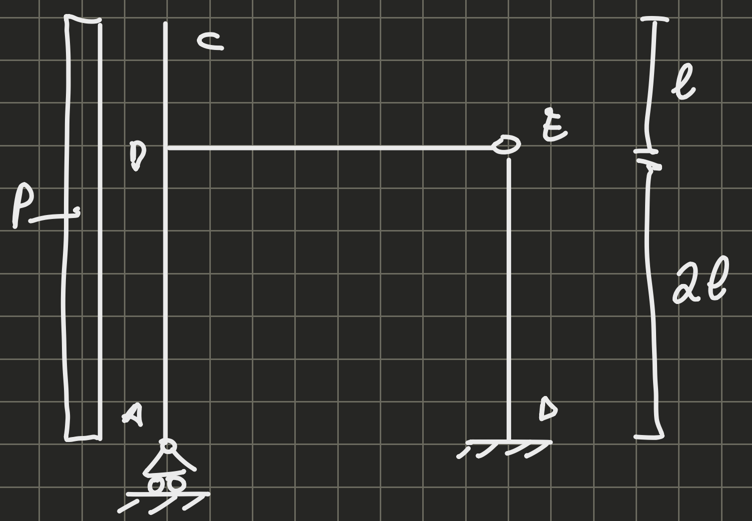
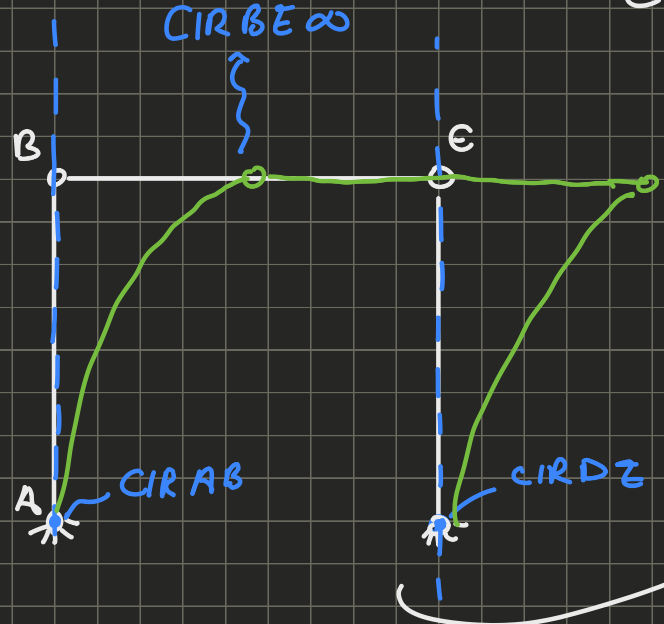
Esercitazione 6 - MDS su telai

TdE 26/7/2023

Esercizio 1.1



Analisi Cinematica



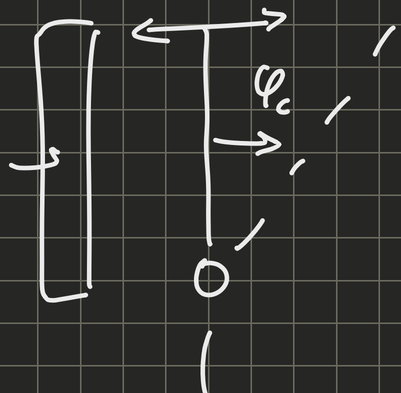
C'è labilità locale in BC, possiamo metterla a parte e risolverla.

Rotazione intorno al punto all'∞, quindi traslazione, può muoversi lungo x.

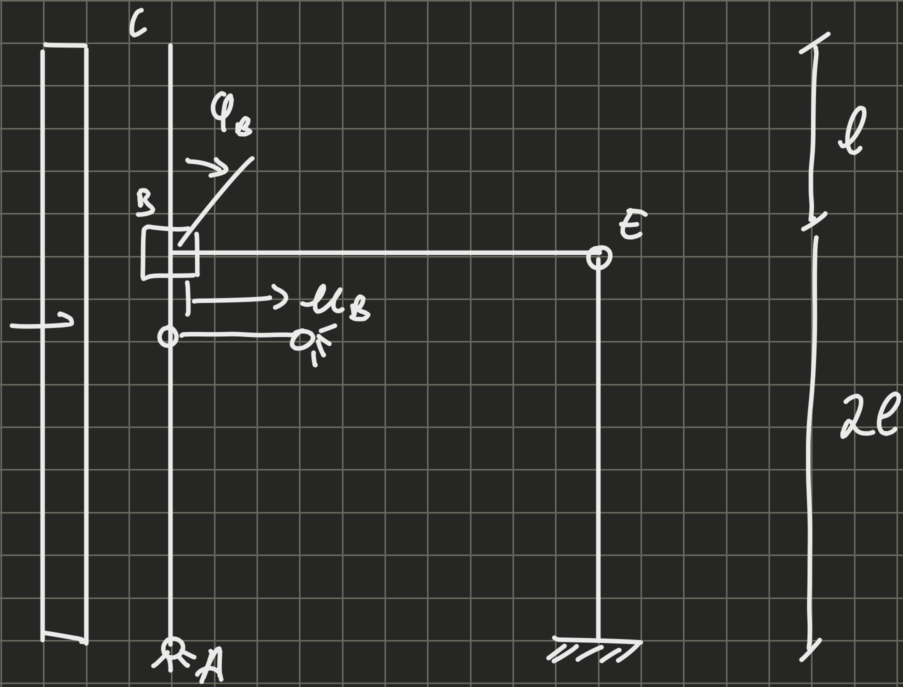
Il telaio è labile.

⟹ il telaio assegnato è a nodi spostabili,

dobbiamo vincolare lo spostamento generalizzato

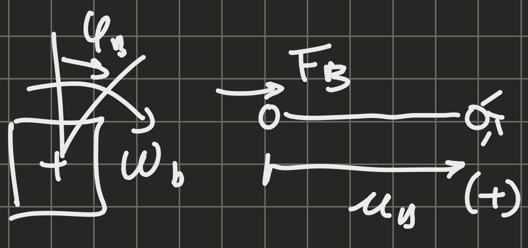


↪ Labilità locale ma non importa perché lo possiamo risolvere e poi metterlo



È a nodi spostabili quindi dobbiamo vincolare lo spostamento in B.

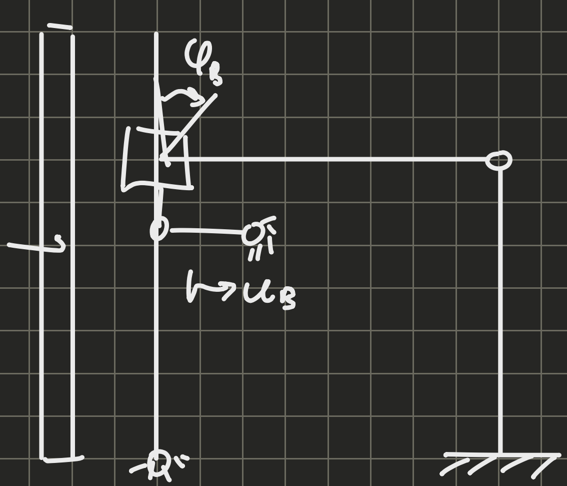
$$\underline{U} = [u_B, \varphi_B]^T$$



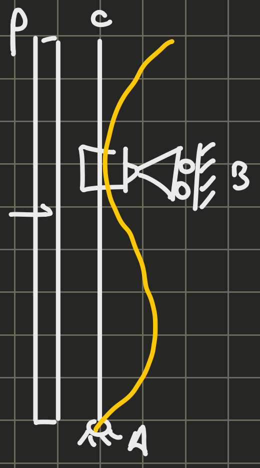
Per equilibrio sono nulle.

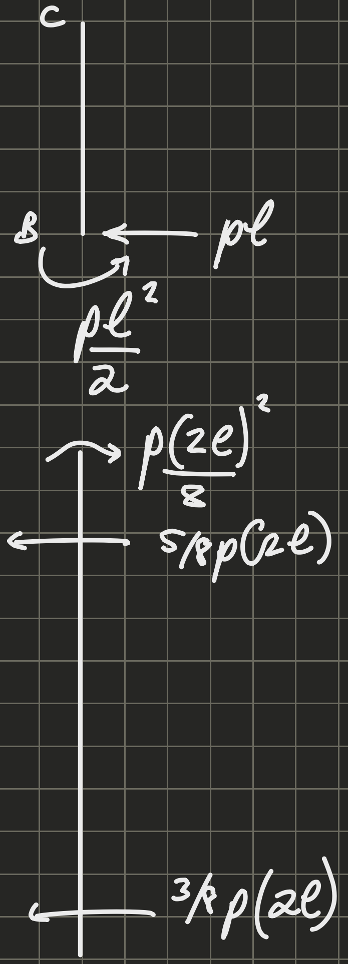
$W_b = 0$, $F_b = 0$

Schema di Calcolo

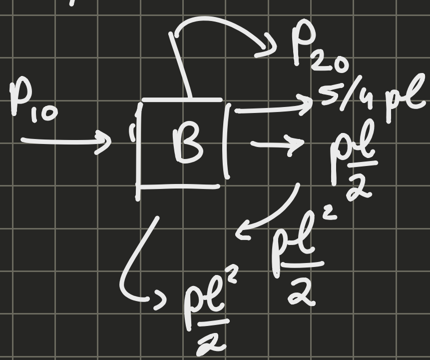


Caso "0" $(p \neq 0,\ u_B = 0,\ \varphi_B = 0)$



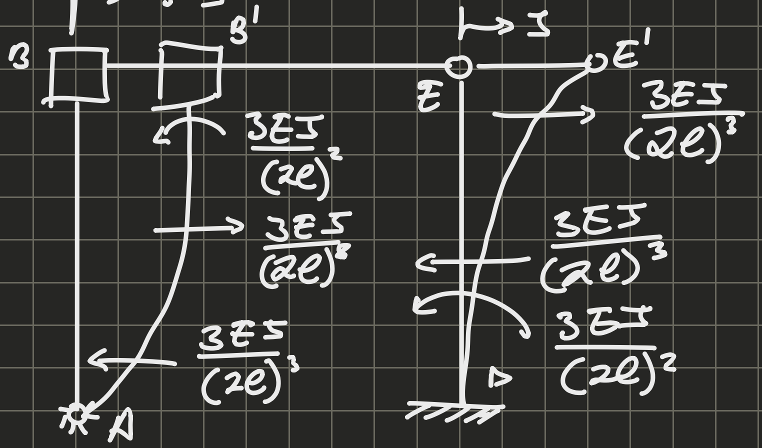


Equilibrio Morsetto in B



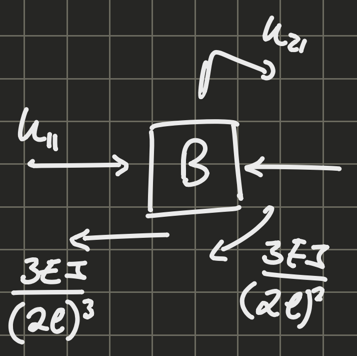
$$\begin{cases} P_{20} = 0 \\ P_{10} = -\frac{9}{4} pl \end{cases}$$

Struttura "1" $(p = 0,\ u_B = 1,\ \varphi_B = 0)$



Dobbiamo considerare questa forza di taglio in B, come azione assiale sulla asta BE

Equilibrio in B



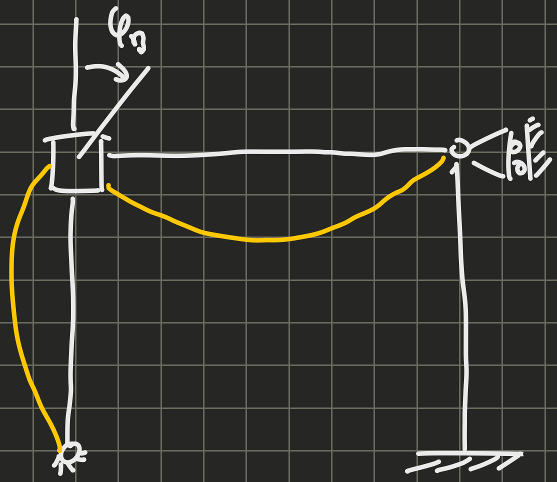
$$N_{BE} = \frac{3EJ}{(2l)^3}$$

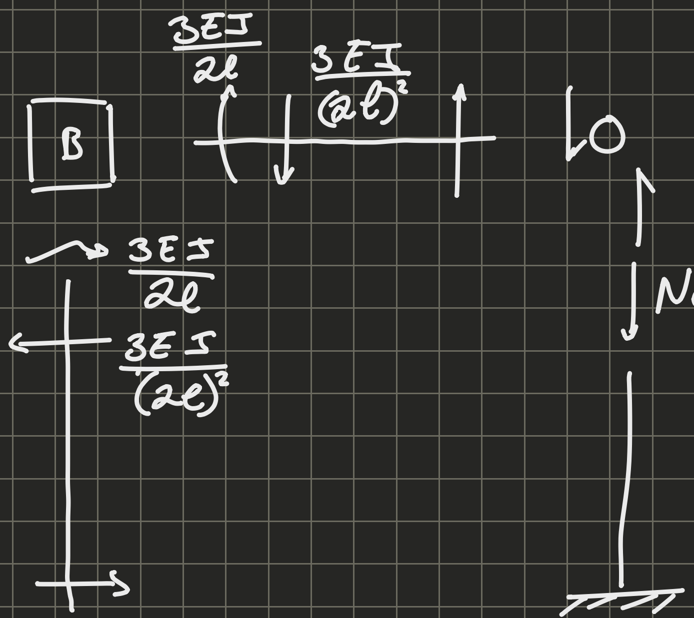
k_{11}
↳ Grado di libertà 1 in "1"
(u_B)

k_{21} → grado di libertà 2 in "1"
(φ_B)

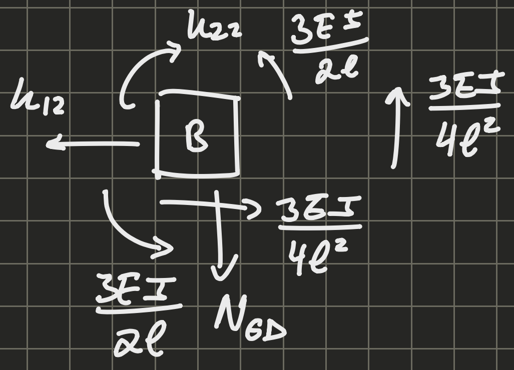
$$\begin{cases} k_{11} = 2\,\frac{3EJ}{(2l)^3} = \frac{3EJ}{4l^2} \\ k_{21} = -\frac{3EJ}{4l^3} \end{cases}$$

Struttura "2" $(p=0,\ u_B=0,\ \varphi_B=1)$





$$N_{CD} = \frac{3EJ}{4l^2}$$



$$k_{22} = \frac{3EJ}{l}$$

$$k_{12} = -\frac{3EJ}{4l^2} = k_{21} \checkmark$$

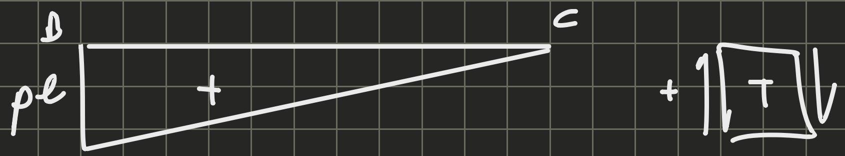
Sistema Risolvente

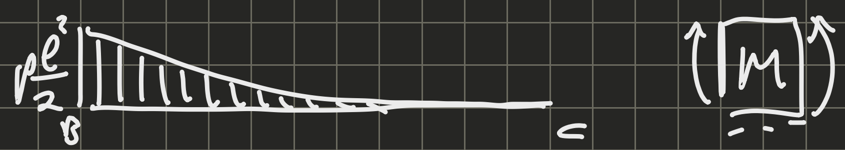
$$\underline{\underline{K}}\,\underline{U} + \underline{P} = \underline{0}$$

$$\frac{3EJ}{4l^3}\begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} u_B \\ \varphi_B l \end{bmatrix} + \frac{3}{4}pl\begin{bmatrix} -3 \\ 0 \end{bmatrix} = \underline{0}$$

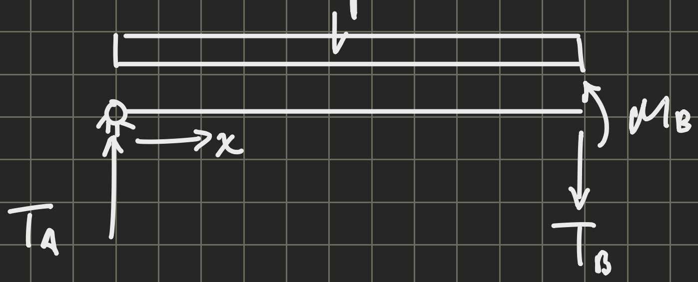
$$u_B = \frac{4pl^4}{EJ} \qquad \varphi_B = \frac{pl^3}{EJ}$$

Diagrammi azioni interne





Asta AB



$$T_A = \underbrace{\frac{3EI}{8l^3}\cdot u_B}_{"1"} - \underbrace{\frac{3EI}{8l^2}\cdot \varphi_B}_{"2"} + \underbrace{\frac{3}{4}pl}_{"0"} = \frac{3}{2}pl\ (+)$$

$$T(x) = T_A - px = \frac{3}{2}pl - px \longrightarrow T_B = T(2l) = -\frac{1}{2}pl$$

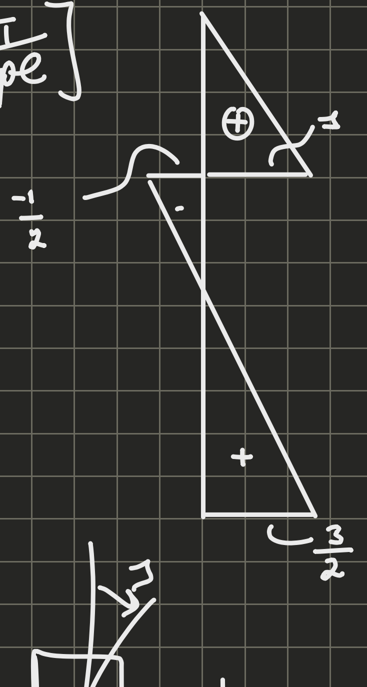
$$M(x) = T_A x - p\frac{x^2}{2} \qquad M(0) = M_A = 0$$

$$M(2l) = M_B = pl^2\ (+) \qquad M(\bar{x}) = 0 \rightarrow \bar{x} = 3l$$

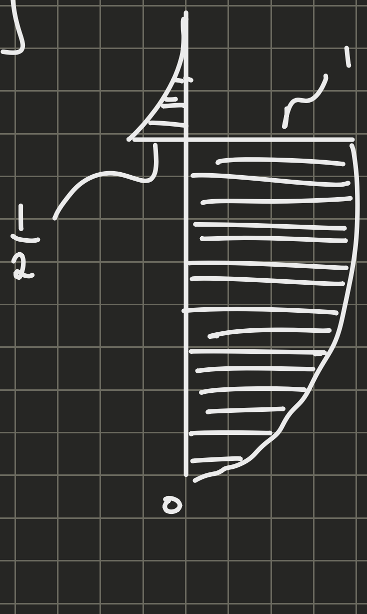
$$M_B = \underbrace{\frac{3EI}{4l^2}u_B}_{"2"} - \underbrace{\frac{3EI}{2l}\varphi_B}_{"4"} - \underbrace{\frac{1}{2}pl^2}_{"0"} = pl^2 \checkmark$$

→ se scriviamo come sovrapposizione

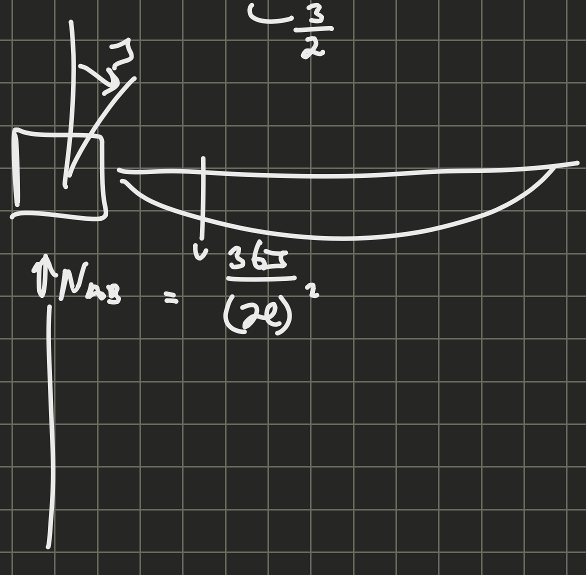
$\left[\frac{T}{pl}\right]$



$\left[\frac{M}{pl^2}\right]$



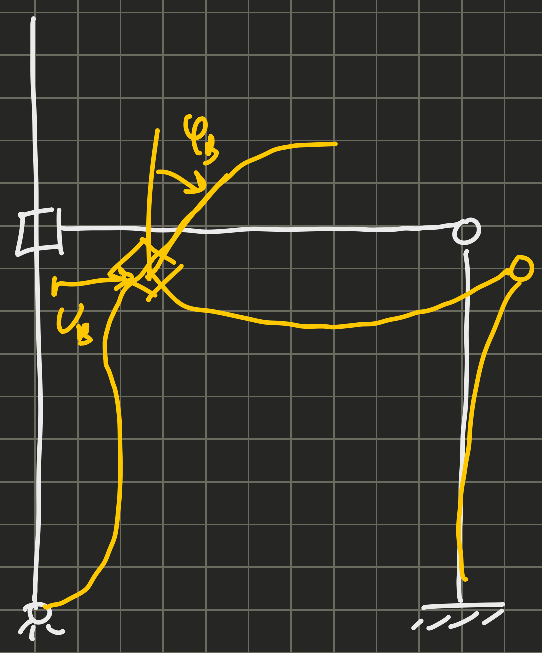
N/pl



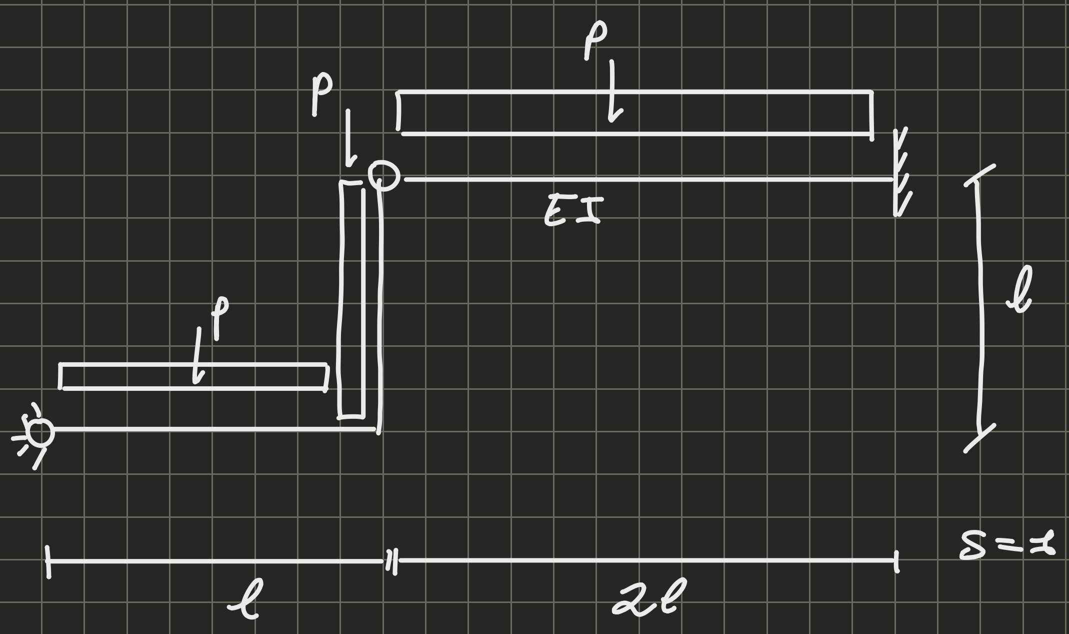
$\uparrow N_{AB} = \frac{3EI}{(2l)^2}$

$$N_{AB} = \frac{3EI}{4l^2}$$

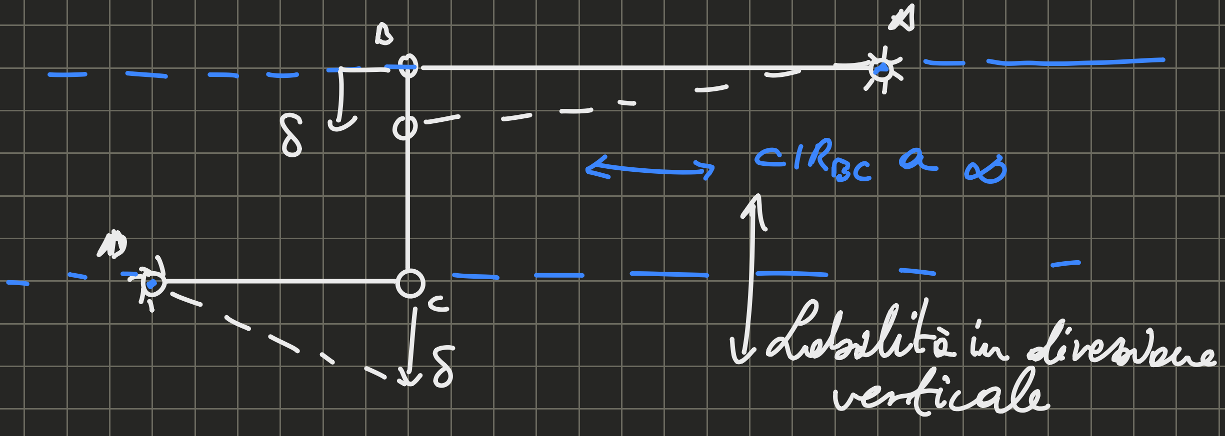
Deformata Qualitativa



TdE 13/02/2023

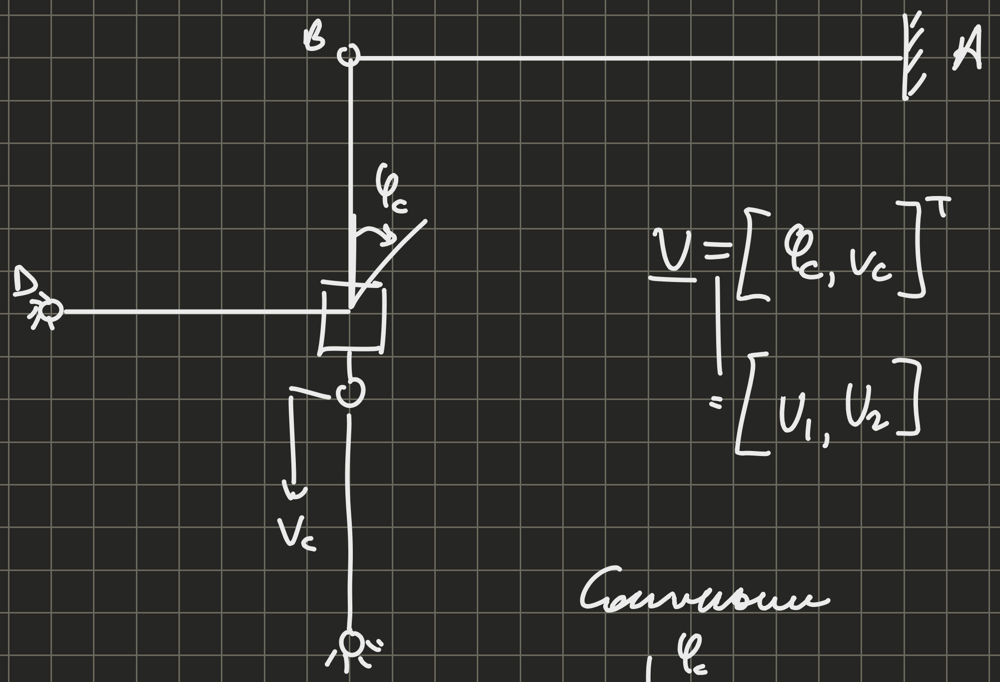


Analisi Cinematica



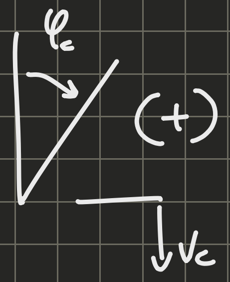
$\Longrightarrow$ Telaio assegnato a nodi spostabili:

Struttura Cinematicamente Determinata

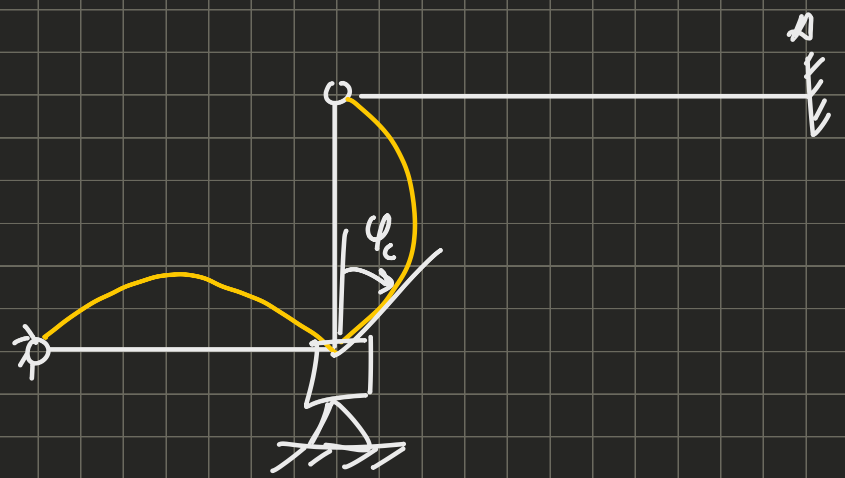


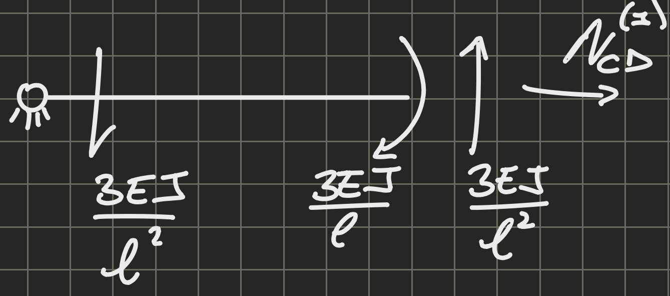
$$\underline{U} = [\varphi_c, v_c]^T = [U_1, U_2]$$

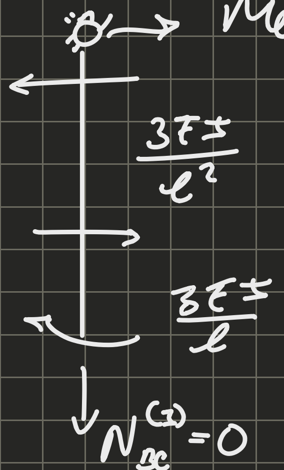
Convenzione



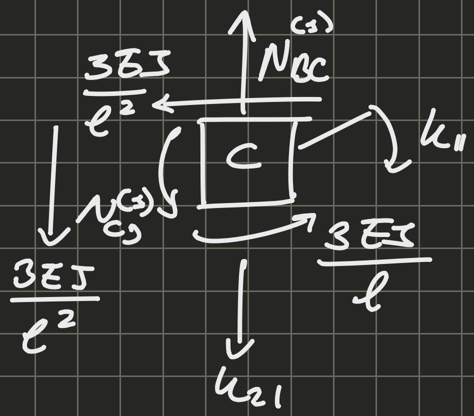
Struttura "1" $\quad p=0,\ U_1 = \varphi_c = 1,\ U_2 = v_c = 0$







Mettiamo a terra perché su AB non c'è deformata



$$
\begin{cases}
k_{11} = 2 \cdot \frac{3EJ}{l} = \frac{6EJ}{l} > 0 \\
N_{CD}^{(1)} = -\frac{3EJ}{l^2} \text{ compressione in CD} \\
k_{21} = -\frac{3EJ}{l^2}
\end{cases}
$$