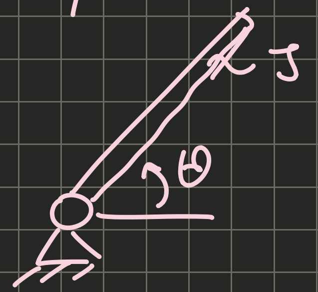
Esercitazione 5 -

Same questions as last time.

(1)

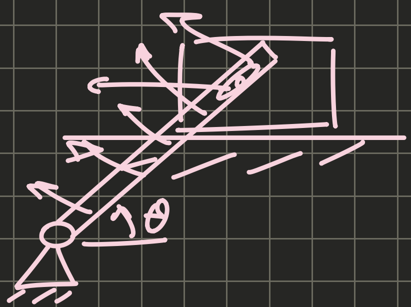


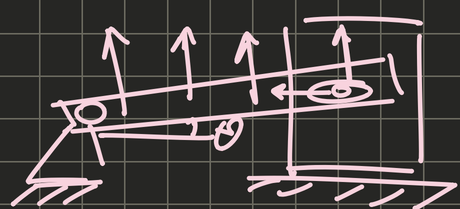
Is J linear or non-linear?

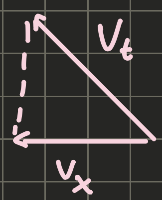
$$F_i = J\ddot{\theta}$$

Linear, since all the points move with the same velocity, so the kinetic energy is linear.

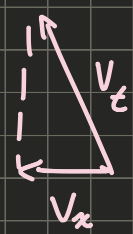
(2)







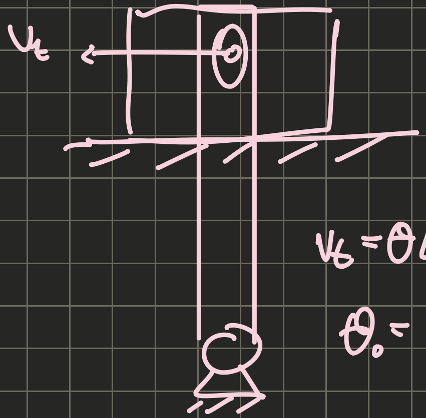
Velocity depends on θ and θ_0.



The inertia is non-linear with respect to θ.

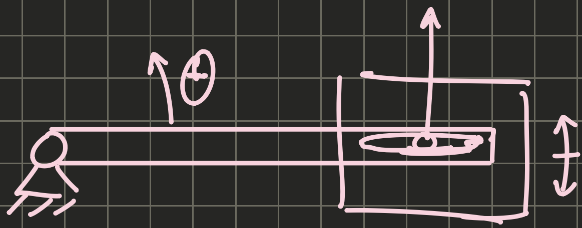
$$E_c = \frac{1}{2} \underbrace{J^*(\theta)}_{} \dot{\theta}^2$$

E_c is linear $\Longleftrightarrow$ J^* is not dependent on θ.



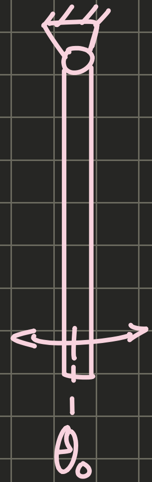
$v_t = \dot\theta L$

$\theta_0 = \frac{\pi}{2}$



$\theta_0 = 0$

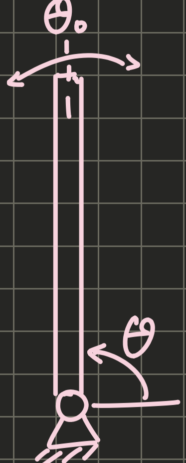
(3)



From EOM, around θ_0

$k_{III} > 0 \rightarrow$ adds stability.
Symmetric since it will push left or right depending on θ.

(4)

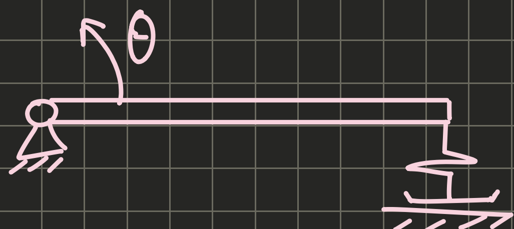


$\theta_0 = \pi$

$k_{II} < 0 \rightarrow$ adds instability.

We can only look at the signs of k_I, k_{II} and k_{III} just in the linear EOM.

(5)



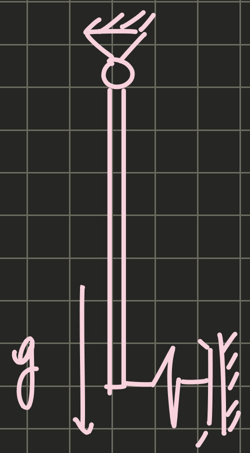
$k_{III} = 0$, the effect of gravity is not symmetric like θ, it always goes down.

$k_I > 0$

↳ usually, since it refers to springs, since for springs the force is always in the same direction as the motion.

$k_{II} = ?$

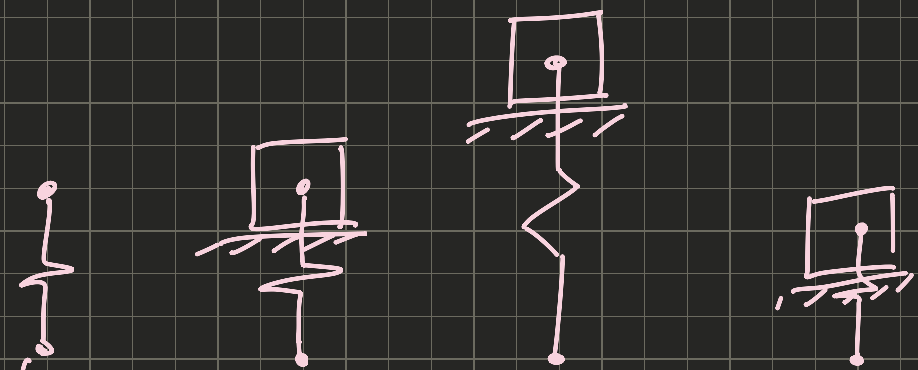
↳ Most difficult to intuitively understand the sign of.



$k_{III} > 0 \to$ symmetric

$k_I > 0$

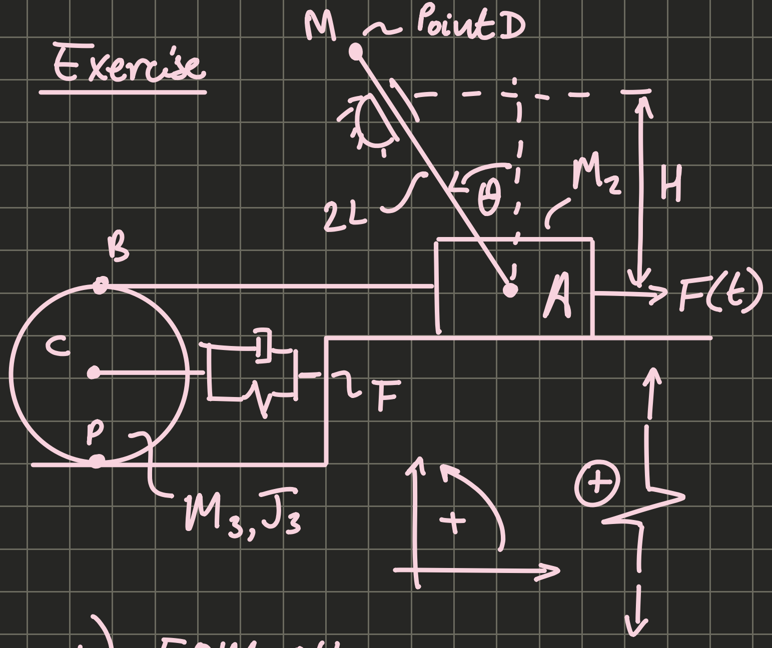
$k_{II} = ?$ If $\Delta l_0 = 0 \Rightarrow k_{II} = 0$

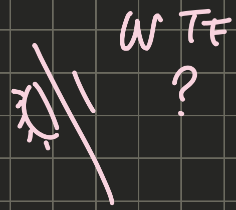


Ideal spring with no mass, cannot vibrate	$\Delta l_0 = 0$	$\Delta l_0 > 0$	$\Delta l_0 < 0$
$\Delta l_0 = 0$	$k_I = 0$	$k_I = 0$	$k_I = 0$
	$k_{II} = 0$	$k_{II} > 0$	$k_{II} < 0$

mass cannot move vertically

Exercise





H → Hint

WTF + HINT → USE HINT

1) EOM NL

2) $\Delta l_0 = ?$ For $\theta_0 = \pi/6$

3) EOM L $\boxed{\theta_0}$

$$F(t) = F_0 + F_t \cdot \cos(\Omega t)$$

degree of constraint.

→ Hinge, the permits sliding ⇒ 1 doc

so only have a reaction force perpendicular to rod.

Steps:

1) 3 bodies → 9 dof → 1 dof left → $\boxed{\theta}$

1 hinge → −2 dof

1 slider → −2 dof

1 rope → −1 dof

1 rotation without sliding → −2 dof

1 → −1 dof

2) $E_c = \frac{1}{2} M_1 V_D^2 + \cancel{\frac{1}{2} J_1 \omega_1^2} + \frac{1}{2} M_2 V_A^2 + \cancel{\frac{1}{2} J_2 \omega_c^2}$

$+ \frac{1}{2} M_3 V_c^2 + \frac{1}{2} J_3 \omega_3^2$

$D = \frac{1}{2} r \Delta \dot{l}^2$

$V = V_k + V_g$

$V_g = M_1 g h_D + \underbrace{M_2 g h_A + M_3 g h_c}_{\text{CONST}}$

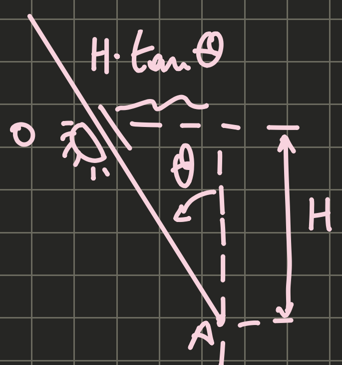
$V_k = \frac{1}{2} k \Delta l^2$

$\delta^* \mathcal{L} = \vec{F} \cdot \delta \vec{s}_F$

3) - $\vec{\omega}_1 = \dot{\theta}\, \hat{k}$

- $\vec{\omega}_2 = 0\, \hat{k}$

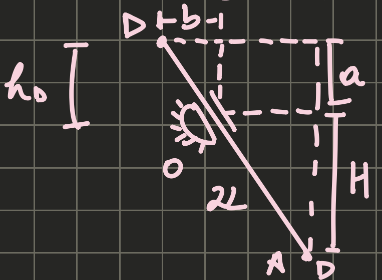
- $V_A = ?$ → Cannot use rivals, use hint.



$(A-O) = H \tan\theta\, \hat{i} - H \hat{j}$

$V_A = \frac{d}{dt}(A-O) = \frac{H}{\cos^2\theta} \dot{\theta}\, \hat{i}$

$h_D = ?$ $\quad V_D = ?$ $\quad$ 1st Approach



$(D-O) = -b\,\hat{i} + a\,\hat{j}$

$a + H = 2L\cos\theta \quad \rightarrow \quad a = 2L\cos\theta - H$

$b = a \tan\theta \qquad b = (2L\cos\theta - H)\tan\theta$

$$= 2L\sin\theta - H\tan\theta$$

$$(D-O) = (H\tan\theta - 2L\sin\theta)\,\hat{i} + \underbrace{(2L\cos\theta - H)}_{h_D}\,\hat{j}$$

$$V_D = \frac{d}{dt}(D-O) = \left[\left(\frac{H}{\cos^2\theta} - 2L\cos\theta\right)\hat{i} - (2L\sin\theta)\,\hat{j}\right]\dot{\theta}$$

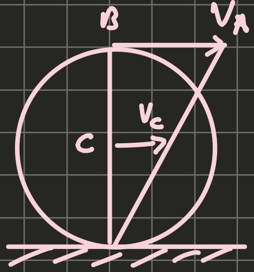
2nd Approach (Vectorial Closing)

$$(D-O) = (A-O) + (E-A) + (D-E)$$

$$= (H\tan\theta\,\hat{i} + H\hat{j}) + (2L\cos\theta\,\hat{j}) + (-2L\cos\theta\,\hat{i})$$

Same result as before

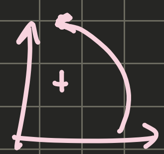
$V_C = ?$



$$V_C = \frac{\vec{V}_A}{2} = \frac{1}{2}\frac{H}{\cos^2\theta}\dot{\theta}\,\hat{i}$$

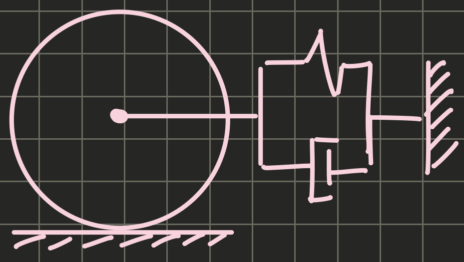
$$|\omega_3|\cdot R = |V_C| \rightarrow |\omega_3| = \frac{|V_C|}{R} = \frac{H}{2R\cos^2\theta}\dot{\theta}$$

$$\rightarrow \vec{\omega}_3 = -|\omega_3|\,\hat{k}$$

↳ Since opposite to 

$$\vec{\delta} s_A = \frac{\delta}{\delta\theta}(A-O)\cdot\delta\theta = \frac{\partial}{\partial\theta}(H\tan\theta\,\hat{i})\,\delta\theta$$

$$= \frac{H}{\cos^2\theta}\,\hat{i}\cdot\delta\theta$$



$\Delta l = ?$

1^{st} Approach

$$x_c = \int_0^T v_c\,dt = x_c + c =$$

$$v_c = \frac{1}{2}\,\frac{H}{\cos^2\theta}\cdot\dot{\theta}$$

1^{st} substitution

$$= \int v_c(\theta)\cdot\theta(t)\,dt = \int v_c(\theta)\,d\theta$$

$$= \int \frac{H}{2\cos^2\theta}\,d\theta = \frac{H}{2}\tan\theta + c$$

2^{nd} approach

$\hookrightarrow x_A = H\tan\theta$

$\Rightarrow x_c = \frac{H}{2}\tan\theta$

right - left (because convention)

$$l_m = x_F - x_c = x_F - \left(\frac{H}{2}\tan\theta + c\right)$$

$$\Delta l = \Delta l_0 + \Delta l_d =$$

$$\Delta l_d = l_m(\theta) - l_m(\theta_0) = \frac{H}{2}\left(\tan\theta_0 - \tan\theta\right)$$

$$\Delta l = \frac{d}{dt}\Delta l = -\frac{H}{2\cos^2\theta}\cdot\dot{\theta}$$

Step 4

$$E_c = \frac{1}{2}\Bigg[M_1\left(\frac{H^2}{\cos^4\theta} + \overbrace{4L^2\cancel{\cos^2\theta} - 4LH\frac{\cos\theta}{\cos^2\theta} + 4L^2\cancel{\sin^2\theta}}^{4L^2}\right)$$

$$+ M_2\left(\frac{H^2}{\cos^4\theta}\right) + M_3\frac{H^2}{4\cos^4\theta} + J_3\left(\frac{H^2}{4L^2\cos^4\theta}\right)\Bigg]\dot{\theta}^2 = \frac{1}{2}J^*(\theta)\,\dot{\theta}^2$$

$$\frac{\partial J^*(\theta)}{\partial \theta} = \left(M_1 + M_2 + \frac{M_3}{4} + \frac{J_3}{4R^2}\right)\frac{H^2}{\cos^5\theta}(-4)(-\sin\theta) - \frac{4M_1LH}{\cos^2\theta}(-1)(-\sin\theta)$$

$$D = \frac{1}{2} r \left(\frac{H}{2\cos^2\theta}\right)^2 \cdot \dot{\theta}^2 = \frac{1}{2} r^*(\theta)\,\dot{\theta}^2$$

$$V = \frac{1}{2} k (\Delta l_0 + \Delta l_d)^2 + M_1 g (2L\cos\theta - H) + M_2 g\, h_A + M_3 g\, h_c$$

$$\delta^* \mathcal{L} = F(t) \cdot \frac{H}{\cos^2\theta} \hat{i}\, \delta^*\theta = \frac{F(t) \cdot H}{\cos^2\theta} \delta\hat{\theta}$$

Step 5 → NL EOM

already found

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{\theta}}\right) - \frac{\partial E_c}{\partial \theta} = J^*(\theta) \cdot \ddot{\theta} + \frac{1}{2} \frac{\partial J^*(\theta)}{\partial \theta} \dot{\theta}$$

$$\frac{\partial D}{\partial \dot{\theta}} = r^*(\theta)\dot{\theta}$$

$$\frac{\partial V_g}{\partial \theta} = M_1 g (-2L\sin\theta)$$

$$\frac{\partial V_k}{\partial \theta} = k\, \Delta l\, \frac{\partial \Delta l}{\partial \theta} = k\left[\Delta l_0 + \frac{H}{2}\tan\theta_0 - \frac{H}{2}\tan\theta\right]\left(\frac{-H}{\cos^2\theta}\right)$$

$$Q_\theta = \frac{\delta^* \mathcal{L}}{\delta\theta} = \frac{F(t)\, dt}{\cos^2\theta} = F_0 \frac{H}{\cos^2\theta} + F_t \frac{H}{\cos^2\theta}$$

$$\text{NLEOM} \quad \frac{d}{dt}\frac{\partial E_c}{\partial \dot{\theta}} - \frac{\partial E_c}{\partial \theta} + \frac{\partial D}{\partial \dot{\theta}} + \frac{\partial V}{\partial \theta} = Q_S + Q_D$$