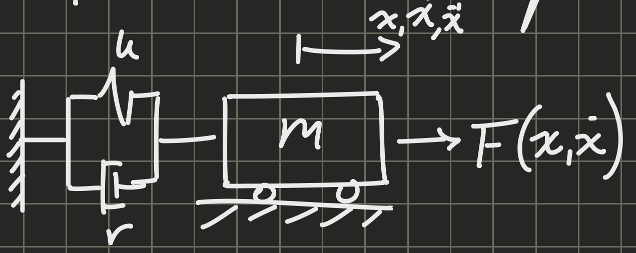
Lezione 6 - State - Dependent Forced Motion

There are time-dependent forces and state dependent forces, we look at time dependent yesterday.



Some examples are fluid dynamic forces $\left(F_D = \frac{1}{2}\rho v^2 C_D(\alpha) S\right)$, forces used by control system which function on the error which is defined as $(\varepsilon = x_{rif} - x)$

The equation of our system will be:

$$m\ddot{x} + r\dot{x} + kx = F(x,\dot{x})$$

↳ generally these forces are non-linear, which is a problem because want to linearize.

For linearizing, we need to find where to linearize. $\Rightarrow \dot{x}, \ddot{x} = 0$

To find the equilibrium position we solve the static equilibrium:

$$k x_0 = F(x_0, 0) \qquad x_0 = \text{equilibrium position}$$

$$F(x,\dot{x}) = \underbrace{F_0(x_0,0)}_{\text{const}} + \underbrace{\frac{\partial F}{\partial x}}_{\text{const}}\Bigg|_{\substack{x=x_0\\ \dot{x}=0}} \overbrace{(x-x_0)}^{\text{displacement}} + \underbrace{\frac{\partial F}{\partial \dot{x}}}_{\text{const}}\Bigg|_{\substack{x=x_0\\ \dot{x}=0}} \dot{x}$$

→ linearized the force around the equilibrium position.

Using a new coordinate system

$$\bar{x} = x - x_0 \quad \text{and} \quad \dot{\bar{x}} = \dot{x}$$

Re-writing our equation of the system

$$m\ddot{\bar{x}} + r\dot{\bar{x}} + k(\bar{x} + \cancel{x_0}) = F_0(x_0, 0) = \cancel{F_0(x_0,0)} + \underbrace{\frac{\partial F}{\partial x}\bigg|_{\substack{x=x_0\\ \dot{x}=0}}}_{-k_F} \bar{x} + \underbrace{\frac{\partial F}{\partial \dot{x}}\bigg|_{\substack{x=x_0\\ \dot{x}=0}}}_{-r_F} \dot{\bar{x}}$$

$$m\ddot{\bar{x}} + r\dot{\bar{x}} + k\bar{x} = -r_F\dot{\bar{x}} - k_F\bar{x}$$

The external forces that are state-dependent act like springs and dampers which add additional in linearized terms. components to our equation.

We don't know the sign of r_F and k_F, they may reduce the forces acting on the system, so the stiffness of the system is not known a priori.

$$m\ddot{\bar{x}} + \underbrace{(r + r_F)}_{r_T}\dot{\bar{x}} + \underbrace{(k + k_F)}_{k_T}\bar{x} = 0$$

Global damping and stiffness $\longrightarrow$ $k_T \gtreqless 0$? and $r_T \lesseqgtr 0$?

If $r_T > 0$, the system is dissipative, but if it is $r = 0$, the dampers balance each other out and there is no dissipation, if $r_T < 0$, the energy of the system increases, the oscillations increase and the everything collapses.

Lyapunov Theorem

- We can study the stability of the linear system, around the equilibrium position and if it is asymptotic, the linear one will be too.
- If non-asymptotically stable at eq. pos, we can draw any conclusion on the non-linear

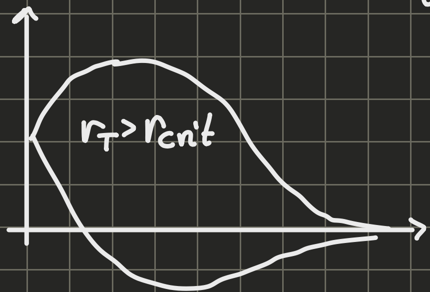
↳ If no stable, then non-linear is also not stable.

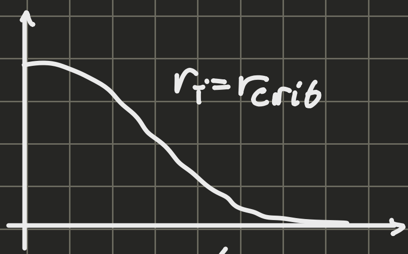
We have to study the stability:

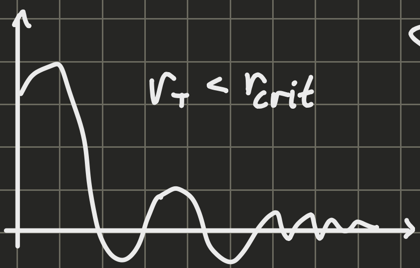
$$\left(\lambda^2 m + r_T \lambda + k_T\right) X_0 \cancel{e^{\lambda t}} = 0$$

If $r_T > 0$, $k_{T} > 0$ → already studied (free motion

3 possible scenarios





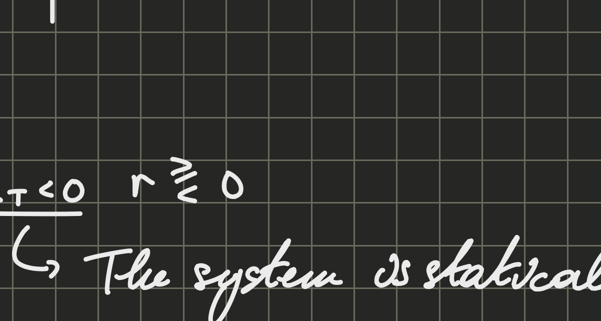


with no $i\omega_0$

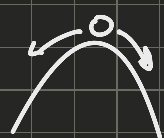
$$\lambda_{1,2} = \underbrace{\alpha}_{\in \mathbb{R}} \pm i\omega_0$$

$k_T > 0$, $r_T = 0$

↳ Solution is purely immaginary, there is no real part to λ



stable but not asymptotically, so we have to study non-linear.

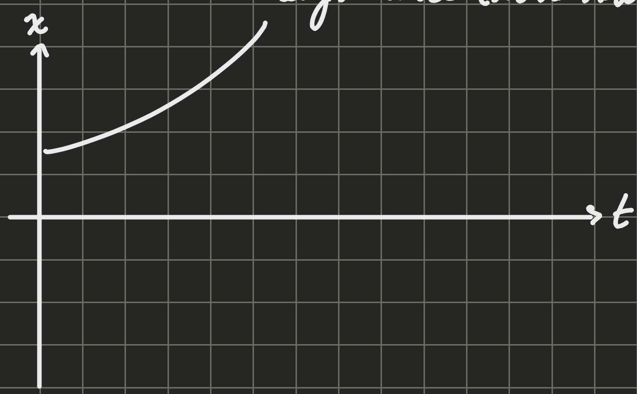


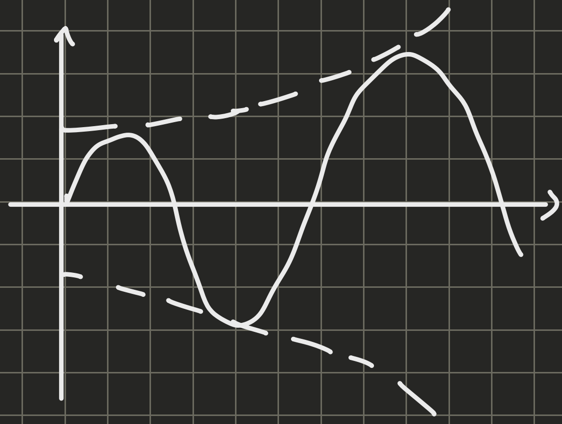
$k_T < 0$ $\quad r \gtreqless 0$

↳ The system is statically unstable, independent of the r_T

$k_T > 0$, $r_T < 0$

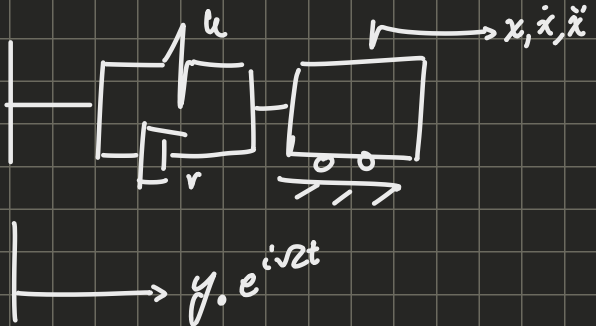
dynamic instability with oscillation





If one of the real parts of the solution is positive the system is not stable

Exercise (winda)



→ The interaction between a system and an irregular surface.

$E_c = \frac{1}{2} m \dot{x}^2$ $\qquad \Delta l = x - y$ $\qquad D = \frac{1}{2} r (\dot{x} - \dot{y})^2$

$D = \frac{1}{2} r \Delta \dot{l}^2$ $\qquad \Delta \dot{l} = \dot{x} - \dot{y}$ $\qquad V = \frac{1}{2} k (x - y)$

$V = \frac{1}{2} k \Delta l^2$

$$m\ddot{x} + r(\dot{x} - \dot{y}) + k(x - y) = 0$$

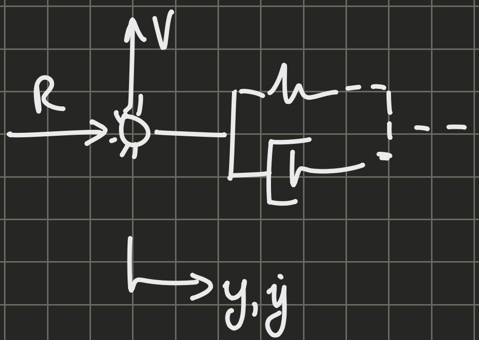
$$m\ddot{x} + r\dot{x} + kx = r\dot{y} + ky = (i\Omega r + k) y_0 e^{i\Omega t}$$ → normal equation we have already seen.

This irregularity basically forces the mass to move.

If we derived based on y, the solution becomes:

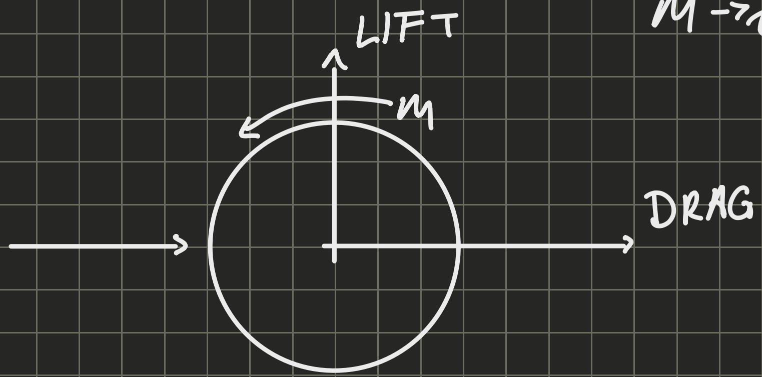
$$-r(\dot{x}-\dot{y}) - k(x-y) = R$$

We we take



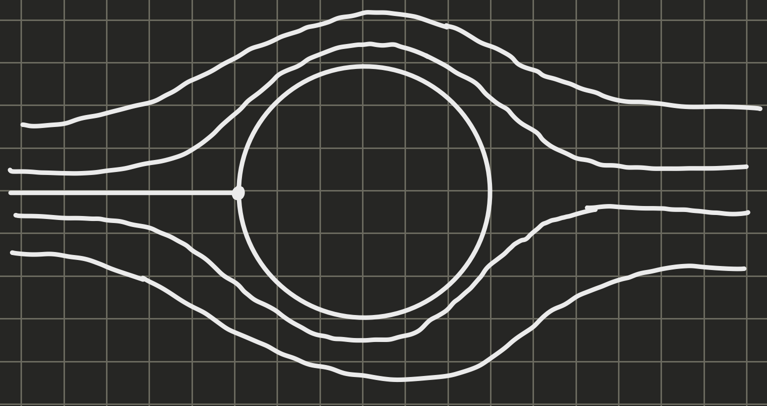
The virtual work of R is non-zero, because the mass is moving.

Aero dynamics (overview for lab on tuesday)



M → torque generated by center of gravity being off center.

A high viscosity or low speed:



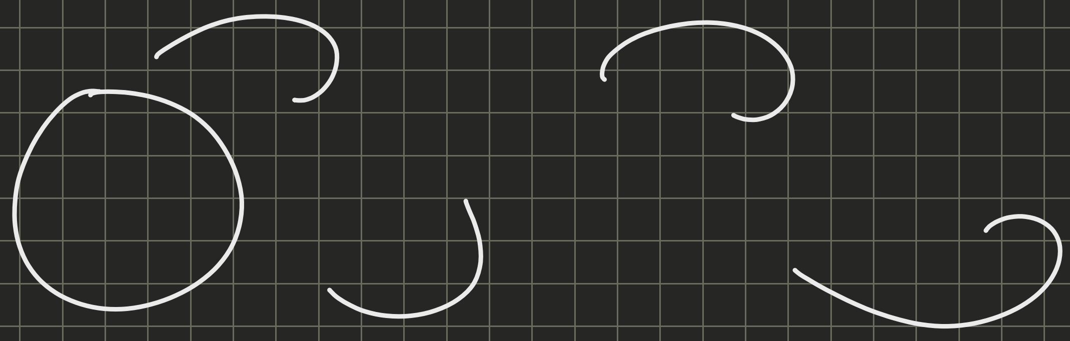
As the speed increases, we generate a wake

chaotic motion.

higher pressure   lower pressure

This pressure difference generates a force which acts on the body

$$F_D = \frac{1}{2}\rho v^2 C_D \cdot S$$

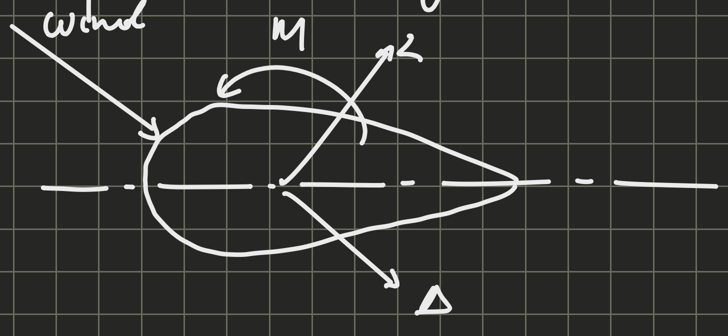


The wake is not constant, but rather it releases von Karman vortices at alternating sides.
We can kind of approximate the frequency of generation

$$f_{ST} = ST \, \frac{V}{D}$$

↳ Strual Number?

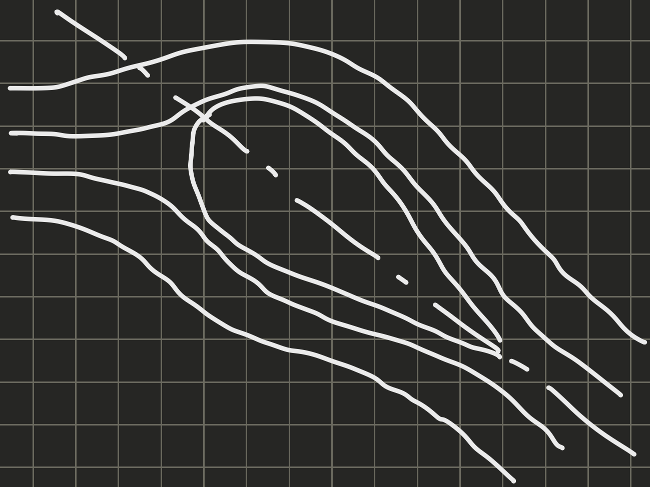
Let's consider now a symmetrical wind



$$L = \frac{1}{2}\rho v^2 C_L(\alpha) S$$

$$D = \frac{1}{2}\rho v^2 C_D(\alpha) S$$

$$M = \frac{1}{2}\rho v^2 C_M(\alpha) S \cdot b$$



Regions of higher
and lower pressure
are generated, causing
a force to be generated.