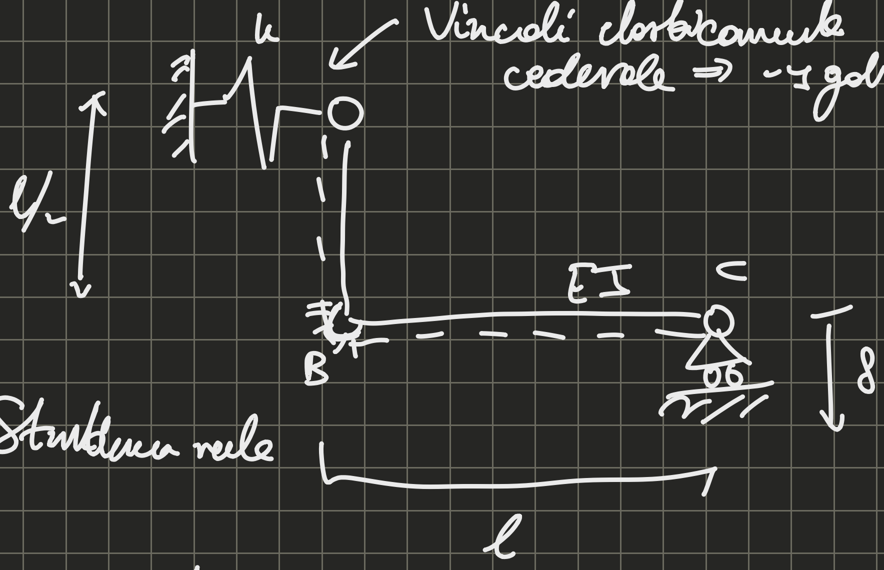
Esercitazione 2

MDF per travi continue iperstatiche
↳ Romani temi d'Esame

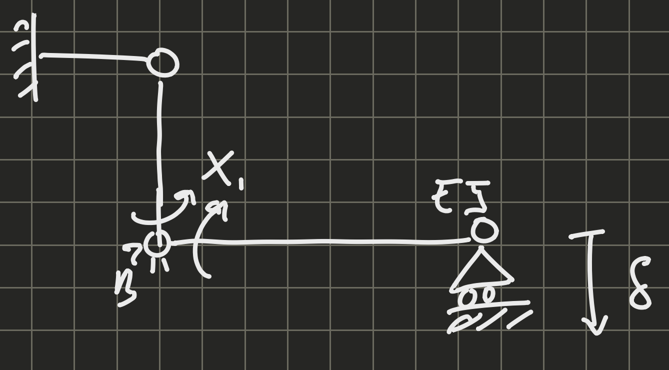
Soluzioni di Travi Continue Iperstatiche (MDF)

Esercizio 1



$s = 1$

Struttura Principale Isostatica



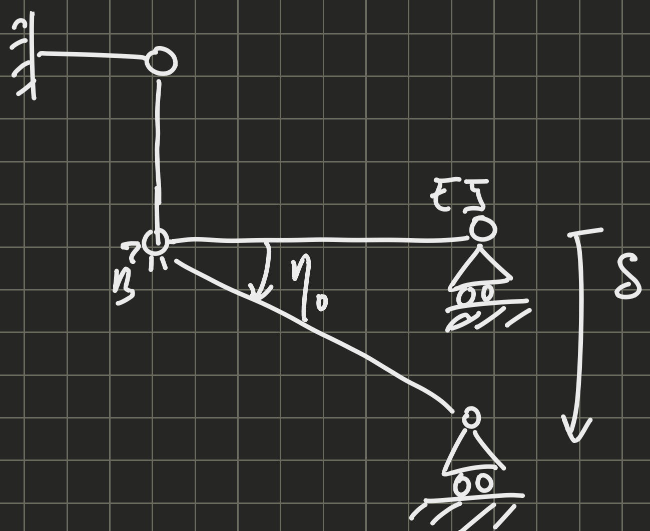
$\Delta \varphi_B = 0$

Equazione di congruenza

↳ perchè nella reale c'è continuità

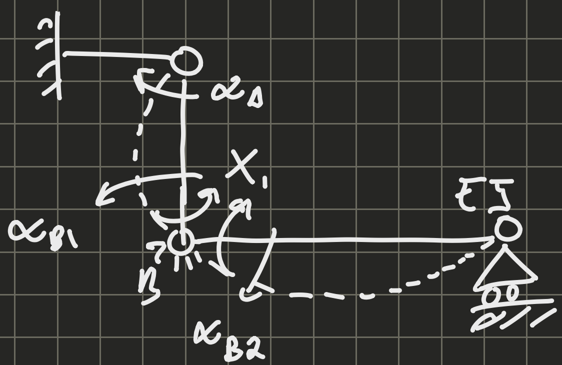
e la abbiamo spaccata e aggiunto un momento di continuità per garantire questo

Struttura Ausiliaria "0" ($\delta \neq 0$, $X_1 = 0$



$$\eta_0 = \frac{\delta}{l}$$

Struttura Ausiliaria "1" ($X_1 = 1$, $\delta = 0$)

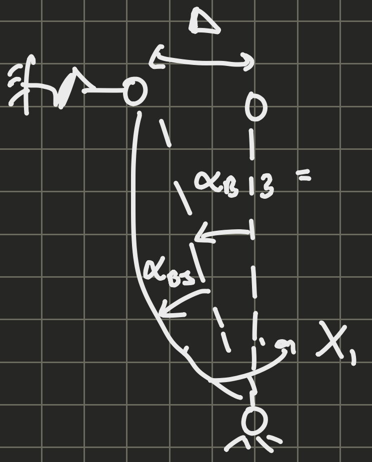


Deformabilità Flessionale delle aste

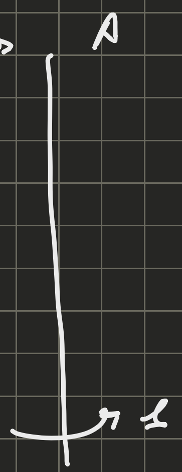
$$\alpha_A = \frac{l}{2} \cdot \frac{1}{6EJ} = \frac{l}{12EI}$$

$$\alpha_{B2} = l \cdot \frac{1}{3EJ} = \frac{l}{3EI}$$

$$\alpha_{B1} = \frac{l}{2} \cdot \frac{1}{3EJ} = \frac{l}{3EJ}$$



$$k = \frac{EI}{l^3}$$



$$\Delta = \frac{2}{lk} = \frac{2l^2}{EI}$$

$$\alpha_{B3} = \frac{2\Delta}{l} = \frac{4l}{EI}$$

$$\eta_1 = \left(\alpha_{B1} + \alpha_{B2} + \alpha_{B3}\right) = \frac{9l}{2EI}$$

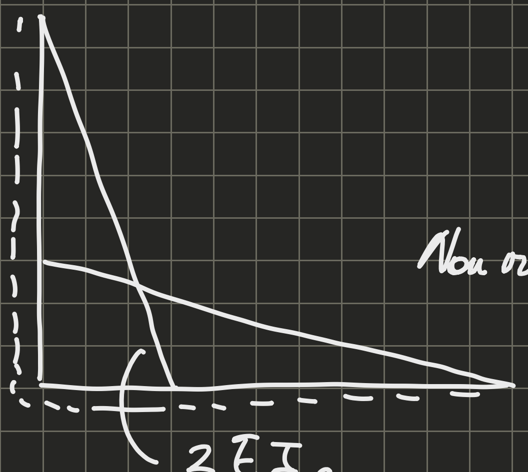
Equazione di Congruenza

$$\eta_1 X_1 + \eta_0 = 0$$

$$\frac{9l}{2EI} \cdot X_1 + \frac{\delta}{l} = 0$$

$$X_1 = \frac{-2EI}{9l^2}\delta$$

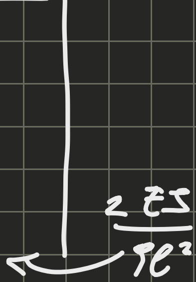
↳ momento opposto quanto ipotizzato



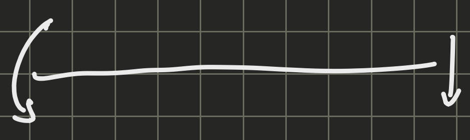
$$\frac{2EI}{9l^2}\delta$$

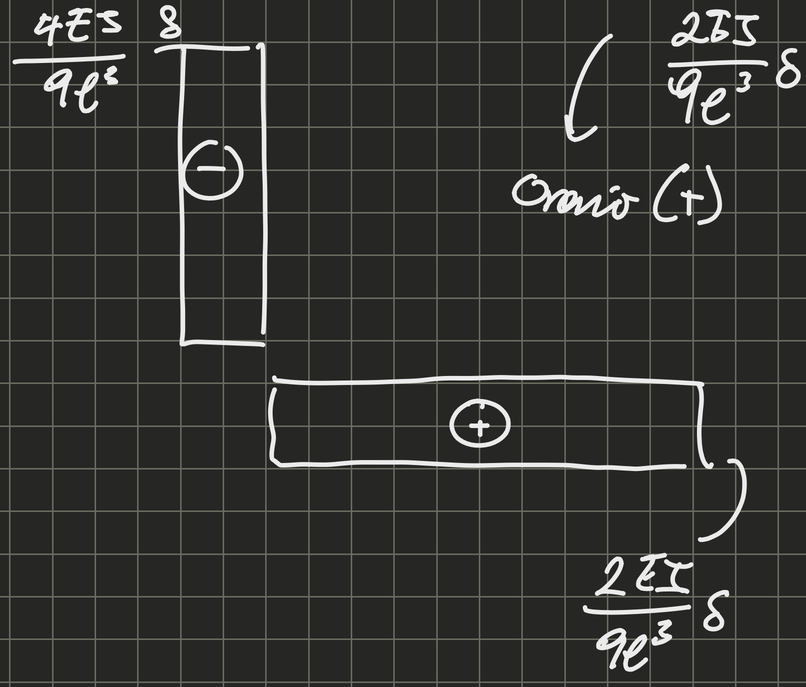
$$\frac{4EI\delta}{9l^3}$$

Non ovvio (-)

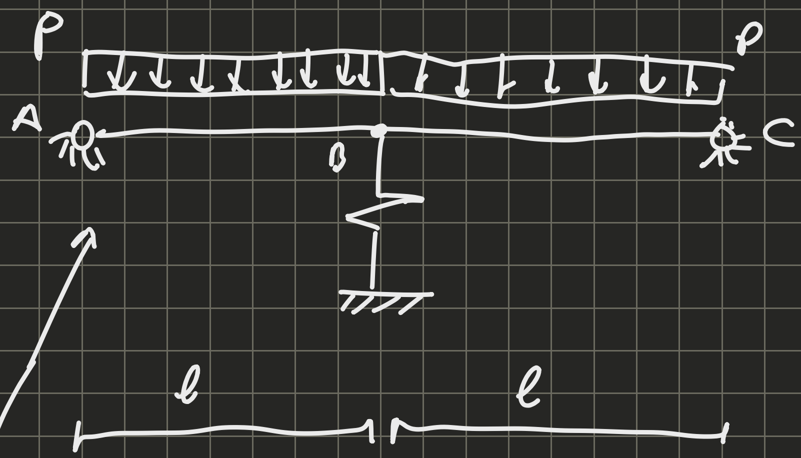


$$\frac{2EI}{9l^2}\delta$$





Esercizio 2



Hp: $GA^* \to \infty$

$k = \frac{\beta EI}{l^3}, \beta \in R^+$

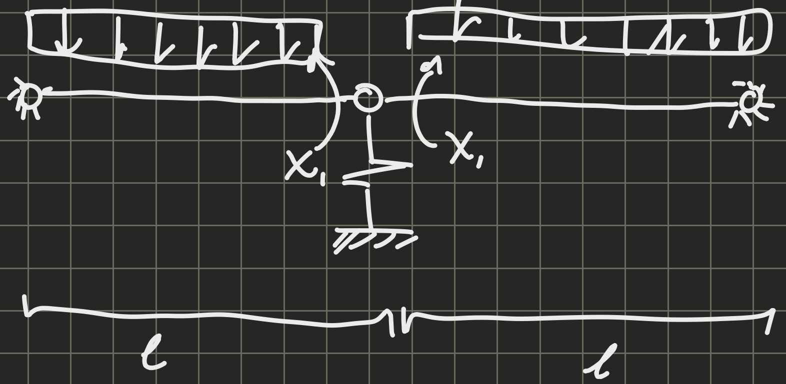
s = 2, ma nel senso orizzontale i vincoli sono uguali, creano carrelli che non ci importano → le travi reagiscono a azioni flessionali non di trazione come le funi quindi non ci importa.

↳ Quindi per semplicità è 1 volta [illegible] iperstatica.

Struttura reale s = 1

Spezziamo la continuità per imporre congruenza.

↳ Spezziamo sempre la continuità ad un vincolo.

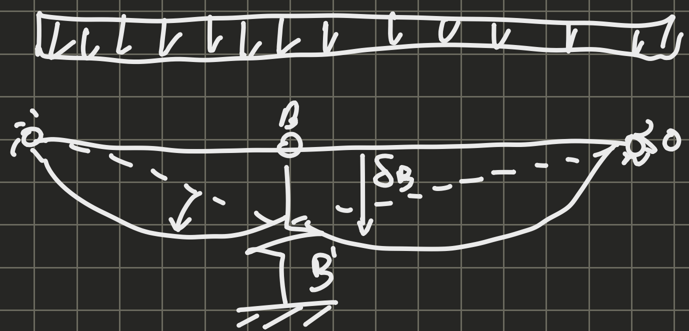


Struttura Principale Isostatica

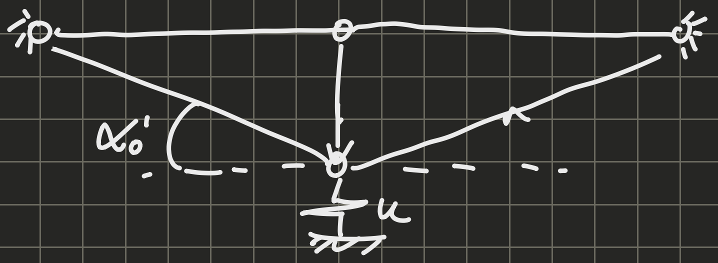
Equazione di congruenza

$\Delta \varphi_B = 0$

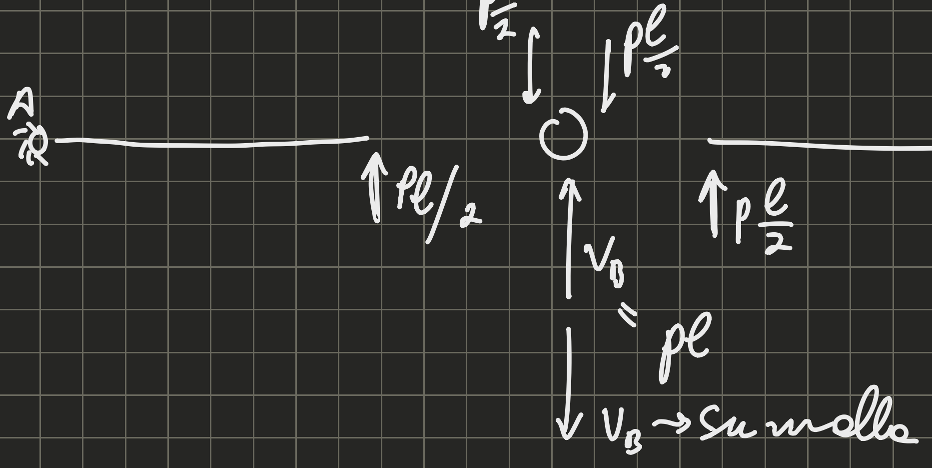
Struttura Ausiliare "0" $(p \neq 0,\ X_1 = 0)$



Abbassamento molla

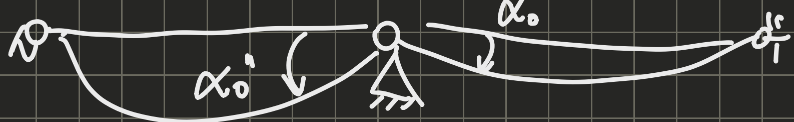


$$\alpha_0' = \frac{\delta_B}{l} = \frac{pl^3}{\beta EI}$$



$$\delta_B = \frac{V_B}{k} = \frac{pl^4}{\beta EI}$$

Effetto flessionale

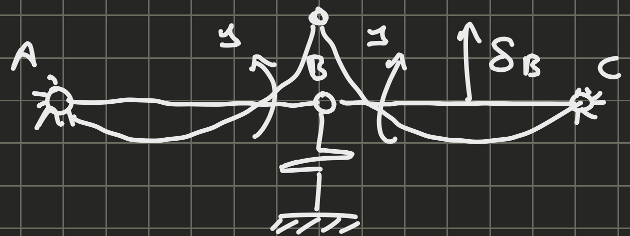


$$\alpha_0'' = \frac{p l^3}{24EI}$$

$\alpha_0 = (\alpha_0'' - \alpha_0') \cdot 2 \rightarrow$ perché sono due lati che contribuiscono al cambio d'angolo.

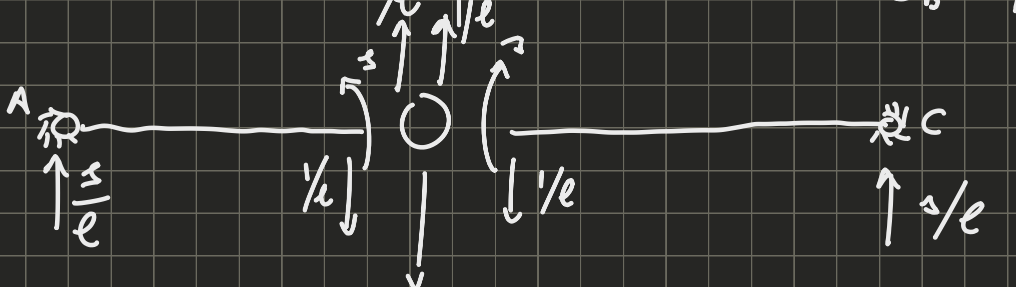
$$= \left(\frac{1}{24} - \frac{1}{\beta}\right) \cdot \frac{pl^3}{EI} \cdot 2 = \frac{\beta - 24}{12\beta} \frac{pl^3}{EI}$$

Struttura Ausiliaria "1" ($p=0, X_1=1$)



Effetto dell'allungamento della molla

$$\delta_B = \frac{V_B}{k} = \frac{2}{l} \cdot \frac{l^3}{\beta EI} = \frac{2l^2}{\beta EI}$$

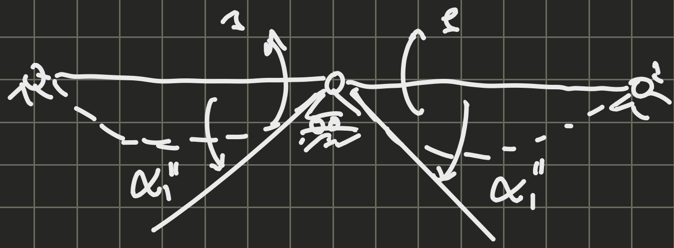


$2/l = V_B$

$\uparrow 2/l = V_B$

$$\Rightarrow \alpha_1' = \frac{2l}{\beta EI}$$

Effetto Flessionale di EJ



$\alpha_1'' = \frac{l}{3EJ} \rightarrow$ da tabelle

$$\alpha_1 = \alpha_1' + \alpha_1'' = \frac{2l}{\beta EJ} + \frac{l}{3EJ} = \frac{6+\beta}{3\beta}\frac{l}{EJ}$$

Equazione di Congruenza

$$\eta_1 X_1 + \eta_0 = 0$$

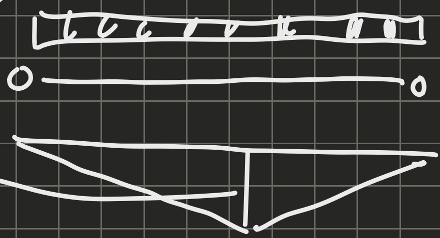
$$\frac{2(6+\beta)}{3\beta}\frac{l}{3EJ} X_1 + \frac{\beta - 24}{12\beta} \cdot \frac{pl^3}{EJ} = 0$$

$$X_1 = \frac{(24-\beta)}{8(6+\beta)} \cdot pl^2$$

Calcolando si trova che è vero.

$\rightarrow$ Assenza di supporto

per $\beta = 0$ $\quad X_1 = \frac{pl^2}{2}$

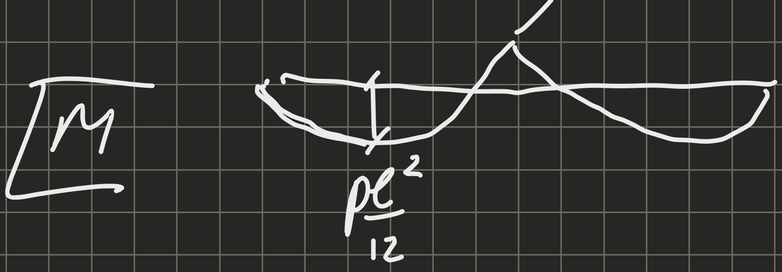


per $\beta = \infty \Rightarrow X_1 = -\frac{pl^2}{8}$

$\hookrightarrow$ Supporto Rigido

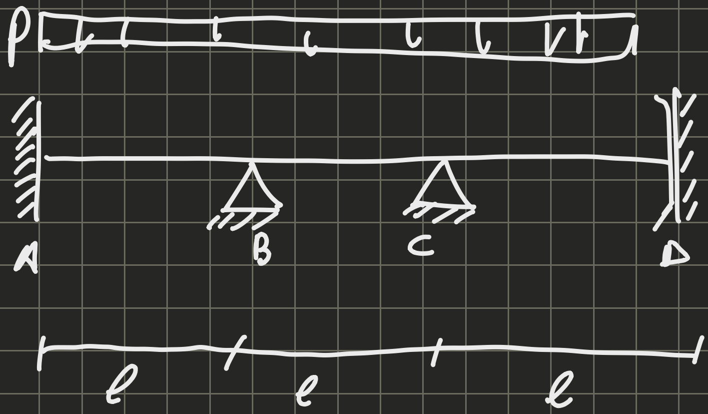
Questo è utile per sistemi di travi e pilastri.

$\frac{pl^2}{8}$



Esercizio 3

Struttura reale

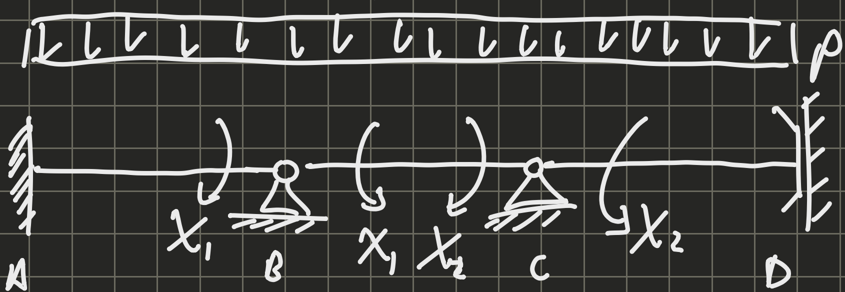


$GA^* \to \infty$

Riduciamo di nuovo il numero di incognite iperstatiche

Abbiamo già A - B tabulato quindi spezziamo solo in B e C

Struttura Principale "Isostatica

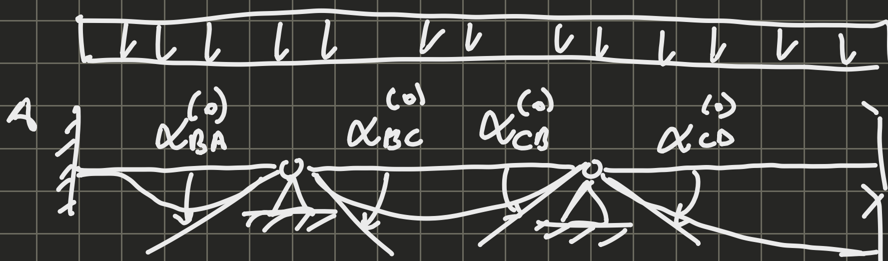


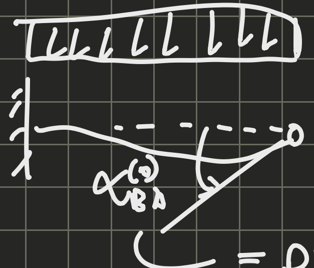
Equazioni di Congruenza

$\Delta \varphi_B = 0$

$\Delta \varphi_C = 0$

Struttura Ausiliare "0"

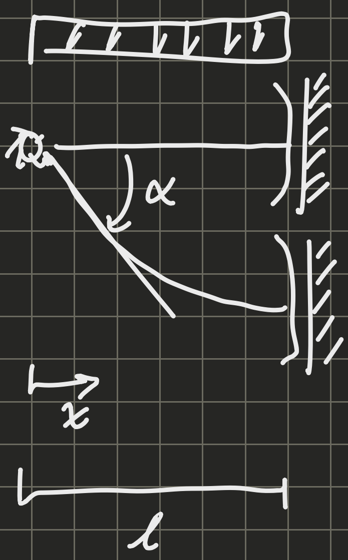




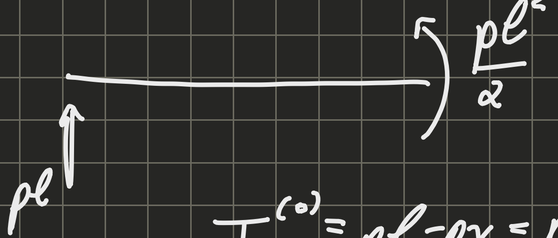
$= p\frac{l^3}{48EI}$ dalle tabelle

$$\alpha_{BC}^{(0)} = \alpha_{CB}^{(0)} = p\frac{l^3}{24EI}$$

$\alpha_{CD}^{(0)}$ non è tabulato quindi abbiamo applicare un piccolo PLV



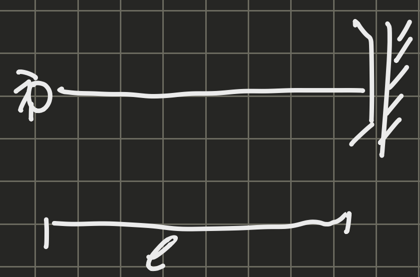
Struttura Reale



$$T^{(0)} = pl - px = pl\left(1 - \frac{x}{l}\right)$$

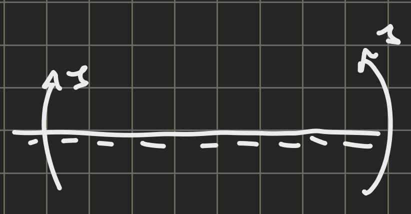
$$M^{(0)} = plx - p\frac{x^2}{2} = p\frac{l^2}{2}\left(2\frac{x}{l} - \left(\frac{x}{l}\right)^2\right)$$

Struttura Ausiliaria → Cinematica



$$T^{(1)} = 0$$

$$M^{(1)} = 1$$



Statica Ausiliaria $\qquad$ Cinematica Reale

$$\xi = \frac{x}{l} \qquad dx = l\, d\xi$$

$L_{ext} = L_{int} \qquad L_{ext} = 1 \cdot \alpha$

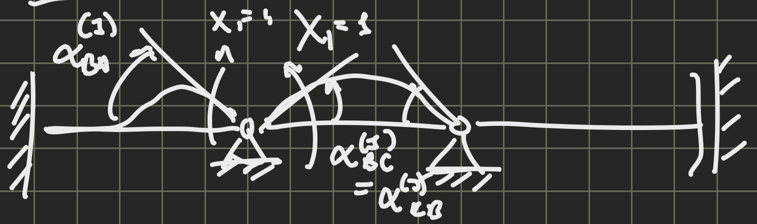
$GdA \to \infty$
$\Rightarrow T = 0$

$$L_{int} = \int_0^l \frac{M^{(1)} \cdot M^{(0)}}{EJ} dx = \frac{pl^3}{2EJ} \int_0^1 (2\xi - \xi^2) d\xi$$

$$L_{int} = \int_0^l \frac{M_i M_j}{EJ} dx \qquad = \frac{pl^3}{2EJ}\left(2 \cdot \frac{1}{2} - \frac{1}{3}\right) = \frac{pl^3}{3EJ}$$

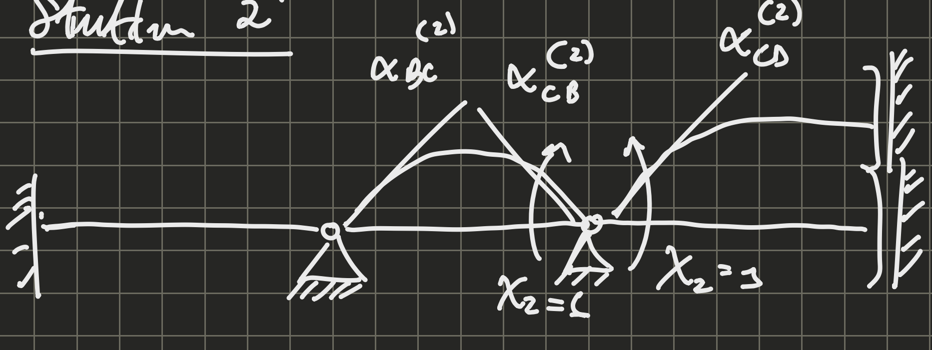
$$\alpha = \frac{pl^3}{3EJ} = \alpha_{CD}^{(0)}$$

Struttura Ausiliaria "1"



$$\alpha_{BA}^{(1)} = \frac{l}{4EJ} \qquad \alpha_{BC}^{(1)} = \frac{l}{3EJ} \qquad \alpha_{CB}^{(1)} = \frac{l}{6EJ}$$

Struttura "2"



Prima con il carico distribuito non era tabulato quindi lo dovevamo risolvere, ora con il momento è tabulato quindi non lo calcoliamo

$$\alpha_{BC}^{(2)} = \frac{l}{6EJ} \qquad \alpha_{CB}^{(2)} = \frac{l}{3EJ} \qquad \alpha_{CD}^{(2)} = \frac{l}{EJ}$$

Sistema Risolvente

$$\Delta\varphi_B = 0 \to \left(\alpha_{BA}^{(1)} + \alpha_{BC}^{(1)}\right) + X_1 + \alpha_{BC}^{(2)} \cdot X_2 - \left(\alpha_{BA}^{(0)} + \alpha_{BC}^{(0)}\right) = 0$$

$$\Delta \varphi_C = 0 \Rightarrow \quad \alpha_{CB}^{(1)} X_1 + \left(\alpha_{CB}^{(2)} + \alpha_{CD}^{(2)} \right) X_2 - \left(\alpha_{CB}^{(0)} + \alpha_{CD}^{(0)} \right) = 0$$