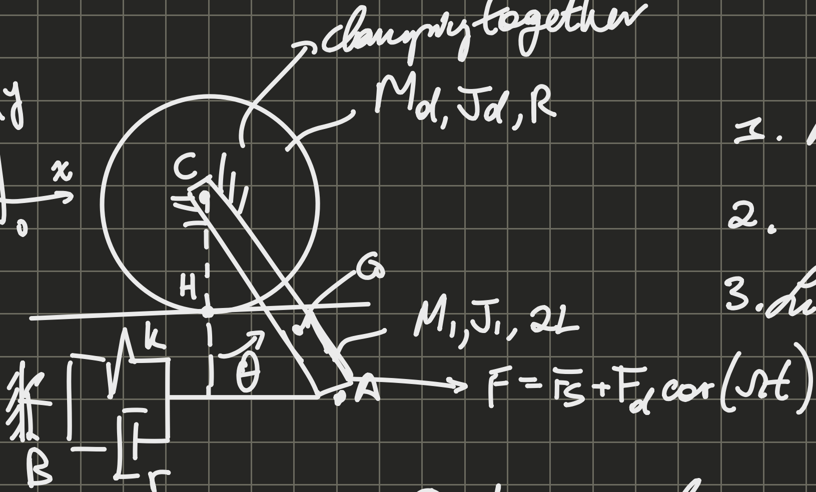
Esercitazione 1 -

Ludovico Dassi

Esercizio 1



1. Non-linear equation of motion
2. $\Delta l_0 = ?\ (\theta = \theta_0)$
3. Linear equation of motion.

Disks are always purely rotating

Step 1: degrees of freedom

$doF = 3 \times 2 = 6$

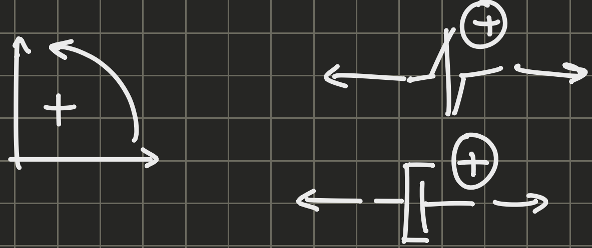
$doc = 1 \text{ clamping } (-3) + 1 \text{ Rotation without Sliding } (-2) = -5$

degrees of costraint

1 dof remaining

independent variable θ

Conventions:



Theorem of König

Step 2:

Asta

$$E_c = \frac{1}{2} M_d V_c^2 + \frac{1}{2} J_D \omega_d^2 + \frac{1}{2} M_A V_G^2 + \frac{1}{2} J_A \omega_A^2$$

$$D = \frac{1}{2} r \dot{\Delta l}^2$$

$$V = V_g + V_k = M_d\, g\, h_D + M_A\, g\, h_G + \frac{1}{2} k \Delta l^2$$

$$\delta^* \mathcal{L} = \left[F_S + F_d \cos(\Omega t)\right] \cdot \underbrace{\delta s_A}_{\text{Infinitesimal change in point A.}}$$

Step 3 → kinematic relations → relating each quantity to each independent variable.

- $\vec{\omega}_A = \dot{\theta}\, \hat{k}$
- $\vec{\omega}_d = \dot{\theta}\, \hat{k} = \vec{\omega}_A \xrightarrow{} \text{pure roll}$
- $\vec{v}_C = \cancel{\vec{v}_H}^{\,0} + \vec{\omega}_D \times (C - H) = -\dot{\theta} R\, \hat{\imath}$
- $\vec{v}_G = \vec{v}_C + \vec{\omega}_A \times (G - C) = -R\dot{\theta}\hat{\imath} + \dot{\theta}\hat{k} \times (L\sin\theta\, \hat{\imath} + L\cos\theta\, \hat{\jmath})$

$$= -R\dot{\theta}\hat{\imath} + \dot{\theta}L\sin\theta\, \hat{\jmath} + \dot{\theta}L\cos\theta\, \hat{\imath} =$$

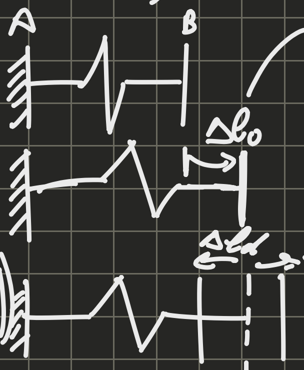
lunghezza molla $= (\dot{\theta}L\cos\theta - R\dot{\theta})\hat{\imath} + \dot{\theta}L\sin\theta\, \hat{\jmath}$

Theory

- $\Delta l = \Delta l_0 + \Delta l_d$

$$\Delta l = l_m(\theta) + l_s$$

$$= \underbrace{(l_m(\theta) - l_m(\theta_0))}_{\Delta l_d} + \underbrace{(l_m(\theta_0) - l_s)}_{\Delta l_s}$$



unloaded → sign of the preload is important.

Stationary

Vibration

$$l_m(\theta) = s_B - s_A$$

returning to exercise:

$$l_m = x_A - x_B$$

$$x_B = \text{const}$$

$x_A = ?$ → Rivals → $v_A \to \int v_A \to x_A$

→ Position vector of A

↳ Position:

$$(A-O) = (C-O) + (A-C) = (C_1 - \overset{\uparrow}{R\theta})\hat{i} + (2L\sin\theta\,\hat{i} - 2L\cos\theta\,\hat{j})$$

negative since disk rolls to the left

$$= (\underbrace{C_1 - R\theta + 2L\sin\theta}) \hat{i} - 2L\cos\theta\,\hat{j}$$

Given the exercise we are only interested in the horizontal so we take this.

↳ Velocity of A (Rivals)

↳ $V_{Ax} = (2L\cos\theta\,\dot{\theta} - R\dot{\theta})\hat{i}$

$$x_A = \int V_{Ax}\,dt = 2L\sin\theta - R\theta + \underbrace{C_1}$$

constant of integration.

$$l_m\ x_A - x_B = (C_1 - R\theta + 2L\sin\theta) - C_2 = C_3 - R\theta + 2L\sin\theta$$

$$\Delta l_d(\theta) = l_m(\theta) - l_m(\theta_0) = (\cancel{C_3} - R\theta + 2L\sin\theta) - (\cancel{C_3} - R\theta_0 + 2L\sin\theta_0)$$

?

$h_C = \text{const}$

$h_G = -L\cos\theta$

$$\dot{\Delta l} = \frac{d}{dt}(\Delta l) = \frac{d}{dt}(\Delta l_d + \Delta l_0) = \frac{d}{dt}\Delta l_d + \boxed{\cancel{\frac{d}{dt}\Delta l_0}^{\,0} =}$$

$$= \frac{d}{dt}(l_m(\theta) - \cancel{l_m(\theta_0)}) = \frac{d}{dt} l_m(\theta)$$

$$\Delta\dot{l} = \dot{\theta}\,2L\cos\theta - R\dot{\theta} = V_{Ax}$$

$$S^* \delta_A = \left[{}^{\delta s_A}/_{\delta\theta}\right] \cdot \delta_\theta = \left[-R + 2L\cos\theta\right] \cdot \cancel{\delta\theta}$$

Step 4

$$\vec{V}_G = (\dot\theta L\cos\theta - R\dot\theta)\hat{i} - (\dot\theta L\sin\theta)\hat{j}$$

$$V_G^2 = (\dot\theta L\cos\theta - R\dot\theta)^2 + (\dot\theta L\sin\theta)^2 = L^2\cos^2\theta + R^2 - 2LR\cos\theta + L^2\sin^2\theta)\dot\theta^2$$

$$\overset{!}{=} (L^2 + R^2 - 2LR\cos\theta)^2\dot\theta^2$$

$$E_c = \frac{1}{2}\left[MR^2 + J_D + J_A + M_A(L^2+R^2) - 2M_A LR\cos\theta\right]\cdot\dot\theta^2 = \frac{1}{2}J^*(\theta)\dot\theta^2$$

$$D = \frac{1}{2}r\,\Delta\dot\ell^2 = \frac{1}{2}r\left[2L\cos\theta - R\right]^2\dot\theta^2 = \frac{1}{2}r^*(\theta)\dot\theta^2$$

$$V = V_k + V_G = \frac{1}{2}k(\Delta\ell_d + \Delta\ell_0)^2 + M_A g(-L\cos\theta) + \overset{\frac{d}{d\theta}=0}{\cancel{M_A g h}}$$

$$\overset{!}{=} \frac{1}{2}k\left[\Delta\ell_0 + (2L\sin\theta - R\theta) - (2L\sin\theta_0 - R\theta_0)\right] + M_A g(-L\cos\theta$$

$$\delta^*\mathcal{L} = \left[F_s + F_d\cos\theta\right](-R + 2L\cos\theta)\cdot\delta\theta$$

<u>Step 5</u>

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot\theta}\right) - \frac{\partial E_c}{\partial\theta} + \frac{\partial D}{\partial\dot\theta} + \frac{\partial V}{\partial\theta} = Q$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot\theta}\right) - \frac{\partial E_c}{\partial\theta} \overset{\text{Theory}}{=} \underbrace{J^*(\theta)}_{\text{know}}\dot\theta + \frac{1}{2}\underbrace{\frac{\partial J(\theta)}{\partial\theta}}_{\text{Need to find}}\cdot\dot\theta^2}$$

$$\boxed{\frac{\partial D(\theta)}{\partial\dot\theta} \overset{\text{Thery}}{=} \underbrace{r^*(\theta)}\dot\theta}$$

$$\frac{\partial V_g}{\partial\theta} = \frac{M_A g(-L\cos\theta)}{\delta\theta} + \frac{\overset{0}{\cancel{M_A g h}}}{\delta\theta}$$

$$\frac{\partial V_u}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{1}{2}k\left[\Delta l_0 + (2L\sin\theta - R\theta) - (2L\sin\theta_0 - R\theta_0)\right]^2\right)$$

$$= \cancel{\frac{1}{2}}k\left[\Delta l_0 + (2L\sin\theta - R\theta) - (2L\sin\theta_0 - R\theta_0)\right]\cdot\cancel{2}\cdot(2L\cos\theta - R)$$

$$Q_\theta = \frac{\partial^* \mathcal{L}}{\partial \theta} = \left[F_s + F_d\cos(\Omega t)\right]\left(-R + 2L\cos\theta\right)\cdot\cancel{\delta\theta}$$

Non-linear

$\Rightarrow$ Equation of Motion