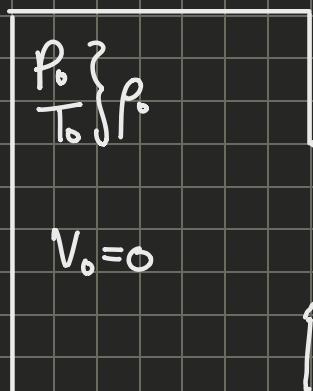


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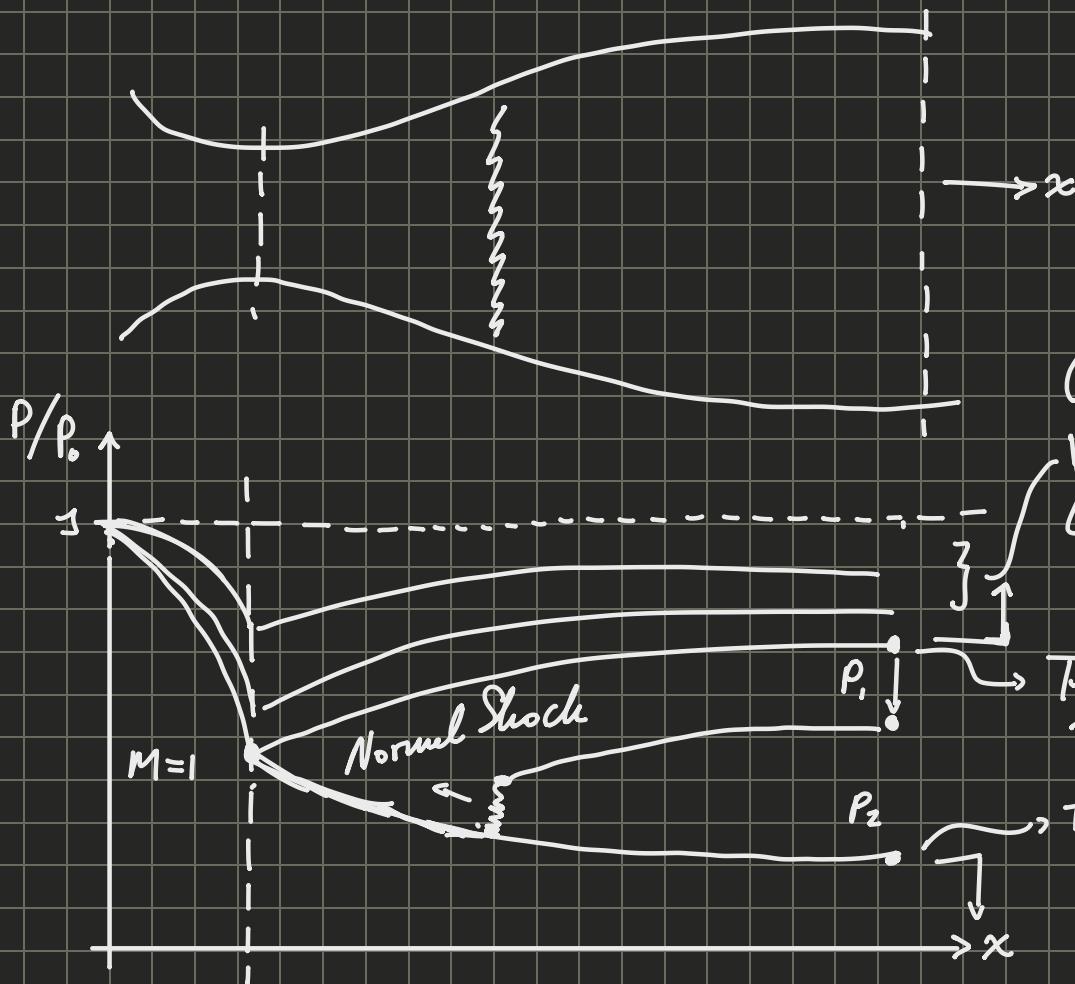
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## De Laval Nozzle



we follow the natural path  
the fluid takes to form this

$P_{\text{EXT}}$



(When  $M < 1$  at throat)

Venturi Duct

↳ Combination of Nozzle and Diffuser → can The fluid chooses to follow the diffuser to match the  $P_{\text{EXT}}$  The fluid can also choose to not match

$M < 0.3$  Venturi Duct

$0.3 < M < 1$  compressible "

$M = 1$  Sonic "

For  $P_{\text{EXT}} \geq P_1$  Venturi (incompressible, compressible, sonic)

For  $P_{\text{EXT}} \leq P_2$ : Full Nozzle

↳ Subsonic Converging Nozzle, supersonic diverging nozzle.

The missing pressure difference will expand after the divergent.

normal shock wave

If  $P_1 < P_{\text{EXT}} < P_2 \rightarrow$  our models do not provide any solution. To get this solution we need to eliminate the isentropic condition.

The fluid will treat a part of the diffuser as a nozzle, then a shock wave creates a discontinuity and then the fluid will act subsonically and see a diffuser so will do to the wanted pressure

The only way to reach the boundary condition  $P_{\text{EXT}}$ , is to not be isentropic and allow discontinuity.

$P_o$  and  $T_o$  are always good boundary conditions.

We need another condition, since we have 3 equations (in 1D)

$P_{\text{THROAT}}$  cannot be used since all  $P_{\text{EXT}} < P_1$ , will have passed subsonic speed at the nozzle, so there will be infinite solutions, all below  $P_1$ .

↳ Same for in THROAT, since every point where  $P_{\text{EXT}} \leq P_1$  will have the same flow rate.

$P_{\text{EXT}}$  allows us to define a well-posed problem.

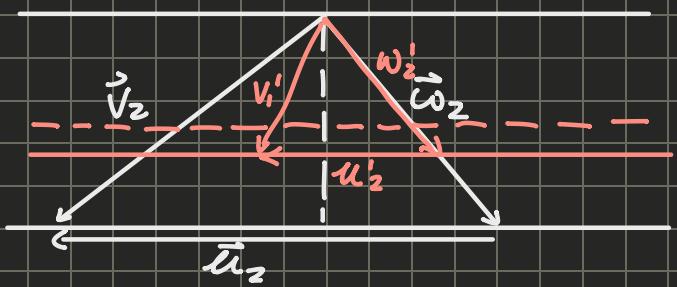
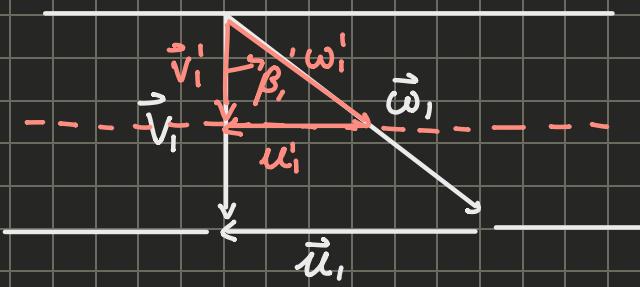
# Analysis of Thermal Machines

(Turbo -) Compressors: Implication of Compressibility

↳ Compressor with the shape of a turbo machine.

We will see the effects of compressibility as a difference to the incompressible case.

Similarity: is  $\varphi$  and  $\psi$  being the same enough now?



$$\underline{\dot{m}'} = \frac{1}{2} \dot{m}$$

$\rho'_1 = \rho_1 \rightarrow$  same density since we are taking from an external environment

$$A'_1 = A_1$$

$$\Rightarrow V'_{1,m} = \frac{1}{2} V_{1,m}$$

↓

$$\underline{V'_1} = \frac{1}{2} \underline{V'_1}$$

$$\underline{\varphi'} = \underline{\varphi} \Rightarrow n' = \frac{1}{2} n, \underline{u'_1} = \underline{u_1}$$

For now it's working by following the rules of hydraulics.

$$\underline{\beta'_1} = \underline{\beta_1}$$

$$\dot{m}' = \dot{m}_2' = \rho_2' V_{zm}^{-1} A_2$$

The machine does not change

$$\dot{m} = \dot{m}_2 = \rho_2 V_{zm} A_2 \quad \left\{ \Rightarrow \rho_2' V_{zm}' = \frac{\rho_2 V_{zm}}{2} \Rightarrow V_{zm}' = \frac{1}{2} V_{zm} \cdot \frac{\rho_2}{\rho_2'} \right.$$

$$\dot{m}' = \frac{1}{2} \dot{m}$$

$\hookrightarrow$  Bad news, since as a consequence of the thermodynamic transformation, the velocity triangles will be altered.

$$l_s = c_p \Delta T_m (\beta^{\frac{\gamma-1}{\gamma}} - 1), \quad l \propto u^2 \quad (u_2 V_{zt})$$

$\hookrightarrow$  Work exchanged

Since we have said  $u'_1 = \frac{1}{2} u_1$   $\hookrightarrow l' \propto \frac{1}{4} l \rightarrow \beta \downarrow \rightarrow \rho_2' \downarrow$  relative to  $\rho_2$ .

This cause  $\alpha'_2 \neq \alpha_2$ , and so similarity is not respected.

The hydraulic approach to similarity

So apply similarity at the outlet goes not give similarity at the outlet and vice versa.

We can use  $\Delta T$  to compensate the change in  $\rho$ .

What is the final approach found to solve it.

We don't derive approaches in this part of the course.

## Dimensionless Approach

We used mass flow rather than volumetric flow rate

$$\varphi \rightarrow m_{AD} = \frac{m \sqrt{RT_{T,IN}}}{P_{T,IN} D^2}$$

We can still use

$$\rightarrow \psi = \frac{\ell_s}{n^2 D^2} = \frac{\gamma}{\gamma-1} \cdot \frac{R}{M} T?$$

$$\ell + q = \Delta h + \frac{\Delta V^2}{2} + g \beta_3 = \Delta \left( h + \frac{V^2}{2} \right) = \Delta h_T = C_p (T_{T,OUT} - T_{T,IN})$$

↑ the full form

~~$\ell = C_p \Delta T$~~  → only if we neglect  $\Delta V^2$

$$= C_p T_{T,IN} \left( \frac{T_{T,OUT}}{T_{T,IN}} - 1 \right)$$

$$\frac{T_{T,1}}{T_1} = \frac{T_{T,IN}}{T_{IN}}$$

$$T_{T,IN} = T_{IN} + \frac{V_{IN}^2}{2C_p}$$

$$= C_p T_{T,IN} \left( \left( \frac{P_{T,OUT}}{P_{T,IN}} \right)^{\frac{n-1}{n}} - 1 \right),$$

$\beta_T \rightarrow$  total pressure ratio

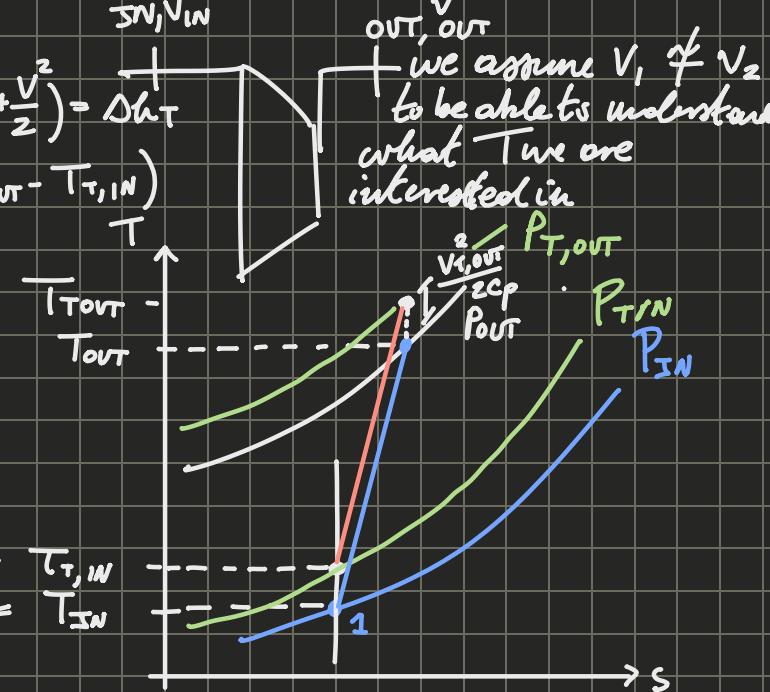
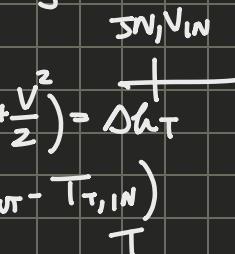
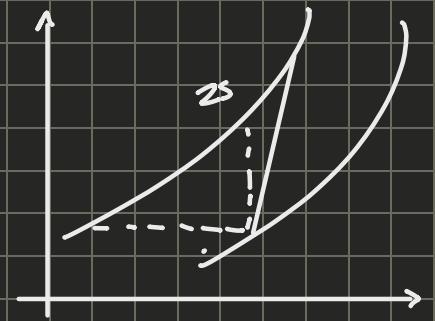
more precise than  $\beta$  since it doesn't ignore  $\Delta V^2$

$$\Delta \equiv \frac{P_{T,OUT}}{P_{T,IN}}$$

$$\ell_s = C_p T_{T,IN} \left( \beta_T^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$$\Rightarrow \psi = \frac{\ell_s}{n^2 D^2} = \frac{\gamma}{\gamma-1} \cdot \frac{R}{M} T_{T,IN} \left( \beta_T^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

The dimensionless parameter we need to add is one for the compressibility, where:  $\rightarrow$  which is the Mach number



$$M_U = \left( \frac{\omega D/2}{\sqrt{\gamma RT_{TIN}}} \right) ; \quad Re = \frac{\omega D^2}{V} ; \quad \gamma$$

Each  $T_{TIN}$  to find  
and also boundary  
condition.  
→  $n$  needs to be scaled, so that is useful.

**Peripheral Mach number**

We have a system of 5 dimensionless parameters.

We have 4 fundamental dimensional quantities, we need to find.

In  $\psi$ ,  $\beta_T$  is the only parameter which doesn't appear in other equations, so instead of using the complicated equation for  $\psi$ , we use  $\beta_T$  since it's also nice for engineers designing.

$$\beta_T = \beta_T (in_{AD}, M_U, Re, \gamma, \cancel{SHAPE})$$

$$\eta = \eta (in_{AD}, M_U, Re, \gamma, \cancel{SHAPE})$$

Generally these are fixed, so we can reduce to 3 dimensionless parameters.

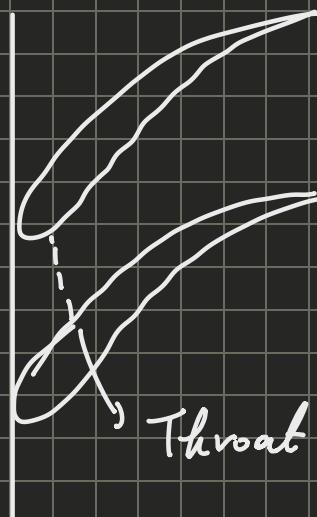
Increasing  $M_U$  increases the range the machine can operate in.

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The curves become vertical since the pressure ratio is decreasing as we change the speed, in some part of the machine, there is a part that chokes because it has reached

mach number.

Once  $M=1$  is reached,  
the curve becomes  
vertical.



Creating a line from where all these points where the curve chochles, we create the chochled-flow line.

In the case of compressors we should be worried about unstable connections, since that can cause issue, this is why there's nothing before the maximum pressure since engineers determined operating before as unsafe.

Cures of compressors become vertical due to chochly, this point is based on  $M_u$ .

The machine curves are cut to only safe operating zones.



like the dimensionless parameters we can extend to and Ds to turbo machines with some limit.

Specific Parameters for Compressors (Thermal Turbines)

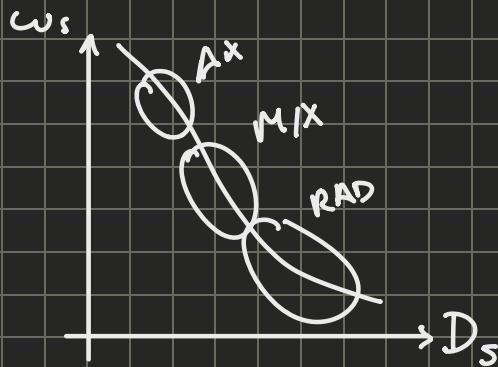
$$\varphi \rightarrow \omega_s$$

$$\psi \rightarrow D_s$$

They are incomplete,  
we need to add (at least)  
the  $M_v$

$$M_v = f(\omega_s, D_s, \dots) ?$$

→ If we find this, we find the  
limitation.



$$M_v = \frac{\omega D / z}{\sqrt{g R T_{TIN}}} = \omega_s \frac{D_s}{\sqrt{\Omega_{IN}}} \cdot \frac{1}{2} D_s \frac{\sqrt{\Omega_{IN}}}{\ell_s^{1/4}} \cdot \frac{1}{\sqrt{g R} \sqrt{T_{TIN}}}$$

$$\begin{aligned} \omega_s &= \omega \cdot \frac{\sqrt{\Omega_{IN}}}{\sqrt[4]{\ell_s^3}} \\ D_s &= D \cdot \frac{\sqrt[4]{\ell_s}}{\sqrt{\Omega_{IN}}} \end{aligned}$$

$$= \omega_s D_s \cdot \frac{1}{2} \sqrt{\frac{g R}{g-1} T_{TIN} \left( \beta_T^{\frac{g-1}{g}} - 1 \right)}$$

$$= \omega_s D_s \cdot \frac{1}{2} \sqrt{\frac{\sqrt{g R}}{g-1} \sqrt{T_{TIN}} \beta_T^{\frac{g-1}{g}}}$$

$$\Rightarrow M_v \propto \omega_s, D_s, f(\beta_T, \gamma)$$

Once we put a point on the Balje diagram, the  
efficiency depends on  $\omega_s$ ,  $D_s$  and  $M_v$ , this is true  
because company different machines using the same  
fluid, if they have different  $\beta_T$  they will have different  
efficiency.

Fixing a point, we also fix the  $M_u$ , and so the

Balje diagram is given for a given pressure ratio.