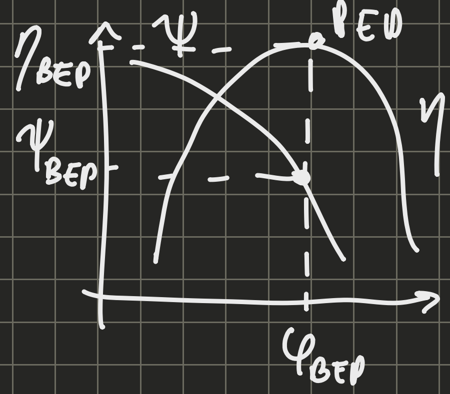
Lezione 12 -

The result that we will see are not results from a theoretical point, they are derived from experiments.

For each machine we know



Mapping the BEP for different machines, he found no trend, as expected.

So what he did was take:

$$\varphi = \frac{Q}{nD^3}$$

$$\Psi = \frac{gH}{n^2 D^2}$$

$$D = \frac{\sqrt{gH}}{n\sqrt{\Psi}}$$

And made dimensionless form in which D doesn't appear

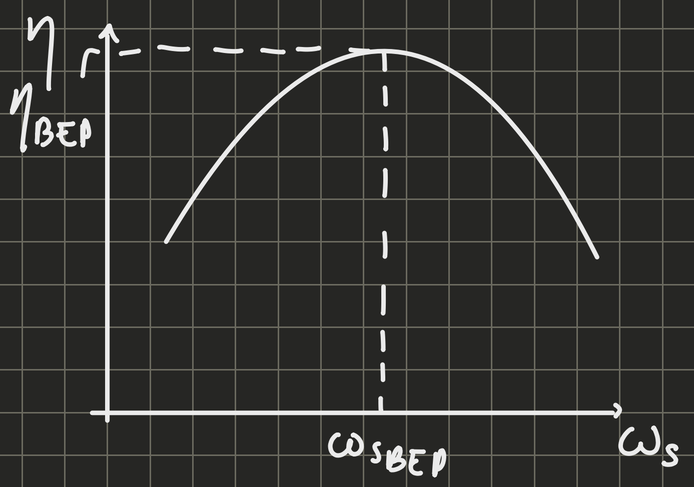
$$= \frac{Q}{n} \frac{n^3 (\sqrt{\Psi})^3}{\sqrt{gH}^3} = n^2 \frac{Q}{(gH)^{3/2}} \cdot \Psi^{3/2}$$

Putting all the dimensionless term on one side:

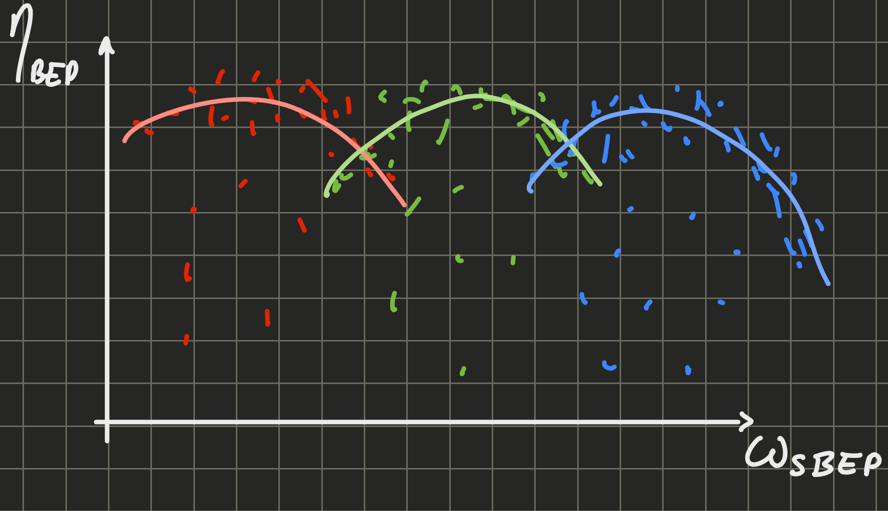
$$\Rightarrow \sqrt{\frac{\varphi}{\Psi^{3/2}}} = \frac{n\sqrt{Q}}{(gH)^{3/4}} = n_s \rightarrow \text{specific angular speed}$$

we can define a counterpart: $\omega_S = \omega \frac{\sqrt{Q}}{(gH)^{3/4}}$

This ω_s was used in spite of φ:



Collecting all the η_{BEP} and $\omega_{s_{BEP}}$ for many machine we can get:



plotting one point for each machine

From the data, ignoring the outliers we can get trends

= radial — mixed flow = axial

Why are we getting this result?

Let's consider a distorted similarity, in which the kinematics are similar (similar triangles), but the geometries are different / non-similar

$$\omega_s = \omega \frac{\sqrt{Q}}{(gH)^{3/4}} \propto \frac{\cancel{U}}{\sqrt{D}} \frac{\sqrt{\cancel{U}Db}}{\cancel{(U^2)^{3/4}}} \propto \sqrt{\frac{b}{D}} \Rightarrow \omega_s \text{ is a shape parameter.}$$

$$\Rightarrow V_i \propto U \rightarrow \begin{cases} V_m \propto U \\ V_t \propto U \\ \omega \propto U/D \end{cases}$$

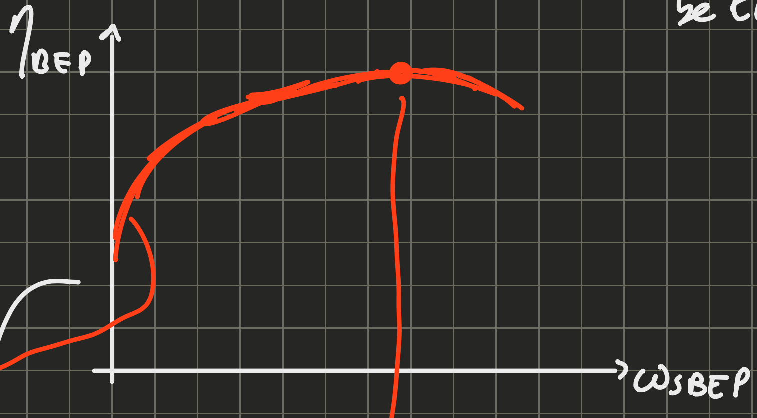
In practice kinematic similarity is not kept but it's close

$Q \propto V_m D b \propto U D b$

$gH \propto U^2$

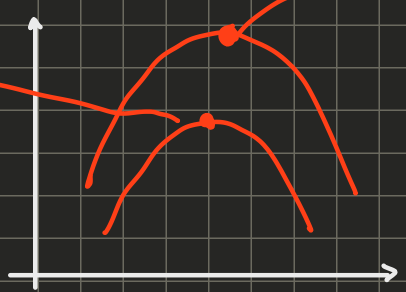
Since the angles have to be the same.

RADIAL MACHINES

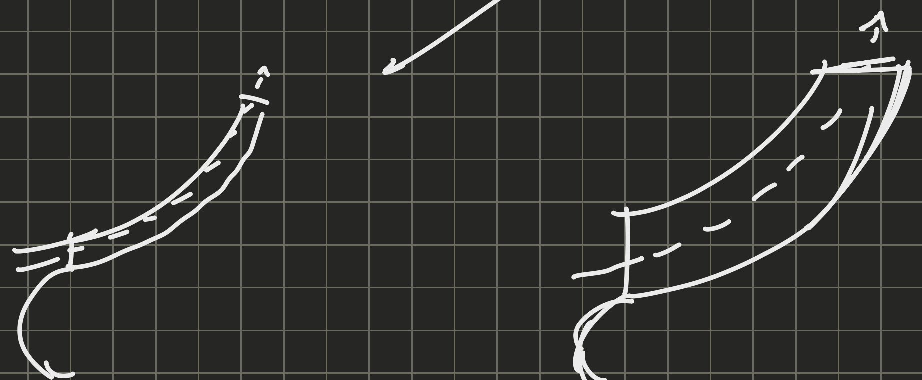


From the next lesson when we write cos we mean $\cos_{BEP}$ and the other parameter will also be considered at the BEP.

η is low at low $\cos_{BEP}$, (meaning that a machine for low $\cos_{BEP}$ has low η), since $\cos_{BEP} \propto \frac{b}{D}$



so for low $\cos_{BEP}$ b will be small relative to D

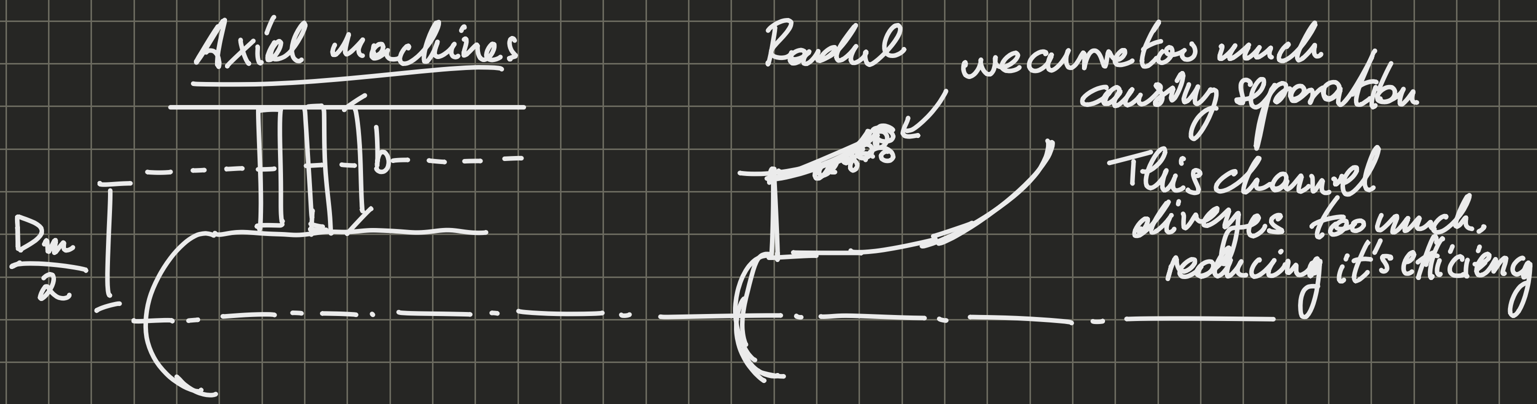


It will be less efficient since there will be more losses, since it's longer relative to it's size.

The longer and thinner is less efficient than a channel which is short and thick.

RADIAL vs. AXIAL machines

Why are axial machines better at higher ω_s.

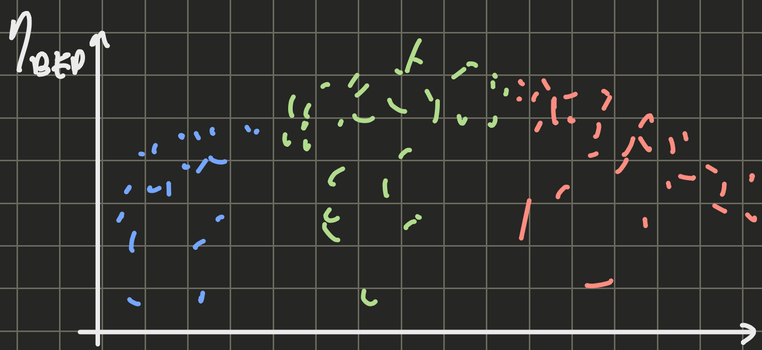


What happens if we eliminate n instead of D?

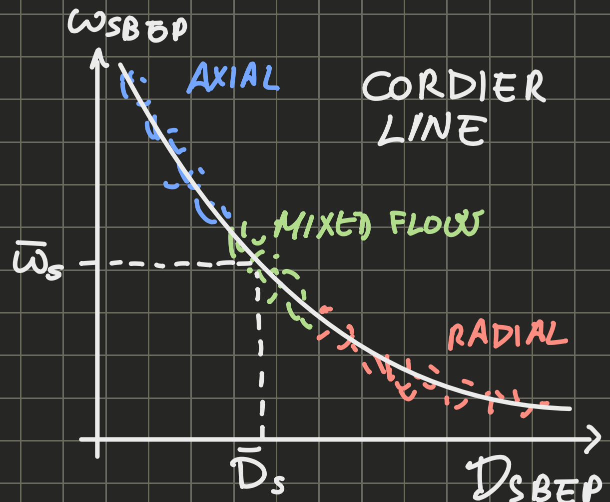
The result gives:

$$\frac{D(gH)^{1/4}}{\sqrt{Q}} = D_s \rightarrow \text{specific diameter}$$

Writing the curves for η on D_s be found the BEP, and representing η_{BEP} and $D_{s,BEP}$



He then found:



This line is still valid today, since which η has increased, the line has no moved.

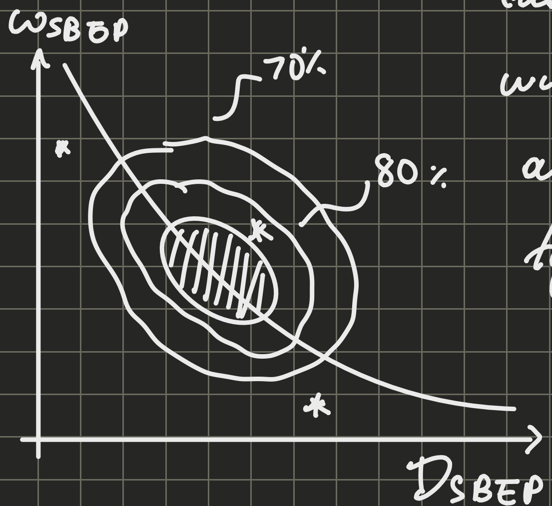
Given Q, H and ω (→ not necessarily free), we can get ω_s, so from this we can get D_s and the type of machine

$$D_s = \frac{D\,(gH)^{1/4}}{\sqrt{Q}}$$ ⇒ we immediately find the optimal size, D, of the machine

This is an idea, the different constraints of our system can force us to function is suboptimal conditions.

The outliers in the graph are machine in which the constraints were very severe and that was the function of our system.

Bolje Diagram: → Added iso efficiency lines, which indicated the cost of not remaining with optimal conditions, and allows to way to advantage of tightening on the constraints and going closer to the optimal condition.



The penalty for axial machines is lower relative to radial machines.

End of Similarity Laws (Technically yesterday)

$\boxed{jk}$

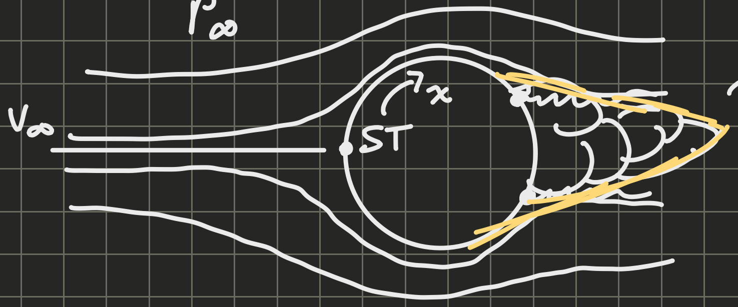
Limits of our similarity laws

When Re is low, it becomes important.

For all blade profiles created a company needs to create loss coefficient diagrams at different Re.

Small machines are penalized by the geometry, and size effects.
geometric similarity is often not observed since given that we can use the surface roughness at both sizes, there is no reason to also scale the roughness. We are not comfortable going below a certain level, since we can keep a penalty at the same level at different dimensions, in the case of the larger dimension, this loss is less significant.
(e.g. we can keep 1mm at the top of a foil, there is no reason to increase scale it, as it only increases losses.
The smaller the machine, the more significant the size effects and constraints are to the machine operation.

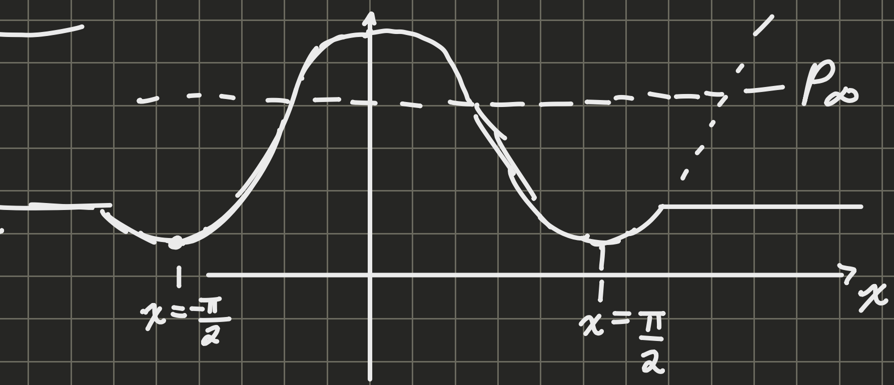
Aero/Hydro-dynamics (recap then cavitation)



Wake of the flow

$\frac{\partial P}{\partial x} \neq 0$

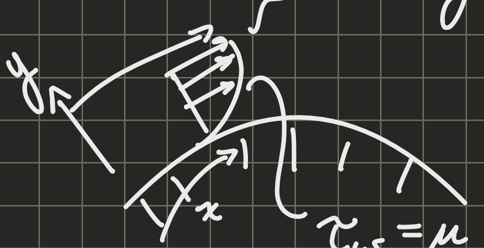
$P_{ST} = P_T = P_\infty + \frac{1}{2}\rho V_\infty^2$



The flow is no longer able to adhere to the surface, detach and generate vorteces.

Since it detaches the pressure is not able to return to the initial pressure

Boundary layer



$\frac{\partial P}{\partial y} \cong 0$

$\tau_w = \mu \frac{\partial V_x}{\partial y}$

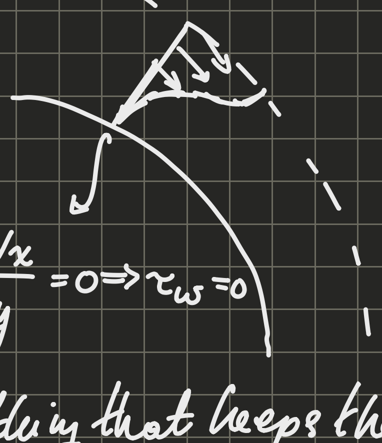
Where $\frac{\partial P}{\partial x} > 0$, it is called <u>adverse pressure gradient</u>

If $\frac{\partial P}{\partial x} > 0 \rightarrow \frac{\partial V_x}{\partial y} \downarrow$

if $\frac{\partial P}{\partial x} < 0 \rightarrow \frac{\partial V_x}{\partial y} \uparrow$

favorable pressure gradient

This matters because:



At some point $\frac{\partial V_x}{\partial y} = 0 \Rightarrow \tau_w = 0$

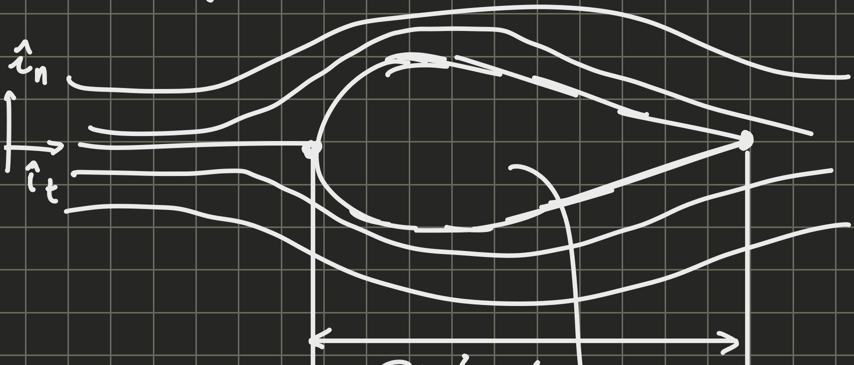
There is nothing that keeps the flow attached to the wall, so it will detached

This cause drag which decelerates our motion massively.

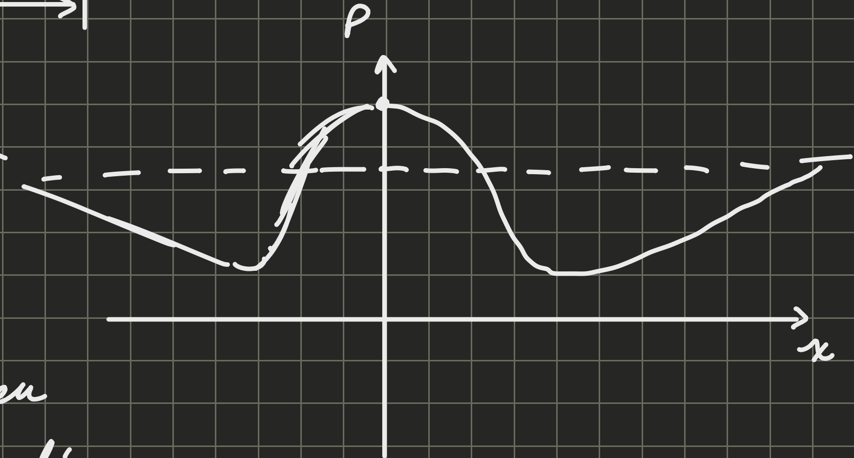
To reduce these resistive effects, we can close the part which is filled by the wake, reducing the curvature and therefore the pressure gradient.

h: length

Airfoil / Aerofoil / Hydrofoil



c: chord



$$\vec{F}_{AERO} = \oint \vec{\sigma}\, h\, dl =$$

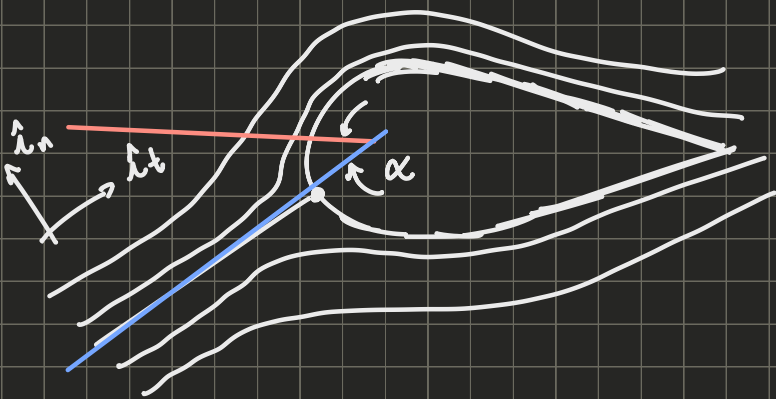
we dont use $\hat{\imath}_x$ or $\hat{\imath}_y$, since the spacial direction doesn't matter, just the direction relative to the flow.

$$= \vec{F}_{AERO,t}\,\hat{\imath}_t + F_{AERO,n}\,\hat{\imath}_n$$

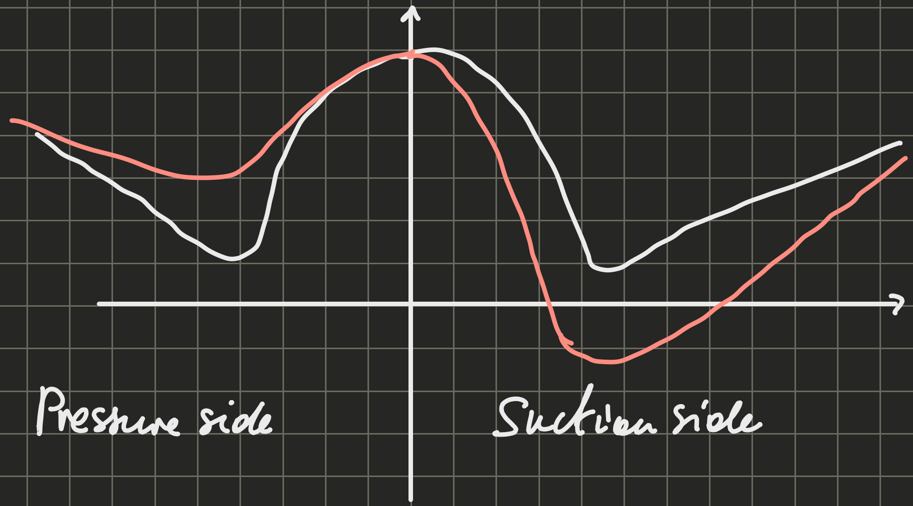
$$F_{AERO,t} = \oint \vec{\sigma}\cdot\hat{\imath}_t\, h\, dl = D \text{ (drag)}$$

$$F_{AERO,n} = \oint \vec{\sigma}\cdot\hat{\imath}_n\, h\, dl = L \text{ lift}$$

Drag is composed of pressure drag and friction losses.



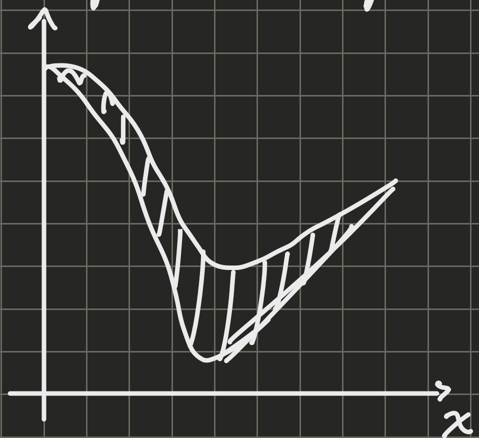
α = angle of attack



In this case, the airfoil will be unbalanced, generating a suction side and pressure side.

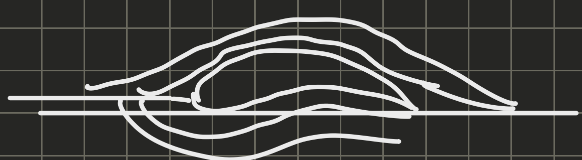
The greater the angle, the greater the suction.

We can represent the pressure difference as:



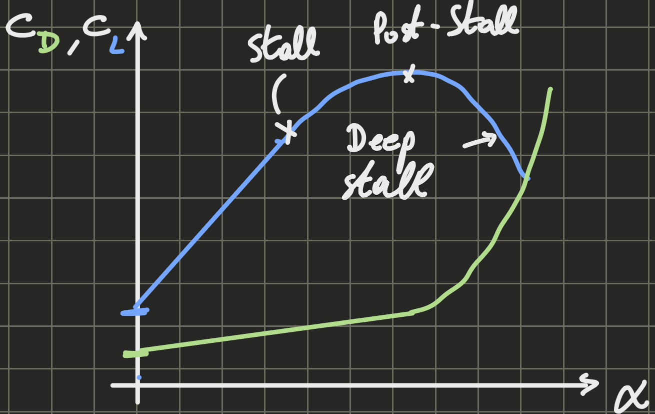
$L \neq 0$, the pressure difference is what allows us to fly.

Cambered Airfoil



Even without an angle of attack, this generates lift.

Defined for each Reynolds number.



← For symmetrical forms

$$C_L = \frac{L}{\frac{1}{2}\rho V_\infty^2 c h}$$

$$C_D = \frac{D}{\frac{1}{2}\rho V_\infty^2 c h}$$

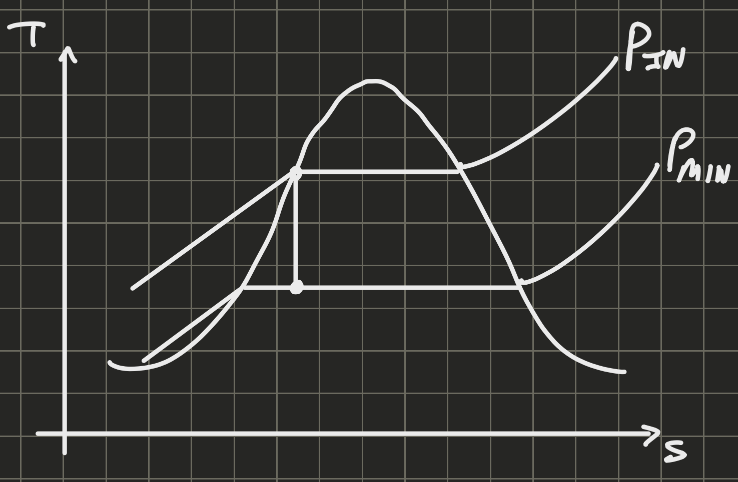
Stall occurs when we have increased α too much and the flow starts to detach.

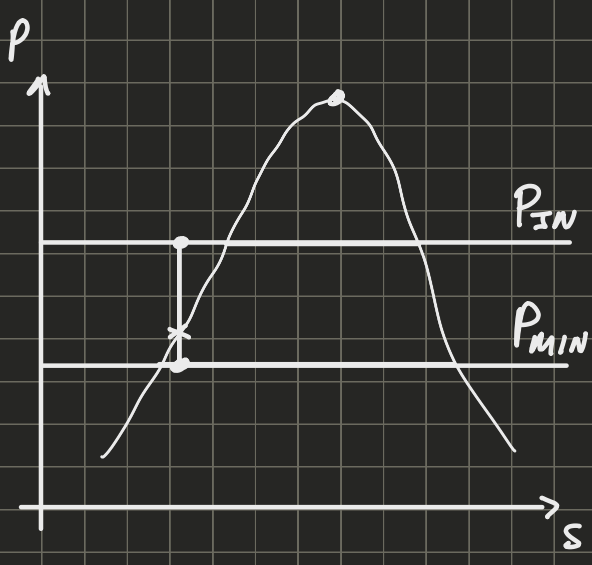
Cavitation

Each blade interacts with each other. The pressure changes between the two sides.

There is a stagnation point. Both sides have a different pressure gradient, in both the pump and the turbine.

The pressure within the blade region can change and be very different from the pressure at inlet and outlet, it can even be lower.





As the stream moves from the inflow, it reaches a region where it is P_{MIN}, passing the saturation line.

The fluid reaches a suction region where it is subjected to phase change and therefore becomes a 2-state problem. Once one bubble forms, the fluid has to go around it too, increasing the suction.

The bubbles do not last long, this is because after detaching from the blade it will enter areas of higher pressure, causing it to eventually re-enter into the single phase region and becoming once again liquid.

If the pressure differential is high (typically with smaller bubbles), the bubble will eventually implode and if such implosion is close to the blade it can cause damage and erosion.