Lezione 4 -

Finish of Thermodynamics of liquids

Ideal Liquid

$$\begin{cases} u = u(T) \rightarrow \frac{du}{dT} = c_L \Rightarrow u = u_0 + \int_{T_{ref}}^{T} c_L(T)\,dT \\ v = cost \end{cases}$$

→ Same as ideal gases

$$du = TdS - P\cancel{dv}^{\,0} \rightarrow ds = \frac{du}{T} = c_L \frac{dT}{T} \rightarrow s = s + \int_{T_{ref}}^{T} c_L(T)\frac{dT}{T}$$

↖ Before enthalpy after everything else

Entropy can be exchanged with an exchange of heat or irreversibility

A change in energy means we can measure the change in entropy

$$c_L = \left(\frac{\delta q}{dT}\right)_L \overset{\text{reversible}}{=} T\left(\frac{\partial s}{\partial T}\right)_L \Rightarrow \underset{\text{ordinary since } u = u(T)}{\frac{du}{dT}} = T\left(\frac{\partial s}{\partial T}\right)_L = c_L$$

↳ liquid (only one since it will be hypho conic

Transformation L

$$h = u + Pv \rightarrow dh = du + d(Pv) = c_L dT + v dP$$

h_f → not really used for liquids

$$h = h_f + \int_{T_{ref}}^{T} c_L(T)\,dT + v(P - P_{ref})$$

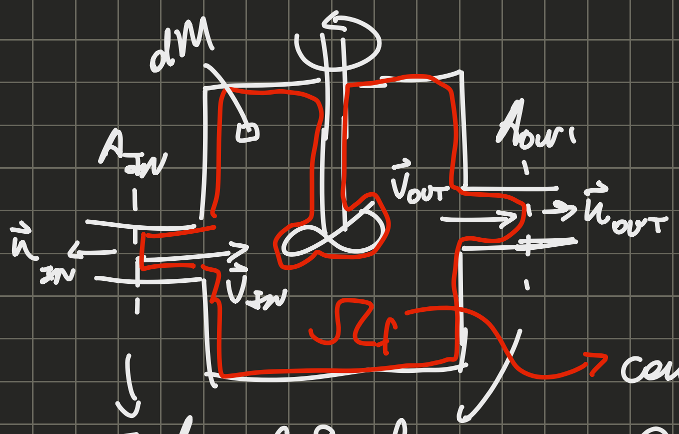
For perfect liquids the $d \rightarrow \Delta$

End of thermodynamic models

Mathematical Operations for open systems

Imagine we want to write a balance of mass, energy or momentum,

Let's think of a box with a machine inside



control volume

Fictional Surfaces to close the open box

$\partial\Omega_f$: boundary of control volume

A_{IN}	A_{OUT}	A_M
$\left(\vec{V}_{IN} \cdot \vec{n}_{IN}\right)$ <0 $= -V_{m,in}$ ↳ Absolute volume normal to the normal vector	$\vec{V}_{OUT} \cdot \vec{n}_{OUT} = V_{m,out}$	$\vec{V} \cdot \vec{n} = 0$ (flow can't cross solid surface

$$M_{cv} = \int_{\Omega_f} \rho \, dV$$

← mass that instantaneously fills the volume

↳ through continuity assumption

What enters can either stay in or go out (we will obtain this system, through more rigorous analysis now.

Fluid body → not controlled it's just a body of the fluid.

$$M(t_0) = \int_{\Omega(t_0)} \rho \, dV \qquad M(t) = \int_{\Omega(t)} \rho \, dV$$

$$\frac{d}{dt} M = \frac{d}{dt} \int_{\Omega(t)} \rho \, dV = 0$$

$$\frac{d}{dt} M_{cv} = \frac{d}{dt} \int_{\Omega f} \rho \, dV \neq 0$$, its not true that $= 0$

Reynolds transport theorem $\boxed{9:12}$ − 9:15

↳ To connect that two conservations

$$t = \tau \rightarrow \Omega(\tau) = \Omega f$$

$$\Downarrow$$

$$M(\tau) = M_{cv}$$

$$\boxed{\frac{dM_{cv}}{dt} \neq \frac{dM}{dt}}$$ → They are not the same since M_{cv} can vary, while M cannot

To know what happen at τ we needs know $\tau \pm dt$

↳ Imagine that we have a field $\varphi = \varphi(\vec{x}, t)$

and $\phi = \int_{\Omega(t)} \varphi \, dV$

We can see the change in volume as the change in infinitesimal volumes in time

Let's evaluate

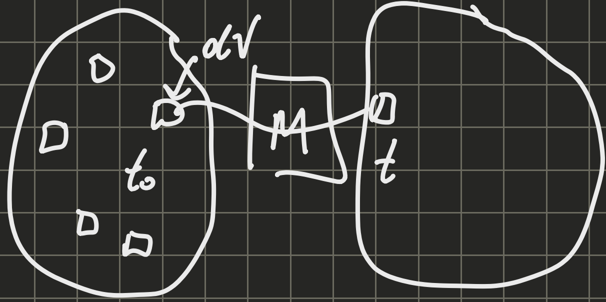
$$\hookrightarrow \frac{d\phi}{dt} = \frac{d}{dt} \int_{\Omega(t)} \varphi \, dV \overset{!}{=} \int_{\Omega(t)} \frac{d}{dt}(\varphi \, dV) = \int_{\Omega(t)} \frac{d\varphi}{dt} \, dV + \int_{\Omega(t)} \varphi \frac{d(dV)}{dt}$$

Time derivative of the elementary volumes

$$\varphi(\vec{x}_0, t_0))$$

$\varphi^L = (\vec{x}_0, t)$

↳ It's sufficient enough to know the initial position and it evolves in time



Since we are engineers we don't care where it starts, we care though where the machine it

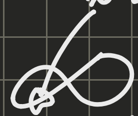
We don't care about the fluid, we only care to evaluate $\frac{dM}{dt}$

What we would like

$\varphi(\vec{x}, t)$

↳ A position that has nothing to do with the fluid

This will $\vec{x}$ if we want to monitor it



$\vec{x}$ is a point in the trajectory of the fluid

$\vec{x} = \vec{x}(\vec{x}_0, t)$

Material derivative

$$\varphi^E(\vec{x}, t) = \varphi^L(\vec{x}(\vec{x}_0, t), t) = \frac{D\varphi}{Dt} \overset{*}{=} \nabla\varphi \cdot \vec{v} + \frac{\partial \varphi}{\partial t}$$

reasoning $\Rightarrow$ * $\frac{\partial\varphi}{\partial t} = \underbrace{\frac{\partial\varphi}{\partial\vec{x}}}_{\nabla\varphi} \cdot \underbrace{\frac{\partial\vec{x}}{\partial t}}_{\vec{v}} + \frac{\partial\varphi}{\partial t}$

This is all just context

↳ Putting the derivative in, it means that we not longer have to take the derivative of the volume, it means that taking the snapshot for dt is valid for both the control volume and the fluid body.

$$\overset{t=\tau}{=} \int_{\Omega(\tau)} \frac{d\varphi}{dt}\, dV + \int_{\Omega(\tau)} \varphi \frac{d(dV)}{dt} \underset{\Omega(\tau)=\Omega_f}{=} \int_{\Omega_f} \frac{d\varphi}{dt}\, dV + \int \varphi \frac{d(dV)}{dt} =$$

We take care of the bad parts at the local level, then we integrate which is much nicer

$$= \dots = \int_{\Omega_f} \underbrace{\frac{\partial \varphi}{\partial t}}\, dV + \underbrace{\int_{\partial\Omega_f} \varphi \vec{v}\cdot\vec{n}\, dA} = \frac{d}{dt} \underbrace{\int_{\Omega f} \varphi\, dV}_{\phi_{CV}} + \int_{\Omega f} \varphi \vec{v}\cdot\vec{n}\, dA$$

↳ only $\frac{\partial \varphi}{\partial t}$ with the eulerian approach, and not in the lagrangian

Flux over border, there are two areas where it is not null

↳ control volume

$$\underbrace{\frac{d\phi}{dt}} = \underbrace{\frac{d}{dt}\phi_{cv}} + \int_{\partial\Omega f} \varphi \vec{v}\cdot\vec{n}\, dt$$

Valid for material body

Valid for control volume.

Mass of fluid = Mass in control volume + flux

Mass Balance

$$\frac{dM}{dt} = 0$$

↳ mass of fluid body

for Reynolds TT we can say: $(\varphi = \rho)$

$$\frac{dM}{dt} = \frac{dM_{cv}}{dt} + \int_{A_{IN}+A_{OUT}} \rho \vec{v}\cdot\vec{n}\, dA = 0$$

$$\rightarrow \frac{dM_{cv}}{dt} + \int_{A_{out}} \rho \, v_{m,out} \, dA = \int_{A_{IN}} \rho \, v_{m,in} \, dA$$

As we said before what enters can either enter of go out.

A process which steady and derivative for the control volume is null, meaning that the mass of the control volume is constant, the mass entering and exiting.

Steady Process $\Rightarrow \frac{dM_{cv}}{dt} = 0$

$$\Rightarrow \int_{A_{out}} \rho \, v_{m,out} \, dA = \int_{A_{IN}} \rho \, v_{m,in} \, dA$$

ρ and $v_{m,out}$ are dependent of fields with gradient in the area, so we take the average valid for the whole cross-section

↳ Lumped Parameter Approach

We will then be able to write:

$$\rho_{out} \underbrace{v_{m,out} A_{out}}_{\dot{m} = \rho v_m A} = \rho_{in} v_{m,in} A_{in}$$

mass flow rate = const

The mass flow rate is not conserved the mass of the control volume is.