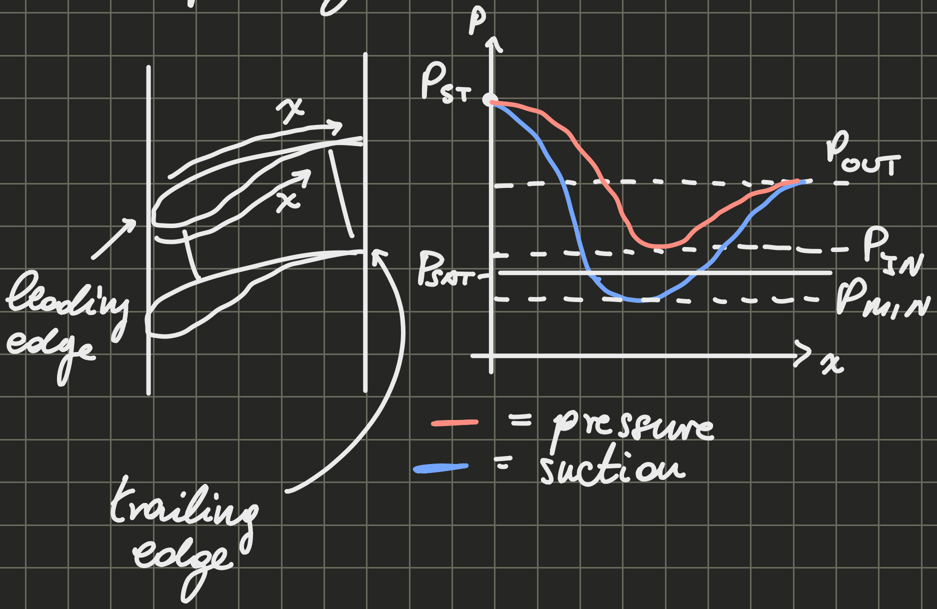
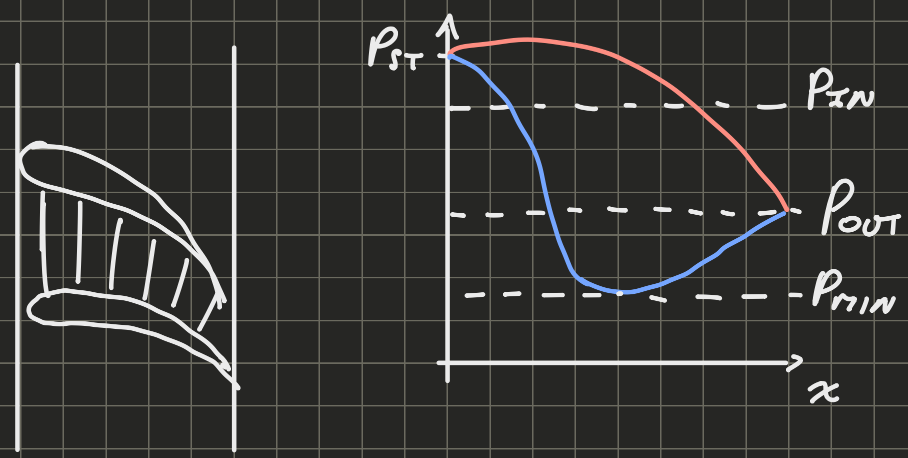
# Lezione 13 – Cavitation + Centrifugal Pumps

## Cavitation

### Operating Machine




### Motor Machine




To avoid cavitation we have to guarantee that the $P_{MIN}$ is higher the saturation of the fluid at the temperature, and the pressure of the dissolved gasses.

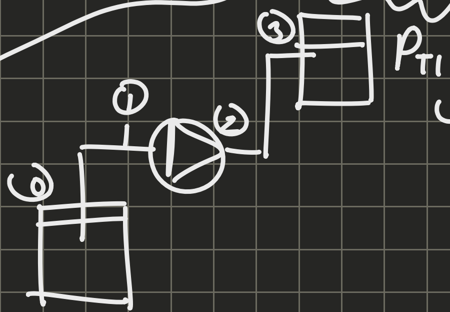
$$P_{MIN} > P_{SAT}(T) + P_{DIS}$$

↳ Pressure of dissolved gasses.

Since the graphs cannot be provided due to legal reasons, we introduce a quantity so a buyer does not just have to hope the machine works.

Net Positive Suction Head → NPSH
↳ measure $P_{MIN}$ in a way that is general

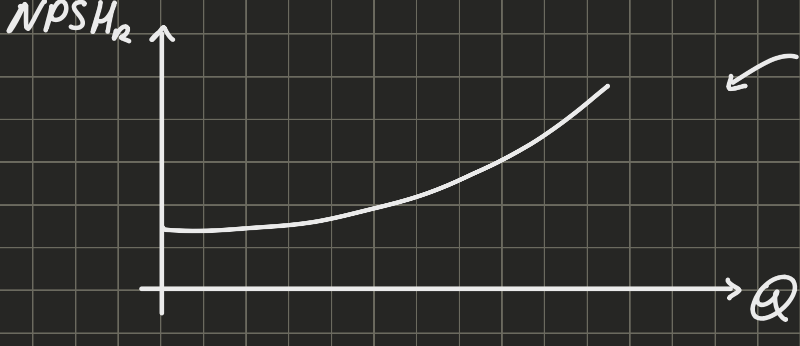
Pumps: $NPSH_R = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{P_{MIN}}{\rho g}$ $\qquad NPSH_R$ = NPSH required




↳ Easy to evaluate since it's plant dependent.

$P_{ST}$ is not $P_{T1}$, since the one we find is where the $\vec{w} = 0$, not $\vec{V} = 0$, therefore it is $P_{ST} = P_1 + \frac{1}{2}\rho w_1^2$

This is useful since it gives us the $\Delta$, so if for some reason $P_0$ and $P_1$ increase, the value does not change.




for a given $\bar{n}$ and $\bar{D}$

$NPSH_A$ → NPSH Available
↳ User can control

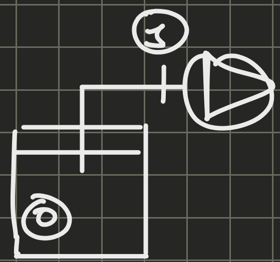
$$NPSH_A = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{P_{SAT} - P_{DIS}}{\rho g}$$

It is sufficient to guarantee that $NPSH_A > NPSH_R$, to say that cavitation will not occur.

Since we get $P_{SAT} + P_{DIS} < P_{MIN}$ → what we said before

## Cavitation Assessment (how to go in practice $NPSH_A > NPSH_R$)

$$NPSH_A = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} - \frac{P_{SAT} - P_{DIS}}{\rho g}$$




It's advantageous to place the pump over the reservoir for maintenance and other access reasons.

Bernulli

$$BME\ 0 \to 1: \cancel{\ell} - \cancel{\ell_w} - y_s = \Pi_1 - \Pi_0$$

$$-y_s = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + g z - \frac{p_0}{\rho g} - \cancel{\frac{v_0}{2g}} - g z_0$$

$$-\frac{y_s}{g} = \frac{p_1}{\rho} + \frac{v_1^2}{2} - \frac{p_0}{\rho} + g\overbrace{(z_1 - z_0)}^{h}$$

$$\Rightarrow \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_0}{\rho g} - h - \frac{y_s}{g}$$

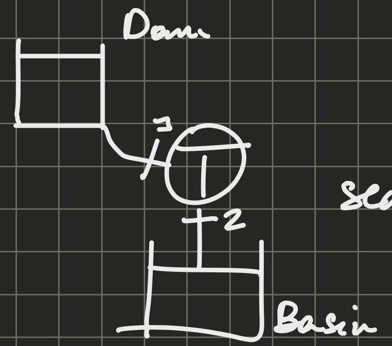
$$= \frac{p_0}{\rho g} - h - \frac{y_s}{g} - \frac{P_{SAT} + P_{DIS}}{\rho g} = \frac{p_0}{\rho g} - h - \xi_s \frac{v_s^2}{2g} - \frac{P_{SAT} + P_{DIS}}{\rho g}$$

consequence of the advantages of elevating the pump.

If $P_0 = P_{ATM} \to h_{MAX} \approx 9{,}81\,m$ − losses

In some cases $h < 0$, since we need to be able to have an $NPSH_A$ such that there is no cavitation.

# Turbines




For turbines it was chosen to use section 2 instead of 1.

For the $NPSH_A$ and $NPSH_R$ the equations are the but the numbers are 2 instead of 1.

## NPSH and similarity

↳ In standard conditions, it is a kinematic quantity, it's like the head.
It's not a surprise

$$NPSH_R = NPSH_R(Q, n, D, SHAPE)$$

looking for a dimensionless parameter, we can work, like the head and work.

$$\sigma = g\frac{NPSH}{n^2 D^2} \rightarrow \sigma = \sigma(\varphi, SHAPE)$$

↳ Thoma's Parameter.

↳ (actual definition : $\sigma = \frac{NPSH_R}{H}$ → useful practically but confusing for students)

→ it is easier for us to use, since

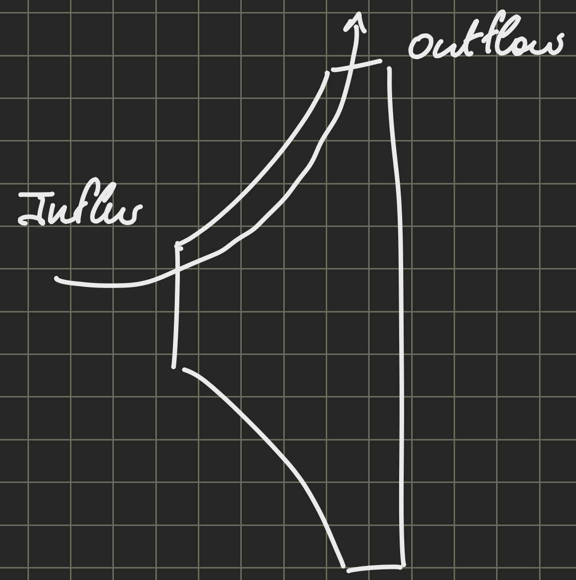
it tells us that if $n' = 2n$, when $NPSH_R' = 4 NPSH_R$

## Experimental Data for Determining the $NPSH_R$

The $NPSH_R$ is not when the head plateaus but when cavitation is visible.

Pumps → Simple, Easy and Effective once we avoid cavitation.

Centrifugal Pumpe. → typically shrouded




The spiral volute brings the flow from all sides to the outlet duct.

Since some flow wants to return, so we introduce a labirith seal.

There will alwy be leaks which induce losses, so we try to seal them.

⇒ constructive blade angles and other aspcts

Curve of a machine

$H = H(Q)$ for a centrifugal pump → characteristic curve

(Given $n$, $D$, SHAPE)

$$H = \frac{l - l_w}{g}$$

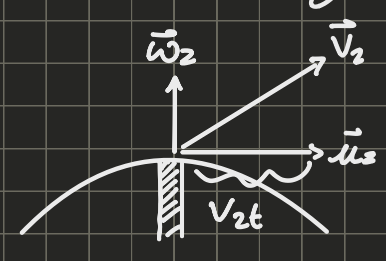
because from duct

$$l(Q) = u_2 v_{2t} - \underbrace{u_1 v_{1t}}_{\to 0} = U_2 (w_{2t} + u_2) \overset{!}{=} u_2 \left( \frac{Q}{\pi D_2 b_2} \tan \beta_2 + u_2 \right)$$

$l_w(Q)$ → general consideration

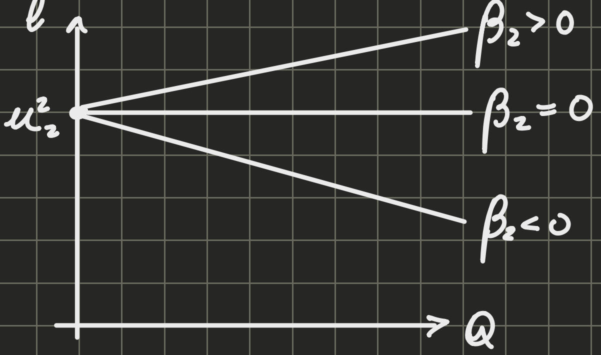
what we did was for a general machine

If $\beta_2 = 0$ ⇒



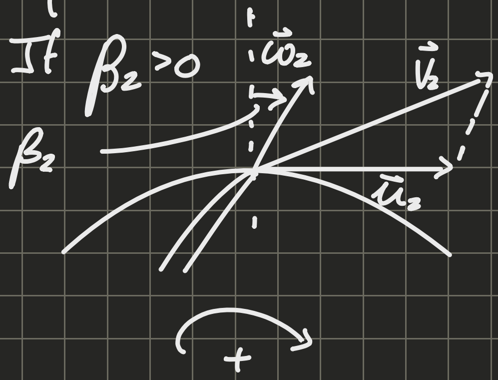

⇒ for $\beta_2 = 0 \to l = u_2^2$

↳ is independent of




forward blades $Q$

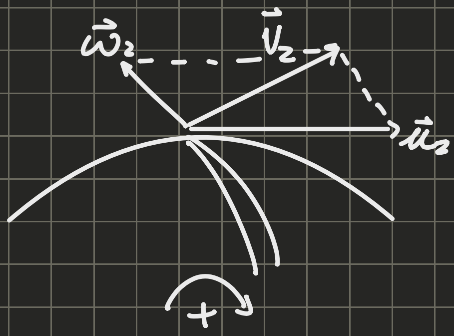
If $\beta_2>0$




$v_{2t} > u_2$

$Q\uparrow \rightarrow w_2\uparrow \rightarrow v_2\uparrow \rightarrow v_{2t}\uparrow$

If $\beta_2<0$




$v_{2t} < u_2$

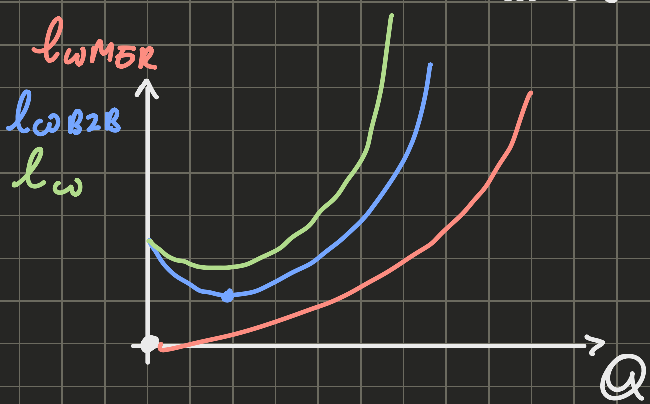
$Q\uparrow \rightarrow w_2\uparrow \rightarrow v_2\downarrow \rightarrow v_{2t}\downarrow$

To answer what the best $\beta_2$ is for our case, we need to quality the $l_w(Q)$

$$l_w = l_{w\,MER} + l_{w\,B2B}$$

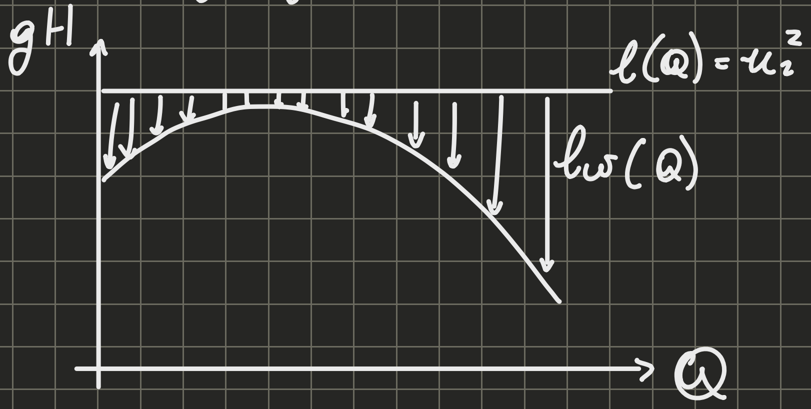
$l_{w\,MER}$: losses in the periodical view

$l_{w\,B2B}$: aerodynamic losses with interaction with blades

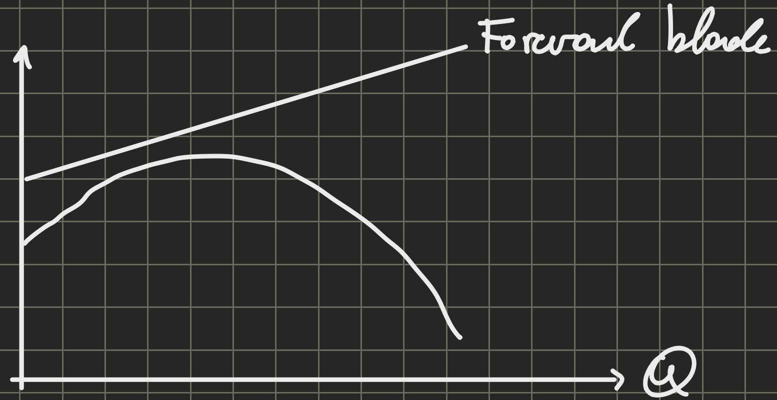


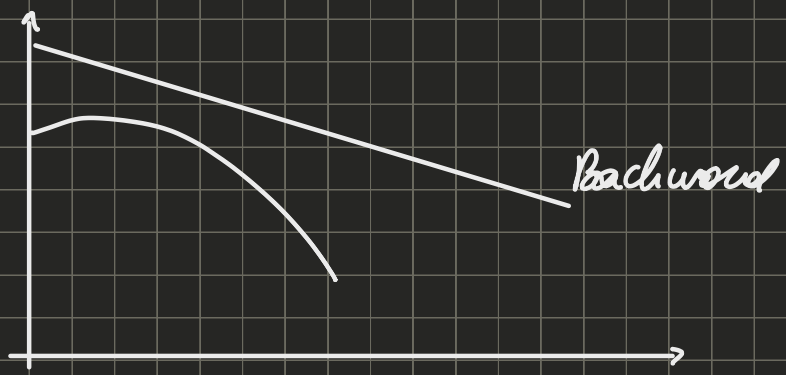

$l_w$ can be interpolated by a parabola with positive concavity.

Putting together

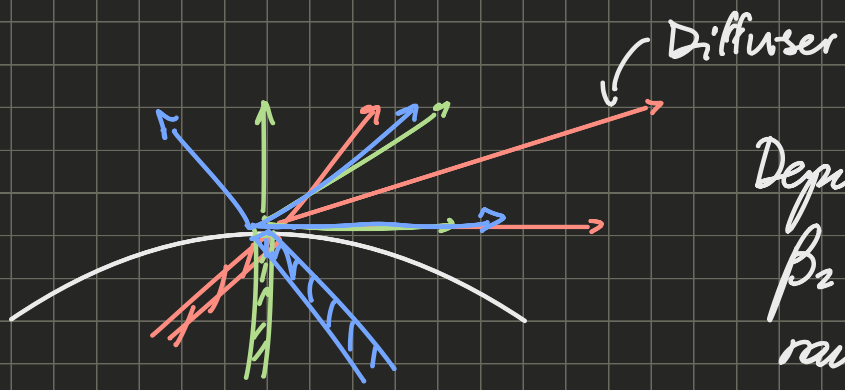



The curve is like many of the curves we have seen since they are arriving from the same function




Is there a relationship between $\ell w$ and $\beta_2$ | design of the blade ?




Depending on the $\beta_2$ we can have a range of possible $v_2$.

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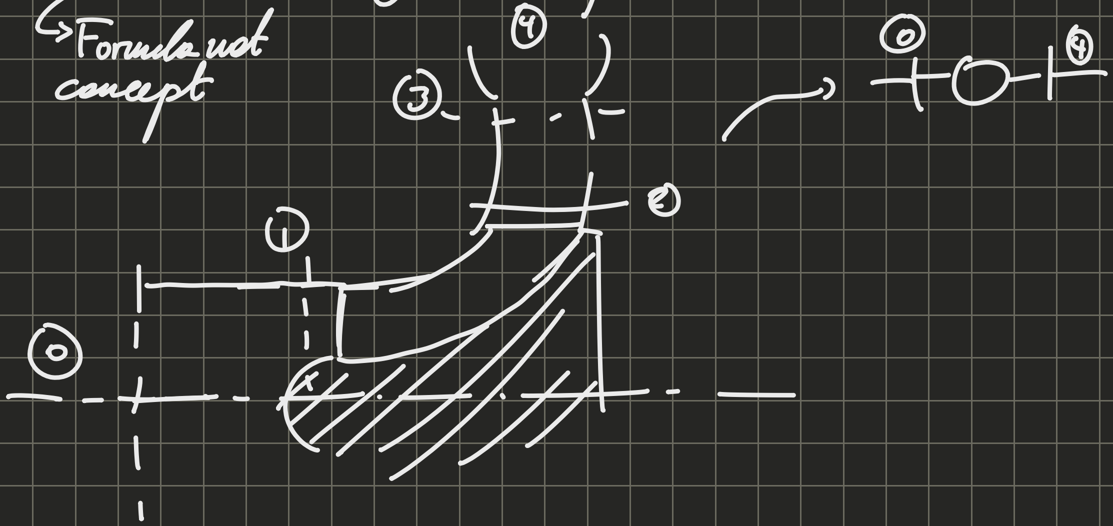
To be able to increase pressure we need to decellerate, the designs that have lower $V_2$ are used more, since they are easier to use. To differ which has higher $V_2$, are less used since they are less efficient and difficult to optimize.

Backward blades are more efficient.

We need a parameter to formalize whether the head becomes mostly pressure or mostly speed, and how this is related to $\beta_2$.

Reaction Degree → part of head which becomes pressure

↳ Formula not concept




$$\chi = \frac{P_2-P_1}{\rho g H} = \frac{gH - \frac{V_2^2-V_1^2}{2}}{gH} = 1 - \frac{1}{2}\frac{V_2^2-V_1^2}{gH} = 1 - \frac{1}{2}\frac{V_2^2-V_1^2}{U_2 V_{2t}} = *$$

chi

Assumptions:

↳ if $l_w = 0 \Rightarrow \begin{cases} gH = l = U_2 V_{2t} \\ gH = T_4 - T_0 = T_2 - T_1 = \frac{P_2-P_1}{\rho} + \frac{V_2^2-V_1^2}{2} \end{cases}$

since in no $l$ is transferred in other parts, this can be written

since head is across entire machine

$\boxed{gH = l - l_w = \Delta T}$

Always remember

$$\Rightarrow \frac{P_2-P_1}{\rho} = gH - \frac{V_2^2-V_1^2}{2}$$

↳ If $V_m = \text{const} \Rightarrow V_2^2 - V_1^2 = (V_{2t}^2 + \cancel{V_{2m}^2}) - \cancel{V_{1m}^2}$

$V_{1t} = 0$ so it does not appear

head

$* \Rightarrow \chi = 1 - \frac{1}{2}\frac{V_{2t}}{u_2}$ → $\beta_2 = 0 \rightarrow \chi = 0,5 \Rightarrow$ half of $Y$ will become increase in pressure and half kinetic energy.

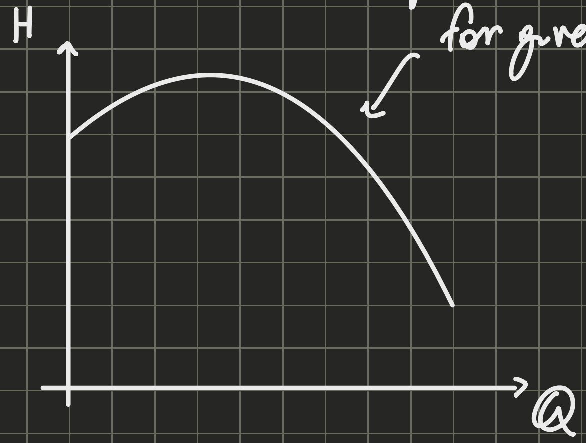
$\beta_2 > 0 \rightarrow \chi < 0,5$ → more head becomes kinetic energy

$\beta_2 < 0 \rightarrow \chi > 0,5$ → most head becomes increase in pressure.

For $\beta_2 > 0$, our impeller can do as much as it can to increase the pressure so we reduce the work on the diffuser.

We have seen that H is parabolic, and can be described by an equation like

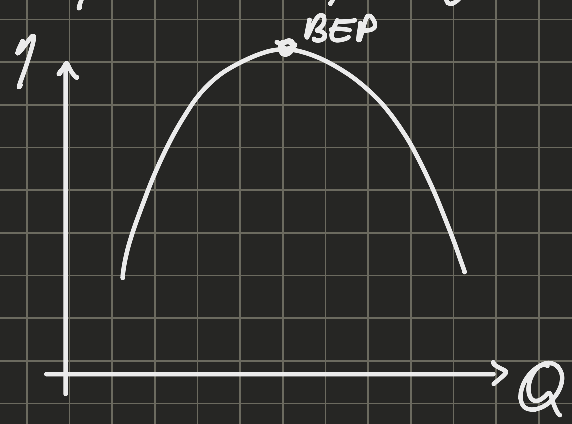
for given $\bar{n}, \bar{D}$, SHAPE




$$H = H(Q) = AQ^2 + BQ + C \quad (*)_2$$

↳ In exercises we write

$\eta = \eta(Q) \rightarrow \eta = \frac{gH}{l}$ ⇒ we can get it's parabola




$$\eta = \eta(Q) = DQ^2 + EQ + F$$

↳ describing parabola

If I know, SHAPE, D and n, and we have the H-Q curve, we will be able to find the dimensionless form and be able to find the curve for all machines of the family, not just that one machine.

dimensionless

$$H = H(Q, n, D, SHAPE) \rightarrow \underline{n = \bar{n},\ D = \bar{D}}$$

Through similarity we can make the dimensionless and redimensionalise | $H = H(Q)$ and $\eta = \eta(Q)$ $(*)_3$ as a function of n and D.

A, B, C, D, E, F are functions of $n$ and $D$ and we will use this,
(in exercises we are given values, without specifying the dependence to n and D)

$$\psi = \frac{gH}{n^2D^2}, \; \varphi = \frac{Q}{nD^3}$$

$$H = \frac{\psi n^2 D^2}{g}, \; Q = n D^3 \varphi$$

$\rightarrow$ at the specific $n = \bar{n}$ and $D = \bar{D}$ for the machine

$$H = \frac{\psi}{g}\bar{n}^2\bar{D}^2, \; Q = \varphi \bar{n}\bar{D}^3$$

these are what we are plotting, they are the specific values for our machine.

(*)$_2$ plugging back in:

$$\frac{\psi}{g}\bar{n}^2\bar{D}^2 = A\bar{n}^2\bar{D}^6\varphi^2 + B\bar{n}\bar{D}^3\varphi + C \Rightarrow \psi = \underbrace{gA\bar{D}^4}_{\alpha}\varphi^2 + \underbrace{gB\frac{\bar{D}}{\bar{n}}}_{\beta}\varphi + \underbrace{\frac{gC}{\bar{n}\bar{D}}}_{\gamma}$$

$$\psi = \alpha\varphi^2 + \beta\varphi + \gamma$$

A, B and C have to cancel out the units of the other machines

$\alpha$, $\beta$ and $\gamma$ are dimensionless coefficients of the shape

A, B, C hide their relationship to the selected $\bar{n}$ and $\bar{D}$

(*)$_3$ we can retrieve the general $H(Q)$ and $\eta(Q)$ from one specific case, by making them dimensionless and dimensionalizing for general values of $n$ and $D$