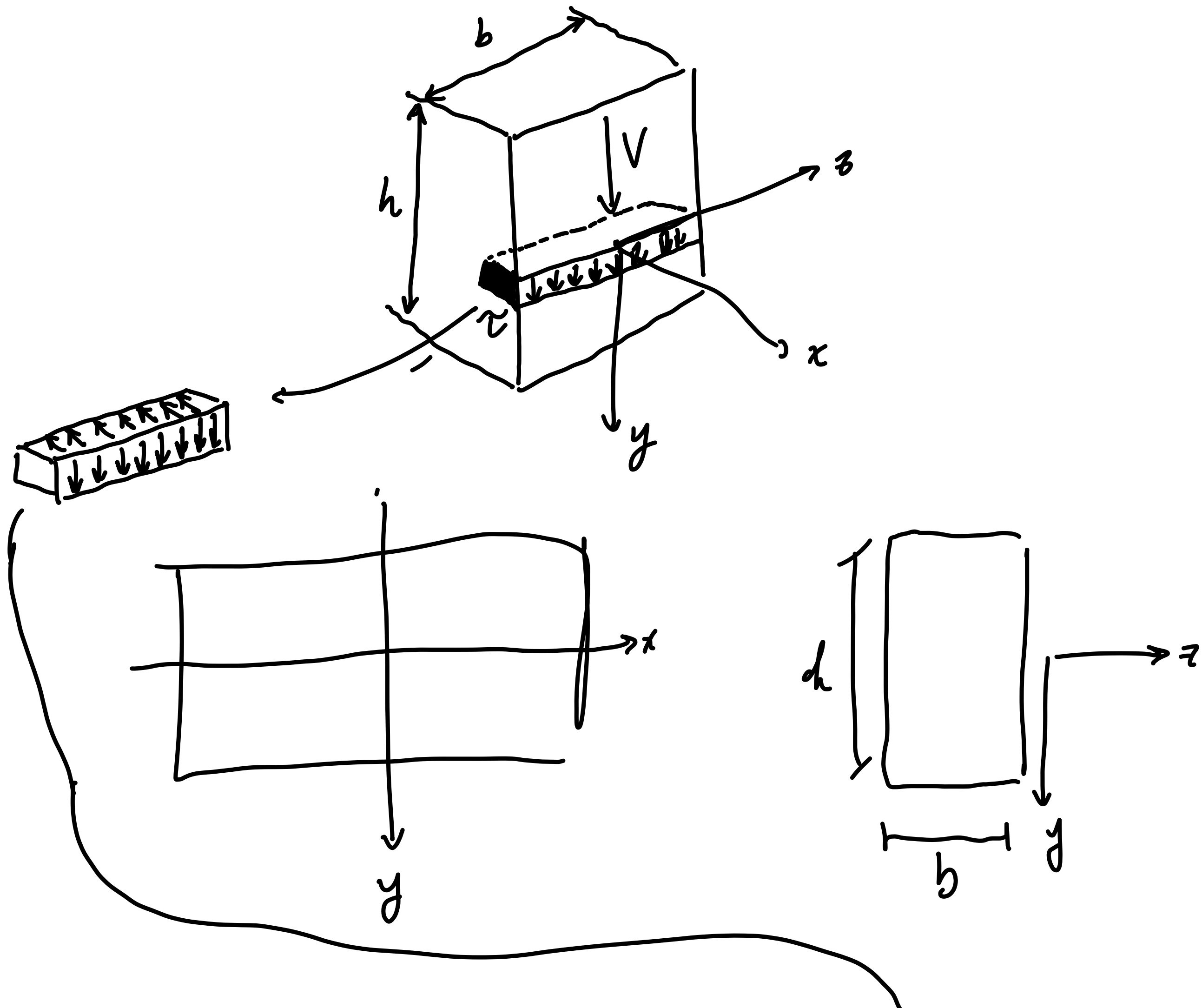
Casi di De Saint-Venant: Taglio

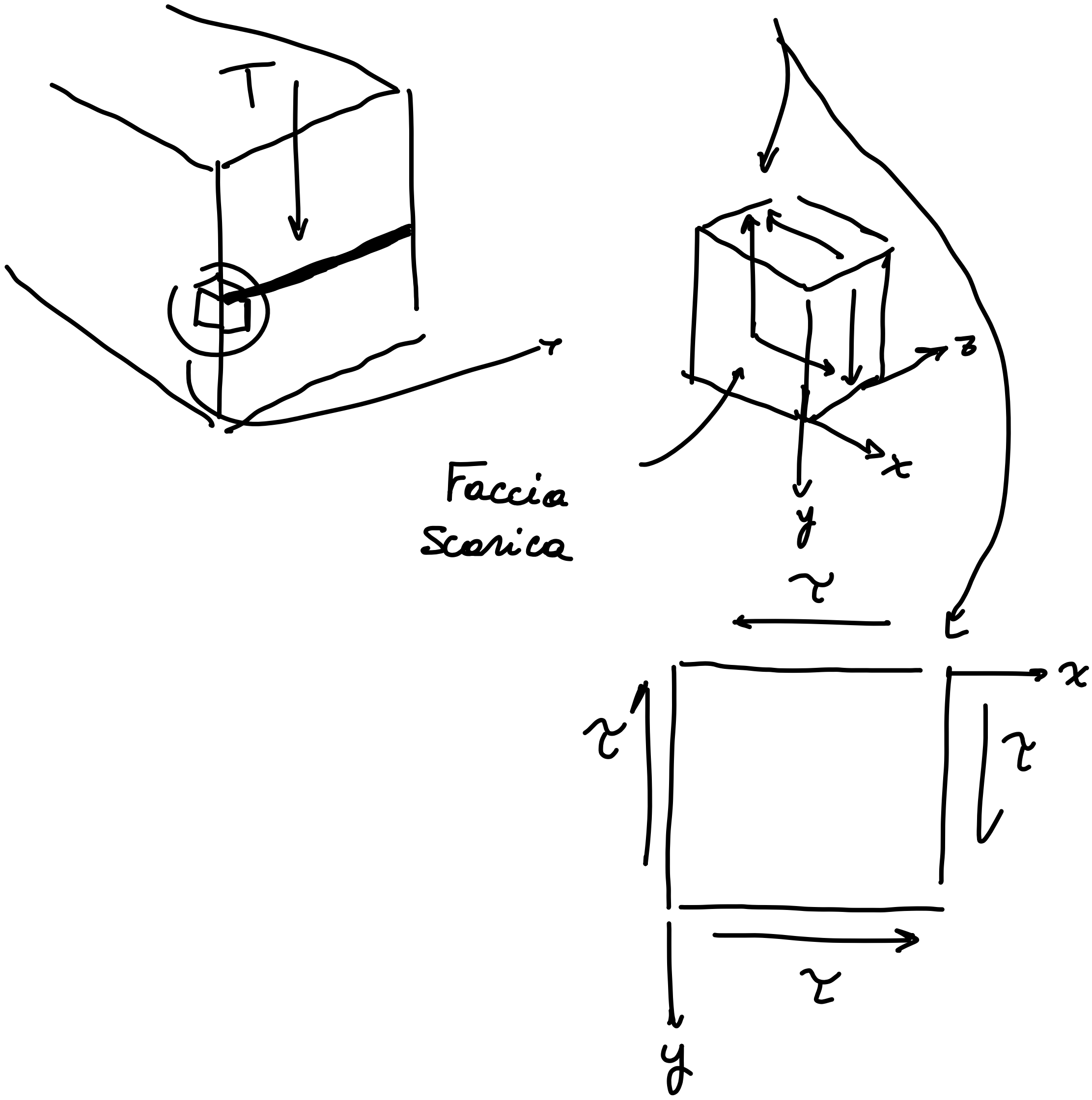
↳ Già visto forza normale e momento flettente

Azione di taglio è azione normale che causa scorrimento fra due sezioni.

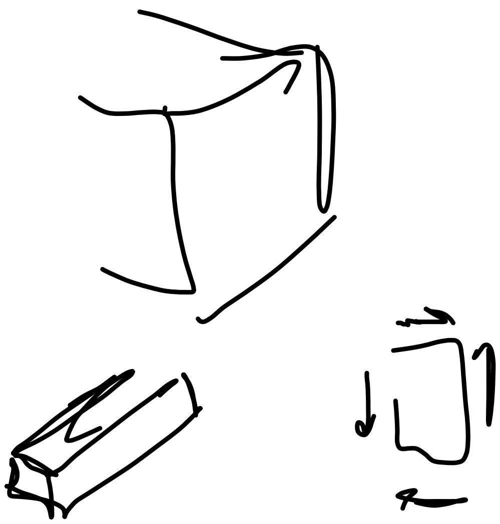
$T = -\frac{dM}{dx}$ se $M = cost$ allora $T = 0$

o $\frac{dM}{dx}$ dipendendo su convenzioni di segno

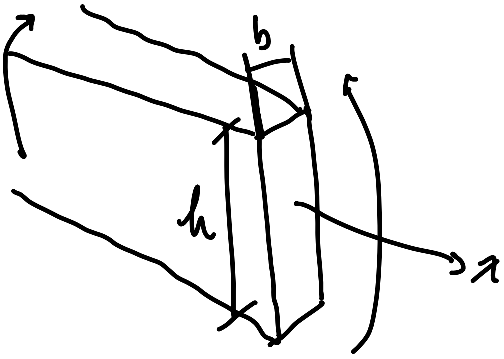


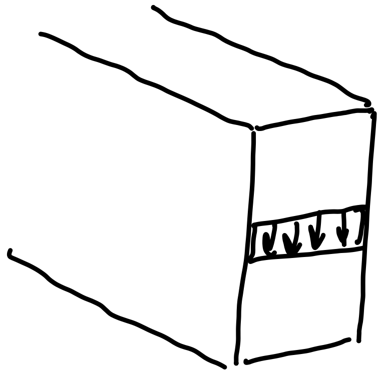


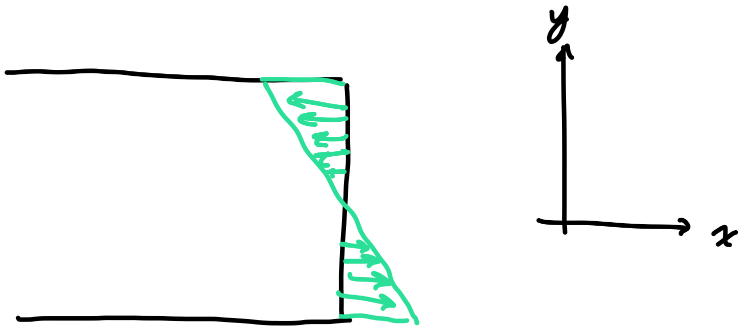
Concentraremo su trave a sezione rettangolare



Taglio è sempre associato a momento flettente







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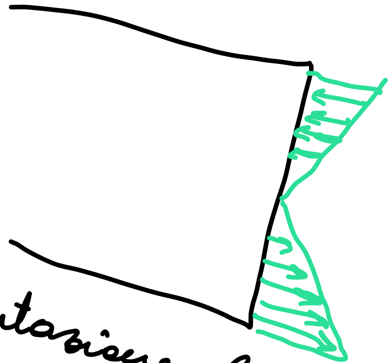
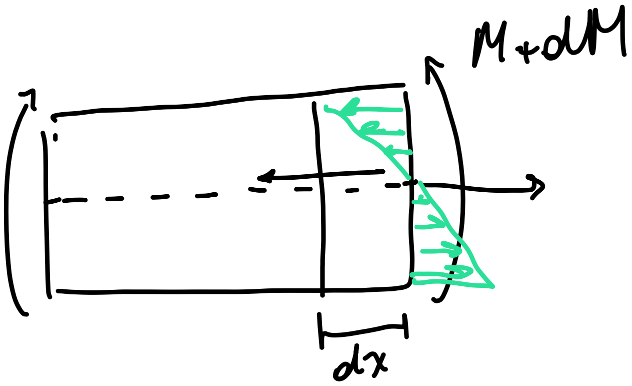


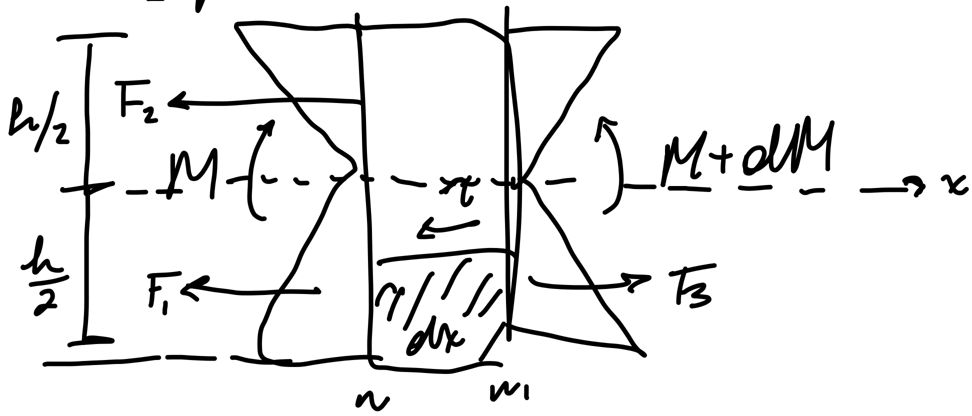
Rappresentazione diversa e basta

Per τ sulla faccia x-y

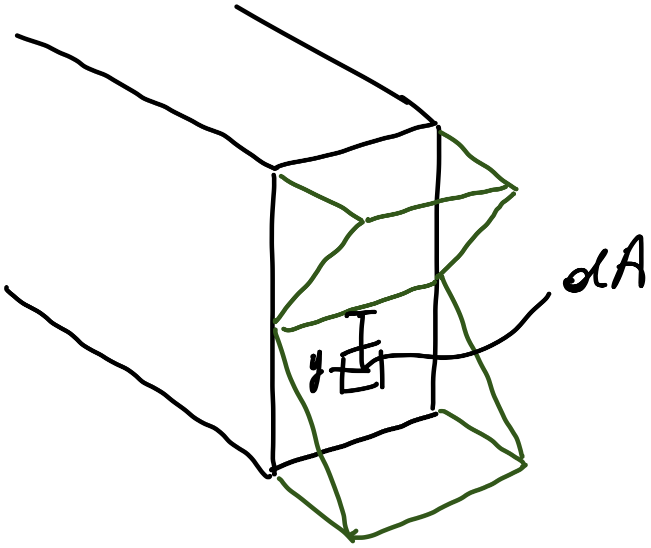
solo



Equilibrio lungo x



$$\sigma_x = \cancel{\frac{N}{A}}^{\,0} + \frac{M}{J}\, y \; dA$$



$$F_1 = \int_{A(y_1)} \frac{My}{J_{zz}}\, dA$$

$$F_3 = \int_{A(y_1)} \frac{(M+dM)\,y}{J_{zz}}\, dA$$

$$F_2 = \tau\, b\, dx$$

Facendo l'equilibrio

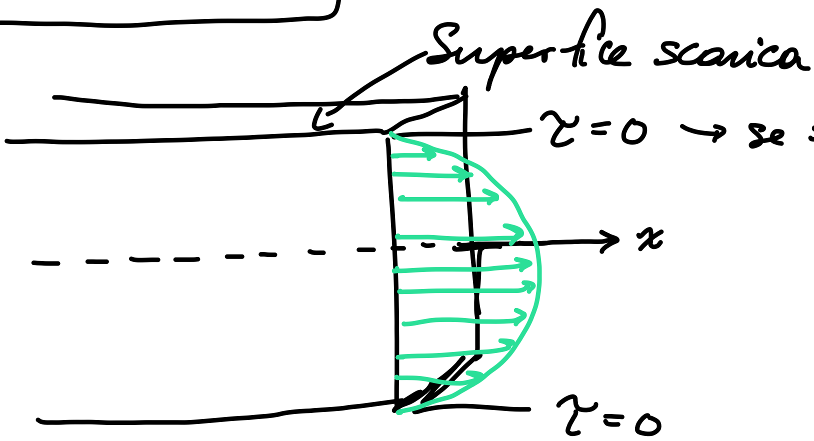
$$F_2 = F_3 - F_1$$

$$\tau\, b\, dx = \int_{A(y_1)} \frac{(M+dM)\,y}{J}\, dA - \int_{A(y_1)} \frac{My}{J}\, dA$$

$$\tau = \frac{dM}{dx} \left(\frac{1}{Jb}\right) \cdot S(y_1)$$

Formula di Jurawski

$$\boxed{\tau = \frac{T\, S(y_1)}{J_{zz} \cdot b}}$$



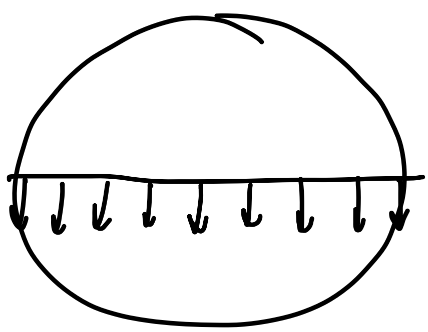
$\to$ se $\neq 0$, ci sarebbe esser tau bilanciante sulla superficie scarica

$$S = \underbrace{b\left(\frac{h}{2} - y_1\right)}_{\text{Area}} \underbrace{\left(y_1 + \frac{\frac{h}{2} - y_1}{2}\right)}_{\text{Braccia}} = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau_{media} = \frac{T}{A}$$

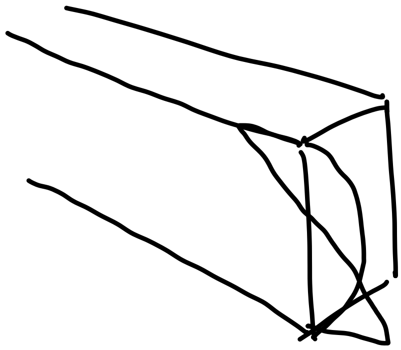
$$\tau_{MAX} = \frac{3}{2}\frac{T}{A}$$

Sezione Circolare



$$\tau_{MAX} = \frac{4}{3}\frac{T}{A}$$

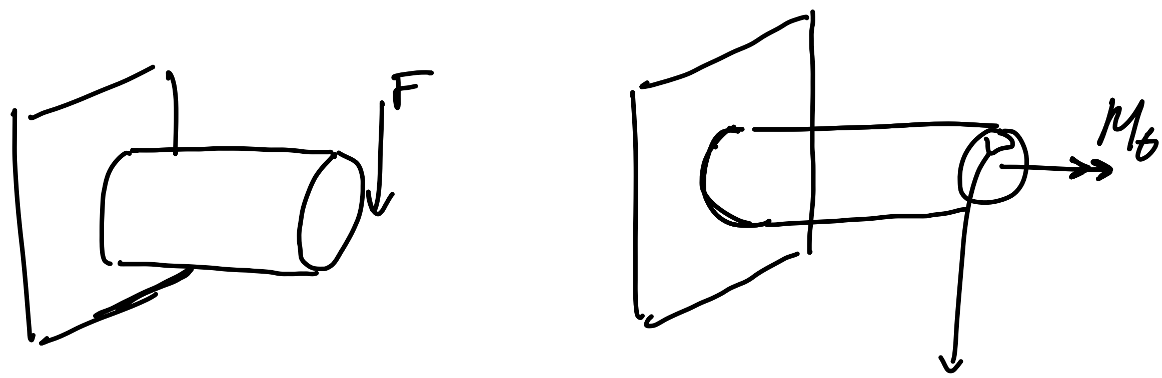
Consideriamo

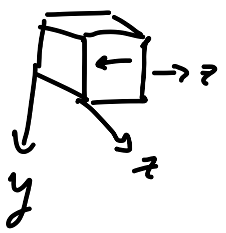


$\tau_{media} = \frac{T}{A}$

$\tau_{MAX} = \frac{3}{2}\frac{T}{A}$

Torsione - Caso di De Saint-Venant

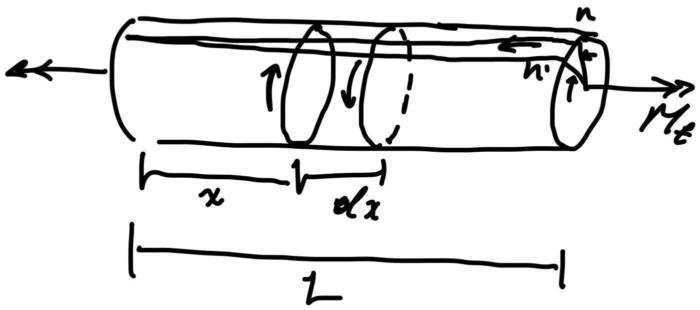


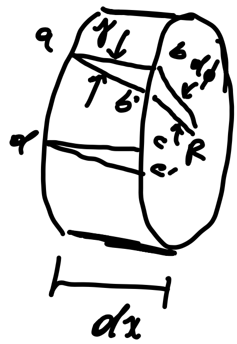


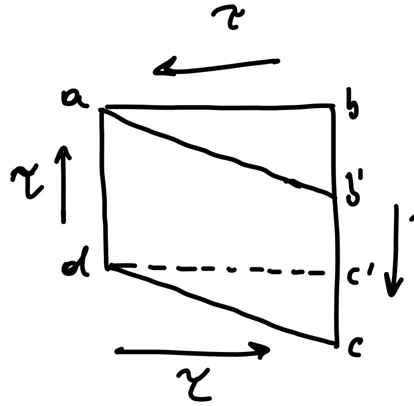
Superfici esterne sono scariche

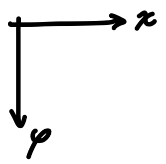
Con M_t:

- Le sezioni rimangono piane
- segmenti radiali rimangono radiali
-









$$\gamma\, dx = R\, d\varphi$$

$$\gamma = R \frac{d\varphi}{dx}$$

$\frac{d\varphi}{dx}$ angolo torsione unitario

Modulo di Rigidità

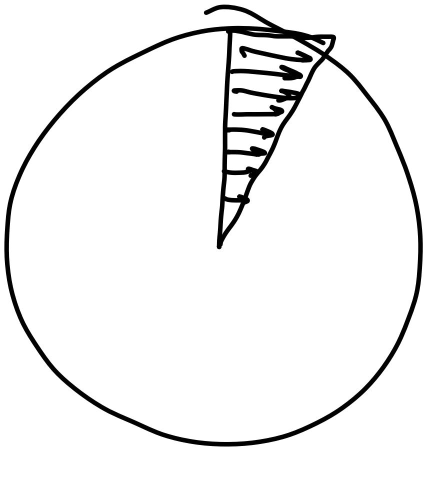
Costante posto

$$\tau = G\, \gamma = G R \phi$$

Costante

definisce τ, più lontano dal centro più alto

$$\gamma = r \phi \qquad \tau = G r \phi$$



$$M_t = \int_A \tau\, dA \cdot r = \int_A G r \phi r\, dA$$

$$= \int_A G\phi r^2 dA = G\phi \underbrace{\int_A r^2 dA}_{J_p} = G\phi J_p$$

$$M_t = G\phi J_p$$

$$\tau = G\phi r$$

$$\Rightarrow \tau = \frac{M_t r}{J_p}$$

$$\tau_{MAX} = \tau(r=R) = \frac{M_t \overset{2D}{R}}{\underset{\frac{\pi D^4}{32}}{J_p}} = \frac{16 M_t}{\pi D^3}$$

$$\phi = \frac{M_t}{G J_p}$$

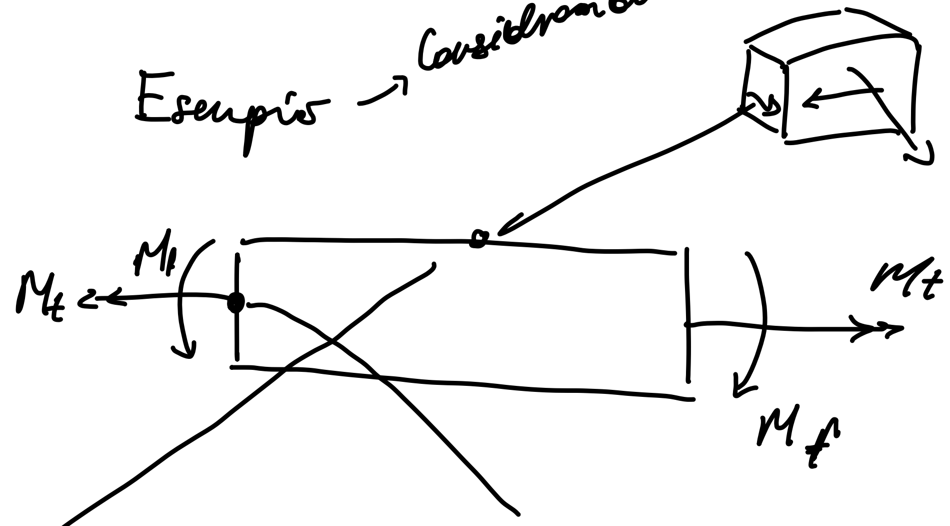
$$\varphi = \frac{M_t L}{G J_p}$$

rotazione

alle due sezioni estreme

$\frac{G J_p}{L}$ è detta rigidezza torsionale

Esempio → Considerazione che dobbiamo fare con M_t
↳ superficie esterna è scarica



$$\tau_z = \frac{16 M_t}{\pi d^3}$$

$$\sigma_f = \frac{32 M_f}{\pi d^3}$$

$$\tau_T = 0$$