Lezione 20 – Completion of Thermodynamics and Gas Dynamics (Gas in ducts and Mach number)

Recall:

Adiabatic Irreversible Compression:

We got the expressions for the work

$$l_s = \frac{\gamma}{\gamma-1}\frac{R}{M}T_1\left(\beta^{\frac{\gamma-1}{\gamma}}-1\right)$$

additional heat loss we treat as reversible

n, since polytropic, takes into account q_{REV}

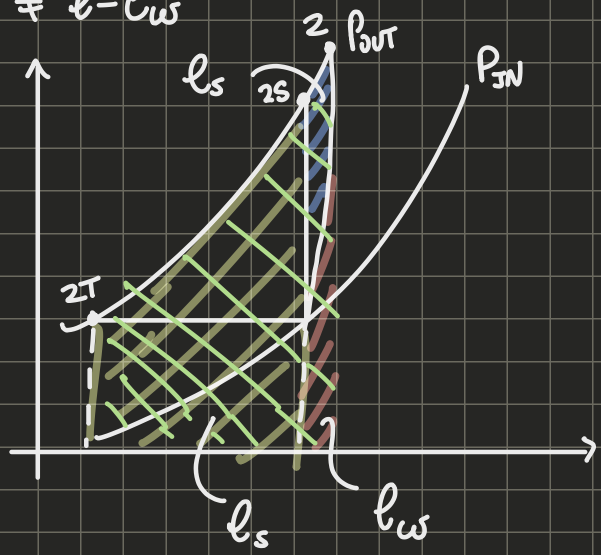
$$l = \frac{\gamma}{\gamma-1}\frac{R}{M}T_1\left(\beta^{\frac{n-1}{n}}-1\right) \qquad [n>\gamma]$$

$$l - l_w = \frac{n}{n-1}\frac{R}{M}T_1\left(\beta^{\frac{n-1}{n}}-1\right)$$

↳ Technical work $\int_1^2 v\,dP$, since Δv^2 and $\Delta z \approx 0$

We also commented that:

$l_s \neq l - l_w$



Not really correct to draw, since it is irreversible so it shouldn't be drawn.

Reversible indirect effect of wasted work

↓

Re-heat

↓

$$\Longrightarrow\ l = l_s + l_w + l_{RH}$$

$$l_{RH} = (l - l_w) - l_s$$

$$= \int_1^2 v(s,P)\,dP - \int_1^{2S} v(s_1,P)\,dP$$

$>$

↳ re-heat is cause by difference in specific volume due to heating, which causes it to be larger than the isoentropic case.

Since we have an additional work, there are more parameters of efficiency/definition of efficiency:

$$\eta_{IS} = \frac{l_s}{l} = \frac{\beta^{\frac{\gamma-1}{\gamma}}-1}{\beta^{\frac{n-1}{n}}-} \rightarrow \text{Isoentropic efficiency}$$

But this is no consistent/equivalent to what we have used for pumps, but there is a efficiency that resembles it:

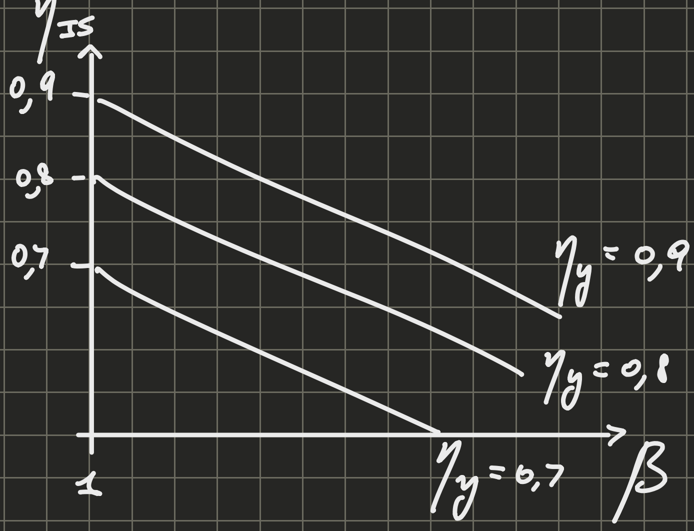
$$\eta_y = \frac{l-l_w}{l} = \frac{n}{n-1} \cdot \frac{\gamma-1}{\gamma} \rightarrow \text{Polytropic Efficiency}$$

For compressors: $\eta_y > \eta_{IS}$

η_{IS} is preferred since:

$$\eta_{IS} = \frac{\beta^{\frac{\gamma-1}{\gamma}}-1}{\beta^{\frac{\gamma-1}{\gamma\cdot\eta_y}}-1}$$

$$\frac{n-1}{n} = \frac{\gamma-1}{\gamma} \cdot \frac{1}{\eta_y}$$



If $\beta\uparrow$, $\Delta(\rho, v)\uparrow$

$$\Rightarrow \int_1^2 v(s,P)\,dP > \int_1^{2s} v(s,P)\,dP$$

$$\Rightarrow l_{RH}\uparrow \Rightarrow \eta_{IS}\downarrow$$

If $\beta \to 1$; $\rho, v = \text{const}$

$$\Rightarrow \int_1^2 v(s,P)\,dP \longrightarrow \int_1^{2s} v(s_1,P)\,dP$$

The compressor becomes a fan

$\Rightarrow l_{RH} \to 0$ and $\eta_y \to \eta_{IS}$

→ As soon as the compressibility increases, the losses appear and both v increase, so increase in $\int_1^2 v\,dP$ so increase in l_{RH} and decrease in η_{IS}

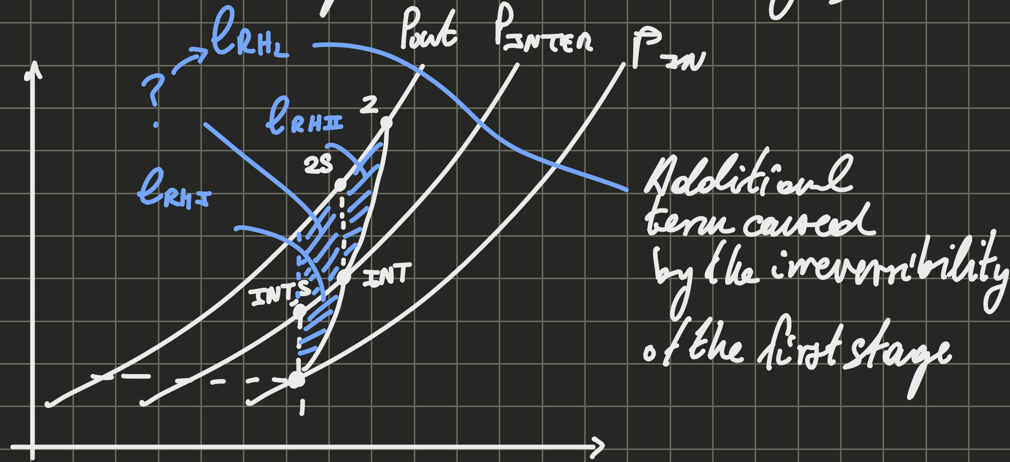
η_y is preferred since it considers l_{RH}, and so it does not penalize higher β, since they will have greater l_{RH}, but this is a consequence of aerodynamics and is not an indicator of the equality of the machine.

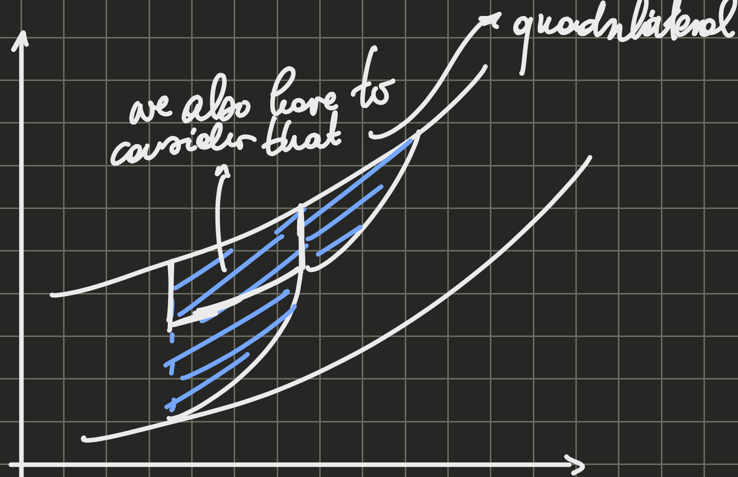
Since $\eta_y = 1 - \frac{l_w}{l}$, it shows only what is in the hands of the designer, which doesn't include l_{RH}, since that is a consequence of thermodynamics and is something the designer cannot control.

Adiabatic Irreversible Compression (Multistage)

$$\frac{P_{OUT}}{P_{IN}} = \frac{P_{INTER}}{P_{IN}} \cdot \frac{P_{OUT}}{P_{INTER}}$$

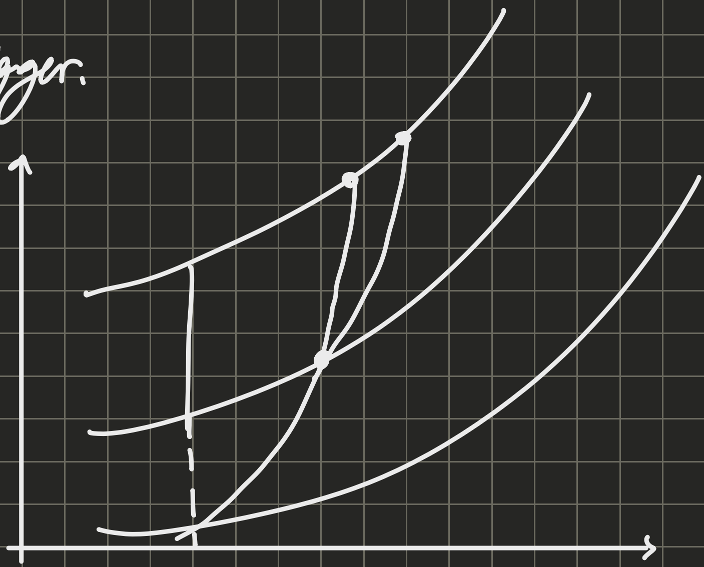
(2 stages)





⇒ when combining stages. $l_{RH} \neq l_{RH1} + l_{RH2}$

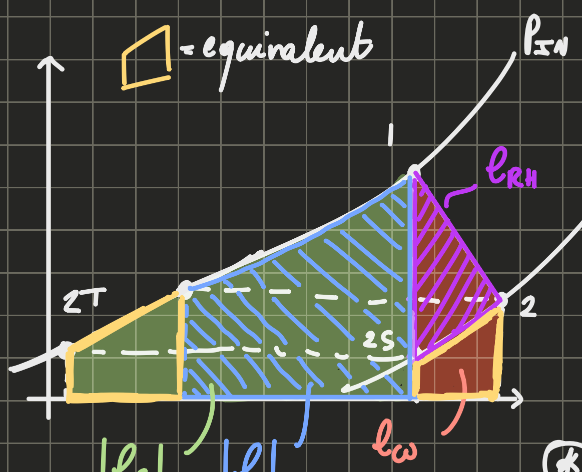
The additional reheat, is a legacy of the first transformation not being isoentropic, so the isoentropic work exchanged, from the fact we start from 1NT rather than 1NTS, is bigger.



In a multistage machine, the irreversibility causes an additional loss at the stages after. So to optimise, we want to make sure to optimise the initial stages, so that we don't pay in the latter stages.
↳ clearly.

Adiabatic Irreversible Expansion ($P_{IN} > P_{OUT}$)

$\beta = \frac{P_{IN}}{P_{OUT}}$
↳ same variable, different definition, will still be > 1.



Isobar work from 1 to 2.

(+) by thermodynamics

We once again use an isobaric between 1 and 2T to map the isoentropic work and also the work for a going to its isotherm, (*) but in this case the area to the right of 1 that takes us away from 2S is l_w.

In gas expansion: part of the l_w is recovered by l_{RH}, so it is useful.

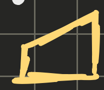
$$\eta_{IS} = \frac{|l|}{|l_s|} = \frac{\beta^{\frac{n-1}{n}}-1}{\beta^{\frac{\gamma-1}{\gamma}}-1}$$

(*) = the losses, l_w, are what cause us to diverge from 2s to 2.

$$\eta_y = \frac{l}{l - l_w} = \frac{-|l|}{-|l| - l_w} = \frac{|l|}{|l| + l_w}$$

$$= \frac{\gamma}{\gamma-1} \cdot \frac{n-1}{n}$$

since $l_s = |l| + l_w - l_{RH}$

Since $l_s - l$ is the work which we effectively lose during the process, but l_w is bigger. We find that the two areas  are equivalent, so the triangle above is not lost, it is recovered at the end of the process thanks to the reheat.

This also tells us that in expansion l_{RH} counteract l_w.

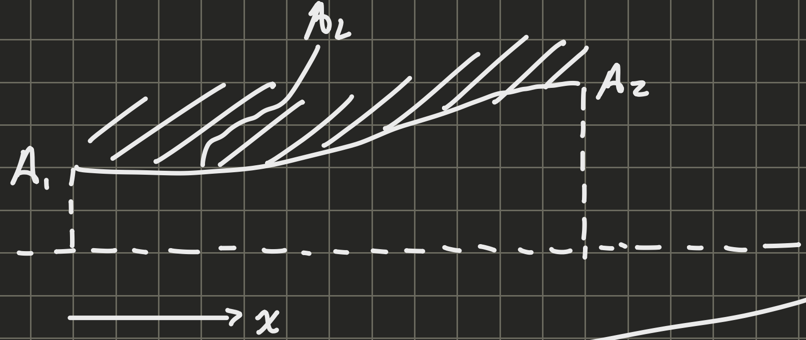
For turbines $n < \gamma$, and $\eta_{IS} > \eta_y$

In multistage turbines, the legacy re-heat acts to increase the efficiency.

→ The triangle is still wasted but it is recovered through l_{RH} so it is a positive effect in turbines.

Gas Dynamics → Analysis of compressible fluid flow.

Compressible flow in ducts



To be able to study nozzles and diffusers

Assumptions:

→ Duct with area change ($\frac{dA}{dx} \neq 0$, but very low)

→ $\vec{V} = V_x \hat{i}_x = V_x$: mono-directional flow

→ $V_r \approx 0$ → radial components

→ $V_t \approx 0$, no swirling

→ we say all quantities are functions of x, so our flow is mono-dimensional

→ $q \approx 0$ → heat exchange negligible (in general, not always

→ $\Delta z = 0$

We assume we have friction (irreversible).

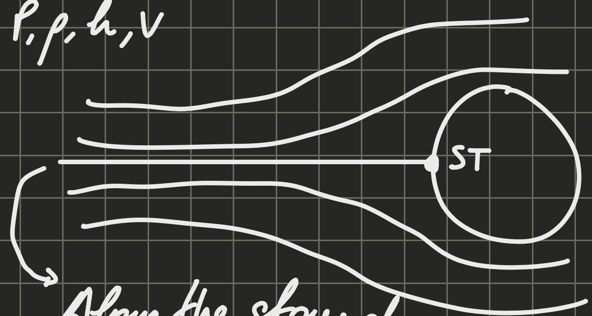
Balances: $l = 0$, no machine and no work on duct

These quantities are conserved.

$$\begin{cases} \dot{m} = \text{const} \Rightarrow \rho_1 v_1 A_1 = \rho_2 v_2 A_2 = \rho(x)\, v(x)\, A(x) \\ \cancel{l}^{\,0} + \cancel{q}^{\,0} = \Delta h + \frac{\Delta v^2}{2} + g\underbrace{\cancel{\Delta z}}_{0} \Rightarrow h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} = h(x) + \frac{v(x)^2}{2} = h_T(x) \\ \dot{m}(\vec{v}_2 - \vec{v}_1)\hat{i}_x + (P_2 \vec{n}_2 A_2 + P_1 \vec{n}_1 A_1)\hat{i}_x = \int_{A_2} \vec{\sigma}\, \hat{i}_x \, dA \end{cases}$$

B. MOM → Only useful for sizing duct, it doesn't help us.

P, ρ, h, v



Along the stagnation streamline, the energy is conserved

$$h + \frac{v^2}{2} = h_{st} + \underbrace{\cancel{\frac{v_{ST}^2}{2}}}_{0} \triangleq h_T$$

→ overall measure of the energy of the fluid

Instead of the B. MOM, we use the equation for a polytropic transformation, since we need another equation that is conserved:

$$\frac{P(x)}{\rho(x)^n} = \text{const}$$

Balance of Energy:
- → Adiabatic (no need for isoentropic)
- → no work

→ no difference between isoentropic or frictional decelleration.

Our system therefore is:

$$\begin{cases} \rho(x)\, v(x)\, A(x) = \text{const} \ (\dot{m}) \\ h(x) + \frac{v^2(x)}{2} = \text{const} \ (h_T) \\ P(x) / \rho(x)^n = \text{const} \end{cases}$$

we need boundary conditions to be able to completely solve it.

$$+\begin{cases} P/\rho = RT \\ \Delta h = c_p \Delta T \end{cases} \Rightarrow \text{solution.}$$

Stop this for now, we define Mach number to give a functional form to our solution.

<u>Definition of speed of "sound"</u>

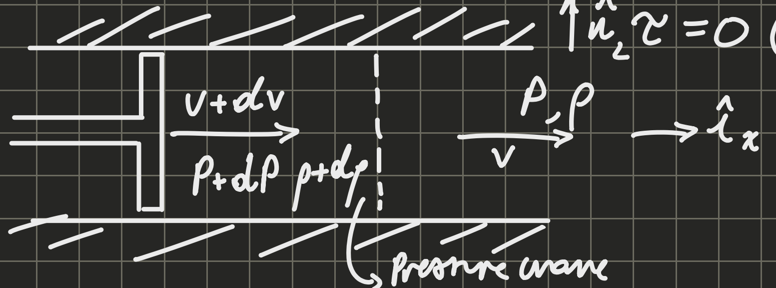
↳ speed at which propagation waves move in a fluid.

↳ if something moves faster that this speed, then there a parts which never know that there has been change.

"sound" → low amplitude pressure waves

low $\Delta P \to dP, d\rho, dV$ low $\to \Delta s \cong 0$ ($\sim$ isoentropic)

Let's imagine a straight cylindrical duct.



$\uparrow \hat{n}_2 \; \tau = 0$ (since $\Delta s = 0$)

$$\hat{n}_L \perp \hat{i}_x \Rightarrow \int_{A_L} \vec{\tau} \cdot \hat{i}_x \, dA = 0$$

The fluid accelerates because it is informed by the pressure wave that travels through the duct.

The wave changes the fluid to adhere to the new boundary conditions. After some time the whole fluid knows of the change.

We want to determine the speed of the pressure wave:

a : speed of sound

The single form of the mass balance cannot be used since we are not in steady state, but can be linearised easily.
But if we move with speed of the wave, the problem is steady from that perspective.
If we are integral with the motion, we can write the balances like we have seen.

From an inertial frame:

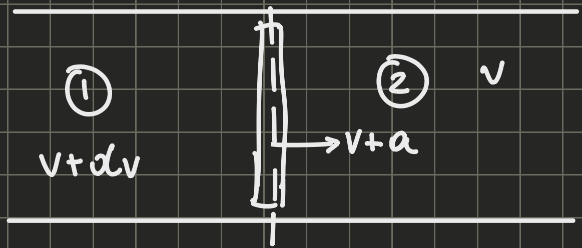
since fluid is already in motion

$u = v + a$

$w_1 = v_1 - u$

$= \cancel{v} + dv - \cancel{v} - a = dv - a$

$w_2 = v_2 - u = v - v - a = -a$



Mass Balance:

$$\rho_1 w_1 \cancel{A_1} = \rho_2 w_2 \cancel{A_2} \Rightarrow (\rho + d\rho)(dv - a) = \rho(-a)$$

≈0 → order of magnitude lower.

$$\Rightarrow \rho\, dv - \cancel{a\rho} + \cancel{d\rho\, dv} - d\rho\, a = -\cancel{\rho a}$$

$$\Rightarrow \rho\, dv = a\, d\rho$$

Momentum Balance on $\hat{i}x$:

As we said $\int \sigma \hat{i}_x dA = 0$

$$\dot{m}(\vec{w}_2 - \vec{w}_1) + P_2 A_2 - P_1 A_1 = 0$$

$$\Rightarrow -\rho a(-\cancel{a} + dv + \cancel{a})\cancel{A} + (\cancel{P} - \cancel{P} - dP)\cancel{A} = 0$$

$$\Rightarrow \rho a\, dv = dP \rightarrow dv = \frac{dP}{\rho a}$$

$$\Rightarrow \frac{dP}{a} = a\, d\rho$$

The velocity has dissapeared, so a only depends on $\frac{dP}{d\rho}$

$$\Rightarrow a^2 = \left(\frac{\partial P}{\partial \rho}\right)_s \Rightarrow a = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s}$$

→ valid for any fluid, since we need introduce a thermodynamic model for the type of fluid.

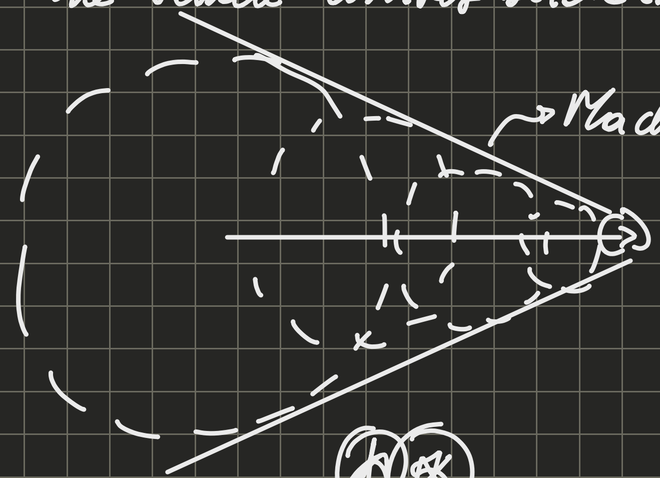
$$M = \frac{V}{a}$$ → Mach Number

V → flow velocity

If $M=0$, the sound is heard radially.

If $M<1$, we are in subsonic conditions, the sound is still heard radially but there might be some delay. (A)

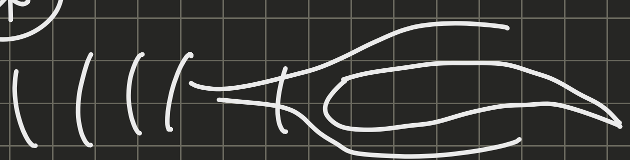
If $M>1$, we are in supersonic conditions, the vehicle travels faster the that the sound, so the vehicle arrives before it is heard



→ Mach cone, this is the area within which the sound has had time to travel, so it's the area within which the vehicle is heard.

(A) (✱)

(✱)



The fluid knows the vehicle is coming, so it is able to change to adhere to the vehicle.

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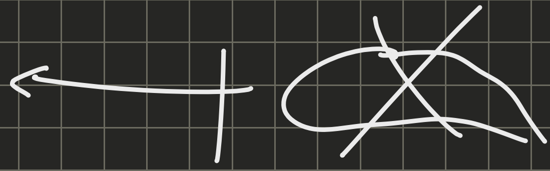
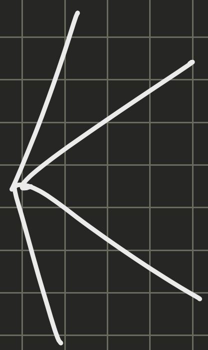


The fluid is not able to move so it sticks and increase drag.



→ so we developed this shape to reduce drag as much as possible.