Lezione 18 - Volumetric Pumps.

Continuing on from last lesson.

↳ We found a plot in when we have mapped the application range of different machines in terms of H and Q rather than dimensionless parameters.

At the top right, it's unfeasable to have a since machine, it is possible with machines in series and parallel.

For H >500, the sole solution is a Pelton.
Inbetween Francis, and at low H and high Q we have kaplan.
There are areas of transition between the regions where one machine is solely the best option.

Finish hydraulic turbomachinery.

Volumetric (Operating) Machines → pumps

↳ No exercises, just theory is possible.

↳ Machines that change volume, how is it possible with a density which is constant.

Volumetric Pumps (positive displacement)

↳ take a portion of fluid and displaces to the delivery by pulling, using the delivery is at a different pressure.

Low flow velocity, static interactions more significant than dynamic ones in the exchange of work.

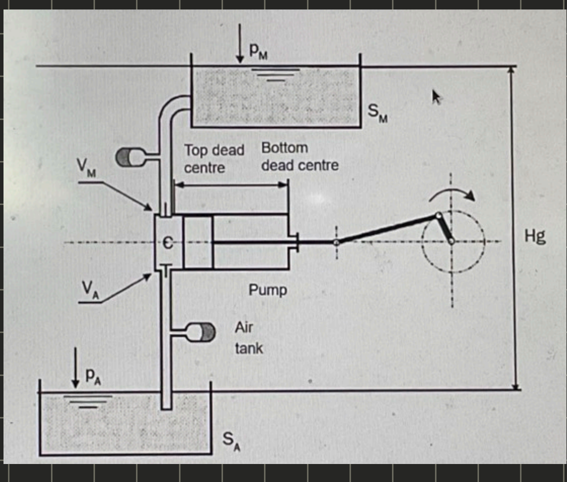
- Low velocity / limited displacement
  ↳ good for low Q

- no dynamics, no cavitation, no stall
  ↳ can exchange a lot of work with imperilling the machine, so good for high H.

Optimal configuration for very low ω_s
↳ since low Q and high H.
↳ For categorization

Two architectures
↳ Reciprocating pumps (very low ω_s)
↳ rotary pumps (low ω_s)

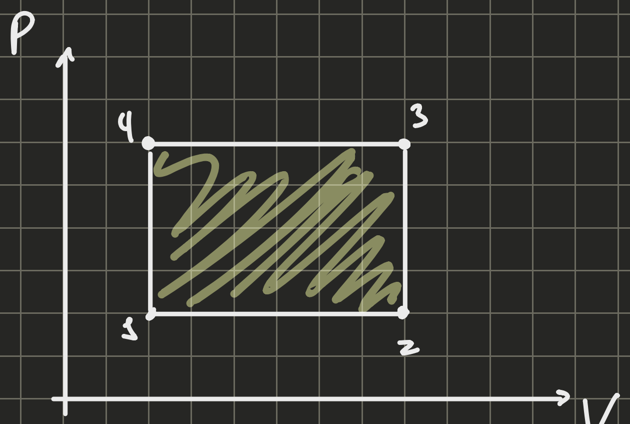
Working principle of reciprocating pump



We open one valve to let in fluid as the piston goes back, once the piston is at the bottom dead center, we close the valve and open the other, and the piston pushes up (the pressure having increased since the delivery duct usually has a higher pressure and geodetic level)

the pressure increases istantaneously (V constant) as it comes into contact with the fluid in the delivery duct which is already at a higher pressure.

Work of Volumetric machine



volume (non-specific)

= work

In practice this is not the case, as there are some losses and there is some transition.

The istantaneous change in P in reality sees a change in ρ, and so there is some oscillation in P.

The flow rate is highly periodic due to the operational cycle.

$$Q = \pi \frac{D^2}{4} r \left(\sin\varphi + \frac{r}{2l}\sin 2\varphi\right)\omega$$

Q is directly proportional to ω.

We have an average flow rate (Q_m)

$$Q = \eta_v Q_{id} = \eta_v V_z e \frac{n}{60}$$

$n \rightarrow$ number of effects

$\eta_v \rightarrow$ volumetric efficiency. $\rightarrow$ emptying and filling is not complete

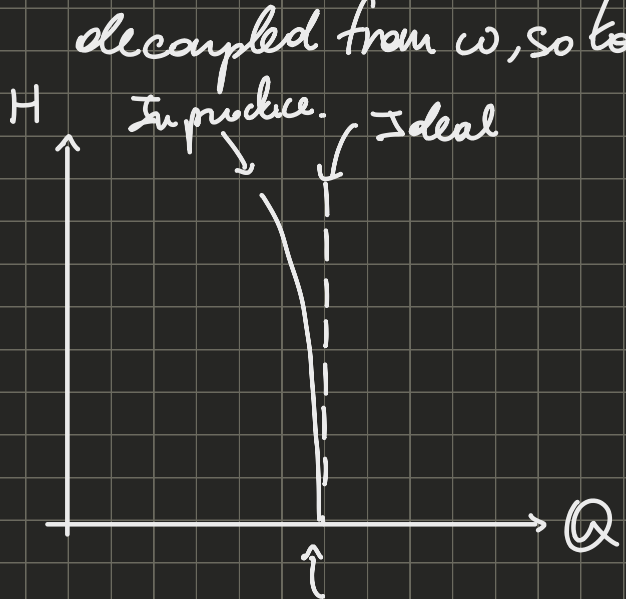
If we have multiple pistons all phase out, we can have a steadier and higher average flow rate.

<u>What is the power exchanged by machine?</u>

We usually say $\dot{L} = l\,\dot{m}$, to do this we need to know how l and Q are connected, in pumps it was through the constructive angles in Euler equation. ($\dot{m} \propto Q$)

Head of the machine does not depend (theoretically) on the flow rate.

To increase Q we increase ω, but Q is (ideally) decoupled from ω, so too is it from Q.



The work is the head that we add to the fluid. So the H is also ideally independent of Q.

There are more losses at higher Q.

$$W_{id} = (P_m - P_A)\,Q_{id} = \underbrace{(P_m - P_A)}_{L_{id}} V z e \frac{n}{60}$$

$\frac{n}{60} \rightarrow \frac{\#\text{strokes}}{t}$

$$W = \frac{(P_m - P_a)}{\eta_y \eta_o} Q = \frac{\eta_v}{\eta_y \eta_o} (P_m - P_a) V_{ze} \frac{n}{60}$$

↳ Efficiency values.

Efficiency is around 60%.

<u>Rotary Pumps</u> (gear, scroll, screw, lobe, etc.)

↳ key advantage, no valves.
↳ For higher Q. Flow rate is more steady.

Can be used for more complicated fluids, like blood and highly viscous fluids.

<u>Wind Turbines</u> (<u>Last topic of hydraulics</u>)

↳ This is a very specific machine, there is no stator or duct forces to utilize.
↳ They use the same theory of ship propellers, so the theory was already developed.

↗ Hub is movable to match wind direction.
Hub + Long and Tapered, Blades on a high tower.
Twisted

Since there is no duct there are things that we can no longer say about the fluid, and we are not able evaluate Q as easily.

There is a strong link between l of the machine and the Q that passes through the work.

Increasing Q, we decrease l, in turbines, so we need to find the condition of optimal efficiency.
We need to search for the maximum $\dot{L}$, rather than l and Q.

(Although the rotor theory still holds, it will be easier to go through slightly different means to calculate $\dot{L}$)

Characteristics of winds
- Wind is created by non-uniform heating.
- Wind is based on weather, so we cannot predict it perfectly, we need to use a statistical data before we install a wind turbine.

A wind is favorable if it is strong enough for a relatively long period.
- there are places that typically have favorable winds, especially offshore since there is less disturbance.
- but can only be <60 m in depth, unless we decide to use floating platforms.

The atmospheric boundary layer has an effect of the velocity profile with the distance to the ground with the $1/7^{th}$ power law:

$$\frac{V}{V_{ref}} = \left(\frac{y}{y_{ref}}\right)^{1/7}$$

This is why we prefer to increase the height of the rotor, to increase the velocity and create a more uniform velocity gradient over the turbine.

we will show that $\dot{L} \propto V^3$

The typical velocity range we can exploit is 3-30 $\frac{m}{s}$

The specific work is ~1/10 hydraulics, we have a low density

$\Rightarrow$ we need huge rotor cross sections to compensate, to increase Q and increase $\dot{L}$

Onshore wind is the most economically sensible technology, it is able to pay itself back in 1 year usually.

The mainstream wind turbine is called the horizontal axis wind turbine (HAWT)

↳ the two main issues are:

↳ low angular speed → gearbox required

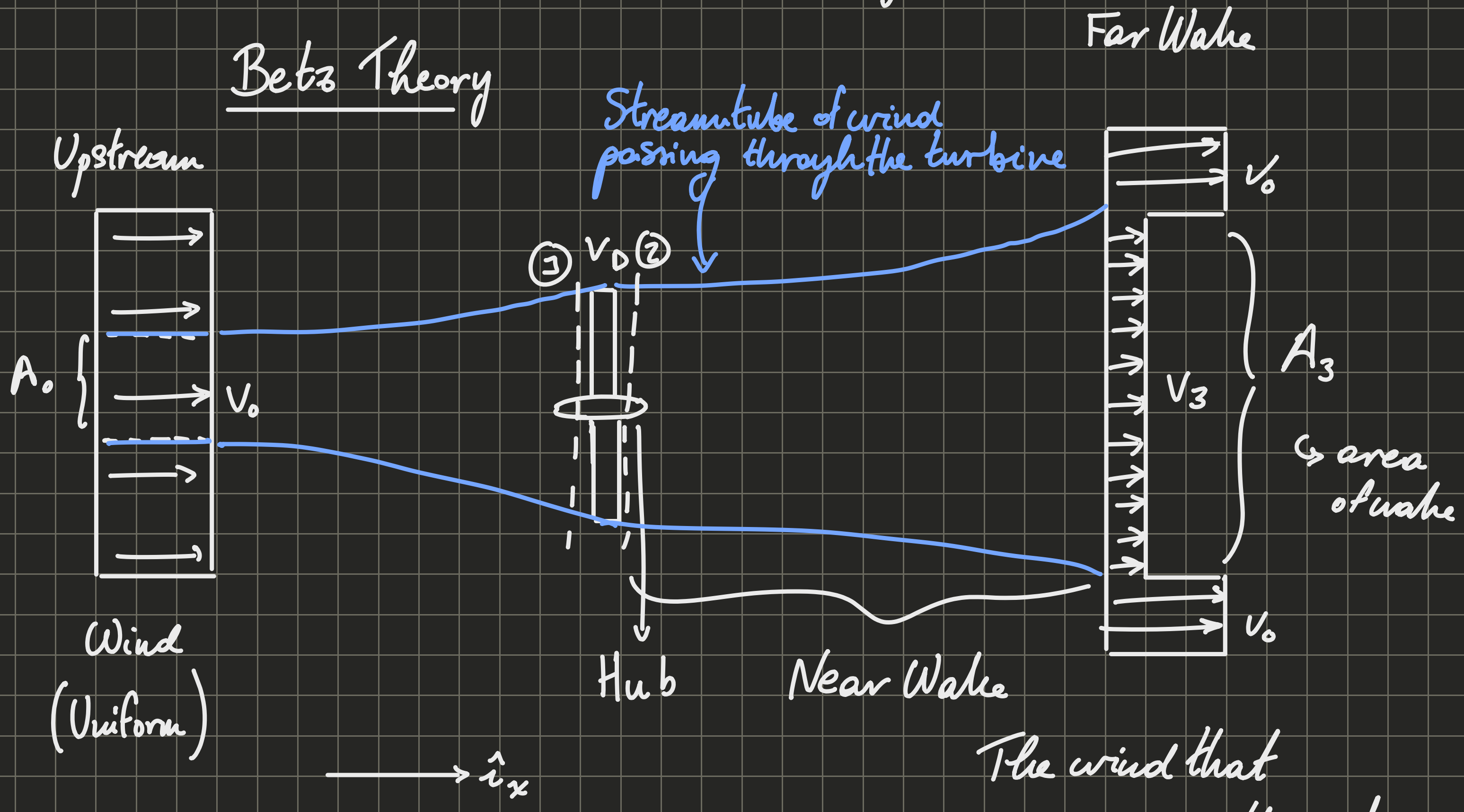
↳ generator at top of tower → cost, maintenance and load.

Throughout history other designs have been proposed but they were all worse and have fallen out of use.

The velocity at the tip of the blade will be the same as that of the wind, so large wind turbine spin slower, and we said that we want larger blades to be able to increase Q.

Wind Turbines (Practical Theory)

Betz Theory



The wind that has passed through the rotor has lost only the kinetic energy, no pressure or geodetic height is lost.

We expect: $V_3 < V_0$ and $V_3 < V_D < V_0$

We associate a flow rate to each velocity:

$$\dot{m}_3 = \rho V_3 A_3 = \dot{m}_D = \rho V_D A_D$$

↳ Area of "dish"

$$= \dot{m}_0 = \rho V_0 A_0$$

↳ Purely fictional, the fraction of the initial flow that is ready to enter the dish.
↓ Turbine

$A_0 < A_3$, since the flow rate needs to be conserved.

area

We can also say $A_0 < A_D < A_3$

Measuring A_3 is very easy, A_0 is difficult since there is no physical differentiator.

The streamtube, collects all the streamlines operating in the flow that crosses the disks.

<u>Assumption we make:</u>

- Incompressible Fluid
- Steady Flow
- Inviscid Flow → No viscosity

$\Rightarrow$ ⊛$_1$

- $v_t \neq 0$ (of course) but very low so we can neglect (not very wrong from a practical standpoint) → we need a new approach

$\hookrightarrow v_t \cong 0 \Rightarrow \vec{V} = V_x \hat{i}_x$

↳ we cannot rely on Euler equation

To find the l we can do a BME between 1 and 2, two sections on either side of the rotor:

$$\text{BME } 1\to2: \quad l - \underbrace{l_w}_{\to 0} = T_2 - T_1 = \frac{P_2}{\rho} + \cancel{\frac{V_2^2}{2}} + \cancel{g z_2} - \frac{P_1}{\rho} - \cancel{\frac{V_1^2}{2}} - \cancel{g z_1}$$

(inviscid)

sections are infinitely close

$$\Rightarrow l = \frac{P_2 - P_1}{\rho} \quad ⊛_2$$

⊛$_1$ $\Rightarrow$

$$\begin{cases} \text{BME } 0\to1: \ \cancel{l}^{\,0} - \cancel{l_w}^{\,0} = T_1 - T_0 = \frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} = \frac{P_0}{\rho} + \frac{V_0^2}{2} + \cancel{g z_0} \\ \text{BME } 2\to3: \ \cancel{l}^{\,0} - \cancel{l_w}^{\,0} = T_3 - T_2 = \frac{P_3}{\rho} + \frac{V_3^2}{2} + \cancel{g z_3} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{g z_2} \end{cases}$$

Summing the two equations:

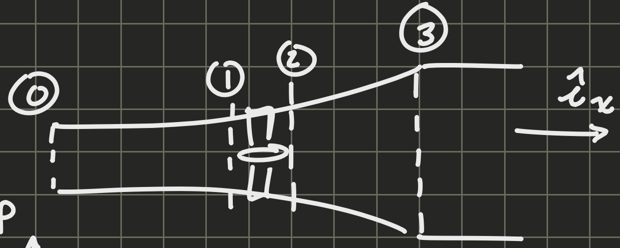
$$\frac{P_1}{\rho} + \cancel{\frac{V_1^2}{2}} + \cancel{\frac{P_3}{\rho}} + \frac{V_3^2}{2} = \cancel{\frac{P_0}{\rho}} + \frac{V_0^2}{2} + \frac{P_2}{\rho} + \cancel{\frac{V_2^2}{2}}$$

$$P_3 = P_0 = P_{ATM} \qquad V_1 = V_2 = V_D$$

$$\Rightarrow \frac{P_2 - P_1}{\rho} = \frac{V_3^2 - V_0^2}{2} \quad (*)_2 \Rightarrow \Delta l = \Delta E_k \text{ of wing across turbine (obvious)}$$

$$l = \frac{P_2 - P_1}{\rho} = \frac{V_3^2 - V_0^2}{2}$$

Porous Dish



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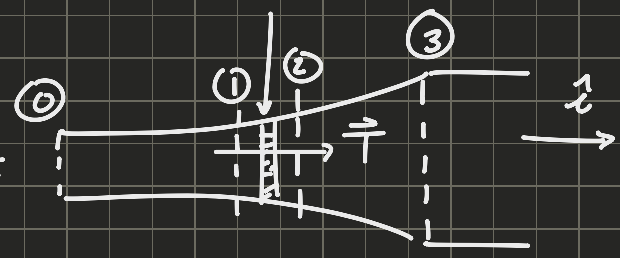
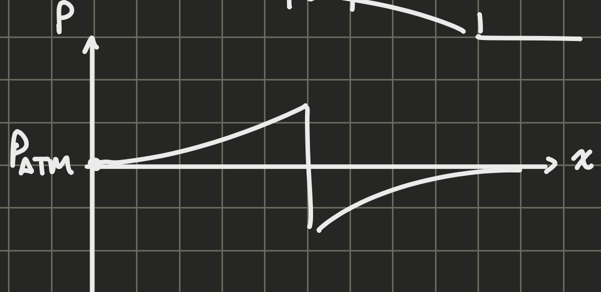
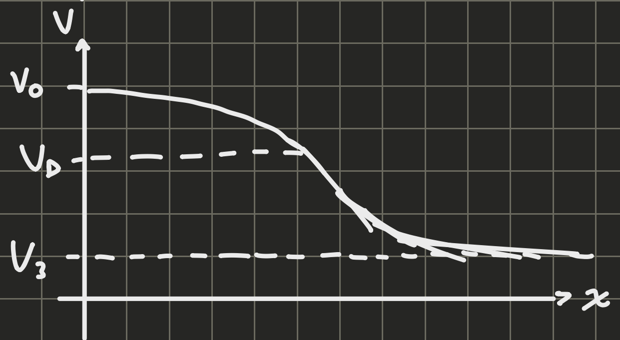




Near wake, where $P < P_{ATM}$,
far wake where $P = P_{ATM}$ first after rotor



We can achieve an equivalent concentrated loss to the loss of the rotor by swapping it with porous dish.

For the porous dish $l = 0$, but l_w performs the loss such that $-l_w = \dfrac{P_2 - P_1}{\rho}$

↳ l_w is by definition positive, so we need the ⊖

$\dot{L} = \vec{T}\cdot\vec{v}_D = \dot{m}\,\ell$

↳ Thrust on porous disk

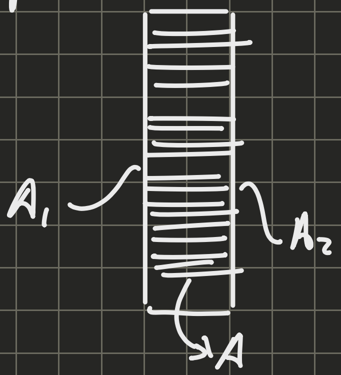
↳ Mechanical power lost by fluid.

We will look at maximumising the $\dot{L}$ of the porous disk and from there find the equivalent for the turbine

To start

B. MOM $1 \to 2$ → Balance of Linear Momentum

We create a control volume around the disk and through it's pores, contouring at material boundaries.



Note: We are using LPA to simplify.

$$\dot{m}\left(\vec{v}_2 - \vec{v}_1\right) + P_2\vec{n}_2 A_2 + P_1\vec{n}_1 A_1 = \vec{T}$$

A vectoral form is no useful so we project along $\hat{i}_x$ by multiplying:

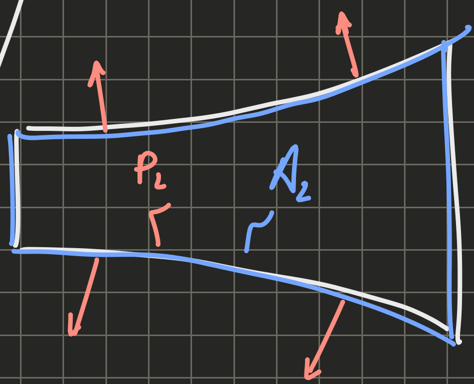
$$\Rightarrow \dot{m}\underbrace{\cancel{\left(v_2 - v_1\right)}}_{\to 0} + \left(P_2 - P_1\right)A_D = T \Rightarrow T = \left(P_2 - P_1\right)A_D$$

↳ This is negative since this is the thrust on the fluid, not on the tower.

We also need: B.MOM $0 \to 3$ along $\hat{i}_x$: For final theory (tomorrow)

We extend the control volume to cover the entire streamtube:

$$\dot{m}\left(\vec{V}_3-\vec{V}_0\right)+P_3\vec{n}_3A_3+P_0\vec{n}_0A_0+\int_{A_L}P_L\vec{n}_L\,dA=\vec{T}$$



The outside produces forces acting perpendicular to the surface due to pressure differences, these constitute the F_{AERO}

Projecting along $\hat{i}_x$:

$$\Rightarrow \dot{m}(V_3-V_0)+\underset{P_{ATM}}{P_3}A_3-\underset{P_{ATM}}{P_0}A_0+\int_{A_L}\underset{P_{ATM}}{P_L}\,\vec{n}_L\,\hat{i}_x\,dA=T$$

$P_{ATM}\rightarrow$ Assumption

$\hookrightarrow$ The pressure is not constant along the section, it is P_{ATM} at the sides and the changing P from before at the core.

Theory will fail because of this

$\hookrightarrow$ So there is a narrow range of validity, we are lucky that the optimal condition lies in this window.

$$\dot{m}(V_3-V_0)+P_{ATM}\Big(\cancel{A_3-A_0}+\underbrace{\int_{A_L}\vec{n}_L\cdot\hat{i}_x\,dA}_{\cancel{-(A_3-A_0)}}\Big)=T\Rightarrow T=\dot{m}(V_3-V_0)$$

Putting it together:

$$\begin{cases} T = (p_2 - p_1) A_D = \dfrac{V_3^2 - V_0^2}{2} A_D \rho \\ T = \dot{m}(V_3 - V_0) = \rho V_D A_D (V_3 - V_0) \end{cases}$$

We will continue from here in the next lecture to develop the theory further.