Lezione 14 -

We illustrated Heuristically that the characteristic curves of a pump could be written as:

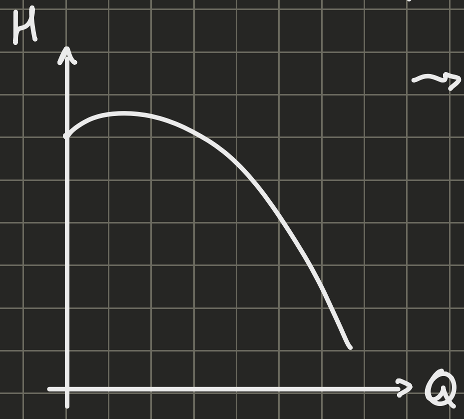
$$\left.\begin{array}{l} H = AQ^2 + BQ + C \\ \eta = EQ^2 + FQ + G \end{array}\right\} \begin{array}{l} \text{SHAPE} \\ n = \bar{n} \\ D = \bar{D} \end{array} \longrightarrow \Psi = \alpha \varphi^2 + \beta \varphi + \gamma$$

valid for a SHAPE $\forall n, D$

$\Rightarrow$ describes the family of the machine.

α, β, γ depend only on SHAPE

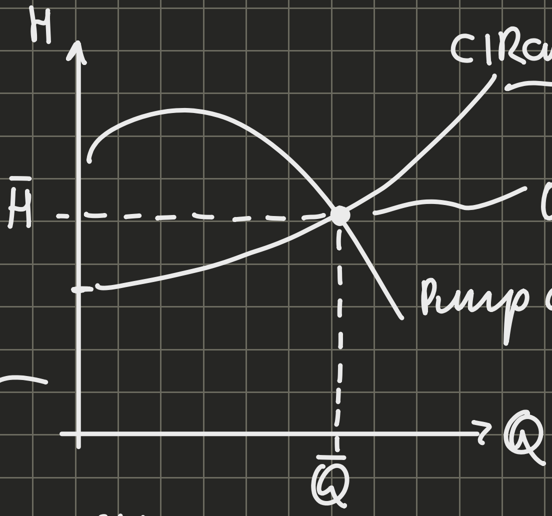
A, B, C are dependent on n and D too.



$\rightarrow$ for a certain SHAPE, n, D

$\rightarrow$ Machine curve

Curve of a Circuit

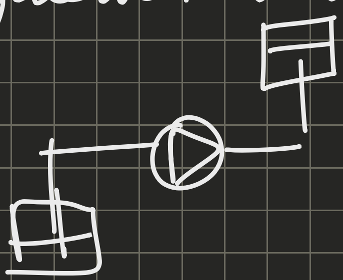


CIRcuit curve $\longrightarrow$ losses grow quadratically with Q.

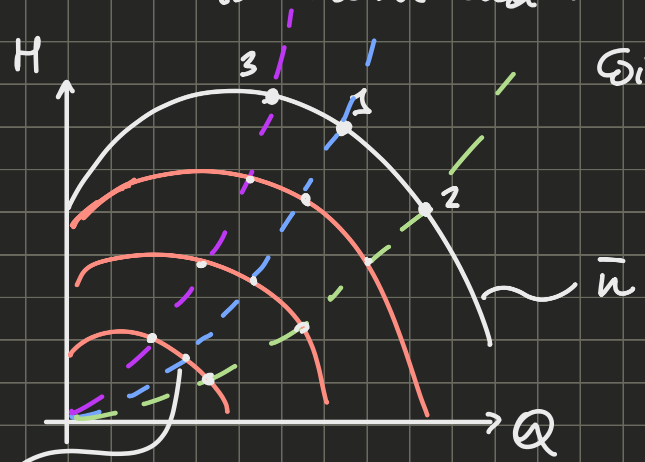
Operating condition when plugged in, automatically found.

Pump characteristic curve at $\bar{n}$ and $\bar{D}$

Let's imagine we are the owner of the plant with system like:



What happens if we change n?
to the curve and introduce



Given: D, SHAPE

n is free

If we identify a point, we know the non-constructive angles.

What happens if we reduce n while maintaining similarity conditions

Similarity

$$\begin{cases} \varphi_1 = \varphi_2 & (*)_1 \\ \psi_1 = \psi_2 & (*)_2 \end{cases}$$

We find the associated point after the change, and do so for every point.

$$(*)_1 \Rightarrow \frac{Q_1}{n} = \frac{Q_2}{n} \rightarrow \text{since D is given} \Rightarrow \frac{Q_2}{Q_1} = \frac{n}{\bar{n}}$$

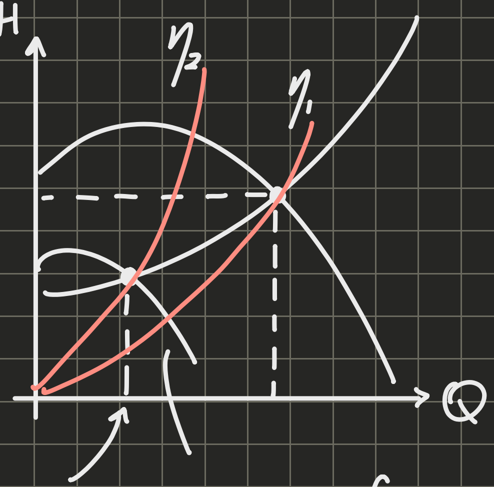
$$(*)_2 \Rightarrow \frac{\cancel{g}H_1}{\bar{n}^2} = \frac{\cancel{g}H_2}{n^2} \Rightarrow \frac{H_2}{H_1} = \left(\frac{n}{\bar{n}}\right)^2 = \left(\frac{Q_2}{Q_1}\right)^2$$

$\Rightarrow$ All points in hydraulic similarity with 1, will be on on a parabola passing through the origin.

All the points on the line will have the same shape triangles and efficiency, φ, ψ will also be constant.

Through each parabola and each point we can get the curves at any n we are interested in.

— are the curves of the same machine run at a different speed.



New operating condition.

new curve of pump at lower Q.

These two points are not in similarity

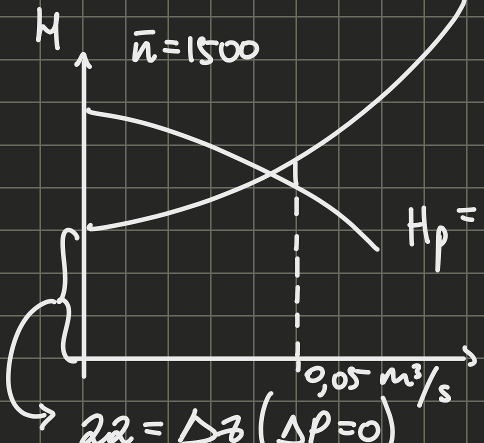
— = similarity curve.

Since they are not similar, the η will be different, φ and ψ will also be different, so to determine our system we need to recalculate the intersection point.

Determining the curve associated with the new n is not the most efficient one, so to speed it up we pass through similarity and the dimensionless forms.

Hydraulic Similarity

↳ Exercise 4



$22 = \Delta z \; (\Delta P = 0)$

Always positive

$H_c = 22 + 1200\, Q^2$

The 1200 comes from the fact that it has to pass through the point at $Q = 0{,}05$

(we will not be asked to do this, this exercise is just old)

Must always be bigger than that, otherwise there is no intersection, it can happen if we forget a g, if it happens we can just write a note that say that we see the problem but don't know where from, from there we

$H_p = 30 - 5000\, Q^2$

Always negative

$\eta_p = -250\,Q^2 + 17Q + 0,5$

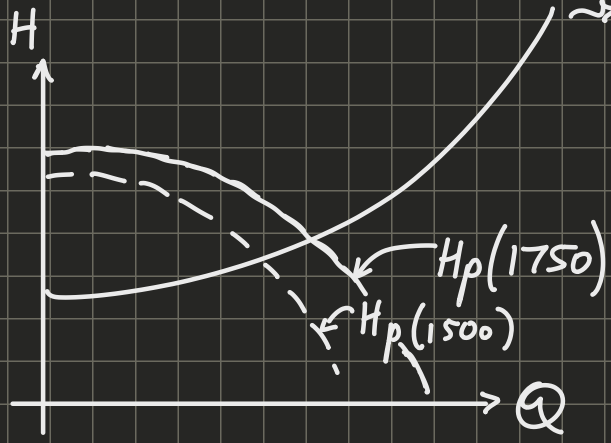
can invert a number that is meaningful.
⇝ calculated η should always be < 1.

$$\begin{cases} \bar{H}_0 = 23,5 \text{ m} \\ \bar{Q} = 0,036 \text{ m}^3/\text{s} \\ \eta(\bar{Q}) = \bar{\eta} = 0,788 \end{cases}$$

$$\dot{L} = \dot{m}\, l = \bar{\dot{m}} \frac{g\bar{H}}{\bar{\eta}} = \rho \frac{\bar{Q}\bar{H}g}{\bar{\eta}} = 10.5\text{kW}$$

$\rho = 1000 \frac{\text{kg}}{\text{m}^3}$

$n_1 = 1500 \longrightarrow n = 1750$



⇝ H_c is always the same.

We know H_p, if we are able to write it dimensionally we will be able to the find the new curve.

We know that the operational point will not follow the similarity curve of the original point.

We can use similarity to generalise the curve, so we use it find the new curve, allowing us to define the curve based on whichever n we want.

General Procedure

$$H = AQ^2 + BQ + C = (\text{for } \bar{n}, \bar{D})$$

$$\psi = \frac{gH}{\bar{n}^2 \bar{D}^2} \qquad \varphi = \frac{Q}{\bar{n}\bar{D}^3}$$

$$= \psi \bar{n}^2 \bar{D}^2 = gA\bar{n}^2\bar{D}^6\varphi^2 + gB\bar{n}\bar{D}^3\varphi + gC$$

$$\psi = \frac{gA\bar{n}^2\bar{D}^6}{\bar{n}^2\bar{D}^2}\varphi^2 + \frac{gB\bar{n}\bar{D}^3}{\bar{n}^2\bar{D}^2}\varphi + \frac{gC}{\bar{n}^2\bar{D}^2}$$

$$= gA\bar{D}^4\varphi^2 + \frac{gB\bar{D}}{\bar{n}}\varphi + \frac{gC}{\bar{n}^2\bar{D}^2}$$

$$\underset{\alpha}{\sim} \qquad \underset{\beta}{\overset{\bar{n}}{\sim}} \qquad \underset{\gamma}{\sim}$$

$\Psi = \alpha\,\varphi^2 + \beta\varphi + \gamma$ → since each dimensional curve is a product of similarity, we know that these dimensionless coefficients have to stay the same, whereas the dimensional curve will change to adhere to the the similarity of it's points and to the new condition.

$$\begin{cases} \alpha = gA\bar{D}^{-4} \\ \beta = gB\dfrac{\bar{D}}{\bar{n}} \\ \gamma = \dfrac{gC}{\bar{n}^{2}\bar{D}^{-2}} \end{cases} \Longrightarrow \begin{cases} A = \dfrac{\alpha}{g\bar{D}^4} \\ B = \dfrac{\beta\bar{n}}{g\bar{D}} \\ C = \dfrac{\gamma}{g}\bar{n}^{-2}\bar{D}^{2} \end{cases}$$

α, β, γ are family coefficients

A, B, C are the realisations of these at $\bar{n}, \bar{D}$ and shape.

→ We render this dimensional but generalised for $\bar{n}$ and $\bar{D}$

$$\underbrace{\varphi = \frac{Q}{nD^3} \qquad \Psi = \frac{gH}{n^2D^2}}_{\{\text{care}\}}$$

$\bar{n}, \bar{D}$ → given

n, D → any arbitrary value

$$\Longrightarrow \frac{\cancel{g}H}{n^2D^2} = \cancel{g}A\bar{D}^{-4}\frac{Q^2}{n^2D^6} + \cancel{g}B\frac{\bar{D}}{\bar{n}}\frac{Q}{nD^3} + \frac{\cancel{g}C}{\bar{n}^2\bar{D}^{-2}}$$

specific given

general (arbitrary

We obtained it by assuming α, β, γ are conserved due to similarity which they are. This gives us this condition.

$$\Longrightarrow \boxed{H = A\left(\frac{\bar{D}}{D}\right)^4 Q^2 + B\left(\frac{n}{\bar{n}}\right)\left(\frac{\bar{D}}{D}\right)Q + C\left(\frac{n}{\bar{n}}\right)^2\left(\frac{D}{\bar{D}}\right)^2}$$

realised coefficient for $\bar{n}, \bar{D}$

↳ how size scales head

↳ $H = H(Q, n, D, \text{SHAPE})$

↳ We have finally been able to achieve it.

↳ we achieved this through similarity

Since in general we use the same machine and only change n,

$$\frac{\bar{D}}{D} = \frac{D}{\bar{D}} = 1$$

We can go the same for η

Already dimensionless

$$\eta = EQ^2 + FQ + G = \underbrace{E\bar{n}^2\bar{D}^6}_{\mathcal{E}} + \underbrace{F\bar{n}\bar{D}^3}_{\sigma}\varphi + G$$

↳ This curve will hold for any n, D

$$Q = \varphi \bar{n} \bar{D}^3$$

$$\varphi = \frac{Q}{nD^3} \Rightarrow \eta = E\left(\frac{\bar{n}}{n}\right)^2\left(\frac{\bar{D}}{D}\right)^6 Q^2 + F\left(\frac{\bar{n}}{n}\right)\left(\frac{\bar{D}}{D}\right)^3 Q + G$$

The circuit curve always remains the same.

The curve moves to the right and up, the η reduces in concavity (wider), we find that the efficiency at the new operating point, we move away from the BEP.

$$H = 30 - 5000\,Q^2$$

$$H = 30\underbrace{\left(\frac{n}{\bar{n}}\right)^2}_{\frac{1750}{1500}} - 5000\,Q^2 = 40{,}8 - 5000\,Q^2$$

↳ increased speed increases head.

$$H_{CIR} = 22 + 1200\,Q^2$$

operating condition $\Rightarrow$ $\begin{cases} H = 25{,}6\,m \\ Q = 0{,}055\ m^3/s \end{cases}$ → we increased Q as expected.

$$\eta = -250\left(\frac{1500}{1750}\right)^2 Q^2 + 17\left(\frac{1500}{1750}\right) Q + 0,5$$

$$= -183\, Q^2 + 16,37\, Q + 0,5$$

$\eta(0,055) = 0,745 \rightarrow$ it decreased as the graphs showed.

$\Longrightarrow \dot{L} = 18,6$ kW

↳ twice as before

$$\dot{L} = \rho \frac{\bar{Q} \bar{H} g}{\bar{\eta}}$$

$\rightarrow$ since $\bar{Q}$ grows, $\bar{H}$ will grow due to more losses and $\bar{\eta}$ has reduced

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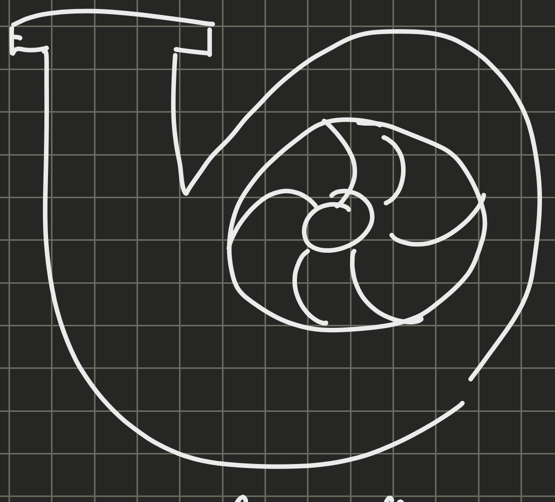
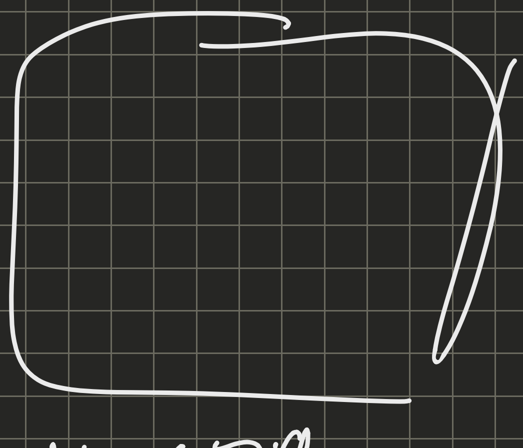
Increasing β_2 to ~ -60 to -70, $\chi = 0,8 - 0,9$, this means that we will not need the diffuser as the pump does everything itself already. This means that the machine will be more compact and useful in a greater number of use cases

↳ and useful,

Diffusers



Free or vaneless diffuser, no longer needs the blades of the diffuser.



Vaned Diffuser, uses blades to diffuse.

↳ Allow us to achieve a higher head for the pump, since χ will be lower so V_{2t} will be higher, l will be higher and so will H. So pumps that need to produce high head do not usually use vaneless diffusers.

work exchanged

The diffuser recieves a flow with angle α_2 (non-constructive, function of operating condition)

Vaneless → wider use, with penalty in efficiency, but less head and work.
Vaned → less used but useful when more head/work needs to be transferred.
→ Usually more flat curve of efficiency. → most industrial pumps are vaneless.

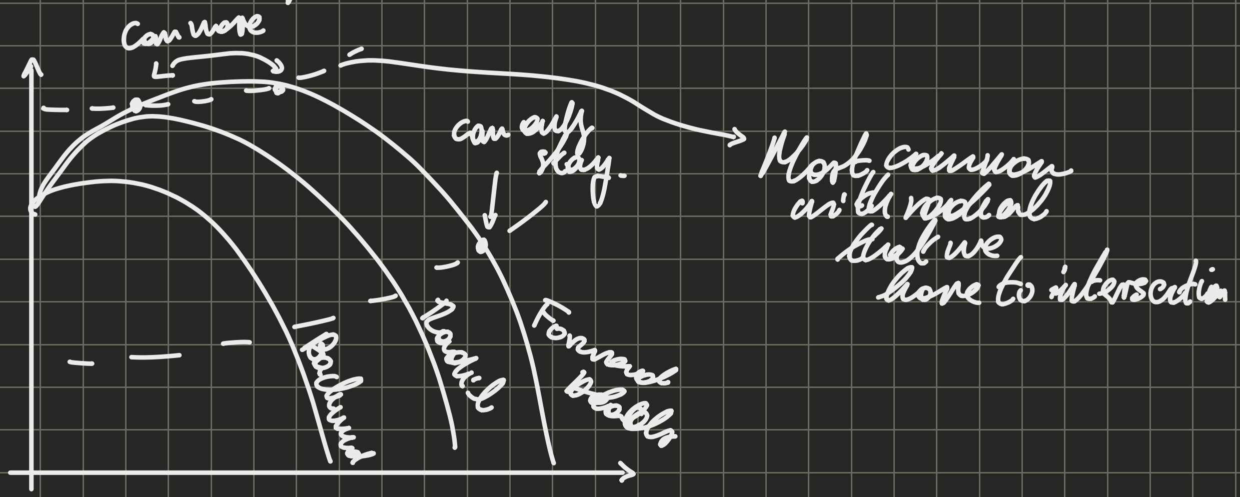
Even in vaneless we still diffuse by widening the path along the flow.

Developers make different families of pumps that cover different ranges of head and flow rate, then from there the catalogue them.

The curves they draw have different blades sizes, so they are not part of the same family.
↳ Geometric similarity is violated

Stable vs. Unstable Pump Operation.

On the descending part of curves → stable equilibrium
On the ascending part of the curve → unstable equilibrium
↳ This is unstable cause it means our curves can have two points with our machine curve.



Stable Condition: A perturbation from our stable point will not be met with a change in head produced as the system cannot produce more under the same system conditions. When the perturbation ends our system returns to it's stable condition.

Unstable Condition: An perturbation is met with an increase in head as we know that our system has a second point. This will cause the stable condition to change to the new one, and vice versa if the perturbation reduces the head.

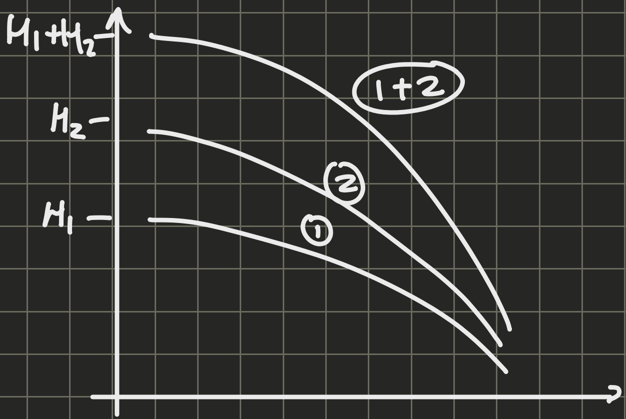
Multistage pumps

↳ can be put in parallel or series.
↳ this allows us to increase the head at each stage
↳ parallel allows us to increase the flow rate at the same head.

Pumps in Series:

$$H_{eq} = H_1 + H_2$$

$$Q_{eq} = Q_1 + Q_2$$



Pump in Parallel

$$Q_{eq} = Q_1 + Q_2$$

$$H_{eq} = H_1 + H_2$$