Lezione 9 – Study of rotors from a non-inertial frame.

general, including rotating

↳ Explains why w decreasing causes v AND P to increase

omega = angular velocity
w = relative velocity.

Non-inertial frames

velocity of non-inertial observer.

$\vec{v} = \vec{w} + \vec{u} \longrightarrow \vec{u} = \vec{\omega} \times r\hat{i}_r = \omega\, \hat{i}_x \times r\hat{i}_r = \omega r\, \hat{i}_\theta$

↳ Velocity percieved by non-inertial observer

$$\frac{d\vec{v}}{dt} = \frac{d\vec{w}}{dt} + \frac{d}{dt}\left(\vec{u}\right) = \frac{d\vec{w}}{dt} + \frac{d}{dt}\left(\vec{\omega} \times r\,\hat{i}_r\right) = \frac{d\vec{w}}{dt} + \overbrace{\underbrace{\vec{\omega} \times \vec{\omega} \times r\hat{i}_r}_{-\omega^2 r\hat{i}_r}}^{\text{Centripetal}} + \underbrace{2\omega \times \vec{w}}_{\text{Coriolis}} + \underbrace{\cancel{\frac{d\vec{\omega}}{dt}} \times r\hat{i}_r}_{0}$$

omega ↑ ω ↓

↳ Acceleration percieved

since steady flow/ operation

Acceleration of inertial = Acceleration perceived + 2 other factors.

3 but one is 0

Momentum Balance

→ Momentum needs to be balanced in an inertial case, we can rewrite it for the non-inertial

$$\frac{d}{dt}\int_{\Omega(t)} \rho \vec{v}\, dV = \vec{F}$$

→ The equation is always the same, it can't change

We can plug the equation to get it with form referring to $\vec{w}$

$$= \frac{d}{dt}\int_{\Omega(t)} \vec{v}\,(\overbrace{\rho\, dV}^{dM}) = \int_{\Omega(t)} \frac{d\vec{v}}{dt}(\rho\, dV) + \underbrace{\vec{v}\, d(\cancel{dM})}_{0} = \int_{\Omega(t)} \rho\frac{d\vec{v}}{dt} dV =$$

↳ Still in Lagrangian base

$$= \int_{\Omega(t)} \rho\left(\frac{d\vec{w}}{dt} + \left(-\omega^2 r\hat{i}_r\right) + 2\vec{\omega} \times \vec{w}\right) dV =$$

$$\overset{!}{=} \int_{\Omega(t)} \rho \frac{d\vec{w}}{dt}\, dV + \int_{\Omega(t)} \left(\left(-\omega^2 r \hat{i}_r\right) + 2\omega \times \vec{\omega} \right) \rho \, dV$$

$$\frac{d}{dt} \int_{\Omega(t)} \rho \vec{w}\, dV$$

we did the inverse of that step before

we can do this since mass is conserved

$$= \frac{d}{dt} \int_{\Omega(t)} \rho \vec{w}\, dV + \int_{\Omega(t)} \underbrace{\left(-\omega^2 r \hat{i}_r\right) \rho \, dV}_{\vec{f}_{CENTIF}} + \int_{\Omega(t)} \underbrace{2\vec{\omega} \times \vec{w} \rho \, dV}_{\vec{f}_{CORIOLIS}} = \vec{F}$$

$$\frac{d}{dt} \int_{\Omega(t)} \rho \vec{w}\, dV = \vec{F} + \underbrace{\int_{\Omega(t)} \rho \omega^2 r \hat{i}_r \, dV + \int_{\Omega(t)} \rho 2 \vec{w} \times \vec{\omega}\, dV}$$

Additional forces we feel when we are on the rotor without knowing

Before energy balance, we will look at these forces

f_{centif} is conservative since it only depends on r,

$f_{CORIOLIS}$ is not conservative

→ We will be able to define a potential

→ We will find out how to get with it.

$$\vec{f}_{centrif} = \omega^2 r \hat{i}_r \rightarrow e_c : \nabla e_c = -f_{centrif}$$

↳ centrifugal energy

so that we can bring it to the other side

$$\nabla e_c = \begin{bmatrix} \frac{\partial e_c}{\partial x} \\ \frac{\partial e_c}{\partial r} \\ \frac{1}{r}\frac{\partial e_c}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 0 \\ -\omega^2 r \\ 0 \end{bmatrix} \Rightarrow e_c \text{ only depends on } r, \text{ not } x \text{ or } \theta$$

↳ cylindrical

$$\Rightarrow \frac{\partial e_c}{\partial r} = \frac{d e_c}{dr} = -\omega^2 r$$

$$\boxed{e_c = \cancel{e_{c_0}}_{\to 0} + \int_{r_0=0}^{r} -\omega^2 r\,dr = -\frac{1}{2}\omega^2 r^2 = -\frac{u^2}{2}}$$

↳ since $|u| = |\omega r|$

Energy Balance (Rotating)

We define: $E_W = \int_{\Omega(t)} \rho \left(u + \frac{W^2}{2} + gz - \frac{u^2}{2} \right) dV$

↳ relative energy

↳ internal energy

Wall

$$\frac{d}{dt}\int_{\Omega(t)} \rho \left(u + \frac{W^2}{2} + gz - \frac{u^2}{2} \right) dV = \dot{L}_W + \dot{Q}$$

→ Coriolis contribution of power

$$\dot{L}_W = \int_{\Omega(t)} \underbrace{\rho\, 2(\vec{W} \times \vec{\omega}) \cdot \vec{W}\, dV}_{\to 0} + \int_{\partial\Omega(t) = \partial\Omega(\tau) = \partial\Omega_f = A_W + A_{IN} + A_{OUT}} \vec{\vec{\sigma}} \cdot \vec{W}\, dA =$$

↳ Power of body forces, we have to consider since Coriolis is non-conservative

since $\vec{W} \times \vec{\omega} = \perp \vec{W} \Rightarrow \cdot \vec{W} = 0$

$$= \underbrace{\int_{A_{IN}} \vec{\vec{\sigma}} \cdot \vec{W}_W\, dA}_{\to 0} + \int_{A_{IN}+A_{OUT}} \vec{\vec{\sigma}} \cdot \vec{W}\, dA = \int_{A_{IN}+A_{OUT}} -P\vec{n}\vec{\omega}\, dA + \underbrace{\int_{A_{IN}+A_{OUT}} \vec{\tau}\, \vec{\omega}\, dA}_{\to 0} =$$

Mechanical power exchanged by the moving surface, perceived by rotating observer.

$\vec{W}_W = \vec{V}_W - u = \vec{u} - \vec{u} = 0$

$$= \int \rho \underbrace{(-Pv)}_{-P \cdot \frac{\rho}{\rho}} \vec{W} \cdot \vec{n}\, dA$$

The adherence condition (for us) is always valid,

$\Rightarrow \vec{V}_W = \vec{u}$

We don't perceive any movement since we are moving with it, so we perceive no power exchange

→ We apply the RTT

$$\frac{d}{dt}\int_{\Omega_f}\left(\cancel{\quad}\right)dV + \int_{A_{IN}+A_{OUT}} \rho\left(u+\frac{w^2}{2}+gz-\frac{u^2}{2}\right)\vec{w}\cdot\vec{n}\,dA =$$

$\to$ 0 since we assume steady flow, and this is un steady due to the $\frac{d}{dt}$

$$= \int_{A_{IN}+A_{OUT}} \rho(-Pv)\,\vec{w}\cdot\vec{n}\,dA + \dot{Q}$$

+ $\to$ to get $u+Pv=h$

Energy Balance $\Longrightarrow$

$$\boxed{\int_{A_{IN}+A_{OUT}} \rho\left(h+\frac{w^2}{2}+gz-\frac{u^2}{2}\right)\vec{w}\cdot\vec{n}\,dA = \dot{Q}}$$

In general turbomachinery is bad at exchanging heat so generally $\dot{Q}$ is very low.

$= \vec{w}_m = v_m \Rightarrow$ doesn't change from the inertial mass flow rate

LPA

$$\Longrightarrow \left(h+\frac{w^2}{2}+gz-\frac{u^2}{2}\right)_{OUT} \underbrace{\int \rho\,\vec{w}\cdot\vec{n}\,dA}_{\dot{m}_{OUT}} + \left(h+\frac{w^2}{2}+gz-\frac{u^2}{2}\right)_{IN} \underbrace{\int \rho\,\vec{w}\cdot\vec{n}\,dA}_{-\dot{m}_{IN}} = \dot{Q}$$

Percieved mass flow

$= -w_n = -v_m$

$$\dot{m} = \dot{m}_{OUT} = \dot{m}_{IN}$$

$$\Delta h + \frac{\Delta w^2}{2} + g\Delta z - \frac{\Delta u^2}{2} = q$$

Across a rotor we can write:

$$\begin{cases} \text{Abs. Energy Balance} & l+q = \Delta h + \dfrac{\Delta v^2}{2} + g\Delta z \\ \text{Relative Energy Balance} & q = \Delta h + \dfrac{\Delta w^2}{2} + g\Delta z - \dfrac{\Delta u^2}{2} \end{cases}$$

$\hookrightarrow$ The work exchanged have been killed.

$\hookrightarrow$ Change in centrifugal potential kinda like rotating kinetic energy.

$1-2 \Rightarrow l = \frac{\Delta v^2}{2} - \frac{\Delta w^2}{2} + \frac{\Delta u^2}{2}$ → Alternative definition of l, which only uses magnitudes of the velocities.

In the exercise we did we get v to increase with w decreases.

l = absolute − relative + rotating.

~9:30 Centrifugal machines have $\frac{\Delta u^2}{2}$ positive so it explaines why they are typically operating machines.

Radial machines have higher ΔP since they also use u.
This allows us to classify radial and axial machines,
we see how much $\frac{\Delta u^2}{2}$ is relative to $\frac{\Delta v^2}{2} - \frac{\Delta w^2}{2}$
Although $\frac{\Delta u^2}{2} \simeq 2\text{-}3\%$ in axial machines, we take it as 0, in this course.

Mechanical Energy Balance for Rotating Observer:

$$q = \int_1^2 T dS = \underset{\text{l wasted}}{l_w}$$

$$\cancel{\int_1^2 T dS} - l_w = \underbrace{\cancel{\int_1^2 T dS} + \int_1^2 v dP}_{\Delta h} + \frac{\Delta w^2}{2} + \underset{\approx 0}{g \Delta z} - \frac{\Delta u^2}{2}$$

$$\boxed{\int_1^2 v dP = -\frac{\Delta w^2}{2} + \frac{\Delta u^2}{2} - l_w}$$

⇒ Demonstrates that the pressure increase as w decreases.

Fans are rotating diffusers, they (pressure increases, velocity decrease) reduce relative velocity to increase the pressure

Mass Balance

Energy Balance

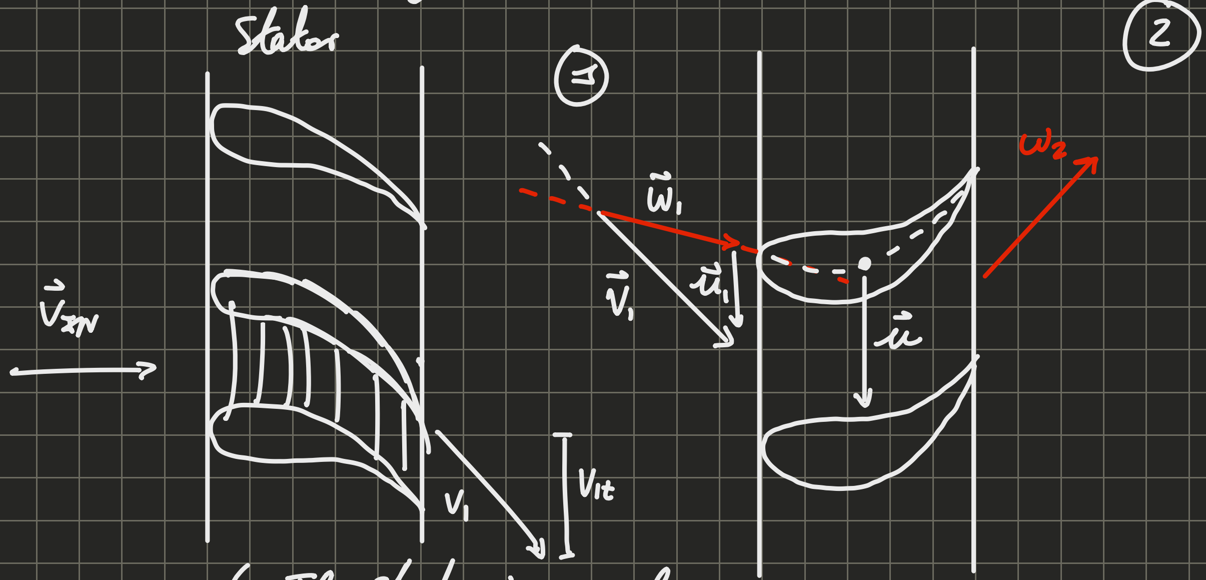
Rotating Energy Balance

Mechanical Energy Balance

Euler Equation.

<u>Turbine Stage</u> $(\ell < 0) \Rightarrow V_{1t} > V_{2t}$ since $\ell = u_2 V_{2t} - u_1 V_{1t}$

To be able to have V_{1t} we introduce a stator to deflect the flow before we get it to the rotors.



↳ The stator is a nozzle, the flow accelerates and pressure decreases