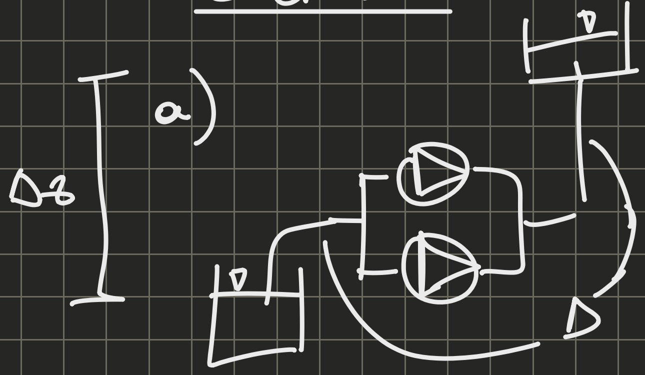
Lezione 15 - (Training

Exercise 6

a)



Pump in series → same flow rate
↳ sum head

$\Delta z = 4\,m$

$n = 1500$

$$\begin{cases} H_p = -100Q^2 - 20Q + 12.5 \\ \eta_p = -0{,}2Q^2 - Q + 1 \end{cases}$$

$D = 0{,}4\,m$

$\dot{Q} = ?$

$H = ?$

$\dot{L} = ?$

CIRCUIT

$$H = \Delta z + \cancel{\frac{\Delta p}{\rho g}}^{\,0} + 3Q^2$$

$$y = 25\,\frac{V^2}{2} = 25\left(\frac{Q}{\pi D^2/4}\right)^2 = \frac{1}{2}\;\frac{25{,}4}{2\pi D^2}\;Q^2$$

$$H_c = 80{,}77\,Q^2 + 4$$

Pumps

$Q = Q_{p1} + Q_{p2} = 2Q_p$

$H = H_1 = H_2$

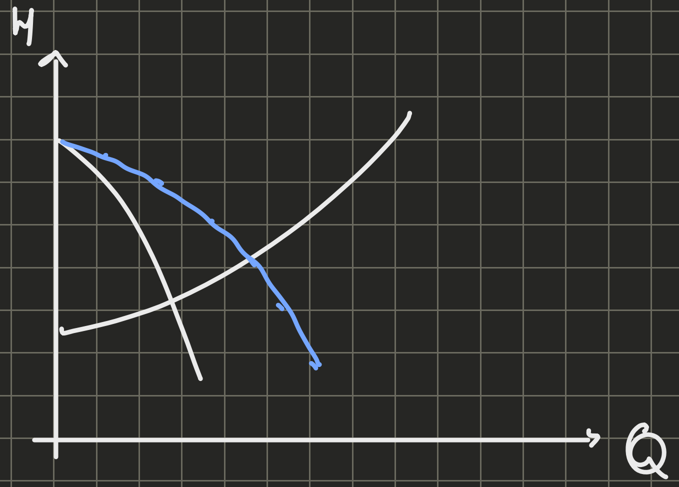
The pressure after both pumps has to be the same since there will be a backflow going into one branch if the pressure is not identical therefore

→ $Q_{p1} = Q_{p2} = Q_p$, since pumps are identical the $Q_p = \frac{1}{2}Q$, and the pumps take the same Q.

$$\Rightarrow H_{p1} = H_{p2} = 100\left(\frac{Q}{2}\right)^2 - 70\left(\frac{Q}{2}\right) + 12,5$$

$$\begin{cases} H = -25Q^2 - 10Q + 12,5 \\ H_c = 80,77Q^2 + 4 \end{cases}$$

$$\Rightarrow \begin{cases} Q = 0,24\frac{m^3}{s} \rightarrow Q_{p1} = Q_{p2} = 0,12\frac{m^3}{s} \\ H = 8,66\ m = H_{p1} = H_{p2} \end{cases}$$



— = system of two parallel pumps

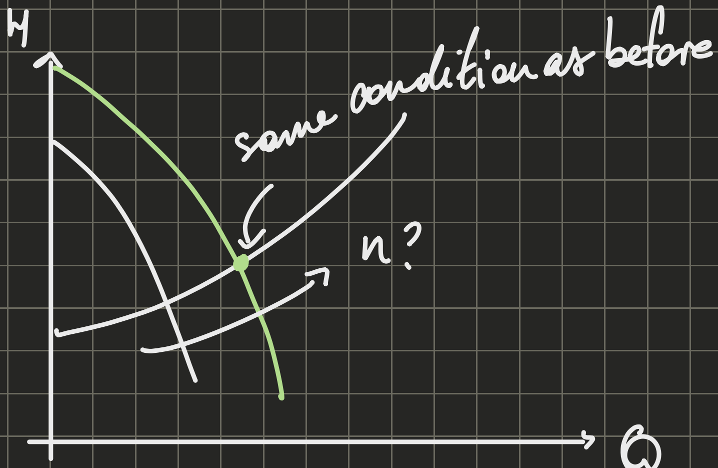
Systems of pumps will allow us to increase the flow rate without buying a specialised heavy flow machine.

$$\dot{L} = \dot{L}_{p1} + \dot{L}_{p2}$$

$$= \rho Q_{p1} \frac{gH_{p1}}{\eta_{p1}} + \rho Q_{p2} \frac{gH_{p2}}{\eta_{p2}} = 23,3\ kW$$

$$\eta_{p1} = -0,2(0,12)^2 - 0,12 + 1 = 0,88 = \eta_{p2}$$

If one breaks but we want to maintain the same operating condition, how much will the n be to operate at this condition?



we cannot search for it through similarity, but we can use the similarity concept to find it.

$$H = -100\,Q^2 - 20\,Q + 12{,}5$$

$$\eta = 0{,}2\,Q^2 - Q + 1$$

$$H(n) = -100\,Q^2 - \frac{n}{1500} \cdot 20\,Q + \left(\frac{n}{1500}\right)^2 \cdot 12{,}5$$

$$H_c = 80{,}77\,Q^2 + 4$$

$$\Rightarrow H(n) = H_c \Rightarrow -100\,Q^2 - 20\,\frac{n}{1500}\,Q + \left(\frac{n}{1500}\right)^2 12{,}5 =$$

$$= 80{,}77\,Q^2 + 4$$

Q is the same as before $\rightarrow Q = 0{,}24\ \frac{m^3}{s}$

$\Rightarrow n = 1925$ rpm

$$\eta(n) = -0{,}2\left(\frac{1500}{n}\right)^2 Q^2 - \left(\frac{1500}{n}\right) Q + 1$$

the efficiency can only close or open with n

$$\eta(1925) = 0{,}806$$

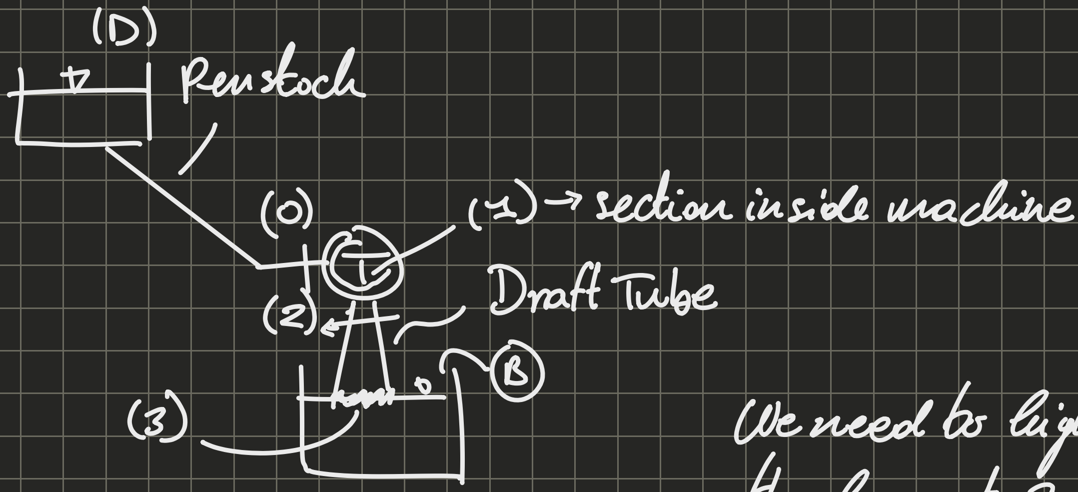
$$\dot{L} = \rho Q\, g \frac{H}{\eta} = 25{,}3\ kW$$

Q, H are the same since the operation condition is the same, but η is lower since we only have one pump, so $\dot{L}$ has to be higher

Start of Theory

Hydraulic Turbines

Hydro-Electricity Systems



We need to highlight the losses at 3 since not accounting for them well can reduce the efficiency by 10%.

BME (D → B)

$$+|l| + l_w + \overset{\text{Penstock}}{y_P} + \underset{\text{Draft Tube}}{y_{DT}} = (T_B - T_D) = -g(z_B - z_D)$$

Technically $P_{ATM,B} \neq P_{ATM,D}$, but we are not making money from this so we consider them to be equal.

since $|l|$

$$|l| = g(z_D - z_B) - y_P - y_{DT} - l_w$$

(*) $$\boxed{g \underset{\text{motorhead}}{H_m} = T_0 - T_2}$$

BME 0 → B $$l - l_w - \overset{\text{Draft Tube}}{y_{DT}} = T_B - T_0$$

$$\Rightarrow -|\ell| - \ell_w - y_{DT} = T_B - T_0$$

$$|\ell| + \ell_w = T_0 - T_B - y_{DT}$$

BME $0 \to 2$

$$\ell - \ell_w = T_2 - T_0 \Rightarrow T_0 - T_2 = g H_m = |\ell| + \ell_w$$

For a turbine:

$$\eta_T = \frac{|\ell|}{g H_m} = \frac{|\ell|}{|\ell| + \ell_w} = \frac{g(z_D - z_B) - \overset{\text{Penstock losses}}{y_P} - y_{DT} - \ell_w}{g(z_D - z_B) - y_P - y_{DT}}$$

$$= 1 - \frac{\ell_w}{g H_m}$$

The manufacturer gives us the machine and the draft tube, the draft tube is a part of the machine, but also considering the draft tube losses is not very useful so we use another explanation.

$$(**)\quad g H_m = T_0 - T_B + T_D - T_D = (T_D - T_B) - (T_0 - T_D)$$

$$= g(z_D - z_B) - (\underbrace{T_0 - T_D}_{y_P})$$

$$= g(z_D - z_B) - y_P$$

Definition of efficiency

$$\eta = \frac{|l|}{gH_m} = \frac{g(z_D - z_B) - y_p - l_w - y_{DT}}{g(z_D - z_B) - y_p} = 1 - \frac{l_w}{gH_m} - \frac{y_{DT}}{gH_m}$$

By playing with the definition of H_m, we managed to include y_{DT} in the definition of the efficiency, which is useful since the manufacturer will also sell us the draft tube.

$$y_{DT} = y_{m_D} + y_{m_C} = \xi \frac{V_3^2}{2} + \frac{V_3^2}{2} = (\xi_{DT} + 1)\frac{V_3^2}{2}$$

concentrated loss coefficient at exit

$$= 1 - \frac{l_w}{gH_m} - \xi_{DT}\frac{V_3^2}{2gH_m} - \frac{V_3^2}{2gH_m}$$

The draft tube will be a divergent since we want to reduce V_3 as much as possible to reduce the losses.

Advantage of increasing cross section

Mass Balance

$2 \to 3$

$$\frac{V_2 \pi D_2^2}{4} = \frac{V_3 \pi D_3^2}{4} \Rightarrow V_3^2 = V_2^2 \left(\frac{D_2}{D_3}\right)^4$$

$V_2 = V_{2m} \to$ in machine with draft tube

we try to make sure V_2 is purely meridional, so we don't have swirls which have low pressure in the middle, possibly causing cavitation.

The fact that the draft tube releases in a basin at P_{ATM}, and the fact that the P increases along the tube, it imposes P_2 to be lower than P_{ATM}.