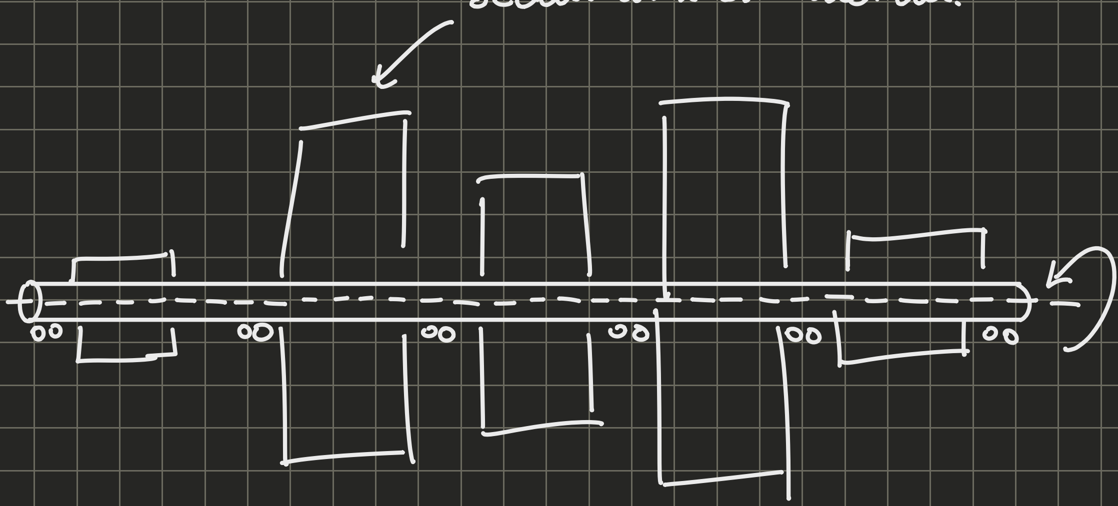
# Jeffcott Rotor

Rotor is a part of a machine, it is an axle that spins around it own axis of rotation.

section with own inertia.

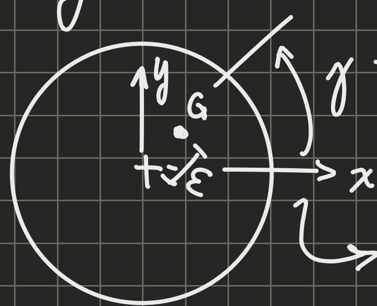


Problem with these large beams:

↳ Center of mass must be on axis of rotation,

  ↳ impossible to do for every single section.

Reality:




$\gamma$ → Phase angle of eccentricity.

$\varepsilon$ and $\gamma$ are different for every section.

→ reference system on axis of rotation.

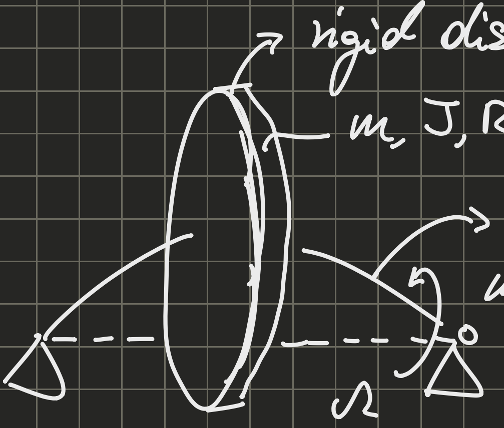
$\varepsilon$ can be small, but with large masses and velocities, it can become a problem.

Since it's large, the bearings will never all be perfectly aligned, this cause high pressures to be produced on one side of the bearing, this causes forces to be different and so can cause further issues.

There can also be issues with friction and heat with older bearings. The heat can also cause varying dilatations of the material. This dilation also causes a change in shape of the shape, so further disallignment in the bearings.

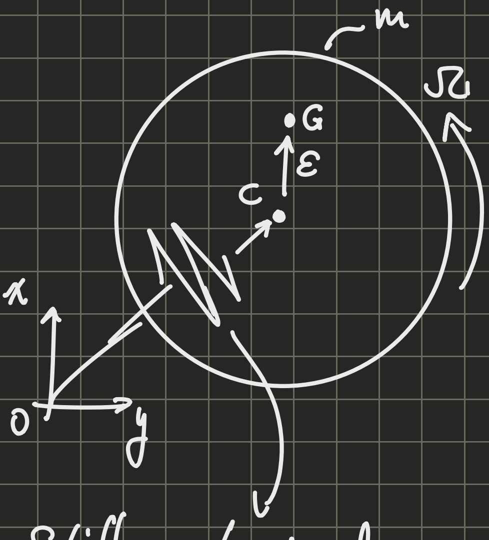
All the problems produce effect that can be modelled with eccentricity ($\varepsilon$).

To solve problems with $\varepsilon$, we need to solve a single system. the Jeffcott rotor.




Beam of negligible mass and isotropic.

Dish cannot sway in directions other than it's axis of rotation.




Stiffness/spring of the system.

Position of center of gravity

$$(G-O) = (G-C) + (C-O)$$

$$x_G = x_c + \varepsilon \cos \Omega t$$

$$y_G = y_c + \varepsilon \sin \Omega t$$

$$\ddot{x}_G = \ddot{x}_c - \Omega^2 \varepsilon \cos \Omega t$$

$$\ddot{y}_G = \ddot{y}_c - \Omega^2 \varepsilon \cos \Omega t$$

We don't care about both degrees of freedom, so my equation of motion for one of the two will be:

$$m\ddot{x} + kx = -\Omega^2 m \varepsilon \cos \Omega t$$

The equation is the same for both degrees of freedom.

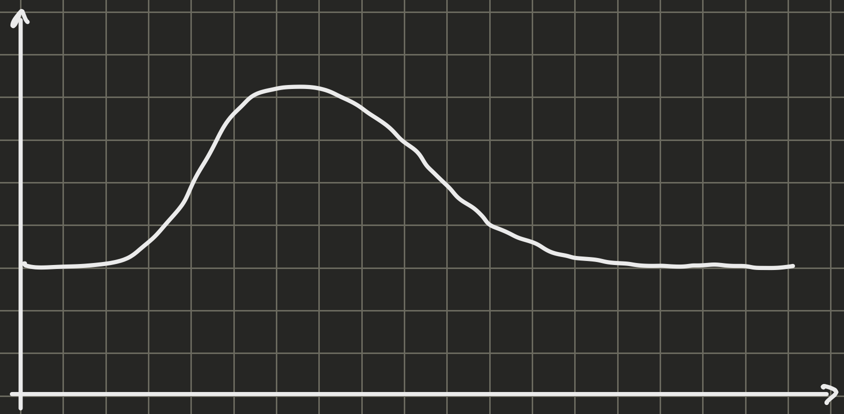
This basically shows that the force that is a function of $\Omega^2$.

$$\omega_y = \omega_x = \sqrt{\frac{k}{m}}$$

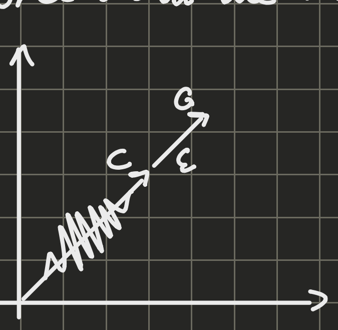
If $\Omega = \omega$, we have resonance basically.

When we spin a Jeffcott rotor, if $\Omega$ is the same as one of it's natural frequencies, we have a resonance problem, so very large vibration.

Our transfer function will look like:

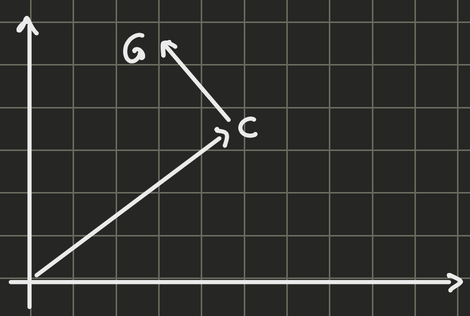


In quasi-static regime, we will have an alignment of the force with the motion of the geometric center, like:




The center of gravity is pushed out by the inertial force.

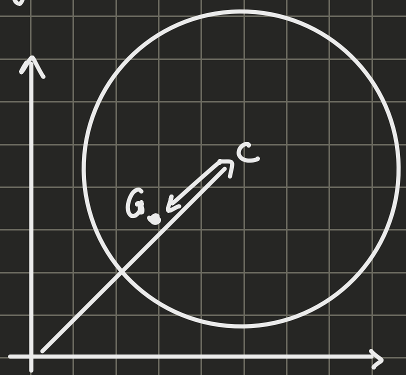
At some point, with $\Omega$, we get $\gamma = 90°$:




stiffness of the system

In this case, the spring cannot do anything, so we don't have damping, and our system cannot stop the vibrations which become bigger and bigger.

All real components are designed to work in the seismographic zone, $\gamma$ becomes 180:

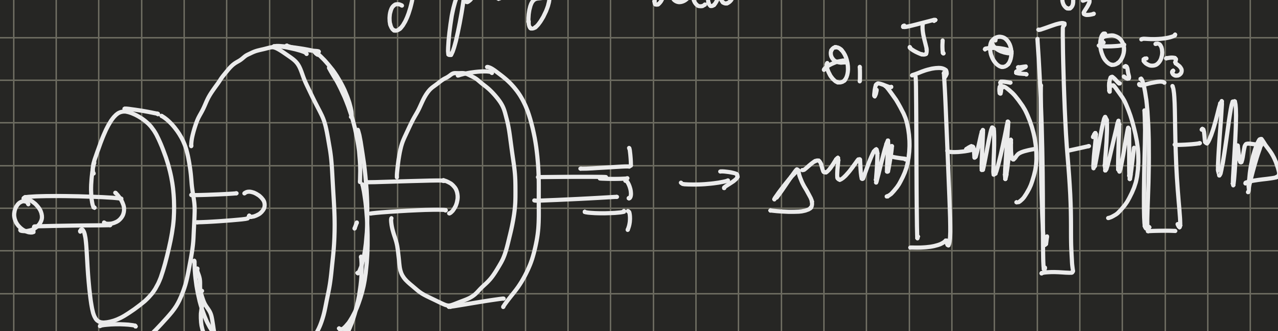



The system pushed again the deformations and self-centers itself

$\frac{x}{x_0} \to 1$ in this case since we have $\Omega^2$

When we have $\omega >> \Omega$, the system self-centers.
The problem here becomes the switch on and switch off transients.

For some rotors can have torsional vibration problems, this is true for wind turbines. The shaft has a torsional stiffness and this can be modelled by springs in between elements:




$$E_c = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{3} J_3 \dot{\theta}_3^2$$

$\Delta l_1 = -\theta_1 \quad \Delta l_2 = \theta_1 - \theta_2 \quad \Delta l_3 = \theta_3 - \theta_2$