Esercitazione 7 - Axial Compressors.
↳ Missed 6 and he didn't record.

2 exercises → 1 ideal and second more interesting.

<u>Exercise 1</u>

$\dot{m} = 50\ kg/s$ It's a middle stage ⇒ v is not negligible.

$n = 5000\ rpm$

air → $\gamma = 1.4$
↳ $R = 287\ \frac{J}{kg\,K}$
↳ $c_p = 1004.5$

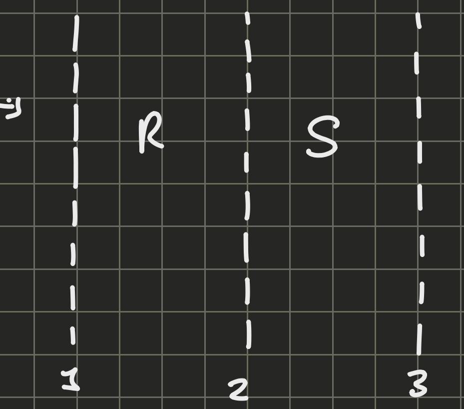
$\chi = 0{,}5$ ⇝ common for compressors.

Designed using repeated stage stategy.

$\Longrightarrow \vec{V}_1 = \vec{V}_3$

$1.2 < \beta_{ST} < 1.4$ ⇝ so to have a big β we need multiple stages.

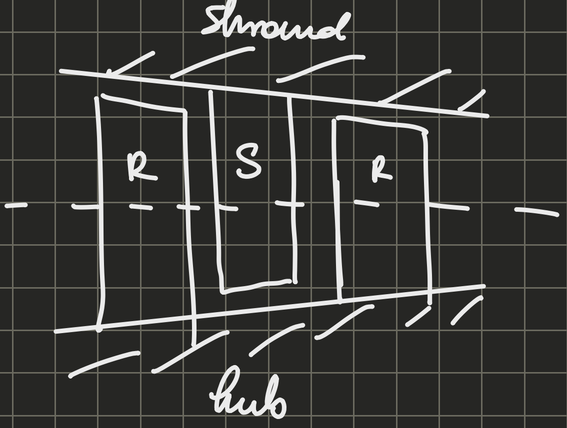
$V_m = V_x = const = 130\ m/s$



$$\begin{cases} T_{T1} = 293\ K \\ P_{T1} = 10^5\ Pa = 1\ bar \end{cases}$$

$\alpha_1 = 30°$

$D_m = const = 0.765\ m$



$$R_m = \frac{R_{sh} + R_{hub}}{2}$$

If R_{sh} is decreasing, R_{sh} needs to increase to maintain R_m constant.

1) $b_1 = ?$

$$\dot{m} = \rho_1 v_m (\pi D_m b_1)$$

To calculate ρ_1, not ρ_{T1}, we need P_1 and T_1, not P_{T1} and T_{T1}.

$$v_x = V_1 \cos\alpha_1 \longrightarrow V_1 = \frac{V_x}{\cos\alpha_1} = 150.1 \ \frac{m}{s}$$

$$\underbrace{T_{T1} = T_1 + \frac{V^2}{2c_p}}_{\text{From definition of } h_T} \longrightarrow T_1 = T_{T1} - \frac{V_1^2}{2c_p} = 281.8\,K$$

$$\frac{T_{T1}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2 \rightarrow \text{Find } M_1$$

$$\frac{P_{T1}}{P_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_{T1}}{T_1} = \frac{P_{T1}}{P_1}^{\frac{\gamma-1}{\gamma}} \rightarrow$$ This is how it was derived, and we can use these.

This is the way the total quantities are found, through isoentropic relationships.

$$P_1 = P_{T1} \cdot \left(\frac{T_1}{T_{T1}}\right)^{\gamma/\gamma-1} = 0{,}87 \text{ bar}$$

$$\Rightarrow \rho_1 = \frac{p_1}{RT_1} = \frac{0{,}87 \cdot 10^5}{287 \times 281.8} = 1.08 \ \frac{kg}{m^3}$$

$$b_1 = \frac{\dot{m}}{\rho_1 \pi D_m V_x} = 0.148 m$$

b) Velocity Triangles

$$\begin{cases} x: & V_{1x} = W_{1x} = 130 \ m/s \\ t: & V_{1t} = W_{1t} + u_m \end{cases}$$

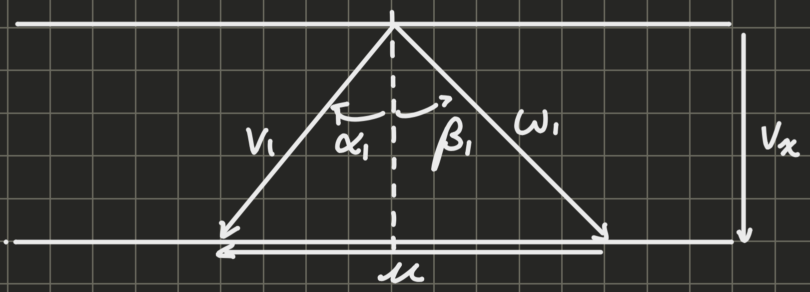
$$u_m = n \frac{D_m}{2} \cdot \frac{2\pi}{60} = 203 \ \frac{m}{s}$$

↳ Since at D_m

$$V_{1t} = V_x \tan\alpha_1 = 75.1 \ m/s$$

$$W_{1t} = V_{1t} - u_m = -125.2 \ m/s$$

$$\beta_1 = \tan^{-1} \frac{W_{1t}}{W_{1x}} = -43.9^\circ$$



$$\ell = u(V_{2t} - V_{1t}) \rightarrow \text{Compressor} \Rightarrow V_{2t} > V_{1t}$$

2) $\begin{cases} x: & V_{2x} = W_{2x} = 130 \ m/s \end{cases}$

$\llcorner t:$

$\hookrightarrow$ We are not given information for the angle.

We can use the definition of the reaction degree:

$$\chi = \frac{\Delta h_R}{\Delta h_{TT} = l}$$ $\leftarrow$ static Δh, not total

$$= \frac{h_2 - h_1}{h_{T3} - h_{T1}} = \frac{h_2 - h_1}{h_3 - h_1 + \cancel{\frac{V_3^2 - V_1^2}{2}}} = \frac{h_2 - h_1}{h_3 - h_2 + h_2 - h_1} = \frac{\Delta h_R}{\Delta h_R + \Delta h_S}$$

$\hookrightarrow$ since stator does exchange work $\quad \frac{V_3^2 - V_1^2}{2} \to 0$

$= 0.5$

$\Rightarrow \Delta h_S = \Delta h_R$

Since l in the stator is 0, h_T is constant.

$R \to \Delta h_T =$

$S \to \Delta h_T = 0 \to h_{T2} = h_{T3}$

$R \to q = \Delta I \Rightarrow \Delta I = 0$ is adiabatic $\to I_2 = I_1$

$\hookrightarrow$ rothalpy $I = h + \frac{\omega^2}{2} - \frac{u^2}{2}$

If $I_2 = I_1 \to \Delta I = \Delta h + \frac{\Delta w^2}{2}$ $\to$ since u = const

$$h_1 + \frac{\omega_1^2}{2} = h_2 + \frac{\omega_2^2}{2}$$

$$\Delta h_R = h_2 - h_1 = \frac{\omega_1^2 - \omega_2^2}{2}$$

$$\chi = \frac{\Delta h_R}{\Delta h_R + \Delta h_S = l} = \frac{\Delta h_R}{l}$$ $\to$ More useful to find triangles than $\Delta h_R + \Delta h_S$

$$\Delta h_R = \frac{w_1^2 - w_2^2}{2}$$

$$l = \frac{\Delta V^2}{2} - \frac{\Delta w^2}{2} = \frac{V_2^2 - V_1^2}{2} + \frac{w_2^2 - w_1^2}{2}$$

$$\chi = \frac{\omega_1^2 - \omega_2^2}{\omega_1^2 - \omega_2^2 + V_2^2 - V_1^2} = 0{,}5$$

$$\rightarrow \omega_1^2 - \omega_2^2 = V_2^2 - V_1^2$$

Since v_m = const, we can write:

$$\omega_{1t}^2 - \omega_{2t}^2 = V_{2t}^2 - V_{1t}^2$$

$$\downarrow$$

$$\tan^2\beta_1 - \tan^2\beta_2 = \tan^2\alpha_2 - \tan^2\alpha_1$$

$|\beta_1| = |\alpha_2|$ ✓

$|\beta_2| = |\alpha_1| \rightarrow \beta_2 = \alpha_1 \Rightarrow |\beta_1| \neq |\alpha_2|$

$ \rightarrow \beta_2 = -\alpha_1$

$$\beta_2 = -\alpha_1 = -30^\circ$$

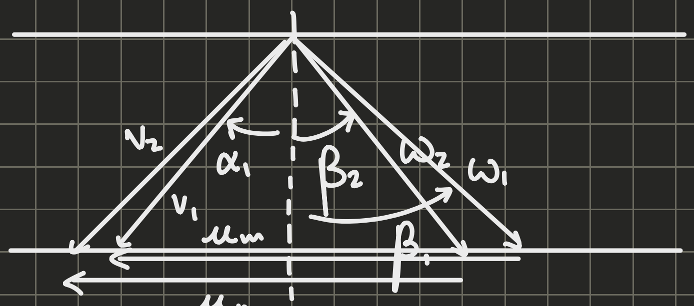
$$\omega_{2x} = 130\ m/s \rightarrow \omega_t = \omega_{2x} \tan\beta_2 = -75.1\ m/s$$

$$V_{2t} = u_m + \omega_{2t} = 125.2\ m/s$$

$$\alpha_2 = \tan^{-1}\left(\frac{V_{2t}}{V_{2x}}\right) = 43.9^\circ \rightarrow \alpha_2 = -\beta_1$$

In repeated stage $\alpha_2 = -\beta_1$ and $\beta_2 = -\alpha_1$

We can draw the velocity triangles:



Since we need $V_3 = V_1$, $\Delta\alpha_S = -\Delta\alpha_R$

↳ Stator ↳ Rotor

c) β_{ST}, β_R, β_S , with the machine being ideal.

↳ Stage

↳ isoentropic relation usable.

↳ We don't use β_T, since the machine is ideal, so there are no losses so calculation β_s is redundant or it will be 1. So normal β is more interesting.

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} \rightarrow \text{this is only valid since the machine is ideal.}$$

isoentropic transformation

$$\frac{P_3}{P_2} = \left(\frac{T_3}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$T_1 = 281.8\,K$ Definition of h_T

$$T_{T2} = T_{T1} + \ell/c_p = 303\,K \longrightarrow T_2 = T_{T2} - \frac{V_2^2}{2c_p} = 286.8\,K$$

V_2 = known $= \sqrt{V_{2t}^2 + V_{2x}^2}$

$$\ell = u\,(V_{2t} - V_{1t}) = 10.04 \quad kJ/kg$$

$$\beta_R = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 1.063$$

$$T_{T3} = T_{T2} = 303\,K \rightarrow T_3 = T_{T3} - \frac{V_1^2}{2c_p} = 291.8\,K$$

$V_3 = V_1 = 150.1\ \frac{m}{s}$

$$\beta_S = \left(\frac{T_3}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = 1.062$$

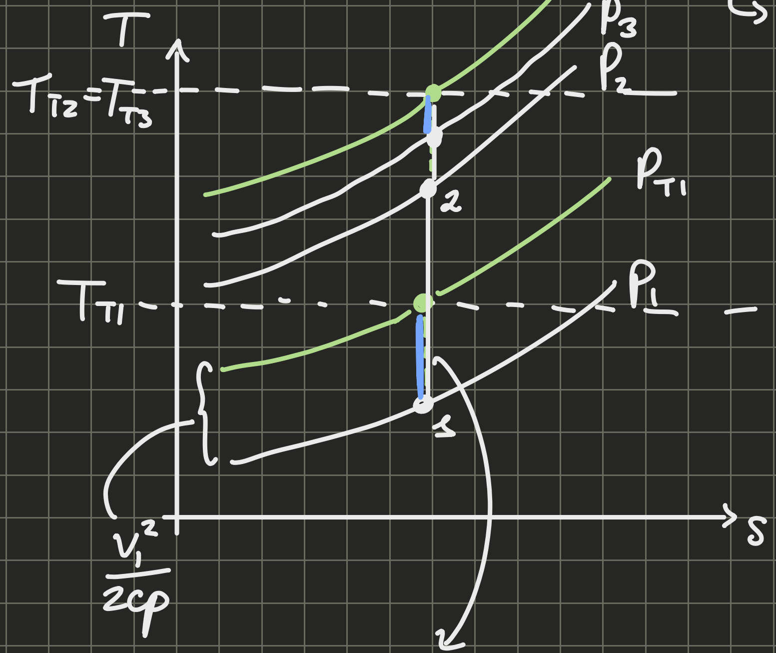
$$\beta_{ST} = \beta_R \cdot \beta_S = 1.13$$

Since $\chi = 0.5 \Rightarrow \Delta h_R = \Delta h_S \rightarrow$ It makes sense why β_R and β_S are similar.

$$c_p(T_2 - T_1) \qquad c_p(T_3 - T_2)$$

$$c_p T_1 \left(\beta_R^{\frac{\gamma-1}{\gamma}} - 1\right) = c_p T_2 \left(\beta_S^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$P_{T2} = P_{T3}$ ↳ since ideal



While $P_{T2} = P_{T3}$ and $T_{T2} = T_{T3}$ because $l=0$ and the machine is ideal, the V changes so $P_2 \neq P_3$.

| = same length because $V_1 = V_3$

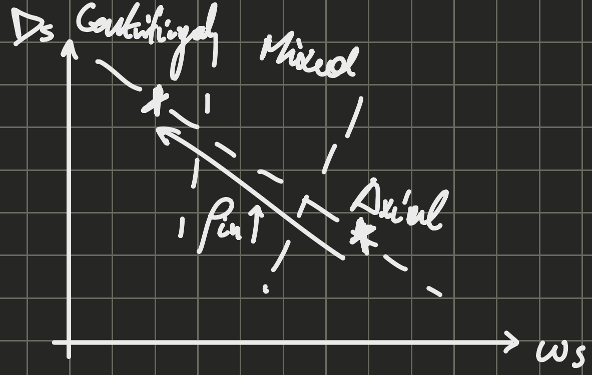
Isoentropic, since machine is ideal

d) Find ω_s

$$\omega_s = \omega \frac{\sqrt{Q_{in}}}{(\Delta h_{is})^{3/4}} = \omega \frac{\sqrt{\dot m}}{\sqrt{\rho_{in}}\,(\Delta h_{is})^{3/4}} = 3.55$$

↳ l

$$D_s = D \frac{(\Delta h_{is})^{1/4}}{\sqrt{Q_{in}}} = \frac{D_m \sqrt{\rho_{in}}\,(\Delta h_{is})^{1/4}}{\sqrt{\dot m}} = 1.12$$



$\rho_{in}\uparrow$ $D_s\uparrow$ $\omega_s\downarrow$

The change in ρ means we have to change our machine along the flow as we move within the Balje diagram.

Exercise 2

$n = 5000$ rpm

$c_{p,air} = 1004{,}5$ J/kgK

$\gamma_{air} = 1{,}4$

$P_{T1} = 8$ bar

$T_{T1} = 600$ K

$V_1 = 150$ m/s $= V_x$

$\alpha_1 = 0$

$b_1 = 35$ mm

$D_m = 0{,}85$ m

$$\Delta\beta_{rot} = |\beta_2 - \beta_1| = 35°$$

$\eta_{TT} = 0.85$

1) Velocity Triangles

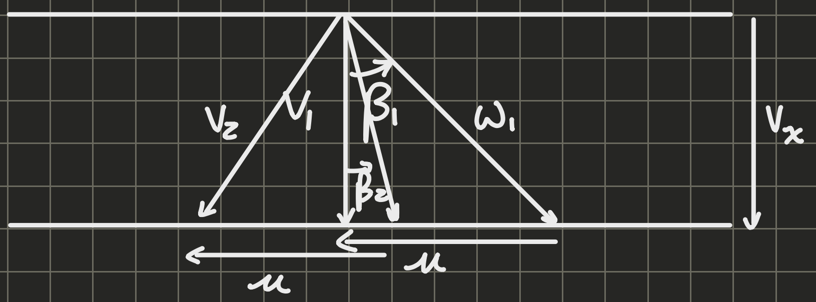
$$\begin{cases} x: \; V_{1x} = w_{1x} = 150 \text{ m/s} \\ t: \; u_m = n \cdot \frac{2\pi}{60} \cdot \frac{D_m}{2} = 222{,}53 \text{ m/s} \end{cases}$$

$\alpha_1 = 0 \rightarrow V_{1t} = 0$

$w_{1t} = V_{1t} - u_m = -222.53 \ m/s$

$\beta_1 = \tan^{-1}\left(\frac{w_{1t}}{w_{1x}}\right) = -56.2°$

$w_1 = \sqrt{w_{1x}^2 + w_{1t}^2} = 268.36 \ \frac{m}{s}$



$\beta_2 = -21.02$

$$\begin{cases} x: & V_{2x} = w_{2x} = w_{1x} = 150 \ m/s \\ t: & w_{2t} = w_{2x} \tan\beta_2 = -57,63 \ m/s \end{cases}$$

$w_1 = \sqrt{w_{1x}^2 + w_{1t}^2} = 164.9 \ \frac{m}{s}$

$\alpha_2 = \tan^{-1}\left(\frac{V_{2t}}{V_{2x}}\right) = 47.7° \rightarrow V_2 = 222.91 \ m/s \rightarrow \beta_2 = -21.02$

2) $\beta_{TT,ST} = \frac{P_{T3}}{P_{T1}}$

$\downarrow$

Stage

$\cancel{\frac{P_{T3}}{P_{T1}} = \left(\frac{T_{T3}}{T_{T1}}\right)^{\gamma/\gamma-1}}$ $\rightarrow$ Not usable since the machine is not ideal, we cannot use isoentropic transformation to go between points when the machine is not ideal.

$$l = u\left(V_{2t} - V_{1t}\right) = 36{,}69 \ \frac{kJ}{kg}$$

$$\eta_{TT} = \frac{h_{T3,is} - h_{T1}}{\Delta h_{TT} = l} = \frac{h_{T2,is} - h_{T1}}{l} = \frac{c_p\left(T_{T2,is} - T_{T1}\right)}{l}$$

Total to total efficiency.

$h_{T3,is} = h_{T2,is}$ if $l = 0$, like in the stator.

η_{TS} were we $h_{T3,is}$ not h_{T3}

Total to static

$$\longrightarrow T_{2,is} = T_{T1} + \frac{\eta_{TT}\, l}{c_p} = 631{,}05\,K$$

$$T_{2,is} = T_{T3,is}$$

$$\beta_{TT} = \left(\frac{T_{T3,is}}{T_{T1}}\right)^{\frac{\gamma}{\gamma-1}} = 1.19$$

c) $\dot{L} = ?$, $\eta_{TS} = ?$

$$\dot{L} = \dot{m}\, l$$

$$T_1 = T_{T1} - \frac{V_1^2}{2c_p} = 588{,}8\,K$$

$$\frac{T_{T1}}{T_1} = \left(\frac{P_{T1}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow P_1 = P_{T1}\left(\frac{T_1}{T_{T1}}\right)^{\gamma/\gamma-1} = 7{,}49\text{ bar}$$

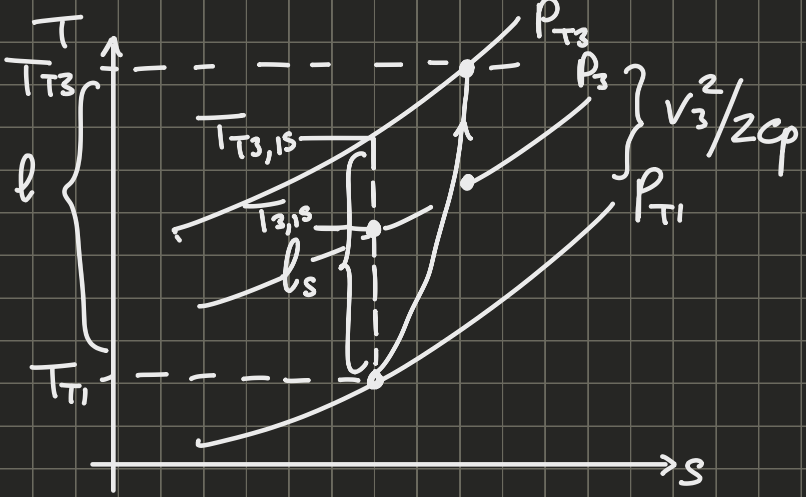
$$\rho_1 = \frac{P_1}{R T_1} = 4.43 \ kg/m^3$$

$$\dot{m} = \rho_1 \left(\pi D_m b_1\right) V_{1a} = 62{,}13 \ kg/s$$

$$\dot{L} = 2.28 \text{ MW}$$

Different from before since static, not total.

$$\eta_{TS} = \frac{h_{3,is} - h_{T1}}{\Delta h_{TT} = l}$$



Since $T_{T3,is} > T_{3,is}$

$\eta_{TT} > \eta_{TS}$

$$\frac{p_3}{p_{T1}} = \left(\frac{T_{3,is}}{T_{T1}}\right)^{\gamma/\gamma-1}$$

$T_{T3} = T_{T2}$

$$T_{T3} = T_3 + \frac{V_3^2}{2c_p}$$

$\longrightarrow T_3 = 625{,}33\ K$

$p_{T3} = p_{T1} \times \beta_{TT,ST} = 9{,}54$ bar

$$\frac{p_{T3}}{p_3} = \left(\frac{T_{T3}}{T_3}\right)^{\gamma/\gamma-1}$$ → we have $T_{T2} = T_{T1} + \frac{\ell}{c_p}$

$p_3 = 8{,}97$ bar

$$h_{3,is} - h_{T1} = c_p\, T_{T1}\left(\underbrace{\frac{T_{3,is}}{T_{T1}}}_{\left(\frac{p_3}{p_{T1}}\right)^{\frac{\gamma-1}{\gamma}}} - 1\right) = 20.03\ \frac{kJ}{kg}$$

Since isoentropic we can connect to β

$\longrightarrow \eta_{TS} = 0.546$

$$\ell_{is} = c_p\, T_{T1}\left(\beta_{TT}^{\frac{\gamma-1}{\gamma}} - 1\right)$$ → only valid for isoentropic case, not real case.

$$T_{T3,1S} - T_{3,1S} = \frac{V_{3,1S}^2}{2c_p}$$

d) b_2, b_3 ? $\frac{P_{T2}}{P_{T3}} = 1.04$

$$P_{T2} = 1.04\, P_{T3} = 9.93 \text{ bar}$$

$$T_2 = T_{T2} - \frac{V_2^2}{2c_p} = 611.8 \text{ K}$$

$$\frac{P_{T2}}{P_2} = \left(\frac{T_{T2}}{T_2}\right)^{\gamma/\gamma-1} \longrightarrow P_2 = 8.64 \text{ bar} \rightarrow \rho_2 = \frac{P_2}{R T_2} = 4.92 \frac{kg}{m^3}$$

$$\dot{m} = \rho_2 \pi D_m b_2 v_x \longrightarrow b_2 = 31.5 \text{ mm}$$

$T_{T3} = T_{T2}$, T_3 and P_3 already calculated in last step

$$\rho_3 = \frac{P_3}{R T_3} \Rightarrow \rho_3 = 4{,}99 \frac{kg}{m^3}$$

$b_3 = 31$ mm $< b_2 \rightarrow$ consistent with compressors.