Lezione 12 - Problema Elastico

$$\begin{cases} \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) = f_x \\ -\left(\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}\right) = f_y \end{cases}$$

$$\left.\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & 0 \\ -\nu/E & 1/E & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} \right\} \text{Legge di Hooke}$$

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

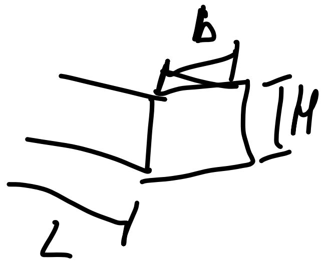
$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\frac{\partial^2 \varepsilon_x}{\partial y \partial z} = \frac{1}{2} \frac{\partial}{\partial x}\left(\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{zy}}{\partial x}\right)$$

Come risolvere problema elastico?

Ipotesi:

Travi snelle: $l \gg b$ e $l \gg h$

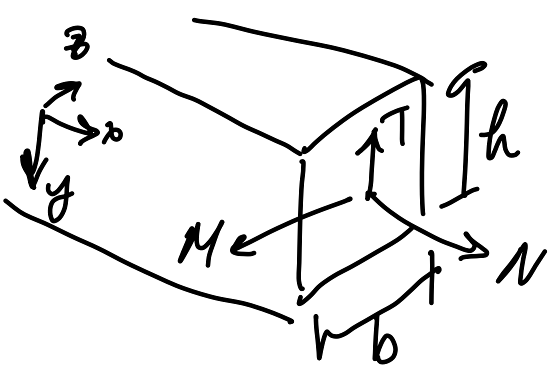


- Materiale Omogeneo, Isotropo, Elastico lineare

In campo lineare:
$\sigma = E\,\varepsilon$

- Forze di Volume nulle
- Forze di Superficie sulle basi
- Analizziamo sufficientemente lontano dai carichi
- Corpo non vincolato, ma in equilibrio
- Sistema di riferimento principale di inerzia

Consideriamo una trave caricata solo su x e y:



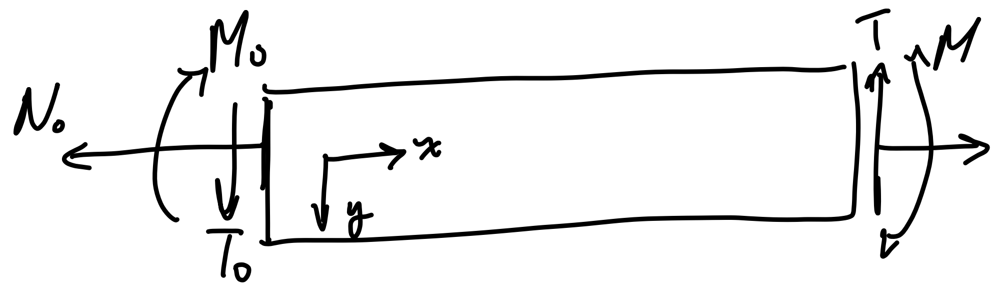
Usando il principio di San Venant:

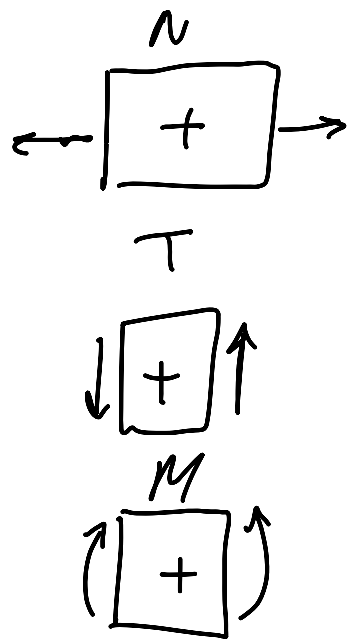
$$N = b\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \, dy$$ } Integrale dello sforzo su

superfice

$$T = b \int_{-h/2}^{h/2} \tau_{xy} \, dy$$

$$M = b \int_{-h/2}^{h/2} y \, \sigma_x \, dy =$$





$$(1) \begin{cases} N = N_0 \\ T = T_0 \\ M = M_0 - T_0 x \end{cases}$$

Saltiamo dei passaggi con San Benand:

Troviamo queste equazioni importanti

$$\sigma_x = a + a_2 y - x b_2 y$$

$$(2) \begin{cases} N = aA \\ M = (a_2 - x b_2) J_{zz} \end{cases}$$

→ Momento d'inerzia

↓

Inerzia per la rotazione

$$a = \frac{N}{A} = \frac{N_0}{A}$$

$$a_2 = \frac{M_0}{J_{zz}} \rightarrow \sigma_x = \frac{N}{A} + \frac{M_0 \, y}{J_{zz}} - x \frac{T_0}{J_{zz}} \cdot y =$$

$$b_2 = \frac{T_0}{J_{zz}} \qquad M_0 = M + T_0 \cdot x$$

$$\sigma_x = \frac{N}{A} + \frac{M}{J_{zz}} \cdot y + \cancel{x \frac{T_0}{J_{zz}} y} - \cancel{x \frac{T_0}{J_{zz}} y}$$

$$\sigma_x = \frac{N}{A} + \frac{M}{J_{zz}} y$$

Linea elastica Assiale

Prendiamo una trave e sottoponiamo a N

$$\sigma_x = \frac{N}{A} \qquad \sigma_y = \tau_{xy} = 0 \qquad \begin{cases} \varepsilon_x = \dfrac{\sigma_x}{E} \\ \varepsilon_y = -\nu \dfrac{\sigma_x}{E} \\ \gamma_{xy} = 0 \end{cases}$$

Coefficiente di Poisson

$$\varepsilon_x = \frac{\partial u}{\partial x} \Rightarrow u(x,y) = \frac{N}{EA} x + C_1$$

costante di integrazione

$$\frac{\partial u}{\partial x} = \frac{N}{EA}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \Rightarrow v(x,y) = -\nu \frac{N}{EA} y + C_2$$

$$\left.\begin{array}{l} u(x,0) = \dfrac{N}{EA} + C_1 \\ v(x,0) = C_2 \end{array}\right\}$$ In questo caso

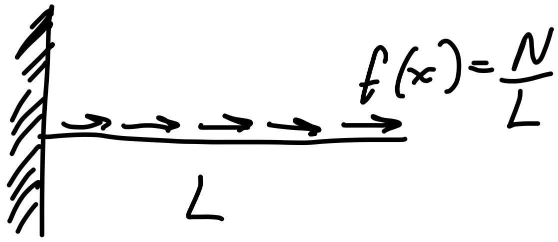
σ_x e ε_x uniformi sulla sezione

→ Consideriamo un carico distribuito

$$-\frac{dN}{dx} = f(x)$$

$$N = \sigma_x A = E \cdot A \cdot \varepsilon_x = E \cdot A \cdot \frac{\partial u}{\partial x} \Rightarrow EA \frac{\partial^2 u}{\partial x^2} = f(x)$$

Esempio



$$\frac{\partial^2 u}{\partial x^2} EA = f(x) = \frac{N}{L}$$

$$\frac{\partial u}{\partial x} = -\frac{N}{EAL} x + C_1$$

$$u = -\frac{N}{EAL} \frac{x^2}{2} + C_1 x + C_2$$

Condizioni al Contorno

$$\begin{cases} u(0) = 0 \\ N(L) = 0 \end{cases}$$

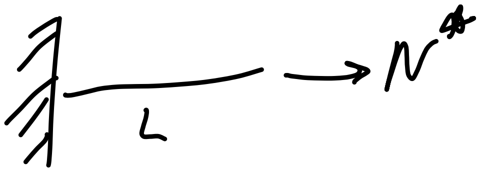
$$EA\left(-\frac{N}{EAL} x + C_1\right)\Bigg|_{x=L} = 0$$

$$\Rightarrow -\frac{N}{EA} + C_1 = 0 \Rightarrow C_1 = \frac{N}{EA}$$

$$u(0) = C_2 = 0$$

$$N(x) = EA\varepsilon_x = EA \frac{\partial u}{\partial x}$$

Altro Esempio



$$-EA\,\frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial u}{\partial x} = C_1$$

$$u = C_1 x + C_2$$

Condizioni al Contorno

$$\begin{cases} u(0) = 0 \rightarrow C_2 = 0 \\ N(0) = N^* \end{cases}$$

$$EA\,\frac{\partial u}{\partial x}\Big|_{x=0} = N^*$$

$$EA \cdot C_1 = N^* \Rightarrow C_1 = \frac{N^*}{EA}$$

$$u(x) = \frac{N^*}{EA}\,x$$

Da ricordare

Ipotesi di San Benand

$-EA\,\dfrac{\partial^2 u}{\partial x^2} = f(x)$ e procedura per i $\underbrace{\text{due esempi}}$

sono gli unici due esempi.

San Benand → Saint-Venant