2}{2} \right]}{-\alpha} = -5,92 \text{ MW} < 0 \checkmark$$

$\Delta S = S_{out} - S_{in} > 0$ since q and S_{in} both > 0

$$\delta q = \int T ds - \int T ds_{irr} \rightarrow \underbrace{\delta q}_{\geq 0} + \underbrace{\int T \delta s_{irr}}_{= \dot{L}_w > 0} = \int T ds$$

$$\Delta U_h = \Delta U_{IRR} \Rightarrow \dot{L}_w = T \delta s_{IRR}$$

$$\Delta S = S_{out} - S_{in} = \underbrace{\delta}_{\text{in}}(P, T) = C_p \ln \frac{T_{out}}{T_{in}} - R \ln \frac{P_{out}}{P_{in}}$$

For perfect
gas

$$\Delta S(S, T) = 1004 \ln \left(\frac{653}{293} \right) = 831,6$$

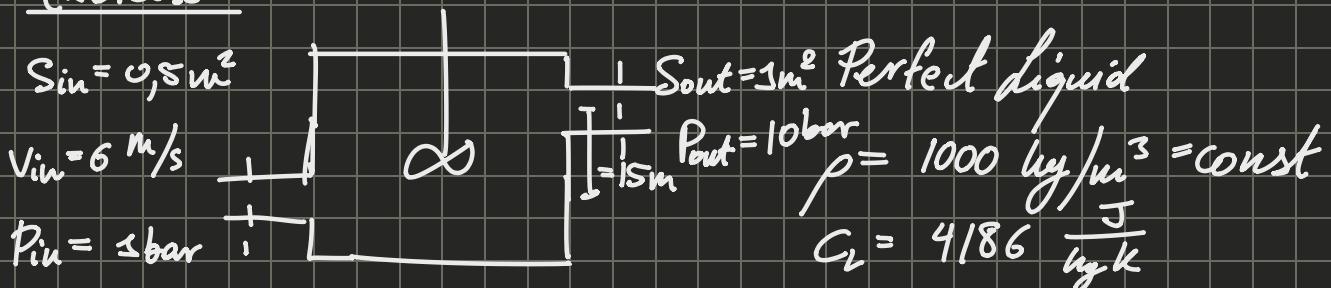
↪ This is why it's important to use Kelvin

To compare if the process is good or not we can compare it to the Carnot efficiency:

$$\eta_c = 1 - \frac{T_c}{T_h}$$

But since we don't have T_c we cannot do the comparison.

Exercise 4



The machine is considered adiabatic, so $\dot{Q} = 0$

(Thermo machines are also adiabatic.)

$$\Delta z = 0$$

$\Delta T = T_{out} - T_{in} = 0,15 \text{ K} \Rightarrow$ since $\dot{Q} = 0$, this is due to the irreversibility

$$\dot{L} = ?$$

$$\eta_{Hypal} = 0$$

$$m = \rho_{in} V_{in} S_{in} \rightarrow v_{in} = 3000 \frac{\text{kg}}{\text{s}} \Rightarrow v_{out} = 3 \frac{\text{m}}{\text{s}}$$

since $\rho = \text{const}$

$$\rho \dot{Q}_{in} = \rho \dot{Q}_{out} \Rightarrow \frac{\dot{Q}_{in}}{v_{in} \cdot S} = \frac{\dot{Q}_{out}}{v_{out} \cdot S}$$

$$m \left[h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(\Delta z) \right] = \frac{\dot{L}}{15 \text{ m}} \quad (\dot{Q} = 0)$$

$$\hookrightarrow c_L(T_{out} - T_{in}) + V(P_{out} - P_{in})$$

$$\rightarrow \dot{L} = 5 \text{ MW}$$

$$q = \int_{in}^{out} T ds = \int T dS_{inout} \Rightarrow q = \int_{in}^{out} T ds - l_w$$

After note:

for perfect liquid

$$ds = c_L dT$$

then we get

$$\dot{Q} + \dot{L}_w = m c_L (T_{out} - T_{in}),$$

since $\dot{Q} = 0$, we get

$$h_{out} - h_{in} + \frac{\Delta V^2}{2} + g \Delta z = l + q$$

By replacing q we isolate the mechanical terms:

$$l - l_w = \int_{in}^{out} v dP + \frac{V_{out}^2 - V_{in}^2}{2} + g \Delta z$$

→ If perfect liquid is easy, if gas not because of the relation between p and P .

Technical work \rightarrow The mechanical power of the enthalpy.

Theoretical work

this course we only do for liquids

$$\rightarrow \dot{L} - \dot{L}_w = \left(\frac{P_{out}}{\rho} + \frac{V_{out}^2}{2} + g z_{out} \right) - \left(\frac{P_{in}}{\rho} + \frac{V_{in}^2}{2} + g z_{in} \right)$$

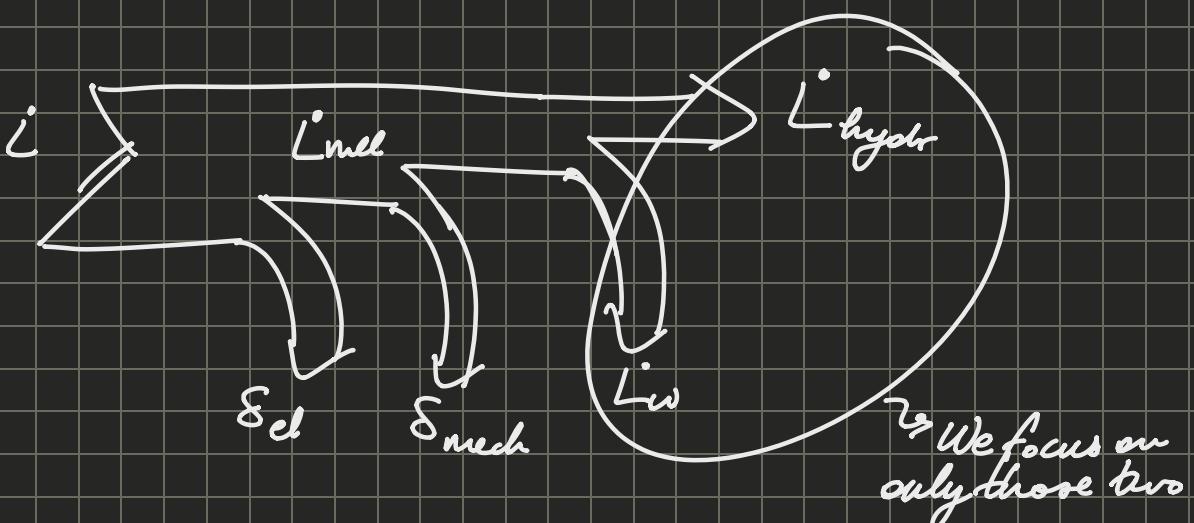
↑
r I → capital I am
↳ Bernoulli trinomial

Since we have \dot{L} we have to convert to the power form rather than its current specific form.

$$\dot{L} - \dot{L}_w = \left[\left(\frac{P_{out}}{\rho} + \frac{V_{out}^2}{2} + g z_{out} \right) - \left(\frac{P_{in}}{\rho} + \frac{V_{in}^2}{2} + g z_{in} \right) \right] \dot{m}$$

$$\rightarrow \dot{L}_w = 1,9 \text{ MW}$$

~~We get the term that we removed immediately, so it's easier to just remember that.~~



$$\eta = \frac{\dot{L}_{turb}}{\dot{L}} = \frac{\dot{L} - \dot{L}_w}{\dot{L}} = 0,622 = 62,2\%$$