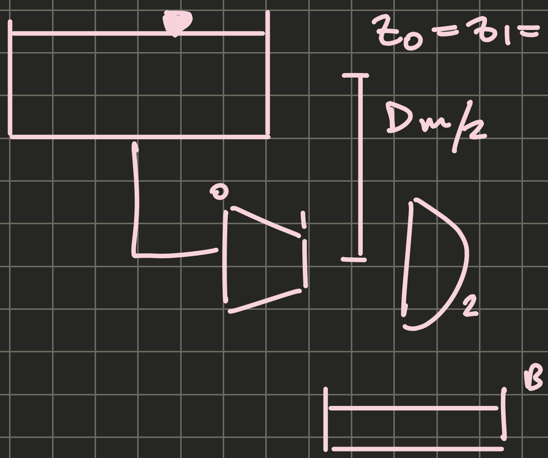
Esercitazione in Italiano

Esercizio 1



$z_0 = z_1 = z_2 = z_B$ $\qquad \Delta z = z_D - z_B$

$D = D_0 = 2\,m$

$L = 1750\,m$

$\lambda = 0{,}04 \qquad \xi = 20$

Ideal Nozzle : $\varphi = 1,\ \psi = 1$

$n = 600$ rpm

$D_m = 2\,m \quad i = 3 \quad d = D_1 = 0{,}22\,m$

$\beta_2 = -68°$

a) $L', H_m, \eta = ?$

Since it's axial

$$\downarrow$$

$$u = u_1 = u_2 = \frac{2\pi n}{60} \cdot \frac{D_m}{2} = 62{,}83\ m/s$$

BME $D \to 1$

Penstock $\qquad \rightarrow$ since $P_1 = P_D = P_{ATM}$, since ρ = const and given by CGV.

$$\cancel{\ell} - l_w - y_p = \cancel{\frac{P_1 - P_D}{\rho}} + \frac{V_1^2 - \cancel{V_D^2}}{2} + g(z_1 - z_D)$$

Hypothesis: $l_w = 0 \rightarrow$ Ideal Nozzle

$$\Rightarrow -y_p = \frac{V_1'^2}{2} + g(z_1 - z_D)$$

$$y_{CF} = y_{DIST} + y_{CONC} = \left(\frac{\lambda L}{D_0} + \xi\right)\frac{V_0^2}{2} \quad (*)_1$$

$\rightarrow$ No solvable since both V_0 and V_1' are unknown

Mass Balance $\quad Q = \frac{\pi D_0^2}{4} V_0 = i\,\frac{\pi D_1^2}{4} \cdot V_1 \underset{\varphi = \frac{V_1}{V_1'}}{=} i\,\frac{\pi D_1^2}{4}\,\varphi V_1'$

$$(*)_1 = \left(\frac{\lambda L}{D} + \xi\right) \frac{8}{\pi^2 D_0^4} \frac{\iota^2 \pi D_1^4}{16} \varphi^2 v_1'$$

Given this Q we can write

$$y_p = \left(\frac{\lambda L}{D} + \xi\right) \iota^2 \varphi^2 \left(\frac{D_1}{D_0}\right)^4 \frac{v_1'^2}{2}$$

$$\frac{v_1'}{2} + \left(\frac{\lambda L}{D} + \xi\right) \iota^2 \varphi^2 \left(\frac{D_1}{D_0}\right)^4 \frac{v_1'^2}{2} = g\underbrace{(z_0 - z_1)}_{\Delta z}$$

$$v_1' = \sqrt{\frac{2g(z_0 - z_1)}{\underbrace{1 + \left(\frac{\lambda L}{D} + \xi\right)\iota^2\varphi^2\left(\frac{D_1}{D_0}\right)^4}_{a}}} = 135,26 \text{ m/s}$$

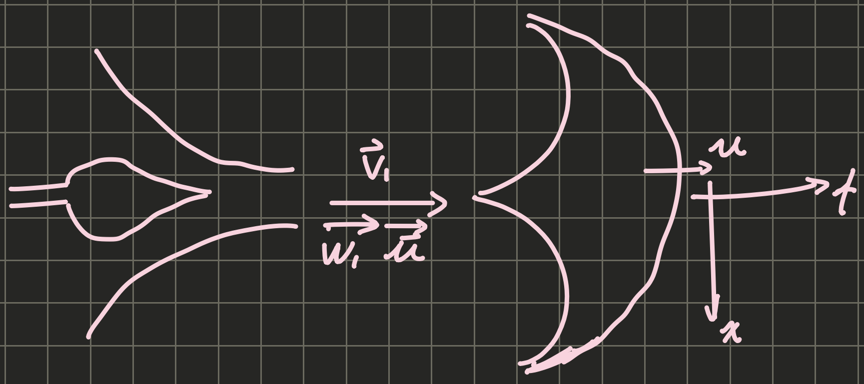
Velocità Torricelliana
↳ Velocità senza perdite

$a > 1 \Rightarrow$ Velocità Torricelliana is the maximum possible speed, we can use that value as a check.

At this point there are many ways with which they can indirectly give us φ.

$v_1 = \varphi v_1' = 135,26 \rightarrow$ In this case since $\varphi = 1$, they are the same

View on nozzle



$w_1 = v_1 - u = 72,43$ kg/s

$v_1 = v_{1t}$, $w_1 = w_{1t}$

$w_2 = \psi w_1 = 72,43$ m/s

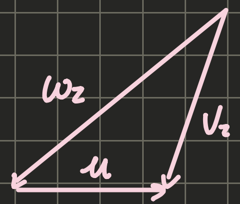
Since the machine is ideal, $\psi = 1$

$w_{2x} = v_{2x} = w_2 \cos\beta_2 = 27,13$ m/s

$w_{2t} = w_2 \sin\beta_2 = -67,16$ m/s

$V_{2,t} = w_{2,t} + u = -4{,}33 \ m/s$

$\alpha_2 = \tan^{-1}\left(\frac{V_{2t}}{V_{2x}}\right) = -9°$



$\dot{L} = \rho Q \ell = -135 \ MW$

$\ell = u\,(V_{2,t} - V_{1,t}) = -8770 \ \frac{J}{kg} < 0$ ✓ Correct since turbines are motor machines.

$Q = i\,\frac{\pi D_1^2}{4} V_1 = 15{,}43 \ m/s$

$\eta = \frac{|\ell|}{\underset{\hookrightarrow ?}{gH_m}} = 0{,}959 = 1 - \frac{\ell w}{gH_m} - \frac{V_2^2}{2gH_m}$ Kinetic Energy we weren't able to convert into work.

$gH_m = T_0 - T_B = \big(\underbrace{T_0 - T_D}_{-y_p}\big) + \big(\underbrace{T_D - T_B}_{g\Delta z}\big)$

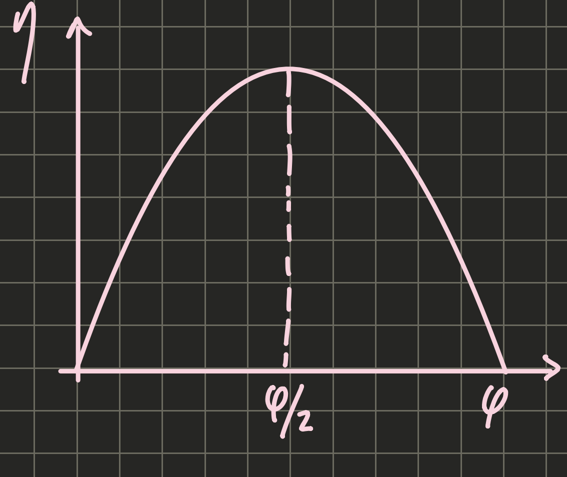
$T_i = \frac{P_i}{\rho} + \frac{V_i^2}{2} + g z_i$

$= g\Delta z - y_p = 9148 \ J/kg$

We can also say: $gH_m = \frac{V_1'^2}{2}$

We can go a check on H_m, since $H_m \leq \Delta z$, always.

Optimized?



$k_p = \frac{u}{V_1'} = 0{,}465 \neq \frac{\varphi}{2} = 0{,}5$

$\Rightarrow$ Machine is not optimized.

$k_p = \frac{u}{V_1'}$

b) $\frac{S_{1b}}{S_{1a}} = 0,4$ $\dot{L} = ?$ $\eta = ?$ $H_m = ?$

$\varepsilon = 0,4$ Things are the just with added ε.

BБM $D \to 1$

$$\frac{v_1'^2}{2} + y_p = g\Delta z \Rightarrow v_1' = \sqrt{\frac{2g\Delta z}{1 + \left(\frac{\lambda L}{D} + \xi\right) i^2 \varphi^2 \varepsilon^2 \left(\frac{D_1}{D_0}\right)^4}} = 139{,}27\ \frac{m}{s}$$

$$y_p = \left(\frac{\lambda L}{D} + \xi\right)\frac{V^2}{2} = \left(\frac{\lambda L}{D} + \xi\right) i^2 \varphi^2 \varepsilon^2 \left(\frac{D_1}{D_0}\right)^4 \frac{v_1'^2}{2}$$

Increases but not by too much, this is because $\left(\frac{D_1}{D_0}\right) \approx .1$, so to the power of 4 it becomes not too significant, so changing D_1 doesn't change v_1' much.

$v_1' = 139{,}27\ m/s$

$\omega_1 = v_1 - u = 76{,}44\ m/s$

$\omega_2 = \Psi \omega_1 = 76{,}44\ m/s$

$\omega_{2x} = V_{2x} = \omega_2 \cos(\beta_2) = 28{,}62\ m/s$

$\omega_{2,t} = \omega_2 \sin(\beta_2) = -70{,}87\ m/s$

$V_{2t} = \omega_{2t} + u = -8{,}04\ m/s$

$\alpha_2 = \tan^{-1}\left(\frac{V_{2t}}{V_{2x}}\right) = -15{,}7°$

$l = u(V_{2,t} - V_{1t}) = 9243{,}2$

$Q = i\varepsilon \frac{\pi D_i^2}{4} V_1 = 6{,}35\ m^3/s$

$\dot{L} = \rho Q l = -58{,}7\ MW$

$H_m = \frac{V_1^2}{2g} = 989{,}9\ m$ $\eta = \frac{|l|}{g H_m} = 0{,}953$

Practically constant with a change in D_1.

$\eta = \underbrace{0{,}956}_{\text{one before}} \Rightarrow$ it's impossible for it to be optimized.

c) n_c : $\eta_c = \eta_a$ in the condition of b

$h_{p,a} = h_{p,c}$ → once the machine is fixed, the h_p is the thing η depends on.

$$\frac{V_{1a}'}{n_a} = \frac{V_{1c}'}{n_c} \Rightarrow n_c = \frac{V_{1c}'}{V_{1,a}'} n_a \Rightarrow \frac{\cancel{2\pi} n_c}{\cancel{60}} \cdot \frac{\cancel{D_m}}{\cancel{2}} = \frac{V_{1c}'}{V_{1a}'} \cdot \frac{\cancel{2\pi} n_a}{\cancel{60}} \cdot \frac{\cancel{D_m}}{\cancel{2}}$$

$$n_c = \frac{V_{1c}'}{V_{1a}'} n_a = 616 \text{ rpm}$$