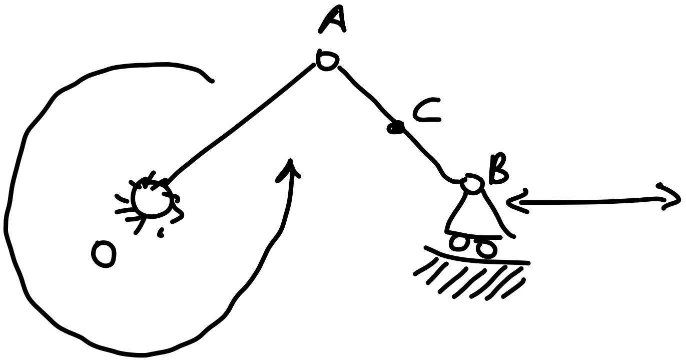
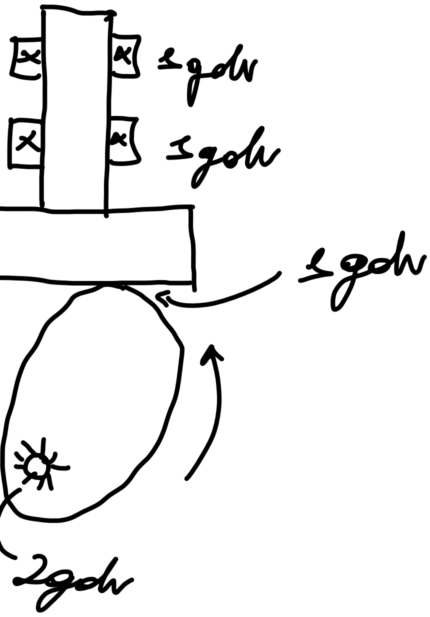
# Esercitazione 1 -

## Panoramica




La traiettoria di C sarà parte rettilinea e in circolare




Vogliamo acquisire tecniche per studiare cinematismi.

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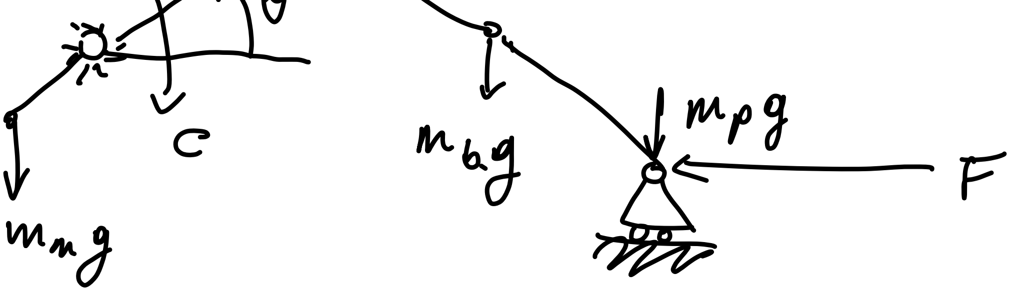
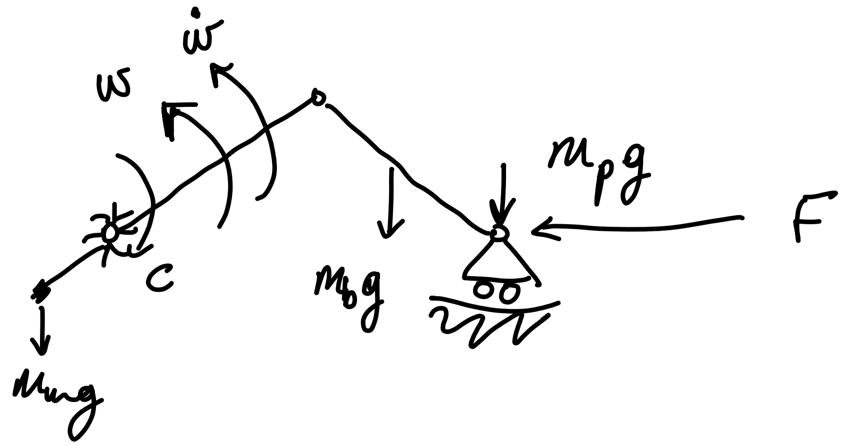


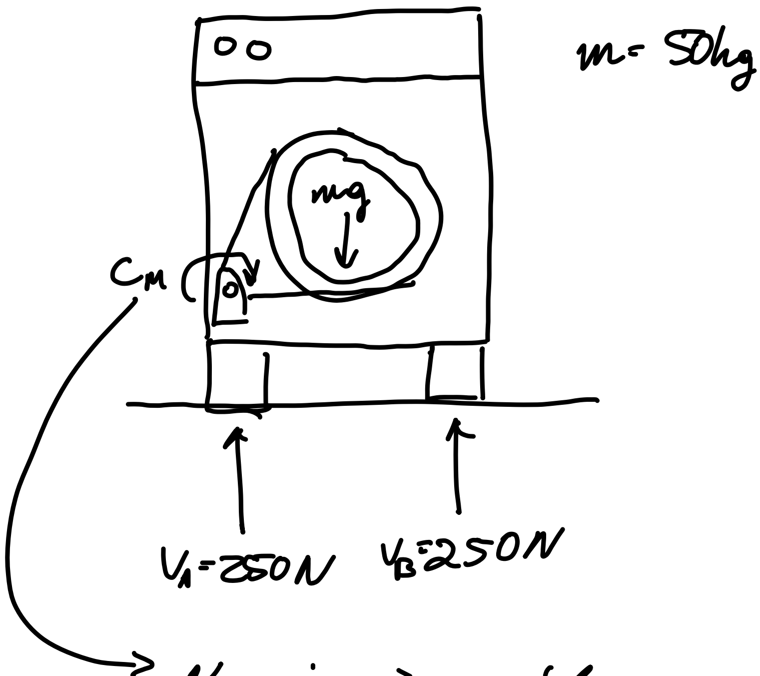

DATI $m, F, \theta \Rightarrow C$

o DATI $m, F, C \Rightarrow \theta$

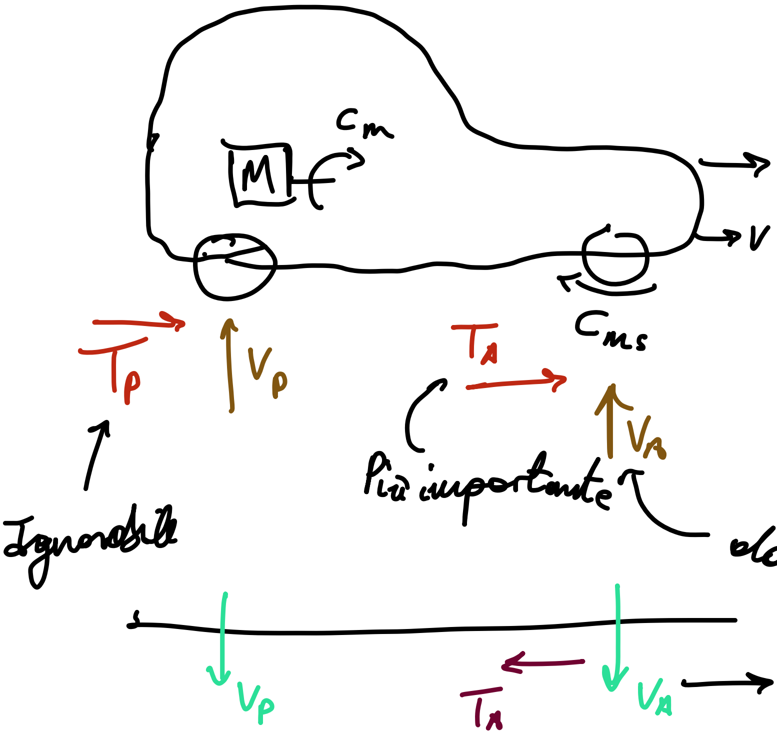
# Principio Lavori Virtuali




E.g. Lavatrice




Non si può considerare perchè è interna




$C_m > C_{ms}$

Potenza

$C_m \omega_m > C_{ms}\, \omega_r$

$\omega_r$ → ruote

Dipende potenza non la coppia

dal terreno a auto

dalla auto su terreno

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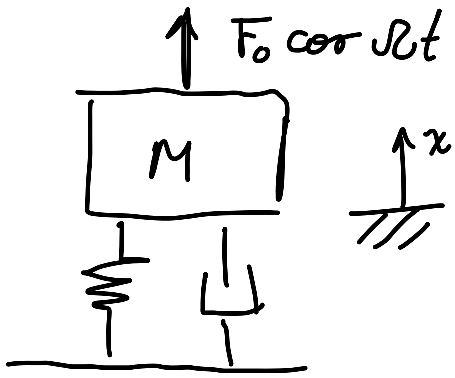


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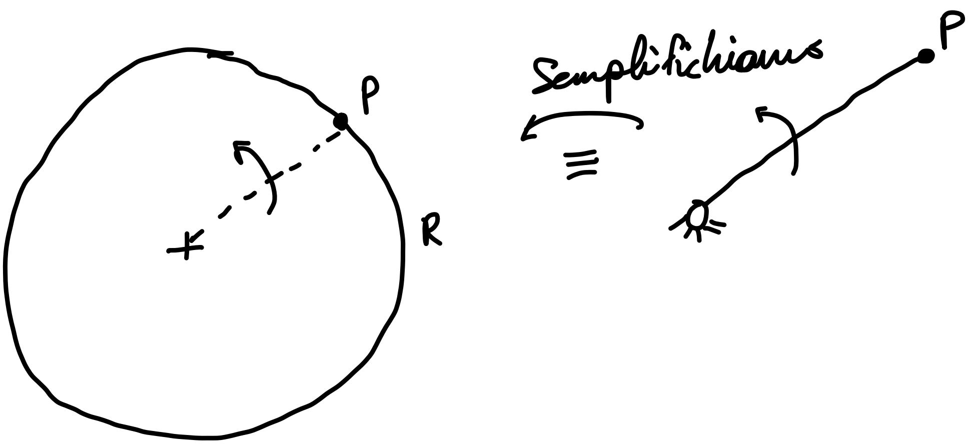
## Moto Circolare



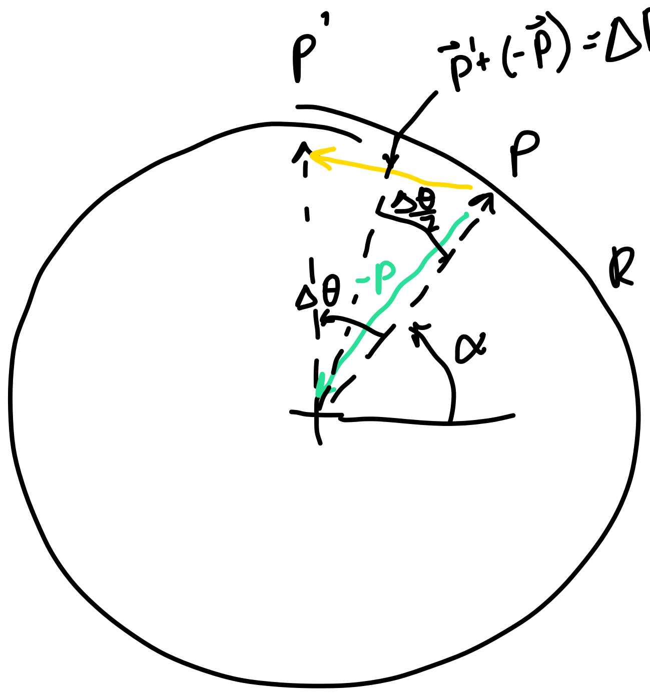
$$\theta = [rad] \circ [grad]$$

$$\dot{\theta} = \omega = \left[\frac{rad}{s}\right] \circ \left[\frac{giri}{min}\right]$$

$$\omega = \frac{10 \text{ giri}}{min}$$

3000 giri/min
1500 giri/min
750 giri/min
} Motori Tipici

Inverter → cambia la frequenza

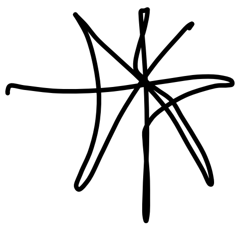



$$V_p = \lim_{\Delta t \to 0} \frac{\vec{P}' - \vec{P}}{\Delta t}$$

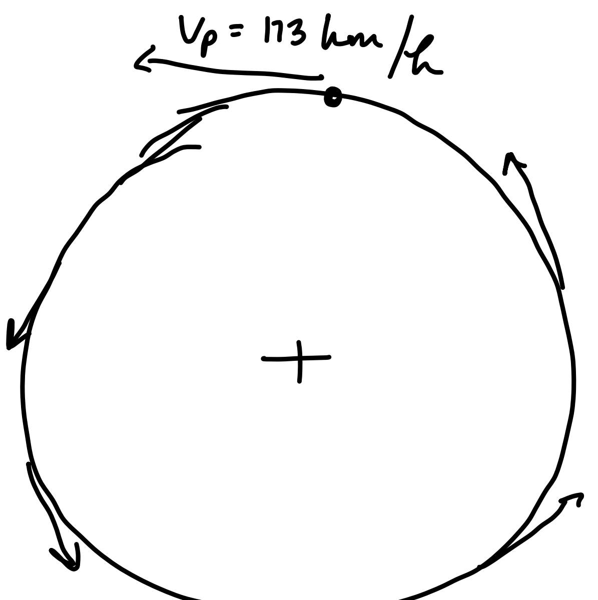
$$= \lim_{\Delta t \to 0} \frac{\vec{P}' + (-\vec{P})}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{2R \sin\frac{\Delta\theta}{2}}{\Delta t}$$



$$= \frac{2R\frac{\Delta\theta}{2}}{\Delta t} = R\frac{\Delta\theta}{\Delta t} = R\omega$$



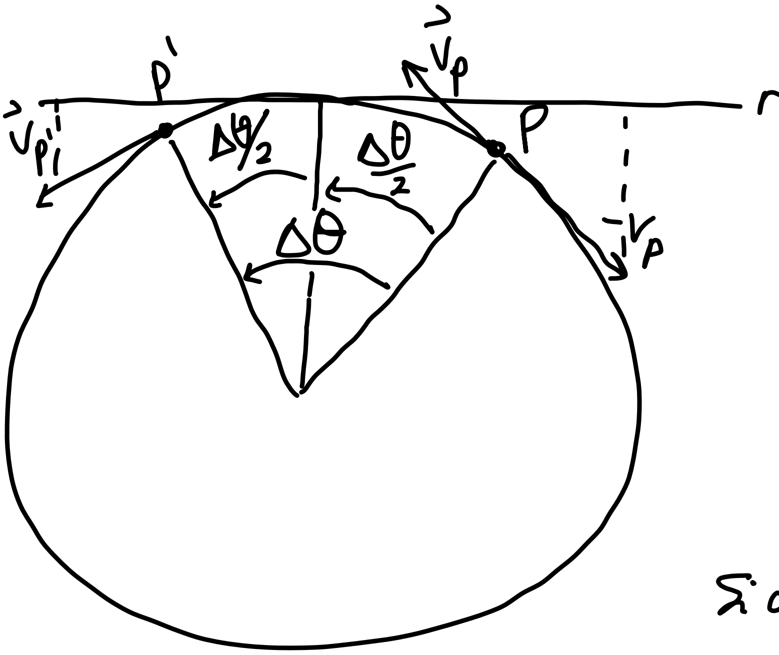

Dato:

$$\omega = 1200 \frac{giri}{min}$$

$$V_p = R\omega = .25 \cdot 1000 \cdot \frac{2\pi}{60}$$

$= 31{,}4\ m/s$

$= 113\ km/h$




$$\vec{a}_n = \lim_{\Delta t \to 0} \frac{\vec{v}_p' - \vec{v}_p}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\vec{v}_{p'} + (-\vec{v}_p)}{\Delta t} =$$

Si cancellano le componenti che sono proiettabili su r e si sommano quelle verso il centro

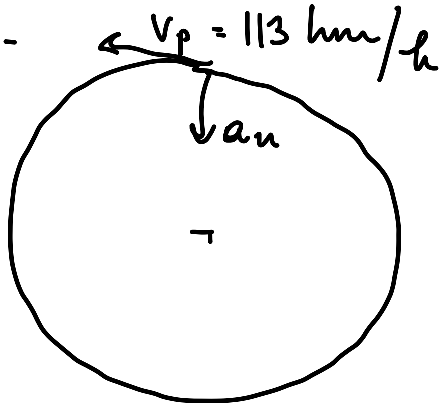
Perchè si sommano

$$= \lim_{\Delta t \to 0} \frac{2 R W \sin \frac{\Delta\theta}{2}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\cancel{2} R w \frac{\Delta\theta}{\cancel{2}}}{\Delta t} = R w \frac{\Delta\theta}{\Delta t} = R w^2$$

$$\boxed{\vec{a}_n = R w^2} = \frac{R^2 w^2}{R} = \boxed{\frac{v_p^2}{R}}$$

e.g. -

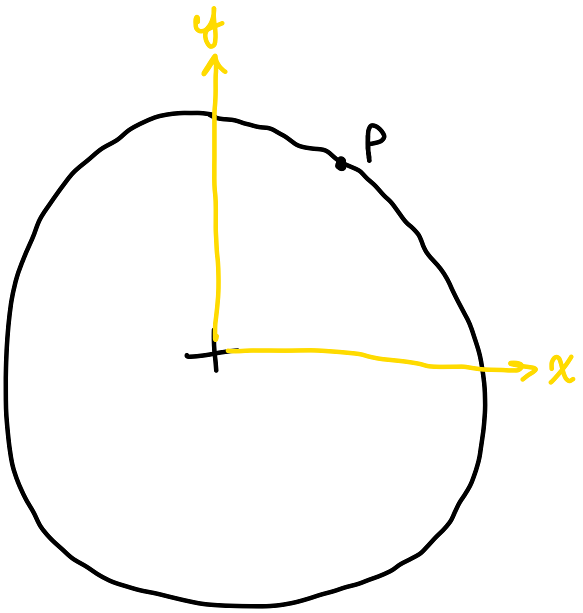


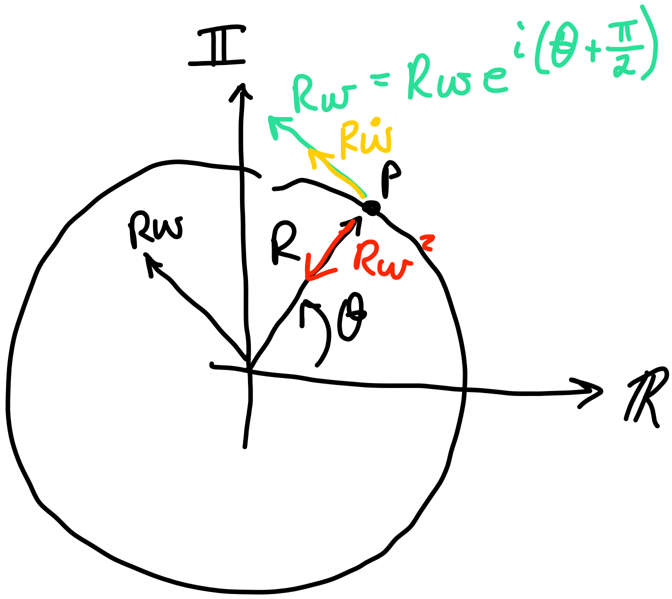

$R = 0{,}25\,m$

$w = 1200$ giri/min

$$a_n = w^2 R = \left(\frac{1200 \cdot 2\pi}{60}\right)^2 \cdot 25 =$$

$= 3944\ m/s^2 = 579\ g$




SISTEMA
OXY

↓

SISTEMA
$OR\mathbb{I}$

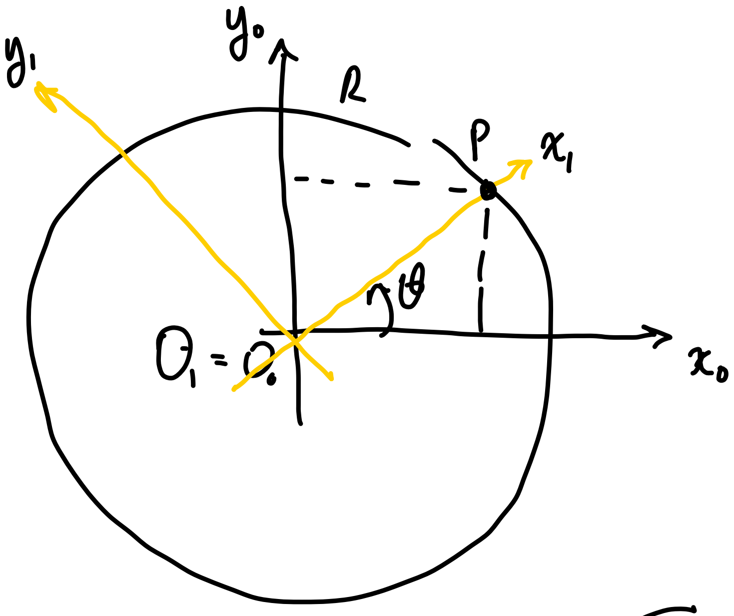
$S_p = Re^{i\theta}$

$$V_p = \frac{ds_p}{dt} = Ri\dot{\theta}e^{i\theta} = R\omega e^{i(\theta+\frac{\pi}{2})}$$

$$(P-O) = x_p\hat{i} + y_p\hat{j}$$

$$\vec{a}_p = \frac{dv_p}{dt} = R\dot{\omega}e^{i(\theta+\frac{\pi}{2})} + R\omega i\dot{\theta}e^{i(\theta+\frac{\pi}{2})} = \underbrace{R\dot{\omega}e^{i(\theta+\frac{\pi}{2})}}_{a_t} + \underbrace{R\omega^2 e^{i(\theta+\pi)}}_{a_n}$$

# Altro modo di risolvere




sistemi riferimento

- Fisso: $O, x_0, y_0$
- Rotante con P: $O_1, x_1, y_1$

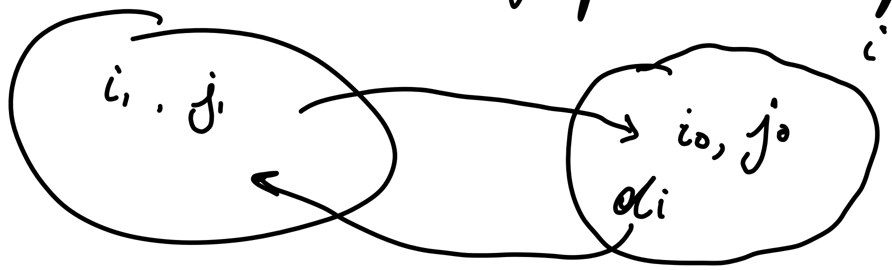
$$(P-O) = R\cos\theta\, i_0 + R\sin\theta\, j_0$$

$$(P-O_1) = R\,\hat{i}_1$$

Perché non cambia mai

$$\vec{v}_P = \frac{d(P-O_1)}{dt} = \cancel{\frac{dR}{dt}}\,\hat{i}_1 + R\,\frac{d\hat{i}_1}{dt}$$

Ci portiamo in $i_0, j_0$ per derivare per semplificare i conti




$$\hat{i}_1 = |\hat{i}_1|\cos\theta\,\hat{i}_0 + |\hat{i}_1|\sin\theta\,\hat{j}_0 = \cos\theta\,\hat{i}_0 + \sin\theta\, j_0$$

$$\hat{j}_1 = |\hat{j}_1|\sin\theta\,\hat{i}_0 + |\hat{j}_1|\cos\theta\,\hat{j}_0 = -\sin\theta\,\hat{i}_0 + \cos\theta\, j_0$$

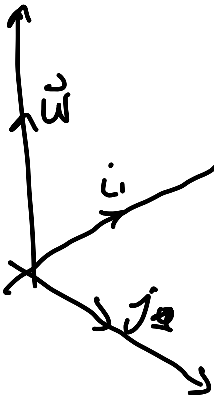
$$\frac{d\hat{i}_1}{dt} = \frac{d}{dt}\left(\cos\theta\,\hat{i}_0 + \sin\theta\,\hat{j}_0\right) = -\dot{\theta}\sin\theta\,\hat{i}_0 + \dot{\theta}\cos\theta\,\hat{j}_0 + \overbrace{\cancel{\cos\theta\,\frac{di_0}{dt}}}^{=0}$$

$$+ \sin\theta \cancel{\frac{dj_0}{dt}} \quad {=0}$$

$$= \left(-\sin\theta\, \hat{i}_0 + \cos\theta\, \hat{j}_0\right)\dot{\theta} = \omega \hat{j}_1 = \vec{\omega} \times \hat{i}_1$$

già sufficiente

ma utile per interpretazione fisica




$$\Rightarrow \vec{v}_P = R\frac{d\hat{i}_1}{dt} = R\,\omega\,\hat{j}_1 = R\left(\vec{\omega} \times \hat{i}_1\right)$$

**Formule Poisson**

$$\frac{d\hat{i}_1}{dt} = \vec{\omega} \times \hat{i}_1 \qquad \frac{d\hat{j}_1}{dt} = \vec{\omega} \times \hat{j}_1$$