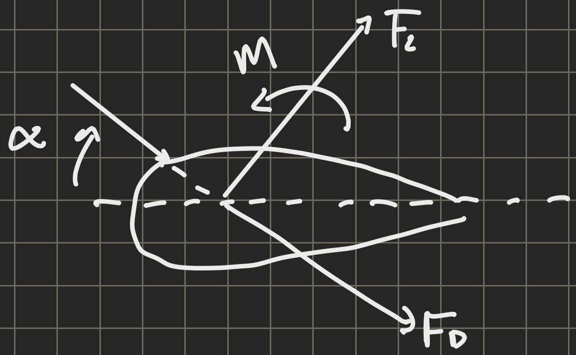
Lezione 7-

Streamlined Body



$$F_D = \frac{1}{2}\rho \underbrace{V_R^2}_{\text{relative velocity}} C_D(\alpha) S$$

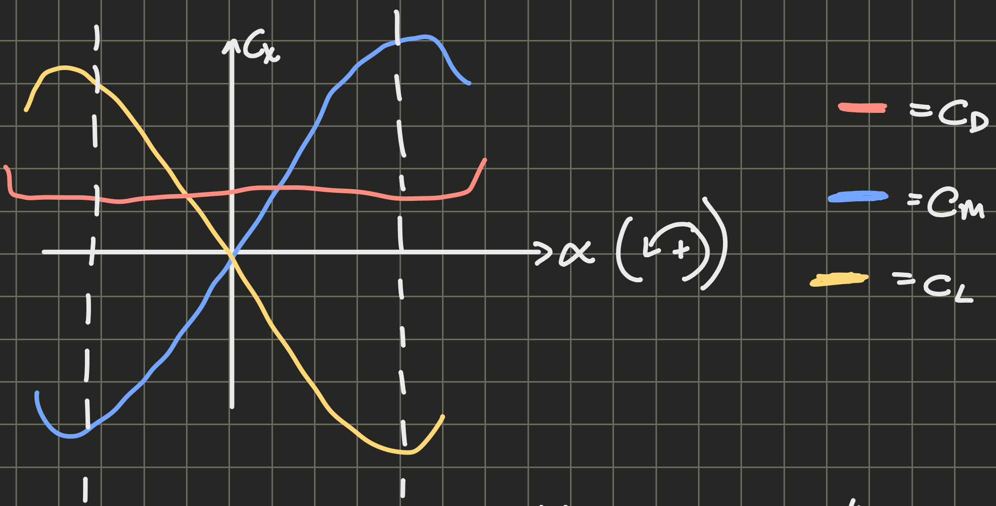
$$F_L = \frac{1}{2}\rho V_R^2 C_L(\alpha) \cdot S$$

$$M = \frac{1}{2}\rho V_R^2 C_M(\alpha) \cdot S b$$

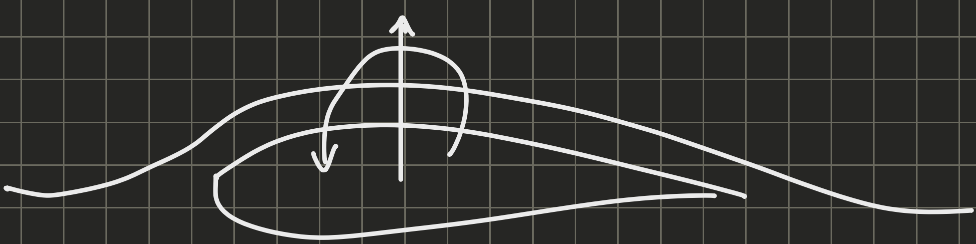
C_D, C_L, C_M are coefficients which are a function of the body shape

→ Calculated through FEM or in a wind tunnel.

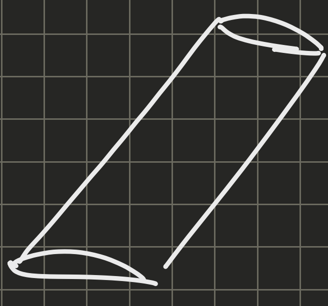
Drag is very small, it is the friction between layers close to the body.

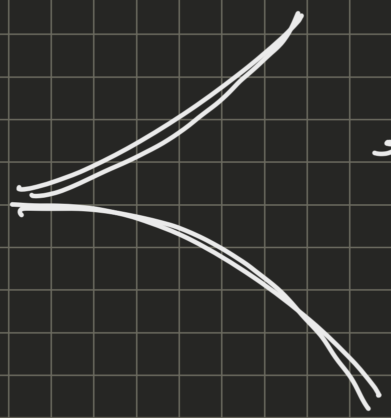


We measure it by changing the angle step by step, and then measure from the wind passing around it.



Plane wind.





→ flapping motion of the wind.

We measure the coefficient by having the body fixed, but our body moves.

We can used for the quasi-static theory:

Reduced Velocity → $V^* = \frac{V}{f \cdot B} > 10$, then we can use the static values.

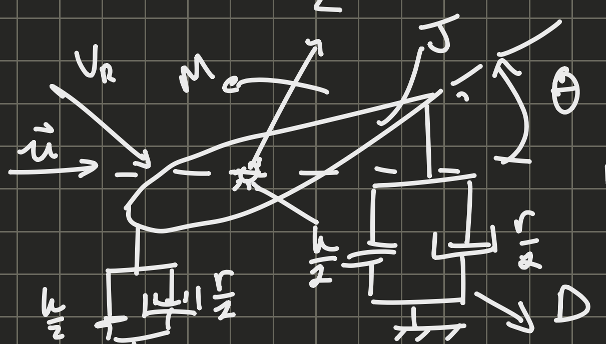
↳ comparison of time $\frac{V}{B}$, time need for particles to cross the profile

If the frequency is much lower than the time needed to cross the body, then the particle is moving so fast it sees the body as if it were fixed

If $v^* < 10$, then angle changes non-negligibly through the motion of the particle.

→ The fluiddynamics are faster than the system.

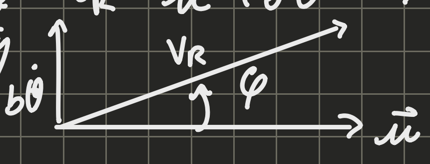
Torsional Case



because $C_M(\alpha)$

$$J\ddot{\theta} + r b \dot{\theta} + k b \theta = M$$

Relative velocity → $\vec{V}_R = \vec{u} + b\dot{\vec{\theta}} \Rightarrow M(\theta, \dot{\theta})$



Spring and dampers describe the general stiffness and damping of the wing.

$$M = \frac{1}{2}\rho V_R^2 c_m(\alpha)\cdot Sb = M(\theta, \dot{\theta})$$

$$\theta = \alpha + \varphi$$

$$\alpha = \theta - \varphi$$

$V_R^2 \simeq u^2$ because $u^2 >> b\dot{\theta} \Rightarrow \varphi$ is very small.

$$q = \frac{1}{2}\rho v^2$$

$$\varphi = \operatorname{atan}\left(\frac{b\dot{\theta}}{u}\right) \simeq \frac{b\dot{\theta}}{u}$$

For small φ

$$\Rightarrow J\ddot{\theta} + rb\dot{\theta} + kb\theta = q\cdot Sb\, c_m(\alpha)$$

$$c_m(\alpha) \simeq \cancel{c_m(\alpha)\Big|_{\alpha=0}}^{\to 0} + \frac{\partial c_m}{\partial \alpha}\Big|_{\alpha=0}(\alpha - \alpha_0) + k_m\alpha$$

Taylor

since symmetric, not in plane wings since they are not symmetric

$$J\ddot{\theta} + rb\dot{\theta} + kb\theta = q\cdot Sb\, k_m\alpha = q\cdot S\cdot b\cdot k_m\left(\theta - \frac{b\dot{\theta}}{u}\right)$$

$$\alpha = \theta - \varphi$$

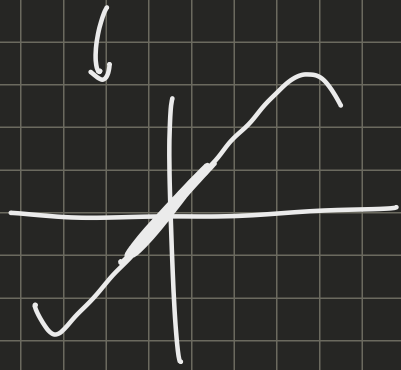
$$\alpha = \theta - \frac{b\dot{\theta}}{u}$$

Flexural Stiffness of Wing

$$J\ddot{\theta} + \left(rb + \underbrace{qSbk_m\frac{b}{u}}\right)\dot{\theta} + (kb - qSbk_m)\theta = 0$$

Grows linearly with speed

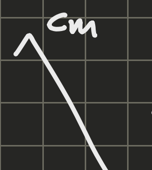
Decreases as speed increases.



↳ In ways this isn't a problem

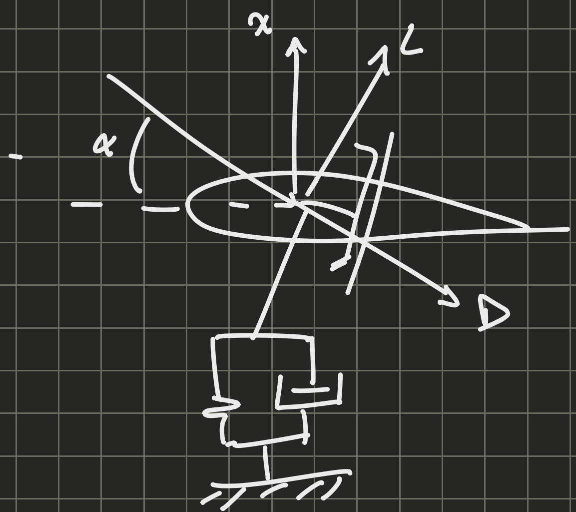
↳ This is mostly a problem for bridges, we have different layouts to deal with torsion in different ways. More stiff they are, more do they cost.

The Tacoma bridge had



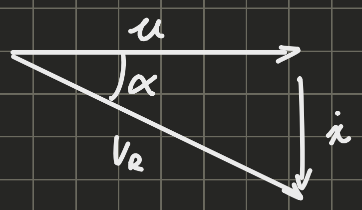
, so hm was negative, so as V increased the dampening capability decreases, causing the catastrophe

Flexural Core



$$m\ddot{x} + r\dot{x} + kx \overset{\circledast_1}{=} F_L\cos\alpha - F_D\sin\alpha$$

F is not a function of x, so it doesn't go into the stiffness. But it is a function of $\dot{x}$.



$$\circledast_1 \overset{}{=} qS\left(c_L(\alpha)\cos\alpha - c_D(\alpha)\sin\alpha\right) \quad \circledast_2$$

u^2

$$V_R^2 = u^2 + \dot{x}^2$$

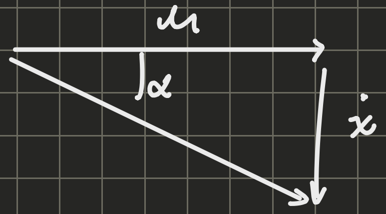
$V_R^2 \simeq u^2$ since $\dot{x}^2$ small $\Rightarrow \cos\alpha = 1$ & $\sin\alpha = \alpha$

$$c_D(\alpha) \simeq c_{D0}\Big|_{\alpha=0} + \cancel{\frac{\partial c_D}{\partial\alpha}}\Big|_{\alpha=0}\alpha$$

→ look at curve $= c_{D0}\underbrace{\alpha}_{=\sin\alpha}$

$$c_L(\alpha) \simeq \cancel{c_{L0}}\Big|_{\alpha=0} + \frac{\partial c_L}{\partial\alpha}\Big|_{\alpha=0}\alpha = h_L\alpha$$

$$\circledast_2 = \circledast_3 \quad qS(h_L\alpha - c_{D0}\alpha) = qS\alpha(h_L - c_{D0})$$



$$\alpha = \operatorname{atan}\left(\frac{\dot{x}}{u}\right) \simeq \frac{\dot{x}}{U}$$

big, negative

small, positive

$$\circledast_3 = qS\left(k_L - C_{D0}\right)\frac{\dot{x}}{u}$$

We only have an effect of the damping and since $-k_L > 0$, then the damping will increase.