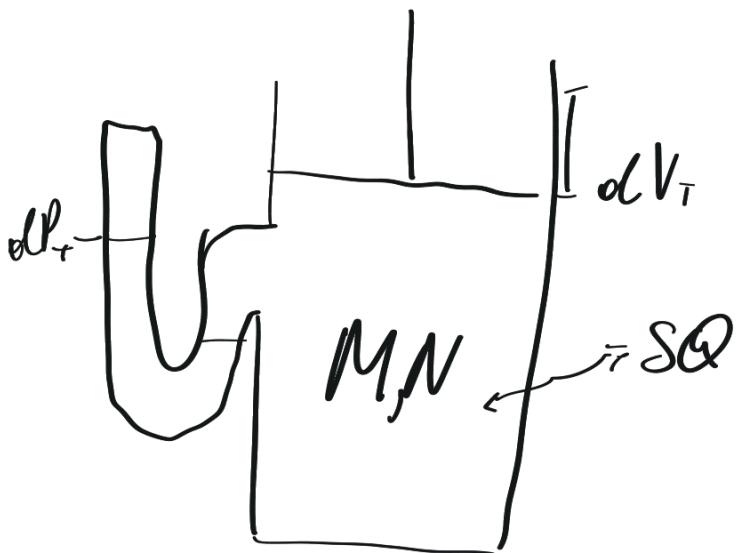


Leczione 8

Ripasso trasformazione isotermica



$$k_T = \frac{-1}{V} \left(\frac{\partial V}{\partial P} \right)_T \quad \text{per gas ideale} \quad k_T = \frac{1}{P}$$

coefficiente
di comprimibilità
isoterma

$$\frac{\alpha_P}{k_T} = \left(\frac{\partial P}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T$$

molari $\overbrace{\qquad\qquad\qquad}^V \cdot \alpha_P = - \left(\frac{\partial \bar{f}}{\partial P} \right)_T$ molare

Cambio dP e dV in una trasformazione isoterna

$$T = T(P, V) \quad dT = \left(\frac{\partial T}{\partial P} \right)_V dP + V \left(\frac{\partial T}{\partial V} \right)_P dV = 0$$

$$\frac{k_T}{\alpha_P} dP + \frac{1}{V \alpha_P} dV = 0$$

$$dP = - \frac{\alpha_P}{k_T} \cdot \frac{1}{V \alpha_P} dV = - \frac{1}{k_T} \cdot \frac{dV}{V}$$

$$\int_1^2 dP = P_2 - P_1 = - \int_1^2 \frac{1}{k_T} \frac{dV}{V}$$

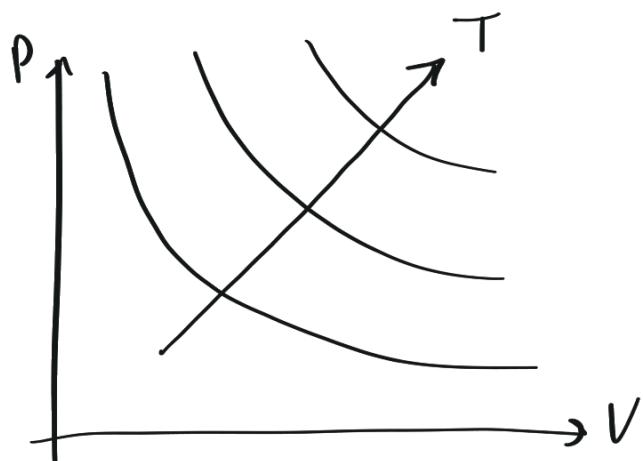
$$dP = -\frac{1}{k_T} \frac{dV}{V} \text{ per gas ideale (G.I.)}$$

gas ideali

$$k_T = \frac{1}{P}$$

$$\frac{dP}{P} + \frac{dV}{V} = 0 \quad \begin{matrix} \text{valori} \\ \text{degge di} \end{matrix}$$

$PV = \text{cost}$ Boyle, legge dell'isoterma dei gas ideali



Studiamo il fatto che le isoterme trionano in funzione di T e P così $\partial T = 0$

$$u = u(P, T)$$

\hookrightarrow molari

$$du = \left[\left(\frac{\partial u}{\partial P} \right)_T \right] dP + \left(\frac{\partial u}{\partial T} \right)_P dT = \left(\frac{\partial u}{\partial P} \right)_T dP$$

sappiamo che

$$du = \underbrace{T dS}_{SQ} - \underbrace{P dV}_{SL}$$

$$\left(\frac{\partial u}{\partial P} \right)_T = T \left(\frac{\partial S}{\partial P} \right)_T - P \left(\frac{\partial V}{\partial P} \right)_T = -T \left(\frac{\partial V}{\partial T} \right)_P + P_V k_T$$

IV Maxwell

$$\frac{v}{\nu} \left(\frac{\partial \nu}{\partial P} \right)_T = v k_T$$

$v \alpha_P$

$$= -Tv \alpha_P + Pv k_T$$

$$= \nu (P k_T - T \alpha_P) =$$

$$du = \nu (P k_T - T \alpha_P) dP$$

la sciamo per uno attimo e cerchiamo ds

$$S = S(P, T) \Rightarrow ds = \left(\frac{\partial S}{\partial P} \right)_T dP + \left(\frac{\partial S}{\partial T} \right)_P dT$$

come questo

$$ds = \left(\frac{\partial S}{\partial P} \right)_T dP = - \left(\frac{\partial \nu}{\partial T} \right)_P dP$$

IV Maxwell

$$= -v \alpha_P dP$$

k_T e α_P sono
misurati in laboratorio

Prendiamo in considerazione i GAS IDEALI

↳ non esistono
ma sono
quasi come
nulli per

$$du = \nu (P k_T - T \alpha_P) dP$$

$$ds = -v \alpha_P dP$$

per i gas ideali $\alpha_P = \frac{1}{T}$ $k_T = \frac{1}{P}$

$$du = \nu \left(P \cdot \frac{1}{P} - T \cdot \frac{1}{T} \right) dP = 0 \text{ per gas ideale in}$$

trasformazione isotermica $\Rightarrow u=u(T)$

$$ds = -\frac{V}{T} dP \quad P_V = RT \rightarrow \frac{V}{T} = \frac{R}{P}$$

$$ds = -R \cdot \frac{dP}{P}$$

In trasformazione isotermica

$$\Delta S_{12} = -R \ln \frac{P_2}{P_1}$$

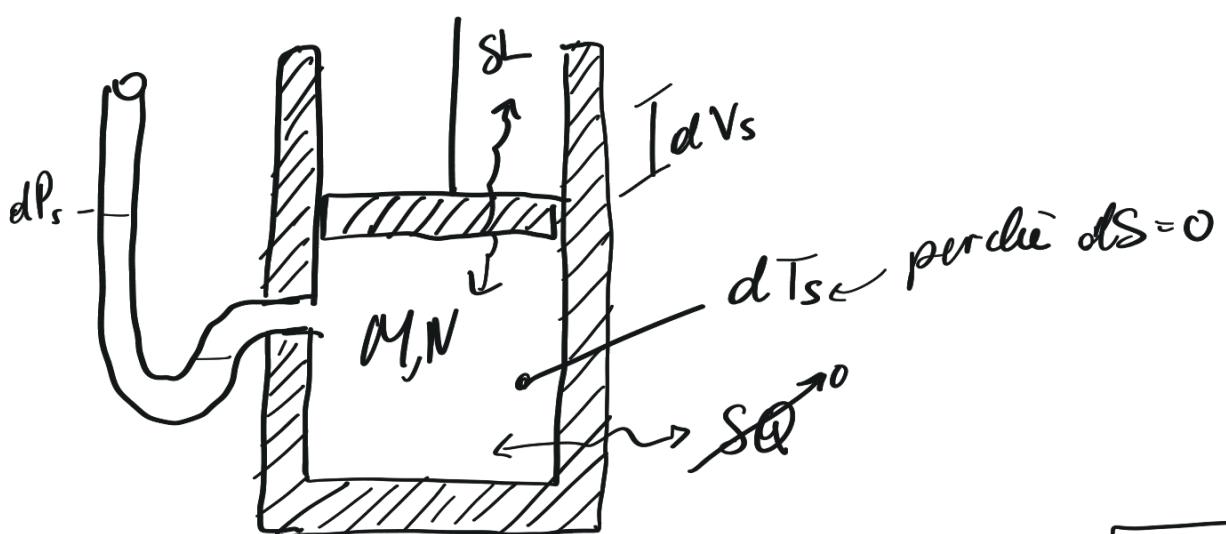
$$\Delta S_{12} = -MR^* \ln \frac{P_2}{P} \quad \text{cost nell'isoterma}$$

$$\text{Dato che } P_1 V_1 = P_2 V_2 \Rightarrow \frac{P_2}{P_1} = \frac{V_1}{V_2}$$

\Rightarrow

Scopo trovare che è ds una trasformazione

trasformazione adiabatica per gas quasi-ri
quasi-statica



ADIABATICA QUASI-STATICA $\rightarrow SQ = TdS = 0 \Rightarrow S = \text{cost}$

$$S = S(P, V) \rightarrow ds = \left(\frac{\partial S}{\partial P}\right)_V dP + \left(\frac{\partial S}{\partial V}\right)_P dV = 0$$

E usiamo S perché $ds = 0$ quindi troviamo la relazione

fro Pev

$$\text{I Maxwell} \quad \left(\frac{\partial P}{\partial S}\right)_V = - \left(\frac{\partial T}{\partial V}\right)_S$$

$$\text{III Maxwell} \quad \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$= - \underbrace{\left(\frac{\partial V}{\partial T}\right)_S}_{\text{segno cambia perché non sono posti}} dP + \left(\frac{\partial P}{\partial T}\right)_S dV = 0$$

$$\left(- \left(\frac{\partial V}{\partial T}\right)_S \stackrel{\text{PROPS JACOBI}}{=} - \frac{\partial(V,S)}{\partial(T,S)} \cdot \frac{\partial(V,T)}{\partial(V,T)} - \frac{\partial(V,S)}{\partial(V,T)} \cdot \frac{\partial(V,T)}{\partial(T,S)} \right)$$

$$= \frac{\partial(S,V)}{\partial(T,V)} \cdot \frac{\partial(V,T)}{\partial(S,T)} = \frac{1}{T} \left(\frac{\partial S}{\partial T} \right)_V \cdot \boxed{\left(\frac{\partial V}{\partial S} \right)_T} = \frac{C_V k_T}{T \alpha_P}$$

$$\left(\frac{\partial S}{\partial V} \right)_P = \frac{\partial(S,P)}{\partial(V,P)} \cdot \frac{\partial(T,P)}{\partial(T,P)} = \frac{\partial(S,P)}{\partial(T,P)} \cdot \frac{\partial(T,P)}{\partial(V,P)} =$$

$$= \boxed{\left(\frac{\partial S}{\partial T} \right)_P} \cdot \frac{1}{T} \cdot \boxed{\left(\frac{\partial T}{\partial V} \right)_P} \cdot \frac{V}{V} = \frac{C_P}{T} \cdot \frac{1}{V \alpha_P}$$

$$- \left(\frac{\partial V}{\partial T}\right)_S dP + \left(\frac{\partial P}{\partial T}\right)_S dV = \frac{C_V k_T}{T \alpha_P} dP + \frac{C_P}{T V \alpha_P} dV = 0$$

perché -0 quindi non ha effetto

$$dP = - \frac{T}{C_V k_T} \cdot \frac{C_P}{T \cdot V} dV = - \frac{C_P}{C_V} \cdot \frac{1}{k_T} \frac{dV}{V}$$

per gas ideali:

$$\gamma_{\text{monoatomic}} = \frac{5}{3} = 1,66$$

$$\hookrightarrow \frac{C_P}{C_V} = \kappa = \gamma = \begin{cases} 5/7 = 1,4 & \text{biatomici} \\ 4/3 = 1,33 & \text{poliatomici} \\ N_A & \text{non atomici} \end{cases}$$

$$\frac{1}{\kappa_T} = P \rightarrow dP = -\gamma \frac{P dV}{V}$$

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0 \rightarrow PV^\gamma = \text{cost}$$



ISOENERGETICO



$$dV = \delta Q - \delta L = T dS - P dV = 0$$

$$\hookrightarrow dV = 0 \Rightarrow \delta Q = \delta L \rightarrow T dS = P dV$$

non molto facile anche in laboratorio

$$T = T(u, v) \rightarrow \text{perdici uno dei due} \rightarrow 0$$

$$dT = \left(\frac{\partial T}{\partial u} \right)_v du + \left(\frac{\partial T}{\partial v} \right)_u dv$$

$$dT = \left(\frac{\partial T}{\partial u} \right)_v dv$$

scambio di calore

$$\left(\frac{\partial T}{\partial v} \right)_u = \frac{\partial (T, u)}{\partial (v, u)} \quad \frac{\partial (v, T)}{\partial (v, T)} = - \frac{\partial (u, T)}{\partial (v, T)} \cdot \frac{\partial (T, v)}{\partial (u, v)}$$

$$= - \left(\frac{\partial u}{\partial v} \right)_T \cdot \left(\frac{\partial T}{\partial u} \right)_v = - \frac{\left(\frac{\partial u}{\partial v} \right)_T}{\left(\frac{\partial u}{\partial T} \right)_v} \cdot du$$

$$= T \left(\frac{\partial s}{\partial v} \right)_T - P \left(\frac{\partial v}{\partial v} \right)_T = T dS - P dv$$

si cancellano ∂u e du
e si usano ds e dv
perché $\partial v / \partial v = \text{cost}$

$$\frac{\alpha p}{k_T}$$

$$T \left(\frac{\partial s}{\partial T} \right)_V - P \left(\frac{\partial v}{\partial T} \right)_V$$

$$= \frac{P - T \left(\frac{\partial s}{\partial v} \right)_T}{C_V} = \frac{P - T \frac{\alpha p}{k_T}}{C_V}$$

$$dT = \left(\frac{\partial T}{\partial v} \right)_u dv = \frac{P - T \frac{\alpha p}{k_T}}{C_V} dv$$

$$dT = \frac{P - T \frac{\alpha p}{k_T}}{C_V} dv$$

per i gas ideali

$$u = u(T) \Rightarrow \text{dato} \text{ isoenergetico} dT = 0$$

$$T = T(u)$$

$$h = h(T)$$

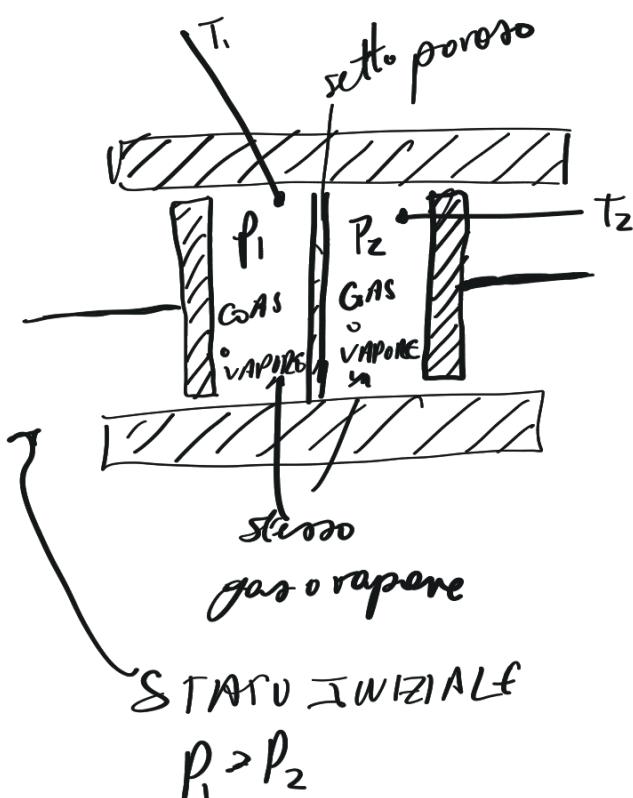
$$\text{Gas ideale} \Rightarrow \alpha_p = \frac{1}{T} \text{ e } k_T = \frac{1}{p}$$

$$\Rightarrow dT = \frac{R - \frac{T \cdot p}{C_V}}{C_V} dv = 0$$

Esansione libera di un gas ideale $\Rightarrow dT = 0$

Fine del calcolo termodinamico

PROCESSO ISOENTROPICO (H-cost) COEFFICIENTE DI JULE-TOMPSON
 \hookrightarrow persistente
climato



1 mole di gas
e vapore a P_1, V_1

STATO FINALE $P_1 > P_2$

The diagram shows the cylinder after the expansion. The piston has moved downwards, compressing the gas/vapor mixture. The left section is now labeled P_1 and the right section is labeled P_2 . The bottom of the cylinder is labeled T_1 and T_2 . The horizontal line "stesso gas o vapore" is still present.

1 mol di gas o vapore
a pressione P_2, V_2

$$\delta = \left(\frac{\partial T}{\partial P} \right)_H - \left(\frac{\Delta T}{\Delta P} \right)_H$$

coefficiente
di Joule-Thompson

$$L_1 = \int_{V_1}^0 P_1 dV = -P_1 V_1$$

↳ lavoro per spostare un mole di gas da sinistra a destra

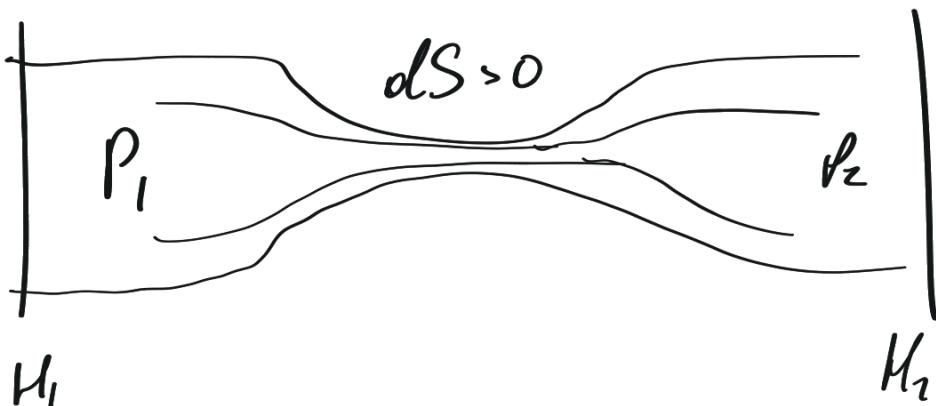
$$L_2 = \int_0^{V_2} P_2 dV = P_2 V_2$$

$$L_{\text{tot}} = L_1 + L_2 = -P_1 V_1 + P_2 V_2$$

$$\Delta U_{\text{tot}} = V_2 - V_1 = \underbrace{P_2 V_2}_{\text{III}} - L_{12} \Rightarrow V_2 - V_1 = -(P_1 V_1 + P_2 V_2)$$

L_{tot}

$$\underbrace{V_2 + P_2 V_2}_{H_2} = \underbrace{V_1 + P_1 V_1}_{H_1} \Rightarrow H_1 = H_2$$



Abbassando lontano $H_1 = H_2$

$$\delta = \left(\frac{\Delta T}{\Delta P} \right)_H \geq 0$$

↳ Ha effetto nella refrigerazione

se $\delta > 0$ e $\frac{\partial P}{\partial T} < 0 \rightarrow dT < 0$ raffreddamento

se $\delta = 0 \quad \nabla dP \Rightarrow dT = 0$ (gas ideale)

$$\hookrightarrow H \cdot H(T)$$

$\delta < 0$ e $\frac{\partial P}{\partial T} < 0 \rightarrow dT > 0$ Riscaldamento

Ricavare δ

$$\delta = \left(\frac{\partial T}{\partial P} \right)_h \stackrel{\text{JACOBIANO}}{=} \frac{\partial(T, h)}{\partial(P, h)} \cdot \frac{\partial(T, P)}{\partial(h, P)} =$$

$$= \frac{\partial(T, h)}{\partial(T, P)} \cdot \frac{\partial(T, P)}{\partial(P, h)} = - \frac{\partial(h, T)}{\partial(P, T)} \cdot \frac{\partial(T, P)}{\partial(h, P)} =$$

scambio
variabili;
grado(-)

$$= - \left(\frac{\partial h}{\partial P} \right)_T \cdot \left(\frac{\partial T}{\partial h} \right)_P = - \frac{\left(\frac{\partial h}{\partial P} \right)_T}{\left(\frac{\partial h}{\partial T} \right)_P}$$

$$(dh) = T dS + v dP \quad \text{sostituiamo}$$

$$\delta = - \frac{T \left(\frac{\partial S}{\partial P} \right)_T + v \left(\frac{\partial P}{\partial T} \right)_V^{-1}}{c_P + v \left(\frac{\partial P}{\partial T} \right)_P^0} = - \frac{T \left(\frac{\partial V}{\partial T} \right)_P + v}{c_P}$$

Maxwell

$v \alpha_p$

c_P

$$\delta = \frac{T v \alpha_p - v}{c_p}$$

$\delta(T, P), c_v(T, P), c_p(T, P), \alpha_p(T, P), \kappa_T(T, P)$

Tutti dipendenti di T e P

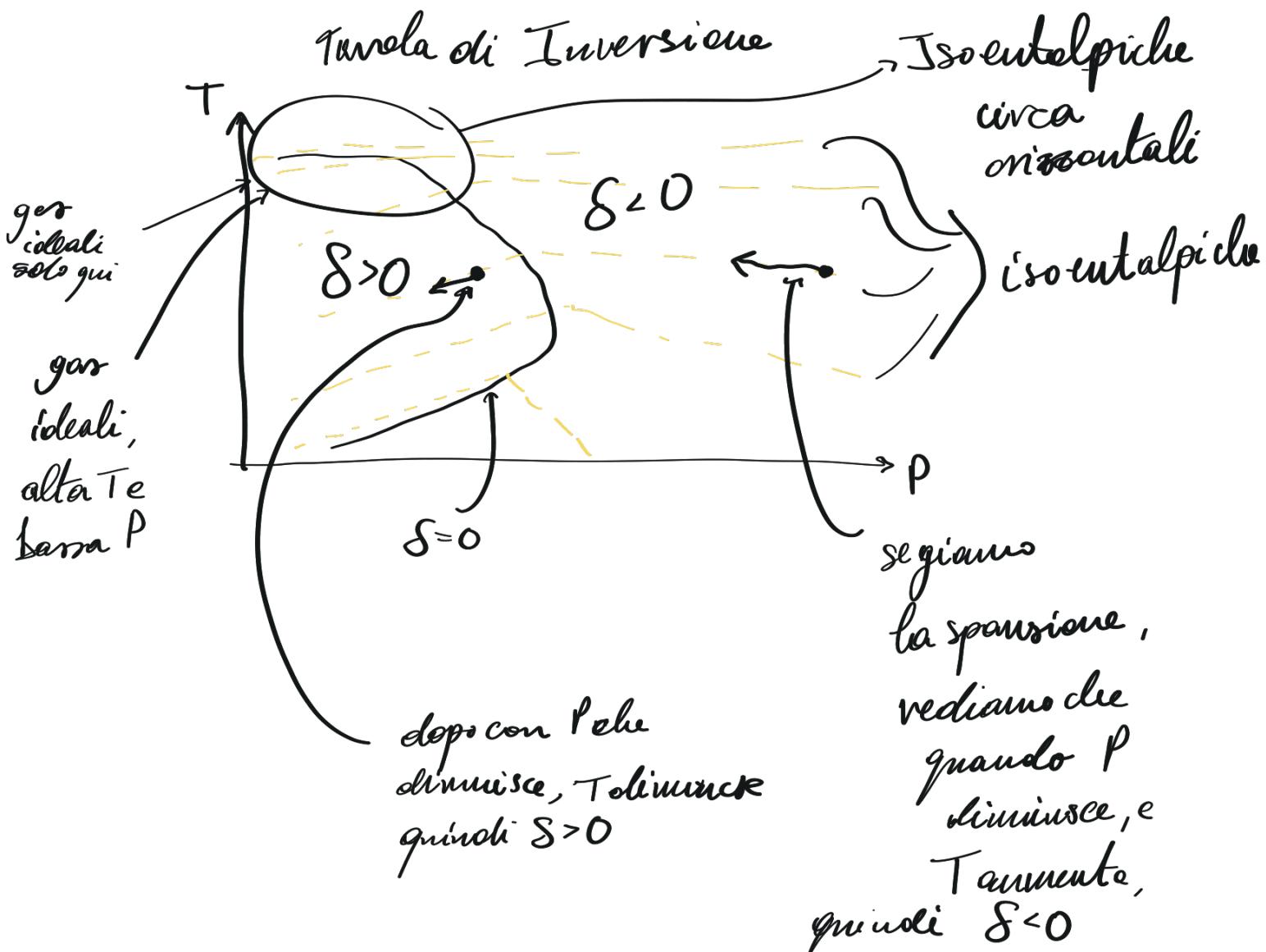
per i gas ideali

$$\alpha_p = \frac{1}{T} \Rightarrow S = \underbrace{T \cdot \frac{1}{T} \frac{v - v_0}{c_p}}_{=0} \Rightarrow H = H(T)$$

$S = 0$ se isentalpica

se $H = \text{cost}$ allora $dT = 0$

Tavola di Inversione



Ogni fluido ha una curva, se si vuole usare come refrigerante si vuole stare a sinistra.

Trasformazione Politropica

Requisiti

- ① GAS IDEALE
- ② QUAZI-STATICA

$$PV^n = \text{cost}$$

n indice della politropica

$$n = \frac{C_x - C_p}{C_x - C_v}$$

$$C_x = \frac{1}{N} \left(\frac{\delta Q}{dT} \right)_x$$

T_{una}
qualsiasi
variabile
termodinamica

T, P, S, V

Esempio trasformazione isotermica ($dT=0$)

$$C_x = C_T \rightarrow \infty \Rightarrow n = 1$$

$$PV = \text{cost}$$

$$dV = N C_v * dT = 0 \quad dV = \delta Q - \delta L = 0 \Rightarrow \delta Q = \delta L$$

$$dM = M C_p * dT = 0$$

-180 BARA $P = \text{cost}$ $dP = 0$

$$C_x = C_p \rightarrow n = 0 \quad P = \text{cost}$$

- ISOCORIA $V=0$ $\partial V=0$

$$C_D = C_F \Rightarrow n = \infty$$

- ADIA BATICA $S = \text{const}$