"Lecture with numbers" (It's a lecture, but formally its a training)
↳ Theory clarified with numbers

Exercise 1

↳ Axial fan with objective of accellerating the fluid.
↳ Text on WeBeep

Isoentropic means there is no loss.
↑ since

1. Ideal Work Exchanged
2. Total pressure rise
3. Pressure at stator exit
4. Discharge section at atmospheric change

Through this exercise we will develop some theoretical understanding and aspects.

Data:

$\hat{i}_m = \hat{i}_m \rightarrow$ Axial $\rightarrow \vec{V}_m = \vec{V}_x$

$D_m = 0{,}2\,m$

$b = 0{,}05\,m$

in steady its constant

Volumetric Flow $\rightarrow Q = \frac{\dot{m}}{\rho} = 0{,}5\ \frac{m^3}{s}$

↳ even in steady flow its not constant unless ρ is constant.

$n = 3000\,rpm =$

$v_x =$ constant

$|\Delta\beta| = 20° \rightarrow$ We need to know direction for change

↳ Deflection.

$\rho = const = 1{,}2\,kg/m^3$

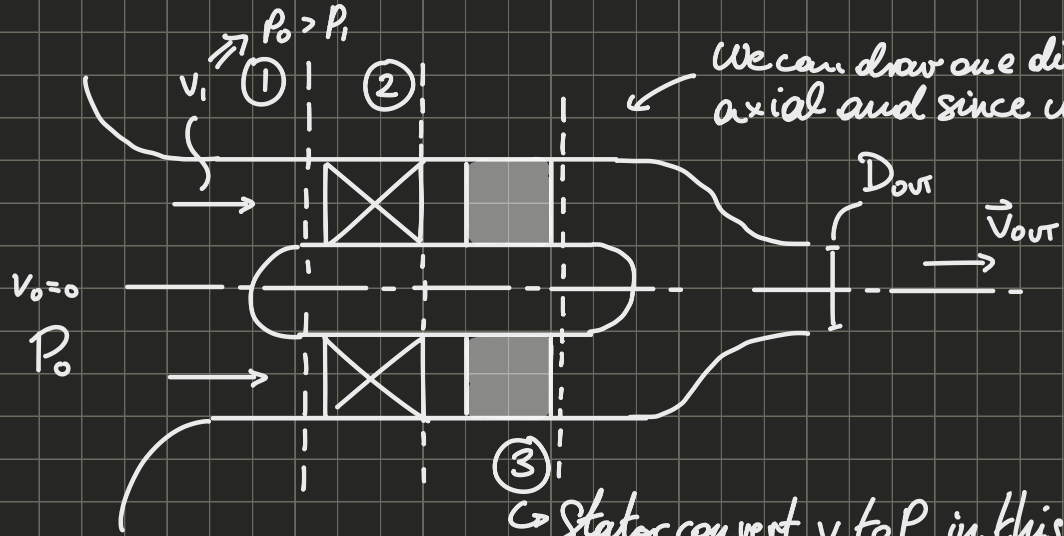
Requests:

↳ l, ΔP_T, $P_{OUT,rotor}$, $P_{OUT,STATOR}$, $D_{OUT,NOZZLE}$?

↳ Ugello.

We need to translate this to a structure

We can draw one diameter since it's axial and since we have D_m.



↳ Stator convert v to P, in this case since v_x=cost

⇒ There is only ΔP due to the work added, since $P_2 > P_0$, the fluid is pushed out to the right.

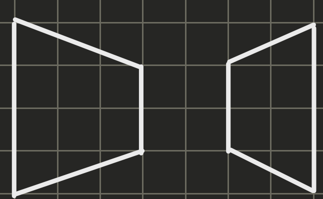
$\alpha_3 = 0$ $\quad l_w = 0 \quad g\Delta z = 0$

↳ Absolute flow angle at stage 3.

Stready Flow ⇒ $\dot{m}$ = const

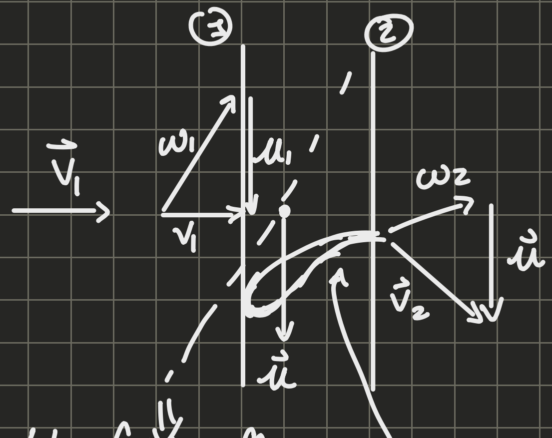
Flow incompressible ⇒ ρ = const ⇒ Q = const

$$Q = v_x \pi D_m b \rightarrow Q, v_x, D_m = \text{const} \Rightarrow b = \text{const}$$

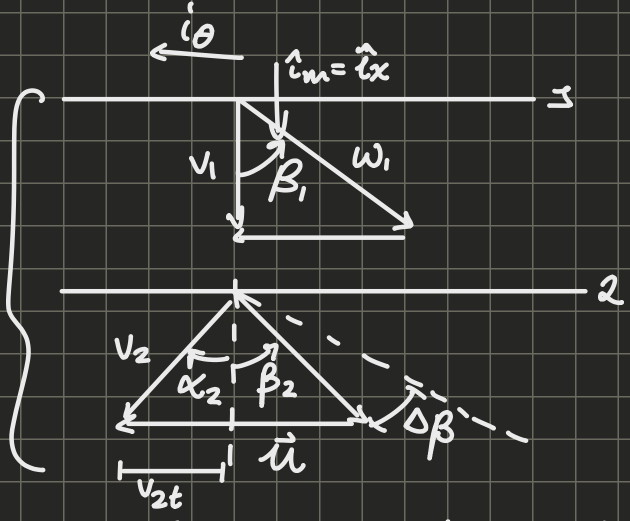


Compressor Turbine

↳ in compressors $\rho \neq$ const, we general we want v_x and D_m = const, so we compensate the change in ρ with the change in b to keep $\dot{m}$ constant. → This is important for later lessons and we will repeat this then.



Should be one but I made it small



the foil will needs to cambered for ω.

Deflected to be axial so ω becomes more axial, and since $V_x = \text{const}$, we have to deflect it to become more axiel.

The rotor sees $\vec{\omega}_1$ and $\vec{\omega}_2$, whereas the inertial observer sees V_1 and V_2.

We need to remember the deflection we need to make the flow to make it more axial or radial.

Operating machine → more axial $\vec{\omega}$

Motor machine → more axial $\vec{\omega}$

We want to make ω less inclined and therefore the blade too

In axial machine, $\vec{u}$ does not change.

$$\ell = u_2 V_{2t} - \cancel{u_1 V_{1t}}^{\,0} = u \cdot V_{2t}$$

If the flow is from a duct or reservoir it is 0.

$$u = u_m = \frac{\omega D_m}{2} = 31{,}4\ \frac{m}{s}$$

$$\omega = \frac{2\pi n}{60} = 314\ rad/s$$

→ To calculate it will take a bit.

$$V_x = \frac{Q}{\pi D_m b} = 15{,}9$$

$$V_{1x} = V_x\ ;\ V_{1t} = 0$$

$$\omega_{1x} = V_{1x} = V_x\ ;\ \omega_{1t} = \cancel{V_{1t}}^{\,0} - u = -u$$

Finding β_1, and knowing $\Delta\beta$ we can find β_2

$$\beta_1 = \operatorname{atan}\left(\frac{\omega_{1t}}{\omega_{1x}}\right) = -63^\circ$$

$\beta_2 = f(\beta_1, \Delta\beta)$

$= \beta_1 + \Delta\beta = -43°$

↳ Due to $w_{1t} = -u$, since we take a sign 0 as down

0° → convention we use.

↳ since $\beta_1 < 0$ and we want it to be closer to the 0.

↳ The angle should reduce in an operating machine and increase in a motor machine.

$$w_{2x} = \underbrace{v_{2x} = v_{1x}}_{v_x = cost.} \rightarrow |\vec{w}_2| = \frac{w_{2x}}{\cos\beta_2} = 21{,}8\ m/s \ ; \ w_{2t} = w_{2x}\tan\beta_2 = -14{,}9\ m/s$$

$$v_{2t} = w_{2t} + u = 16{,}5\ m/s$$

↳ Needs to be positive in operating machines.

$$l = u v_{2t} = 518.1\ J/kg$$

$\alpha_1 = 0$ since $v_{1t} = 0$

$$|\vec{v}_2| = \sqrt{v_{2x}^2 + v_{2t}^2} = 23\ m/s \rightarrow \alpha_2 = \mathrm{atan}\left(\frac{v_{2t}}{v_{2x}}\right) = 46°$$

The stator raddrizza la velocità, conservando l'energia, quindi la energia meccanica che abbiamo

The stator straightens the fluid flow without wasting energy, therefore the mechanical energy that we added will have to be converted into pressure.

If we took $\vec{u}$ in the other direction it still works (as long as the blades adapt) and we will just flip the sign.

⓪ $V_0 = 0$, $P_0 = 1\,\text{bar} = 10^5\,\text{Pa}$

① $v_1 = v_x = 15.9$; $P_1 = ?$

Always specify the zones where we balance

Mechanical Energy Balance $0 \to 1$

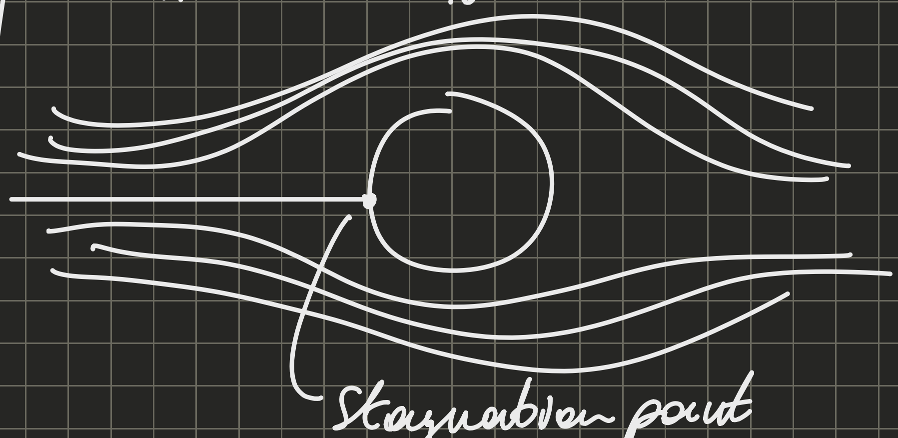
Between 0 and 1 no work is exchanged and no friction.

$$\cancel{q - w_s} = \underbrace{\int_0^1 v\,dP}_{} + \underbrace{\frac{\Delta v^2}{2}}_{} + \underbrace{\cancel{g\Delta z}}_{}$$

$$0 = \frac{P_1 - P_0}{\rho} + \frac{v_1^2 - v_0^2}{2} + g\cancel{(z_1 - z_0)}$$

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_0}{\rho} + \frac{v_0^2}{2}$$

$$\underbrace{P_1 + \frac{1}{2}\rho v_1^2}_{P_{T1}} = \underbrace{P_0 + \cancel{\frac{1}{2}\rho v_0^2}}_{P_{T0}} \Rightarrow P_1 = P_0 - \frac{1}{2}\rho v_1^2 = 99848\,\text{Pa}$$



The total pressure is the isentropic pressure of the fluid were we to stop the fluid. It's a local quantity we can define everywhere, as if we put a measuring device in which caused the fluid to stagnate.

$$P_{T\infty} = P_\infty + \frac{1}{2}\rho v_\infty^2$$

We can formula conservation concepts of the total pressure (in all cases where $\Delta z = 0$)

This is valid if it isentropic and incompressible case.

The total pressure is not a formula but a concept.

The meccanica energy balance tells us that without work, wasted work and $\Delta z = 0 \Rightarrow$ total pressure is conserved.

② $\vec{v}_2 : |\vec{v}_2|, \alpha_2, P_2 = ?$

Mechanical Energy Balance $1 \to 2$:

$$\rho(l - l_w) = \frac{P_2 - P_1}{\cancel{\rho}} \rho + \rho\frac{v_2^2 - v_1^2}{2} + \cancel{\rho g \Delta z}^{\,0}$$

$$\rho l - \rho l_w = \underbrace{P_2 + \frac{1}{2}\rho v_2^2}_{P_{T2}} - \underbrace{\left(P_1 + \frac{1}{2}\rho v_1^2\right)}_{P_{T1} = P_{T0} = P_0}$$

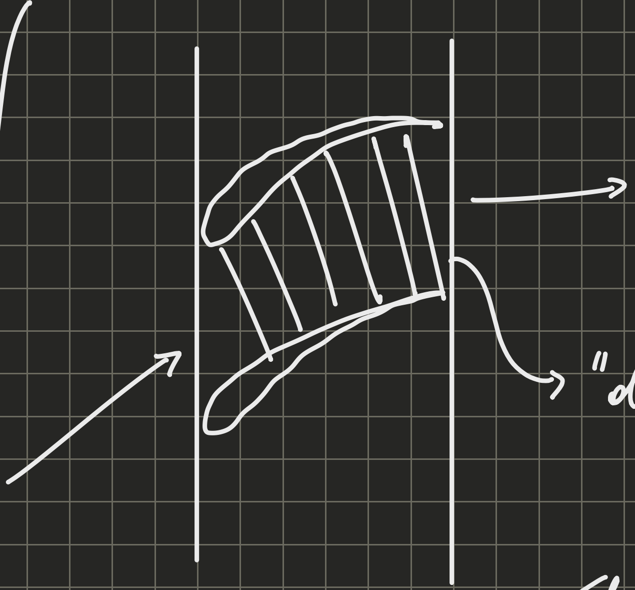
$$\rho l - \underbrace{\rho l_w}_{\to 0 \text{ since isentropic}} = P_{T2} - P_{T1}$$

$$\Delta P_T = P_{T2} - P_{T1} = \rho l = 621 \text{ Pa}$$

$$P_{T2} = P_{T1} + \Delta P_T = P_{T0} + \Delta P_T = \underbrace{P_0}_{\text{Initial mechanical energy}} + \underbrace{\Delta P_T}_{\text{Added from work put in.}}$$

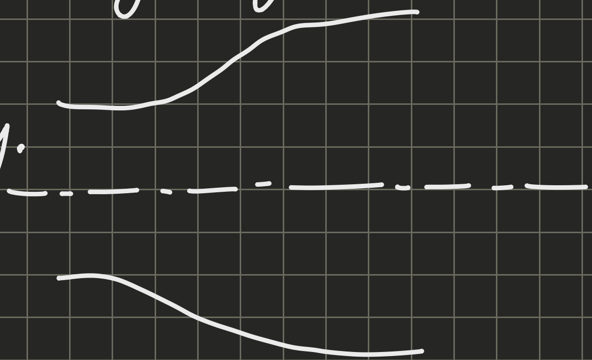
$$P_{T2} = 100621 \text{ Pa}$$

$$P_{T2} = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P_2 = P_{T2} - \frac{1}{2}\rho v_2^2 = 100303{,}6 \text{ Pa}$$



"duct" generated by 2 blades

They diverge



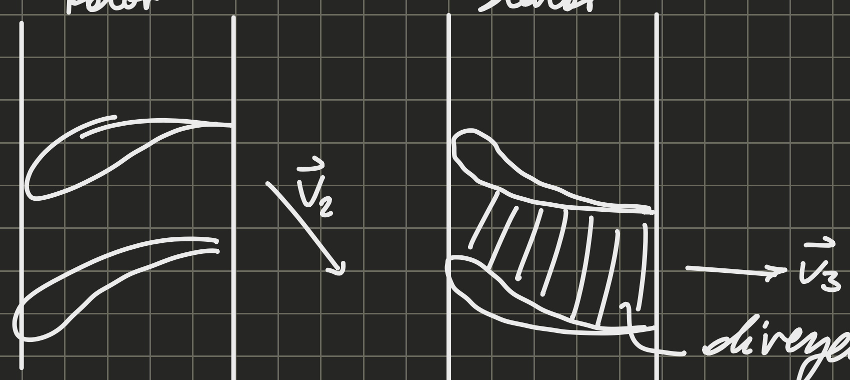
The flow is both accelerating a decellerating.
v is accelerating, while the w is decellerating.

→ There is both a change in velocity and pressure.

Bernoulli's theorem is not true when $l \neq 0$, the mechanical energy applied by the rotor will increase the v and P.

If we consider a rotating observer we will the work exchange and also find that the decrease in w will lead to an increase in P. (We will do this on monday)

③



In the stator both the absolute velocity reduces and the relative does not exist since $u=0$.

$V_{3t} = 0\ m/s$; $V_{3x} = V_x$

$|\vec{V}_3| = |\vec{V}_1|$

$P_{3T} = P_{2T}$ since inviscid stators don't exchange work

$\Rightarrow P_3 = P_{T2} - \frac{1}{2}\rho V_3^2 = 100470\ Pa$

The mechanical energy exchanged has become the change in pressure.

Mechanical Energy Balance 3→OUT

$$P_{T3} = P_{T,OUT} = P_{OUT} + \frac{1}{2}\rho V_{OUT}^2 = P_0 + \frac{1}{2}\rho V_{OUT}^2$$

$$P_{T3} = P_{T2} = P_{T1} + \rho \ell = P_{T0} + \rho \ell$$

In reality there will be friction.

$$\Rightarrow \underbrace{P_{T0}}_{\cancel{P_0}} + \cancel{\rho}\ell = \cancel{P_0} + \frac{1}{2}\cancel{\rho} V_{OUT}^2 \Rightarrow \boxed{\ell = \frac{1}{2} V_{OUT}^2}$$

↳ It's a snow cannot, we have accelerated the fluid by putting work