Esercitazione 5

$$E(X) = 1 \cdot .1 + 2 \cdot .2 + 3 \cdot .3 + 4 \cdot .4 = 3$$

$$Var(X) = E(X^2) - E(X)^2 = (1 \cdot .1 + 4 \cdot .2 + 9 \cdot .3 + 16 \cdot .4) - 3^2 = 1$$

$$\bar{X}_{50} = \frac{X_1 + \dots + X_{50}}{50}$$

$n \geq 50$ $\qquad \bar{X}_n \sim N\left(3, \frac{1}{50}\right)$

$$P(2{,}8 < X \leq 3{,}1) =$$

$$= P(X \leq 3{,}1) - P(X \leq 2{,}8) =$$

$$= P\left(\underbrace{\frac{X-3}{\sqrt{1/50}}}_{Z} \leq \frac{3{,}1-3}{\sqrt{1/50}}\right) - P\left(\underbrace{\frac{X-3}{\sqrt{1/50}}}_{Z} \leq \frac{2{,}8-3}{\sqrt{1-50}}\right) =$$

$$= P(Z \leq .707) - P(\cancel{Z \leq} -1{,}414) =$$

$$= \phi(.707) - (1 - \phi(1{,}414)) =$$

$$= .76115 - (1 - .92073) = .68188$$

$$E(x) = \int_0^1 x \cdot \frac{2}{7}(10x^3+1)\,dx$$

$$= \int_0^1 \frac{2}{7}(10x^4 + x)\,dx$$

$$= \frac{2}{7}\left[2x^5 + \frac{x^2}{2}\right]_0^1$$

$$= \frac{2}{7}\left(2 + \frac{1}{2}\right) = \frac{10}{14} = \frac{5}{7}$$

$$Var(x) = E(X^2) - E(X)^2 = \frac{4}{7} - \left(\frac{5}{7}\right)^2 = \frac{28}{49} - \frac{25}{49} = \frac{3}{49}$$

$$E(X^2) = \frac{2}{7}\int_0^1 x^2(10x^3 + 1)\,dx$$

$$= \frac{2}{7}\int_0^1 10x^5 + x^2\,dx$$

$$= \frac{2}{7}\left[\frac{10}{6}x^6 + \frac{x^3}{3}\right]_0^1 = \frac{2}{7}\left[\frac{10}{6} + \frac{1}{3}\right] = \frac{4}{7}$$

b) $S = X_1 + \dots + X_{210}$

$E(S) = n\mu = 150$

$Var(S) = n\sigma^2 = 12{,}857$

$$P(S > 152{,}2) = 1 - P(S \leq 152{,}2)$$

$$= 1 - P\left(\frac{S-150}{\sqrt{12{,}857}} \leq \frac{152{,}2 - 150}{\sqrt{12{,}857}}\right)$$

$$= 1 - \phi(.61355) = .2709$$

0, independent

a) $Var(X+Y) = Var(X) + Var(Y) + 2\cancel{Cov(X,Y)}$

$$= \frac{4}{12} + \frac{4}{12} = \frac{8}{12} = \frac{2}{3}$$

$E(X+Y) = E(X) + E(Y) = 2$

0,

b) $Cov(X,Y,Y) = \cancel{Cov(X,Y)} + Cov(Y,Y) = Var(Y) = \frac{1}{3}$

c) $S = X_1 + \dots + X_{141}$

$P(S < 161)$

$E(S) = n\mu = 147$

$Var(S) = n\sigma^2 = 49$

$$P(S<161) = P(S\leq 161) = P\left(\frac{S-147}{\sqrt{47}} \leq \frac{161-147}{7}\right)$$

$$= \Phi(2) = .97725$$

5.4

$p = .75$

$$X \sim Bin(10, .75) \rightarrow Y \sim Bin(10, .25)$$

$$P(X \geq 7) = P(Y < 3) = P(Y=0) + P(Y=1) + P(Y=2)$$

$$= .05631 + .1877 + .28156$$

$$= 0,52557$$

5,5

a) $S \sim Poi(400)$

b) $S \sim N(n\lambda, n\lambda) = N(400, 400)$

$$P(S\leq 390) = P\left(\frac{S-400}{20} \leq \frac{390-400}{}\right)$$

$$= P(Z \leq -0,5) = 1 - P(Z \leq 0,5) = .30854$$

c) $P(S > 390) = 1 - P(S \leq 390) = .5$

$$P(S \leq 390) = .5$$

$$P\left(\frac{S-n\lambda}{\sqrt{n\lambda}} \leq \frac{390-n\lambda}{\sqrt{n\lambda}}\right) < .5$$

$$\frac{390 - n\lambda}{\sqrt{n\lambda}} = 0$$

$$390 - 4n > 0$$

$n > \frac{390}{4} = 97{,}5 \approx 98$, since if we increase n, the probability than $P(S < 390)$ decreases, therefore $P(S \geq 390)$ increases.

d) $X_i \sim Poi(256) \rightarrow n\lambda = 256$

$$P(X > 270) = 1 - P(X \leq 270)$$

$$= 1 - P\left(\frac{X - 256}{\sqrt{256}} \leq \frac{270 - 256}{\sqrt{256}}\right) = 1 - \phi(0{,}875) \approx 1 - \phi(.875)$$

$= 0.18943$