Esercitazione 2 - C'erano scioperi quindi questo è da reato non Tanzi

1) Cinematica Punto nel piano:
Traiettoria Parabolica:

Definiti

$$\vec{P}(t=0_s) = 0\,[m] \quad \begin{cases} x(t=0)=0 \\ y(t=0)=0 \end{cases}$$

$$\vec{v}(t) = 2\hat{\imath} + 4t\hat{\jmath}\,[m/s]$$

Trovare:

i) Traiettoria $y(x)$

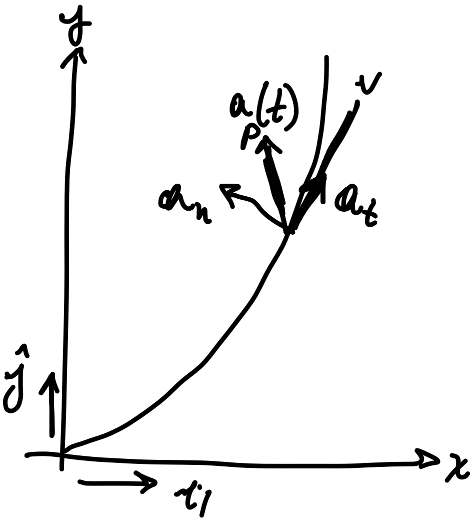
ii) $\vec{p}, \vec{n}, \vec{a}$ per $t = 2s$

iii) $\vec{a}_t, \vec{a}_n$ per $t = 2s$

$$\dot{x}(t) = 2 \quad \dot{y}(t) = 4t \quad \begin{cases} x(t) = \int_0^t \dot{x}(d\tau)\,d\tau + \cancelto{0}{x(0)} = 2\tau = 2t \\ y(t) = \int_0^t \dot{y}(\tau)\,d\tau + \cancelto{0}{y(0)} = \int_0^t 4\tau\,d\tau = \frac{4t^2}{2} = 2t^2 \end{cases}$$

$$\vec{p}(t) = 2t\hat{\imath} + 2t^2\hat{\jmath}$$

$$\begin{cases} x(t) = 2t \\ y(t) = 2t^2 \end{cases} \quad \begin{cases} t = \frac{x}{2} \\ y = \frac{2x^2}{4} = \frac{x^2}{2} \end{cases} \Rightarrow \boxed{y(x) = \frac{x^2}{2}}$$



- vettore posizione

$$\vec{p}(t=2s) = 4\hat{i} + 8\hat{j}\ [m]$$

- vettore velocità

$$\vec{v}(t=2s) = 2\hat{i} + 8\hat{j}\ [m/s]$$

Verifica di tangenza

$$\frac{d(y(x))}{dx} = x(t) = 2t$$

$$\tan\beta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t$$

Accelerazione

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \ddot{x}(t)\hat{i} + \ddot{y}(t)\hat{j}$$

$$= 0\hat{i} + 4\hat{j}$$

$\vec{a}(t=2s) = 4\hat{j}$ [m/s²]

$$\hat{t} = \frac{\vec{v}}{|\vec{v}|} = \frac{2\hat{i}+8\hat{j}}{\sqrt{4+64}} = \frac{2i}{\sqrt{68}} + \frac{8}{\sqrt{68}}\hat{j}$$

$$|\vec{a}_t| = \vec{a}\cdot\hat{t} = (0\hat{i}+4\hat{j})\cdot\left(\frac{2}{\sqrt{68}}\hat{i}+\frac{8}{\sqrt{68}}\hat{j}\right) =$$

$$= \frac{32}{\sqrt{68}}$$

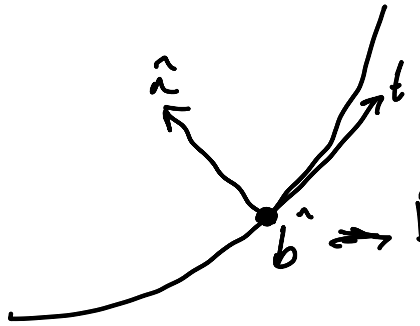
$$\vec{a}_t = |\vec{a}_t|\cdot\hat{t} = \frac{32}{\sqrt{68}}\left(\frac{2}{\sqrt{68}}\hat{i}+\frac{8}{\sqrt{68}}\hat{j}\right) = \frac{16}{17}\hat{i}+\frac{64}{17}\hat{j}$$

1° metodo

$$\vec{a} = \vec{a}_t + \vec{a}_n$$

$$\vec{a}_n = \vec{a} - \vec{a}_t = (0\hat{i}+4\hat{j}) - \left(\frac{16}{17}\hat{i}+\frac{64}{17}\hat{j}\right) = \frac{-16}{17}\hat{i}+\frac{4}{17}\hat{j}$$

<u>2° metodo</u>



$\to \hat{b} = (0\hat{i} + 0\hat{j} + 1\hat{k})$

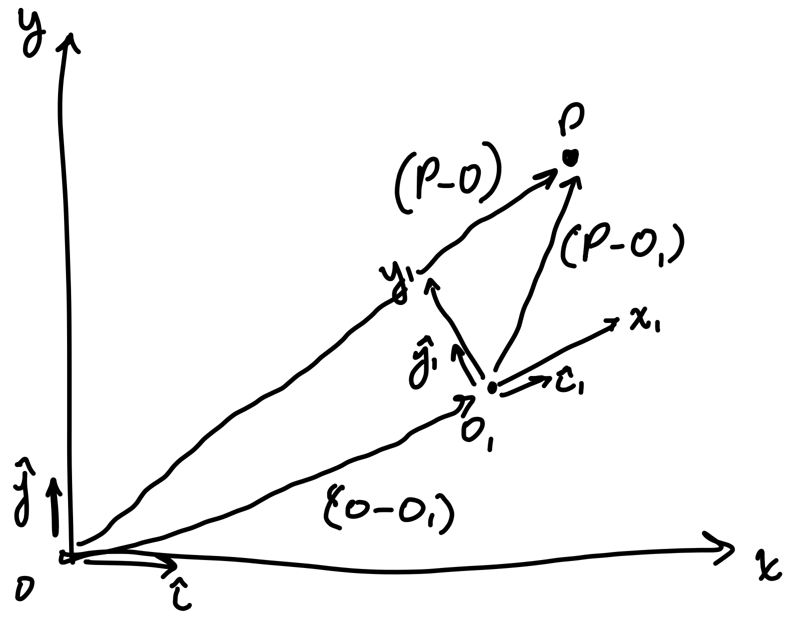
↳ non sempre ma di solito

$$\hat{n} = \hat{b}\times\hat{t} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ \frac{2}{\sqrt{68}} & \frac{8}{\sqrt{68}} & 0 \end{vmatrix} = \frac{-8}{\sqrt{68}}i + \frac{2}{\sqrt{68}}\hat{j}$$

$$|\vec{a}_n| = \vec{a}\cdot\hat{n} = (0\hat{i}+4\hat{j})\cdot\left(\frac{-8}{\sqrt{68}}\hat{i}+\frac{2}{\sqrt{68}}\hat{j}\right) = \frac{8}{\sqrt{68}}$$

$$\vec{a}_n = |\vec{a}_n|\cdot\hat{n} = \frac{-16}{17}\hat{i}+\frac{4}{17}\hat{j} \checkmark$$

2) Teorema del moto relativo



$$(P-O) = (O_1-O) + (P-O_1) = \underbrace{(O_1-O)}_{\text{Rif Assoluto}} + \underbrace{x_1\hat{i}_1 + y_1\hat{j}_1}_{\text{Rif. Mobile}}$$

velocità

$$\frac{d(P-O)}{dt} = \frac{d(O_1-O)}{dt} + \frac{d(P-O_1)}{dt} = \underbrace{\vec{v}_{O_1} + \vec{\omega}\times(x_1\hat{i}_1+y_1\hat{j}_1)}_{\vec{V}_{TRA}} + \underbrace{\dot{x}_1\hat{i}_1+\dot{y}_1\hat{j}_1}_{\vec{V}_{REL}}$$

$$\Rightarrow \vec{v}_P = \vec{V}_{P\,TRA} + \vec{v}_{P\,REL}$$

Accelerazione

$$\frac{d^2(P-O)}{dt^2} = \underbrace{\vec{a}_{O1}}_{a_{TRATRA}} + \overbrace{\dot{\vec{\omega}} \times (x_1\hat{i}_1 + y_1\hat{j}_1) + \vec{\omega} \times \left(\vec{\omega} \times (x_1\hat{i}_1 + y_1\hat{j}_1)\right)}^{a_{TRAROT}}$$

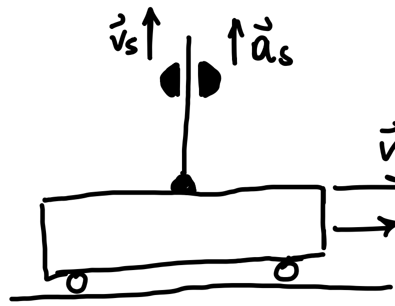
$$+ \underbrace{\ddot{x}_1\hat{i}_1 + \ddot{y}_1\hat{j}_1}_{\vec{a}_{REL}} + \underbrace{2\vec{\omega} \times (\dot{x}_1\hat{i} + \dot{y}_1\hat{j}_1)}_{\vec{a}_{CO}}$$

$$\vec{a}_P = \vec{a}_{TRA} + \vec{a}_{REL} + \vec{a}_{CO}$$

~~Tipi di~~

3) Esercizi di Applicazione Base

①



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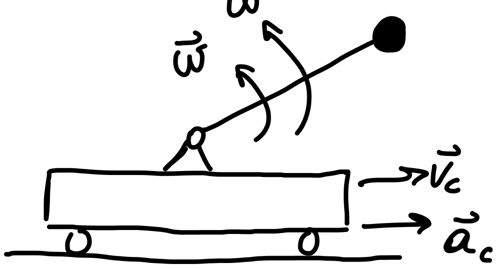

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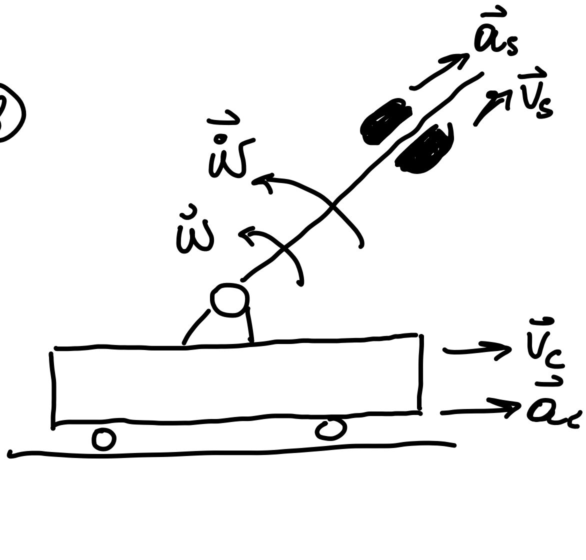

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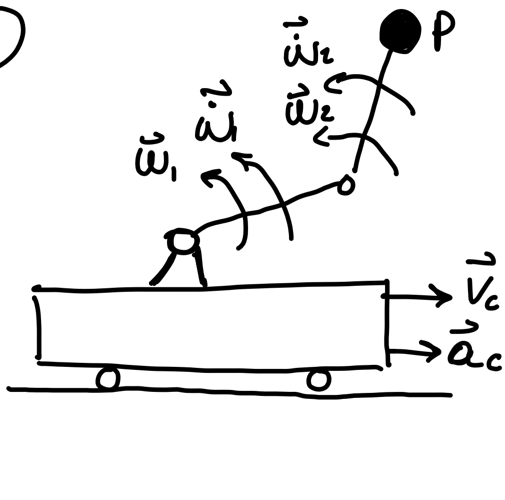


Modellistica dinamatica

↳ Problema di tipo fisico ⟹ Bisogna creare modello semplificato.

PROBLEMA

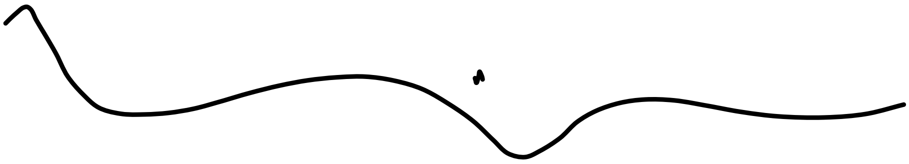
MODELLO

NUMERI COMPLESSI

TEOREMA DEI MOTI RELATIVI

METODO GRAFICO

METODO ANALITICO

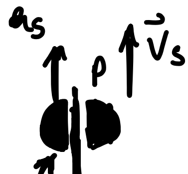


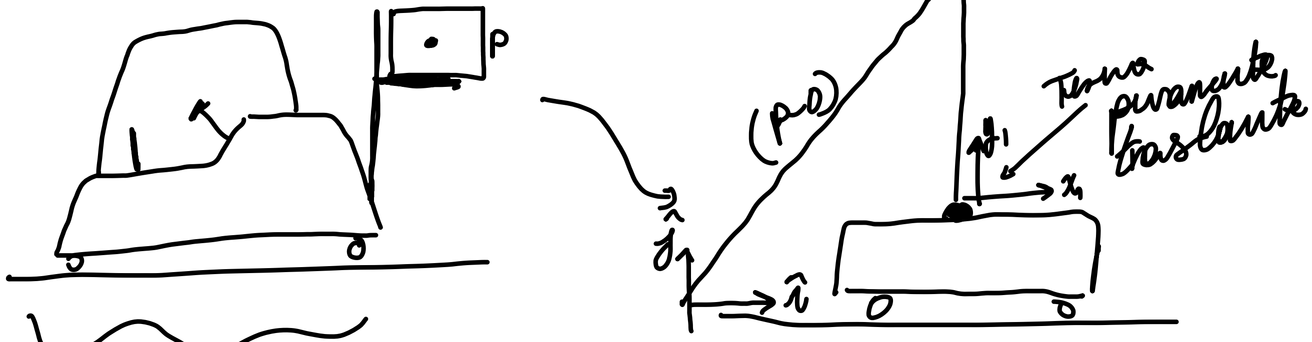
$\Downarrow$

$\vec{p}, \vec{v}, \vec{a}$ — OBIETTIVI $\rightarrow$ COMPRENSIONE DELLO SCHEMA MECCANICO

Primo esercizio   Esercizio Analitico moti relativi base

Teorema moti relativi analitico





Problema fisico

Modello Meccanico

(x_1, y_1, z_1) terna mobile traslante

Posizione

$$(P-O) = (O_1-O) + (P-O_1)$$

Velocità

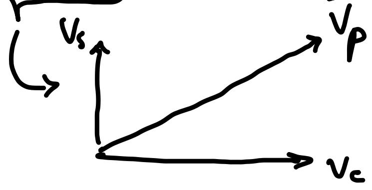
$$\vec v_P = \frac{d(P-O)}{dt} = \frac{d(O_1-O)}{dt} + \frac{d(P-O_1)}{dt} = \frac{d}{dt}(x\hat i + y\hat j) + \frac{d}{dt}(x_1\hat i_1 + y_1\hat j_1)$$

$$= \frac{dx}{dt}\hat i + x\,\cancel{\frac{d\hat i}{dt}}^{\to 0} + \frac{dy_1}{dt}\hat j_1 + y_1\,\cancel{\frac{d\hat j_1}{dt}}^{\to 0}$$

perché $= \vec\omega \times \hat j_1$

ma $\vec\omega = 0 \Rightarrow \frac{d\hat j_1}{dt} = 0$

$$= \dot x\,\hat i + \dot y_1\,\hat j_1 = \underbrace{v_c\,\hat i + v_s\,\hat j_1}$$

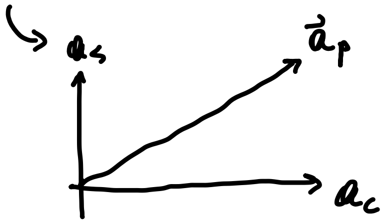


idem

Accelerazione

$$\frac{d(\vec v_P)}{dt} = v_c\hat i + v_s\hat j_1 = \frac{dv_c}{dt}\hat i + \cancel{\frac{d\hat i}{dt}}^{\to 0} v_c + \frac{dv_s}{dt}\hat j_1 + v_s\,\cancel{\frac{d\hat j_1}{dt}}^{\to 0}$$

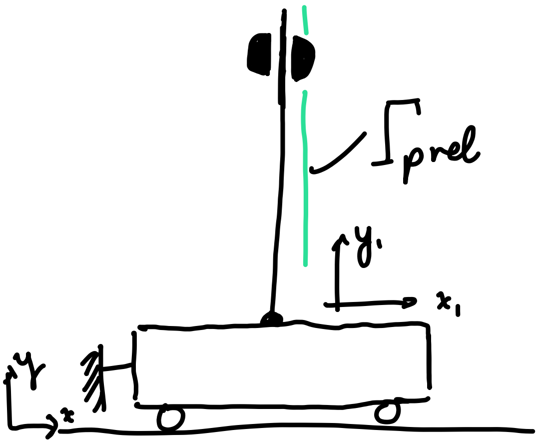
$$= \underbrace{a_c\,\hat i + a_s\,\hat j_1}$$



Approcio Grafico

a) Traccio le traiettorie dei moti

I) MOTO RELATIVO

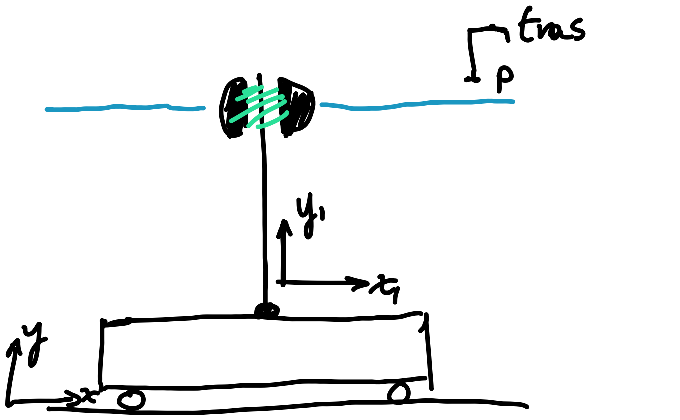


Terna mobile traslante

Solidale al carrello

cent rota in O_1

II) MOTO TRASCINAMENTO

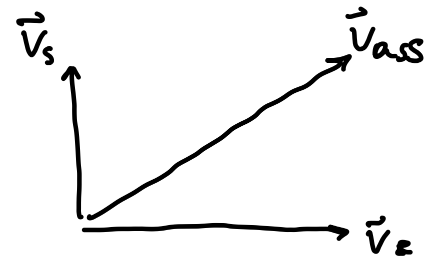


b) Tabella velocita

Tabella delle velocità

	$\vec{V}_p^{ASS}$	$\vec{V}_p^{tras}$	$\vec{V}_p^{rel}$
M	?	V_c	V_s
D	?	$// \hat{i}$	$// \hat{j}_1$

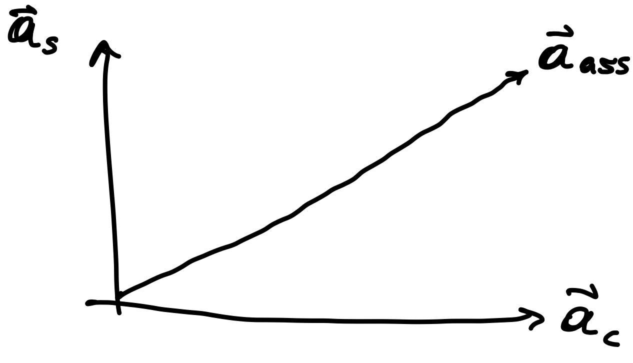
Triangolo delle velocità



c) Studio le accelerazioni

	$\vec{a}_{ASS}$	$\vec{a}_P^{TRAS}$	$\vec{a}_P^{rel}$	$\vec{a}_P^{CO}$
M	?	a_c	a_s	X
D	?	$// \hat{i}$	$// \hat{j}$,	X

Disegnare triangolo



Esercizio Numerico da fare a casa 20/2/15

Nota la legge di moto

$x(t) = \frac{t}{2}$

$y(t) = \cos(\pi t)$

Trovare

I) Traiettoria $y(x)$

II) a_t & a_n

per $t = 2s$