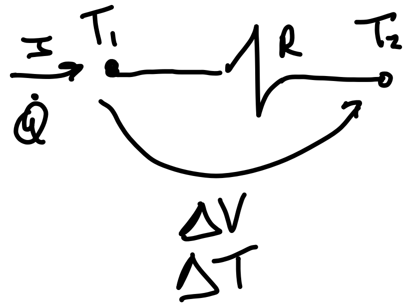
Esercitazione 9 - Conduzione

Analogia Elettrica



$$\Delta V = IR$$

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

$$R_{k\,piana} = \frac{S}{kA}$$

$$R_{k\,cilindro} = \frac{\ln \frac{r_2}{r_1}}{2\pi L k}$$

$$R_{k\,sfera} = \frac{r_e - r_i}{4\pi r_e r_i k}$$

$$R_{conv} = \frac{1}{h\,Area}$$

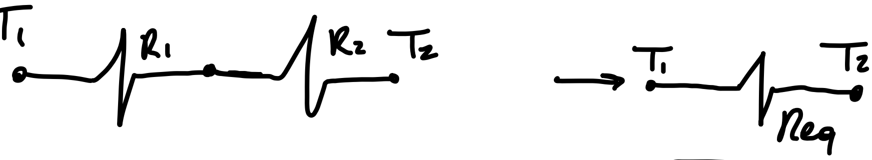
h coefficiente di scambio termico convettivo

Lavoreremo con il flusso $\dot{q} = \frac{\dot{Q}}{A}$

$$\Downarrow$$

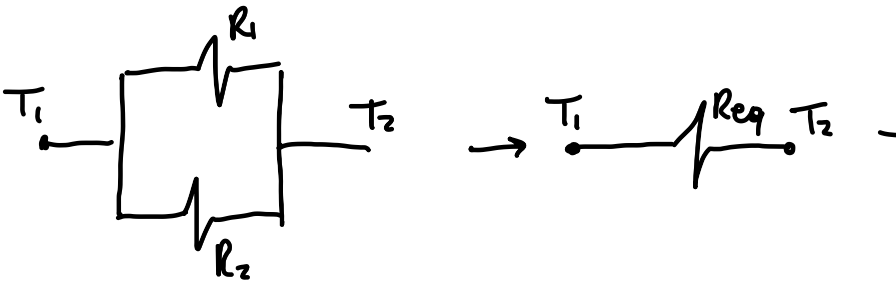
$$\dot{q}'' = \frac{\dot{q}}{A}$$

Analogia Serie → Due pareti



$$\boxed{R_{eq} = R_1 + R_2}$$

Analogia Parallelo → Due pareti



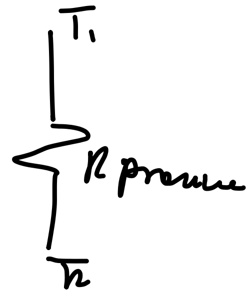
$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Resistenza di Contatto

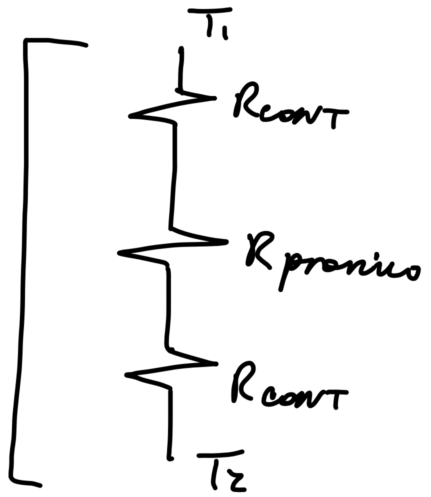
$$R_{CONT} = \frac{1}{h_{CONT} \cdot Area}$$

Problemi Area di Contatto:

$$R_{k\ piana} = \frac{S}{kA}$$



In realtà:



R_k piana
↓
misurata da strumento

Come ricavare R_{CONT}:

Equazione energia, stato stazionario, 1D, σ ↓ $\dot{q}'''$ geometria piana

$$\boxed{\frac{d^2T}{dx} = \frac{-\dot{q}'''}{k}}$$

$$\frac{dT}{dx} = \frac{-\sigma}{k}x + C_1 \rightarrow \boxed{q(x) = -k\frac{dT}{dx}}$$

$$\boxed{= \sigma x - kC_1}$$

$$\boxed{T(x) = \frac{-\sigma}{k}\frac{x^2}{2} + C_1 x + C_2}$$

↓

da condizione Lu contorno

senza σ $\rightarrow$ $\frac{d^2T}{dx^2} = 0 \rightarrow \frac{dT}{dx} = C_1$
generazione

$$\boxed{q(x) = -k\frac{dT}{dx} = -kC_1} \qquad \boxed{T(x) = C_1 x + C_2}$$

- Geometria Cilindrica

↳ con generazione

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\sigma}{k} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -r\frac{\sigma}{k} \qquad r\frac{dT}{dr} = -\sigma\frac{r^2}{2} + C_1$$

$$\frac{dT}{dr} = \frac{-\sigma}{k}\cdot\frac{r}{2}\cdot\frac{C_1}{r} \rightarrow \boxed{q(r) = -k\frac{dT}{dr} = \frac{\sigma r}{2} - \frac{kC_1}{r}}$$

$$\boxed{T(r) = \frac{-\sigma}{4k}r^2 + C_1\ln(r) + C_2}$$

Esercizio 1

$S = 4mm$

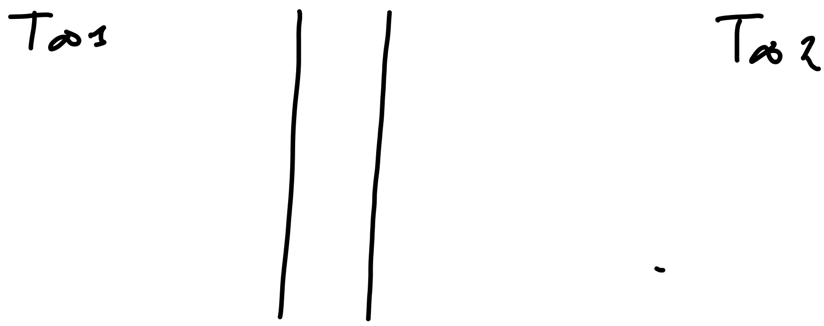
$T_{\infty 1} = 293{,}15K$ $\qquad k_A = 0{,}02 \frac{W}{mK}$

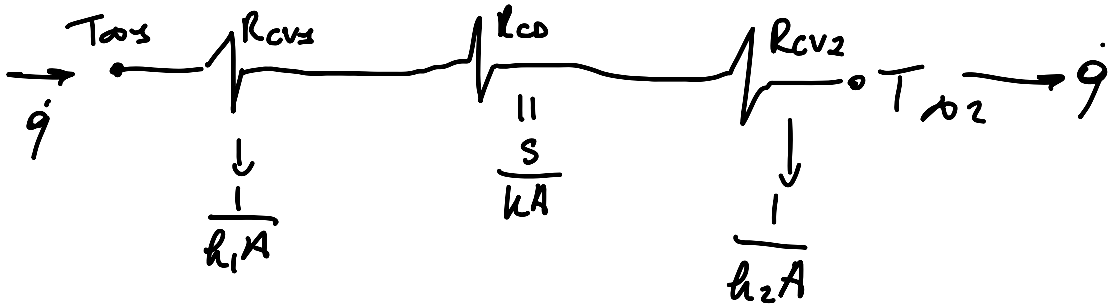
$T_{\infty 2} = 273{,}15K$ $\qquad h_1 = 7\frac{W}{m^2K}$

$k_v = 0{,}6\left[\frac{W}{mK}\right]$ $\qquad h_2 = \frac{20W}{m^2K}$ } → in moto quindi più grande

↳ Conduttività

$h = \left[\frac{W}{m^2K}\right]$



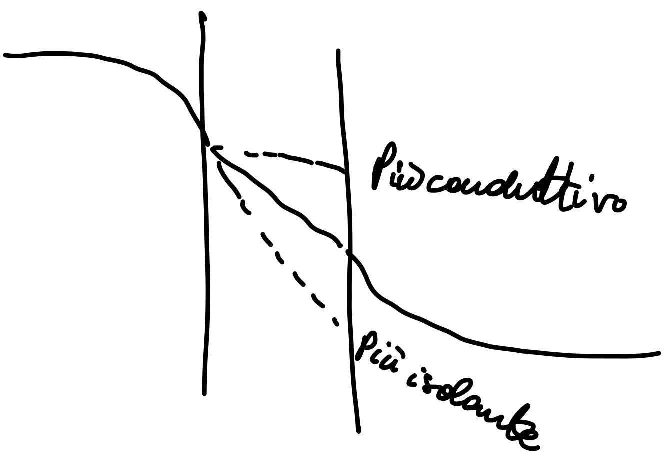


$$\dot q'' = \frac{\dot q}{A} = \frac{T_{\infty1}-T_{\infty2}}{A\left(\frac{1}{h_1A}+\frac{s}{kA}+\frac{1}{h_2A}\right)} = \frac{T_{\infty1}-T_{\infty2}}{\frac{1}{h_1}+\frac{s}{k}+\frac{1}{h_2}} = 100,24\ \frac{W}{m^2}$$

Flusso
(dipendente dal area)

$$\dot Q = \frac{T_{\infty1}-T_{\infty2}}{R_{CV1}+R_{CD}+R_{CV2}} = \frac{T_1-T_2}{R_{CD}} = \frac{T_{\infty1}-T_2}{R_{CV1}+R_{CD}} = \dots$$

Scegliere quello più equivalente



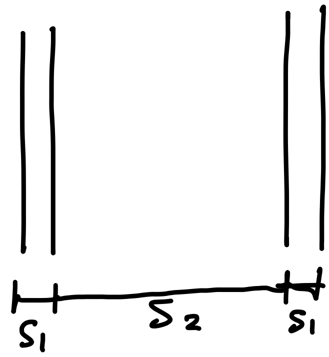
Esercizio 2 - Doppi vetri

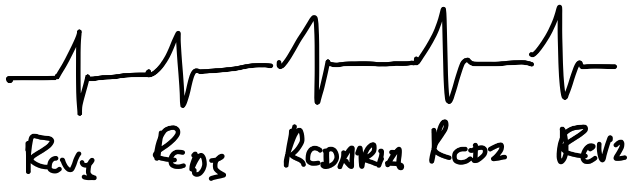
$s_1 = 0,004\,m$

$h_1 = \frac{7\,W}{m^2 K}$

$h_2 = \frac{20\,W}{m^2 K}$

$s_2 = 0,015\,m$

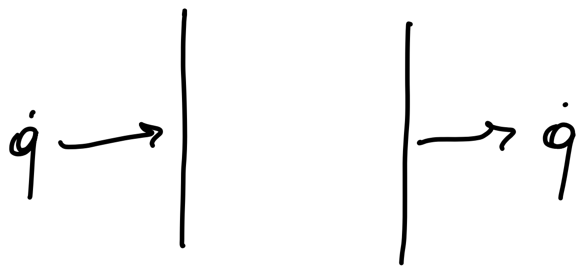




$$\dot{q}'' = \frac{\dot{q}}{A} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1} + \frac{s_1}{k} + \frac{s_2}{k} + \frac{s_1}{k_v} + \frac{1}{h_2}} = 20,92\,\frac{W}{m^2}$$

$\frac{s_2}{k_a} = \frac{0,015}{0,02}$ → sopra ai 0,02 inizia la convezione naturale
↳ peggio del doppio vetro, meglio del singolo

$\frac{s_1}{k_v} = \frac{0,004}{0,6}$ → si può aumentare sotto vuoto,
a quel punto c'è solo l'irraggiamento



sono uguali perché se no si riscalderebbe infinitamente

Aria sotto 15-20mm di spessore non ha convezione

Esercizio 1 - Non Teorica

$r_1 = 0{,}04\,m$ $\quad k_1 = 47\,\frac{W}{mK}$ $\quad r_2 = 0{,}0455$

$s_1 = 0{,}0055$ $\quad k_2 = 0{,}5\,\frac{W}{mK}$ $\quad r_3 = 0{,}1355\,m$

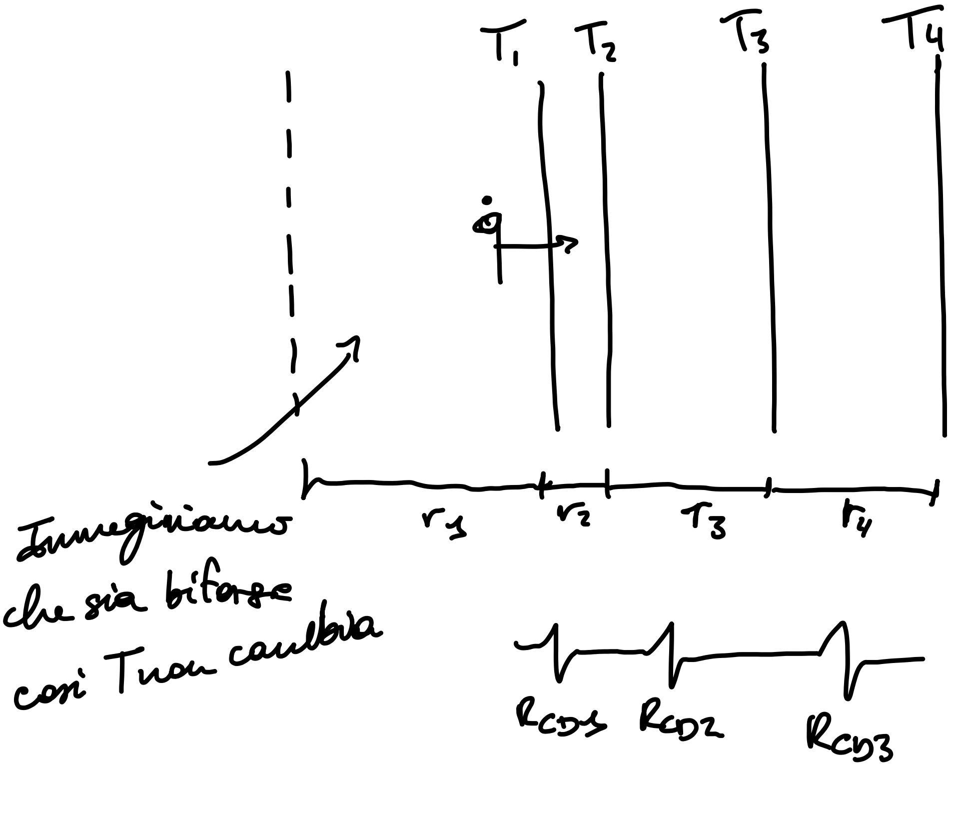
$s_2 = 0{,}09\,m$ $\quad k_3 = 0{,}25\,\frac{W}{mK}$ $\quad r_4 = 0{,}1755\,m$

$s_3 = 0{,}04$

Non abbiamo lunghezza quindi faremo per unità di lunghezza

$T_1 = 523{,}15\,K$

$T_4 = 293{,}15\,K \rightarrow$ superficie esterna dell'isolante 3



$\dot{q}'$

Abbiamo T_1 e T_4 quindi usiamo quelli:

$$\frac{\dot{q}}{L} = \frac{T_1 - T_4}{L\left(\frac{\ln\frac{r_2}{r_1}}{2\pi k_1 L} + \frac{\ln\frac{r_3}{r_2}}{2\pi k_2 L} + \frac{\ln\frac{r_4}{r_3}}{2\pi k_3 L}\right)} = 448,8 \ \frac{W}{m}$$

Per trovare ogni temperatura usiamo $\frac{\dot{q}}{L}$ uguale

$$\frac{\dot{q}}{L} = \frac{T_3 - T_4}{R_{CD3}} \rightarrow T_3 = 93,91\,°C$$

$$\frac{\dot{q}}{L} = \frac{T_2 - T_3}{R_{CD2}} \rightarrow T_2 = 249,8\,°C$$

Esercizio 2

$T_{\infty 1} = 77K$

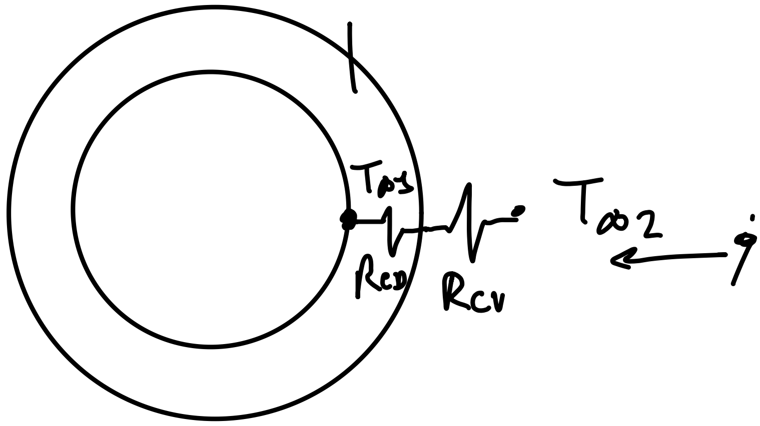
$D_2 = 0,5m$

$k = 0,0017 \ \frac{W}{mK}$

$T_{\infty 2} = 300K$

$h = 20 \frac{W}{m^2K}$

$s = 0,025 \ m$



$$R_{CD} = \frac{1}{4\pi k}\left(\frac{1}{r_{in}} - \frac{1}{r_{est}}\right)$$

$$R_{CV} = \frac{1}{hA}$$

$$\dot{q} = \frac{\Delta T}{R_{TOT}} = \frac{T_{\infty 2} - T_{\infty 1}}{\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{hA}} = 13,06 W$$

Esercizio 3

$T_{N_2} = 77{,}15\,K$

$D = 0{,}3\,m$

$H = 0{,}5\,m$

$s = 0{,}025\,m$

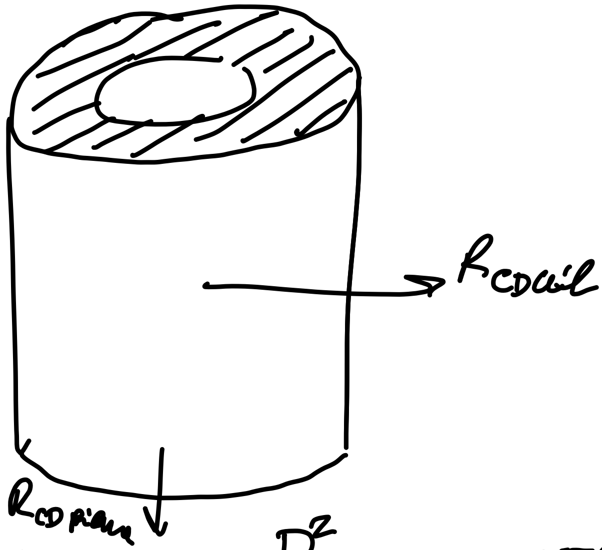
$k = 0{,}0035\,\frac{W}{mK}$

$h = 5\,\frac{W}{m^2K}$

$T_{AMB} = 298{,}15\,K$

$h_{ev\,N_2} = 200\,\frac{kJ}{kg}$

$\rho_{N_2} = 804\,\frac{kg}{m^3}$

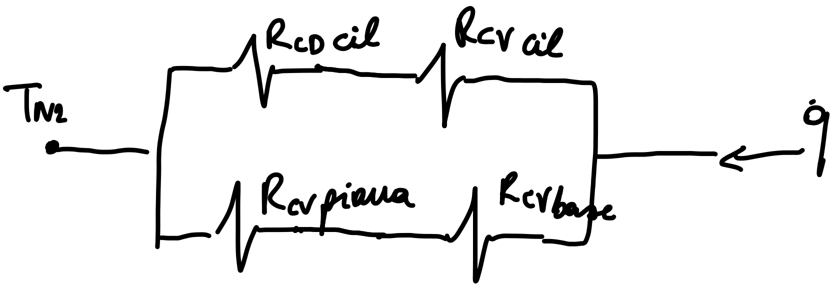


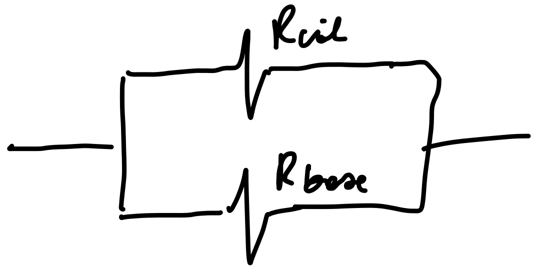
$V_{CONT} = \pi\,\frac{D^2}{4}\,H = 0{,}0353\,m^3$

$M_{N_2} = \rho \cdot V = 28{,}42\,kg$

$Q_{ev} = h_{ev\,N_2} \cdot M_{N_2} = 5683\,kJ$

$T = 1g = 86400\,s$





$$R_{eq} = \frac{1}{\frac{1}{R_{cil}} + \frac{1}{R_{base}}} = \frac{R_b \cdot R_e}{R_e + R_b} = 12{,}63\,\frac{K}{W}$$

$$R_{laterale} = R_{cd\,cil} + R_{cv\,cil} = \frac{\ln \frac{r_{est}}{r_{int}}}{2\pi H k} + \frac{1}{2\pi r_e^2 \cdot H \cdot h}$$

$$= 14{,}38\ \frac{K}{W}$$

$$R_{base} = R_{cd\,piano} + R_{cv\,base} = \frac{s}{A \cdot k} + \frac{1}{h \cdot A} \rightarrow$$

$$= 103{,}88\ \frac{K}{W}$$

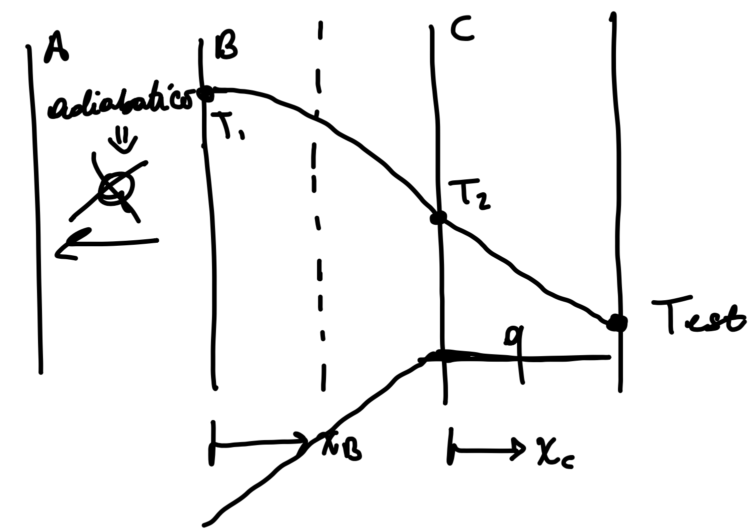
$$\dot{q}_{TOT} = \frac{T_{AMB} - T_{N2}}{R_{eq}} = 17{,}5\ W$$

$$Q_{TOT} = \dot{q}_{TOT} \cdot t = 17{,}5 \cdot 86400\,s = 1512\ kJ$$

$$M_{N2\,ev} = \frac{Q_{TOT}}{h_{ev\,N2}} = 7{,}56\ kg \rightarrow V_{N2\,ev} = \frac{M_{N2\,ev}}{\rho} = 0{,}0094\ m$$

$$V\% = \frac{V_{N2}}{V_{cont}} \cdot 100 = 26{,}86\ \%$$

Esercizio 7 esercitazione 9



$\sigma = 800\ \frac{kW}{m^3}$

$T_{est} = 293{,}15\ K$

$k_B = 2{,}5\ \frac{W}{mK}$ $s_B = 0{,}02\ m$

$k_C = 4\ \frac{W}{mK}$ $s = 0{,}025$

STRATO B

$$\frac{d^2T}{dx^2} = \frac{-\sigma}{k_B}$$

$$q_B = -k_B \frac{dT_B}{dx} = \sigma x - k_B C_1$$

$$T_B(x) = -\frac{\sigma}{k_B}\frac{x^2}{2} + C_1 x + C_2$$

$q_B(0) = 0$ A è adiabatico

$$\Rightarrow -k_B C_1 = 0 \rightarrow C_1 = 0$$

$$q_B(s_B) = \frac{T_2 - T_{est}}{R_{cond\,C}} = \frac{k_C}{s_C}(T_2 - T_{est}) =$$

$$q_B(s_B) = \sigma\, s_B \qquad T(s_B)$$

$$\sigma \cdot s_B = \frac{k_C}{s_C}(T_2 - T_{est})$$

$$\sigma \cdot s_B = \frac{k_C}{s_C}\left[\left(\frac{-\sigma}{k_B} \cdot \frac{s_B^2}{2} + C_2\right) - T_{est}\right]$$

$$C_2 = \sigma \cdot s_B\left(\frac{s_C}{k_C} + \frac{s_B}{2k_B}\right) + T_{est}$$

IN B:

$$T_B(x_B) = \frac{-\sigma}{2k_B}x^2 + \sigma \cdot s_B\left(\frac{s_C}{k_C} + \frac{s_B}{2k_B}\right) + T_{est}$$

$$q_B(x_B) = \sigma x$$

STRATO B

$$\frac{d^2 T_c}{dx} = 0 \qquad q_c(x) = -k_c \frac{dT_c}{dx} = -k_c C_1$$

$$T(x_c) = C_1 x + C$$

$$q_c(0) = q_B(s_B)$$

$$-k_c C_1 = \sigma s_B \rightarrow C_1 = \frac{\sigma s_B}{k_c}$$

$$T_c(s_c) = T_{est} \rightarrow C_1 s_c + C_2 = T_{est}$$

$$C_2 = T_{est} + \frac{\sigma \cdot s_B \cdot s_c}{k_c}$$

$$q_c(x_c) = \sigma s_B$$

$$T(x_c) = -x \left(\frac{\sigma s_B}{k_c}\right) + \frac{\sigma s_B s_c}{k_c} + T_{est}$$

$$T_1 = T_B(0) = 224°C$$

$$T_2 = T_B(s_B) = 160°C$$