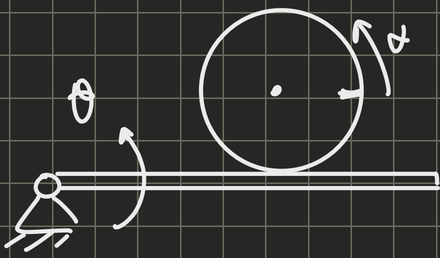
Esercitazione 8 - Difference between absolute and relative rotations



θ is always absolute, while α can be either.

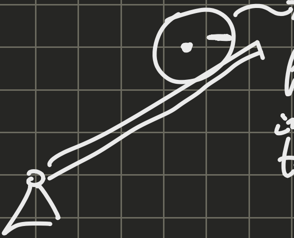
Λ_m	θ	α_{ABS}
V_{1x}	0	0
V_{1y}	L	0
V_{2x}	0	$-R$
V_{2y}	L	0
ω_1	1	0
ω_2	0	1

Λ_m	θ	α_{REL}
V_{1x}	0	0
V_{1y}	L	0
V_{2x}	$-R$	$-R$
V_{2y}	L	0

	θ	α_{REL}
ω_1	1	0
ω_2	1	1

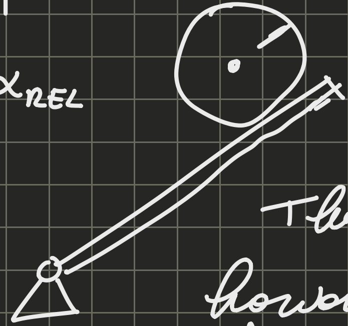
Because it stays parallel.

For α_{ABS},



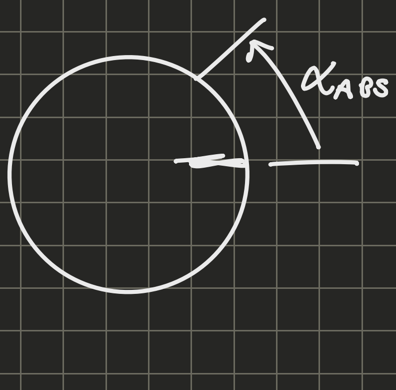
Always horizontal in absolute terms

With α_{REL}



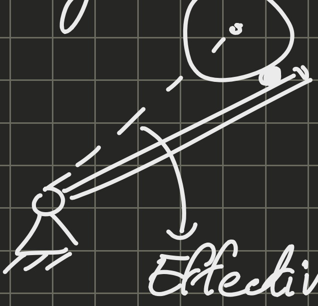
The line is horizontal to the beam.

With absolute, the absolute rotation of the body is always null, so α_{ABS} cancels our θ.



With α_{REL}, it's like the bodies are welded together.

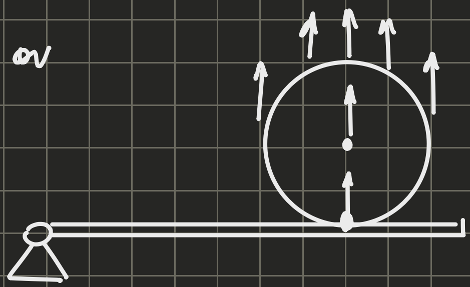
Since it's welded, all the points turn around the hinge:



Effective distance

Can be solve with Rivals

or



The velocity of the disk is the same velocity of the point of contact, since the α compensates for any horizontal motion.

$$(G_i - O) = (G_i - P) + (P - O)$$

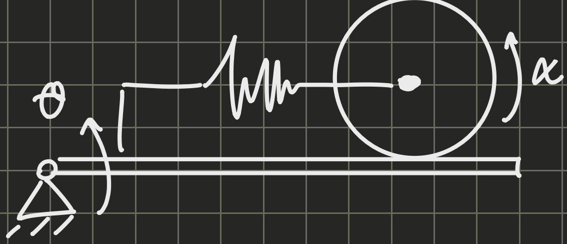
$$\cancel{\alpha_{ABS}\, \hat{k} \times (G_i - P)} + \dot{\theta}\, \hat{k} \times (P - O)$$

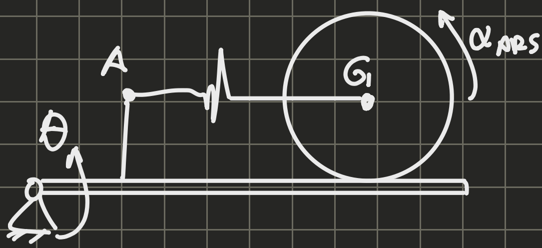
The only contribution of α_{ABS} is null, and only the effect on P by $\dot{\theta}$ stays.

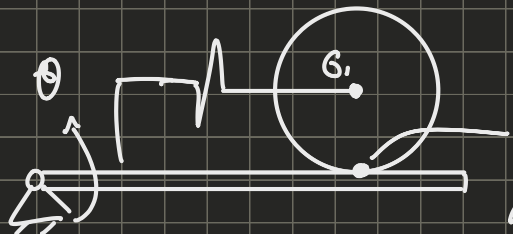
$\alpha_{ABS} = \alpha_{REL} + \dot{\theta}$ → we can change from one matrix to another.

$$\vec{\omega}_2 = \alpha_{ABS}\, \hat{k} = (\alpha_{REL} + \dot{\theta})\, \hat{k}$$

Can also be done for velocities:







	θ	α_{ABS}
Δl_1	R	$-R$

	θ	α_{REL}
Δl_1	0	$-R$

we rotate the disk in the opposite way to α_{REL} to keep $\alpha_{ABS} = 0$

Since they are welded as the beam rotates, the length of the spring does not change

$$\Delta l_1 = \Delta l_0 + \Delta l_d$$

$$\Delta l_d = l_m(\theta) - l_m(\theta_0)$$

$$l_m(\theta) = G_1 - A$$

$$\alpha_{ABS} = \theta + \alpha_{REL}$$

$$0 = \theta - R$$

$$\theta = R$$

For α_{REL}:

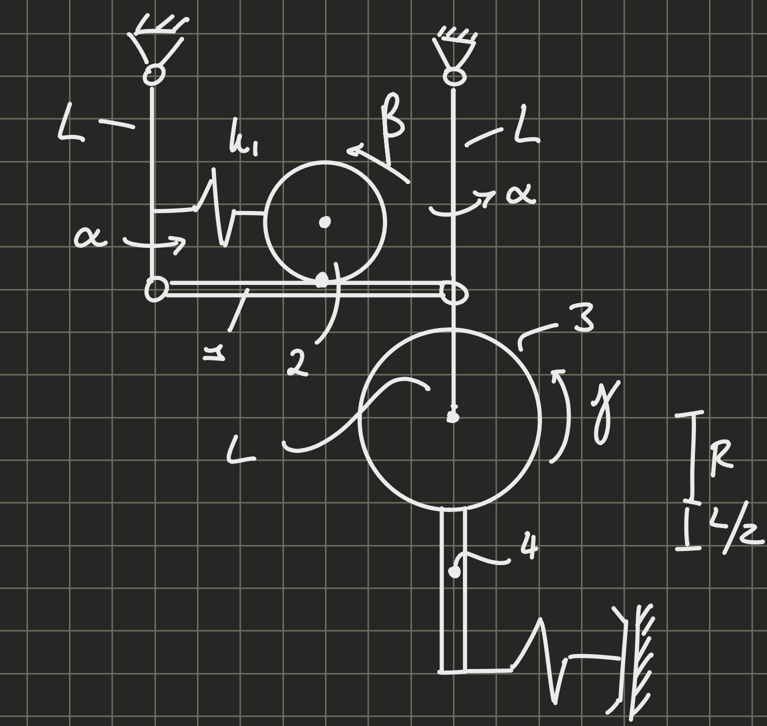
$$\Delta l_1 = -R \cdot \alpha_{REL}$$

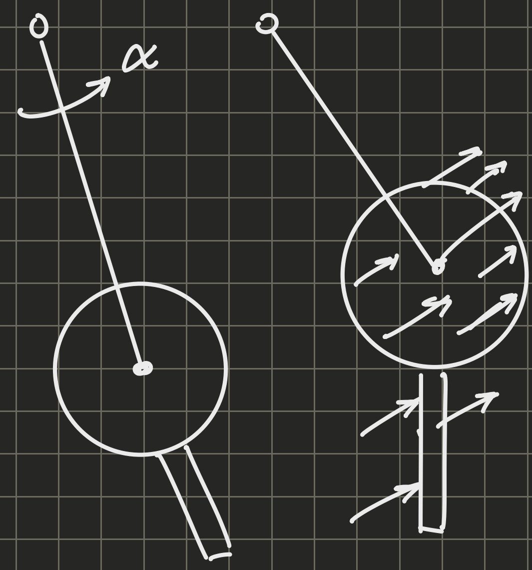
Same, which is good.

For α_{ABS}:

$$\Delta l_1 = R\theta - R\alpha_{ABS} = R\theta - R(\theta + \alpha_{REL}) = -R\alpha_{REL}$$

TdE with absolute and relative rotations





$[\Lambda_m]$	α	β_{REL}	γ_{REL}
V_{1x}	L	0	0
V_{1y}	0	0	0
ω_1	0	0	0
V_{2x}	L	$-R$	0
V_{2y}	0	0	0
ω_2	0	1	0
V_{3x}	0	1	0
V_{3y}	$2L$	0	0
ω_3	1	0	1
V_{4x}	$\left(2L+\frac{L}{2}+R\right)$	0	$R+L/2$
V_{4y}	0	0	0
ω_4	1	0	1

$[\Lambda_m]$	α	β_{ABS}	γ_{ABS}
V_{1x}	L	0	0
V_{1y}	0	0	0
ω_1	0	0	0
V_{2x}	L	$-R$	0
V_{2y}	0	0	0
ω_2	0	1	0
V_{3x}	0	1	0
V_{3y}	$2L$	0	0
ω_3	1	0	1
V_{4x}	$2L$	0	$R+L/2$
V_{4y}	0	0	0
ω_4	0	0	1

Velocity is the same as center of disk

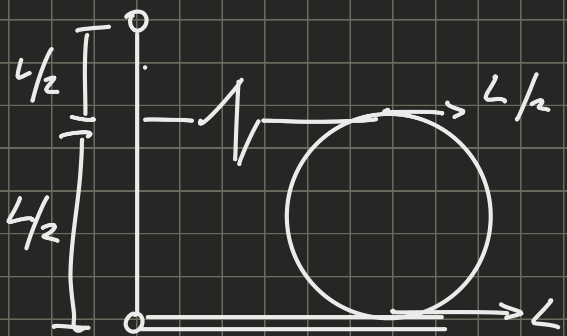
$$\beta_{ABS} = \beta_{REL} + \cancel{\varphi_1}$$

$[\Lambda_u]$	α	β_{REL}	γ_{REL}
Δl_1	$L - \frac{L}{2}$	$-L/2$	0
Δl_2	$-(3L+R)$	0	$-(R+L)$

$[\Lambda_u]$	α	β_{ABS}	γ_{ABS}
Δl_1	$L - L/2$	$-L/2$	0
Δl_2	$-2L$	0	$-(R+L)$

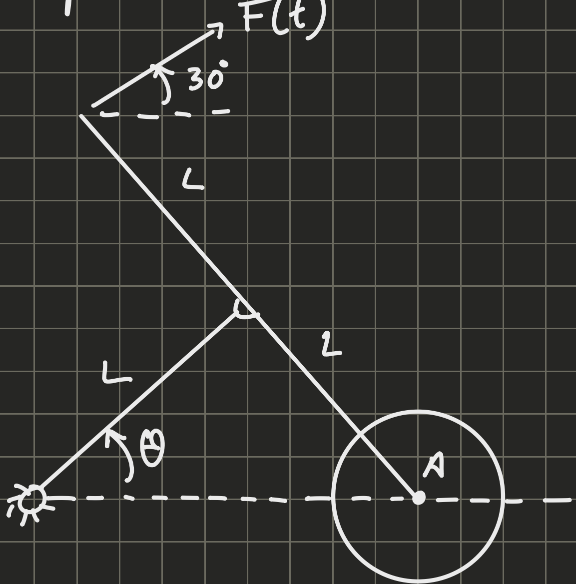
L not $L/2$ since the spring is not in 4.

$\beta_{REL} = \beta_{ABS}$ because $\omega_1 = 0$



Examples of particular mechanisms

Example 1



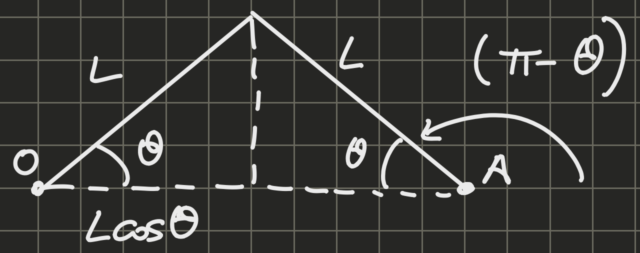
EOM NL

$\delta^* \mathcal{L} = ?$

$$\delta^* \mathcal{L} = \underbrace{\vec{F}(t) \cdot \delta \vec{S}_c}_{\text{Interesting part}}$$

$F_0 = F_0 + F_1 \cdot \cos(\Omega t)$

To find δs_c we need to solve kinematics, using isosclese triangle



$$(B-O) = L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}$$

$$(A-O) = 2\,(B-O)_x = 2L\cos\theta\,\hat{i}$$

$$(C-O) = (A-O) + (C-A)$$

$$= 2L\cos\theta\,\hat{i} + 2L\cos(\pi-\theta)\,\hat{i} + 2L\sin(\pi-\theta)\,\hat{j}$$

$$\rightarrow (C-O) = \underline{2L\sin\theta\,\hat{j}}$$

$$\delta \vec{s}_c = \frac{\delta s_c}{\delta\theta}\cdot\delta\theta = (2L\cos\theta\cdot\delta\theta)\,\hat{j}$$

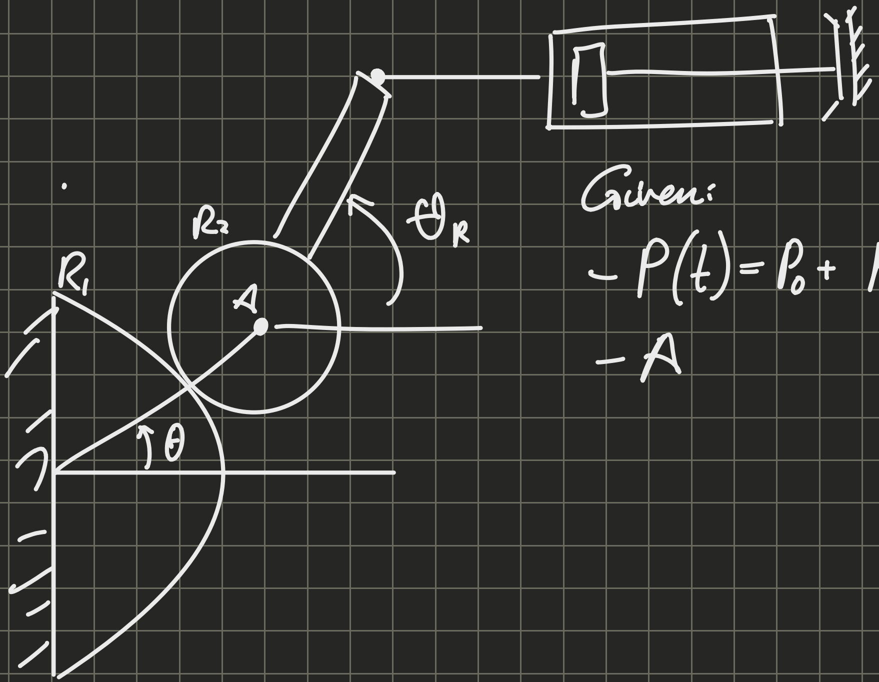
changing to a vector.

$$\delta^* \mathcal{L} = \left| \left[F_0 + F_1\cos(\Omega t)\right]\left(\cos(30)\,\hat{i} + \sin(30)\,\hat{j}\right) \right| \cdot$$

$$\cdot \left| \left(2L\cos\theta\,\hat{j}\right) \right| = \left(F_0 + F_1\cos(\Omega t)\right)\sin 30 \cdot 2L\cos\theta\;\delta\theta$$

We just consider the vertical work of the force.

Other example:



Given:
- $P(t) = P_0 + P_1 \cos(\Omega t)$
- A

Kinematics:

$$(A-O) = (R_1+R_2)\cos\theta\,\hat{i} + (R_1+R_2)\sin\theta\,\hat{j}$$

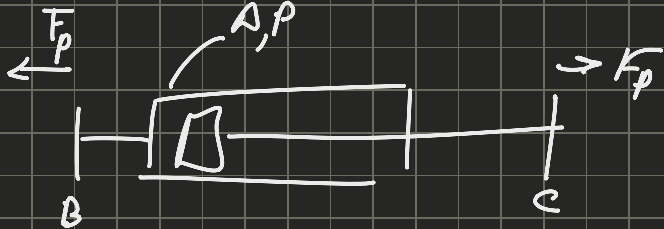
$$= R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$v_A = R\cdot\dot{\theta} = (R_1+R_2)\dot{\theta}$$

$$\omega_2 = \frac{(R_1+R_2)\dot{\theta}}{R_2}$$

$$(B-O) = (A-O) + (B-A) = (R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}) + (L\cos(\theta_2)\hat{j} + L\sin(\theta_2)\hat{i}$$

$$\theta_2 = \theta_{R0} + \frac{R_1+R_2}{R_2}\theta$$



$$\delta^* \mathcal{L} = (-PA\,\hat{i})\,\delta\vec{s}_B + \cancel{(PA\,\hat{i})\,\delta\vec{s}_C}$$ since $v_C = 0$

$\dot{v}_c = 0$

$$\delta \vec{x} = \underbrace{\frac{\delta x}{\delta q}}_{=0} \cdot \delta q$$

For the equilibrium condition we can be given different data:

- $P, A \xrightarrow{\text{calculate}} \theta_0$

$\theta_0, P \longrightarrow A$

$\theta_0, A \longrightarrow P$