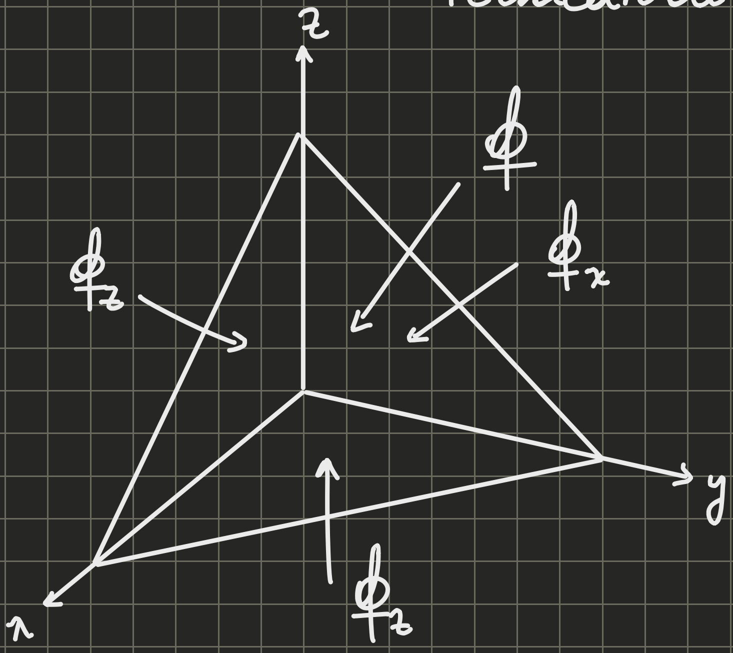
Tetraedro di Cauchy



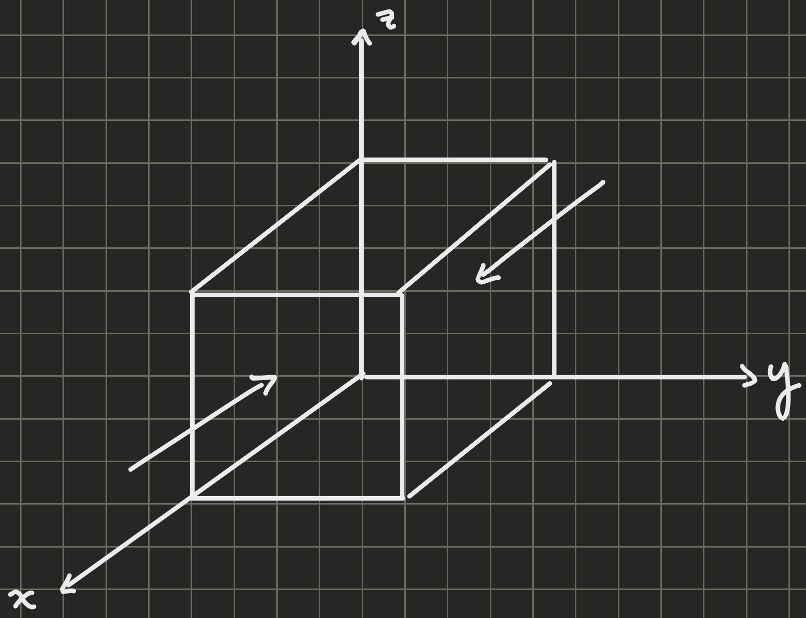
$$\phi A = \phi_x A_x + \phi_y A_y + \phi_z A_z$$

$$\phi = \phi_x \frac{A_x}{A} + \phi_y \frac{A_y}{A} + \phi_z \frac{A_z}{A}$$

$$= \phi_x n_x + \phi_y n_y + \phi_z n_z$$

$$\phi = \underline{\underline{\phi}}\, \underline{n}$$

Equilibrio Statico indefinito e legge di Stevino



$$\cancel{p\, dy dz \hat{i}} - \left(\cancel{p} + \frac{\partial p}{\partial x} dx\right) dy dz \hat{i}$$ per ogni direzione

$$-\frac{\partial p}{\partial x} dW - \frac{\partial p}{\partial y} dW - \frac{\partial p}{\partial z} dW = -\text{grad}\, p\, dW$$

L'azione di massa è:

$$\rho \underline{f}\, dW$$

L'equilibrio dei due è:

$$\rho \underline{f}\, \cancel{dW} - \text{grad}\, p\, \cancel{dW} = 0$$

$$\rho \underline{f} - \text{grad}\, p \overset{!}{=} 0$$

Stevino

Essendo: $\underline{f} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$ grad z, troviamo

$$-\rho g\, \text{grad}\, z = \text{grad}\, p$$

Abbiamo allora che: $\text{grad}\left(z + \underbrace{\frac{p}{\rho g}}_{= \frac{p}{\gamma}}\right) = 0$

$$\Rightarrow z + \frac{p}{\gamma} = \text{costante}$$

Spinta su superficie piana

$$\vec{S} = \int_A p\, \vec{n}\, A = \int_A \gamma z\, \vec{n}\, dA = \int_A \gamma\, x \cos\alpha\, \vec{n}\, dA$$

$$= \gamma \cos\alpha\, \vec{n} \int_A x\, dA = \gamma \cos\alpha\, \vec{n}\, x_G A = \gamma\, \vec{n}\, z_G A = \vec{n}\, p_G A$$

$$\vec{S}\, b = \int_A p\, x\, \vec{n}\, dA$$

$$\cancel{\gamma \cos\alpha\, \vec{n}}\, b \int_A x\, dA = \cancel{\gamma \cos\alpha\, \vec{n}} \int x^2\, dA$$

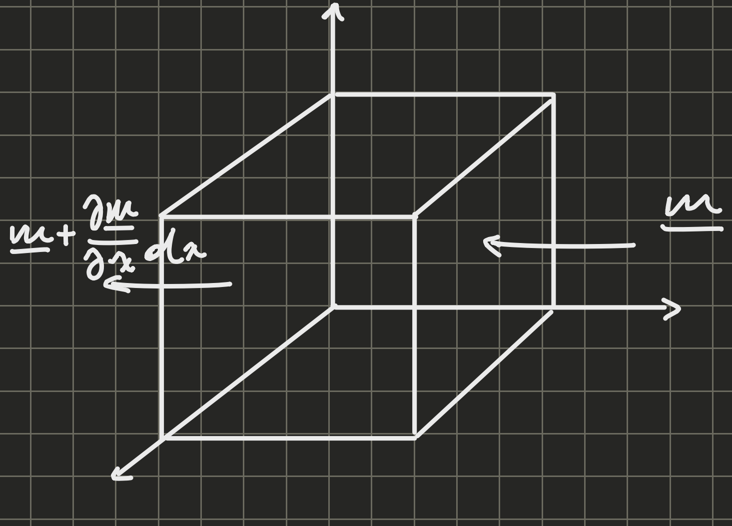
$$b = \frac{\int_A x^2\, dA}{\int_A x\, dA}$$

Equilibrio Statico Globale

$$\underbrace{\int_W \rho \vec{f}\, dW}_{\underline{G}} = \underbrace{\int_W \operatorname{grad} p\, dW}_{-\underline{\Pi}}$$

$$\underline{G} + \underline{\Pi} = 0$$

Equazione indefinita di continuità



Massa Entrante nel tempo dt:

$$\rho u\, dy\, dz\, dt + \rho v\, dx\, dz\, dt + \rho w\, dx\, dy\, dt$$

Massa Uscente in dt

$$\left(\rho u + \frac{\partial \rho u}{\partial x} dx\right) dy\, dz\, dt + \left(\rho v + \frac{\partial \rho v}{\partial y} dy\right) dx\, dz + \left(\rho w + \frac{\partial \rho w}{\partial z} dz\right) dx\, dy\, dt$$

La massa accumulata è:

$$\frac{\partial \rho}{\partial t}\, dx\, dy\, dz\, dt$$

Semplificando tutto si trova:

$$-\frac{\partial \rho u}{\partial x} dx\, \cancel{dy\, dz\, dt} - \frac{\partial \rho v}{\partial y} \cancel{dx\, dy\, dz\, dt} - \frac{\partial \rho w}{\partial z} \cancel{dx\, dy\, dz\, dt} =$$

$$= \frac{\partial \rho}{\partial t}\, \cancel{dx\, dy\, dz\, dt}$$

$$-\frac{\partial \rho u}{\partial x} - \frac{\partial \rho v}{\partial y} - \frac{\partial \rho w}{\partial z} = \frac{\partial \rho}{\partial t}$$

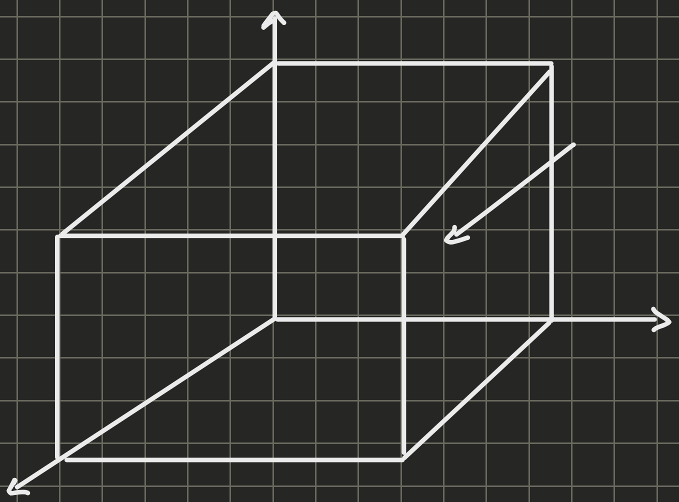
$$-\operatorname{div}(\rho \vec{v}) = \frac{\partial \rho}{\partial t} \implies \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{v}) = 0$$

<u>Equazione Globale Continuità</u>

$$\int_W \frac{\partial \rho}{\partial t}\, dW = -\int_W \operatorname{div}(\rho \vec{v})\, dW$$

$$\frac{\partial M}{\partial t} = \int_A \rho \vec{v} \cdot \vec{n} \, dA = \rho Q$$

Equazione indefinita dell'equilibrio dinamico



Equilibrio di azioni di massa e superficie

$$\rho \vec{f} \cancel{dx dy dz} - \underbrace{\rho \vec{a} \, dx \cancel{dy dz}}_{\text{Azione di inerzia}} + \left(-\frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_y}{\partial y} - \frac{\partial \phi_z}{\partial z} \right) \cancel{dx dy dz} = 0$$

$$\rho(\vec{f} - \vec{a}) = div(\underline{\phi})$$

Teorema di Bernoulli

↳ Tante ipotesi, incluso fluido incomprimibile e stazionario

$$\rho(\vec{f} - \vec{a}) = div(\underline{\phi}) \qquad \underline{\phi} = p \underline{\underline{I}}$$

$$\Rightarrow \rho(\vec{f} - \vec{a}) = grad \, p$$

Rispetto alla terna intrinseca (s, n, b):

$$\begin{cases} -\rho g \dfrac{\partial z}{\partial s} - \rho \dfrac{dV}{dt} = \dfrac{\partial p}{\partial s} \quad \circledast \\ -\rho g \dfrac{\partial z}{\partial n} - \rho \dfrac{V^2}{R} = \dfrac{\partial p}{\partial n} \\ -\rho g \dfrac{\partial z}{\partial b} = \dfrac{\partial p}{\partial b} \end{cases}$$

$$-\rho g \frac{\partial z}{\partial s} - \rho \cancelto{0}{\frac{\partial V}{\partial t}} - V \frac{\partial V}{\partial s} - 0 \cancel{\frac{\partial V}{\partial n}} - 0 \cancel{\frac{\partial V}{\partial b}} = \frac{\partial p}{\partial s}$$

$$-\frac{\partial z}{\partial s} - \frac{\partial}{\partial s} \frac{V^2}{2g} = \frac{\partial}{\partial s}\left(\frac{p}{\gamma}\right)$$

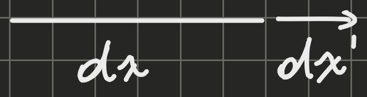
$$\frac{\partial}{\partial s}\underbrace{\left(z + \frac{p}{\gamma} + \frac{V^2}{2g}\right)}_{H} = 0 \Rightarrow z + \frac{p}{\gamma} + \frac{V^2}{2g} = \text{costante}$$

Essendo H costante troviamo che un fluido ideale non dissipa energia quando è in flusso, questo è vero perché H può esser visto come $\frac{\text{energia}}{\text{peso}}$ quindi troviamo che la sua energia non cambia in flusso.

Velocità di Deformazione

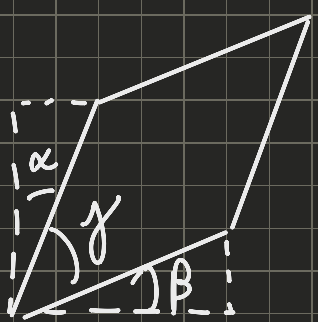
$$\mathrm{grad}\,\underline{v} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} = \underline{\underline{D}} + \underline{\underline{\Omega}}$$

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \\ & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \\ \text{SIMM.} & & \frac{\partial w}{\partial z} \end{bmatrix}$$



$$\varepsilon = \frac{dx' - dx}{dx} = \frac{1}{dx}(u_2 - u_1)\,dt$$

$$\frac{d\varepsilon}{dt} = \frac{\partial u}{\partial x}$$



$$\alpha = \frac{v_2 - v_1}{dx}\,dt$$

$$\beta = \frac{u_2 - u_1}{dy}\,dt$$

$$\frac{d\gamma}{dt} = -\frac{d\alpha}{dt} - \frac{d\beta}{dt} = -\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

Navier Stokes

Prendiamo $\underline{\underline{\Phi}} = p\underline{\underline{I}} + \underline{\underline{\Phi}}_D$

$$\underline{\underline{\Phi}}_D = \underline{\underline{\Phi}} - p\underline{\underline{I}} = -\mu' \operatorname{div}(\underline{v})\underline{\underline{I}} - 2\mu \underline{\underline{D}}$$

Usando l'equazione indefinita dell'equilibrio dinamico

$$\begin{cases} \rho(\vec{f} - \vec{a}) = \operatorname{div} \underline{\underline{\Phi}} \\ \underline{\underline{\Phi}} - p\underline{\underline{I}} = -\mu' \operatorname{div}(\underline{v})\underline{\underline{I}} - 2\mu \underline{\underline{D}} \end{cases}$$

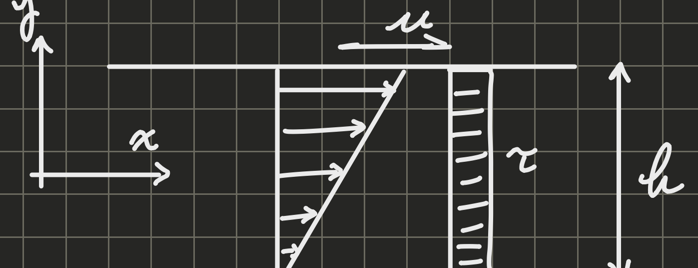
$$\rightarrow \rho(\vec{f} - \vec{a}) = \operatorname{grad} p - \operatorname{grad}(\mu' \operatorname{div}(\underline{v})) - \operatorname{div}(2\mu \underline{\underline{D}})$$

$$= \operatorname{grad} p - \mu \nabla^2 \underline{v}$$

$\hookrightarrow$ Flusso incomprimibile.

$$\rho(\vec{f} - \vec{a}) = \operatorname{grad} p - \mu \nabla^2 \underline{v}$$

Couette



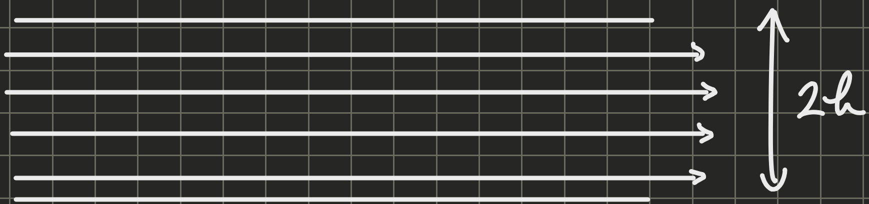
$$\rho\left(\cancel{f_x}-\cancel{\frac{\partial u}{\partial t}}-u\cancel{\frac{\partial u}{\partial x}}-\cancel{v}\frac{\partial u}{\partial y}\right)=\cancel{\frac{\partial p}{\partial x}}-\left(\cancel{\frac{\partial^2 u}{\partial x}}-\frac{\partial^2 u}{\partial y^2}\right)\mu$$

$$\mu\frac{\partial^2 u}{\partial y^2}=0\rightarrow\mu\frac{\partial u}{\partial y}=A\rightarrow\mu u(y)=Ay+B$$

la velocità prende forma lineare

$$u(0)=0\rightarrow B=0$$
$$u(h)=U\rightarrow Ah=U\rightarrow A=\frac{U}{h}$$
$$u(y)=U\frac{y}{h}\qquad y\in[0,h]$$

<u>Poiseuille</u>



$$\rho\left(\cancel{f_x}-\cancel{\frac{\partial u}{\partial t}}-u\cancel{\frac{\partial u}{\partial x}}-\cancel{v}\frac{\partial u}{\partial y}\right)=\frac{\partial p}{\partial x}-\left(\cancel{\frac{\partial u^2}{\partial x^2}}+\frac{\partial u^2}{\partial y^2}\right)\mu$$

$$\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial y}=\frac{1}{\mu}\frac{\partial p}{\partial x}y+C$$

$$u(y)=\frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2}+Cy+D$$

$$u(0) = 0 \Rightarrow D = 0$$

$$u(2h) = 0 \Rightarrow C = -\frac{1}{\mu}\frac{\partial p}{\partial x} h$$

$$\rightarrow u(y) = \frac{1}{\mu}\frac{\partial p}{\partial x}\left(\frac{y^2}{2} - hy\right)$$

Equazione Globale dell'equilibrio dinamico:

$$\underbrace{\int_W \rho\vec{f}\,dW}_{\underline{G}} - \underbrace{\int_W \rho\vec{a}\,dW}_{\underline{I}+\underline{M}} = \underbrace{\int_W div(\underline{\underline{\Phi}})\,dW}_{-\overline{\overline{\Pi}}}$$

Sforzo per la accellerazione locale e convettiva

Ideale

$$\underline{\underline{\Phi}} = p\,\underline{\underline{I}}$$

$$\downarrow$$

$$-\left(\overline{\overline{\Pi}}_p\right)$$

Reale

$$\underline{\underline{\Phi}} = p\,\underline{\underline{I}} - \underline{\underline{\Phi}}_D$$

$$\downarrow$$

$$-\left(\overline{\overline{\Pi}}_p + \overline{\overline{T}}\right)$$

Termine di pressione

Termine di Sforzi di Attrito dati dalla viscosità

Reynolds

La media di Reynolds è usata per analizzare flussi turbolenti, permettendoci da isolare variazioni dalla media, poi possiamo lavorare sul flusso media e aggiungendo poi le deviazioni.

Navier-Stokes:

$$\rho(\vec{f} - \vec{a}) = \text{grad}\, p - \mu \nabla^2 \underline{v}$$

Reynolds:

$$\rho\left(\vec{f}_m - \frac{\partial \underline{v}_m}{\partial t} - u_m \frac{\partial \underline{v}_m}{\partial t} - v_m \frac{\partial \underline{v}_m}{\partial y} - w_m \frac{\partial \underline{w}_m}{\partial z}\right) = \text{grad}\, p_m - \mu \nabla^2 \underline{v}_m + \text{div}\, \underline{\underline{\phi}}_R$$

$$\underline{\underline{\phi}}_R = \begin{bmatrix} (u'u')_m & (u'v')_m & (u'w')_m \\ & (v'v')_m & (v'w')_m \\ \text{SIMM} & & (w'w')_m \end{bmatrix}$$