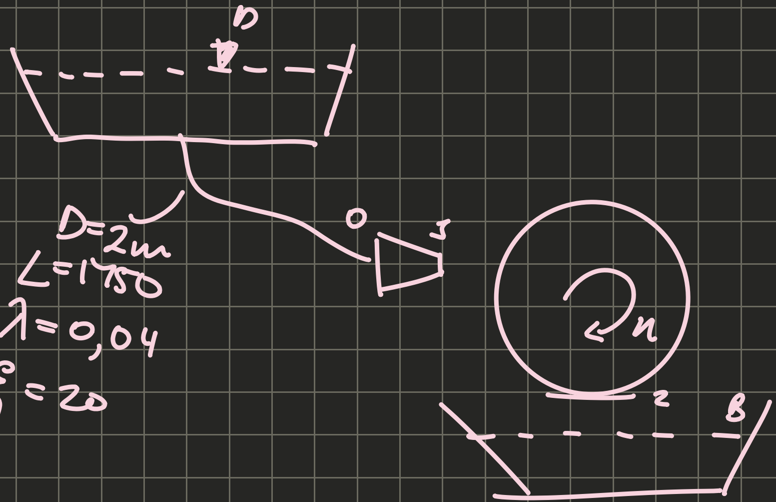
Esercitazione 4 – Hydraulic Turbines and Ducts

Exercise Pelton



$z_D - z_B = 1000\,m$

$z_0 \approx z_1 \approx z_2 \approx z_B$
No draft tube (only for Pelton)

$D = 2m$
$L = 1750$
$\lambda = 0{,}04$
$\xi = 20$

$\rho = 1000\ \frac{kg}{m^3}$

$\beta_2 = -68°$

$n = 600$ rpm

nozzles $\rightarrow i = 3$

$D_m = 2m$
↳ mean diameter of machine

$d = 0{,}22\,m$

1) Velocity Triangles

2) Calculate gH_m?

3) $\dot{L}$?

4) Is the condition optimized?

ideal machine
↳ ⇒ optimized.
↳ $\varphi = 1$, $\psi = 1$

Machine = Stator + Rotor
↳ Nozzle
↳ lw_N ↳ φ
↳ Turbine
↳ $lw_B \rightarrow \psi = \frac{\omega_2}{\omega_1}$

BME $(D \rightarrow 1)$

→ since $P_1 = P_D = P_{ATM}$

$$\cancel{l}^{\,0} - lw - y_P = \cancel{\frac{P_1 - P_D}{\rho}}^{\,0} + \frac{V_1^2 + \cancel{V_D^2}}{2} + g(z_1 - z_D)$$

$lw = lw_N$

y_P ↳ Penstock

$$\rightarrow \frac{v_1^2}{2} = g(z_D - z_1) - l_{wN} - y_p$$

we want v_1 to find the velocity triangles.

$$l_{wN} \rightarrow \varphi = \frac{v_1}{v_1'} \rightarrow \text{ideal velocity} \Rightarrow v_1 = \varphi v_1'$$

y_p must be considered since it is not part of the nozzle.

Problem since we now need to find v_0 and v_1

$$y_p = \left(\xi + \frac{\lambda L}{D}\right)\frac{v_0^2}{2} = (*)_1$$

$$Q_0 = Q_1 \longrightarrow \frac{\pi D^2}{4} v_0 = \frac{\pi d^2}{4} v_1 i = \frac{\pi d^2}{4} \varphi v_1' i$$

Mass Balance

$$\Rightarrow v_0 = \varphi i \left(\frac{d}{D}\right)^2 v_1'$$

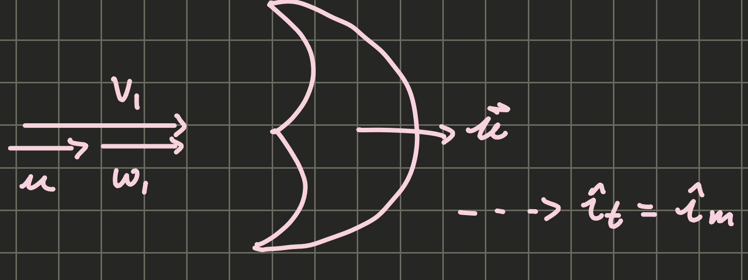
$$(*)_1 = \left(\xi + \frac{\lambda L}{D}\right) \varphi^2 i \left(\frac{d}{D}\right)^4 \frac{v_1'^2}{2}$$

Substitute in BME

$$\frac{v_1'^2}{2} = g(z_D - z_1) - \left(\xi + \frac{\lambda L}{D}\right)\varphi^2 i^2 \left(\frac{d}{D}\right)^4 \frac{v_1'^2}{2}$$

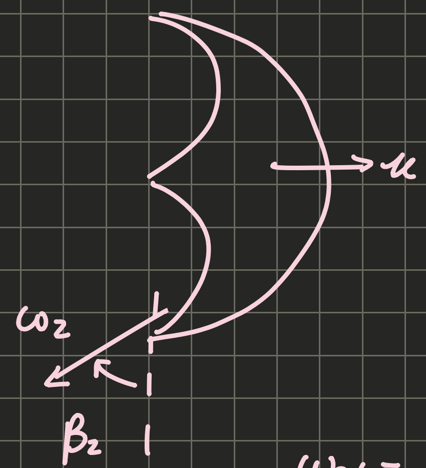
$$\Rightarrow v_1' = \sqrt{\frac{2g(z_D - z_1)}{1 + \left(\xi + \frac{\lambda L}{D}\right)\varphi^2 i^2 \left(\frac{d}{D}\right)^4}}$$

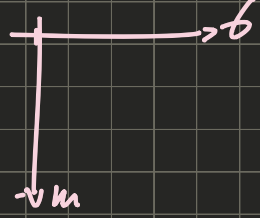
since it's an ideal machine $\rightarrow \varphi = 1 \Rightarrow v_1 = v_1' = 135{,}2$



$u = n \cdot \frac{2\pi}{60} \cdot \frac{D_m}{2} = 62{,}8 \frac{m}{s}$ $\qquad w_1 = V_1 - u = 72{,}4 \frac{m}{s}$

Outlet





$u = 62{,}8\ m/s$

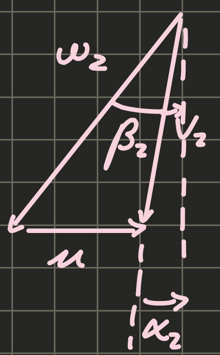
$w_{2t} = V_{2t} - u \rightarrow V_{2t} = -4{,}3\ m/s$

$w_2 \sin \beta_2$

since the machine is ideal, $\psi = \frac{w_2}{w_1} = 1 \Rightarrow w_1 = w_2 = 72{,}4 \frac{m}{s}$

$V_{2x} = w_{2x} = w_2 \cos \beta_2 \Rightarrow V_{2x} = 27{,}2\ m/s$

$\alpha_2 = \tan \frac{V_{2t}}{V_{2x}} = -9°$



1 ✓

3) $\dot{L} = ? \quad \overbrace{\qquad}^{-4{,}3 \frac{m}{s}} = \dot{m} l = 134{,}9\ MW$

$l = \underbrace{u}_{u_1 = u_2} (V_{2t} - \underbrace{V_{1t}}_{= V_1}) = -8760{,}6 \frac{J}{kg}$

$\dot{m} = \rho Q = \rho\, i\, \frac{\pi d^2}{4} v_1 = 15\,400 \frac{kg}{s}$

2) gH_m?

$$\eta = \frac{|\ell|}{gH_m}$$

Assuming we have a completely ideal turbine ($\eta = 1$), they would be the same, but we don't.

$$gH_m = T_0 - \overset{\to T_B}{T_2} = T_D - y_p - T_2 = \frac{\overset{\to 0 \to \text{Both } P_{ATM}}{\cancel{P_D - P_2}}}{\cancel{\rho}} + \frac{\overset{\to 0}{\cancel{V_D^2}} - V_2^2}{2} + g(z_D - z_D) - y_p$$

↳ The energy available to the turbine is the difference between 0 and B.

$$= g(z_D - z_B) - y_p - \frac{V_2^2}{2}$$

With T_B $\boxed{= g(z_D - z_B) - y_p}$ ⇝ The one we use.

BME($D \to 1$):

$$-y_p = \frac{\overset{\to 0}{\cancel{P_1 - P_D}}}{\cancel{\rho}} + \frac{V_1'^2 - \overset{\to 0}{\cancel{V_D^2}}}{2} + g(z_B - z_D) \to \frac{V_1'^2}{2} = \underbrace{g(z_D - z_P) - y_p}_{gH_m}$$

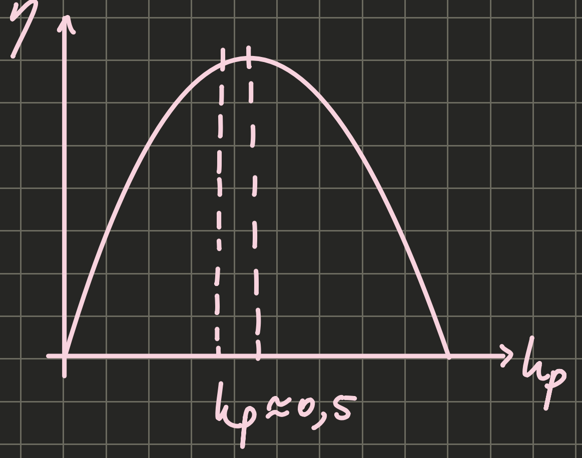
$$\Rightarrow gH_m = \frac{V_1'^2}{2} = 9139{,}52 \text{ J/kg}$$

↳ This is always true, independent on whether the machine is ideal or not.

$$\eta = \frac{|\ell|}{gH_m} = 95{,}9\%$$

Does the machine work in the optimized condition?

↳ What is the definition of optimized condition?



↳ when $u = \frac{v_1}{2}$

$$k_{p,opt} = \frac{\varphi}{2}$$

$$k_p = \frac{u}{v_1'}$$

$$k_p = \frac{u}{v_1'} = 0,464 \neq \underbrace{0,5}_{k_{p,opt}(\varphi=1)}$$

We need to increase u, to reach $k_{p,opt}$

Load regulation in Pelton turbine (second part of exercise)

$$\dot{L} = \underbrace{\dot{m}}_{\rho Q} \, \ell$$

$$\hookrightarrow i \frac{\pi d^2}{4} \underbrace{\varepsilon}_{0<\varepsilon<1} \varphi v_1'$$

coefficient of decrease of d.

$\varepsilon = 0,6$ $\quad n = 600$

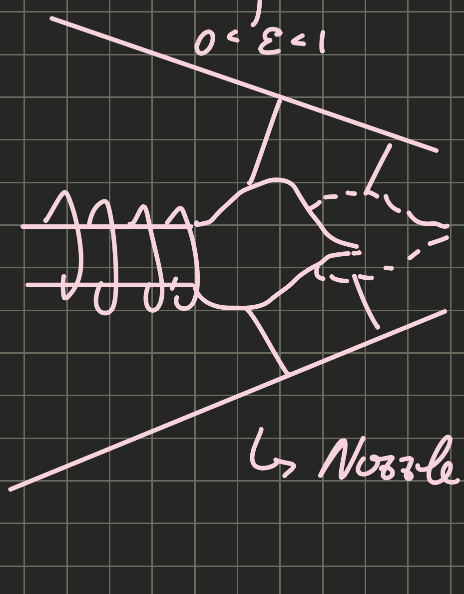
$\xi = 20$

$\dot{L} = ?$

$gH_m = ?$

$\eta = ?$

optimized ?



BME

$(D \to 1)$

$$-y_p = \frac{v_1'^2}{2} + g(z_B - z_D)$$

$$\left(\xi + \frac{\lambda L}{D}\right) \frac{v_0^2}{2} = \left(\xi + \frac{\lambda L}{D}\right) \varphi^2 i^2 \varepsilon^2 \left(\frac{d}{D}\right)^4 \frac{v_1'^2}{2}$$

$$Q_0 = Q_1 \rightsquigarrow \frac{\pi D^2}{4} v_0 = \frac{\pi d^2}{4} i \varepsilon \varphi v_1' \to v_0 = \varphi i \varepsilon \frac{d^2}{D^2} v_1'$$

$$v_1' = \sqrt{\frac{2g(z_D - z_B)}{1+\left(\xi+\frac{\lambda L}{D}\right)\varphi_i^2 \varepsilon^2 \left(\frac{d}{D}\right)^4}} = 139{,}2\ \frac{m}{s}$$

Before we have 135 m/s,

since $\left(\frac{d}{D}\right)^4 \approx 10^{-4}$ $\rightarrow$ this is negligible, so the velocity will be nearly the same.

We are able to modify the power while maintaining relatively good efficiency.

Same process as before

$u = 62{,}8\ m/s$

$w_1 = 76{,}4\ m/s$

$\psi = 1 \rightarrow w_2 = w_1 = 76{,}4\ \frac{m}{s}$ — w_{2x}, $w_{2t} = -70{,}8\ \frac{m}{s}$

$v_{2t} = -8\ \frac{m}{s}$

$\ell = u(v_{2t} - v_{1t}) = -9244.2\ J/kg$

$\dot{L} = \rho Q \ell = -58{,}7\ MW$ $\rightsquigarrow$ Before it was $\approx -134\ MW$, so we have been able to reduce $\dot{L}$ by reducing d.

$\hookrightarrow \frac{i\pi d^2}{4}\varepsilon v_1 = 6{,}3\ \frac{m^3}{s}$

$gH_m = \frac{v_1'^2}{2} = 95{,}4\%$ $\rightsquigarrow$ we have only lost 0,5%, while reducing $\dot{L}$ by a lot.

$k_{p,opt} = \frac{\varphi}{2} = .5$ $\qquad k_p = \frac{u}{V_1'} = 0,451$

↳ We are not optimized.

How can we change u so that the efficiency is the same as before.

$k_{p_a} = 0,464$

↳ k_p of case a.

since now $k_p = 0,451 = \frac{u}{V_1'}$, we need to increase u to have the same k_p as before

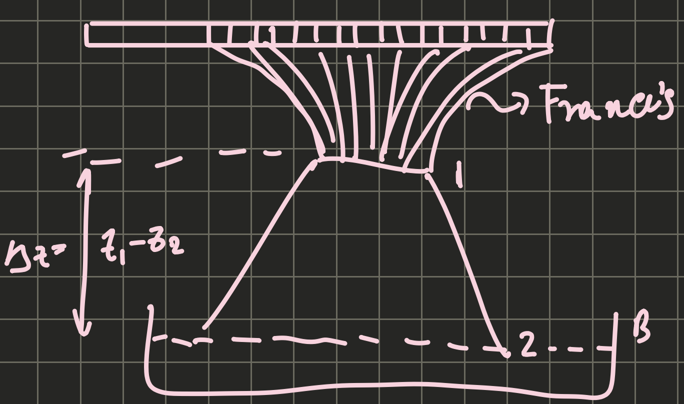
$k_{p_a} = k_{p_b} = 0,464$

$\frac{u}{V_1'} = 0,464 \rightarrow u = 0,464\, V_1' \Rightarrow n = 617\,\text{rpm}$

↳ $u = n \cdot \frac{2\pi}{60} \cdot \frac{D_m}{2}$

↳ speed at which we need to run the turbine to have the same k_p and η.

Exercise 5 - Draft Tube



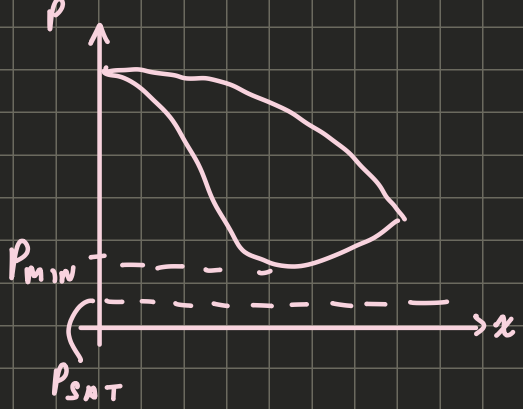
$S_2 = 4\,m^2$

$S_1 = 0,75\,m^2$

$\xi_{diff} = 2$ (evaluated at exit) (excluding outlet loss)

$Q = 10\,\frac{m^3}{s}$

$\frac{P_{1,MIN}}{\rho g} = 0,75\,m$



To not have cavitation

$P_{MIN} > P_{SAT}$

BME $1 \to B$

$$-y_{DT} = \frac{P_B - P_1}{\rho} + \frac{\cancel{V_B^2}^{\,0} - V_1^2}{2} + g\underbrace{(z_B - z_1)}_{?}$$

$$z_1 - z_B = \frac{P_{ATM} - P_1}{\rho g} + \frac{V_1^2}{2g} + \frac{y_d}{g}$$

$$Q_1 = Q = S_1 V_1 \to V_1 = 13{,}3\ m/s$$

$$Q_2 = Q = S_2 V_2 \to V_2 = 2{,}5\ m/s$$

$$y_D = \xi \frac{V_2^2}{2} + \frac{V_2^2}{2} = (\xi + 1)\frac{V_2^2}{2}$$

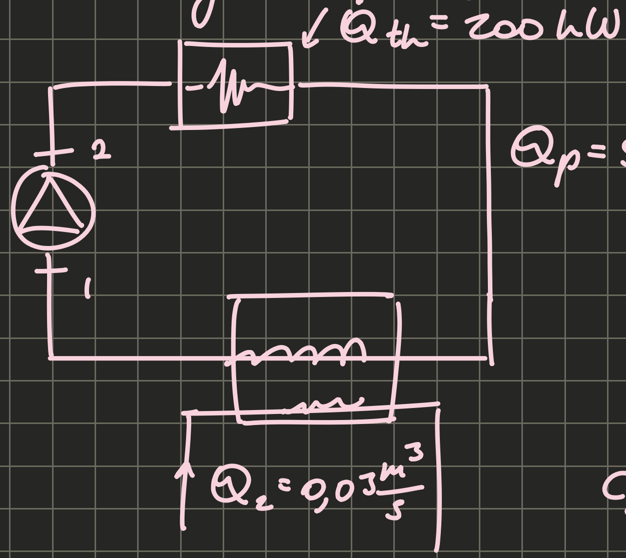
$\hookrightarrow \sum_i \xi_{conc} + \frac{\lambda L}{D}$ → distributed losses

exit losses / discharge

$$\to z_1 - z_B = \frac{P_{atm}}{\rho g} - \frac{P_{1,min}}{\rho g} - \frac{V_1^2}{2g} + (\xi + 1)\frac{V_2^2}{2g} = 1{,}38\ m$$

If the machine is placed at a higher height, there will be cavitation.

Exercise 2 - Hydraulic Ducts



$Q_p = 50\,\frac{l}{s} = \frac{.05\,m^3}{s}$

$\dot{L}_p = ?$

$\Delta T_2 = ?$

$c_L = 4186\,\frac{J}{kg\,K}$

$\eta_{hydr} = 0{,}6$

$\xi = 150$

$D = 100\,mm$

The role of this pump is to overcome the losses and impose Q.

$$\boxed{l - l_w = y}$$

Proof:

BME $2 \to 1$

$$\cancel{l} - \cancel{l_w} - y = \frac{P_1 - P_2}{\rho} + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}$$

$$y = \frac{P_2 - P_1}{\rho} \quad (*)$$

BME $1 \to 2$ — we don't have a duct

$$l - l_w - \cancel{y}^{\,0} = \frac{P_2 - P_1}{\rho} + \cancel{\frac{V_2^2 - V_1^2}{2}} + \cancel{g(z_2 - z_1)}$$

$$l - l_w = \frac{P_2 - P_1}{\rho} \quad (**)$$

$$l - l_w = y$$

$$y = \rho \frac{v^2}{2} \rightarrow Q_p = \frac{\pi D^2}{4} v \rightarrow v = \frac{4 Q_p}{\pi D^2}$$

$$y = \rho \frac{8 Q_p^2}{\pi^2 D^4} = 3072 \text{ J/kg}$$

$$\eta_{hydr} = \frac{l - l_w}{l} \rightarrow l - l_w = \eta_{hydr} l$$

$$\rightarrow \eta_{hydr} l = y \rightarrow l = y / \eta_{hydr} = \frac{3072}{.6} = 5120 \text{ J/kg}$$

$$\dot{L} = \rho Q_p l = 256 \text{ kW}$$

2) $\Delta T = ?$

we need the heat exchanger to remove the heat from the other heat exchanger, and also the heat from l_w, since $l_w = T dS_{IRR}$

$$\delta q = T \Delta s - \underbrace{T \delta s_{IRR}}_{l_w}$$

$$\delta q + \delta l_w = T \Delta S$$

$$\rightarrow q + l_w = \int T dS = c_L \Delta T_p$$

primary fluid

$$\eta = \frac{l - l_w}{l} \rightarrow l_w = (1 - \eta) l$$

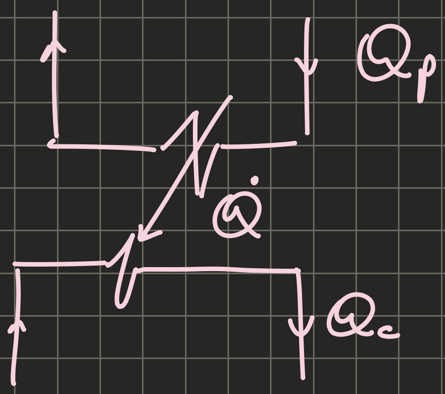
$$\frac{\dot{Q}_{th}}{\dot{m}} + (1 - \eta) l = c_L \Delta T_p$$

$$\Rightarrow \Delta T_p = 2{,}18\,°C$$

$$dh = T ds + v dP$$

$$du = T ds - \cancel{P dv} = c_L \Delta T$$

To compensate ΔT_p we have the secondary heat-exchanger



$\Delta T_p = 2{,}18\,°C$

$\Delta T_s = ?$

↳ secondary fluid

$$c_L \rho Q_p \Delta T_p = c_L \rho Q_c \Delta T_s$$

$$\Delta T_s = \frac{Q_p}{Q_c} \Delta T_p = 3{,}6\,°C$$