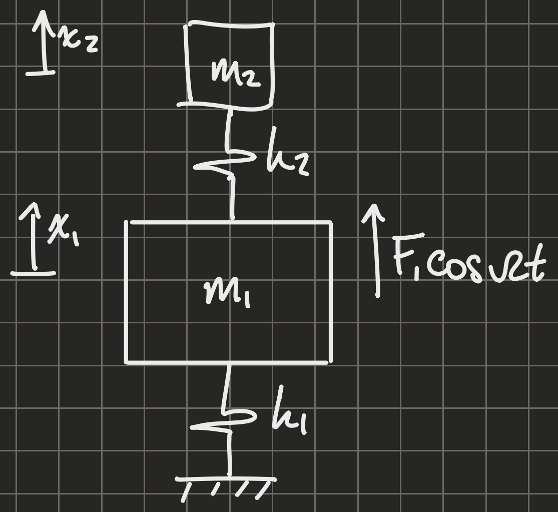
TMDs

Assorbitore Dinamico non smorzato



absorber $\rightarrow \omega_2 = \sqrt{\dfrac{k_2}{m_2}}$

System to protect $\quad \omega_1 = \sqrt{\dfrac{k_1}{m_1}}$

$$\mu = \frac{m_2}{m_1}$$

ω_2 and ω_1 are independent

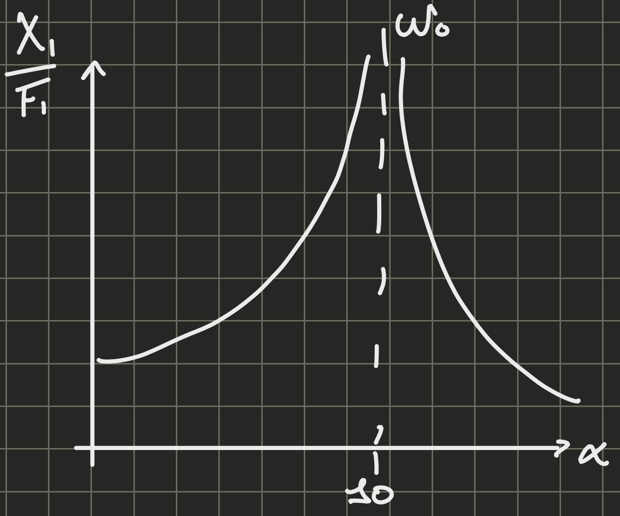
EOM:

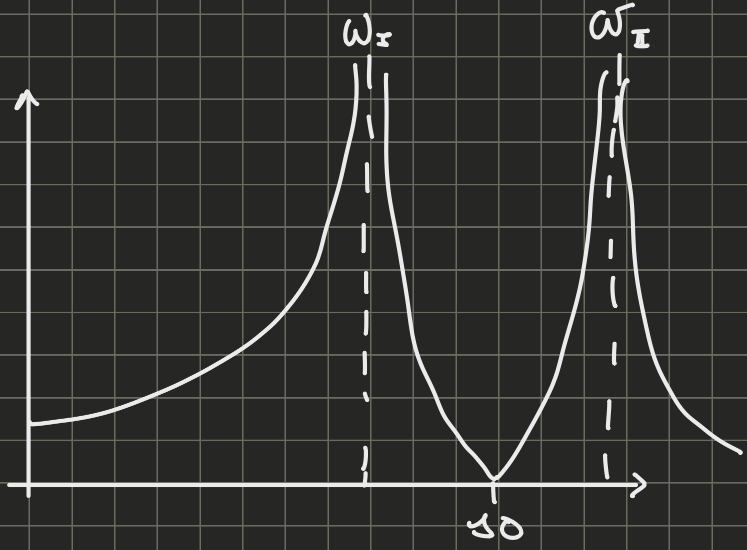
$$\begin{bmatrix} -m_1\Omega^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2\Omega^2 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix}$$

$$\frac{X_1}{F_1} = \frac{-m_2\Omega^2 + k_2}{(-m_1\Omega^2 + k_1 + k_2)(-m_2\Omega^2 + k_2) - k_2^2}$$

$$\frac{X_2}{F_1} = \frac{k_2}{(-m_1\Omega^2 + k_1 + k_2)(-m_2\Omega^2 + k_2) - k_2^2}$$

System response without absorber





Resonance at $\Omega = \omega_0$
(Typical 1 dof response)

We now have 2 dof, we see that the mass M_1 no longer resonates at the frequency it did before, and the system does not displace anymore. The drawback is that now have have more natural frequencies.

If we tune this system, such that:

$$\omega_2^2 = \frac{k_2}{m_2} = \omega_1^2$$

The response of M_1, caused by F_1, annuls when $\Omega = \omega_1 = \omega_2$

$$\frac{X_1}{F_1} = 0$$

$$\frac{X_2}{F_1} = -\frac{1}{k_2} \Rightarrow k_2 X_2 = -F_1 \Rightarrow F_1 \cos \Omega t = -k_2 X_2 \cos \Omega t$$

The absorber acts to create a force equal to the applied force.

This is not a real case, since F_1 is very ideal and we don't have damping.

Dimensionless quantities:

$$a_1 = \frac{\Omega}{\omega_1} \qquad a_2 = \frac{\Omega}{\omega_2} \qquad f = \frac{\omega_2}{\omega_1} \qquad \mu = \frac{m_2}{m_1}$$

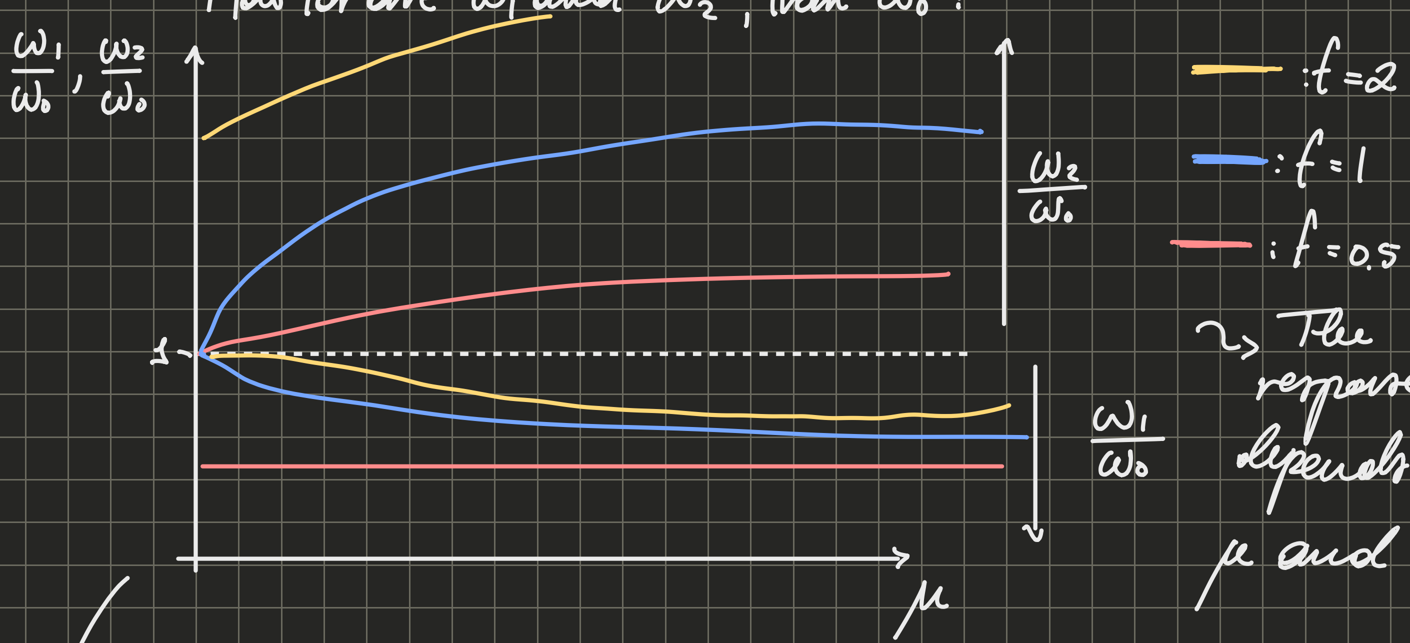
μ → Mass ratio

$$\frac{k_2}{k_1} = \frac{\omega_2^2 m_2}{\omega_1^2 m_1} = f^2 \mu \qquad \delta_{ST} = \frac{F_1}{k_1}$$

δ_{ST} → Static deflection of main system

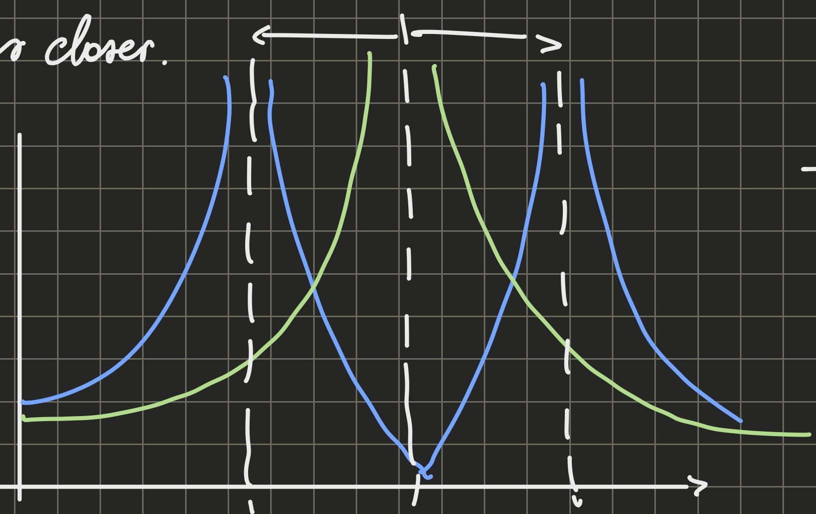
[We skipped some maths]

How far are ω_1 and ω_2 from ω_0?



→ The response depends on μ and f.

As we increase f, the ω_2 moves further from ω_0, and ω_1 moves closer.

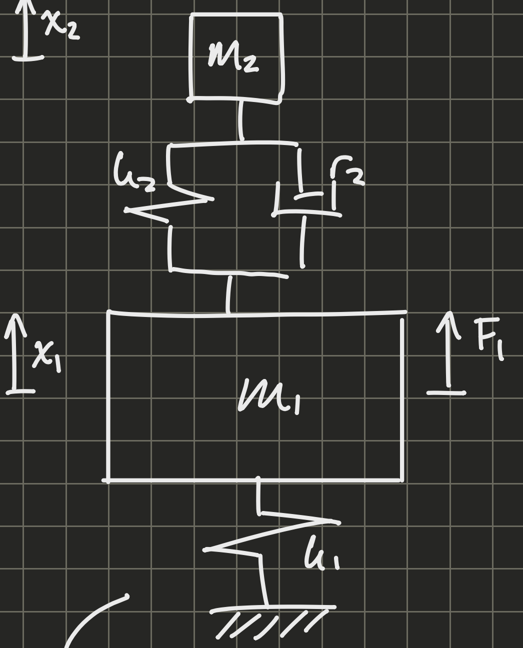


→ What the above graph is measuring.

ω_1 ω_0 ω_2

↳ The graph should be normalized on the x-axis such that ω_0 is where 1 is, meaning $\frac{\Omega}{\omega_0} = 1$, and the other two are proportionally distant relative to Ω.

Dynamic Absorber <u>with</u> damping (TMD)



$$h = \frac{r_2}{2 m_2 \omega_2}$$

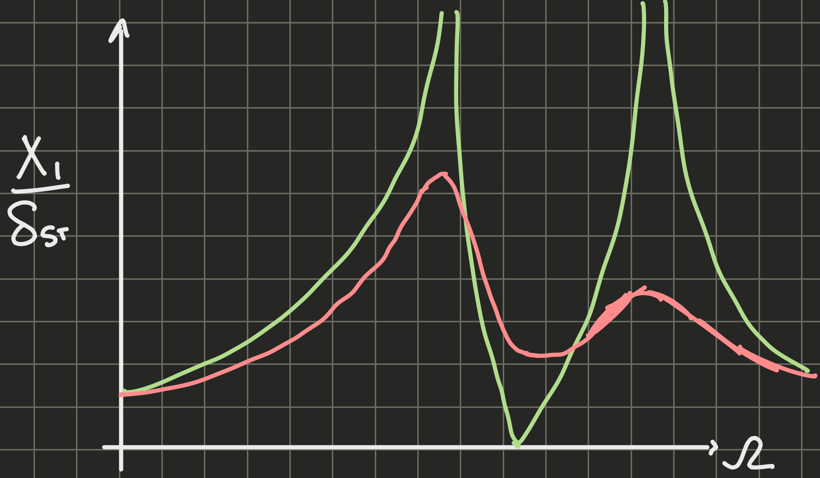
The relative motion of the two masses activates the damper. We have added global damping with connecting a damper to the ground.

↳ This is useful for skyscrapers, since the top is where the most motion is caused, and connecting a damper from there to the ground is useless.

↳ There is no damping, because when looking at buildings for example, the damping is negligible.

Before we worked to kill the forces, now with the TMD we want to make the damper work to increase the global damping.

Comparison of the transfer function between dampened and undampened:



While the behaviour is worse at $\omega = 10$, the overall behaviour of the system is better as the system is globally dampened.

We are increased the capability of reducing the response in a greater range of frequencies other than just one.

How much we dampen, depends on the μ, f like before <u>and</u> the h of our damper and system, the are our design parameters.

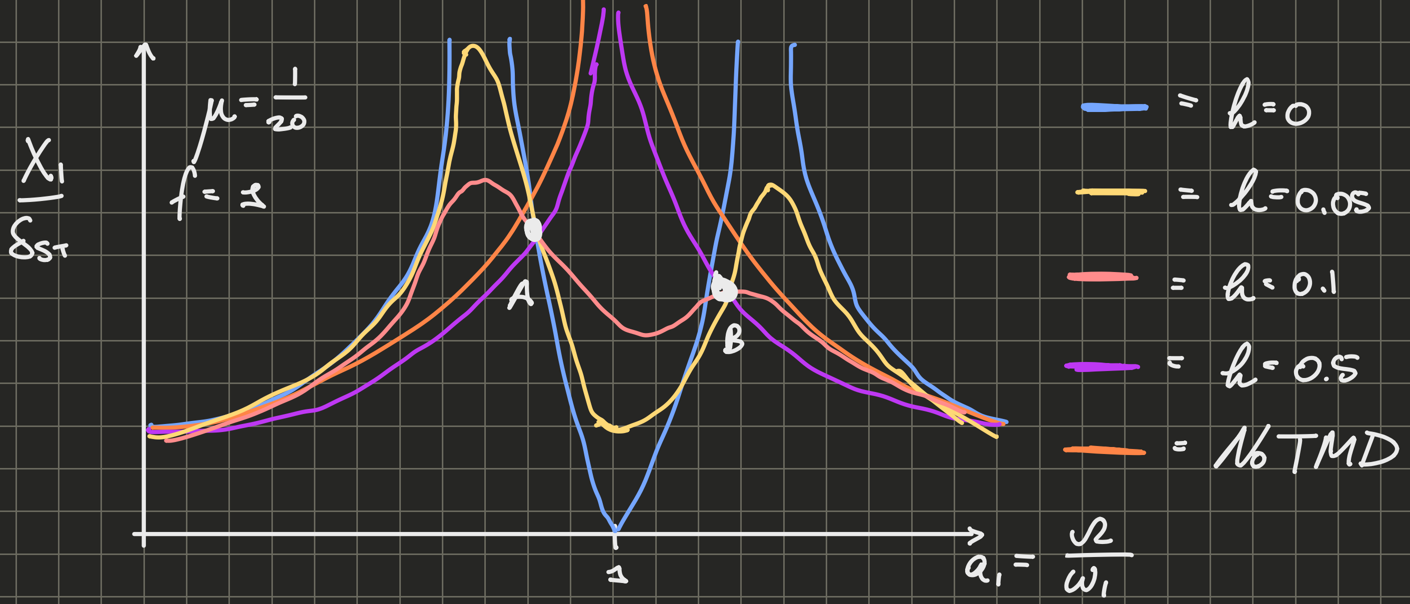
Our design with the TMD is based on how much energy we can dissipate.

$$\frac{X_2}{F_{2eq}} = |H(\Omega)|\, e^{i\varphi(\Omega)}$$

$$E_d = \int_0^T F_{2eq} \cdot \dot{x}_2\, dt = -\pi \cdot |H| \cdot |F_{2eq}|^2 \sin\varphi$$

In resonance $\Omega = \omega_2 = \omega_1$, we are able to dissipate the most energy because:

$$\varphi = -\frac{\pi}{2} \Rightarrow E_d = \pi \cdot |H| \cdot |F_{2eq}|^2$$



We see that as $h \to 0$, the system becomes more like a system with no damper, whereas if we increase h too much it becomes as if we don't have a dynamic absorber.

All curves (should) pass between points A and B.

We are able to optimise our system by changing how, f, μ and h are designed. This allows us to have different curves and different optimal solutions.

TMD review:

Parameters: h, f, μ

We use these parameters to:

- Maximize damping for primary system
- Minimize mass of TMD
- Maximum amplitude of the TMD is acceptable (must be contained)
- Must work for whole range of frequencies