Esercitazione 10

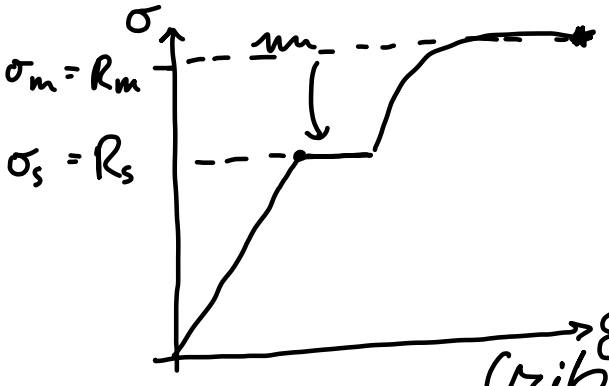
Criteri di Resistenza

Resistenza Statica → componenti rimangono si allineano con le componenti fisse

$$\sigma_{eq} < \sigma_{amm} = \frac{\sigma_{lim}}{\eta}$$

η ← fattore di sicurezza

I materiali duttili ci dicono che si stanno per rompere i materiali fragili rompono a schianto



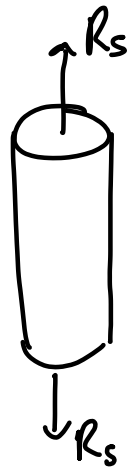
Campo Elastico lineare

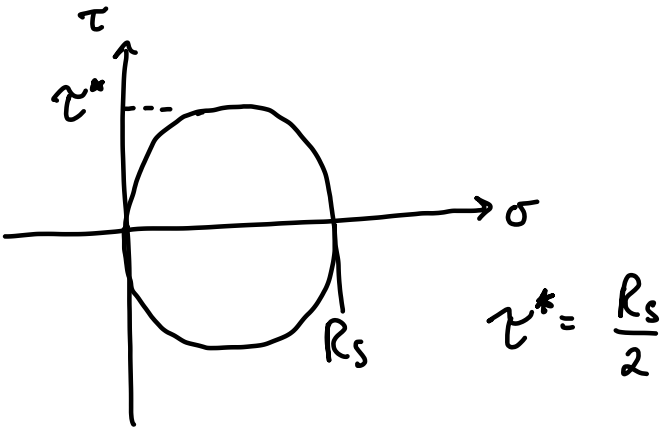
$\sigma = E\varepsilon$

$\eta = 1,5$ per materiali duttili

Criteri per Materiali Duttili

Criterio di GUEST - TRESCA





$\tau^* = \frac{R_s}{2}$

Formulazione criterio originale

$$\tau^{ab} < \frac{\tau_{lim}}{\eta}$$

Ora:

$$\sigma < \frac{R_s}{\eta}$$

σ_{eq} di guest-Tresca

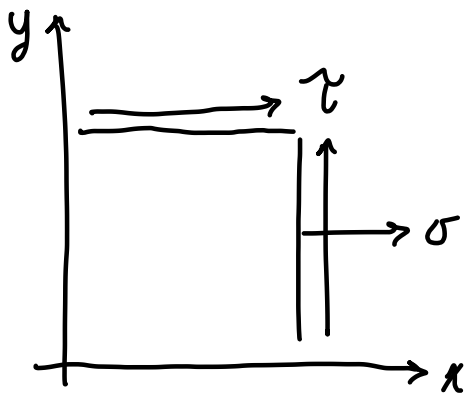
$$\tau^{ab} = \frac{\sigma_I - \sigma_{III}}{2} \leq \frac{R_s}{2} \frac{1}{\eta}$$

$$\sigma_{GT}^{ab} = \sigma_I - \sigma_{III} \leq \frac{R_s}{\eta}$$

Criterio di Von Mises

$$\sigma_{VM}^{ab} = \sqrt{\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - \sigma_I \sigma_{II} - \sigma_I \sigma_{III} - \sigma_{II} \sigma_{III}} \leq \frac{R_s}{\eta}$$

σ_{eq} di Von Mises



$$\begin{cases} \sigma_I = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \\ \sigma_{II} = 0 \\ \sigma_{III} = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \end{cases}$$

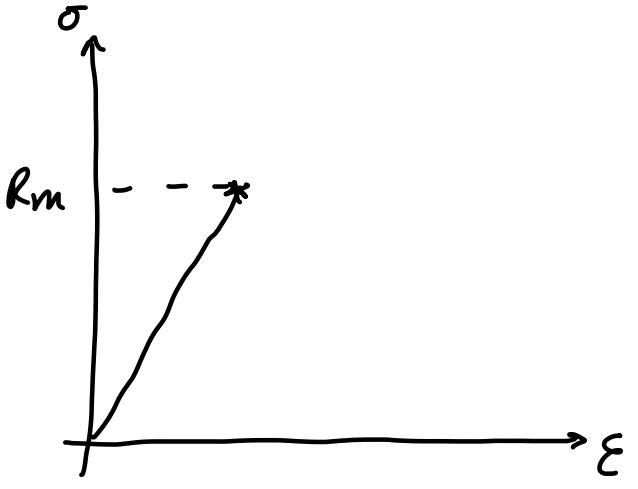
σ_{eq} per i due criteri

$$\sigma^*_{VM} = \sqrt{\sigma^2 + 3\tau^2}$$ } Più vicino a quello reale

$$\sigma^*_{GT} = \sqrt{\sigma^2 + 4\tau^2}$$ } Più conservativo perché esce più alto, quindi è più cautelo

Materiali Fragili

Criterio Galileo - Rankine - Navier



Rottura quando:

$|\sigma_{II}| \leq \frac{R_c}{\eta}$ ← compressione

σ

$|\sigma_I| \leq \frac{R_t}{\eta}$ ← trazione

Per materiali fragili $\eta = 3$

$$\sigma^*_{GRN} < \frac{R_m}{\eta}$$

Verifiche → troviamo che η sia più elevato di 1,5 per duttili o 3 per fragili

↳ In realtà di solito si anticipa e poi si progetta, ma boh.

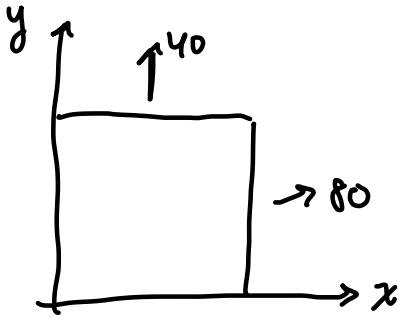
Esercizio 1

$R_s = 345$ MPa

$\sigma_x = 80$ MPa

$\sigma_y = 40$ MPa

σ_z nullo



$\sigma_I = 80$

$\sigma_{II} = 40$

$\sigma_{III} = 0$

$$\sigma^*_{GT} = \sigma_I - \sigma_{III} = 80\,MPa \leq \frac{R_s}{\eta}$$

Per trovare η ci mettiamo nel caso peggiore:

$$\sigma^*_{GT} = \frac{R_s}{\eta} \Rightarrow \eta_{GT} = \frac{R_s}{\sigma^*_{GT}} = 4,31$$

$$\sigma^*_{VM} = \sqrt{\sigma_I^2 + \sigma_{III}^2 - \sigma_I \sigma_{III}} = 69,3\,MPa$$

$$\eta_{VM} = \frac{R_s}{\sigma^*_{VM}} = 4,98$$

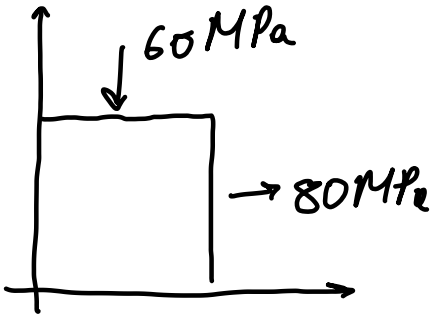
2) $\sigma_x = 80\,MPa = \sigma_I$

$\sigma_y = -60\,MPa = \sigma_{III}$

$\sigma_z = 0 = nullo \quad \sigma_{II} = 0$

$$\sigma^*_{GT} = \sigma_I - \sigma_{III} = 140\,MPa$$

$$\eta_{GT} = 2,46$$



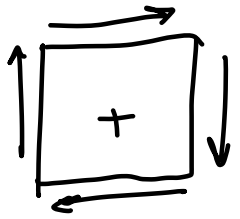
$$\sigma^*_{VM} = \sqrt{\sigma_I^2 + \sigma_{III}^2 - \sigma_I \sigma_{III}} = 121,65\,MPa$$

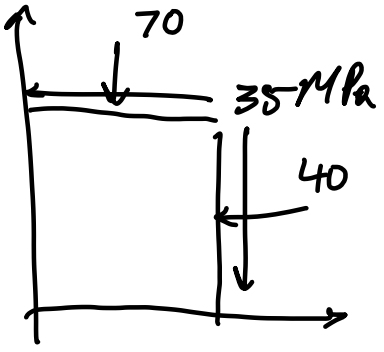
$$\eta_{VM} = 2,83$$

3) $\sigma_x = -40\,MPa$

$\sigma_y = -70\,MPa$

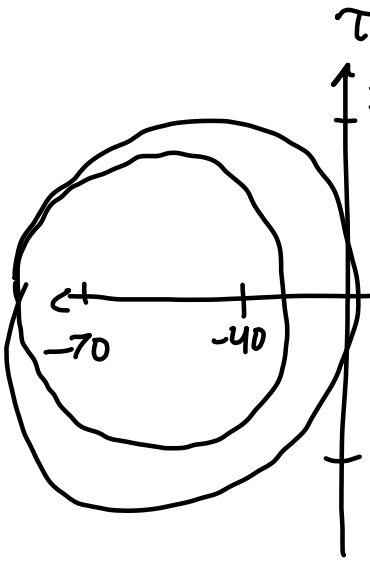
$\tau_{xy} = -35\,MPa$





$A(-40, 35)$

$B(-70, -35)$



$\sigma_I = \sigma$

$C = \frac{\sigma_A + \sigma_B}{2} = -55\,MPa$

$R = \sqrt{(\sigma_A - C)^2 + \tau_{xy}^2} \cong 38\,MPa$

$\sigma_{II} = C + R = 17\,MPa$

$\sigma_{III} = C - R = -93\,MPa$

$\sigma_{GT}^* = \sigma_I - \sigma_{III} = 93\,MPa$

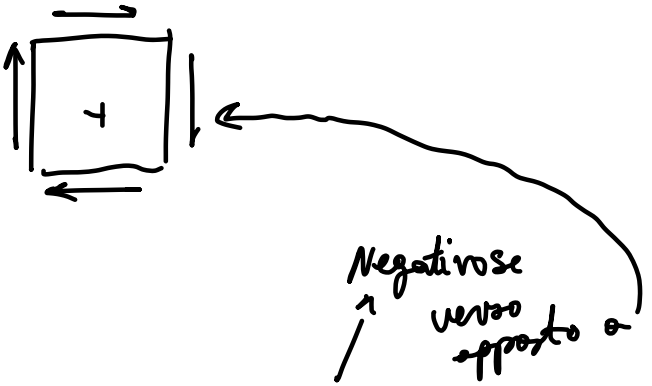
$\eta_{GT} = \frac{R_s}{\sigma_{GT}^*} = 3{,}71$

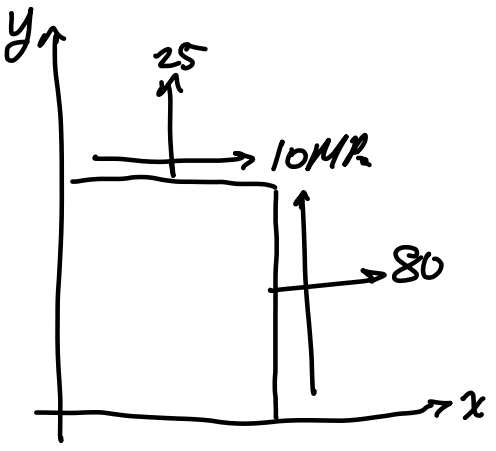
$\sigma_{VM}^* = \sqrt{\sigma_{II}^2 + \sigma_{III}^2 - \sigma_{II}\sigma_{III}} = 85{,}77\,MPa \Rightarrow \eta_{VM} = 4{,}02$

4) $\sigma_x = 80\,MPa$

$\sigma_y = 25\,MPa$

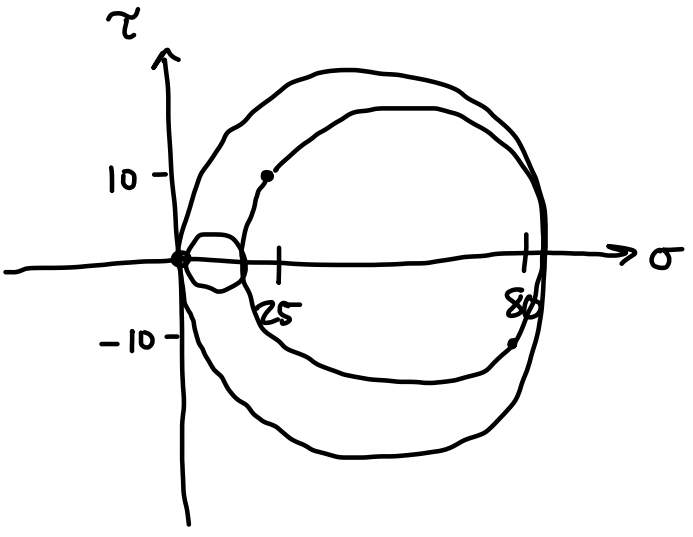
$\tau_{xy} = 10$





$A(80, -10)$

$B(25, 10)$



$\sigma_{III} = 0$

$C = \frac{\sigma_A + \sigma_B}{2} = 52,5\ MPa$

$R = \sqrt{(\sigma_A - C)^2 + \tau_A^2} = 29,26\ MPa$

$\sigma_I = C + R = 81,76\ MPa$

$\sigma_{II} = C - R = 23,24\ MPa$

$\sigma_{GT}^* = \sigma_1 - \sigma_{III} = 81,76\ MPa$

$\sigma_{VM}^* = \sqrt{\sigma_I^2 + \sigma_{III}^2 - \sigma_I \sigma_{III}} = 73\ MPa$

$\Rightarrow \eta_{GT} = 4,22$

$\Rightarrow \eta_{VM} = 4,72$

Stessi problemi ma fragile:

$R_t = 200\ MPa \qquad R_c = 690\ MPa$

1) $\sigma_I = 80 \quad \sigma_{II} = 40 \quad \sigma_{III} = 0$

2) $\sigma_I = 80 \quad \sigma_{II} = 0 \quad \sigma_{III} = -60$

3) $\sigma_I = 0 \quad \sigma_{II} = -17 \quad \sigma_{III} = -93$

4) $\sigma_I = 82 \quad \sigma_{II} = 23 \quad \sigma_{III} = 0$

	η_T	η_c	η_{min}
1)	2,5	N/A	2,5
2)	2,5	11,5	2,5
3)	N/A	7,42	7,42
4)	2,44	N/A	2,44

$\eta_T = \frac{R_T}{\sigma_I}$ $(\sigma_I > 0)$

$\eta_c = \frac{R_c}{\sigma_{III}}$ $(\sigma_{III} < 0)$

↖ Dato che sono materiali fragili, solo (3) soddisfa la verifica

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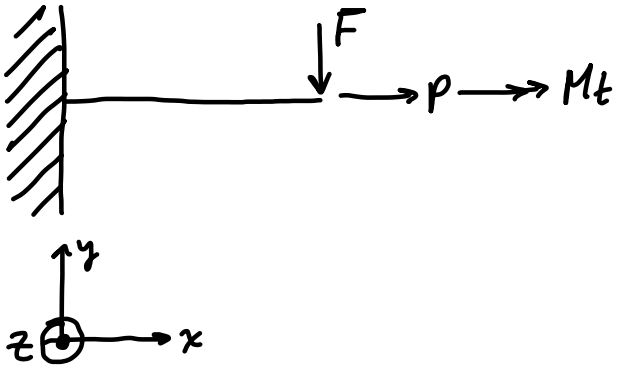
Esercizio 2)



$l = 100\,mm$

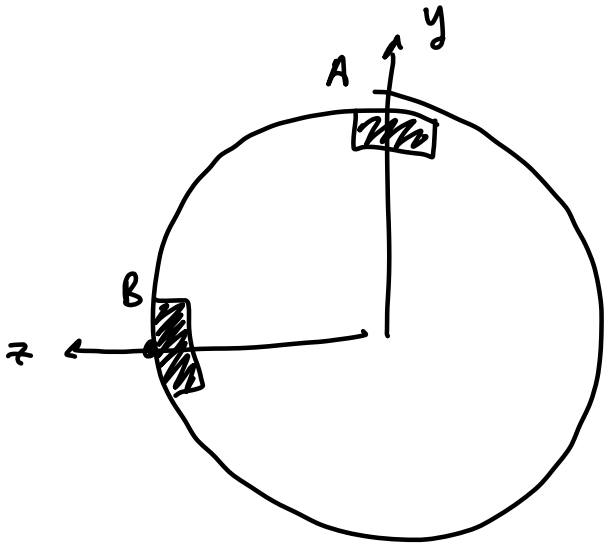
$\sigma_R = 430\,MPa$

$\sigma_S = 275\,MPa$

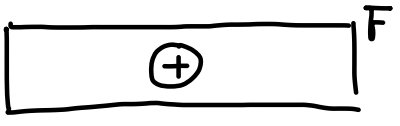
$F = 550N$

$P = 8000N$

$M_t = 30\,Nm$

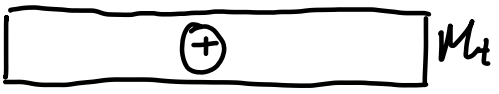


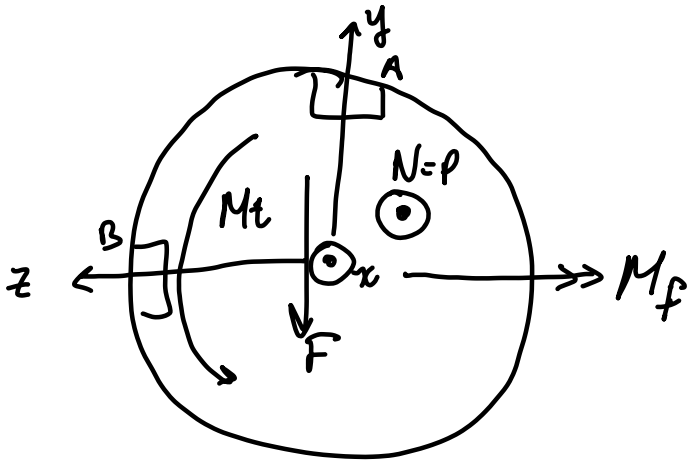
N

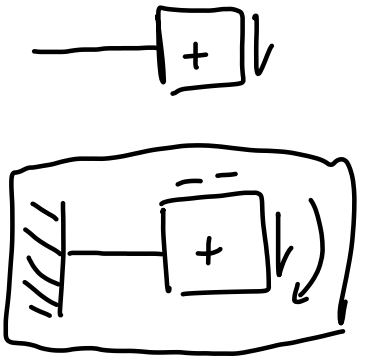


M_f

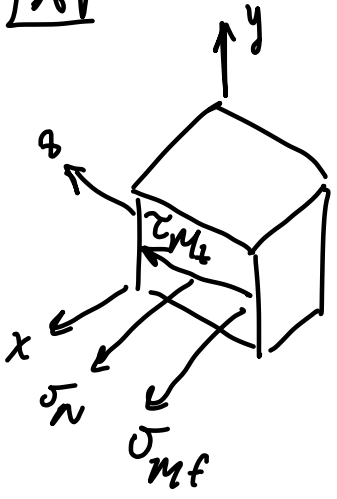
M_t







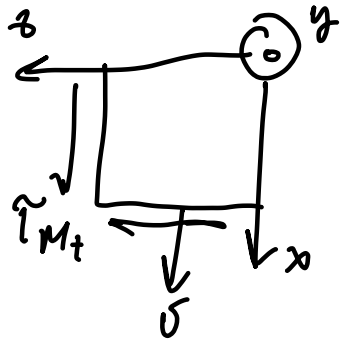
A



$$\sigma_N = \frac{N}{A} = 25,46 MPa$$

$$\sigma_{Mf} = \frac{32 M_f}{\pi D^3} = 70 MPa$$

$$\tau_{Mb} = \frac{16 M_t}{\pi d^3} = 19,1 MPa$$

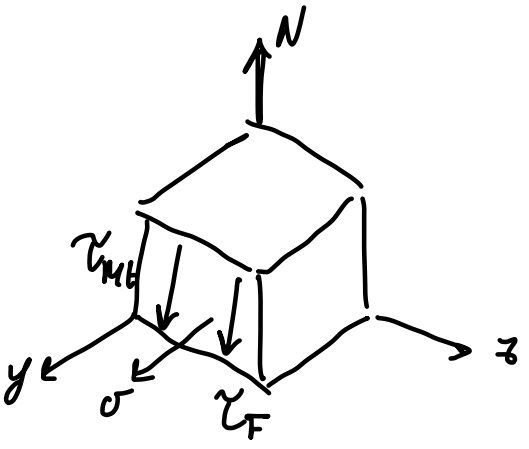


$$\sigma_{GT}^a = \sqrt{\sigma^2 + 4\tau^2} = 102,5 MPa$$

$$\sigma_{VM}^a = \sqrt{\sigma^2 + 3\tau^2} = 100,6 MPa$$

$$\eta_{GT} = \frac{\sigma_S}{\sigma_{GT}^a} = 2,68 \qquad \eta_{VM} = \frac{\sigma_S}{\sigma_{VM}^a} = 2,73$$

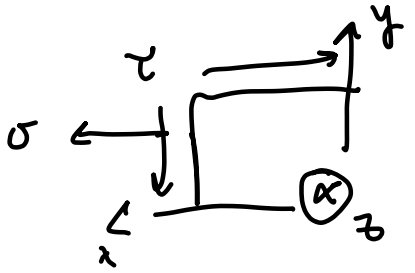
B $M_f = 0$



$$\sigma = \frac{N}{A} = 25,46\,MPa$$

$$\tau_F = \frac{4}{3}\frac{T}{A} = 2,33\,MPa$$

$$\tau_{Mt} = \frac{16 M_b}{\pi d^3} = 19,1\,MPa$$



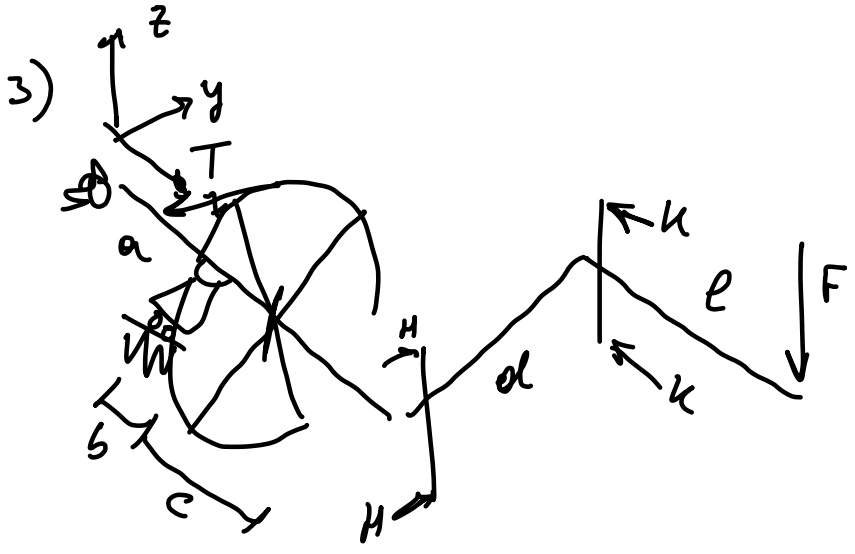
$$\sigma_{GT}^{a} = 49,8\,MPa$$

$$\sigma_{VM}^{a} = 44,96\,MPa$$

$$\eta_{GT} = 5,52$$

$$\eta_{VM} = 6,11$$

3)



Dati:

$a = 50\,mm$

$b = 20\,mm$

$c = 30\,mm$

$d = 200\,mm$

$l = 100\,mm$

Diametro puleggia → $d_p = 300\,mm$

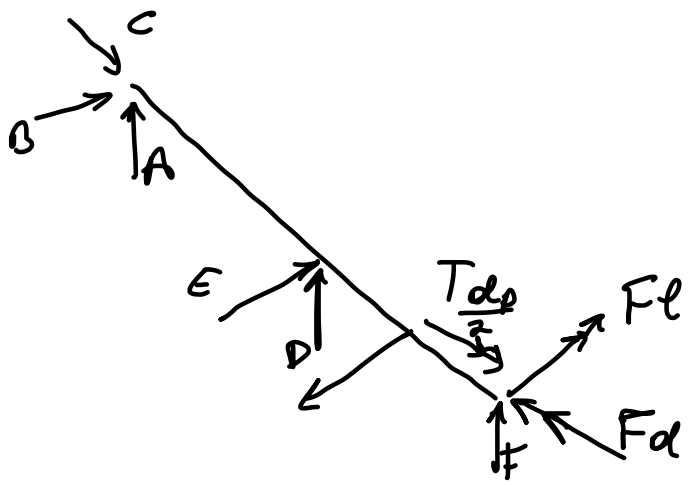
$F = 500\,N$

$d_{MH} = 20\,mm$

$d_{KK} = 16\,mm$

$R_m = 500\,MPa$

$R_s = 300\,MPa$



$$\sum M_{tx}^{p} = 0 = \frac{T d_p}{2} - F d \Rightarrow T = \frac{2Fd}{d_p} = 666{,}7\ N$$

$$\sum F_x = 0 = C$$

$$\sum F_y = 0 = -T + E + B$$

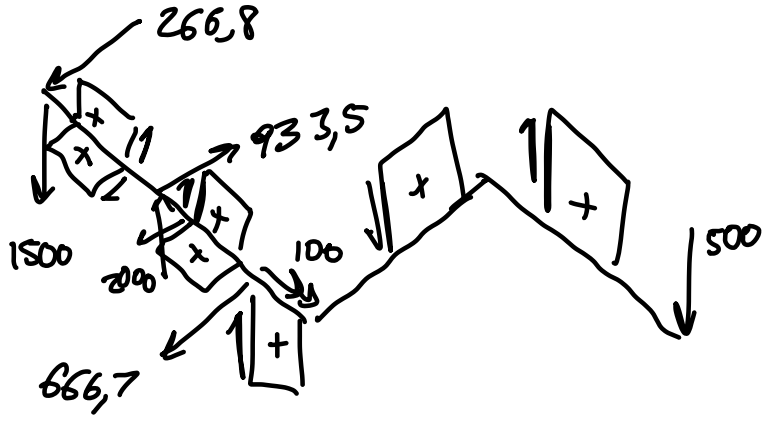
$$\sum F_z = 0 = -F + D + A$$

$$\sum M_z = 0 = Ea - T(a+b) \Rightarrow E = 933{,}3\ N$$

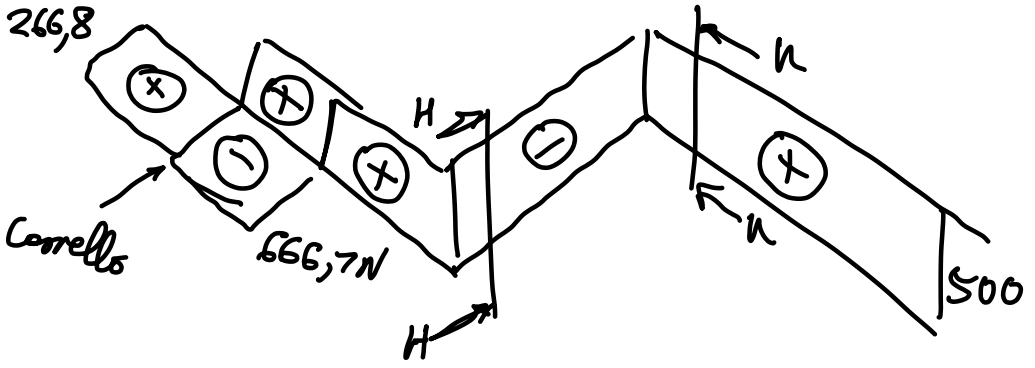
$$\Rightarrow B - E + T = -266{,}8\ N$$

$$\sum M_y = 0 = Da - F(a+b+c+l) \Rightarrow D = 2000\ N$$

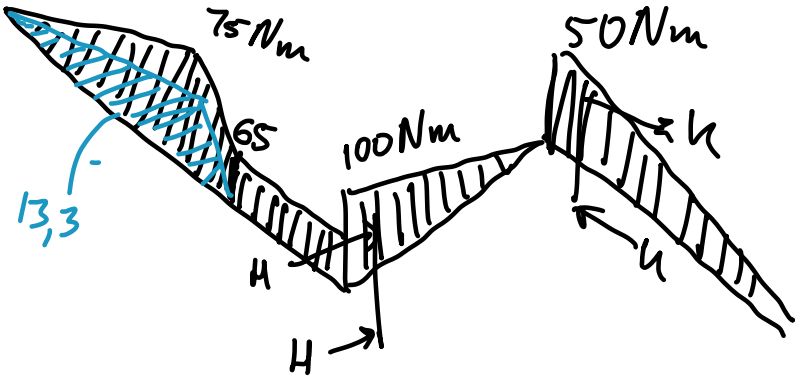
$$\Rightarrow A = F - D = -1500\ N$$



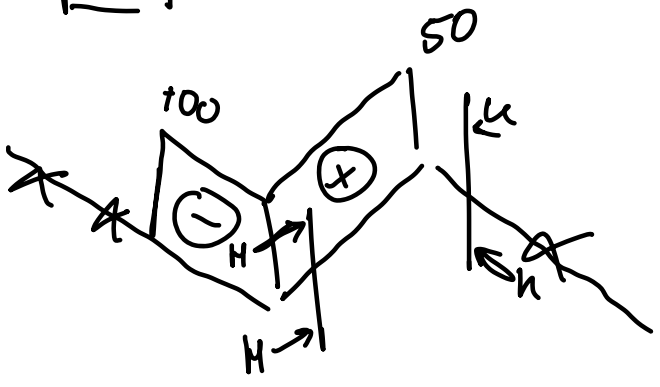
N

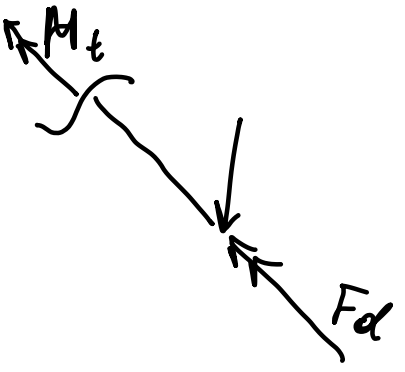


M_f



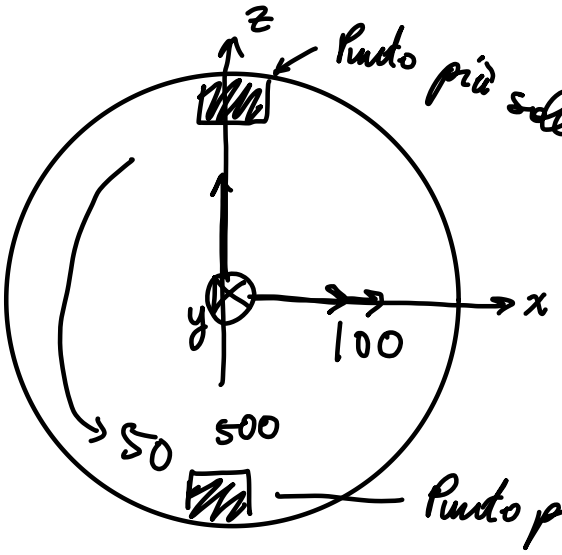
Tutti i calcoli li ha fatti lui





Sezione H-H

Non consideriamo più il taglio



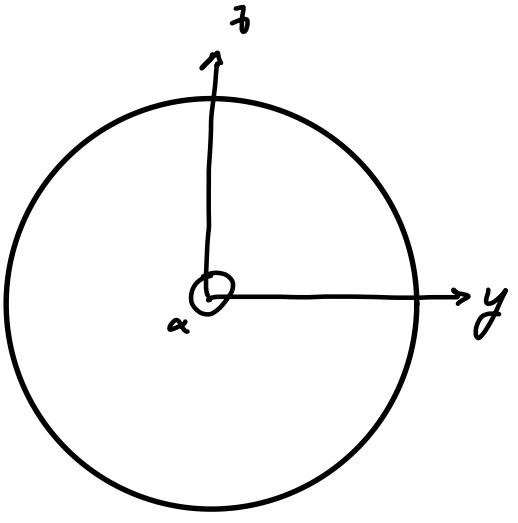
$$\sigma = \frac{32\,M_f}{\pi d_{HH}^3} = 127,3\ MPa$$

$$\tau = \frac{16\,M_t}{\pi d_{HH}^3} = 31,8\ MPa$$

$$\sigma_{GT}^{id} = \sqrt{\sigma^2 + 4\tau^2} = 142,13\ MPa \Rightarrow \eta_{GT} = \frac{\sigma_s}{\sigma_{GT}} = 2,11\ \checkmark$$

$$\sigma_{VM}^{id} = \sqrt{\sigma^2 + 3\tau^2} = 138,7\ MPa \Rightarrow \eta_{VM} = \frac{\sigma_s}{\sigma_{VM}} = 2,16\ \checkmark$$

Sezione K-K



$$\sigma = \frac{32\,M_f}{\pi d_{KK}^3} = 14,34\ MPa$$

$$\sigma_{GT}^{id} = \sigma_{VM} = \sqrt{\sigma^2} = \sigma$$

$$\eta = \frac{\sigma_s}{\sigma} = 2,41$$