Lezione 26 -

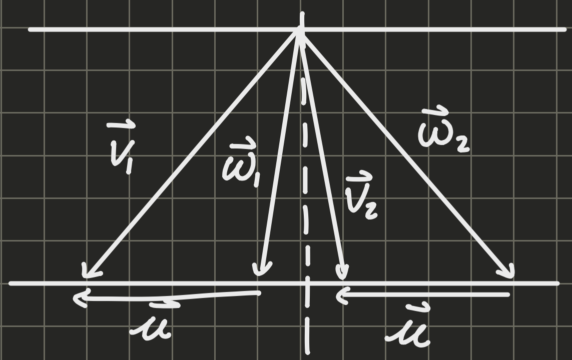
Reaction Stage Axial Turbines with $\chi = 0,5$ & $v_m = \text{const}$

We already define χ for turbines as:

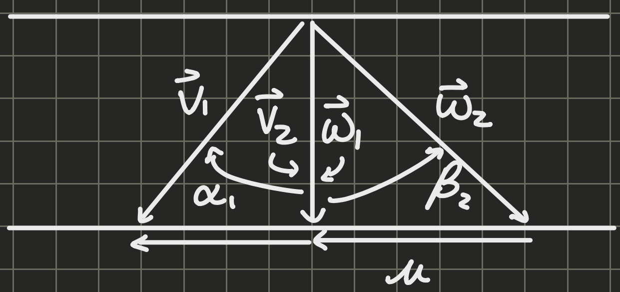
$$\chi = \frac{\omega_2^2 - \omega_1^2}{V_1^2 - V_2^2 + \omega_2^2 - \omega_1^2} = \frac{\omega_{2t}^2 - \omega_{1t}^2}{V_{1t}^2 - V_{2t}^2 + \omega_{2t}^2 - \omega_{1t}^2} = \frac{\tan^2(\beta_2) - \tan^2(\beta_1)}{\tan^2\alpha_1 - \tan^2\alpha_2 + \tan^2\beta_2 - \tan^2\beta_1}$$

There are many ways for $\chi = 0,5$, but the only one that matters is when:

$$\begin{cases} \beta_2 = -\alpha_1 \\ \beta_1 = -\alpha_2 \end{cases} \iff \chi = 0,5$$



We say the stage is optimized $\iff V_{2t} = 0$

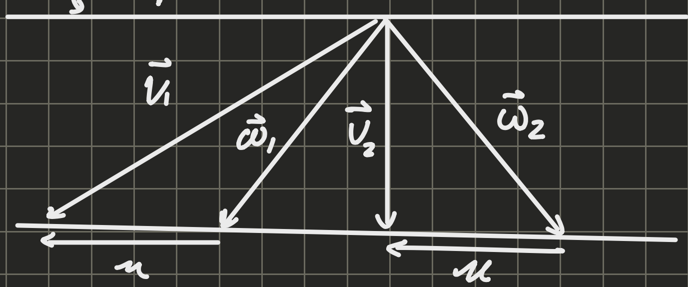


$$k_{p,opt} = \frac{u}{V_1} = \sin\alpha_1$$

$$l_{opt} = -u^2$$

↳ Half that for an impulse stage.

Impulse stage triangles.

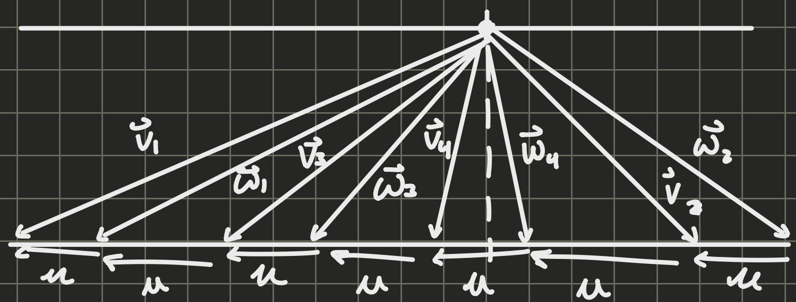


Reaction stages provide higher efficiency than impulse stages, since they deflect less.
a minimizes peak velocity.

Curtis / Velocity Compound Stage $\chi = 0, v_m = \text{const}$

Designed to leave extra work exchanged by not being optimized.

We want v_{2t} to be negative.



$$l = u(v_{2t} - v_{1t})$$

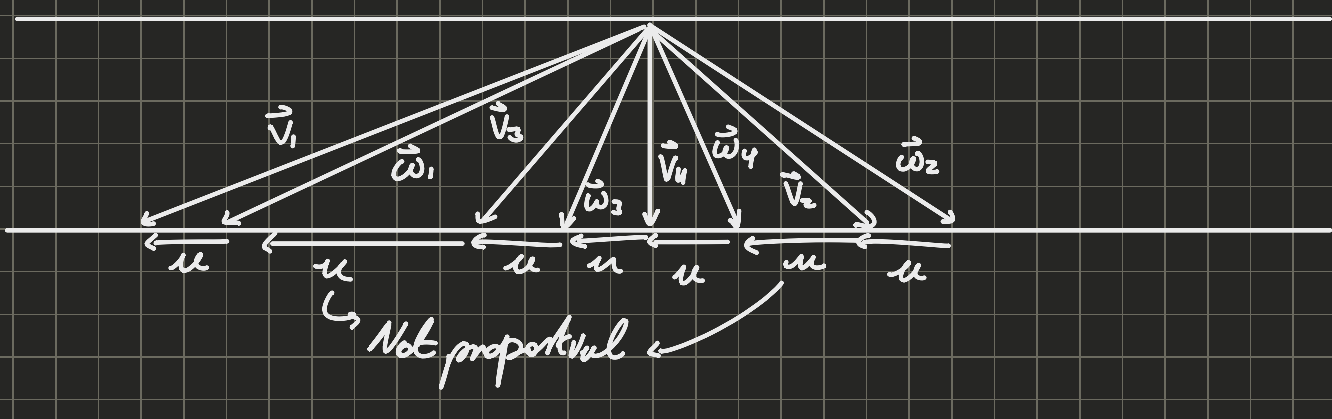
What happens if we added an additional rotor to consume the remaining v_{2t}.

Since v_{2t} is in the wrong direction, we need a stator before the second rotor.

$$l = \underbrace{l_I + l_{II}}_{} = u(v_{2t} - v_{1t}) + u(v_{4t} - v_{3t})$$

Two stages

Optimized $\Longleftrightarrow$ $v_{4t} = 0$



$V_{1t} = 4u$

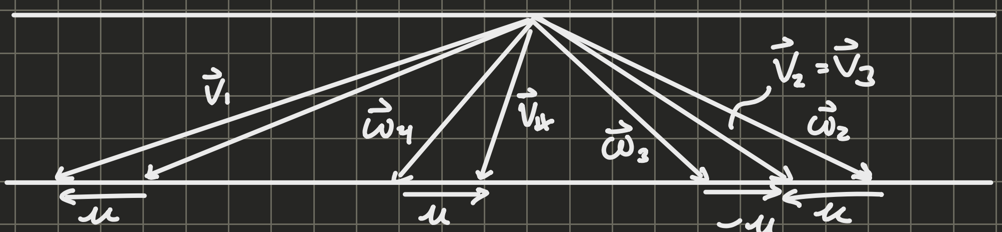
$$\lambda_{opt} = \frac{\sin \alpha_1}{4} = \frac{u}{V_1}$$

$$\boxed{l_{opt}} = (-2u - 4u)u + (0 - 2u)u = \boxed{-8u^2}$$

It has a significant capability of exchanging work.
This stage is very interesting in case for which u is low.

There is a way to remove the stator.

Taking the velocity triangle:



We can use a counterrotating rotor

→ Counter-Rotating Rotor Stage.

Insight into aerodynamic losses:

→ There are several loss mechanisms:

- → Profile Losses due to boundary layers.
- → Endwall losses
- → Tip clearance losses.
- → Shock losses

These depend on Reynolds, Mach and shape.

We can quantify these losses through coefficients:

$$\zeta = \frac{h_{OUT} - h_{OUT,is}}{V_{out}^2/2} \rightarrow \frac{V_{out}^2}{V_{out,is}^2} = \frac{1}{1+\zeta}$$

↳ zeta

The higher the $\Delta\beta$ the higher the losses.

<u>Global layout of a steam turbines</u>

→ How huge steam turbines are designed. Consider change in flow and volume flow rate changes.

→ We also have to consider re-heat.

In first approximation, >10 stages are needed.

Comparison among stages

$$\omega_s \propto \sqrt{b/D_m}$$

<Slides for comments>

$$\omega_s = \omega \frac{\sqrt{Q}}{(\Delta h_{TT,s})^{3/4}} = \frac{\omega}{\sqrt{\rho}} \eta^{3/4} \frac{\sqrt{\dot{m}}}{\ell^{3/4}}$$

We divide our machine in different parts:

- High pressure section → low blade heigh, low D_m and low shape factor.
- Intermediate Pressure section → standard aspect ratio, it is less critical
- Low Pressure section → extremely high blade shape factor

Design choices are also defined by structural integrity considerations.

Limitations on Design

→ Aerodynamic Performance → Too high $\Delta\beta$ is bad.
Mach number at the inlet is critical, we must avoid it reaching 1.
We also limit shape factor:

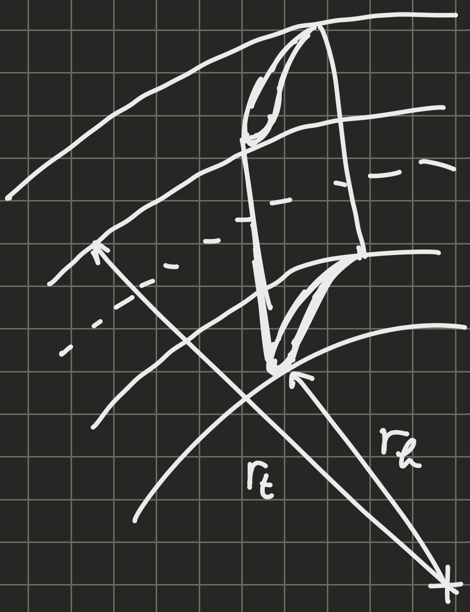
$$0.025 < \frac{b}{D_m} < 0.4$$

→ Endwall losses   too high   optimized.

→ Feasibility of some blade shape, with the being too long.

→ Structural Perspective → aerodynamic forcing due to the flow deflection and, especially, centrifugal stresses in the rotor must be considered and limited.

Structural limitation on high aspect ratio blades.



$$F_c(r) = \int_r^{r_t} \rho_m \omega^2 r A \, dr$$

$$= \rho_m \omega^2 A \int_r^{r_t} r \, dr$$

$$F_{c\,max} = F_c(r = r_h) = \rho_m \omega^2 A \int_{r_h}^{r_t} r \, dr =$$

$$= \frac{1}{2}\rho_M \omega^2 A (r_t^2 - r_h^2)$$

$$= \rho_M \omega^2 A \underbrace{\frac{(r_t + r_h)}{2}} \cdot (r_t - r_h)$$

$$= \rho_M \omega^2 A \, r_m \, b = \rho_M \omega^2 A r_m D_m \frac{b}{D_m}$$

r_m ↳ Mean radius

$$= 2 \rho_M A \omega^2 r_m^2 \frac{b}{D_m} = \boxed{2 \rho_M A u_m^2 \frac{b}{D_m}}$$

$$F_{c\,max} < F_{LIM} = \sigma_{m,LIM} \cdot A$$

$$2 \rho_M \cancel{A} \cdot u_m^2 \frac{b}{D_m} < \sigma_{m,LIM} \cdot \cancel{A} \implies u_m^2 \frac{b}{D_m} < \frac{\sigma_{m,LIM}}{\rho_M \cdot 2}$$

ρ_M ↳ Blade material density.

Our best choice is a very light and resistant material, since it allows a very high $\frac{b}{D_m}$

For turbines we can reach $u_m = 600 \frac{m}{s}$

<u>High - Pressure Section of a Steam Turbine</u>
↳ Important for theory

$$\dot{m}_{HP} = \rho_{HP} \cdot Q_{HP} \implies Q_{HP} = \frac{\dot{m}_{HP}}{\rho_{HP}}$$

ρ_{HP} ↳ high → Q low.

We need a machine for very low Q_{HP}.

$$Q_{HP} = V_m \pi D_m b = V_m \pi \underline{D_m^2} \cdot \frac{b}{D_m}$$

There is a relationship between D_m and u since n is fixed.

$$= V_m \pi \frac{u_m^2}{\omega^2} \frac{b}{D_m} \propto u_m^2 \frac{b}{D_m}$$

↳ Not easily modifiable

$$Q_{HP} \Downarrow \Longrightarrow \begin{cases} \frac{b}{D_m} = \left(\frac{b}{D_m}\right)_{min} \to 0{,}025 \\ u_m \Downarrow \end{cases}$$

INSUFFICIENT Peripheral speed!

$\boxed{15:57}$

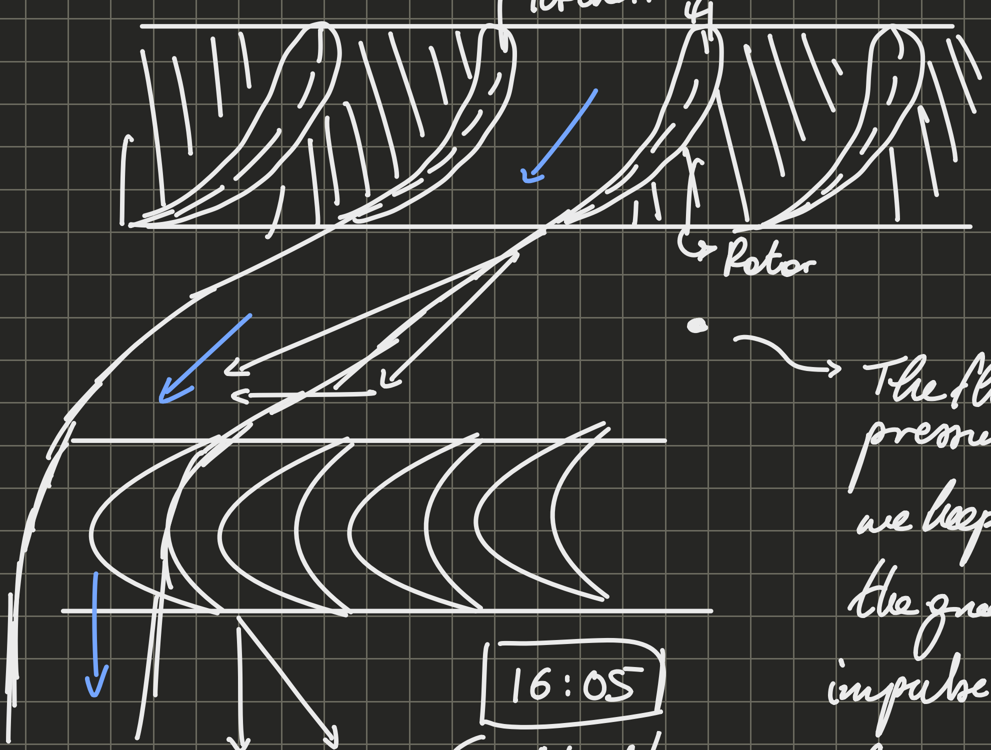
↳ Bagno

Our stage will exchange low work, Why? → impulse stages! → since $l = 2u^2$ or $8u^2$, even if higher losses

↳ optimized for low u.

If neither is enough we can reduce the cross section to reduce further.

Let's assume we leave only this section open. Q_{HP}



The remaining jet will follow that path.

The fluid here feels the pressure gradient, but because we keep the section constant the gradient is not felt. The impulse stage is the only stage where we can maintain the consistency of the jet.

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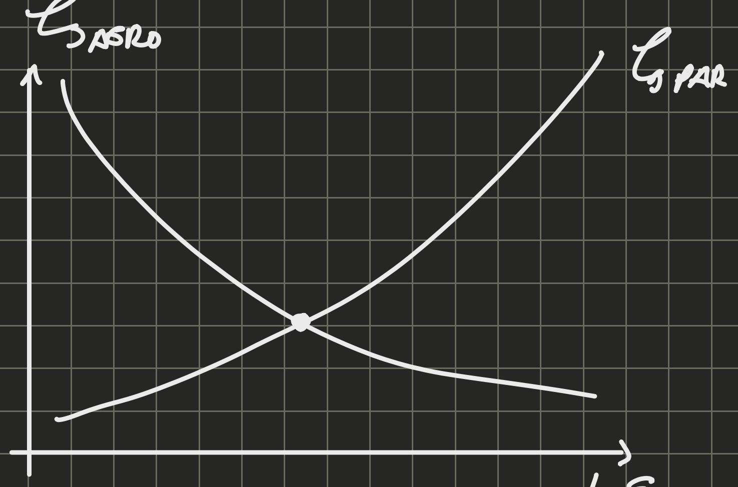
↳ Need to recheck to understand.

We usually keep two opening to maintain symmetry but keeping wake losses.

$$\varepsilon = \frac{A_{OPEN}}{A_{TOT}}$$



In a partial emission stage

$$Q_{HP} = V_m \pi \frac{u_m^2}{\omega^2} \frac{b}{D_m} \cdot \varepsilon$$

Low Pressure of a steam turbine

$$\dot{m}_{LP} = \rho_{LP} \cdot Q_{LP} \Rightarrow Q_{LP} = \frac{\dot{m}}{\rho_{LP}} \uparrow\uparrow\uparrow \text{ very high}$$

$$Q_{LP} = v_m \pi D_m^2 \frac{b}{D_m} = V_m \pi \frac{u_m^2}{\omega^2} \frac{b}{D_m}$$

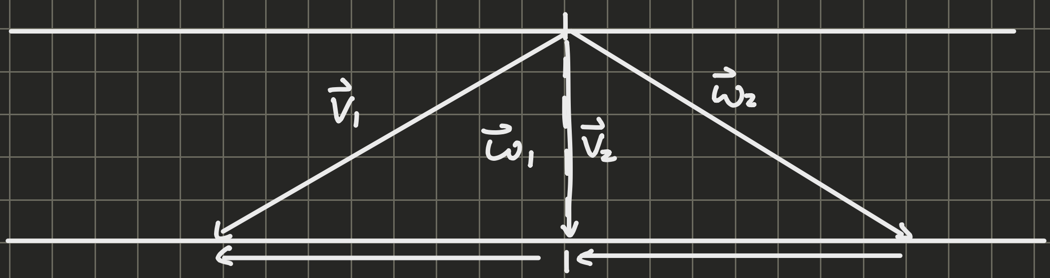
$\chi_{LP} = 0{,}5$! (AT MID AT LEAST !)

$l - l_w = \int v\,dP$

$(l - l_w)_{LP} \int V_{LP}\,dP$ → Very high

So $\int V_{LP}\,dP$ is very high

⇒ The last stage will exchange a lot of work and produce the most enthalpy drop.



$$V_m = \frac{V_{1t}}{\tan\alpha_1} = \frac{u}{\tan\alpha_1} \rightsquigarrow V_m = u_m \cdot \frac{1}{\tan\alpha_1}$$

We need the highest efficiency

↳ We need to use reaction machines.

↳ Also u is high so we can use impulse and have high efficiency

$$Q_{LP} = \frac{1}{\tan\alpha_1} \cdot \pi \frac{u_m^3}{\omega^2} \cdot \frac{b}{D_m}$$

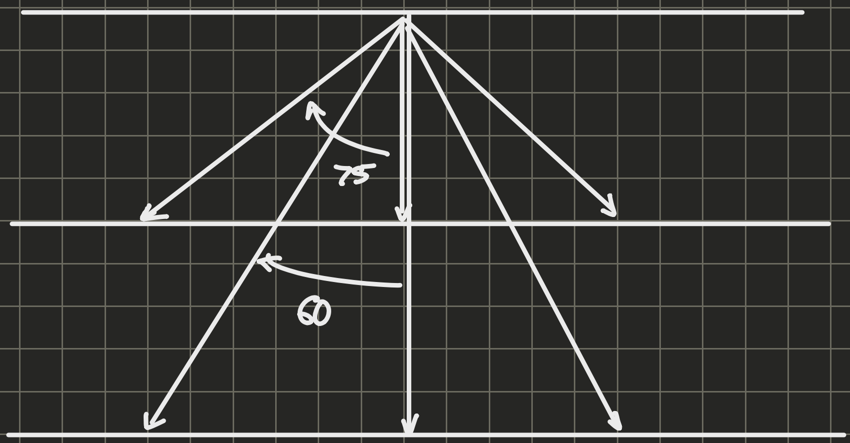
↳ This needs to be maximized

$\alpha_{1MIN} \cong 65°$

$u_{m,max} \cong 600\ m/s \Rightarrow \dot{L}_{MAX} \cong 200MW$

$\left(b/D_m\right)_{MAX} \cong 0{,}4$

Curtis and impulse don't have high efficiency at high u.



We already said steam turbines can reach 1GW, there are some things we are missing.

Increasing v_n we reduce α_1, but v_n also has it's own associated losses.

<! Figure of how turbine works.

In the low pressure, the flow is split in two streams, which operate on two different parts on the same rotor. This parallelisation allows us to increase the fluxes. We only do this in the low pressure part since the high pressure section is penalised by the separation where the volume flow is already so low.

Specific Issue

↳ This parallelisation may not be enough since it can cause the vapor to enter the turbine as saturated, so we have to reduce n to 1500.

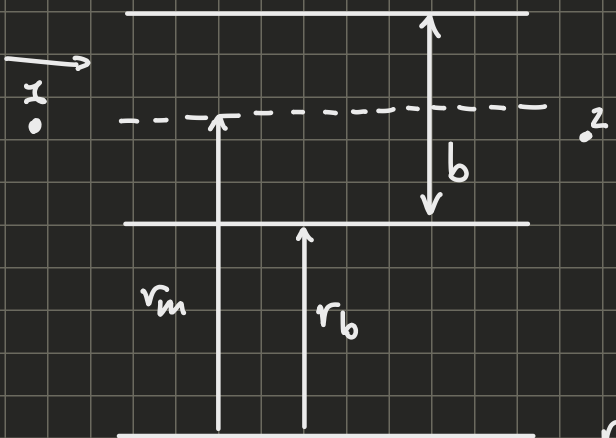
enthalpy drop is low.

To emit high L, we need high $\dot{m}$ and so Q, so we need more fluxes and reduce n

↳ since we have a limitation on u_m, this decrease means we can increase D_m and so the area so Q can be separated more.

The last few blades in the low pressure zone are twisted.

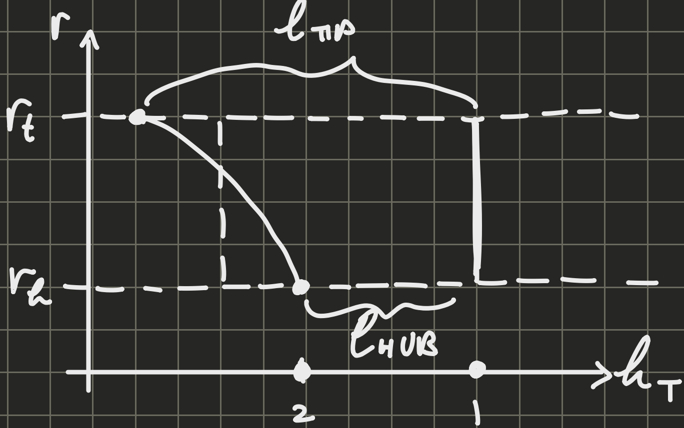
If we keep constant the stage style along the blade, the ability for the blade to exchange work changes.



$u = \omega r$

$l_{HUB} = k\omega^2 r_h^2$

$l_{TIP} = k\omega^2 r_t^2$



This difference will cause vortices and mixing at the exit of the stage. We want l to be near uniform along the span.

$$l = l(r) = u\,\Delta V_t = u(r)\big(\underbrace{V_{2t}}_{\to 0} - V_{1t}\big) = -V_{1t}(r)\cdot u(r)$$

$$l = \text{cost} \iff V_{1t}(r)\cdot u(r) = \text{const} \Rightarrow V_{1t}\cdot\omega r = \text{const}$$

$$\Rightarrow V_{1t} = \frac{k}{r}$$: Free Vortex Blade Design

↳ Applied to both rotors and stators.

$$\left.\begin{array}{ll} \text{Hub:} & V_{1tH} = \dfrac{k}{r_h} \\ \text{TIP:} & V_{1tT} = \dfrac{k}{r_t} \end{array}\right\} \Rightarrow \begin{array}{l} V_{1tH} > V_{1tT} \\ V_{1m} = \text{const}(r) \end{array} \Rightarrow \left\{\begin{array}{l} V_{1H} > V_{1T} \Rightarrow p_{1H} < p_{1T} \\ \beta = \text{const} \end{array}\right.$$

$$\Rightarrow \chi_H < \chi_T$$

Most of the expansion will occur at the hub instead of the tip. l is uniform, but χ is not the same.