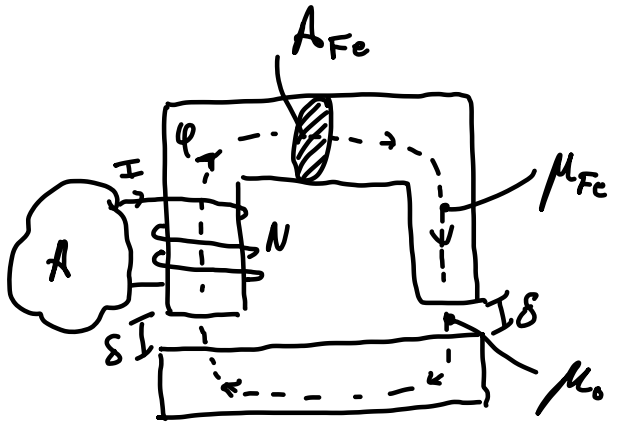
Esercitazione 9 - Circuiti Magnetici



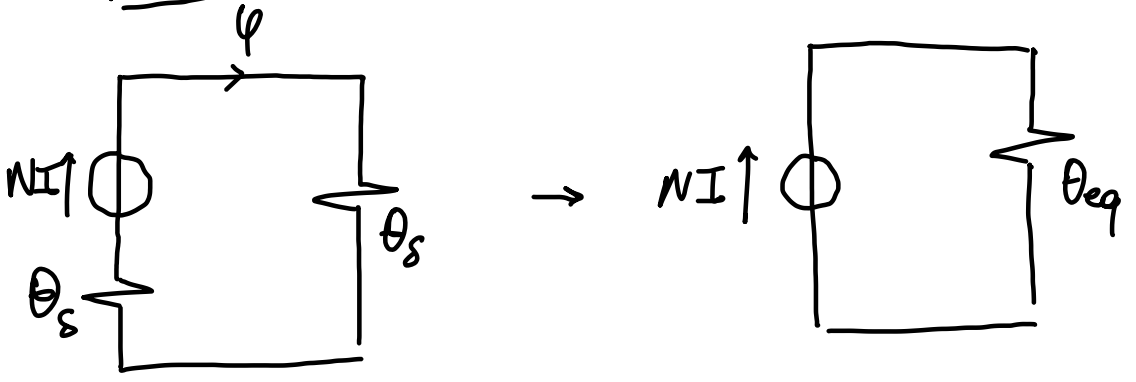
Tipi di Problema:

1. Calcolo L di A equivalente al circuito
2. Calcolo delle Forze

Condizione per i nostri problemi

$\mu_{Fe} \to \infty \quad \mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$

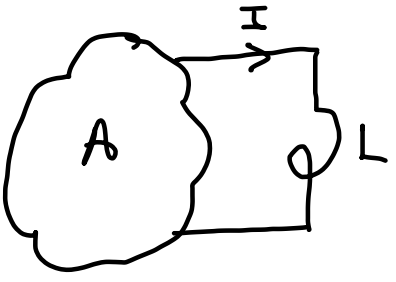
Problema Induttanza



$$\theta_\delta = \frac{\delta}{\mu_0 A_{Fe}}$$

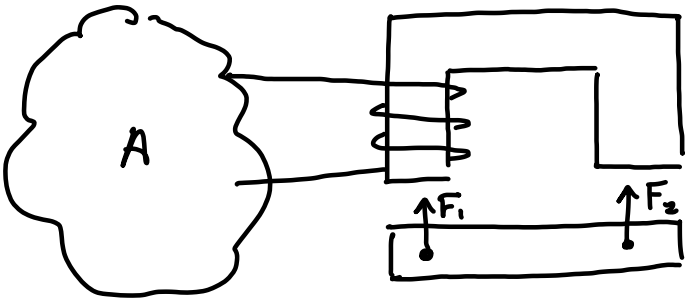
Induttanza vista dal circuito A

$$L = \frac{\Psi}{I} = \frac{N\varphi}{\frac{NI}{N}} = \frac{N^2}{\theta_{eq}}$$



$$W_L = \frac{1}{2} L I^2$$

Problema Forze

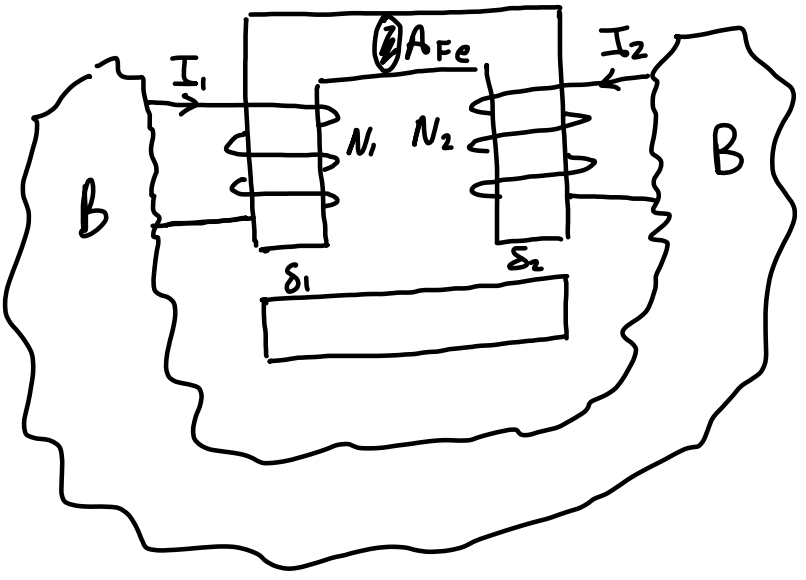


Formula Generale:

$$F = \frac{\varphi^2}{2\mu_0 A_{Fe}}$$

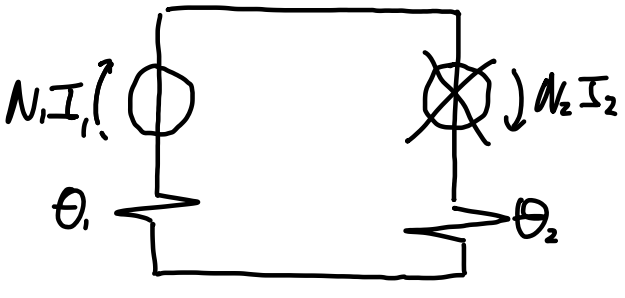
$$F_1 = \frac{\varphi_1^2}{2\mu_0 A_{Fe}} \qquad F_2 = \frac{\varphi_2^2}{2\mu_0 A_{Fe}}$$

Problemi con più avvolgimenti / bobine



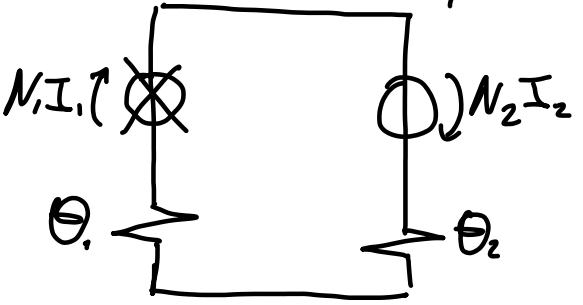
Calcolo di L:

L_1

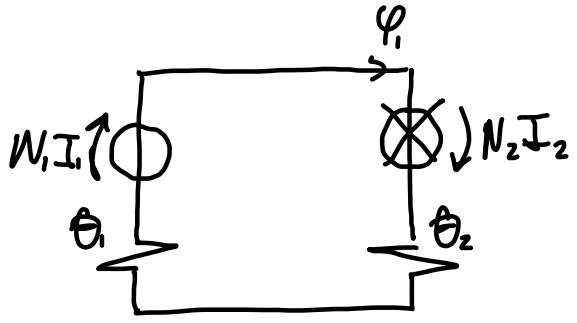


$L = N_1^2 / \Theta_{eq}$

$$L_2 = \frac{N_2^2}{\Theta_{eq}}$$



Mutue Induttanze



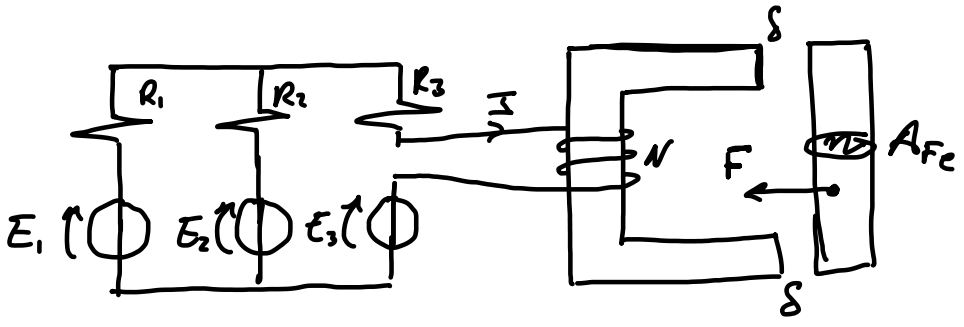
$$L_m = \left.\frac{N_2 \varphi_2}{I_1}\right|_{I_2=0} = \left.\frac{N_1 \varphi_1}{I_2}\right|_{I_1=0}$$

↳ È possibile trovare L_m senza sapere I_1, verrà coperto in un problema

$$W_L = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + L_m I_1 I_2$$

↳ energia accumulata (sempre +)

Esercizio 1



$E_1 = 18V$ $R_1 = 5\Omega$

$E_2 = 26V$ $R_2 = 8\Omega$

$E_3 = 34V$ $R_3 = 12\Omega$

$N = 400$

$\delta = 1mm$

$A_{Fe} = 100\,cm^3$

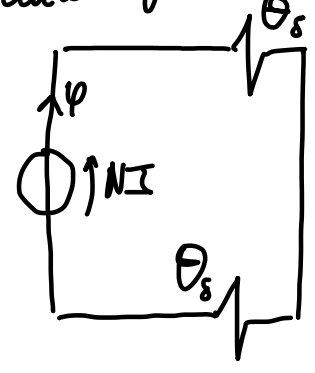
$\mu_{Fe} = \infty$

$\mu_0 = 4\pi \times 10^{-7}$

W_L? F = ?

Risolvendo il sistema magnetico prima risulta in meno errori

Circuito Magnetico:



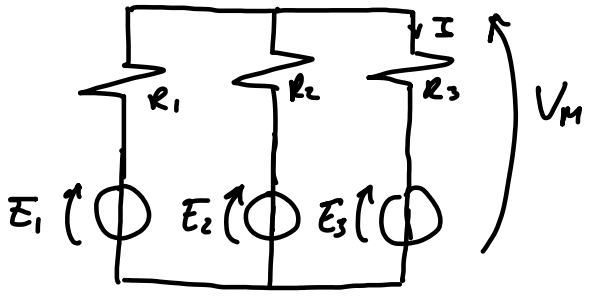
$$\theta_{eq} = 2\theta_\delta = \frac{2\delta}{\mu_0 A_{Fe}} = 1591549\,H$$

$\delta = 1 \cdot 10^{-3}\,m$

$A_{Fe} = 1 \cdot 10^{-2}\,m^2$

$$L = \frac{N^2}{\theta_{eq}} = 1{,}005\,H$$

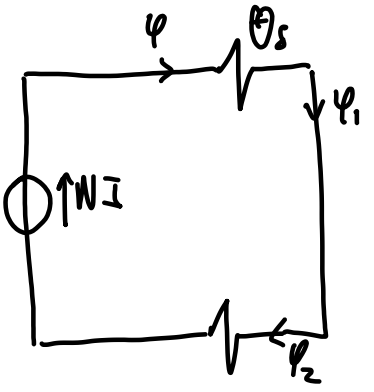
Circuito Elettrico



$$V_M = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 23,71\ V$$

$$I = \frac{V_M - E_3}{R_3} = -0,86\ A$$

$$W_L = \frac{1}{2} L I^2 = 0,37\ J$$



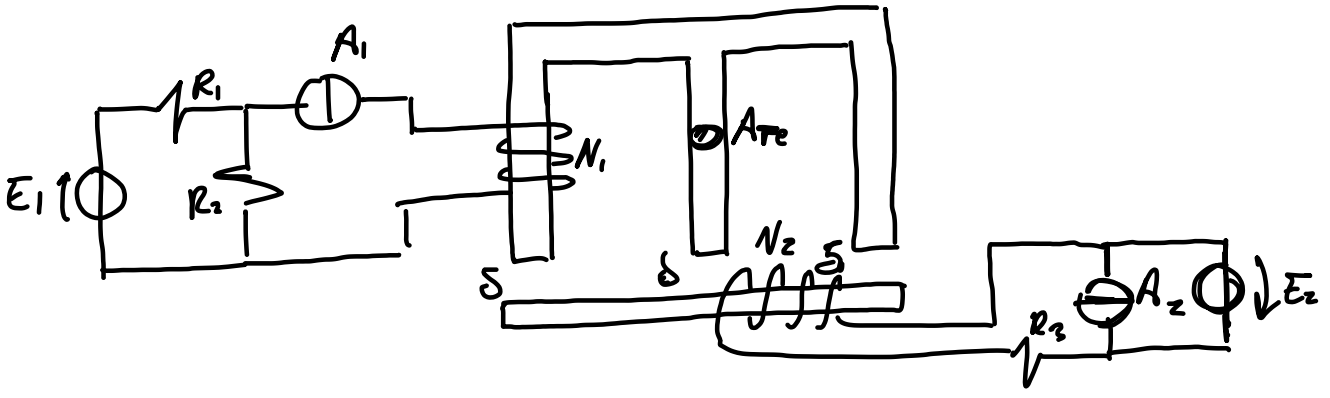
$$F_1 = \frac{\varphi_1^2}{2\mu_0 A_{Fe}}$$

$$F_2 = \frac{\varphi_2^2}{2\mu_0 A_{Fe}}$$

$$F = \frac{\varphi^2}{\mu_0 A_{Fe}} = 371,8N$$

$$\varphi = \frac{NI}{\theta_{eq}} = -2,16\ mWb$$

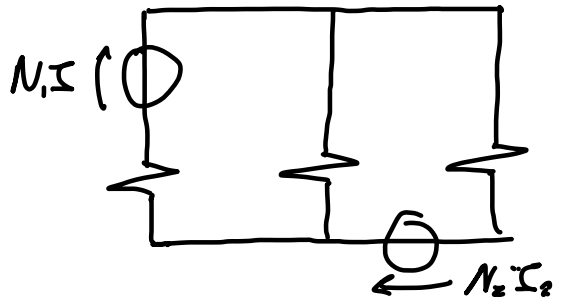
Esercizio 2



$E_1 = 10V$
$E_2 = 15V$
$A_1 = 2A$
$A_2 = 4A$
$R_1 = 3\Omega$
$R_2 = 5\Omega$
$R_3 = 3\Omega$

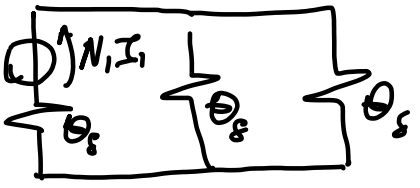
$N_1 = 100$
$N_2 = 200$
$\delta = 1mm$
$A_{Fe} = 60\,cm^2$
$\mu_{Fe} \to \infty$
$\mu_0 = 4\times10^{-7}\,\frac{H}{m}$

Circuito Magnetico



$$\theta_\delta = \frac{\delta}{\mu_0 A_{Fe}} = 132629\,H^{-1}$$

Calcolo L_1

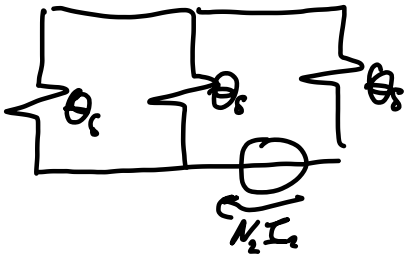


$$\theta_{eq1} = \theta_\delta + \frac{\theta_\delta\theta_\delta}{\theta_\delta+\theta_\delta}$$

$$= \frac{3}{2}\theta_\delta = 198943\,H^{-1}$$

$$L_1 = \frac{N_1^2}{\theta_{eq1}} = 50{,}3\,mH$$

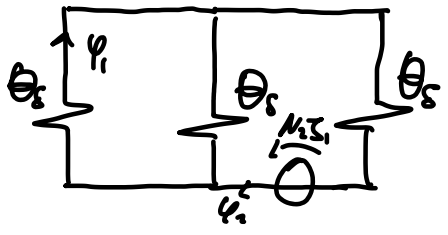
Calcolo L_2:



$$\theta_{eq2} = \frac{3}{2}\theta_\delta = 198943\,H^{-1}$$

$$L_2 = \frac{N_2^2}{\theta_{eq2}} = 201{,}2\,mH$$

Calcolo L_M



$$\boxed{L_M = \frac{N_1\varphi_1}{I_2}} =$$

$$\varphi_2 = \frac{N_2 I_2}{\theta_{eq}}$$

$$\varphi_1 \overset{\text{dato che } \theta_\delta \text{ sono uguali}}{=} \frac{N_2 I_2}{2\theta_{eq2}}$$

$L_1 = 50{,}3\,mH$

$L_2 = 201{,}2\,mH$

$L_m = 50{,}3\,mH$

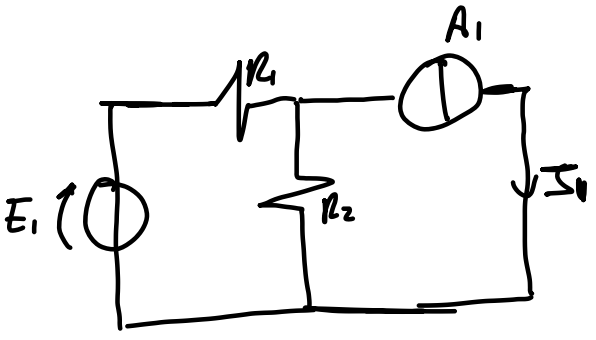
$= \frac{N_1 N_2 I_2}{2\mathcal{R}_{eq} I_2} = \frac{N_1 N_2}{2\mathcal{R}_{eq2}} = 50{,}3\,mH$

Nota per teoria:

Questo è il modo che possiamo calcolare senza sapere I_1 o I_2

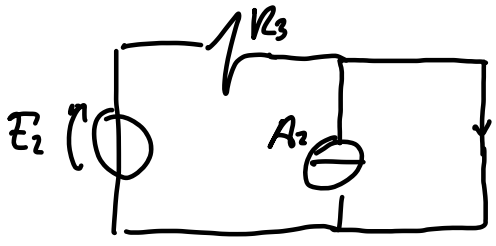
...

Circuito Elettrico Sinistra:



$I_1 = I_2$

Circuito Elettrico Destra:

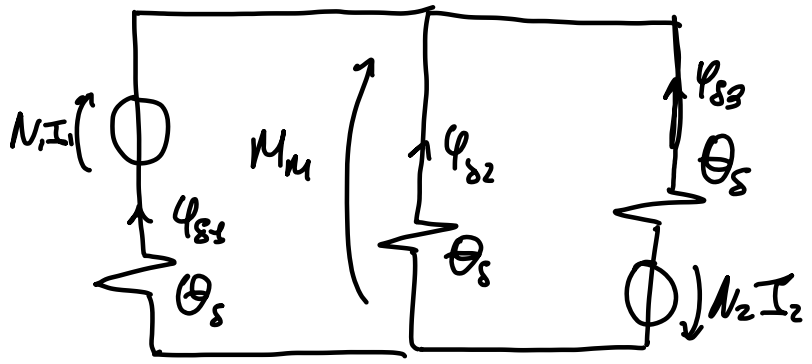


$I_2 = \frac{E_2}{R_3} + A_2 = 9A$

$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + L_m I_1 I_2$

$= 9{,}15\,J$

Calcolo F:



$$M_M = \frac{\frac{N_1 I_1}{\theta_\delta} - \frac{N_2 I_2}{\theta_\delta}}{\frac{1}{\theta_\epsilon} + \frac{1}{\theta_\delta} + \frac{1}{\theta_\delta}}$$

Forza Magneto Motrice

$$= \frac{N_1 I_1 - N_2 I_2}{3} = -533{,}3\ A$$

$$\varphi_{\delta 1} = \frac{N_1 I_1 - M_M}{\theta_\delta} = 5{,}53\ mWb$$

$$\varphi_{\delta 2} = \frac{-M_M}{\theta_\delta} = -4{,}17\ mWb$$

$$\varphi_{\delta 3} = \frac{-N_2 I_2 - M_M}{\theta_\delta} = -9{,}55\ mW$$

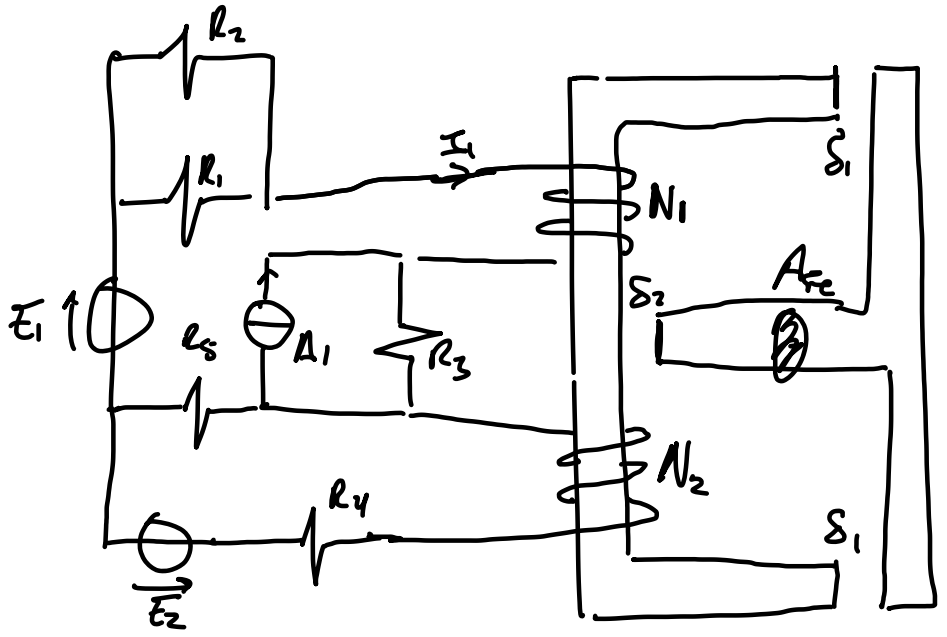
$$F_1 = \frac{\varphi_{\delta 1}^2}{2\mu_0 A_{Fe}} = 2027{,}96\ N$$

$$F_2 = \frac{\varphi_{\delta 2}^2}{2\mu_0 A_{Fe}} = 1153{,}14\ N$$

$$F_3 = \frac{\varphi_{\delta 3}^2}{2\mu_0 A_{Fe}} = 6048{,}05\ N$$

$$F = F_1 + F_2 + F_3$$

Esercizio 3



$R_1 = 10\,\Omega$ $E_1 = 20V$ $A_{Fe} = 10\,cm^2$

$R_2 = 8\,\Omega$ $E_2 = 15V$ $\delta_1 = 2\,mm$

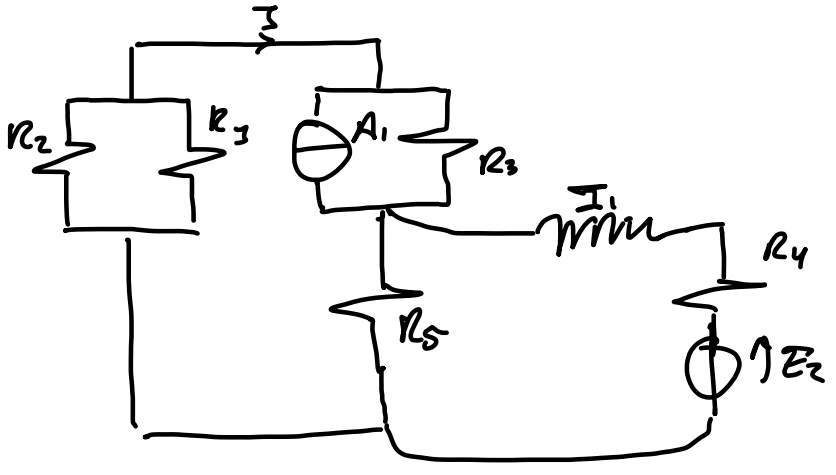
$R_3 = 12\,\Omega$ $A_1 = 10A$ $\delta_2 = 4\,mm$

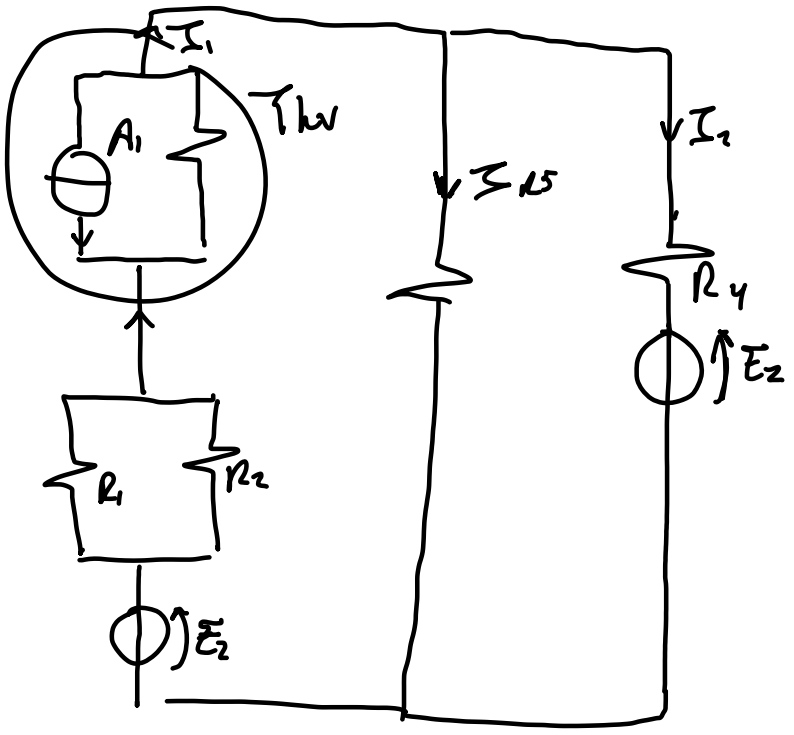
$R_5 = 4\,\Omega$ $N_1 = 100$

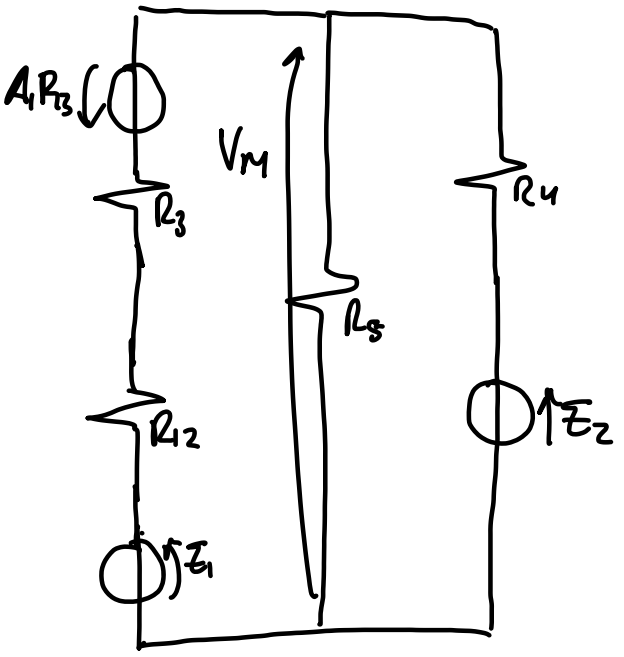
$R_4 = 6\,\Omega$ $W = ?$ $N_2 = 200$

$F = ?$ $\mu_{Fe} \to \infty$

$\mu_0 \to 4\pi \cdot 10^{-7} \frac{H}{m}$







$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 4{,}44\ \Omega$$

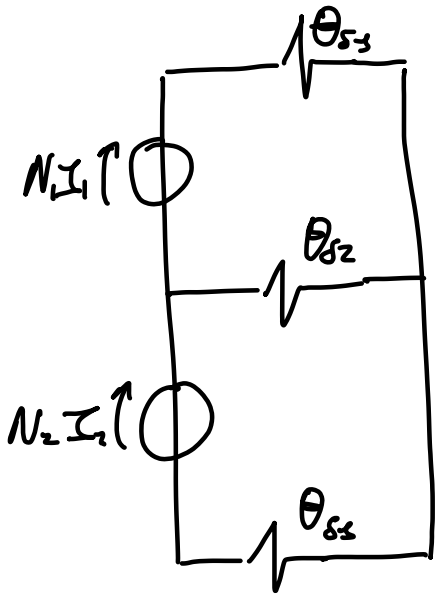
$$V_M = \frac{\dfrac{E_1 - A_1 R_3}{R_{12} + R_3} + \dfrac{E_2}{R_4}}{\dfrac{1}{R_{12} + R_3} + \dfrac{1}{R_5} + \dfrac{1}{R_4}} = -7{,}5\ V$$

$$I_2 = \frac{V_M - E_2}{R_4} = -3{,}75$$

$$I_1 = \frac{E_1 - A_1 R_3 - V_M}{R_{12} + R_3} = \underbrace{I_2 + I_{R5}}_{\text{Solo per vedere questo fatto}} = -5{,}63\ A$$

I sono indipendenti da I in corrente continua,
in corrente alternata sono dipendenti invece

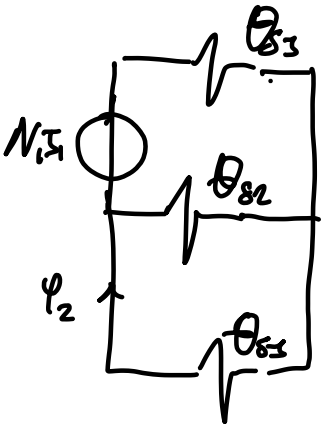
Circuito Magnetico



$$\theta_{s3} = \frac{\delta_1}{\mu_0 A_{Fe}} = 1591549\ H^{-1}$$

$$\theta_{s2} = \frac{\delta_2}{\mu_0 A_{Fe}} = 3183098\ H^{-1}$$

L_1



$$\theta_{eq1} = \theta_{s3} + \frac{\theta_{s1}\theta_{s2}}{\theta_{s1} + \theta_{s2}}$$

$$= \theta_{s1} + \frac{2\theta_{s2}\theta_{s1}}{2\theta_s + \theta_{s1}}$$

$$= \theta_{s1} + \frac{2}{3}\theta_{s1} = \frac{5}{3}\theta_{s1} = 2652581\ H^{-1}$$

$$\boxed{L_1 = \frac{N_1^2}{\theta_{eq1}}} = 3{,}77\ mH$$

$$L_m = \frac{N_2 \varphi_2}{I_1}$$

$$= \frac{N_2 \varphi_1 2}{3 I_1}$$

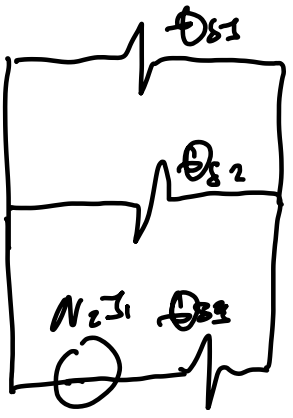
$$\varphi_1 = \frac{N_1 I_1}{\theta_{eq1}} =$$

$$\varphi_2 = \frac{\theta_{s2}}{\theta_{s1} + \theta_{s2}} \cdot \varphi_1$$

$$= \frac{2}{3}\frac{N_2 N_1 I_1}{\Theta_{eq2} I_1} = \frac{\delta_2}{\delta_1+\delta_2}\varphi_2$$

$$= \frac{2}{1+2}\varphi_1 = \frac{2}{3}\varphi_1$$

$$= \frac{2}{3}\frac{N_2 N_1}{\Theta_{eq1}} = 5{,}03\,mH$$

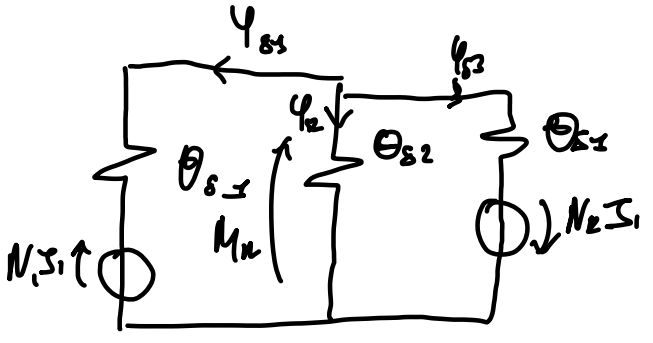


$$\Theta_{eq2} = \Theta_{s1} + \frac{\Theta_{s2}+\Theta_{s3}}{\Theta_{s3}+\Theta_{s2}}$$

$$= \frac{5}{3}\Theta_{s2} = 2562581\ H^{-1}$$

$$L_2 = \frac{N_2^2}{\Theta_{eq2}} = 15{,}1\ mH$$

$$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + L_m I_1 I_2 = 0{,}17\ J$$



$$M_m = \frac{\frac{N_1 I_1}{\Theta_{s1}} + \frac{N_2 I_2}{\Theta_{s2}}}{\frac{1}{\Theta_{s1}} + \frac{1}{\Theta_{s2}} + \frac{1}{\Theta_{s3}}}$$

$$= \frac{\frac{N_1 I_1}{\delta_1} - \frac{N_2 I_2}{\delta_2}}{\frac{1}{\delta_1} + \frac{1}{\delta_2} + \frac{1}{\delta_3}} =$$

$$= \frac{N_1 I_1 - N_2 I_2}{1+\frac{1}{2}+1} =$$

$$= \frac{2}{5}(N_1 I_1 - N_2 I_2) = 74{,}8\ A$$

$$\varphi_{s3} = \frac{N_m - N_1 I_1}{\theta_{s1}} = -401\ \mu Wb$$

$$\varphi_{s2} = \frac{N_m}{\theta_{s2}} = 23,5\ \mu Wb$$

$$\varphi_{s3} = -\varphi_{s1} - \varphi_{s2} = -424,5\ \mu Wb$$

LKC M

$$F = \frac{\varphi_{s3}^2}{2\mu_0 A_{Fe}} + \frac{\varphi_{s2}^2}{2\mu_0 A_{Fe}} + \frac{\varphi_{s3}^2}{2\mu_0 A_{Fe}}$$

$$= \frac{\varphi_{s1}^2 + \varphi_{s2}^2 + \varphi_{s3}^2}{2\mu_0 A_{Fe}} = 135,9\ N$$