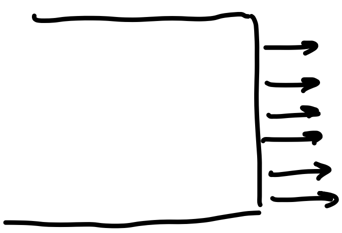
Lezione 13 -

Ultima Lezione:

Equazione di Napier

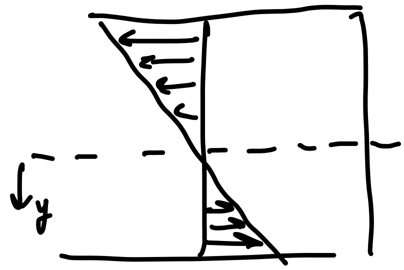
$$\sigma_x = \frac{N}{A} + \frac{M_z}{J_{zz}} y$$

N



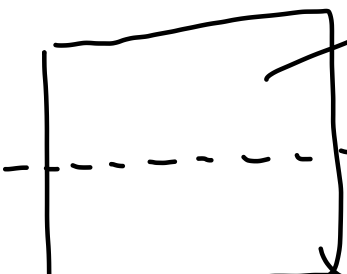
σ_x per N è costante se N è costante

M_f → per ricavare questo il sistema di riferimento è al centro



Distribuzione a farfalla per sforzo da M_f

Dimostrare che dove $\overbrace{M_f = 0}^{\text{asse neutro}}$ è il baricentro per $N=0$

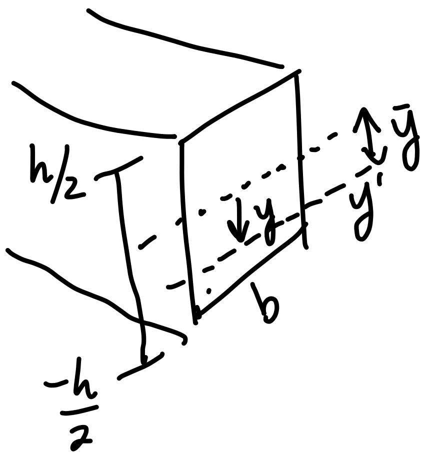


$\sigma < 0 \Rightarrow \varepsilon_x < 0$ compressione

Asse neutro

$\sigma > 0 \Rightarrow \varepsilon_x > 0$ tensione

Per assurdo dobbiamo dimostrare che baricentro è asse di neutro.



$$\sigma_x = \frac{M_z}{J_{zz}} y'$$
↳ Asse neutro

y baricentro

$$y' = y - \underbrace{\bar{y}}_{\text{costante}} \Rightarrow dy' = dy$$

Integriamo: per dimostrare $\bar{y} = 0$

$$N = \int_A \underbrace{\sigma_x}_{} \, dA = 0 \leftarrow \text{per caso dove solo flessione}$$

↳ $\frac{M_z}{J_{zz}}$ per solo flessione

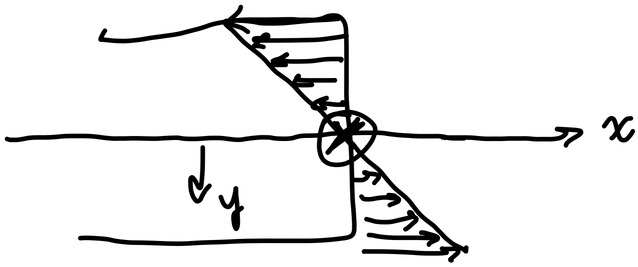
$$0 = \int_A \frac{M_z}{J_{zz}} y' \underbrace{dA}_{b\,dy'} = b \int_{-\frac{h}{2}-\bar{y}}^{\frac{h}{2}-\bar{y}} \frac{M_z}{J_{zz}} y' dy'$$

↑ costanti

$$= \frac{bM_z}{J_{zz}} \int_{-h/2-\bar{y}}^{h/2-\bar{y}} (y - \bar{y}) \, dy = \frac{bM_z}{J_{zz}} \left(\underbrace{\cancel{\left[\frac{y^2}{2}\right]_{-\frac{h}{2}}^{\frac{h}{2}}}}_{0} - \bar{y} \left[y \right]_{-\frac{h}{2}}^{\frac{h}{2}} \right)$$

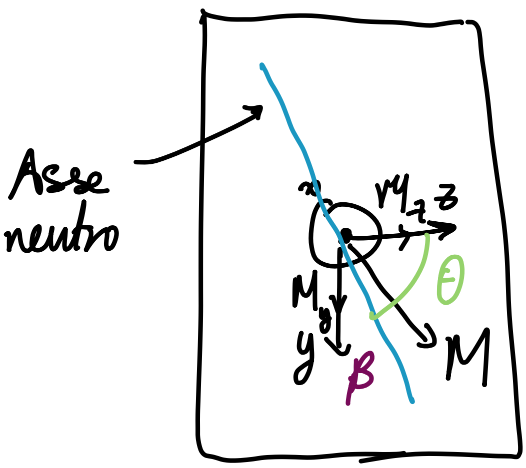
$$= -\frac{bM_z}{J_{zz}} \bar{y} \left(\frac{h}{2} + \frac{h}{2} \right) = 0 \Longrightarrow \bar{y} = 0$$ perché il resto non ha effetto o sono costanti

Possibile all'orale



Asse neutro coincide con asse del momento, se momento applicato su asse principale di inerzia

<u>Caso dove non applicato su asse principale di inerzia</u>



$$\frac{M_z}{J_{zz}} y^* - \frac{M_y}{J_{yy}} z^* = 0$$

y e z dell'asse neutro

$$\frac{y^*}{z^*} = \tan\beta = \underbrace{\boxed{\frac{M_y}{M_z}}}_{\tan\theta} \cdot \frac{J_{zz}}{J_{yy}}$$

$$\tan\beta = \frac{J_{zz}}{J_{yy}} \tan\theta$$

se $\theta = 0$ asse neutro coincide con z

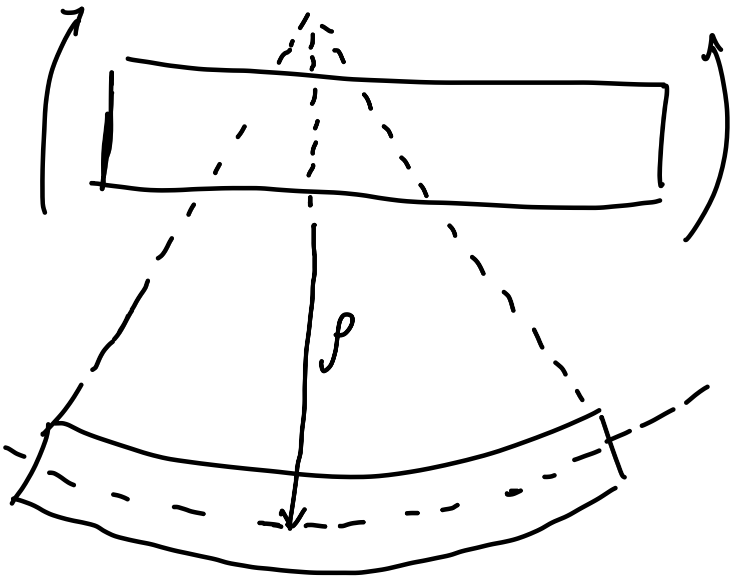
se $\theta = \frac{\pi}{2}$ asse neutro coincide con y

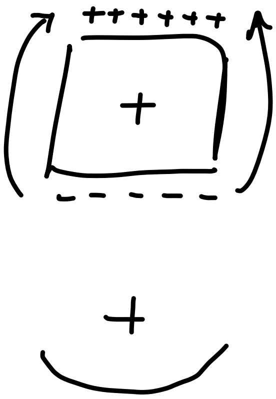
$$\sigma_x = \frac{M}{J} y$$

distanza da asse neutro

Problema Elastico per Flessione ($N=0$)

↳ Linea Elastica Flessionale

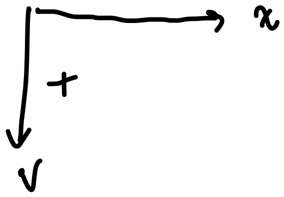


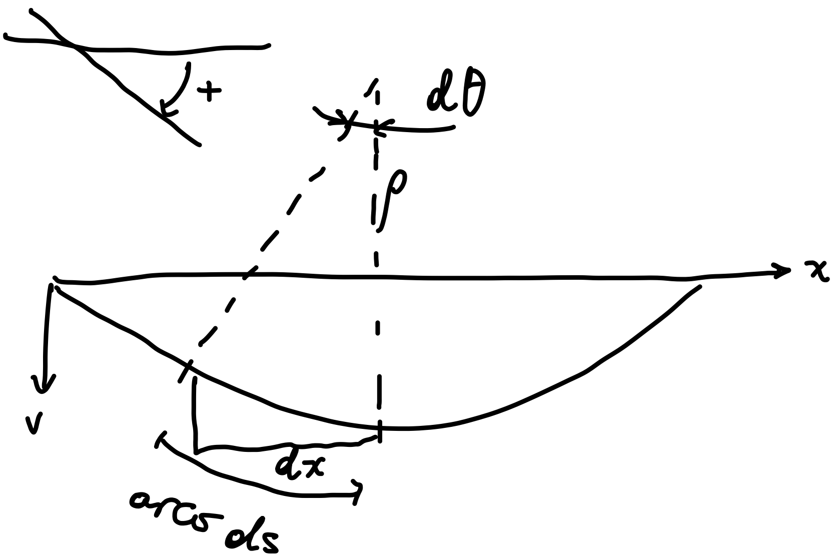


$K \rightarrow$ curvatura

$$K = \frac{1}{\rho}$$

↳ raggio del cerchio osculatore all'asse neutro





$$ds = -\rho\, d\theta \simeq dx$$

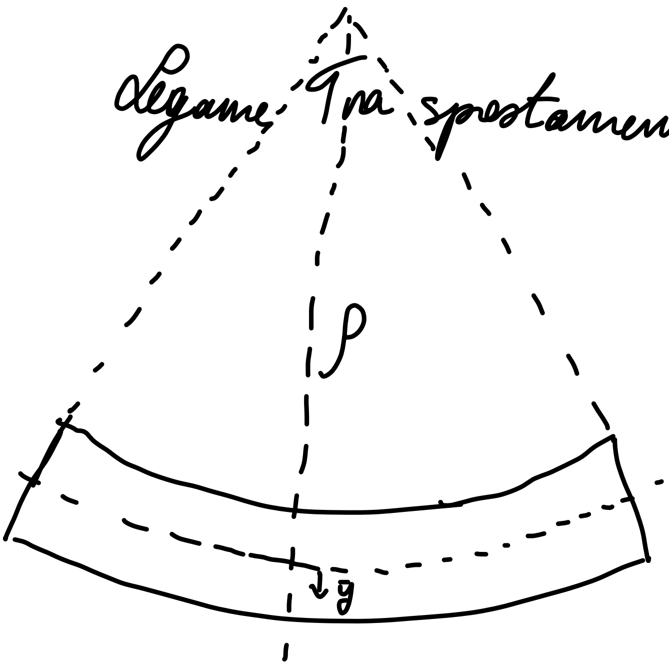
$\frac{dv}{dx} = \tan\theta \simeq \theta$ per angoli piccoli

$$K = \frac{1}{\rho} \simeq -\frac{d\theta}{dx} = -\frac{d^2 v}{dx^2}$$

$$\boxed{K = \frac{d^2 v}{dx^2}} \quad (1)$$

↳ Legame tra curvatura e spostamenti

Legame tra spostamenti e deformazione



La deformazione è funzione della distanza da asse neutro

$$\varepsilon_x(y) = \frac{\Delta L(y)}{l_0} = \frac{L_1(y) - l_0}{l_0}$$

Infinitesimamente $l_0 = dx = -\rho\, d\theta$ per $dx > 0$

Perché andiamo in senso antiorario

$$L_1(y) = -(\rho + y)\, d\theta$$

$$\varepsilon_x(y) = \frac{-(\cancel{\rho} + y)\, d\theta + \cancel{\rho}\, d\theta}{dx} = -\frac{y\, d\theta}{dx}$$

$$\sigma_x = E \varepsilon_x$$

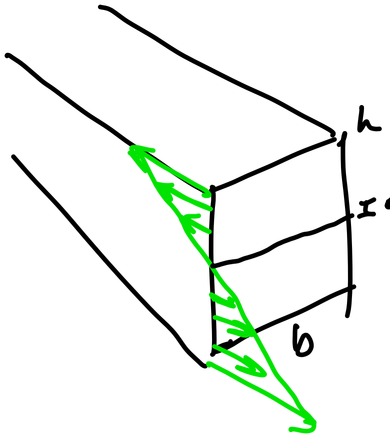
(2)

$$\boxed{\frac{d\theta}{dx} = \frac{d^2 v}{dx^2}}$$

(3)

$$\boxed{\varepsilon_x(y) = -y \frac{d\theta}{dx}}$$

Saint-Venant (Ultima Lezione)



$dA = b\,dy$

$\underbrace{y}_{\text{Distanza da asse neutro}}$

$$M = \underbrace{\int_{-h/2}^{h/2} \sigma_x(y)\, b\, y\, dy}_{\text{Equivalenza elastica per Saint-Venant}}$$

$$= \int_{-h/2}^{h/2} \overbrace{E\,\varepsilon_x(y)}^{\sigma_x(y)}\, b\, y\, dy$$

$$M = Eb \int_{-h/2}^{h/2} \underbrace{\varepsilon_x(y)}_{3}\, y\, dy = -\int_{-h/2}^{h/2} \underbrace{\frac{d\theta}{dx}}_{\text{non varia per } y} y^2\, dy\; Eb$$

$$= -\underbrace{Eb}_{J}\; \overbrace{\frac{d\theta}{dx}}^{2} \underbrace{\int_{-h/2}^{h/2} y^2\, dy}_{\text{Momento d'inerzia di una sezione (capiremo domani)}} \longrightarrow b \cdot \int = J$$

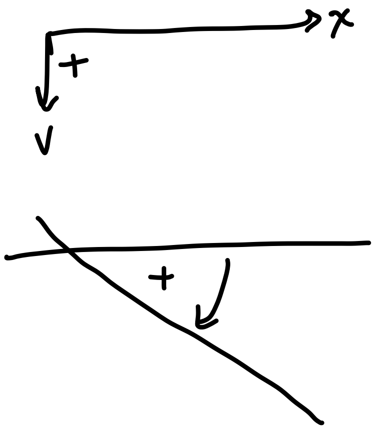
$$= -EJ\,\frac{d^2v}{dx^2}$$

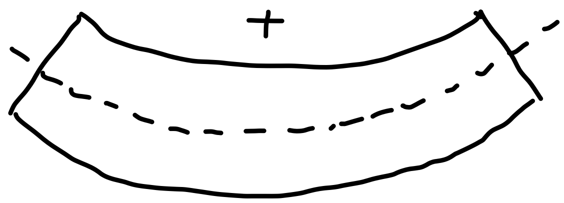
$$\boxed{M = -EJ\,\frac{d^2v}{dx^2}}$$ Equazione della linea elastica flessionale

$$= -EJK$$

Dimostrazione non verrà chiesta

Convenzioni Sulla Linea Elastica





$$M \pm EJ\,\frac{d^2 v}{dx^2}$$

v positiva verso basso

	+ $\cup$	+ $\cap$
v + ↓	−	+
v + ↑	+	−

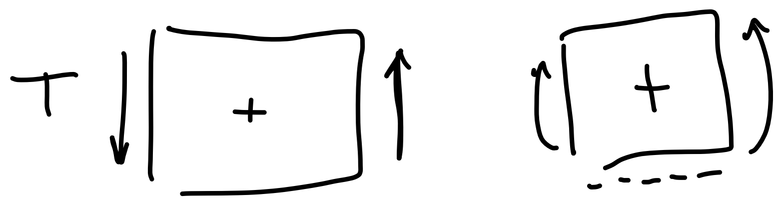
Convenzioni

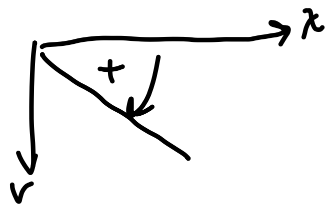
$$\boxed{\frac{dM}{dx} = -T}$$

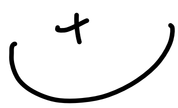
$$\boxed{\frac{dT}{dx} = q}$$

Carico

$$\boxed{\frac{d^2M}{dx^2} = -q}$$



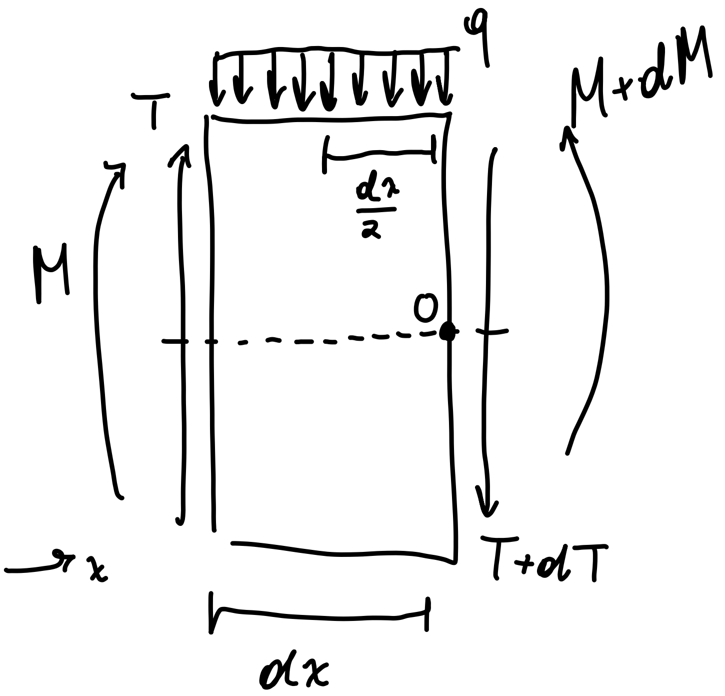




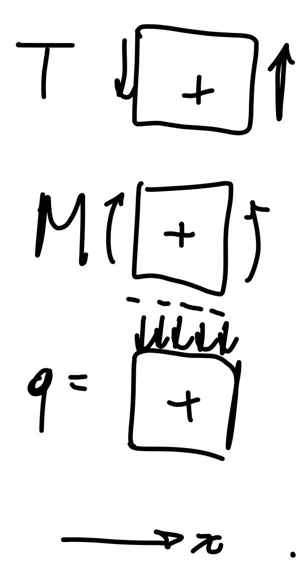
$$\underbrace{EJ\,\frac{d^2 v}{dx^2} = q}_{\text{Per strutture}}$$

iperstatiche

→ Come cambiano in base alle convenzioni (memorizzarne uno)



Convenzioni

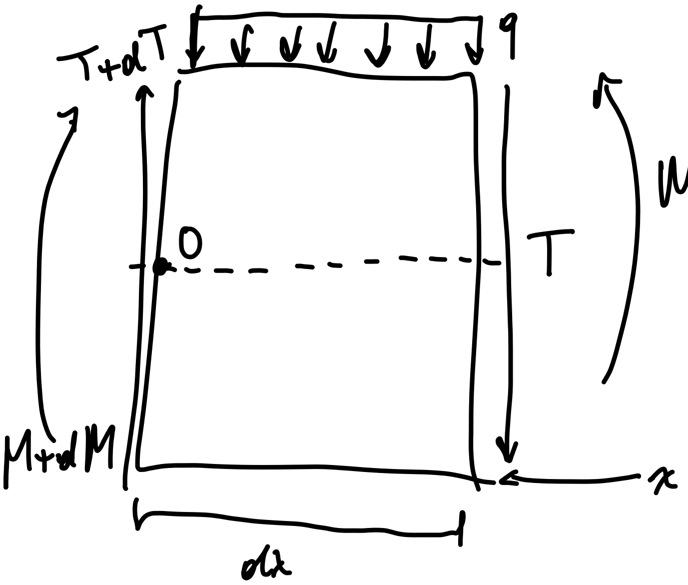


$$\sum M = 0 = -T\,dx - \cancel{M} + \underbrace{\underbrace{q \cdot dx}_{F_{eq}} \cdot \cancel{\frac{dx}{2}}}_{M_{eq}} + \cancel{M} + dM$$

dx^2 → infinitesimo di secondo grado ↓ trascurabile rispetto a primo grado

$$T = \frac{dM}{dx}$$

$$\sum F_y = 0 = \cancel{T} - \cancel{T} - dT - q\,dx \Rightarrow q = \frac{-dT}{dx}$$



$$\sum M = 0 = \cancel{M} - \cancel{M} - dM - T\,dx - q\,dx\,\cancel{\frac{dx}{2}}$$

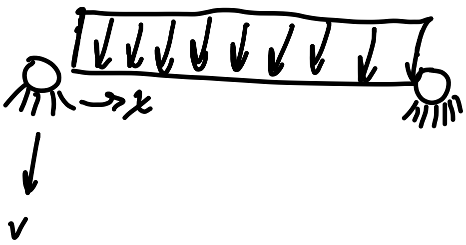
$$-\frac{dM}{dx} = T$$

Cambiano come prendiamo x positivo, cambia la equazione

$$\sum F = 0 = \cancel{-T} + \cancel{T} + dT - q\,dx$$

$$\Rightarrow q = \frac{dT}{dx}$$

Contestualizzazione



Cambiano i risultati da come imponiamo il sistema di riferimento

In casi isostatici come carichi concentrati dobbiamo imporre due sistemi di riferimento