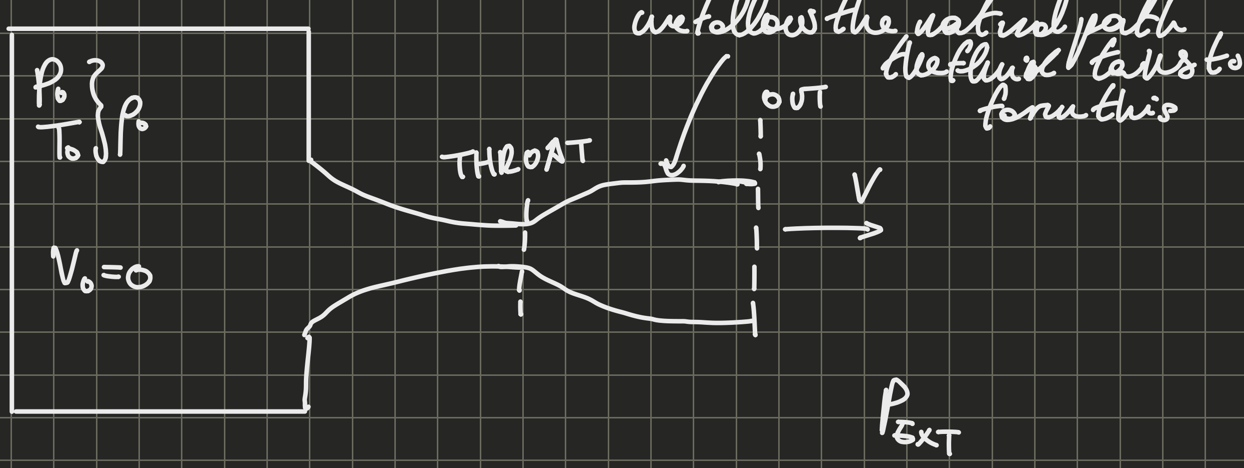
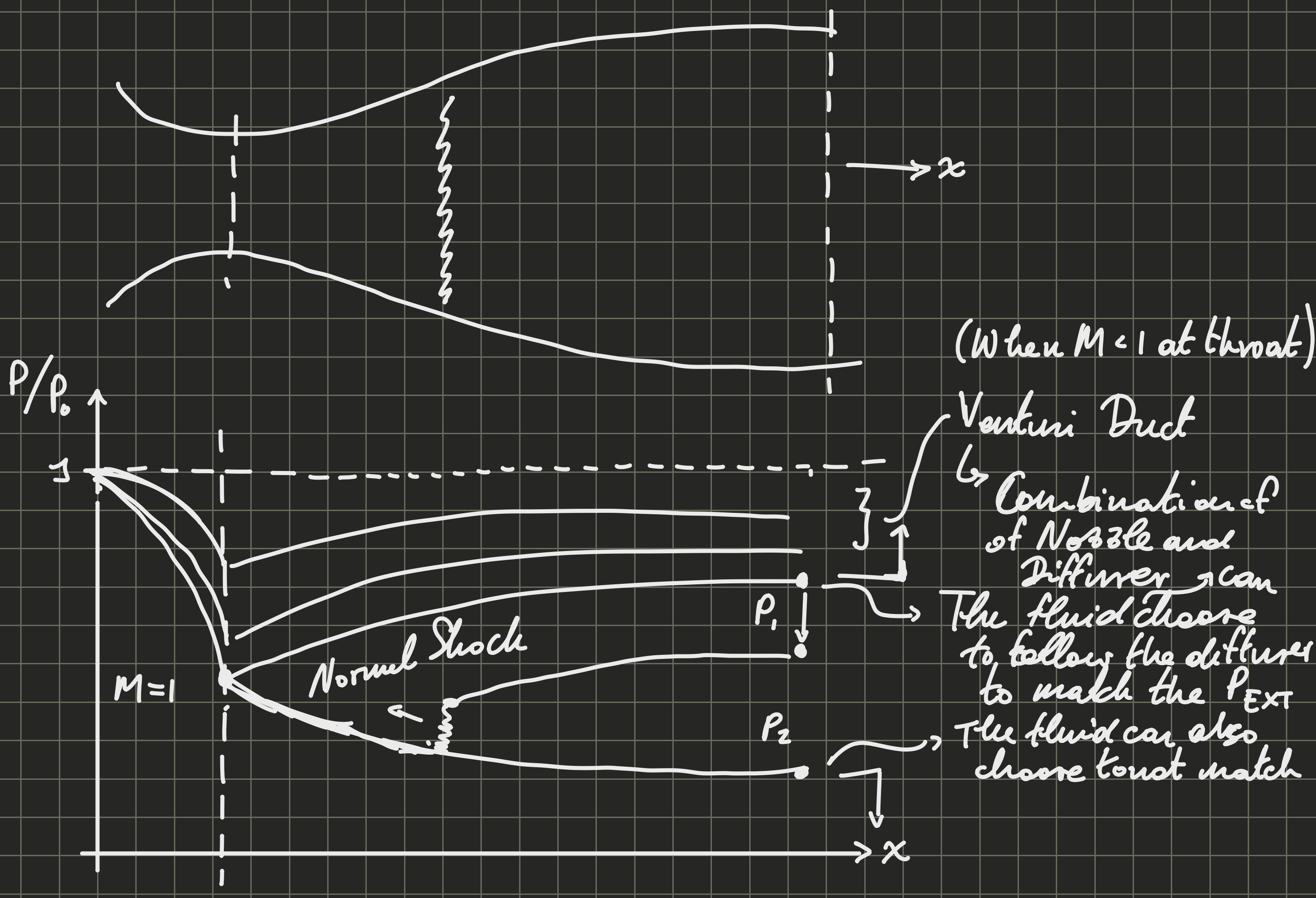
Lezione 22

$\boxed{14:34}$ De-Laval Nozzle




$M<0{,}3$ Venturi Duct

$0{,}3<M<1$ compressible "

$M=1$ Sonic "

For $P_{EXT} \geq P_1$ Venturi (uncompressible, compressible, sonic)

For $P_{EXT} \leq P_2$ : Full Nozzle

↳ Subsonic Converging Nozzle, supersonic diverging nozzle.

The missing pressure difference with expand after the divergent.

normal shock wave

If $P_1 < P_{EXT} < P_2$ → our models do not provide any solution. To get this solution we need to eliminate the isoentropic condition.

The fluid will treat a part of the diffuser as a nozzle, then a shock wave creates a discontinuity and then the fluid will act subsonically and see a diffuser so will do to the wanted pressure

The only way to reach the boundary condition $P_{EXT}$, is to not be isoentropic and allow discontinuity.

$P_0$ and $T_0$ are always good boundary conditions.

We need another condition, since we have 3 equations (in 1D)

$P_{THROAT}$ cannot be used since all $P_{EXT} < P_1$, will have passed subsonic speed at the nozzle, so there will be infinite solutions, all below $P_1$.

↳ Same for $\dot{m}_{THROAT}$, since any point where $P_{EXT} \leq P_1$ will have the same flow rate.

$P_{EXT}$ allows us to define a well-posed problem.

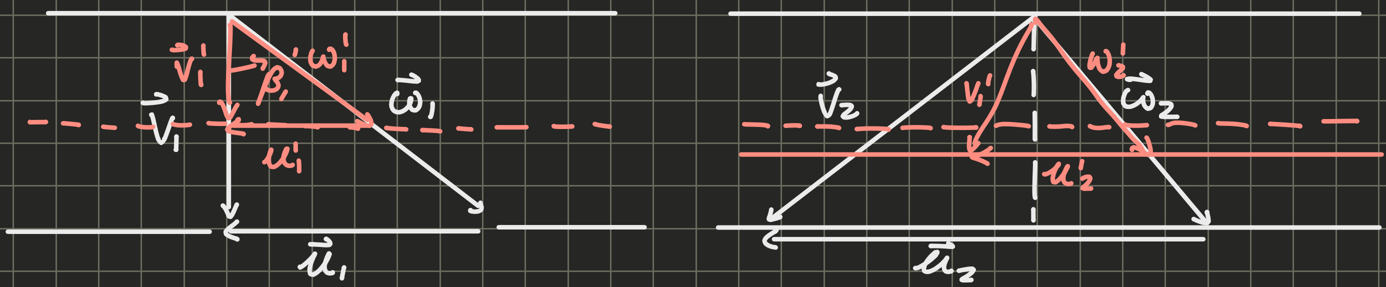
# Analysis of Thermal Machines

## (Turbo-) Compressors: Implication of Compressibility

↳ Compressor with the shape of a turbomachine.

We will see the effects of compressibility as a difference to the incompressible case.

Similarity: is $\varphi$ and $\psi$ being the same enough now?




$$\underline{\dot{m}'} = 1/2\ \dot{m}$$

$\rho_1' = \rho_1$ → same density since we are taking from an external environment

$A_1' = A_1$

$\rightarrow v_{1m}' = \frac{1}{2} v_{1m}$

$\downarrow$

$\underline{v_1'} = \frac{1}{2} v_1'$

$$\varphi' = \varphi \Rightarrow n' = \frac{1}{2} n\ ,\ \underline{u_1'} = u_1$$

For now it's working by following the rules of hydraulics.

$$\underline{\beta_1'} = \beta_1$$

$$\dot m' = \dot m_2' = \rho_2' V_{2m}' A_2$$

↳ The machine does not change

$$\dot m = \dot m_2 = \rho_2 V_{2m} A_2$$

$$\dot m' = \frac{1}{2}\dot m$$

$$\Rightarrow \rho_2' V_{2m}' = \frac{\rho_2 V_{2m}}{2} \Rightarrow \underline{V_{2m}' = \frac{1}{2} V_{2m} \cdot \frac{\rho_2}{\rho_2'}}$$

Bad news, since as a consequence of the thermodynamic transformation, the velocity triangles will be altered.

$$l_s = c_p \Delta T_{in}\left(\beta^{\frac{\gamma-1}{\gamma}} - 1\right), \quad l \propto u^2 \quad (u_2 V_{2t})$$

↳ Work exchanged

In case of perfect similarity is $\frac{1}{4}$, but

Since we have said $u_1' = \frac{1}{2} u_1$

$$l' \tilde{\propto} \frac{1}{4} l \rightarrow \beta \downarrow \rightarrow \rho_2' \downarrow \text{ relative to } \rho_2.$$

This cause $\alpha_2' \neq \alpha_2$, and so similarity is not respected.

The hydraulic approach to similarity

So apply similarity at the outlet goes not give similarity at the outlet and vice versa.

We can use $\Delta T$ to compensate the change in $\rho$.

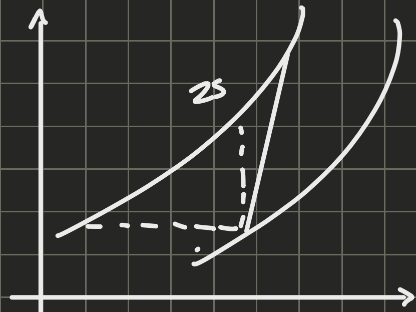
What is the final approach found to solve it.

We don't derive approach in this part of the course.

## Dimensionless Approach

We used mass flow rather than volumetric flow rate

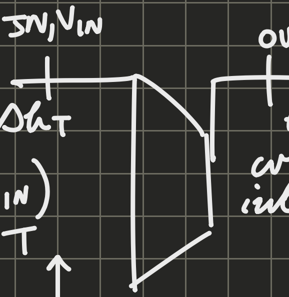
$$\varphi \rightsquigarrow \dot{m}_{AD} = \frac{\dot{m}\sqrt{R\,T_{T,IN}}}{P_{T,IN}\,D^2}$$




We can still use $\rightarrow \psi = \frac{l_s}{n^2 D^2} = \frac{\gamma}{\gamma-1}\cdot\frac{\mathcal{R}}{M}\,T?$

$$l + q = \Delta h + \frac{\Delta V^2}{2} + \underbrace{g\Delta z}_{\to 0} = \Delta\left(h + \frac{V^2}{2}\right) = \Delta h_T$$

$$= c_p(T_{TOUT} - T_{T,IN})$$

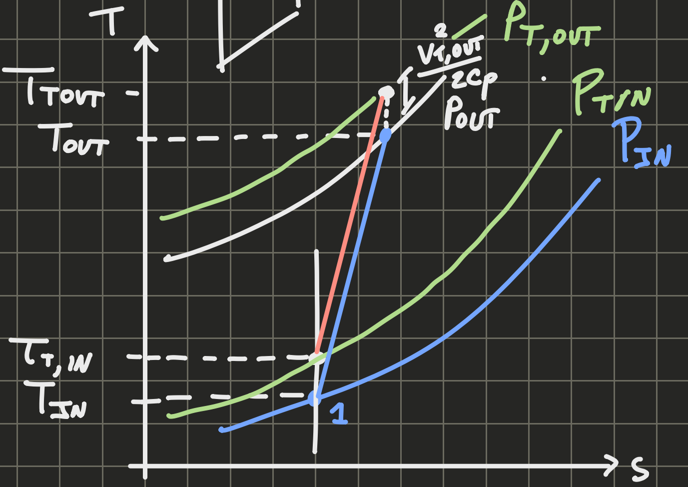



we assume $V_1 \neq V_2$ to be able to understand what T we are interested in

↑ the full form

$l = \cancel{c_p \Delta T}$ → only if we neglect $\Delta V^2$

$$= c_p T_{TIN}\left(\frac{T_{TOUT}}{T_{TIN}} - 1\right)$$




$T_{T1} = T_{T,IN}$

$T_1 = T_{IN}$

$$T_{TIN} = T_{IN} + \frac{V_{IN}^2}{2c_p}$$

$$= c_p T_{TIN}\left(\left(\frac{P_{T,OUT}}{P_{T,IN}}\right)^{\frac{n-1}{n}} - 1\right),$$

$\beta_T$ → total pressure ratio

↳ more precise than $\beta$ since it doesn't ignore $\Delta V^2$

$$\beta_T \triangleq \frac{P_{T,OUT}}{P_{T,IN}}$$

Since are now inside the machine, $V_1 \neq V_2$, so we have to also consider it in the transformation, and therefore we use $\beta_T$, $T_T$ and $P_T$ to model similarity in a compressor.

$$l_s = c_p T_{TIN}\left(\beta_T^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$\Rightarrow \psi = \frac{l_s}{n^2 D^2} = \frac{\frac{\gamma}{\gamma-1}\,\frac{\mathcal{R}}{M}\,T_{T,IN}\left(\beta_T^{\frac{\gamma-1}{\gamma}} - 1\right)}{n^2 D^2}$$

The dimensionless parameter we need to add is one for the compressibility, where: → which is the Mach number

$$M_U = \frac{\omega D/2}{\sqrt{\gamma R T_{TIN}}} \;;\quad Re = \frac{\omega D^2}{\nu} \;;\quad \gamma$$

$\omega D/2$ → Dm or $D_2$ based on machine

$M_U$ → Peripheral Mach number

$T_{TIN}$ → Each $T_{TIN}$ to find and also boundary condition.

→ u needs to be scaled, so that is useful.

$\nu$, $\gamma$ ← we add these too

We have a system of 5 dimensionless parameters.

We have 4 fundamental dimensional quantities, we need to find.

In $\Psi$, $\beta_T$ is the only parameter which doesn't appear in other equations, so instead of using the complicated equation for $\Psi$, we use $\beta_T$ since it's also nice for engineers designing.

$$\beta_T = \beta_T\left(\dot{m}_{AD}, M_U, Re, \cancel{\gamma, \text{SHAPE}}\right)$$

$$\eta = \eta\left(\dot{m}_{AD}, M_u, Re, \cancel{\gamma, \text{SHAPE}}\right)$$

Generally these are fixed, so we can reduce to 3 dimensionless parameters.

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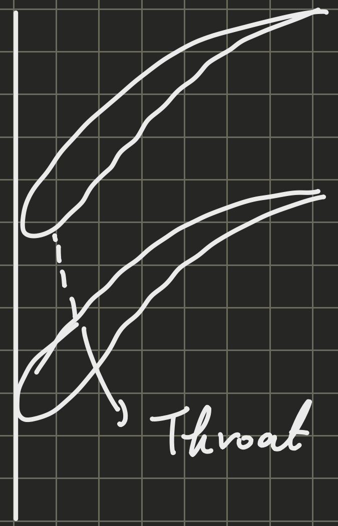
Increasing $M_U$ increases the range the machines can operate in.

$\boxed{16:25}$ The curves become vertical since the pressure ratio is decreasing as we change the speed, in some part of the machine, there is a part that chokes because it has reached mach number.

Once $M=1$ is reached, the curve becomes vertical.




Creating a line from where all these points where the curve chokes, we create the choked-flow line.

In the case of compressors we should be worried about unstable connections, since that can cause issue, this is why there is nothing before the maximum pressure since engineers determined operating before as unsafe.

Curves of compressors become vertical due to choking, this point is based on $M_u$.
The machine curves are cut to only safe operating zones.

Like the dimensionless parameters we can extend ωs and Ds to turbo machines with some limit.

Specific Parameters for Compressors (Thermal Turbines)

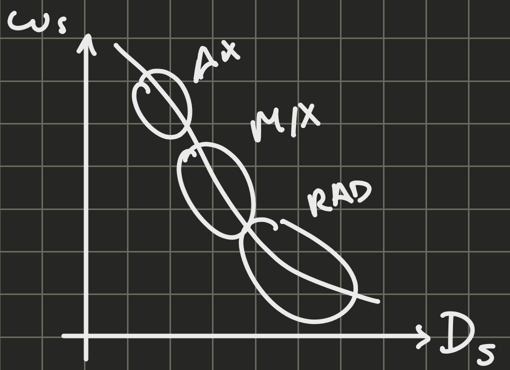
$\varphi \to \omega_s$

$\psi \to D_s$

They are incomplete, we need to add (at least) the $M_v$

$M_v = f(\omega_s, D_s, \dots)$?

$\hookrightarrow$ If we find this, we find the limitation.




$$M_v = \frac{\omega\, D/2}{\sqrt{\gamma R T_{TIN}}} = \omega_s \frac{\ell_s^{3/4}}{\cancel{\sqrt{Q_{IN}}}} \cdot \frac{1}{2} D_s \frac{\cancel{\sqrt{Q_{IN}}}}{\ell_s^{1/4}} \cdot \frac{1}{\sqrt{\gamma R}\,\sqrt{T_{T,IN}}}$$

$$\omega_s = \omega \cdot \frac{\sqrt{Q_{IN}}}{\sqrt[4]{\ell_s^3}}$$

$$D_s = D \frac{\sqrt[4]{\ell_s}}{\sqrt{Q_{IN}}}$$

$$= \omega_s D_s \cdot \frac{1}{2} \frac{\sqrt{\frac{\cancel{\gamma R}}{\gamma - 1} \cancel{T_{TIN}} \left(\beta_T^{\frac{\gamma-1}{\gamma}} - 1\right)}}{\cancel{\sqrt{\gamma R}}\ \cancel{\sqrt{T_{TIN}}}}$$

$$= \omega_s D_s \cdot \frac{1}{2} \sqrt{\frac{\beta_T^{\frac{\gamma-1}{\gamma}} - 1}{\gamma - 1}}$$

$$\Rightarrow M_v \propto \omega_s, D_s, f(\beta_T, \gamma)$$

One we put a point on the Balje diagram, the efficiency depends on $\omega_s$, $D_s$ and $M_v$, this is true because comparing different machines using the same fluid, if they have different $\beta_T$ they will have different efficiency.

Fixing a point, we also fix the $M_u$, and so the Balje diagram is given for a given pressure ratio.

Not really, but it's a simplification that doesn't cost much.