Lezione 10 - Incompressible Fluid Machines

Hydraulics → machines operating with liquids or gases for which

$$\rho = const$$

$$\begin{cases} \dot{m} = \rho v_n A = const \rightarrow \text{Mass Balance} \\ l + q = \Delta h + \frac{\Delta v^2}{2} + g\Delta z \rightarrow \text{Energy Balance} \\ l - l_w = \int_{IN}^{OUT} v\,dP + \frac{\Delta v^2}{2} + g\Delta z \rightarrow \text{Mechanical Energy Balance} \end{cases}$$

We also add:

$$\begin{cases} v = const \rightarrow \rho = const \\ \Delta h = c_L \Delta T + \frac{\Delta P}{\rho} \end{cases}$$

Perfect liquid model

will be more complex in thermal machines when $\rho \neq const$

Put together we get

Volumetric Flow

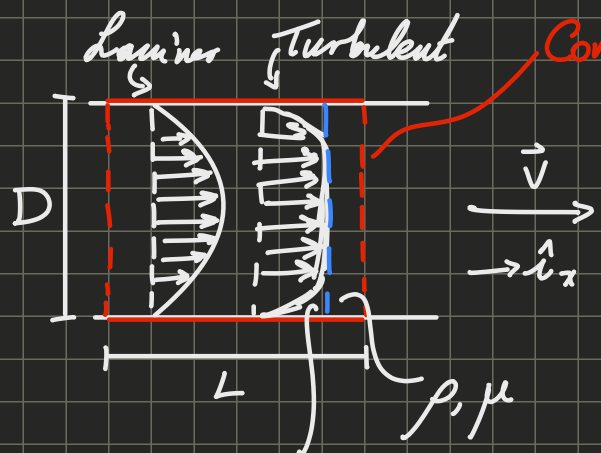
$$\Longrightarrow \text{since } \dot{m}, \rho = const \Rightarrow v_n A = Q = const \Longrightarrow \frac{dv_n}{v_n} = -\frac{dA}{A}$$

should be with the other 3

$$\begin{cases} l + q = c_L \Delta T + \frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z \\ l - l_w = \frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z = \Delta T \longrightarrow T \doteq \frac{P}{\rho} + \frac{v^2}{2} + gz \end{cases}$$

$$\rightarrow q + l_w = c_L \Delta T \rightarrow \text{Thermal Energy Balance}$$

Behaviour of Flow in a duct $(A = const)$ $q \cong 0$



normal velocity

average

Mass Balance

$$\vec{v} = v_n \hat{i}_x = v \hat{i}_x \Rightarrow v = const \Rightarrow v_{in} = v_{out}$$

we are just going to write v_n

Mechanical Energy Balance (M.E.B / B.M.E)

$$\hookrightarrow \underbrace{l}_{\to 0} - l_w = \Delta T = \frac{\Delta P}{\rho} + \underbrace{\frac{\Delta v^2}{2}}_{\to 0} + \underbrace{g \Delta z}_{\to 0}$$

$$v = const \quad l_w = -\frac{\Delta P}{\rho} \triangleq y$$

turbulent flow make it easy to use the LPA as they are near constant in the center.

Perdita di carico is a change in mechanical energy, since everything is constant losses lead to a loss of pressure.

Due to the friction losses in ducts being dependent on the shape of the duct.

$$\Rightarrow l - l_w - y = \Delta T$$

Work lost in machine

Mechanical energy losses due to friction

We can change the balance equations.

$$l - l_w - y = \frac{\Delta}{\rho} + \frac{\Delta v^2}{2} + g \Delta z = \Delta T$$

$$q + l_w + y = c_L \Delta T$$

This allows to distinguish the losses so the machine does not work, it tells us who to go to.

We know that:

$$y = y(v, D, L, \rho, \mu)$$

This is a dynamic problem

$\Rightarrow$ we need to search for 3 relevant scales: mass, time and length scales.

Problem we have in physics:
- Geometric $\rightarrow$ we only use lengths
- Kinematic $\rightarrow$ length scale and time scale
- Dynamic $\rightarrow$ length, time and mass scales
- Thermo mechanic
  - when thermal phenomena are also important

Length $\rightsquigarrow D$

Time $\rightsquigarrow v$

Mass $\rightsquigarrow \rho$

$$y = y(\cancel{v}, \cancel{D}, L, \cancel{\rho}, \mu)$$

we need to rewrite y, L & μ based on these scales.

$$y \rightarrow y = \xi_D \frac{v^2}{2} \longrightarrow \xi_D = \frac{y}{v^2/2}$$

distributed loss coefficient

different than concentrated

$L \rightarrow L/D$

$\mu \rightarrow \rho \frac{vD}{\mu} = Re$

$\rightarrow$ the dimensionless counterparts.

$$\xi_D = \cancel{\xi_D(Re, L/D)}$$

To define the Reynolds number, there is no one style way. It requires defining the dimensional quantities. The definition of the Reynolds number is different based on the dimensions we take.

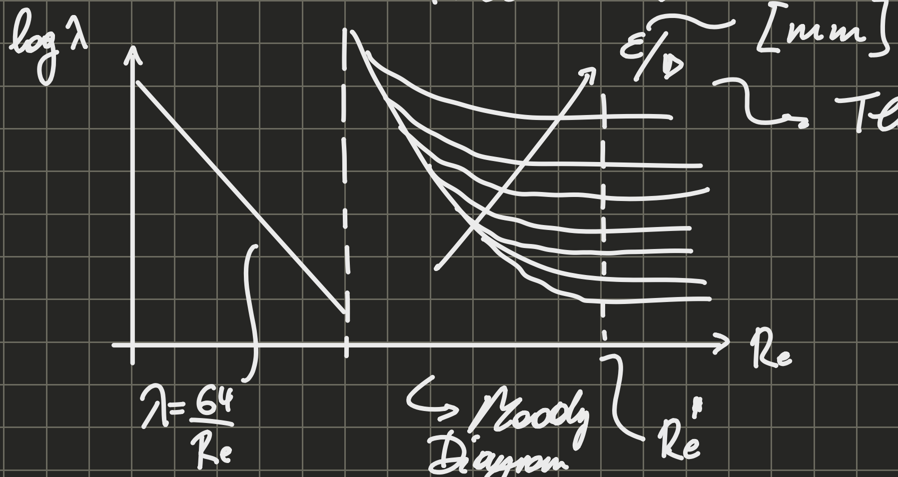
As the Reynolds number increase, the inertial forces non-linearly increase, and our flow develops non-linear instabilities.

Analytical solutions to the turbulent flow problem do not exists, we only have results from experiments.

We did experiments, and put the data into tables for which we found:

$$\xi_D = \cancel{\lambda(Re)} \cdot \frac{L}{D}$$

↳ Friction coefficient



↳ The mistake we make was that we did consider the surface roughness.

Fixing it we get:

$$\xi_D = \xi_D\left(Re, \frac{\varepsilon}{D}, \frac{L}{D}\right)$$

↳ $$\xi_D = \lambda\left(Re, \frac{\varepsilon}{D}\right) \cdot \frac{L}{D}$$

If $Re > Re^* \Rightarrow \frac{\partial \lambda}{\partial Re} = 0$

↳ the flow has become self-similar to the Reynolds number, if we have passed Re^*, we consider it full-turbulent flow.

Full turbulent flow $\Rightarrow y = \xi_D\left(\frac{\varepsilon}{D}, \frac{L}{D}\right)\frac{V^2}{2}$ → independent of Re

↳ For us it's almost always this so we can drop any dependence from Re.

The problem becomes: $y = y(V, D, L, \varepsilon)$ → the problem becomes a kinematic problem.

This is flow in ducts which create distributed losses. In ducts we can have elements which create concentrated losses

↳ The concentrated loss coefficient is $\xi_{c} = \xi_c(Re, SHAPE)$

↳ in fully turbulent flow $\xi_c = \xi_c(SHAPE)$

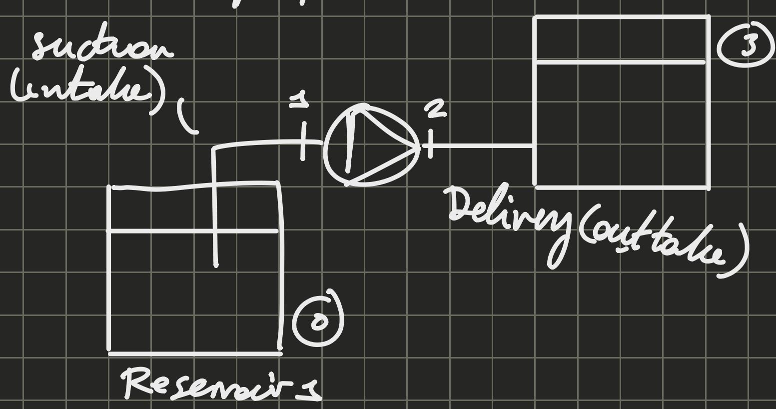
↳ function of only shape

Concentrated loss coefficient.

Other applications of this

Pumping Plant

↳ Device to pump fluid from one reservoir to another with a higher geodetic levels or pressure.



Balance of Mechanics Energy $1 \to 2$

$$l - l_w - y = T_2 - T_1 \Rightarrow l - l_w = T_2 - T_1 \overset{D}{=} gH \qquad [H] = m$$

Defined

$y \to 0$ it is not a duct

↳ Head

gravitational constant $g = 9{,}81\ m/s^2$

$V_3 = V_0 = 0$

BME $0 \to 3$

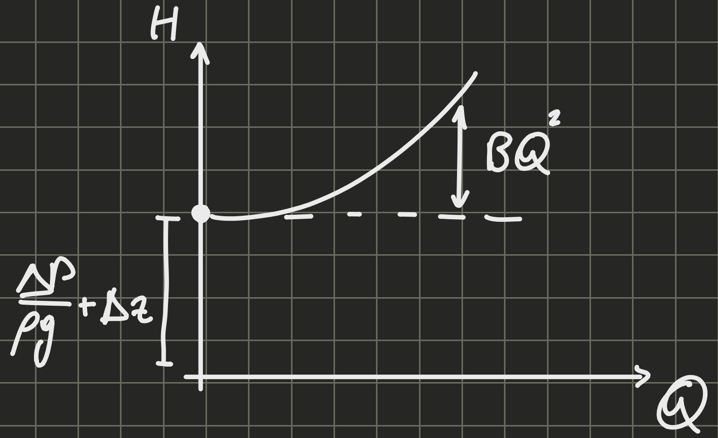
$$\underbrace{l - l_w}_{gH} - y = T_3 - T_0 = \frac{P_3 - P_0}{\rho} + g(z_3 - z_0)$$

Pump exchanges work, with some loss, we also have losses in ducts to be able to write the ΔP and Δz.

$$\text{So: } \cancel{g}H = \frac{P_3 - P_0}{\rho g} + \cancel{g}(z_3 - z_0) + \frac{y}{g} = \frac{P_3 - P_0}{\rho g} + (z_3 - z_0) + BQ^2$$

$$\hookrightarrow (\xi_c + \xi_D)\frac{V^2}{2g} \quad V = \frac{Q}{A}$$

$$\Rightarrow y \propto Q^2$$



$$y = BQ^2$$
$$\downarrow$$
$$B = B(A, \xi_c, \xi_D)$$

High flow, means more losses in the duct, so more head is required for the fluid to flow as such.

→ Characteristic Curve of the Circuit

Pump can exchange work, with some that is lost, and the rest is to be sent to the fluid.

If we design a system and need to get a pump at a specific H and Q, we can find a pump with a characteristic curve that intersect the point and the characteristic of our circuit.

How is the characteristic curve for the pump generated.

Pump: $H = H(Q)$?

$gH = l - l_w$

→ Before this let's ask $l = l(Q)$ and $l_w = l_w(Q)$

l ↳ Euler

l_w ↳ Impossible to obtain analytically

dependent of α_2 which is bad

dependent on β_2 which good.

$$l = u_2 V_{2t} - u_1 V_{1t} =$$
$$= u_2 (w_{2t} + u_2) - u_1 V_{1t}$$

α_1 $\quad \cancel{\alpha_2}$

$\cancel{\beta_1}$ $\quad \beta_2$

In the impeller we go there,

$w_{2t} = w_{2m} \cdot \tan\beta_2$

↳ Associated with work exchayed

↳ Associated with work

We are trying to change the association to get a link between l and Q.

$$w_{2m} = V_{2m} = \frac{Q}{\pi D_2 b_2}$$

$$w_{2t} = \frac{Q}{\pi D_2 b_2} \tan\beta_2$$

$$V_{1t} = V_{1m} \tan\alpha_1 = \frac{Q}{\pi D_1 b_1} \tan\alpha_1$$

two are geometric and 2 not,

α_1 is known since our flow comes from our duct, β_1 comes from v_1 and u_1

β_2 is geometric since it's displayed by the blade shape, the α_2 is not geometric since it comes from u_2 and w_2.

α_1 & β_2 are known as constructive angles.

β_1 and α_2 are dependent on the operational conditions, it is possible to control them though.

$$\Rightarrow l = u_2 \left(\frac{Q}{\pi D_2 b_2} \tan\beta_2 + u_2 \right) - u_1 \frac{Q}{\pi D_1 b_1} \tan\alpha_1$$

$$= u_2^2 \left(\frac{Q}{\pi D_2 b_2 u_2} \tan\beta_2 + 1 - \frac{u_1}{u_2} \frac{Q}{\pi D_1 b_1 u_2} \tan\alpha_1 \right)$$

↳ everything is dimensionless.

since $\omega_1 = \omega_2 = \omega$

$$\frac{l}{u_2^2} = \frac{Q}{\pi D_2^2 u_2} \cdot \frac{D_2}{b_2} \tan\beta_2 + 1 - \frac{D_1}{D_2} \frac{Q}{\pi D_1^2 u_2} \cdot \frac{D_1}{b_1} \tan\alpha_1$$

$$u_2 = \omega R_2 = \omega \frac{D_2}{2} = \frac{2\pi n}{60} \frac{D_2}{2} = \frac{n D_2 \pi}{60}$$

$$\frac{l}{u_2^2} = \frac{l}{n^2 D_2^2} \cdot \left(\frac{60}{\pi}\right)^2 \Rightarrow l \propto n^2 D_2^2 \Rightarrow$$ we can make l dimensionless,

we can use D as the length scale and define:

↳ and n as the time scale

$$\lambda = \frac{l}{n^2 D^2}$$

↳ Only 2 since it's a kinematic problem

$$\frac{Q}{\pi D_2^2 u_2} \cdot \frac{D_2}{b_2} \tan\beta_2 = \frac{Q}{\pi D_2^2 n D_2 \frac{\pi}{60}} \cdot \frac{D_2}{b_2} \tan\beta_2$$

It's dimensionless so we can write $\stackrel{!}{=} \underbrace{\dfrac{Q}{n D^3}}_{\varphi} \cdot f(\text{SHAPE})$

$\varphi \rightarrow$ flow coefficient

We can also do it for the last part and get: $\dfrac{Q}{nD^3} f(\text{SHAPE})$

Given $\ell = \dots$ we dimensionalised and rewrote it as a function of Q, such that:

$$\frac{\ell}{n^2 D^2} = f\left(\frac{Q}{n D^3}, \text{SHAPE}\right)$$

We finally got a relationship of the type:

$$\lambda = \lambda(\varphi, \text{SHAPE})$$

We can use this to get back to the dimensional representation: and the relationship that: $\ell = \ell(Q, n, D, \text{SHAPE})$

This is a kinematic problem, and it was derived from Euler's equation.