Shaft-Hub Connections

↳ Not a proper machine element, rather a set of all possible design solutions to transmit torque and rotation between parts.

A hub is the internal part of an element that interacts with a shaft

There are groups of solutions:
- ↳ Force-based solutions (totally based on friction)
- ↳ Shape-based connection, interactions between profiles
- ↳ pre-loaded shape based (only one solution)
  - ↳ a mix of the two
- ↳ connection with interposed material.

Friction based → e.g. press fit or shrink fit.

Shape based → spline

Mixed → taper key

4th → adhesive bonding.

Friction-based connections

↳ Many different ways to realise them

↳ Coulomb's law: $F_{friction} = \mu F_{normal}$

↳ There many ways we can realise the normal forces, these original friction which transmits torque

Fnoml is under our control, μ is not, since μ varies depending on many parameters.

→ Boil down to three diffrent ways.
- → Shrink fit
- → Bolted connection
- → Saddle key.

We have to make sure we stick to static friction, we don't want relative motion, unlike bearings.

$\mu \rightarrow 0{,}05 - 0{,}2$

Press-fits
→ we realise pressure, such that creates frictional stresses, and distributes torque.

$p_f \rightarrow \tau_{frict}$

The relationship between the pressure and transferred torque is:

$$dM_t = \mu \cdot p_F \cdot L \cdot r^2 \cdot d\alpha$$

$$\rightarrow M_t = 2\pi \cdot r^2 \mu \, p_F \, L$$

How is p_F calculated? → it is designed but that is for a future course

Pressure is caused by shrinking the shaft, so if we want to pressfit we need to increase shaft size to boring back to nominal diameter.

Bolt-based connections

↳ Loading a bolt to squeeze parts around the hub.
or bolts.

pg. 7 the case we are considering is a bit complicated but still analisable.

The p as a function of α has a sinusoidal tendency. The load is simmetric, but at the point where the parts meet it is 0.

We can get p_{MAX} (through F_S and an equilibrium with one of the connections):

$$p_{MAX} = \frac{2F_S}{\pi L r}$$

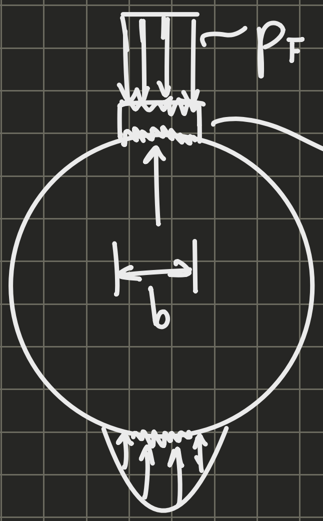
From here we get the transmitted torque as:

$$M_t = \frac{16}{\pi} r^2 L \mu \, p_F$$

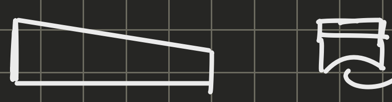
$$\frac{M_t \text{ of Press Fit}}{M_t \text{ of Bolted Conn.}} = 1{,}23$$ ⇒ Press fit is more effective than bolted connections.

<u>But</u> press fits are more complicated and therefore require more work.

Saddle key



Hammered in to wedge it.



The key is hammered in, and by equilibrium friction is generated in two areas.

Since we know the forces are balanced, we don't need to calculate pressure at the bottom, but two times at the top.

We get: $M_t = 2rbL\mu p_F$

↳ Maximum transmittable torque

(also the same in the last to cases)

Typically: $b = \frac{1}{2} r$

$$\frac{\text{Press fit}}{\text{Saddle key}} = \pi \frac{r}{b} = 6{,}28$$ → much better, but saddle key is even easier and cheaper.

There are other ways to do a force bond approach

↳ Conical sections → more pressure more torque can be transferred.

↳ interposition of wedge-like elements to generate forces.

Shape-based connections:

↳ Geometrical interactions

Types:

↳ Pins

↳ Parallel keys

↳ Splines

↳ Feather keys

↳ Polygonal profile.

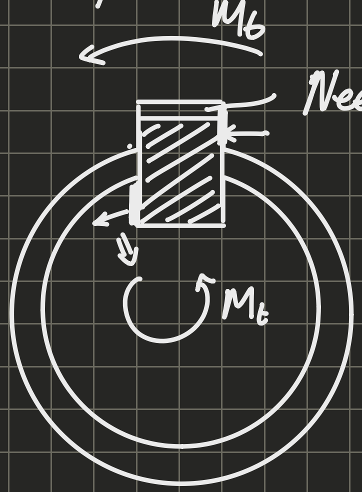
In shape-based connections, requires a very good check on tolerances and geometrical shapes.

Sizes and tolerances must be managed well

Keys are standardized

Parallel / Feather key (Linguetta)

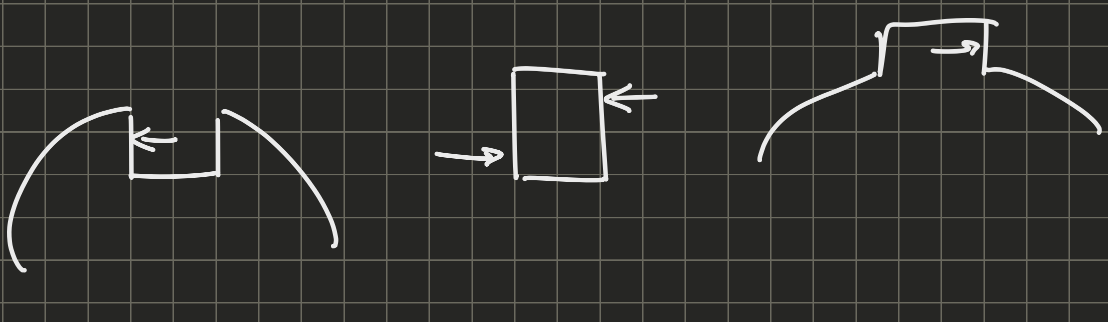
↳



Need to leave space. (Always, it's standardised)

↳ It distinguished it from (chiavette) and shows it's working principle.

No friction, it's all face to face interaction between surfaces.



The shape needed to accomodate fill affect the assembly.

The one problem is that there is nothing impeding axial motion.

The key can take many shapes.

If we need to use 2 key we place them at 120 degrees, and not symmetrical due to vertical tollerance issues also not 90°.

Spline profiles

↳ Shafts with squaretooth.
  ↳ While in general we use 1, 2, or at most 4 parallel keys, if we need more we just place the teeth directly on the shaft, these are splines.

Like multiple parallel keys. Axial movements are still not prevented.

Pins

↳ Dowel pins
  ↳ They prevent any degree of freedom between connected parts.
  ↳ Also used to position parts, since threaded connections are very loose with tollerances.

pg. 16 With dowel pins, we need to use 3 checks

↳ Shear forces

↳ Contact stresses

↳ Hub.

↳ others depending type

↳ depending on pin type.

The stress values are standardized

↳ Hinges

↳ Pins that still allow one degree of freedom.

↳ Hinge pins usually have a thing at the end to keep everything together.

↳ Not only do we have to consider shear and contact stresses but also bending.

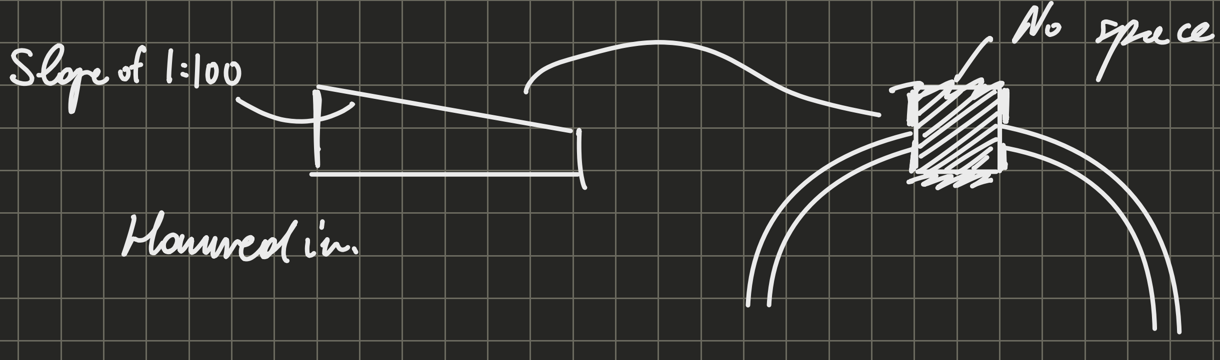
pg. 18 → table of equations

Tables for different materials include values for permissible stresses to be used in the past equations

↳ pg. 19

Preload shape based connection

↳ The only solution in this class is the tapered key (chiavetta)



The transfer of torque is mostly based on friction.

Good: we don't need to prevent axial motions.

Bad: Sometimes the push down causes the shaft to no longer to be concentric to the other element, and in some cases it can be a problem.

Table, indication for choosing connection pg. 22

Failure modes of Parallel keys → also common in spline profiles and others.

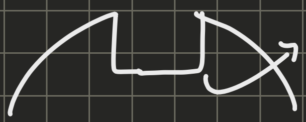
- Shaft failure → Weakest so most failures, usually breaks at corner of keyway
- Hub Failure — To fatigue — ↳ same type of failure for hubs.
- Key Failure → Failure due to shear fatigue
- Pressure on the sides
  - ↳ Contact pressures plastically deform both sides and the key. Excessive stresses cause failure of both

Key selection
- ↳ 2 morphologies of keys, type A (round) and form B (square)
- ↳ We are given the height of the Key and for the slot, this is done so we are sure there is no contact.

Keyway

pg. 5 knowing the diameter we select the key, it's tollerances, and the depth and radius of the keyway.

We have a different check for all 4 points.



→ rounded (defined by table. This is accomodated by corner chamfering of the key (which is also defined)

It is best for the key length to span the whole area of the element, if we are shorter the stresses are concentrated and in some cases the element can start to wobble, but if we take a longer key we might have interference with other elements.

In some cases we need to use a longer key to pass checks, but we need to make sure there is no interference.

The standard only defines a check for contact pressure (on sides), (at few startups) the rest is up to our knowledge.

Contact Pressure check

↳ local pressure (static)

↳ local pressure at max-load (fatigue)

Opposite of shaft where: static was maxim, fatigue was normal

For the shaft with low start-up we did static on maximum load and fatigue on normal, but with many start-up we do both at maximum.

1) Check local pressure on sides

Used for all checks → $$p = \frac{M_t}{z \cdot d/2} \cdot \underbrace{\frac{1}{L-(h-t_i)}}_{1/\text{Contact Area}} \cdot k_{\varphi\beta} \leftarrow p_{lim}$$

$k_{\varphi\beta}$ → adjustment factor

z → number of keys or spline teeth

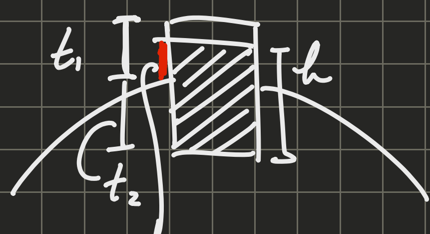
d → nominal hub or shaft diameter

$h \rightarrow$ tall height of key

$t_i \rightarrow$ height from hub or shaft side.

$\hookrightarrow$ $i \Rightarrow$ we need to do three checks, for hub, shaft & key.

$\hookrightarrow$ from table.



$M_t \rightarrow$ fatigue $\rightarrow M_{t,max}$ $\hookrightarrow$ do on the smallest area, this area in this case.

$\hookrightarrow$ static $\rightarrow M_t$

We are not working with the Hertz model since there is plane to plane contact.

$$p_{lim} = f_s \cdot R_{su}$$

$\hookrightarrow$ table pg. 9

$$S_F = \frac{p_{lim}}{p} \geq S_{F,min}$$

$S_{F,min} = 1-1,3$ for ductile. and 1,3 to 2 for brittle

$\hookrightarrow$ Used for things like pullys which are made with brittle materials.

For fatigue

$$p_{max,lim} = f_L(N) \cdot R_{su}$$

$\hookrightarrow$ From table (pg. 10)

We still have to do the static and fatigue on the shaft and hub.

$\hookrightarrow$ We can use Peterson diagram (if we are inside the values) and perform 2 checks each at the two points

Pertersen, indicates as troublesome for bending only and torque one. Or if both then we can use the diagram.

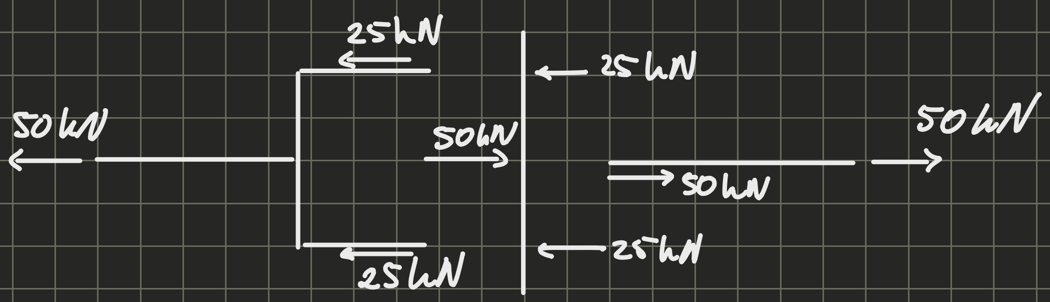
These are US standards so they are not good for us, and so we don't know the error we introduce. So since we can't use them, we either do experiments or FEA (most of FEA done is for this)

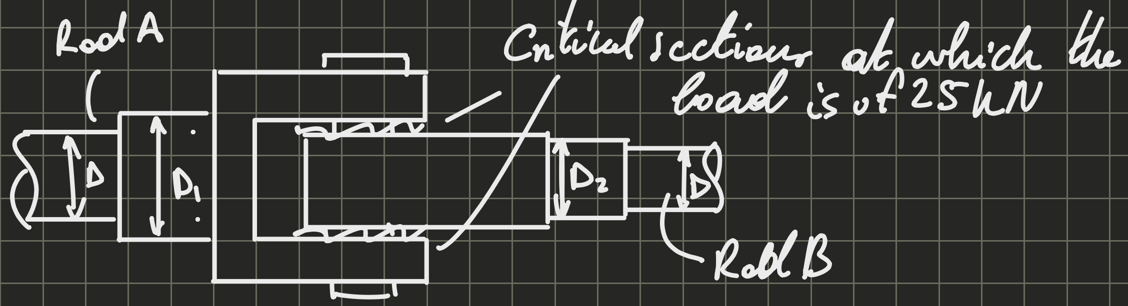
We will be using Pertersen achnowledging that they are erroneus in our case.

<u>Check on a pin-made hinge</u>

↳ The hinge is composed of three parts

<u>Scheme for forces in action</u>





We do a quick pre sizing for the rod:

$$\sigma_a = \frac{4P}{\pi D^2} \leq \frac{R_{sm}}{\eta}$$ → This allows us to choose the material and size the rod.

We get D and from there $D_1 = 1{,}1\,D$

↳ empirical formula

Hinges are typically under high loads and bad environments, so we choose high η since a user will definitely misuse it.

Sizing fork and eye

a and b are defined empirically

$a = D$

$b = 2a$

Size of the diameter of the clevis pin

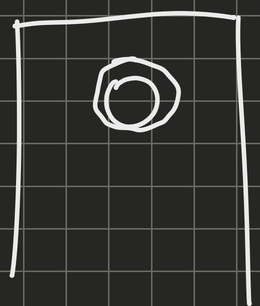
$$\tau_p = \frac{2P}{\pi d^2} \leq \tau_{sam} \to d$$

↳ from table for materials from before

$$M_{f,max} = \frac{P}{4}(a+b) \to d$$

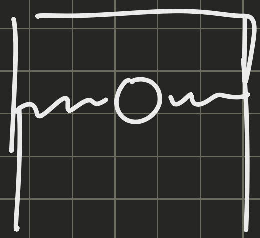
↳ It is subject to shear and bending, due to kind of load applied.
Since we can't put them together to compare them we do checks to presize and calculate the diameter to resist either and we choose the biggest.

Based on empirical approaches we have sized everything, now we have to do final checks.

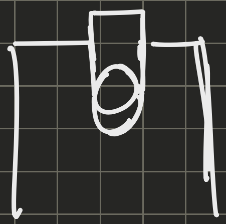


The pin pushes in the whole and creates contact stresses. So we need to do 3 checks to close our design.

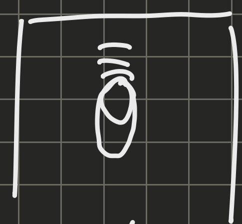
Net section



Shear out



Bearing failure



↳ hole become oval due to plasticization

pg. 30 and 31 → checks are defined by standards

↳ The needed numbers are still in the same tables.