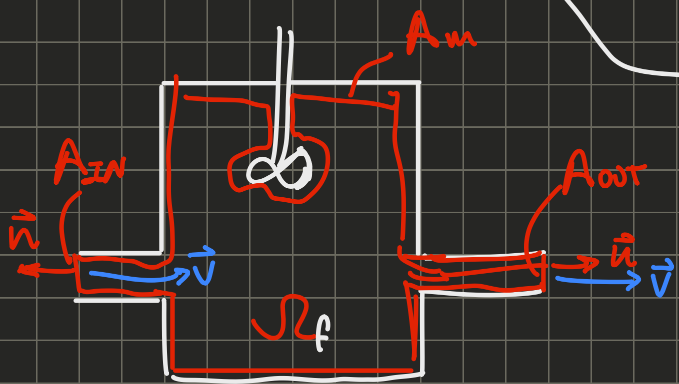
Lezione 5 -

What we did yesterday and will do today should be reviewed as quick as possible.

Mass in the control volume can either accumulate or change through a differential in the mass entering and exiting at the two ports.

Energy Balance

(Balance of Power really since



$$\frac{dM}{dt} = 0$$

M → Mass of the fluid body

where $M = \int_{\Omega(t)} \rho \, dV$

linear momentum

$$\vec{\Pi} = \int_{\Omega(t)} \rho \vec{v} \, dV \longrightarrow \frac{d\vec{\Pi}}{dt} = \vec{F}$$

Newton's law

$\vec{F}$ ↳ summation of all the forces

Volume forces

$$\vec{F} = \int_{\Omega(t)} \rho \vec{f} \, dV + \int_{\partial\Omega(t)} \vec{\sigma} \, dA$$

$\vec{f}$ ↳ force field

$\partial\Omega(t)$ → Surface forces such as pressure and friction.

These laws are for bodies not control volumes.

$$\vec{\sigma} = -\vec{P}_n + \vec{\tau}$$

$\vec{\tau}$ → It's dependent on the speed of deformation of the fluid body (seen in MF) → strain-rate

14:46

In general $\tau \propto \frac{\partial V_i}{\partial x_j} \cdot \mu$

Our equations are instantaneous, we don't need the history, we can take the fluid body as the one which at $t = \tau$ fills the control volume.

At $t=\tau \rightarrow \Omega(\tau) = \Omega_f \Rightarrow$ For RTT we can calculate the $\dot{L}$ and $\dot{Q}$ in the CV without issue.

$$= \int_{\Omega(\tau)} \rho \vec{f}\, dV + \int_{\partial\Omega(\tau)} \vec{\sigma}\, dA$$

$$= \left. \int_{\Omega_f} \rho \vec{f}\, dV + \int_{\partial\Omega_f} \vec{\sigma}\, dA \right\} \ 14:49$$

since we taking snapshot

$$\vec{\Gamma} = \int_{\Omega(t)} \vec{r} \times \rho \vec{v}\, dV \qquad \frac{d\vec{\Gamma}}{dt} = \vec{M}$$

Angular momentum

Moment / Torque of the forces

$$\vec{M} = \int_{\Omega(t)} \vec{r} \times \rho \vec{v}\, dV + \int_{\partial\Omega(t)} \vec{r} \times \vec{\sigma}\, dA$$

For the same reasons as

$$= \int_{\Omega_f} \vec{r} \times \rho \vec{v}\, dV + \int_{\Omega_f} \vec{r} \times \vec{\sigma}\, dA$$

We will not be developing linear/angular momentum momentum balances since they are not too useful (except for one case)

We can only calculate the rate of change of energy, not the energy exchange, so we calculate power.

We will be using the istantaneous exchange of work and heat.

Istantaneous power that can be exchanged by mechanical or thermal power.

$$\frac{dE}{dt} = \dot{L}_o + \dot{Q}$$

overall

There are other methods of exchanging power but the are not useful for us.

We need to find E, $\dot{L}_o$ and $\dot{Q}$.

why this overall

We focus mainly on E and $\dot{L}_o$

$\boxed{E}$

$$E_k = \frac{1}{2} V^2 M \rightarrow \boxed{E_k = \int_{\Omega(t)} \frac{1}{2} \rho \overset{\text{speed}}{V^2} \, d\overset{\text{volm}}{V}}$$

$$E_p = g z M \rightarrow \boxed{E_p = \int_{\Omega(t)} \rho g z \, dV}$$

$$\boxed{U = \int_{\Omega(t)} \rho u \, dV}$$

$$\underbrace{\frac{d}{dt} \int_{\Omega(t)} \rho \left(u + \frac{V^2}{2} + gz \right) dV}_{\frac{dE}{dt}} = \dot{L}_o + \dot{Q}$$

we can say more about this

like volume forces

If we only consider only conservative forces, the mechanical power can be see it as a change in potential energy. Since cannot count it twice, we will not continuing it as a change in work, therefore the potential energy change is considered a just the change in the potential energy, and only count it there, in $\frac{dE}{dt}$.

15:21

these are not dependent on how we design our macchine

$$\dot{L}_o = \int_{\partial\Omega(t)} \vec{\sigma} \vec{v} \, dA \xrightarrow{t=\tau} \int_{\partial\Omega_f} \vec{\sigma} \vec{v} \, dA = \underbrace{\int_{A_m} \vec{\sigma} \cdot \vec{v} \, dA}_{\dot{L}} + \int_{A_{IN}+A_{OUT}} \vec{\sigma} \cdot \vec{v} \, dA =$$

$$\vec{\sigma} = -P\vec{n} + \vec{\tau}$$

$$= \dot{L} + \int_{A_{IN}+A_{OUT}} -P\vec{n} \vec{v} \, dA$$

$$\int_{A_{IN}+A_{OUT}} \vec{\sigma}\,\vec{v}\, dA = \underbrace{\int_{A_{IN}+A_{OUT}} -P\vec{n}\cdot\vec{v}\, dA}_{\text{Pulse power}} + \cancel{\int_{A_{IN}+A_{OUT}} \vec{\tau}\,\vec{v}\, dA} \to 0$$

Non è 0 perché ci sono punti come alla superficie della turbina dove la velocità non è 0.

↳ If we take the A_{IN} and A_{OUT} we can find places in which the velocity is practically uniform and therefore the $\vec{\tau}$ will minimal. The power on the immaterial surface is negligible for the calculations since we take the area advantageously.

$$\frac{d}{dt}\int_{\Omega(t)} \rho\left(u + \frac{V^2}{2} + gz\right) dV = \dot{L}_0 + \dot{Q}$$

Derived from the RTT

$$\frac{d\Phi}{dt} = \frac{d\Phi_{CV}}{dt} + \int_{\partial\Omega_f} \varphi\,\vec{v}\cdot\vec{n}\, dA$$

$$= \frac{d}{dt}\int_{\Omega_f} \rho\left(u + \frac{V^2}{2} + gz\right) dV + \int_{A_{IN}+A_{OUT}} \rho\left(u + \frac{V^2}{2} + gz\right)\vec{v}\cdot\vec{n}\, dA =$$

↳ Flux of material that enters/exits the CV

$$= \dot{L} + \int_{A_{IN}+A_{OUT}} -\rho(vP)\,\vec{v}\cdot\vec{n}\, dA + \dot{Q}$$

multiply and divided by density and $\frac{1}{\rho} = v$, does so too keep the same form.

$$\Rightarrow \frac{d}{dt}\int_{\Omega_f} \rho\left(u + \frac{V^2}{2} + gz\right) dV + \int_{A_{IN}+A_{OUT}} \rho\left(\underbrace{u + Pv}_{h} + \frac{V^2}{2} + gz\right)\vec{v}\cdot\vec{n}\, dA = \dot{L} + \dot{Q}$$

$h \to$ enthalpy

By calculating the enthalpy we don't need to calculate u, P or v.

h is the union of the flux of energy (u) and the flux of mechanical power Pv.

<u>In full</u>: ↳ Power balance for control volume

$$\frac{d}{dt}\int_{\Omega_f} \rho\left(u + \frac{V^2}{2} + gz\right) dV + \int_{A_{OUT}} \rho\left(h + \frac{V^2}{2} + gz\right) v_{n,OUT}\, dA$$

↳ This is not the

$$-\int_{A_{IN}} \rho\left(h+\frac{V^2}{2}+gz\right)v_{m,in}\,dA = \dot{L}+\dot{Q}$$

flux of the first since we use h not u, and h considers the pulse power term.

from the general form by

We want to simplify it using the assumptions.

$$\frac{d}{dt}\int_{VC} \underbrace{\rho\left(u+\frac{V^2}{2}+gz\right)dV}_{\to 0,\ \text{steady flow}} + \int_{A_{OUT}} \rho\left(h+\frac{V^2}{2}+gz\right)v_{m,OUT}\,dA$$

$$-\int_{A_{IN}} \rho\left(h+\frac{V^2}{2}+gz\right)v_{m,in}\,dA = \dot{L}+\dot{Q}$$

we no longer care about what's happening inside we infer it from what happens at the two ends.

If we apply the lumped parameter approach:

$$\Rightarrow \underbrace{\left(h+\frac{V^2}{2}+gz\right)_{OUT}}_{\text{Average value}} \cdot \underbrace{\int_{A_{OUT}} \rho\, v_{m,out}\,dA}_{\dot{m}_{OUT}\ \to \text{mass flow}} - \left(h+\frac{V^2}{2}+gz\right)_{IN} \cdot \dot{m}_{IN} = \dot{L}+\dot{Q}$$

Due to the assumptions the mass balance is also valid

$$\begin{cases} \dot{m}_{IN} = \dot{m}_{OUT} = \dot{m} \\ \left(h+\frac{V^2}{2}+gz\right)\cancel{\dot{m}} - \left(h+\frac{V^2}{2}+gz\right)\cancel{\dot{m}} = \frac{\dot{L}}{\dot{m}}+\frac{\dot{Q}}{\dot{m}} = l+q \end{cases}$$

The assumptions are not strong enough to cause issues in the balances.

$$\boxed{l+q = h_{OUT}-h_{IN}+\frac{V_{OUT}^2-V_{IN}^2}{2}+g(z_{OUT}-z_{IN})} \longrightarrow \text{Energy Balance}$$

While this equation is useful since it includes everything, sometimes we would just the mechanical terms or just the heat terms, this is since it allows us to find the efficiency, writing equations based on mechanical or heat terms makes the wasted products come out.

Clausius: $q = \int_{IN}^{OUT} T\,ds - \underset{\sim}{l_w}$

since $l_w = T\Delta s_{IRR}$

$h_{OUT} - h_{IN} = \int_{IN}^{OUT} T\,ds + \int_{IN}^{OUT} v\,dP$

$\boxed{16:19}$ → Why surprising

$$l + \cancel{\int_{IN}^{OUT} T\,ds} - l_w = \cancel{\int_{IN}^{OUT} T\,ds} + \int_{IN}^{OUT} v\,dP + \frac{\Delta v^2}{2} + g\Delta z$$

$$\boxed{l - l_w = \int_{IN}^{OUT} v\,dP + \frac{\Delta v^2}{2} + g\,\Delta z}$$

→ Mechanical Energy Balance

↳ It states that mechanical energy is not conserved, there is a sink which is the l_w, this is because we are only considering mechanical energy and don't look at the thermal energy.

$$\boxed{q + l_w = \int_{IN}^{OUT} T\,dS}$$

→ Thermal Energy Balance

PUMP

Compressor

$l > 0$
$l_w > 0$

$l > l - l_w =$
$= l > \int v\,dP + \frac{\Delta v^2}{2} + g\Delta z$

↳ what i pay will always be greater than what we effectively transfer to the fluid.

Turbine T $\quad l<0$ $\qquad l - l_w = -|l| - l_w$ → Trick to make the sign appear

Thermal Turbine T $\quad l_w > 0$

$$-|l| - l_w = -\left|\int_{IN}^{OUT} v\,dP + \frac{\Delta v^2}{2} + g\Delta z\right|$$

shows that mechanical energy reduces.

$$\Rightarrow |l| = \left|\int_{IN}^{OUT} v\,dP + \frac{\Delta v^2}{2} + g\Delta z\right| - l_w$$

Turbine efficiency is how much of the mechanical energy of the fluid which we have that we can extract

$g\Delta z$ is typically very small, the change in velocity can also be small.

$$l - l_w = \underbrace{\int_{IN}^{OUT} v\,dP}_{} \xrightarrow{\text{Ideal liquid}} v\Delta P \rightarrow l - l_w = \frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z =$$

Normally this term dominates.

→ Technical work

$$= \Delta\Big(\underbrace{\frac{P}{\rho} + \frac{v^2}{2} + gz}_{\mathcal{T}}\Big) = \Delta\mathcal{T}$$

$\mathcal{T}$ → Bernoulli Trinomial

$$= \int_{IN}^{OUT} v\,dP \xrightarrow{\text{IDEAL GAS}} \int_{IN}^{OUT} RT\,\frac{dP}{P}$$ → we cannot say since it depends on the relationship between T and P, it depends on the transformation,

in ideal liquid we didn't have such an issue since we can only have isocoric transformations.

⇒ we have to define the transformations to calculate the change in energy.

Not all thermodynamic cycles can be realized (e.g. isothermic), thats why many transformations are not used.

Compressors have fans that spin fast this heat the fluid, so if we wanted an isothermal compressors, we can cool the fluid down and have something that is kind of like that transformation.