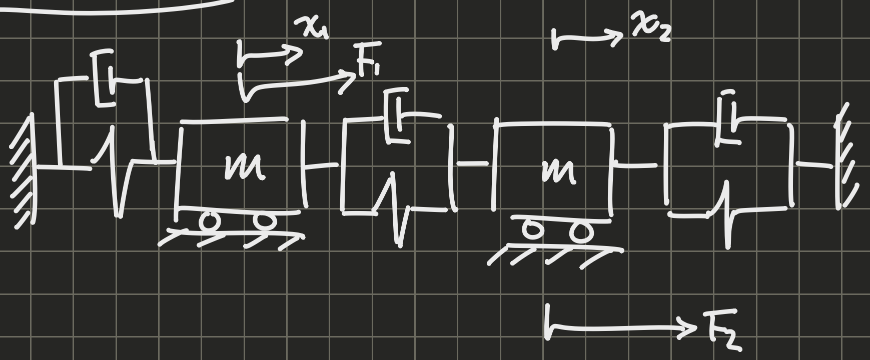
Lezione 12 -



We can use any 2 dof system

Frequency of forcing.

if we only have one

$$[M]\ddot{\underline{x}} + [R]\dot{\underline{x}} + [k]\underline{x} = \underline{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} e^{i\Omega t}$$

$$[M(\Omega)]\underline{x} = \underline{F}$$

In 1dof the FRF is single input, single output.

In n-dof system the FRF connects every input with every output, so multiple inputs and outputs.

Can we change from these degrees of freedom to another system? Like the beam

$$[\phi]\underline{x} = q \qquad [\phi]^T \underline{F}$$

$$\begin{bmatrix} m & \\ & \ddots \end{bmatrix}\ddot{q} + \begin{bmatrix} r & \\ & \ddots \end{bmatrix}\dot{q} + \begin{bmatrix} k & \\ & \ddots \end{bmatrix} q = \underline{Q}$$

coupled

This changes n equations to n independent equations. Thus is simpler to calculate, and doing considerations on $\underline{Q}$ we can reduce the number of equations we are considering.

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix} \begin{matrix} \rightarrow \text{Mode 1, moving together} \\ \rightarrow \text{Mode 2, moving opposite} \end{matrix}$$

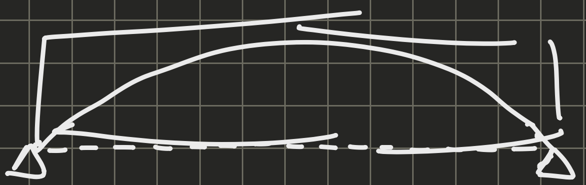
→ Free motion equation are only important in transition state.

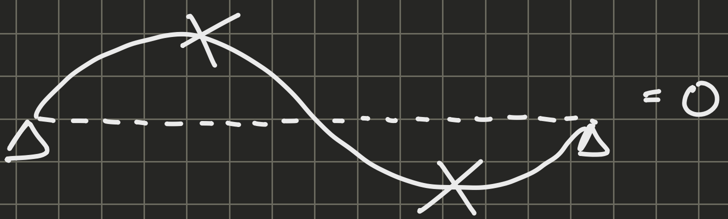
→ These forces are acting only on mode 1 and not mode 2.

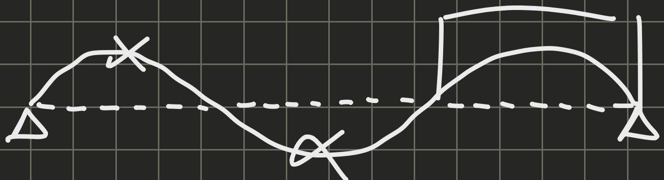
$$\underline{Q} = [\Phi]^T \underline{F}$$

These forces are not forcing the second mode of vibration.

We can discard some equation since they lagrangian component is 0.



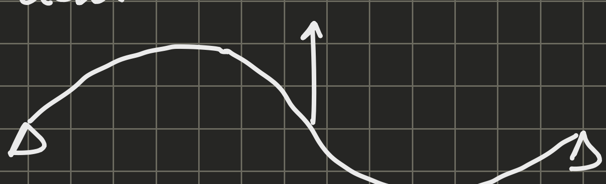




Let's immagine the forcing:

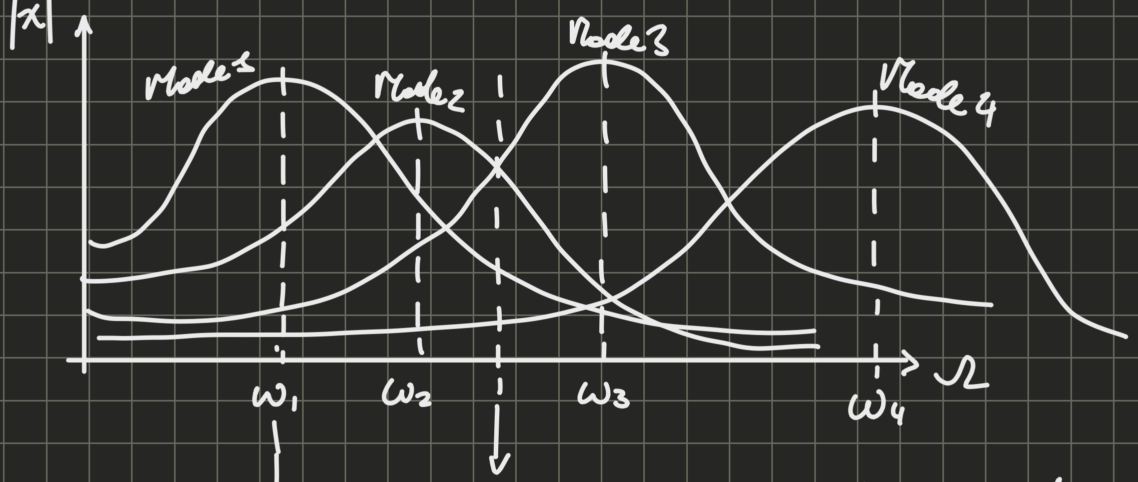
If Forcing

This cannot force even numbers, since the point is not in motion.



This is the power of using the modal approach, it immediately tells us what will happen.

If we are interested in damping we cannot put it in the middle since we would not be able to dampen even modes. So we have to put it near the constraint to capture more modes.



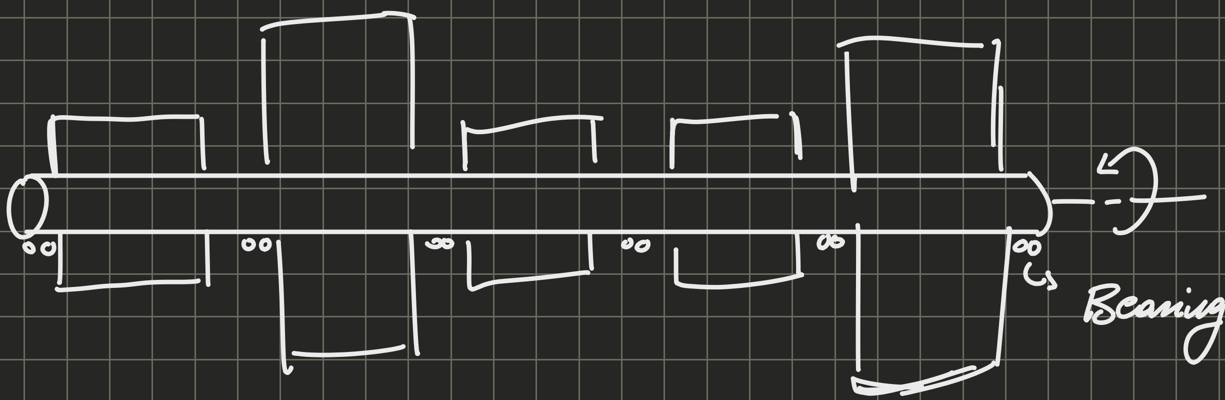
If Q is here, we will only have great contributions but mode 2 and mode 3.

If Q_1 is here since 1 is in resonance, the rest are not interesting.

In general we look at the modes where $\omega < 2\Omega$, since the other modes are in quasi-static state.

The usefulness of switching from $\underline{x}$ to q is that it allows us to simplify the problem not just by diagonalising but by reducing the equations that we consider, where $\omega > 2\Omega$ and where $Q = 0$

<u>Vibrations on Rotors</u>

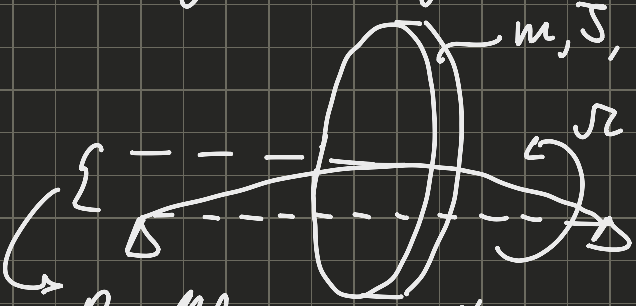


It's important for the center of gravity to be exactly on the axis, but this is near impossible

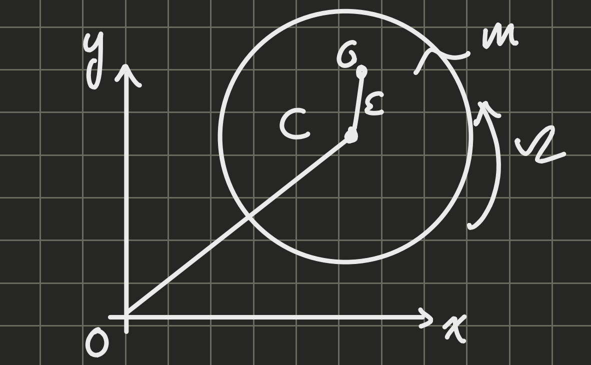
There are many factors which can cause the center of mass to not be alligned with the axis of rotation.
All the factors are producing effects that can be modelled with heavy eccentricity.

What happens with a high-rotational velocity system with excerentricity?

We can use a Jefcott Rotor as a simple example to analyze this system:



Parallel so we don't consider gyroscopic effect.



$$G-O = (G-C)+(C-O)$$

$$x_G = x_C + \varepsilon \cos \Omega t$$

$$y_G = y_C + \varepsilon \sin \Omega t$$

$$\ddot{x}_G = \ddot{x}_C - \Omega^2 \varepsilon \cos \Omega t$$

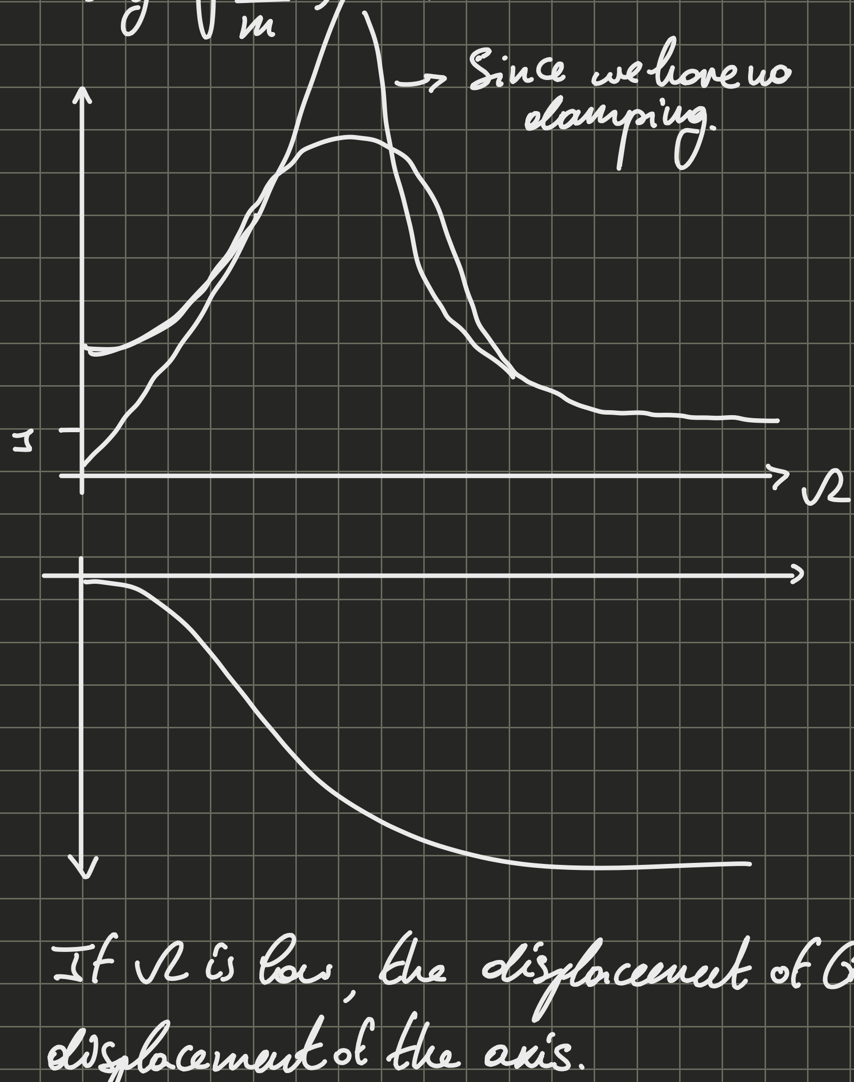
$$\ddot{y}_G = \ddot{y}_C - \Omega^2 \varepsilon \sin \Omega t$$

Our equation of motion along one degree of freedom (we don't care about both), will be:

$$m\ddot{x} + kx = -\Omega^2 m \varepsilon \cos \Omega t$$

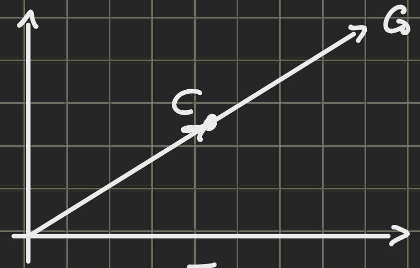
$$\left.\begin{aligned} \omega_x &= \sqrt{\frac{k}{m}} \\ \omega_y &= \sqrt{\frac{k}{m}} \end{aligned}\right\} \text{System response is same}$$

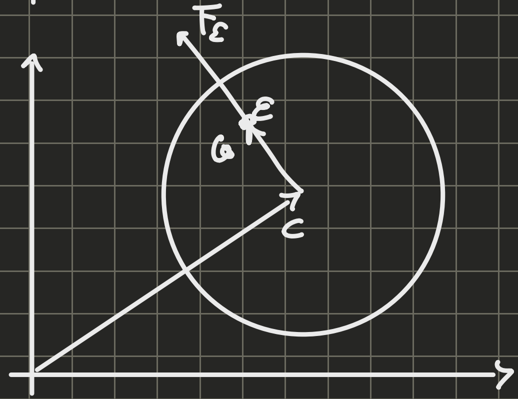
If $\Omega = \omega$, we have resonance



The beam acts like a spring.

If Ω is low, the displacement of G alligns with the displacement of the axis.

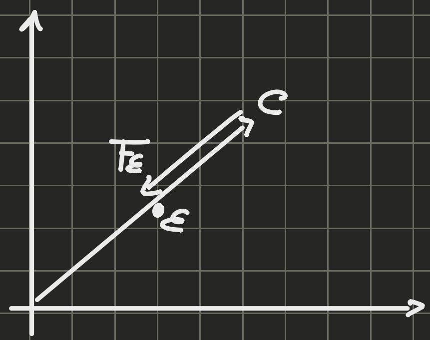




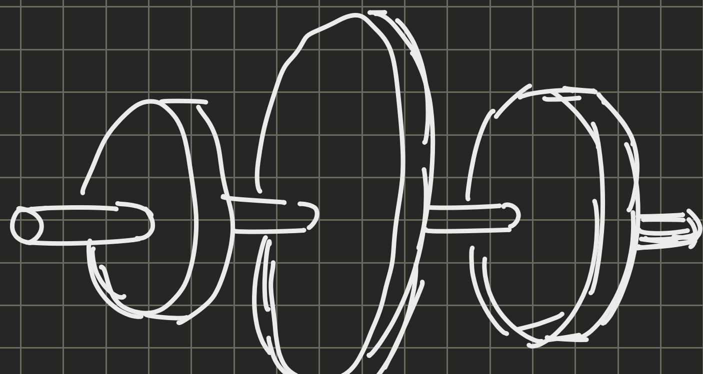
For resonance:

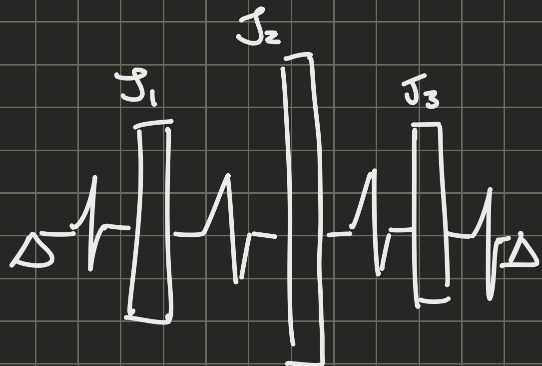
Forces generated by the eccentricity is completely misalligned with the motion of the axis.

When we increase even further, the forces will acts against the misallignment, causing the reallignment of C and the axis.



To avoid the resonance phase many machines have systems to accelerate quickly and decelerate quickly to reduce the transient.
startup and stopping



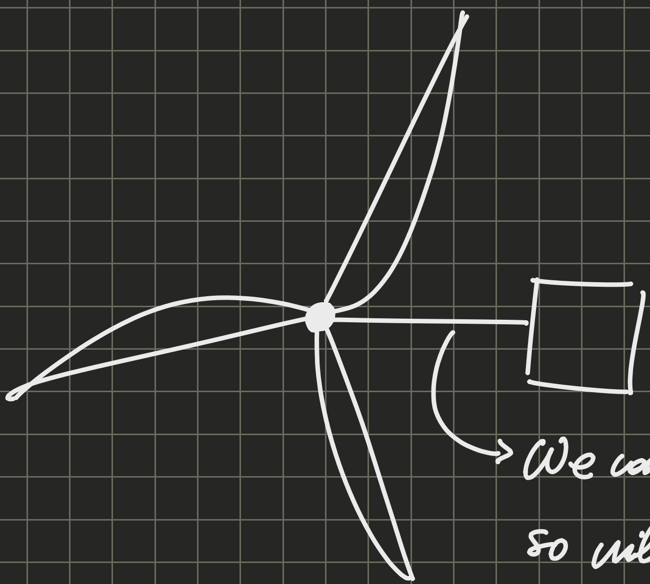


$E_{ci} = \frac{1}{2} J_i \dot{\theta}_i$

$\Delta\theta_1 = \theta$

$\Delta\theta_2 = \theta_2 - \theta_1$

$\Delta\theta_3 = \theta_3 - \theta_2$



We controlling the generator not the hub, so will have a transfer function on the beam in between.