Esercitazione 3 - Hydraulic Similarity + Cavitation

Exercise 1

Centrifugal Pump,

design

$n_d = 1400\,rpm$ $\quad H_d = 39\,m$

$Q_d = 0{,}153\,\frac{m^3}{s}$ $\quad W_d = 68\,kW$

$D_d = 300\,mm$ $\quad \eta_d = \eta_{MAX} \rightarrow BEP$

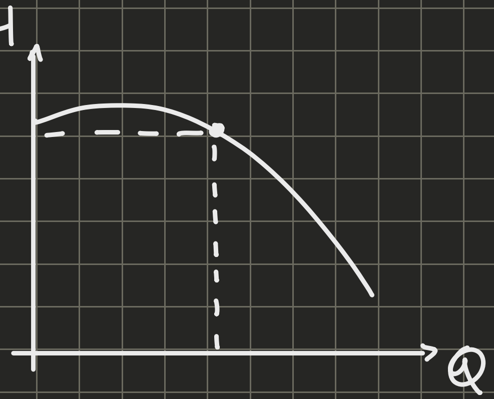
We need to keep η the same, so we need to maintain similarity.

a) $n_a = 1200\,rpm$

$D_a = D_d$

$Q_a = ?$

$H_a = ?$



$$gH = \underbrace{l}_{\propto U^2 \rightarrow f(D,n)} - l_w$$

To non-dimensionalize, we divide by $n^2 D^2$

$$\underbrace{\frac{gH}{n^2D^2}}_{\psi} = \underbrace{\frac{l}{n^2D^2}}_{\lambda} - \underbrace{\frac{l_w}{n^2D^2}}_{\zeta} \rightarrow \psi = \underbrace{\psi(\varphi, Re, SHAPE)}_{\text{dynamic problem}} \; * $$

$\zeta \rightarrow$ zeta (?)

ψ: Head Coefficient

λ: Work Coefficient

φ: Flow Coefficient

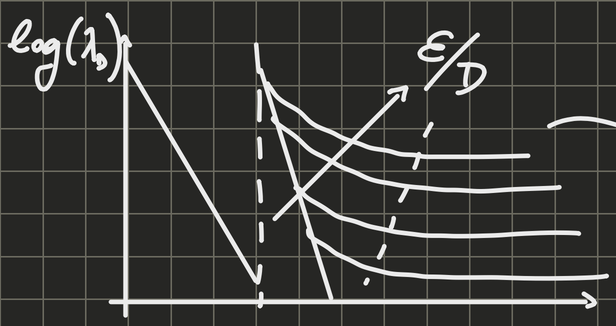
$$\lambda = \lambda\Big(\varphi, \underbrace{\frac{D_2}{b_1}, \frac{D_2}{b_2}, \alpha_1, \beta_2}_{\text{Shape paramters}}\Big) = \lambda(\varphi, SHAPE)$$

$$\varphi = \frac{Q}{nD^3}$$

ξ is like $Y_{loss,friction} = \xi_{JD} \frac{V^2}{2}$ → ρ includes $Y_{friction}$ and other factors.

$$\xi_{JD} = \xi_{JD}(Re, SHAPE) = \lambda_D\left(Re, \frac{\varepsilon}{D}\right) \cdot \frac{L}{D}$$

For high Re: $\frac{\partial \lambda_D}{\partial Re} = 0$



→ self-similarity in Re

(*)$_1$ With high Re, we have self-similarity of Re:

$$\Rightarrow \Psi = \Psi(\varphi, SHAPE) \rightarrow \text{Kinematic problem.}$$

From were we can get efficiency:

$$\eta = \eta(\varphi, SHAPE)$$

$$\eta = \frac{gH}{\ell} \rightarrow \eta = \frac{\Psi}{\lambda} = \frac{\Psi(\varphi, SHAPE)}{\lambda(\varphi, SHAPE)}$$

Fixing φ, Ψ will be the same

Return to exercise:

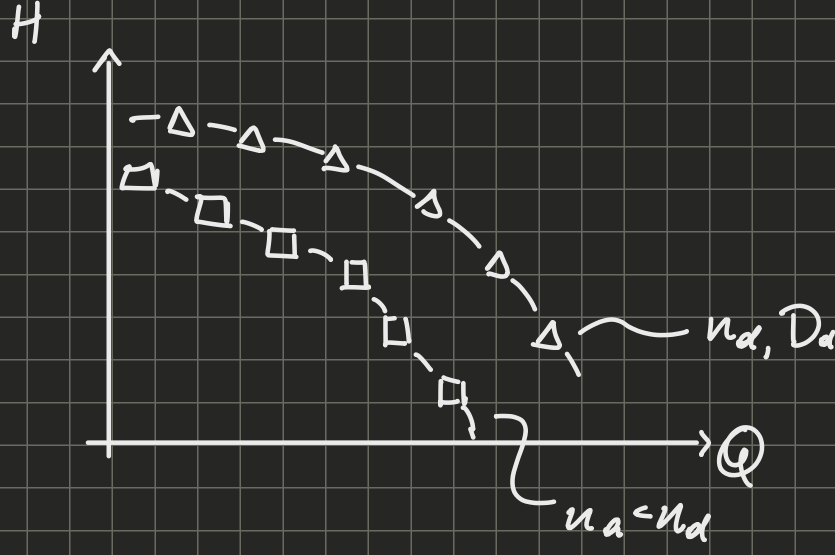
$$\varphi_d = \varphi_a$$

$$\frac{Q_d}{n_d \cancel{D_d^3}} = \frac{Q_a}{n_a \cancel{D_a^3}} \longrightarrow Q_a = \frac{n_a}{n_d} Q_d = 0{,}127 \ \frac{m^3}{3}$$

We respect the similarity by fixing the φ and SHAPE

$$\rightarrow \Psi_d = \Psi_a \rightarrow \frac{gH_d}{n_d^2 \cancel{D_d^2}} = \frac{gH_a}{n_a^2 \cancel{D_a^2}} \rightarrow H_a = \left(\frac{n_a}{n_d}\right)^2 H_d = 26{,}7 \ m$$

$\eta_a = \eta_d = \eta_{MAX}$ → since we have to keep similarity.



b) $n_b = n_d$

$D_b = 200\,mm \; (< D_d)$

Q_b and H_b? such the $\eta = const \Rightarrow$ similarity

$\varphi_b = \varphi_d$

$$\frac{Q_b}{\cancel{n_b} D_b^3} = \frac{Q_d}{\cancel{n_d} D_d^3} \rightarrow Q_b = \left(\frac{D_b}{D_d}\right)^3 Q_d = 1{,}453 \frac{m^3}{s}$$

$$\psi_b = \psi_d \rightarrow \frac{g H_b}{\cancel{n_b^2} D_d^2} = \frac{g H_d}{\cancel{n_d^2} D_d^2} \longrightarrow H_b = \left(\frac{D_b}{D_d}\right)^2 H_d = 17{,}5\,m$$

When we adhere to a similarity condition, we are able to calculate everything

Exercise 3

Hydraulic Turbine

$H_d = 1200\,m$

$Q_d = 4 \frac{m^3}{s}$

→ chosen from Balje chart

→ Pelton Turbine (is most efficient turbine)

$\omega_s = 0{,}093 \rightarrow$ specific speed $\rightarrow \eta = 0{,}88$

$D_s = 16 \rightarrow$ " diameter

$Q_L = ?$

$H_L = 200\,m$

$W_L = 15\,kW$

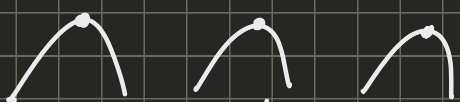
$$\left.\begin{array}{l} \varphi = \dfrac{Q}{nD^3} \\ \psi = \dfrac{gH}{n^2D^2} \end{array}\right\}$$ don't use ω_s or D_s, so we need to use different dimensionless parameters.

$\omega_L, \omega_d = ?$

$D_L, D_d = ?$

↳ lab ↘ real

→ no correllation, since dependent on shape and size



Machines at their BEP

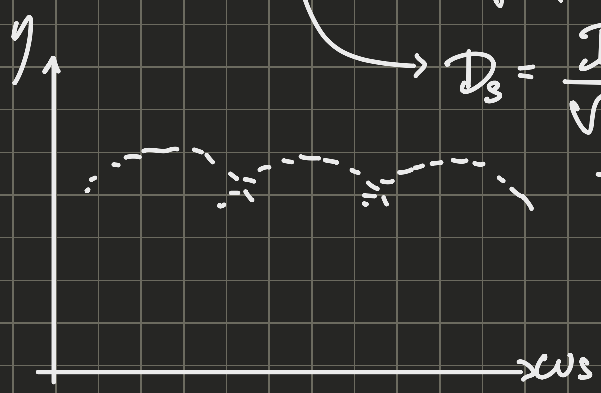
$$\psi = \frac{gH}{n^2D^2} \rightarrow D = \frac{(gH)^{1/2}}{n\sqrt{\psi}} \rightarrow n = \ldots$$

$$\varphi = \frac{Q}{nD^3} \rightarrow \varphi = \frac{Qn^3\psi^{3/2}}{n(gH)^{3/2}}$$

$$\sqrt{\frac{\varphi}{\psi^{3/2}}} = \frac{\sqrt{Q}n}{(gH)^{3/4}} = n_s$$

→ if ω rather than n, it would be ω_s

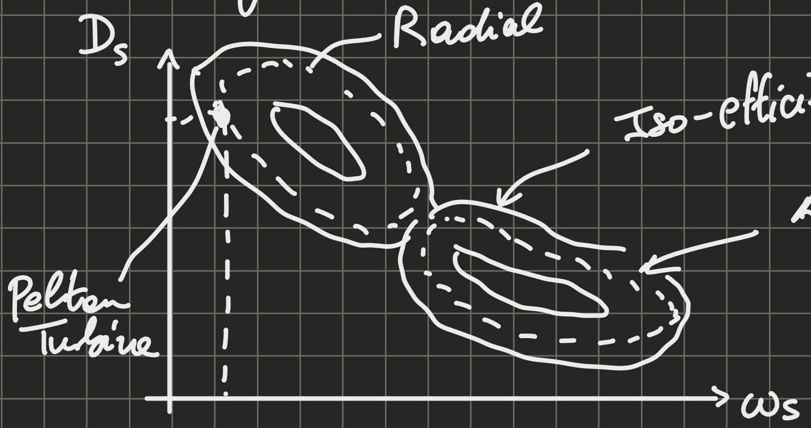
$$D_s = \frac{D\sqrt{gH}}{\sqrt{Q}}$$



→ we are able to isolate the effect of shape

We have found the definition of D_s and ω_s

Balje Chart



$$\omega_d = \frac{\omega_s \left(gH_d\right)^{3/4}}{\sqrt{Q_d}} = 52,5 \ \frac{rad}{s} = 500 \, rpm$$

$$D_d = \frac{D_s \sqrt{Q_D}}{\left(gH_D\right)^{3/4}} = 3,1 \ m$$

$$Q_L = \frac{W_L}{\rho g H_L \eta} = 0,0087 \ \frac{m^3}{s}$$

$$W_L = \rho \, Q_L \cdot l = \rho \, Q_L \, \eta \, g \, H_L$$

$$\eta = \frac{l}{gH_L} \rightsquigarrow l = (gH_L)\eta$$

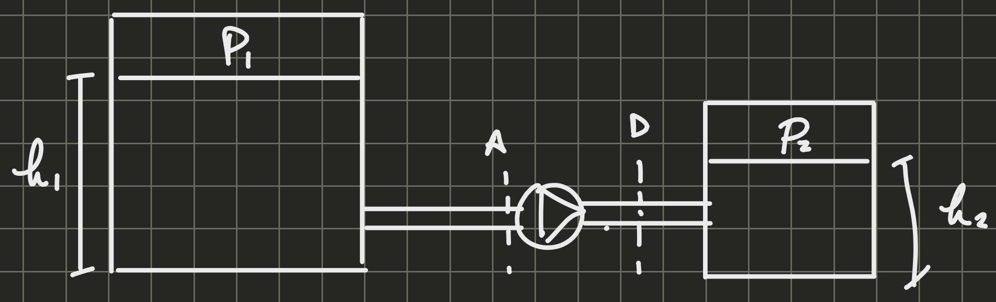
inverse of the pump, since l in turbine is the useful output.

$$\Rightarrow \omega_L = \frac{\omega_s \left(gH_L\right)^{3/4}}{\sqrt{Q_L}} = 293,7 \ \frac{rad}{s} = 2800 \, rpm$$

$$D_L = \frac{D_s \sqrt{Q_L}}{\left(gH_L\right)^{3/4}} = 0,224 \ m$$

Cavitation

Exercise 2

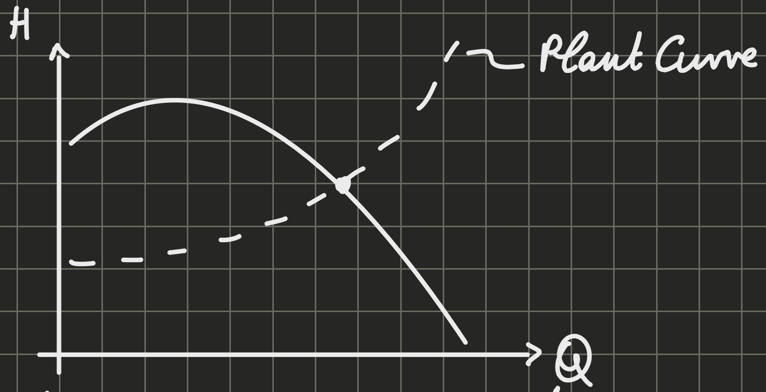


$P_1 = 0,3$ bar $\qquad P_2 = 2,3$ bar

$z_1 = 10m \qquad z_2 = 5m$

$D_1 = 350\,m \qquad D_2 = 350\,mm$

$\xi_{ASP} = 5 \qquad \xi_{del} = 15$



a) H = ? Q = ? Operating Point → intersection of two curves.

BME$(1 \rightarrow 2)$

$$\underbrace{gH_c}_{=l-l_w} - Y_{asp} - Y_{del} = \frac{P_2 - P_1}{\rho} - \overbrace{\cancel{\frac{V_2^2 - V_1^2}{2}}}^{0} + g(z_2 - z_1)$$

$$Y_{asp} = \xi_{asp} \frac{V_{asp}^2}{2}$$

$$Y_{del} = \xi_{del} \frac{V_{del}^2}{2}$$

$$Q_{del} = Q_{asp}$$

$$V_{del}\,\frac{\pi D_{del}^2}{4} = V_{asp}\,\frac{\pi D_1^2}{4} \rightarrow \underbrace{V_{del} = V_{asp}}_{\text{same pipe}} = V$$

$$H_c = \frac{P_2 - P_1}{\rho g} + (z_2 - z_1) + (\xi_{asp} + \xi_{del})\,\frac{V^2}{2g}$$

$$Q = \frac{\pi D^2}{4} V \rightarrow V = \frac{4Q}{\pi D^2}$$

$$= \frac{P_2 - P_1}{\rho g} + (z_2 - z_1) + (\xi_{asp} + \xi_{del})\,\frac{8Q^2}{\pi^2 D^4 g}$$

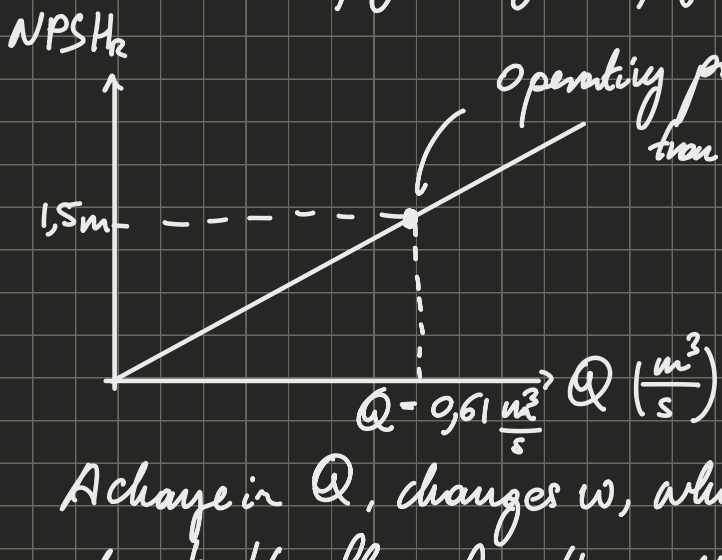
We now have $H_c = H_c(Q) \rightarrow$ plant resistance curve.

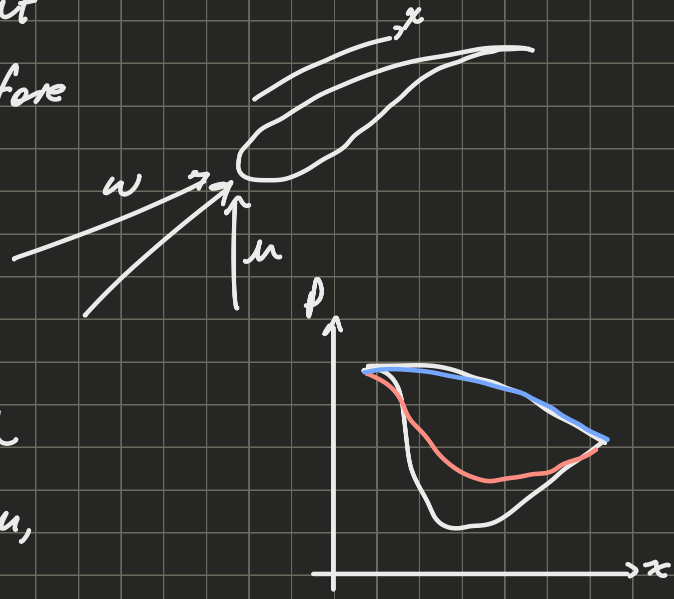
$$H_c = 17{,}68 + 110{,}24\,Q^2$$

We are able to then find the intersection point with the pump curve, where $H = 58\,m$, $Q = 0{,}61\,\frac{m^3}{s}$.

b) Maximum height of installation, to avoid cavitation

$$NPSH_R = \frac{P_{in}}{\rho g} + \frac{V_{in}^2}{2g} - \frac{P_{min}}{\rho g}$$





A change in Q, changes w, which deviates the flow from the optimum, changing P.

We need to find h such that $NPSH_A > NPSH_R$

$$NPSH_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} - \frac{P_{SAT}}{\rho g}$$

Available

Inlet

$$\frac{P_{SAT}}{\rho g} = 1,5\,m$$ → Be careful, sometimes they give P_{SAT} in meters, which already takes into account ρg.

$$BME\ (1 \to A) \to -\frac{Y_{asp}}{g} = \left(\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z\right) - \left(\frac{P_1}{g\rho} + \frac{V_1^2}{2g} + z_1\right)$$

$$Y_{asp} = \xi_{asp}\frac{V_{asp}^2}{2} = \xi_{asp}\frac{8Q}{\pi^2 D_{asp}^4}$$

$$NPSH_A = \frac{P_1}{\rho g} + (z_1 - z_A) - \xi_{asp}\frac{8Q^2}{\pi^2 D^4 g} - \frac{P_{SAT}}{\rho g}$$

$NPSH_A > NPSH_R$ → find z_A such that this is true

$$\Rightarrow z_A < \frac{P_1}{\rho g} + z_1 - \xi_{asp}\frac{8Q^2}{\pi D^4 g} - \frac{P_{SAT}}{\rho g} - NPSH_R$$

$z_A < 0,15\,m$