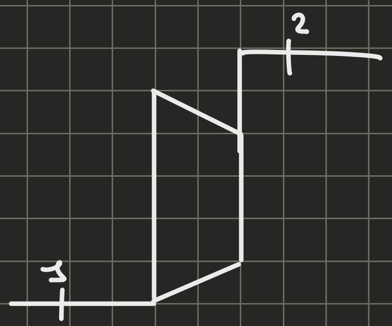
Esercitazione 5 - Thermodynamics and Gas Dynamics

Exercise 1



$\beta = 14$ $\qquad$ $T_2 = ?$

$T_1 = 10°C = 283,15\,K$

$P_1 = 1\,atm = 101325\,Pa$

$\gamma = 1,4 \qquad R = 287\,\frac{J}{kg\,K}$

monatomic gas : $n_{fr} = 3$

Air $\longrightarrow$ biatomic gasses : $n_{fr} = 5$ $\longrightarrow c_v = \frac{5}{2}R,\ c_p = \frac{7}{2}R$

polyatomic gasses : $n_{fr} = 6$

$$\boxed{c_v = \frac{n_{fr}}{2} \cdot R}$$

$$\boxed{c_p = c_v + R} = \frac{n_{fr}+2}{2}R$$

$$\boxed{\gamma = \frac{c_p}{c_v}} = \frac{7}{5} = 1,4$$

$$\frac{c_p}{c_v} = \gamma$$

$$c_p - c_v = R \longrightarrow c_p = \frac{\gamma}{\gamma - 1}R = 1004\,\frac{J}{kg\,K}$$

1) Calculate T_2, with a isothermal transformation

Isothermal $\Rightarrow T_2 = T_1 = const$

$$l + q = \Delta h + \cancel{\frac{\Delta V^2}{2}} + \cancel{g\Delta z}^{\,0} = \Delta h_T$$

$\longrightarrow$ negligible since $V_1 \approx V_2$

$$= \Delta h = c_p (T_2 - T_1)$$

$$\underbrace{l - l_w}_{\text{Technical Work}} = \int v \, dP + \cancel{\frac{\Delta v^2}{2}} + \cancel{g \Delta z}$$

Isothermal Process $\Rightarrow l + q = 0 \Rightarrow l_T = -q$ $\Rightarrow$ we need to remove heat to keep temperature constant and counteract l.

$$\Rightarrow l_T - \underbrace{l_w}_{\to 0 \text{ since isothermal, no irreversibility}} = \int_1^2 v \, dP \quad (\text{A})_1$$

We can use the equation of state

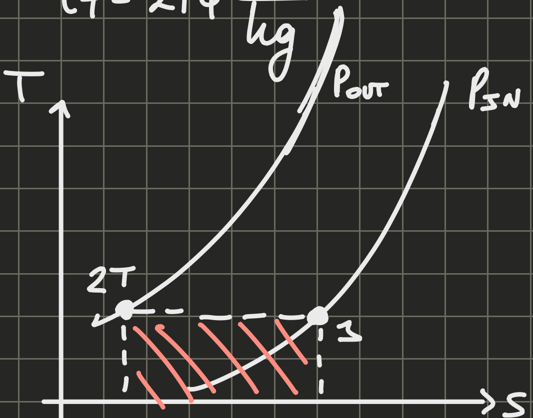
$$Pv = RT \rightarrow v = \frac{RT}{P}$$

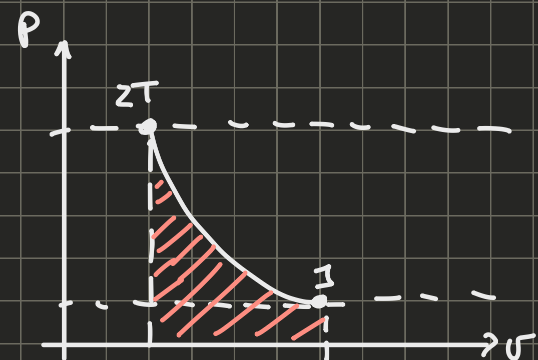
Replacing v in $(\text{A})_1$:

$$l_T = \int_1^2 RT \frac{dP}{P} = RT_1 \int_1^2 \frac{dP}{P} = RT_1 \ln \frac{P_2}{P_1}$$

$\Rightarrow$ we can calculate l_T, since we have T_1 and β

$$l_T = 214 \frac{kJ}{kg}$$





Isothermal transformations are not very realistic since q is typically not very high.

2) Isoentropic → also not realistic but is a good reference.

Clausius Theorem: $\cancel{q} = \underbrace{\int T ds}_{=0 \text{ since isothermal}} - \int T \delta s_{irr}$

$q = 0$ adiabatic

$\Rightarrow \delta S_{irr} = 0$, so isoentropic processes are reversible

$l_s + \cancel{q} = c_p (T_{2s} - T_1)$ → T_2 is always bound by the transformation.

$$\longrightarrow l_s = \frac{\gamma R}{\gamma - 1} T_1 \left(\underbrace{\frac{T_{2s}}{T_1}}_{\text{Unknown}} - 1 \right) \circledast_2$$

We can relate it to the pressure ratio:

$$du = Tds - Pdv$$
$$dh = du + d(Pv)$$
$$\longrightarrow dh = Tds + vdP \longrightarrow ds = \frac{dh}{T} - \frac{v}{T} dP$$

We know that for a perfect gas $ds = c_p \frac{dT}{T} - R \frac{dP}{P}$

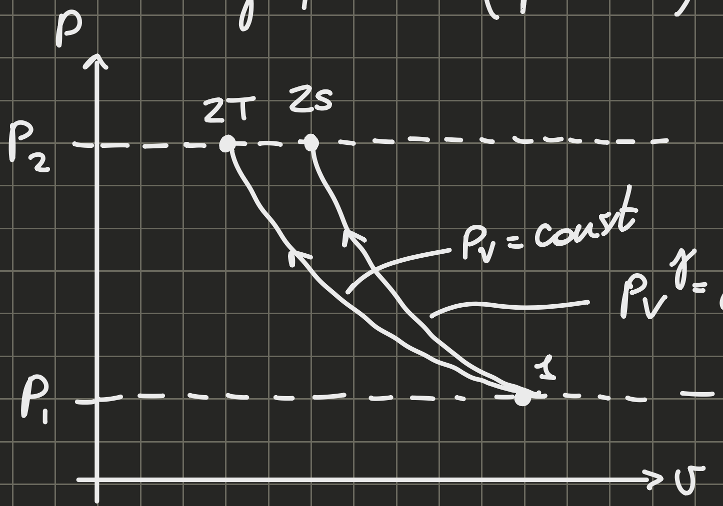
Since we are using an isoentropic transformation:

$$ds = 0 \Rightarrow \frac{dT}{T} = \underbrace{\frac{\gamma - 1}{\gamma}}_{1/c_p} \frac{dP}{P} \rightarrow \ln\left(\frac{T_{2s}}{T_1}\right) = \ln\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow \text{Only for isoentropic}$$

We can put this in the equation ⊛$_2$:

$$l_s = \frac{\gamma}{\gamma-1} R T_1 \left(\beta^{\frac{\gamma-1}{\gamma}} - 1\right) \underset{\text{case}}{\overset{\text{in this}}{=\!=}} 322 \frac{kJ}{kg} > l_T$$



$l_s > l_T \rightarrow$ always if β is the same and T_1 too.

Since we know: $l_s = c_p(T_{2s} - T_1)$ we can find T_{2s}.

We also know $\frac{T_{2s}}{T_1} = \beta^{\frac{\gamma-1}{\gamma}}$ so we can also use that

$T_{2s} = 601{,}5\ K$

3) Real Adiabatic Compression $\eta_s = 0{,}85$ ← Isoentropic efficiency

$$l = c_p(T_2 - T_1) = \frac{\gamma R}{\gamma-1} T_1 \left(\frac{T_2}{T_1} - 1\right)$$

$l - l_w = \int v\,dP$

↳ we again would need to use $Pv^n = const$ to solve the integral

$\frac{T_2}{T_1} = \beta^{\frac{n-1}{n}}$ If we know n, we could do the same as before

Since we have nothing else, we use the definition of the isoentropic efficiency:

$$\eta_s = \frac{l_s}{l} = \frac{\frac{\gamma R}{\gamma-1} T_1 \left(\beta^{\frac{\gamma-1}{\gamma}} - 1\right)}{l}$$

$$\Rightarrow l = \frac{l_s}{\eta_s} = 376{,}5 \ \frac{kJ}{kg}$$

$$T_2 = T_1 + \frac{l}{c_p} = 657{,}8 \ K$$

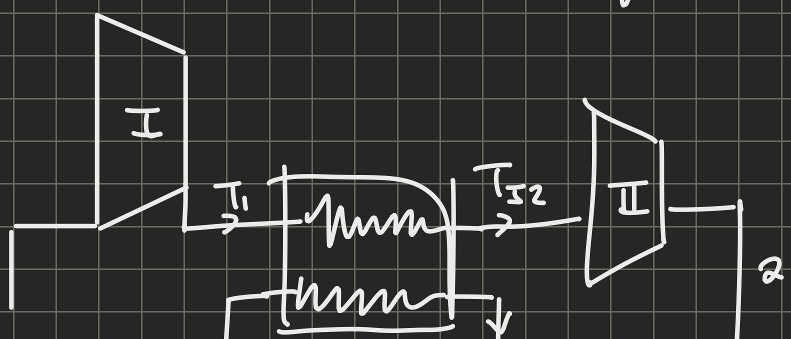
$l > l_s$ because of two reason.

The first is the irreversibilities which come with a real transformation.

Since for the real transformation we use a q_{REV} to model the real q, we gain an additional reheat effect term, which is reversible and is just a consequence of thermodynamics.

$$\underbrace{l}_{\text{Real Work}} = l_s + l_w + \underbrace{l_R}_{\text{Reheat Effect}}$$

4) With intercooler between stages.



$T_{I2} = T_1$

$$\beta = \frac{p_2}{p_1} = \underbrace{\frac{p_2}{p_{I2}}}_{\beta_{II}} \cdot \underbrace{\frac{p_{I1}}{p_1}}_{\beta_I} = \beta_{ST}^2$$

$\beta_I = \beta_{II} = \beta_{ST}$

$$\beta_{ST} = \sqrt{\beta} = 3{,}74$$

$$l_I = l_{Is} = \frac{\gamma R}{\gamma - 1} T_1 \left(\overbrace{\frac{T_{I1}}{T_1}}^{\beta_{ST}^{\frac{\gamma-1}{\gamma}}} - 1 \right) = \frac{\gamma R}{\gamma - 1} T_1 \left(\beta^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

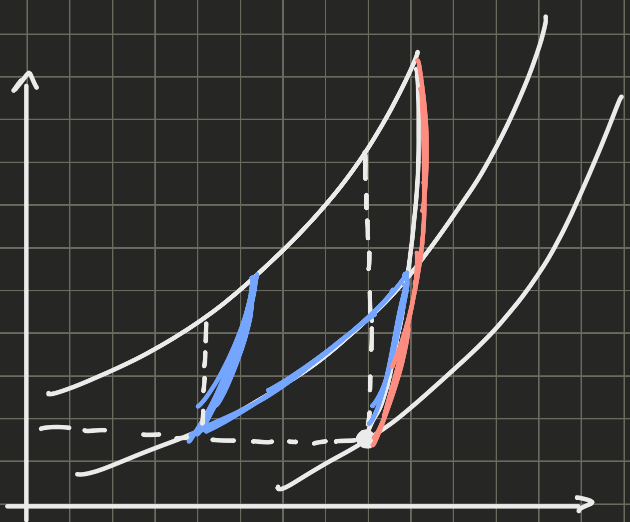
↳ since we are told the compressors are ideal

$$l_{II} = l_{IIs} = \frac{\gamma R}{\gamma - 1} \underset{\substack{\parallel \\ T_1}}{T_{I2}} \left(\beta^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$\Rightarrow l_I = l_{II}$

$$l_I = l_{II} \Rightarrow l_I + l_{II} = \frac{2\gamma R}{\gamma - 1} T_1 \left(\beta_{ST}^{\frac{\gamma-1}{\gamma}} - 1 \right) = 260.2 \frac{kJ}{kg}$$

$l_I + l_{II} < l$, because we won't have legacy re-heat work, which will increase the work we need to do.

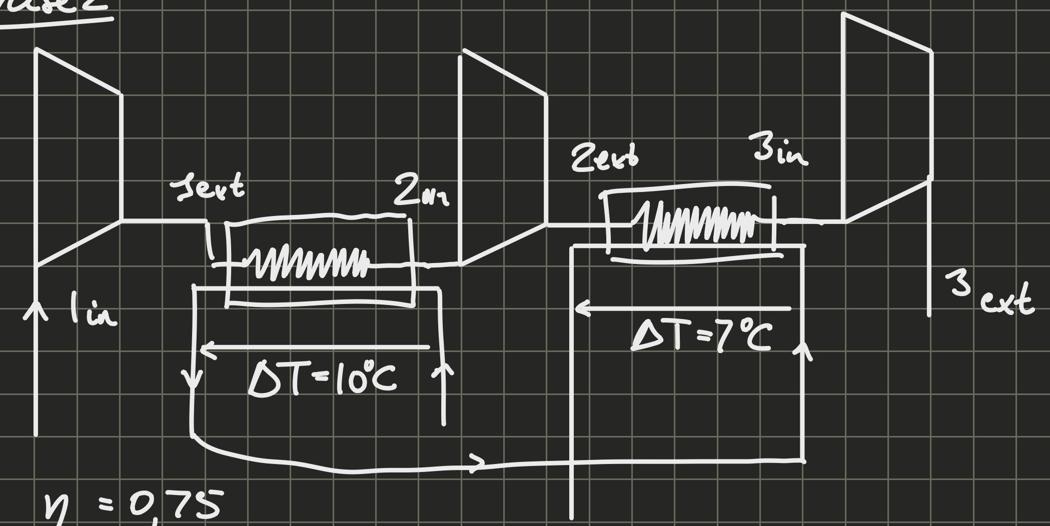


$$l_I + l_{II} = c_p (T_2 - T_1) \rightarrow T_2 = 412{,}5\,K$$

We can also use β_{ST} to find T_2, from T_{I2}:

$$\frac{T_2}{T_{I2}} = \beta_{ST}^{\frac{\gamma-1}{\gamma}}$$

Exercise 2



$$\eta_s = 0{,}75$$

$$\dot{m}_{air} = 15{,}91 \ \frac{kg}{s}$$

$$\dot{L} = \dot{m}_{AIR}\left(l_I + l_{II} + l_{III}\right) = 5250\ kW$$

$T_{1,IN} = 300K$

$P_{Iin} = 1bar = 100000\ Pa$

$\beta_I = \beta_{II} = 2{,}5$

$T_{2in} = T_{1IN}$

$C_{H_2O} = 4186 \ \frac{J}{kgK}$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

γ: 1,4 for air

R: 287 $\frac{kJ}{kgK}$ for air

Stage 1

$$l_I = c_p\left(T_{1OUT} - T_{1IN}\right) = \frac{l_s}{\eta_s} = 120{,}3\ \frac{kJ}{kg} \Rightarrow T_{1,OUT} = 419{,}8\ K$$

$$l_s = \frac{\gamma R}{\gamma-1} T_{1IN}\left(\underbrace{\frac{T_{1,EXT,s}}{T_{1,IN}}} - 1\right) = \frac{\gamma R}{\gamma - 1} T_{IN}\left(\beta_I^{\frac{\gamma-1}{\gamma}} - 1\right) = 90{,}2\ \frac{kJ}{kg}$$

$$\beta_I^{\frac{\gamma-1}{\gamma}}$$

$P_{1OUT} = P_{1IN}\,\beta_I = 2{,}5$ bar

<u>Stage 2</u>

$$l_{IIs} = \frac{\gamma R}{\gamma-1} T_{2IN}\left(\beta_{II}^{\frac{\gamma-1}{\gamma}} - 1\right) = 90{,}2$$

since η_s = const and $l_{sII} < l_{sI} \Rightarrow l_{II} = l_I = 120{,}3\ \frac{kJ}{kg}$

$T_{2OUT} = T_{1OUT} = 419{,}8$ K

↳ since $T_{2IN} = T_{1IN}$ and $l_I = l_{II}$

$P_{2OUT} = P_{2IN} \cdot \beta_{II} = 6{,}25$ bar

Intercoolers → isobaric heat exchangers.

So $P_{OUT} = P_{IN}$

<u>HX I</u> → To find $\dot{m}_{water}$, which will be needed.

no work exchanged; ≈ 0, since we are in ducts

$$\cancel{l} + q = \Delta h + \cancel{\frac{\Delta v^2}{2}} + \cancel{g\Delta z}$$

$q = \Delta h =$

to keep $\dot{Q}$ positive, since heat is leaving the air and entering the water

$$\left.\begin{aligned} -\dot{Q}_{air} &= \dot{m}_{air}\, c_{p,air}\left(T_{2IN} - T_{1OUT}\right) \\ \dot{Q}_{H_2O} &= \dot{m}_{water}\, c_{H_2O}\, \Delta T_1 \end{aligned}\right\}$$

$$\rightarrow \dot{m}_{water}\, c_{H_2O}\, \Delta T_1 = \dot{m}_{air}\, c_{p,air}\left(T_{1,OUT} - T_{2,IN}\right)$$

$$\rightarrow \dot{m}_{water} = \frac{\dot{m}_{air}\, c_{p,air}\left(T_{1,OUT} - T_{2,IN}\right)}{c_{H_2O}\, \Delta T_1}$$

$\rightarrow \dot{m}_{water} = 45{,}7 \ \frac{kg}{s}$

HX2

$$|\dot{Q}_{AIR}| = \dot{m}_{air}\, c_{p,air}\left(T_{2,OUT} - T_{3,IN}\right)$$

$$|\dot{Q}_{wtr}| = \dot{m}_{water}\, c_{H_2O}\, \Delta T_2$$

$$T_{3,IN} = T_{2,OUT} \frac{\dot{m}_{water}\, c_{H_2O}\, \Delta T_2}{c_{p,air}\, \dot{m}_{air}} = 336\,K$$

$$l_{III} = \frac{\dot{L}}{\dot{m}_{air}} - l_I - l_{II} = 89{,}4\ \frac{kJ}{kg}$$

$$l_{III} = c_p\left(T_{3OUT} - T_{3IN}\right) \longleftrightarrow T_{3OUT} = 425\,K$$

$$\eta_s = \frac{l_{IIIs}}{l_{II}} \rightarrow l_{IIIs} = \eta_s\, l_{III} = 67{,}1\ \frac{kJ}{kg}$$

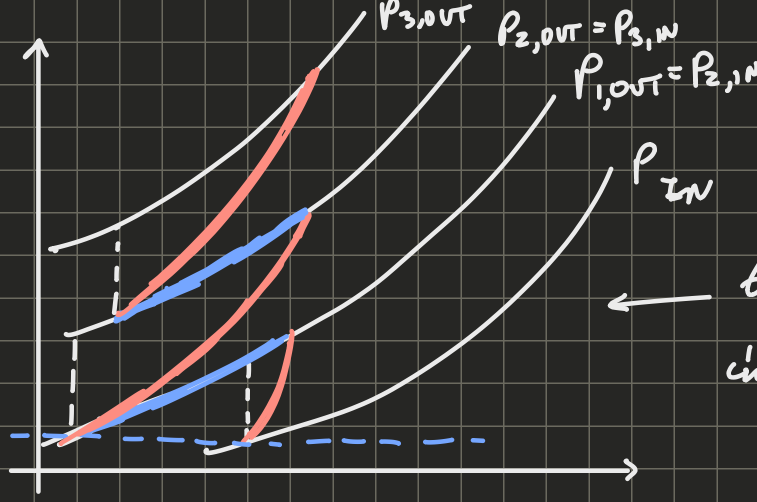
$$l_{IIIs} = \frac{\gamma R}{\gamma - 1} T_{3IN}\left(\beta_{III}^{\frac{\gamma-1}{\gamma}} - 1\right) \longrightarrow \beta_{III} = 1{,}89$$

$$P_{3,OUT} = P_{3,IN}\, \beta_{III} = 11{,}8\ bar$$

The other way is to write:

$$l_{III,s} = c_p\left(T_{3,OUT,S} - T_{3,IN}\right)$$ $\rightarrow$ from here we can get $T_{3,OUT,S}$ and find β since

$$\frac{T_{3,OUT,S}}{T_{3,IN}} = \beta_{III}^{\frac{\gamma-1}{\gamma}}$$



these lines should be less inclined.

Exercise 3 → Gas Dynamics

When we look inside the machine $\frac{\Delta v^2}{2}$ is no longer negligible.

such that: $l = \Delta h + \frac{\Delta v^2}{2} = \Delta h_T$

$\dot{m} = 9 \frac{kg}{s}$

$P_{T0} = 1{,}5$ bar

$T_{T0} = 450$ k

$T_0 = 0{,}5\, T_{T0} = 225$ k

we don't have these, we need to find them

$\dot{m} = \rho v_0 S_0$

need these to solve problem.

$P_T, h_T \Rightarrow$ properties of the flow

h_T is the total enthalpy of the flow found by bringing the flow to a stop, the process which we use is irrelevant because Δh includes both thermal and mechanical component which can assume the kinetic energy of the flow, it will also consider due to the irreversibility of stopping the flow.

$h_T = h + \frac{v^2}{2}$

$$\hookrightarrow \frac{T_T}{T} = 1 + \frac{V^2}{\underset{\frac{c_v \gamma}{\gamma-1}}{2c_p T}} = 1 + \frac{\gamma-1}{2}\frac{V^2}{\gamma RT} = 1 + \frac{\gamma-1}{2}M^2$$

From T_T and M, we can calculate the static temperature T.

$M_0 = 2.24 \rightarrow$ we have supersonic flow at the inlet.

P_T can only be found through iso-entropic cooling of the flow.

$$\rho_0 = \frac{P_0}{RT_0}$$ $P_0 \rightarrow$ we don't have this, but we can find it, since we have P_{T0} and other data.

$$\frac{P_{T0}}{P_0} = \left(\frac{T_{T0}}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$ $\rightarrow$ because P_T is found through isoentropic processes, this is how it's found.

$$\frac{P_{T_0}}{P_0} = \left(1 + \frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}} \longrightarrow P_0 = 0,133 \text{ bar}$$

$$\rho_0 = \frac{P_0}{RT_0} = 0,205 \frac{kg}{m^3}$$

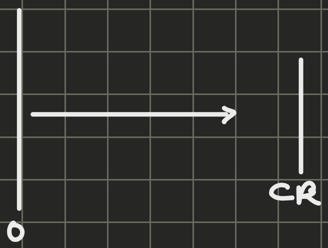
$$M_0 = \frac{V_0}{a_0} = \frac{V_0}{\sqrt{\gamma RT_0}} \longrightarrow V_0 = 672,3 \frac{m}{s}$$

It's also possible to use:

$$\underset{c_p T_T}{\underset{\shortparallel}{h_T}} = \underset{c_p T}{\underset{\shortparallel}{h}} + \frac{V^2}{2}$$ to find V.

$$S_0 = \frac{\dot{m}}{\rho_0 V_0} = 0{,}065\ m^2$$

b) $S = ?\ :\ M_1 = 1$



$$\frac{T_{TCR}}{T_{CR}} = 1 + \frac{\gamma - 1}{2} \underbrace{M_{CR}}_{=1}$$

$\Delta h_T = \ell = 0 \Rightarrow h_T = \text{const} \Rightarrow T_T = \text{const} \Rightarrow T_{TCR} = T_{T0}$

↳ in this case

$\Rightarrow T_{CR} = 375\ K$

same as before $P_{TCR} = P_{T0}$, since $\ell = 0$

$$\frac{P_{TCR}}{P_{CR}} = \left(1 + \frac{\gamma-1}{2} M_{CR}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow P_{CR} = 0{,}79\ \text{bar}$$

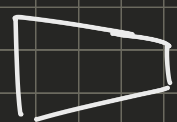
We could also calculate the static temperature ratio and find from there β, such that $\frac{T_{CR}}{T_0} = \beta^{\frac{\gamma-1}{\gamma}}$

$$\rho_{CR} = \frac{P_{CR}}{R T_{CR}} = 0{,}734\ \frac{kg}{m^3}$$

$$V_{CR} = \dot{m}_{CR} \sqrt{\gamma R T_{CR}} = 388{,}2\ \frac{m}{s}$$

$$\rightarrow S_{CR} = \frac{\dot{m}}{\rho_{CR} V_{CR}} = 0{,}0315\ m^2$$

⇑ cross-section needs to decrease



$$\underset{(-)}{\frac{dS}{S}} \Leftarrow \underset{(+)}{(M^2 - 1)} \underset{(-)}{\frac{dv}{v}}$$

→ since we're reducing v, S will also decrease, since M is > 1.

$P_{OUT} = 1\,bar$

$$\frac{\overset{=P_{T0}}{P_{TOUT}}}{P_{OUT}} = \left(1 + \frac{\gamma - 1}{2} M_{OUT}\right)^{\frac{\gamma}{\gamma-1}} \longrightarrow M_{OUT}$$

$$\frac{P_{OUT}}{P_{CR}} = \left(\frac{T_{OUT}}{T_{CR}}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_{OUT} = 400{,}8\,K$$

$\rho_{OUT} = 0{,}869\ kg/m^3$

$$h_{TOUT} = h_{OUT} + \frac{v_{OUT}^2}{2}$$

$$v_{OUT} = \sqrt{2C_p(\underbrace{T_{TOUT}}_{=T_{T0}} - \underbrace{T_{OUT}}_{known})} = 314{,}4\ \frac{m}{s}$$

We find $M<1$, that means that the cross-section will have to enlarge.

$$S_{OUT} = \frac{\dot{m}}{\rho_{OUT} V_{OUT}} = 0{,}0329\,m^2$$