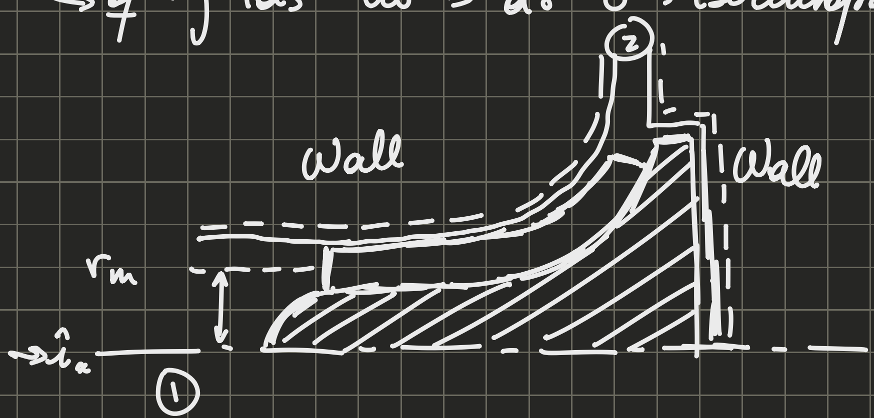
Esercitazione 2 - Application of the Euler equation

Linear moment balance → no (very) useful
Angular momentum balance → Euler equation.

Euler Exercise 2

$\rho = \text{const}$, ideal conditions $\Rightarrow l_w = 0$
(isoentropic)

$q = \int T ds - l_w \Rightarrow ds = 0 \rightarrow$ isoentropic

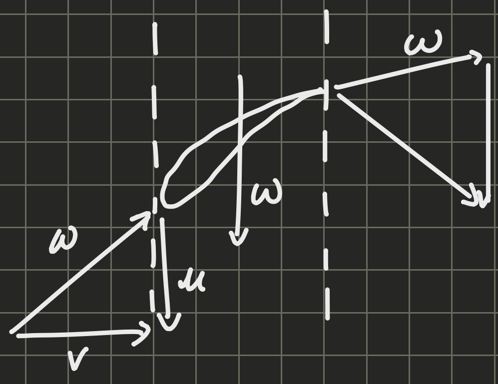


$n = 1500$ rpm
$\hookrightarrow \omega = 157 \frac{rad}{s}$

$Q = \frac{1000 \, m^3}{h} = 0{,}278 \frac{m^3}{s}$

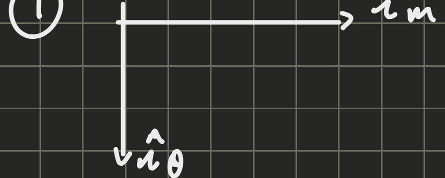
$b_1 = 50$ mm $D_{1m} = 150$ mm $b_2 = 20$ mm , $D_2 = 300$ mm

Stator : α_2
rotor : β_2



We are told the discharge
flow angle is 0, since this
is a rotor which works with ω, means that $\beta_2 = 0$

$l = ?$



At ① $\hat{i}_m = \hat{i}_x$ → meridional = axial

$Q_1 = V_{1x} S_1$ → we get $V_{1x} = 11{,}8\ \frac{m}{s}$

$$S_1 = \frac{\pi (D_{1sh}^2 - D_{1h}^2)}{4} = \pi\, \underbrace{\frac{\overset{\text{shroud}}{D_{1sh}} - \overset{\text{hub}}{D_{1h}}}{2}}_{b_1} \cdot \underbrace{\frac{D_{1sh} + D_{1h}}{2}}_{D_{1m}} = \pi D_{1m}\, b$$

In 2 where $D_{2sh} = D_{2h}$ we can use D_2. → true in the radial case

$$\vec{V} = \vec{u} + \vec{\omega} \longrightarrow \begin{cases} \text{meridional:} & V_m = \omega_m \\ \text{tangential} & V_{1t} = u + \omega_{1t} \end{cases}$$

$\omega_{1x} = V_{1x} = 11{,}8\ \frac{m}{s}$

$V_{1t} = u_1 + \omega_{1t} \Rightarrow \omega_{1t} = -u_1$

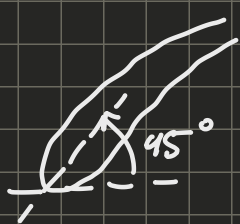
↳ $= \frac{\omega D_{1m}}{2} = 11{,}8\ \frac{m}{s}$

$V_{1t} = 0$, since aren't deflection flows before the rotor.

$V_1 = V_{1x} = 11{,}8\ \frac{m}{s}$ → $\alpha_1 = 0$ → flow is purely axial/meridional

$\omega_1 = \sqrt{\omega_{1x}^2 + \omega_{1t}^2} = 16{,}7\ \frac{m}{s}$ → $\beta = -45°$

↳



② Meridional in section 2 = radial

$$Q = V_{2r} S_2$$

$$S_2 = \pi D_2 b_2$$

↳ diameter is always D_2 along the blade span.

$$V_{2r} = \frac{Q}{S_2} = 14{,}7 \ \frac{m}{s}$$

$$\omega_{2r} = V_{2r} = 14{,}7 \ \frac{m}{s}$$

$$V_{2t} = u_2 + W_{2t} \Rightarrow V_{2t} = u_2 = 23{,}5 \ \frac{m}{s}$$

$$u_2 = \omega \frac{D_2}{2} = 23{,}5 \ \frac{m}{s} > u_1 \text{ since } D_2 > D_{1m}$$

$$\omega_{2t} = \omega_{2r} \tan \underbrace{\beta_2}_{=0} = 0$$

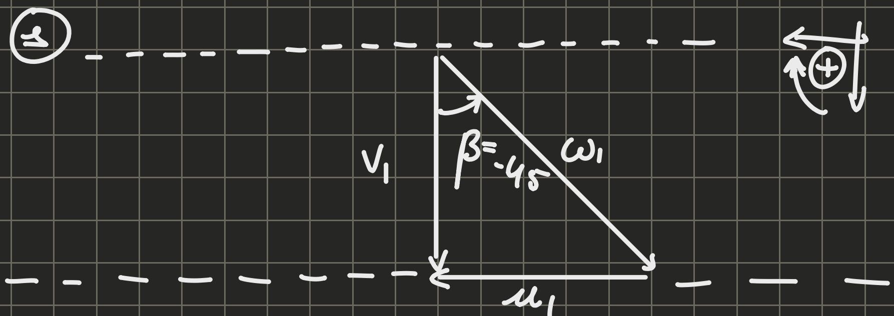
$$V_2 = \sqrt{V_{2t}^2 + V_{2r}^2} = 27{,}8 \ \frac{m}{s}$$

$$\alpha_2 = \tan^{-1}\left(\frac{V_{2t}}{V_{2r}}\right) = 58^\circ$$

$$\beta_2 = 0$$

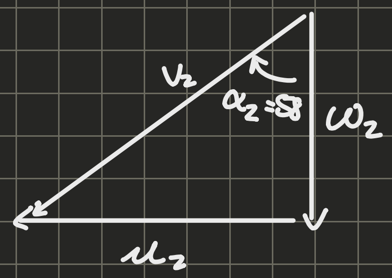
$$\omega_2 = \omega_{2r} = 14{,}7 \ \frac{m}{s}$$

<u>Velocity Triangles</u>



We components of v or w are in the opposite direction of u, the angle will be 0

②



1st way to write Eulerian work:

$$l = u_2 V_{2t} - \overset{0}{u_1 V_{1t}} = u_2 V_{2t} = u_2^2 = 554{,}7 \ \frac{J}{kg}$$

$l > 0$ since we think it's an operating machine.

2nd way to write Eulerian work:

Inertial E.B $\quad \Delta h + \frac{\Delta v^2}{2} + g\Delta z = l + q$

Non-inertial E.B $\quad \Delta h + \frac{\Delta w^2}{2} + g\Delta z - \frac{\Delta u^2}{2} = q$ → since it's rotating with the fluid, it cannot see work being done since the relative motion is 0.

Inertial − Non-inertial → $\boxed{\frac{\Delta v^2}{2} - \frac{\Delta w^2}{2} + \frac{\Delta u^2}{2} = l}$

$$\frac{\Delta v^2}{2} = \frac{(V_2 - V_1)^2}{2} = 316{,}8 \ \frac{J}{kg}$$

$$\frac{\Delta w^2}{2} = \frac{(\omega_2 - \omega_1)^2}{2} = -31{,}4 \ \frac{J}{kg}$$

$$\frac{\Delta u^2}{2} = \frac{(u_2 - u_1)^2}{2} = 206{,}5 \ \frac{J}{kg}$$ → 37% of l → it contribution is considerable

In an axial machine $\frac{\Delta u^2}{2} = 0$, so to achieve the same Eulerian work $\frac{\Delta v^2}{2}$ needs to be a lot, so we would need high deflection, which creates an adverse pressure gradient, so to generate the same work in axial machines we need more stages, which in radial machines we do not.

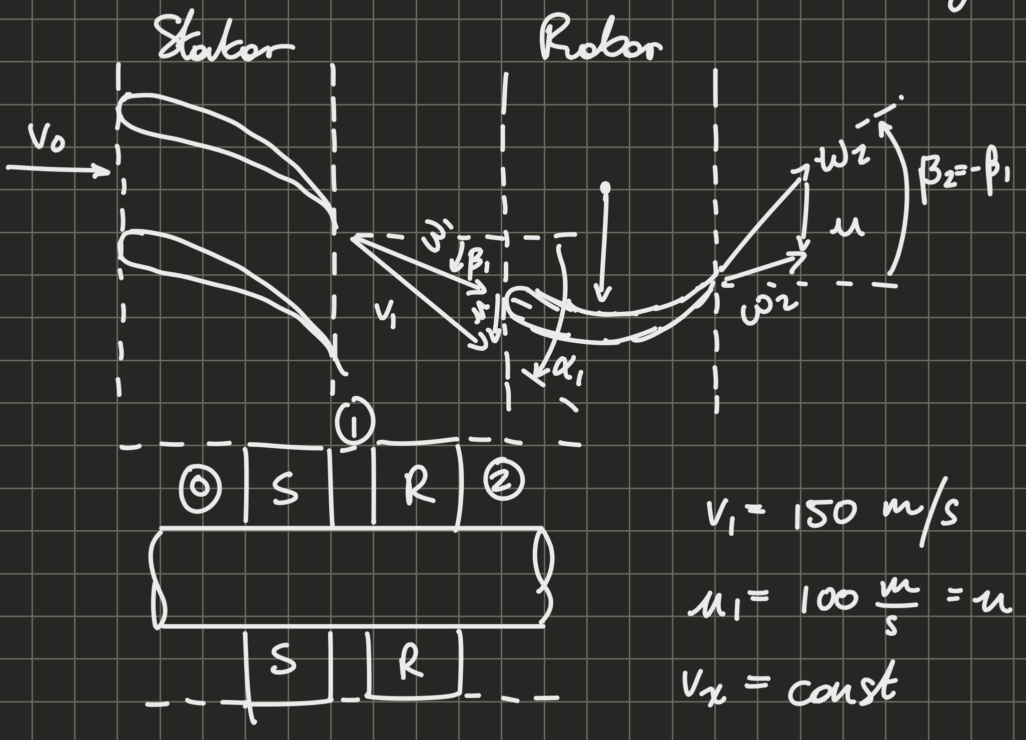
Exercise 3 (Euler)

$$\underset{\text{Axial}}{l \overset{\downarrow}{=} u\,(v_{2t} - v_{1t})}$$

Turbine

↳ $l < 0 \implies v_{1t} > v_{2t} \Rightarrow v_{1t} > 0$

$\Rightarrow$ we need a stator to deflect the velocity



$v_1 = 150$ m/s

$u_1 = 100\ \frac{m}{s} = u$

$v_x = \text{const}$

$\beta_2 = -\beta_1 \rightarrow$ symmetric blade

$\alpha_1 = 74°$ ↳ since the relative velocity is symmetric across.

① $V_1 = 150 \ \frac{m}{s}$

meriodband = axial $\rightarrow V_{1x} = V_1 \cos\alpha_1 = 41{,}3 \ \frac{m}{s}$

$W_{1x} = V_{1x} = 41{,}3 \ \frac{m}{s}$

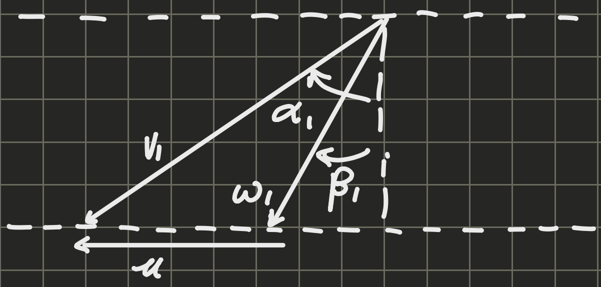
tangential $\quad V_{1t} = V_1 \sin\alpha_1 = 144{,}2 \ \frac{m}{s}$

$V_{1t} = W_{1t} + u \rightarrow W_{1t} = V_1 - u_1 = 44{,}2 \ \frac{m}{s}$

$$W_1 = \sqrt{W_{1x}^2 + W_{1t}^2} = 60{,}5 \ \frac{m}{s}$$

$$\beta_1 = \tan \frac{W_{1t}}{W_{1x}} = 46{,}9°$$

Triangle ①



② $V_{2x} = V_{1x} = 41{,}3 \ m/s$

$W_{2x} = V_{2x} = 41{,}3 \ m/s$

$V_{2t} = W_{2t} + u$

$\beta_2 = -\beta_1 = -46{,}9$

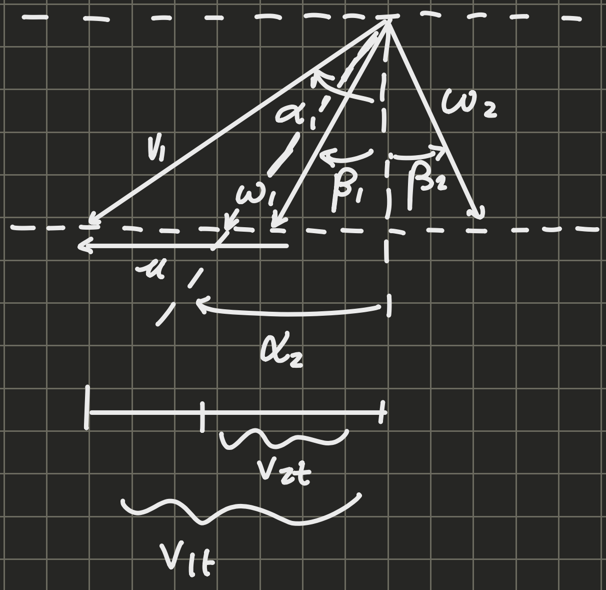
$W_{2t} = W_{2x} \tan \beta_2 = -44{,}2 \ \frac{m}{s} \rightarrow V_{2t} = 55{,}8 \ \frac{m}{s}$

$$V_2 = \sqrt{V_{2t}^2 + V_{2x}^2} = 69{,}5 \ m/s$$

$$\alpha_2 = \tan^{-1}\left(\frac{V_{2t}}{V_{2x}}\right) = 53{,}5°$$

$\omega_2 = 60{,}5 \frac{m}{s}$

Since v_x is constant we can use the same diagram

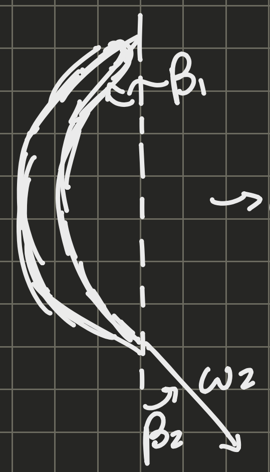


$v_{1t} > v_{2t} \Rightarrow v_{2t} - v_{1t} < 0 \Rightarrow l < 0$

$$l = u\left(v_{2t} - v_{1t}\right) = -8440 \ \frac{J}{kg}$$

Symmetric Blade

$\hookrightarrow \beta_2 = -\beta_1$



→ Crescent Shaped Blades

Also known as impulse blades

$$\Delta I = q = 0$$

$$h_1 + \frac{\omega_1^2}{2} - \frac{u_1^2}{2} = h_2 + \frac{\omega_2^2}{2} - \frac{u_2^2}{2}$$

$h_1 = h_2$ → since $|\omega_1| = |\omega_2|$ and

and $|u_1| = |u_2|$

↳ in an ideal machine $\Rightarrow P_1 = P_2$

Because:

↳ $\Delta h + \cancel{\frac{\Delta w^2}{2}} - \frac{\Delta u^2}{2} + g\Delta z = \cancel{q}$

↳ $\int \cancel{T ds} - l_w$

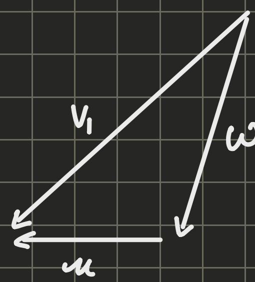
$\int \cancel{T ds} + \int v dP$

$\int v dP = -l_w = 0 \rightarrow dP = 0 \rightarrow P_2 = P_1$

Part B

↳ Non simmetric blade

$|w_2| = 2|w_1|$



$v_{2x} = v_1 = 41,3 \frac{m}{s}$

$w_{2x} = w_{2x} = 41,3$ m/s

$v_{2t} = w_{2t} + u$

↳ in this case the sign of w_{2t} is important.

$|\beta_2| = \cos^{-1}\left(\frac{w_{2x}}{w_2}\right) \Rightarrow |\beta_2| = 70°$

↳ we don't know the sign since we don't know any of the components.

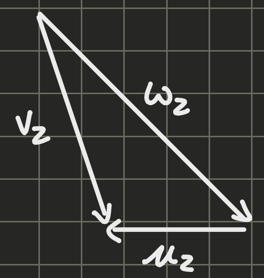
$\beta_2 = -70$ since v_{1t} needs to be larger than v_{2t} for $l < 0$. If $\beta_2 = 70 \Rightarrow v_{2t} < v_{1t} \Rightarrow l > 0$ which is not consistent with the turbine.

$w_{2t} = w_2 \sin \beta_2 = -113,5 \frac{m}{s}$

$V_{2t} = w_{2t} + u = -13,5 \frac{m}{s}$

$\alpha_2 = \tan^{-1}\left(\frac{V_{2t}}{V_{2x}}\right) = -18,1°$

$V_2 = 43,5 \frac{m}{s}$

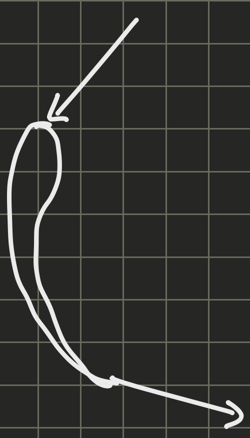


$$\ell = u(V_{2t} - V_{1t}) = -15770 \frac{J}{kg}$$

$$= \underbrace{\frac{\Delta v^2}{2}}_{(-)} - \underbrace{\frac{\Delta w^2}{2}}_{(-)} =$$

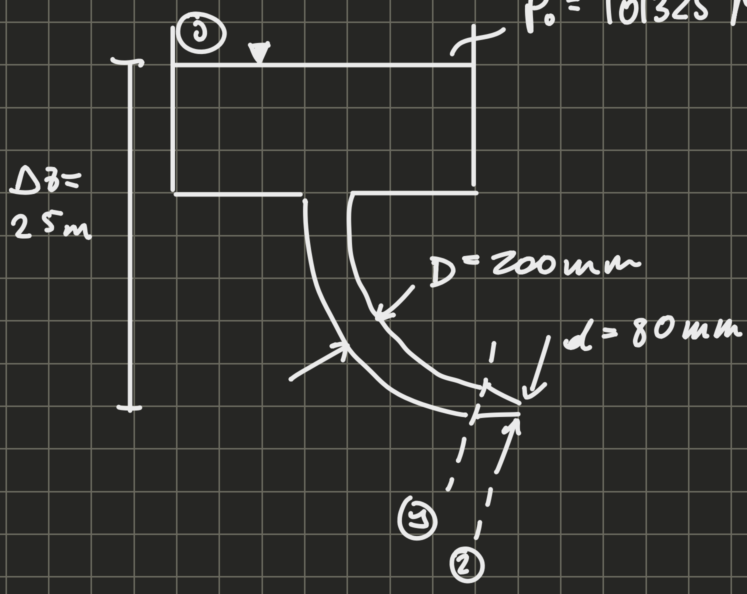
↳ when before it was 0, so $|\ell|$ increased.

The blade therefore goes like



This is not practical since at some point the flow will seperate from the blade.
An exageration of the deflection causes lost work.

Exercise 1 → Hydraulics



$P_0 = 101325\ MPa = 1\,atm \simeq 1\,bar$

$\rho = 1000\ \frac{kg}{m^3}$

Scenario a) no loss

$\dot m = ?\ ,\ P_1 = ?$

M.E.B : 0 − 2 : $\cancel{\mathcal{L} - \ell w - y} = \frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z =$ (0 nomachine)

$0 = \cancel{\frac{P_2 - P_0}{\rho}} + \frac{V_2 - \cancel{V_0}}{2} + g(z_2 - z_0) \Rightarrow V_2 = \sqrt{2g(z_2 - z_0)} = 22{,}15\ \frac{m}{s}$

$\to P_2 = P_0$ since we discharge to the atmosphere

$\to Q = S_2 V_2 = \frac{\pi d^2}{4} \cdot V_2 = 0{,}111\ \frac{m^3}{kg}$

$\dot m = \rho Q = 111\ \frac{kg}{s}$

MEB : 1 → 2 : $0 = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + \cancel{g\Delta z}$

$\Rightarrow \underbrace{\left(\frac{P_2}{\rho} + \frac{V_2^2}{2}\right)}_{P_{T_2}} - \left(\frac{P_1}{\rho} + \frac{V_1^2}{2}\right) = \frac{P_{T2} - P_{T1}}{\rho} \to P_{T2} = P_{T1}$

$P_{Ti} = P_i + \rho \frac{v_i^2}{2}$

↳ Valid for
$\rho = const$

ρ

$$\underbrace{P_2}_{P_0} + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2 \longrightarrow P_1 = 340000\ Pa = 3,4\ bar$$

$$v_1 S_1 = v_2 S_2 \rightarrow v_1 = 3,54\ \frac{m}{s}$$

Between P_{T0} and P_{T1}, total pressure is not conserved because we have a $g\Delta z$ which we didn't directly take into account with the M.E.B from $1 \to 2$.

Scenario B)

loss $\to y$

$\dot{m} = const$

$d_{new} = ?$ → To be able to have $\dot{m} = const$
↳ since with losses v decreases and so $\dot{m}$ will decrease, so we have to increase d.

M.E.B. $0 \to 2$

$$-y = \frac{P_2 - \cancel{P_0}}{\cancel{\rho}} + \frac{V_2^2 - \cancel{V_0^2}}{2} + g(z_2 - z_0)$$

$$\longrightarrow V_2 = \sqrt{2\left[g(z_2 - z_0) - y\right)}$$

↳ As we said, losses reduce velocity.

$$y = y_{LOC} + y_{DIS} = \sum_i \underbrace{\xi_i}_{\text{localized loss coefficients}} \frac{V^2}{2} + \underbrace{\frac{\lambda L}{D}}_{\text{Distributed loss coefficient}} \frac{V^2}{2} = \left[\sum \xi_i + \frac{\lambda L}{D}\right]\frac{V^2}{2}$$

$L = 50\,m$ $\quad 5 \cdot \xi_{90} = 0{,}3$

$\lambda = 0{,}01$ $\quad \xi_{filter} = 0{,}8$

$$y = \left[5\,\xi_{90} + \xi_{filter} + \frac{\lambda L}{D}\right] \frac{v_1^2}{2} \rightsquigarrow \text{same as before.} = 30{,}08\ \frac{J}{kg}$$

$$\to v_{2,new} = 20{,}7\ \frac{m}{s} < 22{,}15\ \frac{m}{s}$$

$$Q = v_{2,new}\ \frac{\pi d_{new}^2}{4} \to d_{new} = \sqrt{\frac{4\,Q}{\pi V_{2,new}}} \approx 83\ mm$$

Scenario C

$D_{new} = 150\,mm$

$\dot{m} = const$

$d_{new} = ?$

$$\to y_{new} = 111{,}1\ \frac{J}{kg}$$

$$V_1 = \frac{Q}{\pi D_{new}^2} = 6{,}28\ \frac{m}{s}$$

$$\longrightarrow V_{2new} = \sqrt{2\left[g(z_0 - z_2) - y\right]} = 16{,}4\ \frac{m}{s} \to d_{new} = \sqrt{\frac{4Q}{\pi V_{2,new}^2}} = 93\,mm$$