Lezione 3

Writing $\ell(Q)$ was through the dimensionlessisation of the Euler equation, and allowed us to find dimensionless terms.

Result of last lesson:

$$\ell = u_2 v_{2t} - u_1 v_{1t}$$

Using:

$n \rightarrow$ time scale

$D \rightarrow$ length scale

$$\ell = \ell(Q, \cancel{n}, \cancel{D}, SHAPE)$$

↳ becomes dimensionless

We find:

$$\lambda = \lambda(\varphi, SHAPE)$$

$$\begin{cases} \varphi = \dfrac{Q}{nD^3} \\ \lambda = \dfrac{\ell}{n^2 D^2} \end{cases}$$

Abstract approach to return to $H(Q)$

$$H = H(Q)\,? \longrightarrow \boxed{gH} = \ell - \ell_w = \ell(Q, n, D, SHAPE) - \ell_w(n, D, \rho, \mu, SHAPE)$$

$$\Rightarrow \boxed{H(Q, n, D, \rho, \mu, SHAPE}$$

What is ℓ_w?

$$y \rightarrow \xi = \frac{y}{V^2/2} = \xi(Re, SHAPE)$$

↳ Dimensionless loss

"Tsi" → Zeta

$$\ell_w \rightarrow \zeta = \ell_w / n^2 D^2 = \zeta(Re, SHAPE); \; Re = \frac{\rho D^2 \omega}{\mu}$$

↳ Physically it's the same a y, but occurs inside the machine

↳ To make dimensionless it's best to use the easiest thing to determine.

$\Rightarrow l_w = l_w(n, D, \rho, \mu, \text{SHAPE})$

↳ We are taking this for granted without much demostation

↳ Head dimensionless coefficient:

$$\psi = \frac{gH}{n^2 D^2}$$

$$\Rightarrow \psi = \psi(\varphi, Re, \text{SHAPE})$$

In this course we always consider Re such that we have a fully-turbulent flow. $\Rightarrow \frac{\partial \psi}{\partial Re} = 0 \begin{cases} \Rightarrow \psi = \psi(\varphi, \text{SHAPE}) \\ \text{and } gH = g^{+}(Q, n, D, \text{SHAPE}) \end{cases}$

Hydraulic similarity

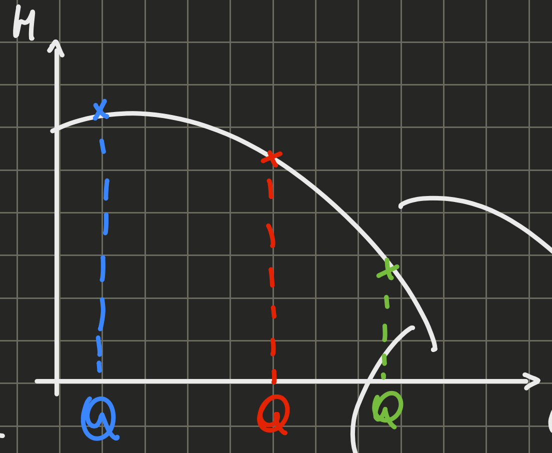
$\Rightarrow$ ↳ Incompressible flows

$\Rightarrow$ ↳ Fully-turbulent flow (dynamic → kinematic)

SHAPE ASSIGNED ⇝ we focus on a family of geometrically similar machines.

D assigned $\Rightarrow$ size of the machine defined, since we have already given SHAPE, everything else it defined

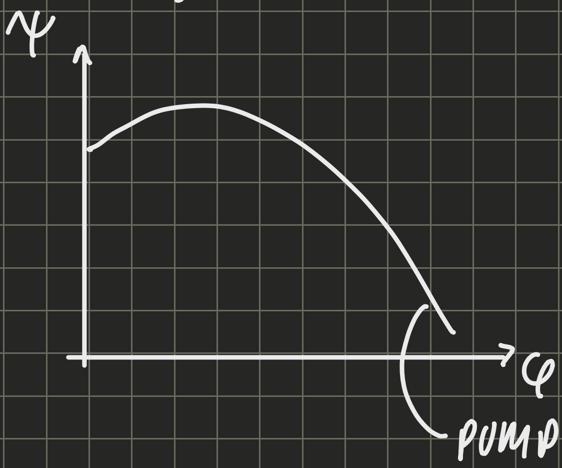
Q, n → represent the operating condotion of the machine



SHAPE, D and u assigned
=> only once have done this will we have a dimensional relationship

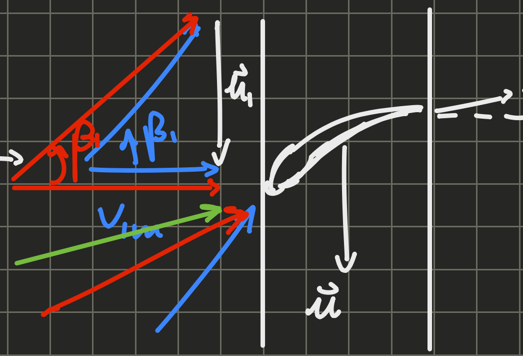
→ valid for specific machine, of shape and size and operating condition.

In dimensionless terms only the SHAPE needs to be assigned since $\psi = \psi(\varphi, \text{SHAPE})$



→ these curves are true for a family of machines, independent of size and operating condition

↳ This is very useful for us.



Once we know Q and the size we know V_{1m}

We only design the blade to one β_1, so if we change Q, there will only one condition (nominal condition) that works.

β_2 is constructive. The more we have to deflect the flow, the more energy do we have spent. So varying Q, the head also changed.

Off the nominal point, the triangles change, and performances.

Using l_w to define efficiency.

Hydrodynamic efficiency of pump

$$\boxed{\eta_p} = \frac{gH}{\ell} = \frac{\ell - l_w}{\ell} = \boxed{1 - \frac{l_w}{\ell}}$$

$$\eta_p = \frac{\psi}{\lambda} = \eta_p(\varphi, \text{SHAPE})$$

since both are their dimensional terms divided by $n^2 D^2$.

IF NOT FULLY-TURBOLENT FLOW: $\eta_p(\varphi, Re, \text{SHAPE})$

Result of dimensionless analysis:

$\lambda = \lambda(\varphi, \text{SHAPE})$ $\boxed{9:24}$

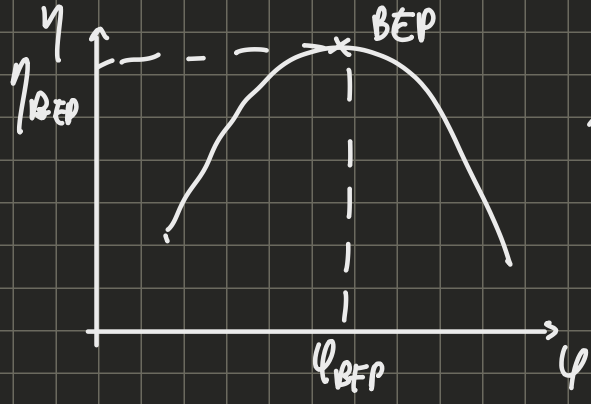
$\psi = \psi(\varphi, \cancel{Re}, \text{SHAPE})$ → if Re is low we need to add corrective term.

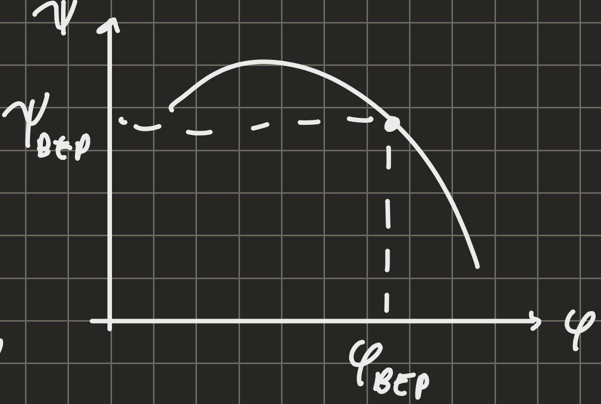
$\eta = \eta(\varphi, \cancel{Re}, \text{SHAPE})$

$\rightsquigarrow \psi = \psi(\lambda, \text{SHAPE})$ → we can do this as long as we only have 2 dimensionless terms.
↳ independent

$\rightsquigarrow \eta = \eta(\lambda, \text{SHAPE})$ or $\eta = \eta(\psi, \text{SHAPE})$

We can represent the relationships:





B.E.P = efficiency point

We typically try to design a machine to it's BEP.

Given a machine for which $H = H(Q, n, D, SHAPE)$

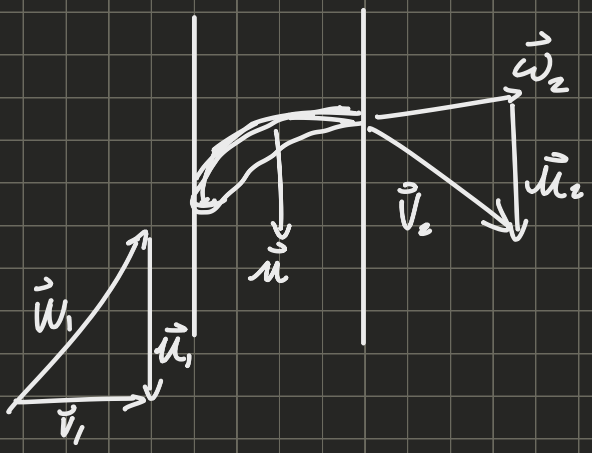
$\psi = \psi(\varphi, SHAPE)$

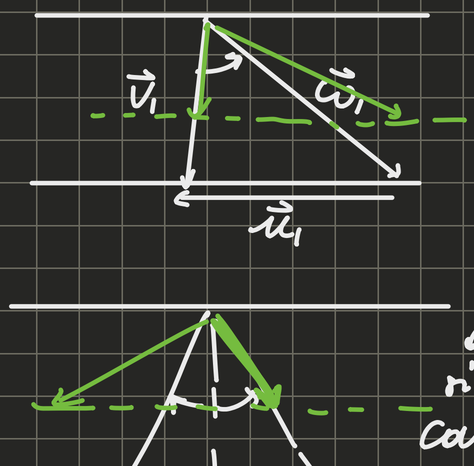
Is it possible to change Q, n, D such that they change but ψ doesn't?

Yes. If true these conditions are called similarity corresponding conditions.

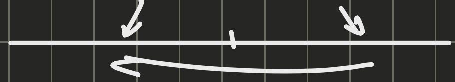
$$\begin{cases} \varphi, \varphi' \\ \psi, \psi' \end{cases} \text{such that} \begin{cases} \varphi = \varphi' \\ \psi = \psi' \end{cases} \Rightarrow$$ similarity or corresponding conditions

Example to help understand



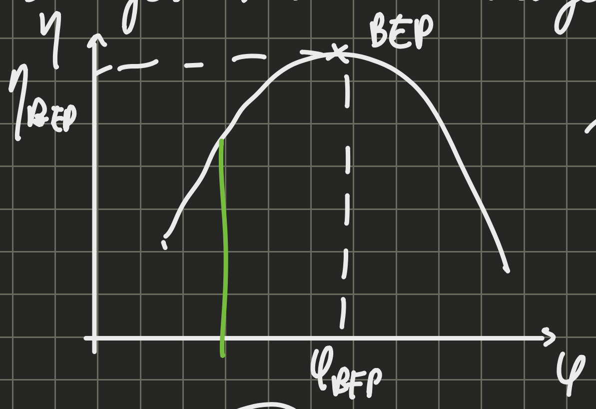


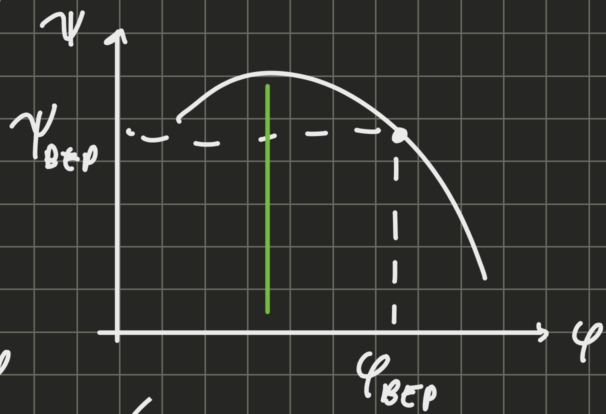
α_1 & β_2 don't change since they are constructive



What happens if $Q' = Q/2$

keeping the machine as is, without changing D or n, everything changes as we have changed conditions.



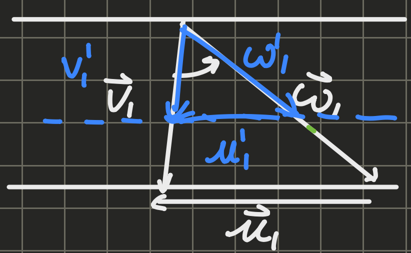


↳ we exchange more work, but we waste a lot more work.

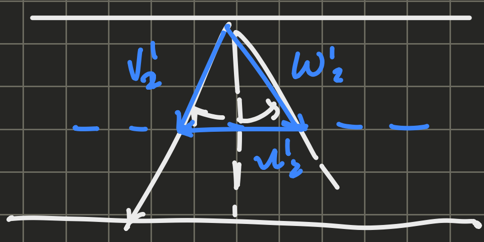
$Q' = Q/2$, $\underbrace{D' = D}$ → we want to find n so we have a similarity condition.

we keep the same machine so we don't have to get another one.

if similarity: $\varphi' = \varphi \Rightarrow \dfrac{Q'}{n'\cancel{D^3}} = \dfrac{Q}{n\cancel{D^3}} \Rightarrow \dfrac{n'}{n} = \dfrac{Q'}{Q} \Rightarrow n' = n/2$



→ to have similarity we need to halve the speed.



The β_1 & α_2 are the same
⇒ through similarity conditions,

$\beta_1' = \beta_1$ & $\alpha_2' = \alpha_2$ we can guarantee that our non-constructive angles are conserved.

Conserving the shape of the triangles, we also conserve performance parameters ($\varphi' = \varphi$ & $\psi' = \psi$)

The velocity triangles are different, meaning the work be lower since the factor was 2, the work will be 4 times lower.

The head is proportional to the work only if the efficiency is constant $\Rightarrow$ since $\underbrace{\frac{\ell'}{\ell} = \frac{1}{4}}_{\text{In this core}}$ $\Rightarrow$ $\underbrace{\frac{H'}{H} = \frac{1}{4}}_{\text{In this case.}}$

Other approach

$Q' = Q/2$; $D' = D$, and shape assigned. (necessary)

$$\left.\begin{aligned} &\varphi' = \varphi \\ &\psi = \psi(\varphi, \text{SHAPE}) \\ &\psi' = \psi'(\varphi', \text{SHAPE}) \end{aligned}\right\} \Rightarrow \psi' = \psi \Rightarrow H' = \left(\frac{n'}{n}\right) H$$

since $\varphi' = \varphi$ and the shape is assigned.

quicker derivation

Similarity condition $\Rightarrow$ similarity in the velocity triangles.

Once again, everything will become more complicated with thermal machines where $\rho \neq$ const.

The next 2 hours we will be using an approach which isn't very rigorous, but the result works.