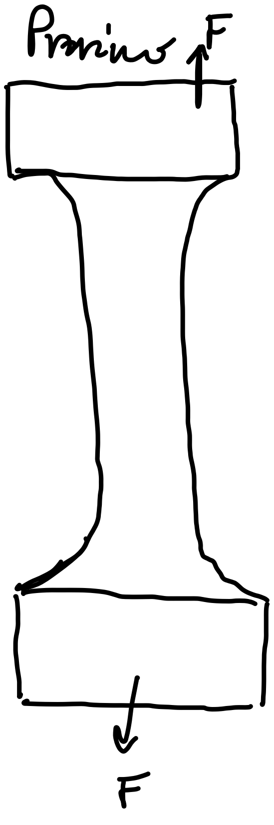
Esercitazione 1 -

Dr. Leonardo Caprio

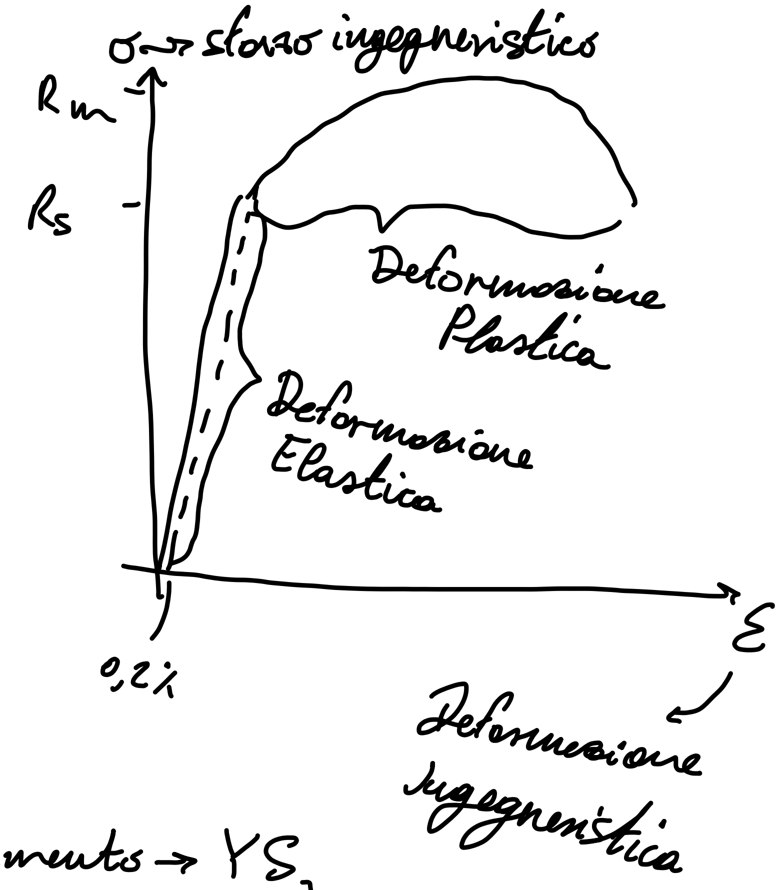
Complementi sul comportamento in campo plastico dei materiali

Prova di trazione

Provino

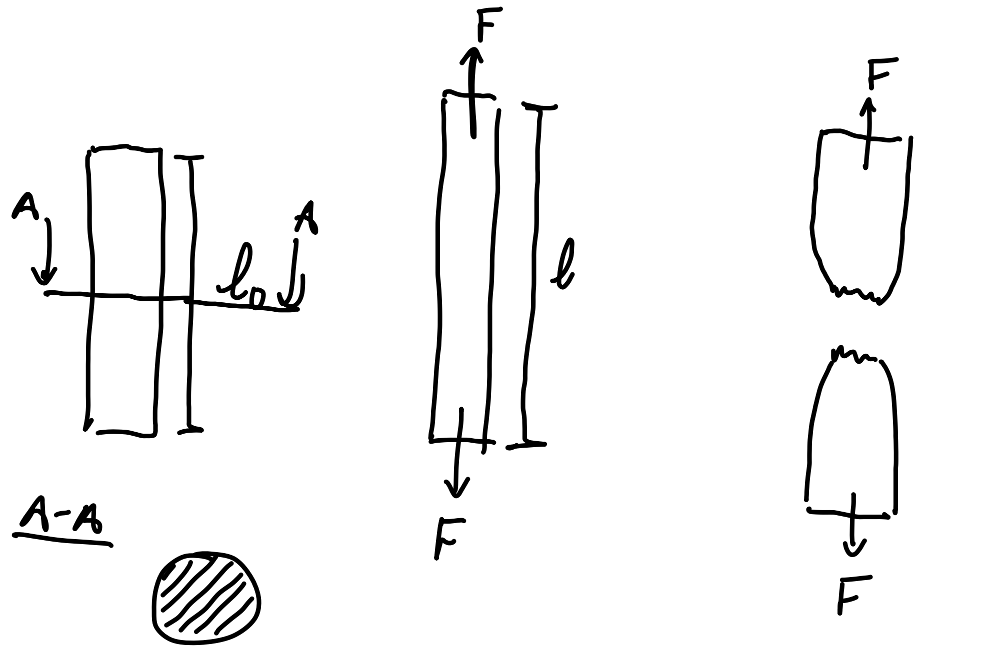


Risultato



R_s → sforzo di snervamento → YS → Yield Stress

R_m → resistenza meccanica → UTS → ultimate tensile strength



$$\sigma = \frac{F}{S_0}$$

↳ Sezione nominale

$$\varepsilon = \frac{L - L_0}{L_0}$$

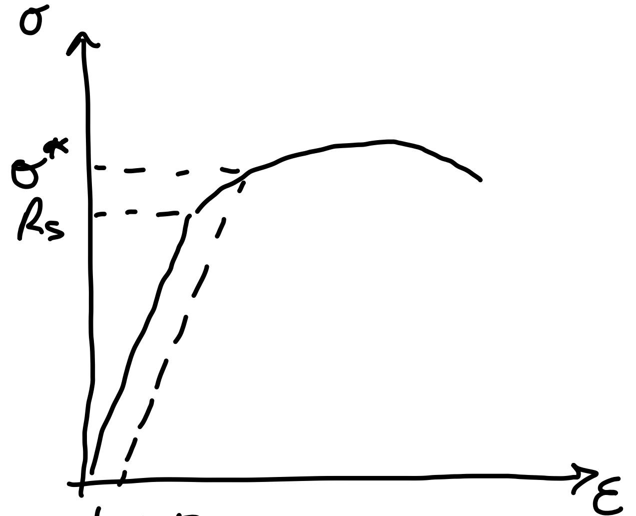
Deformazione Elastica

$\sigma = E\varepsilon$

$E \rightarrow$ modulo Young

$\varepsilon \rightarrow$ deformazione ingegneristica

$\sigma \rightarrow$ sforzo ingegneristico

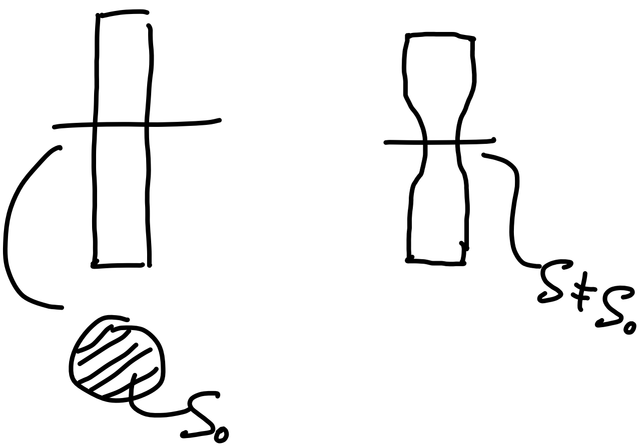


↳ Per noi considereremo trascurabile

Campo Plastico

$$\varepsilon = \int_{L_0}^{L} \frac{dL}{L} = \ln\left(\frac{L}{L_0}\right)$$

$\varepsilon \rightarrow$ deformazioni reale



Tensione Reale σ

$$\sigma = \frac{F}{S}$$

S - area reale

Sforzo Ingegneristico

$$s = \frac{F}{S_0}$$

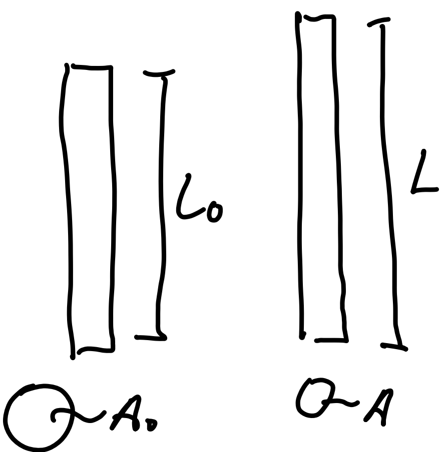
$\hookrightarrow$ Diminuisce

Dopo R_m continuerà ad aumentare

Deformazioni Plastiche

$V = cost$

$$A_0 \cdot L_0 = A \cdot L \qquad \frac{L}{L_0} = \frac{A_0}{A}$$



$$\varepsilon = \frac{L - L_0}{L_0} = \frac{L}{L_0} - 1$$

$$\frac{L}{L_0} = \varepsilon + 1 = \frac{A_0}{A}$$

$$\sigma = \frac{F}{A} \qquad s = \frac{F}{A_0}$$

$$F = \sigma A = s A_0$$

$$\sigma = s \frac{A_0}{A}$$

reale

$$\sigma = s(1+e)$$

ingegneristico

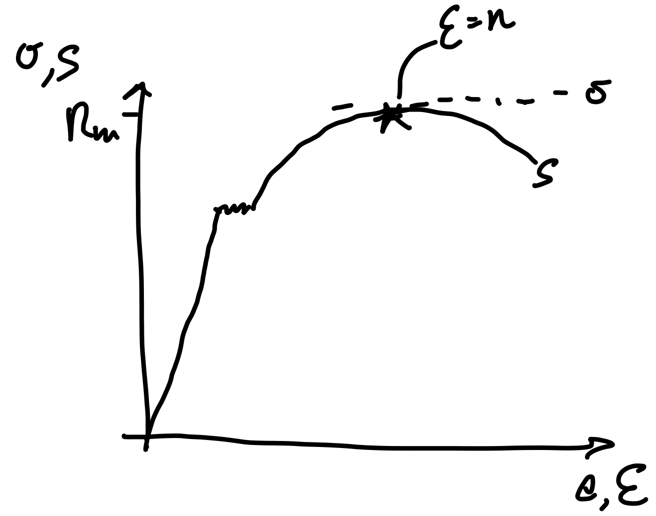
$$\varepsilon = \ln(1+e)$$

<u>Campo Elastico</u>

$$e \approx \varepsilon$$

Campo Plastico

$$e \not\approx \varepsilon$$



$$\sigma = k\varepsilon^n$$

k fattore di resistenza

n = coefficiente di incrudimento

Generalmente Tabulati

$$dF = 0$$

$$F = \sigma S$$

$$dF = \sigma\, dS + S\, d\sigma = 0$$

$$\frac{dF}{\sigma} = \frac{-dS}{S}$$

$$dV = 0$$

$$d(S \cdot L) = 0$$

$$L dS + S dL = 0$$

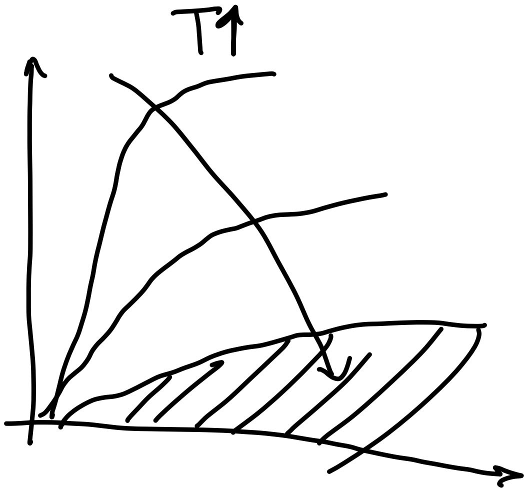
$$\frac{-dS}{S} = \frac{dL}{L} = d\varepsilon$$

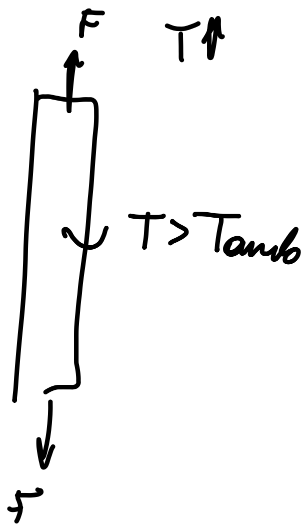
$$\frac{d\sigma}{\sigma} = d\varepsilon$$

$$\frac{d\sigma}{d\varepsilon} = \sigma = k\varepsilon^{n} = n k \varepsilon^{n-1}$$

$$\Rightarrow \sigma = \varepsilon$$

Effetti di T





Duttilità ↑ Tenacità ↑ Carico di snervamento R_s ↓

Durezza ↓   Modulo di Elasticità E ↓

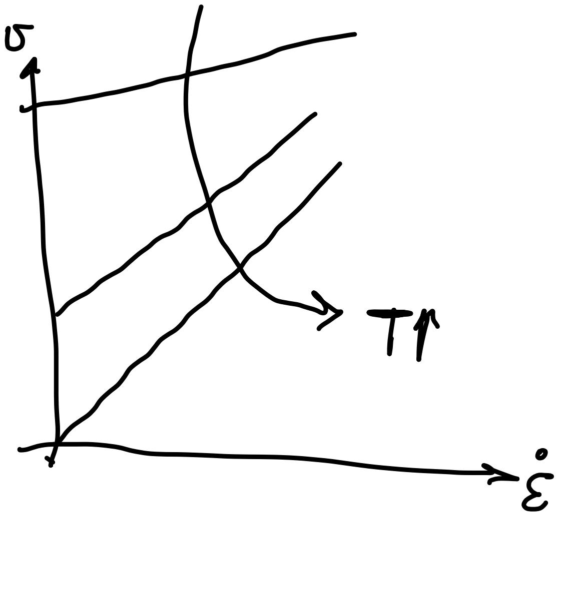
Deformabilità ↑

Velocità di Deformazione

$$v = \frac{dL}{dt} \qquad \frac{v}{L} = \frac{dL}{L}\,dt = \frac{d\varepsilon}{dt} = \dot{\varepsilon}$$

$$\dot{\varepsilon} = \frac{v}{L} \qquad \sigma = k\,\varepsilon^n\,\dot{\varepsilon}^m$$

$$\dot{\varepsilon}^m \approx 1$$

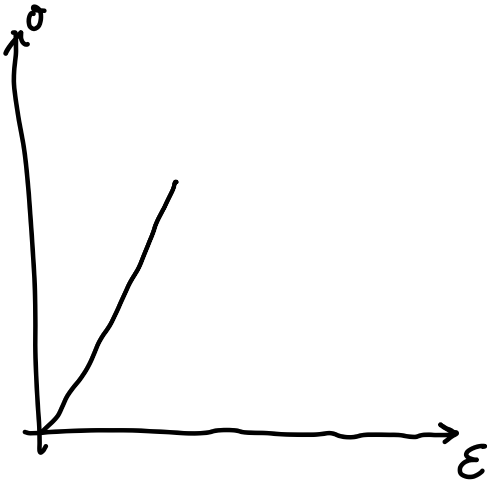


Per aver materiale deformabile serve $\geq 2/3$ o velocità di deformazione basse.

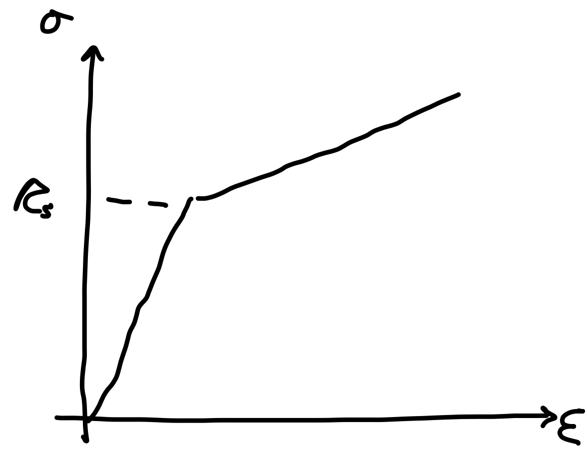
Modelli Costitutivi

Elastico Perfetto

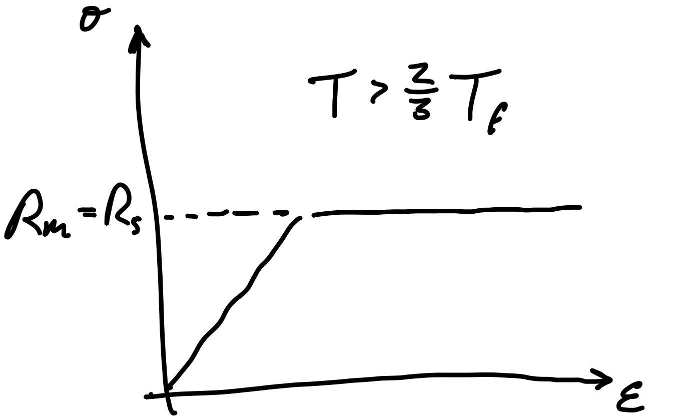
Elastico-Plastico



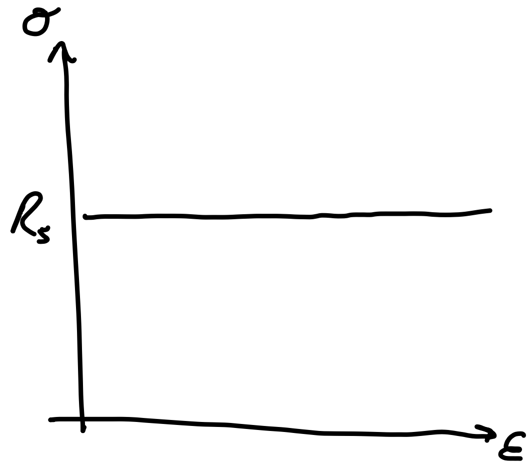
con incrudimento



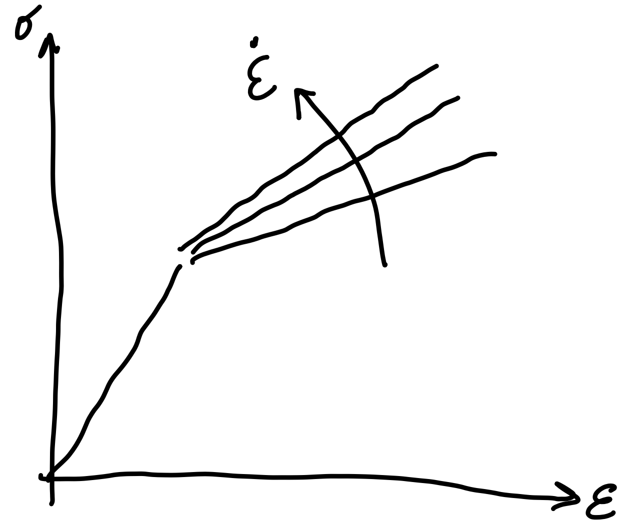
Elasto-Plastico Perfetto



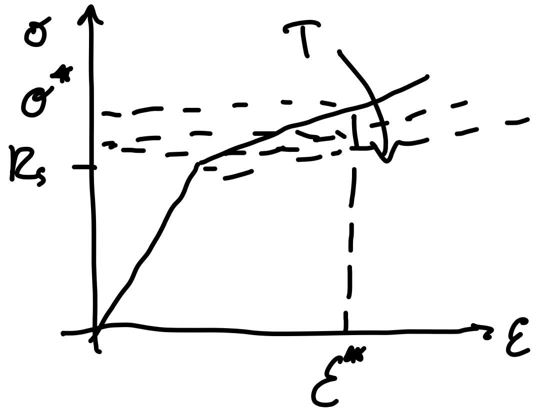
Rigido Plastico Perfetto



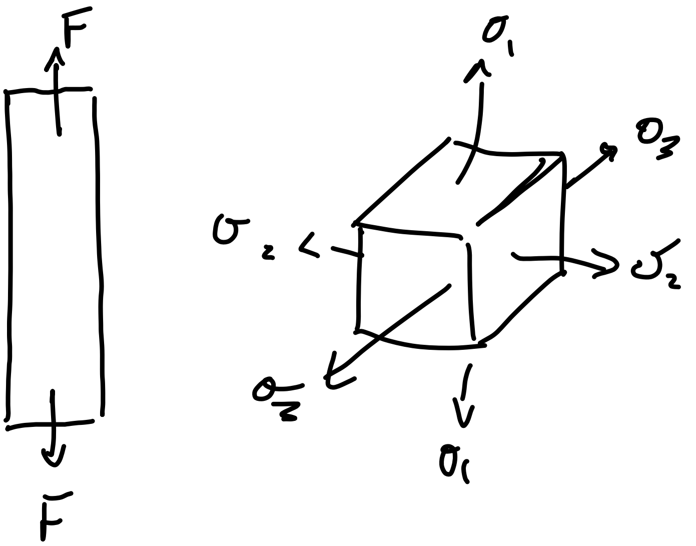
Velocità di Deformazione



Temperatura



<u>Tensione di Flusso</u> $\Rightarrow$ Criterio di Von Moises



Prova trazione $\sigma_2 = \sigma_3 = 0$

$$Y_f = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

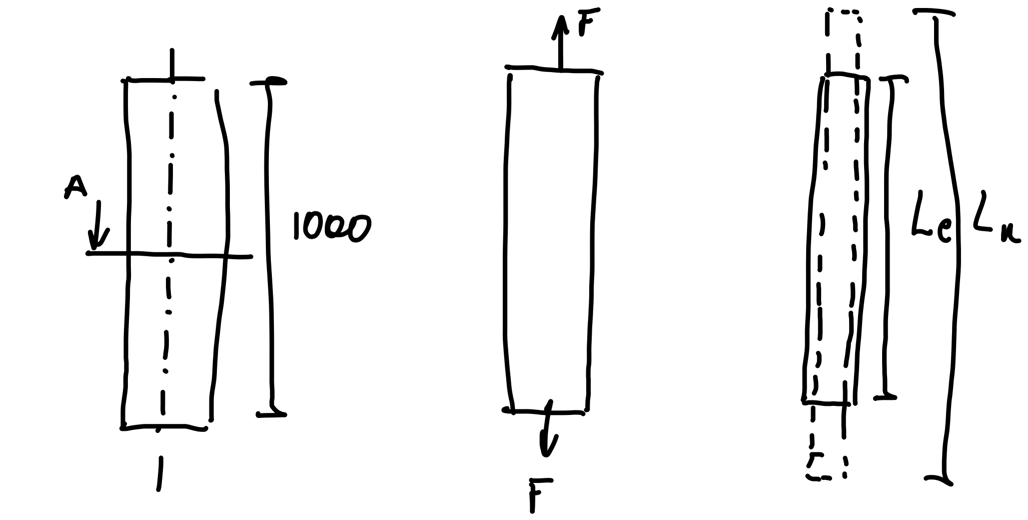
$$Y_f = \frac{1}{\sqrt{2}} \sqrt{\sigma_1^2 + \sigma_1^2} \Rightarrow Y_f = \sigma_1$$

$$\sigma = k\varepsilon^n$$

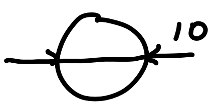
$$Y_f = k\varepsilon^n$$

<u>Complemento sui materiali [chap3]</u>

ES 1



Sezione A-A



Allungamento $\Delta L = L_u - L_e$

$F = 100\ N \quad L_e = 1000\ mm$

$d = 10\ mm$

$E = 200\ \frac{kN}{mm^2}$

$$\sigma = \frac{F}{S_e} = \frac{F}{\pi \frac{d_e^2}{4}} = 1,27\ \frac{N}{mm^2}$$

$$\varepsilon = \frac{\sigma}{E} = \frac{1,27}{200\,000} = 6,4 \cdot 10^{-6}$$

$$\varepsilon = \frac{\Delta L}{L_e} = \frac{L_u - L_e}{L_e} \Rightarrow \Delta L = \varepsilon L_e = 6,4\ \mu m$$

se Al:

18 μm

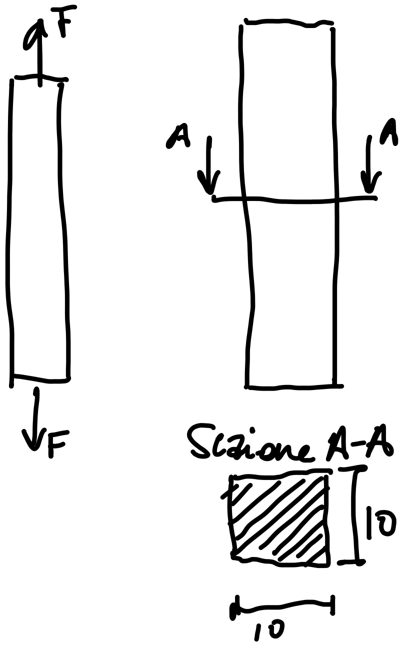
se gomma:

1,3 mm

se plastica:

210 μm

Esercizio 2



$R_m = UTS = 1200 \frac{N}{mm^2} = 1200 MPa$

$S_e = 10 \times 10 = 100 mm^2$

$F_m = R_m S_e = 1200 \cdot 100$

$F_m = 120000 N$

$= 120 kN$

Esercizio 3

$D_{ACC} = 125 mm$ $\qquad E_{ACC} = 200000 \frac{N}{mm^2}$

$D_{ALL} = 250 mm$ $\qquad E_{ALL} = 70000 \frac{N}{mm^2}$

$S_{ACC} = 2 \times \pi \frac{D_{ACC}}{4} = 24543,7 mm^2$

$S_{ALL} = \frac{\pi D_{ALL}^2}{4} = 49087,4 mm^2$

① $e_{ALL} = e_{ACC}$

$F_{TOT} = F_{ACC} + F_{ACC}$

$$S_{ALL} = E_{ALL} \cdot e_{ALL} \Rightarrow e_{ALL} = \frac{S_{ALL}}{E_{ALL}}$$

$$S_{ACC} = E_{ACC} e_{ACC} \qquad e_{ACC} = \frac{S_{ACC}}{E_{ACC}}$$

$$\frac{S_{ALL}}{E_{ALL}} = \frac{S_{ACC}}{E_{ACC}} \qquad \frac{S_{ALL}}{S_{ACC}} = \frac{E_{ALL}}{E_{ACC}}$$

$$F_{ALL} = s_{ALL} S_{ALL}$$

$$F_{ACC} = s_{ACC} S_{ACC}$$

$$F_{TOT} = F_{ALL} + F_{ACC}$$

$$= s_{ALL} S_{ALL} + S_{ACC} S_{ACC}$$

$$= S_{ACC} \left(\frac{S_{ALL}}{S_{ACC}} S_{ALL} + S_{ACC} \right)$$

$$F_{TOT} = S_{ACC} \left(\frac{E_{ALL}}{E_{ACC}} S_{ALL} + S_{ACC} \right)$$

$$S_{ACC} = \frac{F_{TOT}}{\frac{E_{ALL}}{E_{ACC}} S_{ALL} + S_{ACC}} = 21,57 \frac{N}{mm^2}$$

$$S_{ALL} = \frac{E_{ALL}}{E_{ACC}} \times S_{ACC} = 7,55 \frac{N}{mm^2}$$

<u>Esercizio</u> 4

Parametro di E

$l_e = 30mm$ $\qquad$ $l_u = 40\ mm$

$b_e = 40mm$ $\qquad$ $b_u = 50mm$

$h_e = 50\,mm$ $h_u = 30\,mm$

$\Delta L = L_u - L_e = 40 - 30 = 10\,mm$

$\Delta b = b_u - b_e = 50 - 40 = 10\,mm$

$\Delta h = h_u - h_e = 30 - 50 = -20\,mm$

Deformazione Ingegneristiche

$e_L = \frac{\Delta L}{L_e} = 0,33$

$e_b = \frac{\Delta b}{b_e} = 0,25$

$e_h = \frac{\Delta h}{h_e} = -0,4$

Rapporto di Deformazione

$\frac{l_u}{l_e} = 1,33$

$\frac{b_u}{b_e} = 1,25$

$\frac{h_u}{h_e} = 0,6$

Sono simili ma

Deformazione Reale

$\varepsilon_L = \ln\left(\frac{l_u}{l_e}\right) = \ln(1,33) = 0,287$

$\varepsilon_b = \ln\left(\frac{b_u}{b_e}\right) = \ln(1,25) = 0,223$

$$\varepsilon_h = \ln\left(\frac{h_u}{h_e}\right) = \ln(0{,}6) = -0{,}51$$

Deformazione Plastica $V = \text{costante}$

$$l_c b_c h_c = l_e b_e h_e$$

$$\frac{l_u b_u h_u}{l_e b_e h_e} = 1$$

$$\varepsilon_l + \varepsilon_b + \varepsilon_h = 0$$

$$\ln\frac{l_u}{l_e} + \ln\frac{b_u}{b_e} + \ln\frac{h_u}{h_e} = \ln\left(\frac{l_u b_u h_u}{l_e b_e h_e}\right) = \ln 1 = 0$$

$$0{,}287 + 0{,}223 - 0{,}51 = 0$$

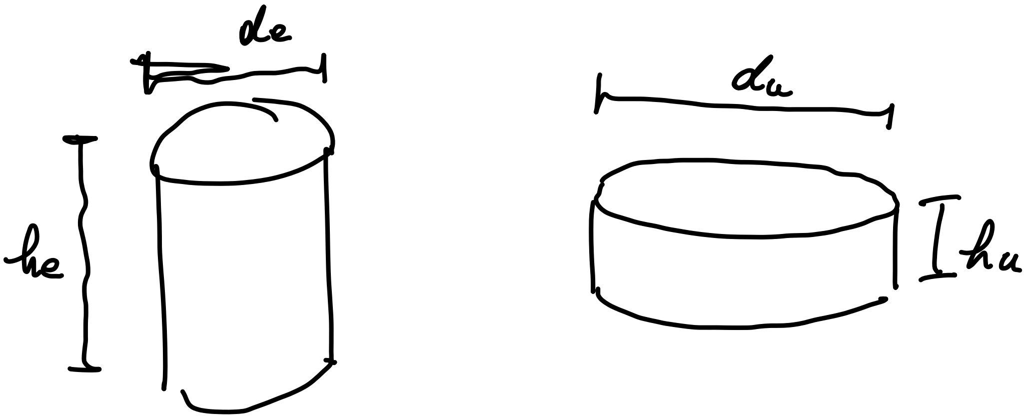
$$e_l = \frac{\Delta l}{l_u} = \frac{l_u - l_e}{l_u} = \quad 1 + e_l = \frac{l_e + l_u - l_e}{l_e} = \frac{l_u}{l_e}$$

$$1 + e_l = \frac{l_u}{l_e}$$

$$1 - \frac{l_u b_u h_u}{l_e b_e h_e} = (1 + e_l)(1 + e_b)(1 + e_h) = 1 + e_l + e_b + e_h +$$

$$\Rightarrow e_l + e_b + e_h = 0$$

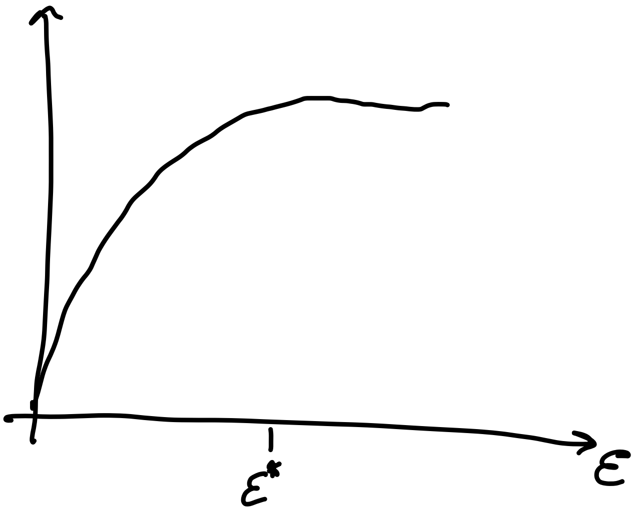
Esercizio 5 - Deformazione a freddo



$d_e = 10\,mm$ $\qquad h_u = \frac{h_e}{2} = 10\,mm$

$h_e = 20\,mm$

$$|\varepsilon| = \left| \ln\left(\frac{h_u}{h_e}\right)\right| = \left|\ln(0{,}5)\right| = 0{,}693$$



$\sigma^* \cong 610 \frac{N}{mm^2}$

Per conservazione di volume

$S_u h_u = S_e h_e$

$$S_u = S_e \cdot \frac{h_e}{h_u} = \frac{\pi d_e^2}{4} \cdot \frac{h_e}{h_u} = 157{,}1\ mm^2$$

$$F = S_u \sigma^* = 157{,}1 \cdot 610$$

$$F = 95819 = 95{,}8\ kN$$

Trovare Lavoro compiuto in deformazione

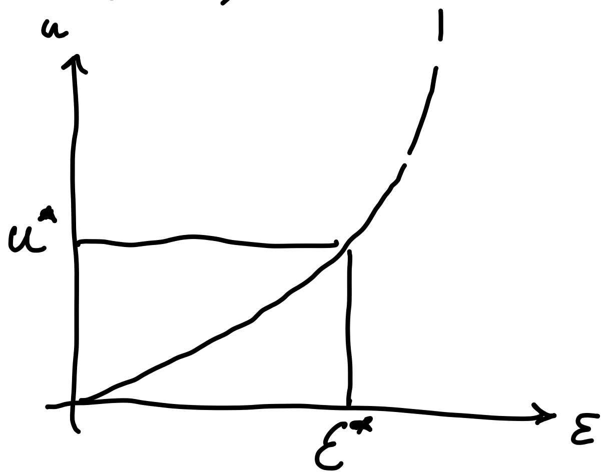
$$u = \frac{L}{V} \qquad L = uV$$

Da grafico ($a\ \varepsilon^*$)

L - lavoro

u - lavoro per unità di V

V - volume



$$u^* = 335\ \frac{Nmm}{mm^3}$$

$$L = uV = u \cdot \pi \frac{d_e^2}{4} h_e$$

$$= 339 \times \frac{\pi \cdot 10^2}{4} \cdot 20$$

$$= 558\ J$$