Esercitazione 7 -

Last practical lesson with full exercise for multiple dof system.
From next only the matrices

Continuing exercise from last time

L EOM:

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot\theta} - \frac{\partial E_c}{\partial \theta} + \frac{\partial D}{\partial \dot\theta} + \frac{\partial V}{\partial \theta} = Q$$

we usually partition the free and constrained dof.

By skipping the theoretical passages we end up with:

$$[M]\ddot{q} + [R]\dot{q} + [k]q \quad \text{with } q = \left[\begin{array}{c} \theta \\ \hline z \end{array}\right]$$

θ → free (no constraint)

z → constrained

$$\left[\begin{array}{c|c} \sim & \sim \\ \hline \sim & \sim \end{array}\right]\left[\begin{array}{c} \ddot\theta \\ \hline \ddot z \end{array}\right] + \left[\begin{array}{c|c} \sim & \sim \\ \hline \sim & \sim \end{array}\right]\left[\begin{array}{c} \dot\theta \\ \hline \dot z \end{array}\right] + \left[\begin{array}{c|c} \sim & \sim \\ \hline \sim & \sim \end{array}\right]\left[\begin{array}{c} \theta \\ \hline z \end{array}\right] = \left[\begin{array}{c} Q_L \\ \hline Q_u \end{array}\right]$$

→ can be a matrix, or just an element

In 3dof system with 1 constrained:

e.g.

$$\left[\begin{array}{cc|c} \sim & \sim & \sim \\ \sim & \sim & \sim \\ \hline \sim & \sim & \sim \end{array}\right]\left[\begin{array}{c} \sim \\ \sim \\ \hline \sim \end{array}\right]$$

we split the matrix in the same way.

$$\begin{array}{c} \text{I} \\ \text{II} \end{array}\left[\begin{array}{c|c} [M_{LL}] & [M_{LU}] \\ \hline [M_{UL}] & [M_{UU}] \end{array}\right]\left[\begin{array}{c} \ddot q_L \\ \hline \ddot q_U \end{array}\right] + \left[\begin{array}{c|c} [R_{LL}] & [R_{LU}] \\ \hline [R_{UL}] & [R_{UU}] \end{array}\right]\left[\begin{array}{c} \dot q_L \\ \hline \dot q_U \end{array}\right] + \left[\begin{array}{c|c} [k_{LL}] & [k_{LU}] \\ \hline [k_{UL}] & [k_{UU}] \end{array}\right]\left[\begin{array}{c} q_L \\ \hline q_U \end{array}\right] = \left[\begin{array}{c} Q_L \\ \hline Q_U \end{array}\right]$$

L → libero V → vincolato
↳ not V

→ We have split the contributions of the free and constrained effects.

q_L and q_V can be scalar or vectors.

q_V is known, Q_L is known
but q_L and Q_V are unknown

We need to solve the two rows, one is an ODE, the second is an algebraic equation.

(ODE) I ROW

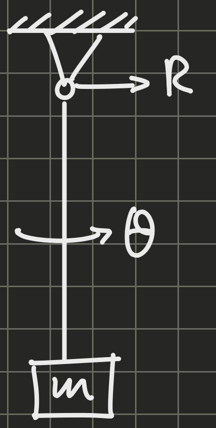
$$[M_{LL}]\underbrace{\ddot{q}_L}_{?} + [R_{LL}]\underbrace{\dot{q}_L}_{?} + [K_{LL}]\underbrace{q_L}_{?} = -\Big(\underbrace{[M_{LV}]\ddot{q}_V + [R_{LV}]\dot{q}_V + [K_{LV}]q_V}_{\text{known}}\Big) + \underbrace{Q_L}_{\text{known}}$$

→ solving we find q_L

II ROW (Algebraic equation)

$$Q_V = [M_{VL}]\underbrace{\ddot{q}_L}_{\text{we have found these}} + [R_{VL}]\dot{q}_L + [K_{VL}]q_L + [M_{VV}]\underbrace{\ddot{q}_V}_{\text{known}} + [R_{VV}]\dot{q}_V + [K_{VV}]q_V$$

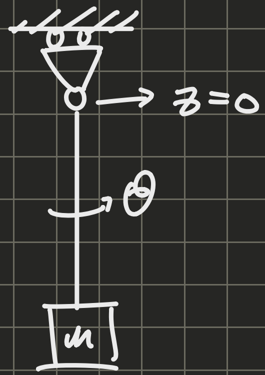
→ we get Q_V → this is how we get the reaction forces where the degrees of freedom are constrained.



→ How to calculate R?

↳ Solve the system for t

↳ or change the system to:



in this case z=0

imposing a motion of z, we get the reaction forces of the system.

$z(t) = z_0 \cos(\Omega t)$

LEOM → FRF

From LEOM we can write FRF for a degree of freedom

$z \rightarrow \theta$

F_L

INPUT → $\boxed{\text{FRF}}$ → θ

↳ can be z, or some force.

OUTPUT is always θ for us.

$$q_V = \bar{q}_{V0}\cos(\Omega t) \xrightarrow{L} q_L = \bar{q}_{L0}\cos(\Omega t)$$

→ use first row of system, substitute and calculate FRF

$$\underbrace{\left(-\Omega^2[M_{LL}] + i\Omega[R_{LL}] + [K_{LL}]\right)}_{[A]} \bar{q}_{L0}\, \cancel{e^{i\Omega t}} = \overbrace{\left(-\Omega^2[M_{LV}] + i\Omega[R_{LV}] + [K_{LV}]\right)}^{[B]} \bar{q}_{V0}\, \cancel{e^{i\Omega t}}$$

↳ complex since i

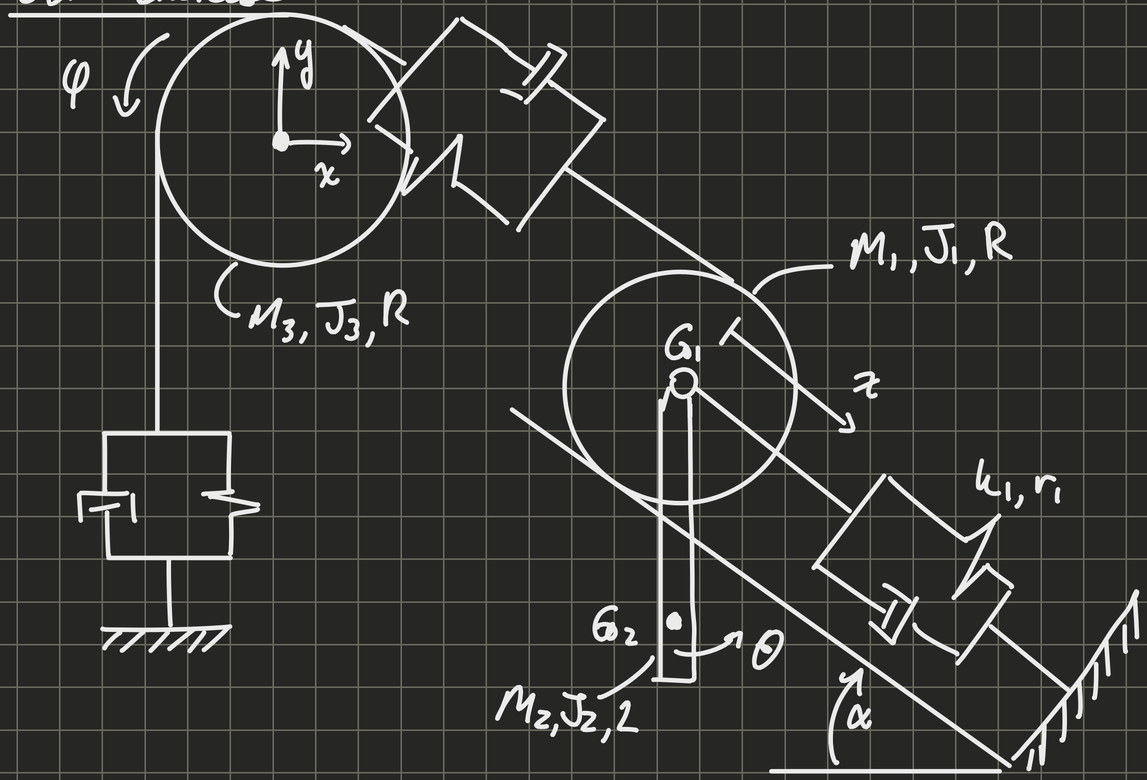
↳ we don't care for t since we are interested in the steady state

[A] depends on the system

[B] depends on the position of the input sum of all FRF between inputs and outputs

$$[A]\,q_{L0} = [B]\,q_{v0} \rightarrow q_{L0} = [A]^{-1}[B]\,q_{v0} = [\underbrace{G(\Omega)}_{\text{mechanical impedence matrix of system}}]\,q_{v0}$$

Other exercise



$$q = [x, \theta, \varphi]^T$$

↳ constrained variable → just for practice, nothing changes.

$\varphi = \varphi_0 \cos(\Omega t)$

Step 1 dof

3 bodies	3×3	9 dof
2 hinges	2×2	−4
1 roll w/slide	1×2	−2

3dof

Step 2

- $E_c = \frac{1}{2} \underline{\dot{y}_m}^T [m_1] \underline{\dot{y}_m}$

$$[m_1] = diag\left(\underbrace{M_1, J_1}_{V_{G1}, W_1}, \underbrace{\overset{\downarrow V_{G2x}}{M_2}, \overset{\downarrow V_{G2y}}{M_2}}_{V_{G2}}, \underset{\uparrow \omega_2}{J_2}, \underset{\uparrow \omega_3}{J_3}\right)$$

since V_{G2} has a complex trajectory it's easier to split the motion in two directions.

- $D = \frac{1}{2} \underline{\dot{\Delta l}}^T [r_1] \underline{\dot{\Delta l}}$
- $V_k = \frac{1}{2} \underline{\Delta l}^T [k_1] \underline{\Delta l}$
- $V_g = P^T \cdot \underline{h}_G$
- $\dot{\delta}^* \mathcal{L} = \underline{F}^T \cdot \underline{\delta y_F}$

$r_i = diag[r_1, r_2, r_3]$

$k_i = diag(k_1, k_2, k_3)$

$P^T = [M_1 g, M_2 g, M_2 g]^T$

$$\left.\begin{aligned} \underline{\Delta l} &= [\Delta l_1, \Delta l_2, \Delta l_3]^T \\ \underline{\dot{\Delta l}} &= [\dot{\Delta l}_1, \dot{\Delta l}_2, \dot{\Delta l}_3]^T \\ \underline{h} &= [h_1, h_2, h_3]^T \end{aligned}\right\}$$ Always vertical vectors

Step 3

- $V_{G1} = \dot{x}$

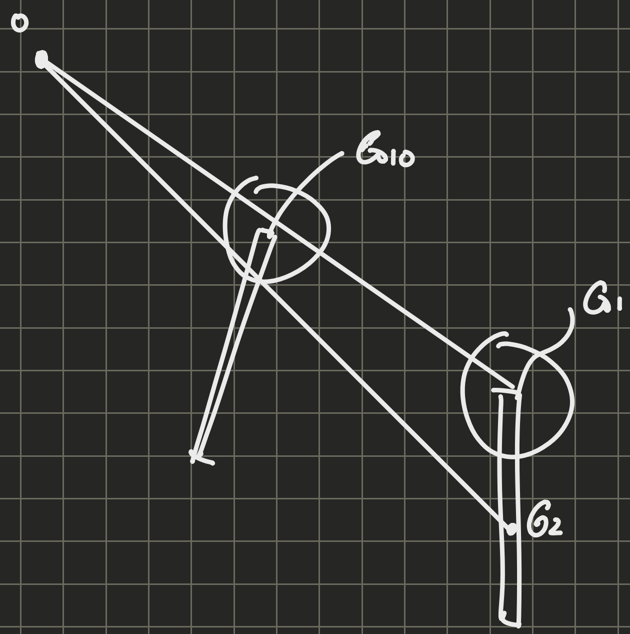
- $\omega_1 = -\frac{\dot{x}}{R}$

- $\omega_2 = \dot{\theta}$

- $\omega_3 = \dot{\varphi}$

$V_{G2} = ?$

$V_{G2} =$



(NL) Vectorial Closing

$$(G_{10} - O) = (r_x \hat{i} + r_y \hat{j})$$

$$(G_1 - G_{10}) = x\cos\alpha\, \hat{i} - x\sin\alpha\, \hat{j}$$

↳ constant, shown in diagram

$$(G_2 - G_1) = \frac{L}{2}\sin\theta\, \hat{i} - \frac{L}{2}\cos\theta\, \hat{j}$$

$$(G_2 - O) = (G_{10} - O) + (G_1 - G_{10}) + (G_{20} - G_1)$$

$$= \left(\tau_x + x\cos\alpha + \frac{L}{2}\sin\theta\right)\hat{i} + \left(\tau_y - x\sin\alpha - \frac{L}{2}\cos\theta\right)\hat{j}$$

$$V_{G2} = \frac{d}{dt}(G_2 - O) = \left(x\cos\alpha + \frac{L}{2}\cos\theta + \dot{\theta}\right)\hat{i} + \left(\underbrace{-x\sin\alpha + \frac{L}{2}\sin\theta\,\dot{\theta}}_{y_{G2} = h_2}\right)\hat{j}$$

$[\Lambda_m]$	$\dot{x}$	$\dot{\theta}$	$\dot{\varphi}$
V_{G1}	1	0	0
ω_1	$-1/R$	0	0
V_{G2x}	$\cos\alpha$	$\frac{L}{2}\cos\theta$	0
V_{G2y}	$-\sin\alpha$	$\frac{L}{2}\sin\theta$	0
ω_2	0	1	0
ω_3	0	0	1

$\downarrow$ linearize

$[\Lambda_m]_0$	$\dot{x}$	$\dot{\theta}$	$\dot{\varphi}$
~	1	0	0
~	$-1/R$	0	0
~	$\cos\alpha$	$\frac{L}{2}$	0
~	$-\sin\alpha$	0	0
~	0	1	0
~	0	0	1

since α is a constant, it's already linear

$\theta_0 = 0$

$\Delta l = ?$

since x compresses h_1

$\Delta l_1 = \Delta l_{01} - x$

$\Delta l_2 = \Delta l_{02} + 2x + \varphi R$

$\Delta l_3 = \Delta l_{03} - R\varphi$

Most likely asked

$[\Lambda_u]$	x	θ	φ
Δl_1	-1	0	0
Δl_2	2	0	R
Δl_3	0	0	$-R$

$$[\Lambda_u]_0 = [\Lambda_k]_0$$

is evaluated at equilibrium

because h_i and r_i are all in parallel sets

$$- h_{G1} = \tau_2 - x\sin\alpha \rightarrow [H_{G1}] = \phi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$- h_{G2} = \tau_2 - x\sin\alpha - \frac{L}{2}\cos\theta \rightarrow [H_{G2}] = \begin{bmatrix} \frac{\partial^2 h_2}{\partial x^2} & \frac{\partial^2 h_2}{\partial x \partial \theta} & \frac{\partial^2 h_2}{\partial x \partial \varphi} \\ \frac{\partial^2 h_2}{\partial \theta \partial x} & \frac{\partial^2 h_2}{\partial \theta^2} & \frac{\partial^2 h_2}{\partial \theta \partial \varphi} \\ \frac{\partial^2 h_2}{\partial \varphi \partial x} & \frac{\partial^2 h_2}{\partial \varphi \partial \theta} & \frac{\partial^2 h_2}{\partial \varphi^2} \end{bmatrix}$$

$$- h_{G3} = \text{const} \rightarrow [H_{G3}] = \phi$$

Hessian of h_{G2}

$$[H_{G2}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & L/2\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Eval. at Equilibrium

$$q_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad [H_{G2}]_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & L/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\delta^* \mathcal{L} = \underline{F}^T \delta y_F = \underbrace{\underline{F}^T [\Lambda_F]_0}_{Q^T} \delta \bar{q}$$

Q^T → Lagrangian component of external forces.

$[\Lambda_F]$	x	θ	φ
Sy_3	0	0	1

$$E_c = \frac{1}{2} \underline{\dot{q}}^T [M] \underline{\dot{q}} \quad \text{with } [M] = [\Lambda_{M0}]^T [\diagdown m_i \diagdown] [\Lambda_{M0}]$$

$$D = \frac{1}{2} \underline{\dot{q}}^T [R] \underline{\dot{q}} \quad \text{with } [R] = [\Lambda_{R0}] [\diagdown r_i \diagdown] [\Lambda_{R0}]$$

$$V_k = \frac{1}{2} \underline{q}^T [k] \underline{q} \quad \text{with } [k] = [\Lambda_{k0}] [\diagdown k \diagdown] [\Lambda_{k0}]$$

$$V_g = \underline{P}^T \underline{h}$$

$$V = V(\underline{q}_0) + \left.\frac{\partial V}{\partial \underline{q}}\right|_{\underline{q}_0} (\underline{q} - \underline{q}_0) + \frac{1}{2} \underline{q}^T \underbrace{\left. [\Lambda_{k0}]^T [\diagdown k_i \diagdown] [\Lambda_{k0}] \right|_{\underline{q}_0}}_{[k_I]} \underline{q}$$

$$+ \frac{1}{2} \underline{q}^T \left[\sum_{i=1}^{N_g} M_i g [H_{h_i}] \right]_{\underline{q}_0} \underline{q} \longrightarrow [k_{II}]$$

$$+ \frac{1}{2} \underline{q}^T \left[\sum_{i=1}^{N_k} k_i \, \Delta l_{0i} [H \Delta l_i] \right]_{\underline{q}_0}$$

$\hookrightarrow [k_{III}]$

$\longrightarrow$ Hessian for Δl_i $\rightarrow$ we didn't calculate since they were all linear, so it would be a null matrix.

$$Q^T = \underline{F}^T [\Lambda_F]$$