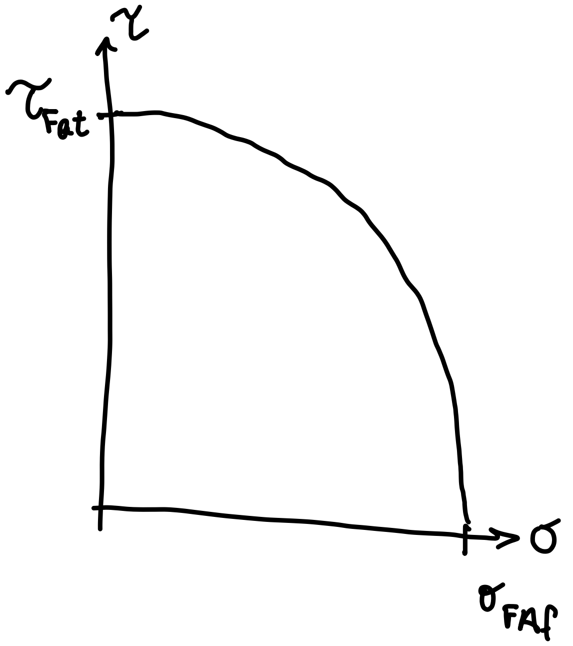
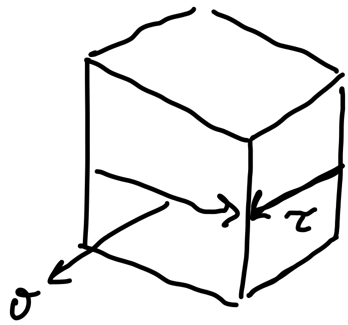
Esercitazione 13 -

Fatica Multi-assiale



$$\left(\frac{\sigma_a}{\sigma_{Faf}}\right)^2 + \left(\frac{\tau_a}{\tau_{Fat}}\right)^2 = 1$$



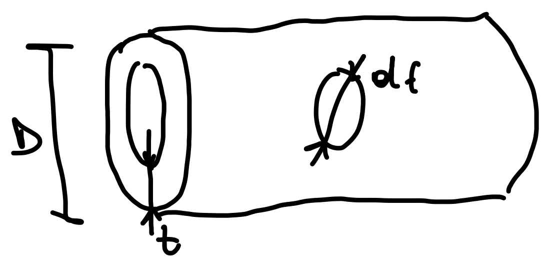
$$\sigma_{GOP}^{*} = \sqrt{\sigma_a^2 + H^2 \tau^2}$$

Casi:

- $\tau = cost = \tau_m$ $\quad H = \dfrac{\sigma_{lim}}{0{,}77 R_m}$ $\quad \left(\tau_{lim} = \dfrac{R_s}{\sqrt{3}}\right.$

- $\tau = \tau_a$ $\quad H = \dfrac{\sigma_{lim}}{\tau_{lim}}$

Esercizio 1



$D = 40\,mm$

$t = 4\,mm$

$d_f = 6\,mm$

$b_2 = b_3 = 0{,}85$

$M_f = M_{f0} \sin \omega t$

$M_t = M_{t0} \sin \omega t$

$M_{f0} = 120\,Nm$

$M_{t0} = 100\,Nm$

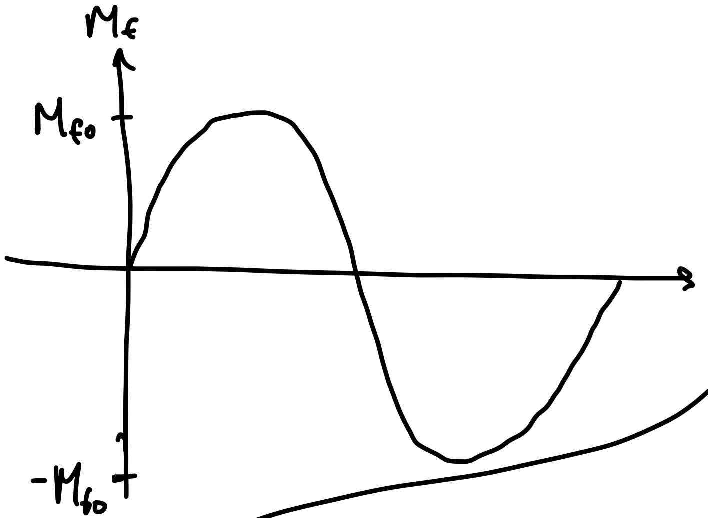
$K_{f,f} = 2{,}4$

$K_{f,t} = 2{,}2$

$R_m = 450\,MPa$

$R_s = 280\,MPa$

$d = D - 2t = 32\,mm$



$M_{fm} = 0$

$M_{fa} = M_f^{MAX} - M_{fm} = M_{f0}$

$\Rightarrow$ non ci serve Haigh, possiamo trovarlo calcolando solo σ'_{Faf}

$$\sigma'_{FAf} = \frac{0{,}5\, R_m\, b_2 b_3}{K_{f,f}} = 67{,}7\,MPa$$

$$\tau'_{Fat} = \frac{0{,}25\, R_m\, b_2 b_3}{K_{t,f}} = 36{,}9\,MPa$$

$$H^2 = \left(\frac{\sigma'_{FAf}}{\tau'_{Fat}}\right)^2 = 3{,}37$$

per sezioni cave sempre sul diametro più grande

$$\sigma_a = \frac{32\, M_{f0}\, D}{\pi\,\underbrace{(D^4 - d^4)}} = 32{,}3\ MPa$$

↳ persezioni cave

$$\tau_a = \frac{16\, M_{t0}\, D}{\pi (D^4 - d^4)} = 13{,}5\ MPa$$

$$\sigma^*_{GP} = \sqrt{\sigma_a^2 + H^2 \tau_a^2} = 55{,}8\ MPa$$

$$\eta = \frac{\sigma'_{Faf}}{\sigma^*_{GP}} = 1{,}21 < 2$$ X verifica non passata

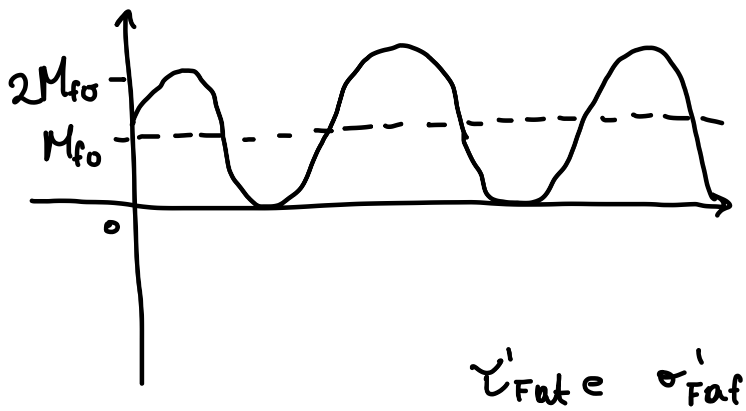
Stessi passi della verifica monoassiale si aggiunge solo σ^*_{GP}

Esercizio 2

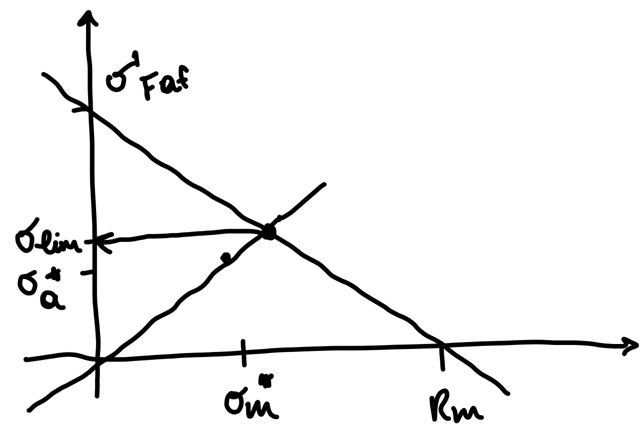
Stesso esercizio ma $M_f = M_{f0}(1 + \sin \omega t)$

$M_t = M_{t0}(1 + \sin \omega t)$

Dato che medio $\neq 0 \Rightarrow$ serve Haigh



τ'_{Fat} e σ'_{Fat} non cambiano



$$\frac{\sigma_a}{\sigma'_{Fat}} + \frac{\sigma_m}{R_m} = 1$$

$$\sigma_a = \frac{\sigma_a^*}{\sigma_m^*} \cdot \sigma_m$$

$*$ = del nostro stato di sforzo

Sostituendo

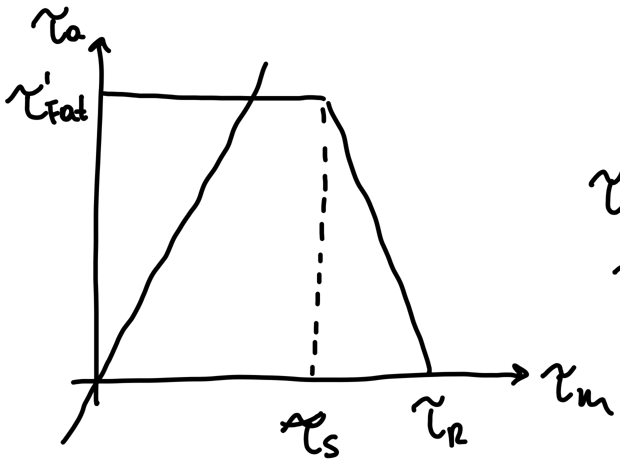
Dato che $\sigma_a^* = \sigma_m^*$

$\Rightarrow \sigma_a = \sigma_m = \sigma_{lim}$

$$\sigma_a = \left(\frac{1}{\sigma'_{Fat}} + \frac{1}{R_m} \right)^{-1}$$

$= 58{,}85$ MPa

$\sigma'_{Fat} = \sigma_{lim}$ se $\sigma_m = 0$



se $\sigma_m \neq 0 \Rightarrow \sigma'_{Fat} \neq \sigma_{lim}$

$\tau_R = 0{,}77\, Rm$

$\tau_S = Rs / \sqrt{3}$

ma σ'_{Fat} utile per calcolare σ_{lim}

Quasi sempre $\tau_{lim} = \tau'_{Fat}$

$$H^2 = \left(\frac{\sigma_{lim}}{\tau'_{Fat}}\right)^2 = 2{,}54$$

$\sigma_a = 32{,}3 \text{ MPa} = \sigma_m$

$\tau_a = 13{,}5 \text{ MPa} = \tau_m$

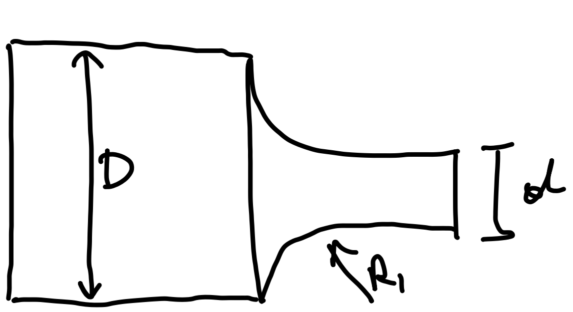
$$\sigma^*_{GP} = \sqrt{\sigma_a^2 + H^2 \tau_a^2} = 38{,}83 \text{ MPa}$$

$$\eta = \frac{\sigma_{lim}}{\sigma^*_{GP}} = 1{,}52 < 2$$ X verifica non passata

1,5 minimo per statiche (3 se fragile)

2 minimo per dinamico

Esercizio 3



$D = 30\,mm$ $\quad K_{f,f} = 1{,}86$

$d = 25\,mm$ $\quad b_2 = 0{,}925$

$M_t = 150\,Nm$ $\quad b_3 = 0{,}96$

$M_f = 100 \sin \omega t$

$R_m = 450\,MPa$

$R_s = 280\,MPa$

$$M_t = \text{cost} \Rightarrow \tau = \text{cost} \Rightarrow \tau_{lim} = 0{,}77\, R_m = 346{,}5\,MPa$$

$$\tau_m = \frac{16 M_t}{\pi d^3} = 48{,}9\,MPa$$

$$M_f = M_{f0} \sin \omega t \Rightarrow \sigma_m = 0 \Rightarrow \sigma_{lim} = \sigma'_{FAT} = \frac{0{,}25 R_m b_2 b_3}{K_{f,f}} = 107{,}4\,MPa$$

$$\sigma_a = \frac{32 M_{f0}}{\pi d^3} = 65{,}2\,MPa$$

$$H^2 = \left(\frac{\sigma_{lim}}{\tau_{lim}} \right)^2 = 0{,}096$$

$$\sigma^*_{GP} = \sqrt{\sigma_a^2 + H^2 \tau_m^2} = 66{,}94\,MPa$$

$$\eta = \frac{\sigma_{lim}}{\sigma^*_{GP}} = 1{,}6 < 2$$

se

Massimo sforzo equivalente di GP

$$\eta = \frac{\sigma_{lim}}{\sigma^*_{GP}} = 2 \Rightarrow \tilde{\sigma}^*_{GP} = \frac{\sigma_{lim}}{2}$$

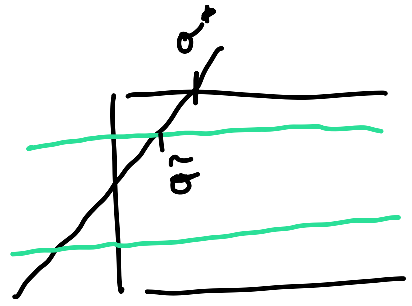
$$\tilde{\sigma}^*_{GP} = \sqrt{\left(\frac{32 M_{f0}}{\pi \tilde{d}^3} \right)^2 + H^2 \left(\frac{16 M_t}{\pi \tilde{d}^3} \right)^2} = \frac{\sigma_{lim}}{2} \rightarrow$$

Possiamo trovare $\tilde{d}$ tale

$\Rightarrow \tilde{d} = 20{,}2\ mm$

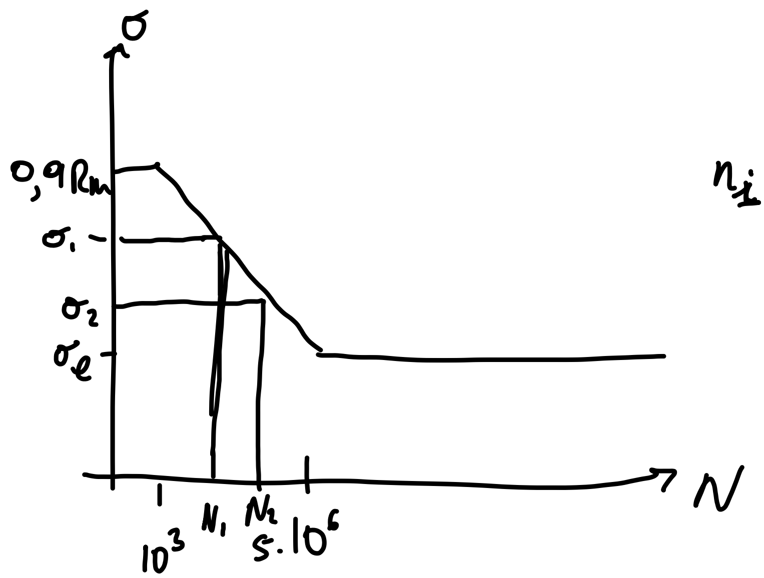
↳ Ha senso perchè diminuendo riduciamo la distanza dall'asse neutro, quindi riduciamo lo sforzo massimo

che possiamo avere $\tilde{\sigma}^*_{GP}$



$\tilde{\sigma} < \sigma^*$

<u>Bode</u>



$n_1 < N_1$

$n_2 < N_2$

regola di Miner

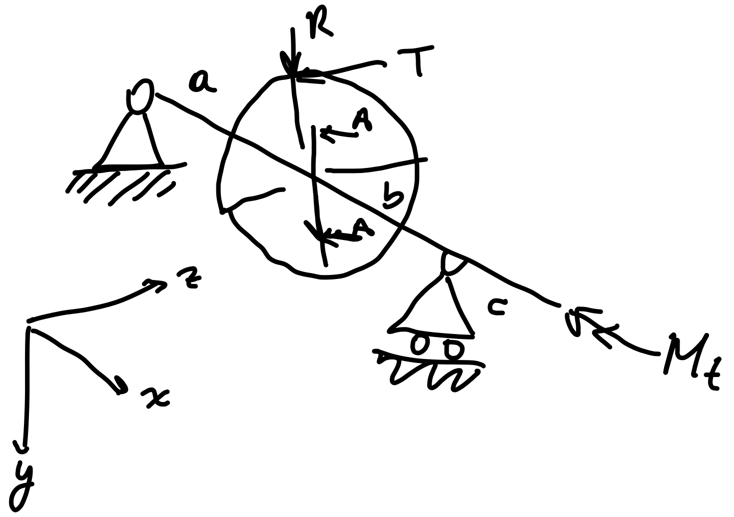
$\frac{n}{N} = 1 \rightarrow$ quando $= 1$ si rompe

$$D = \sum_i \frac{n_i}{N_i}$$ quando $= 1$ abbiamo cedimento

↑

Danneggiamento

Esercizio 4



$d = 200\ mm$

$d_r = 450\ mm$

$a = 270\ mm$

$b = 250\ mm$

$c = 400\ mm$

$K_t\ @_{A-A} = 2,2$

$R = 62\ kN$

$T = 171\ kN$

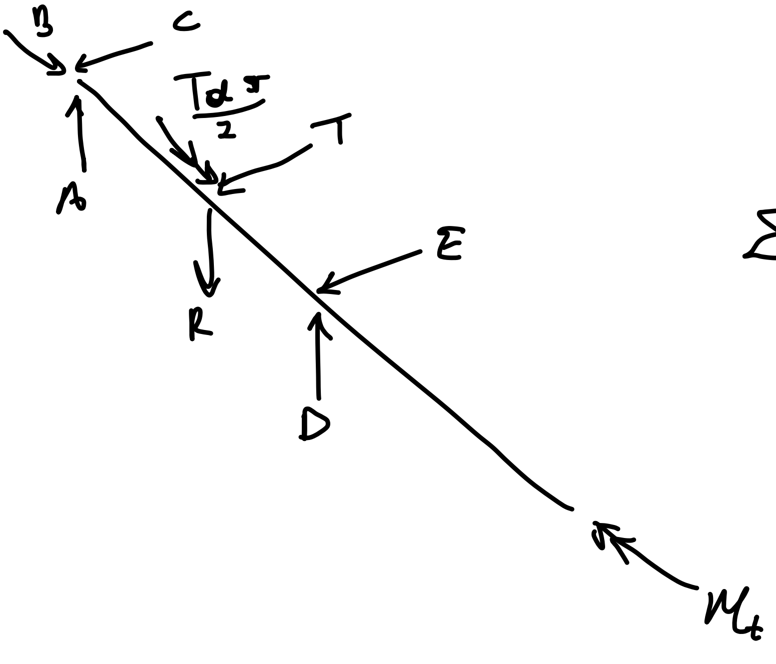
$R_m = 635\ MPa$

$R_s = 413\ MPa$

$\sigma_{Faf} = 305\ MPa$

$q = 0,9$

$b_2 = b_3 = 0,85$



$$\sum M_x = 0 = 0 - M_t + \frac{T d_r}{2}$$

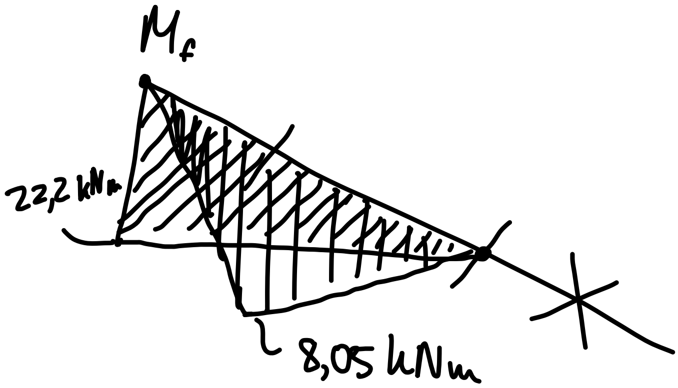
$$\Rightarrow M_t = \frac{T d_r}{2} = 76,95\ kNm$$

$$\sum M_y = 0 = E(a+b) + Ta \Rightarrow E = \frac{-Ta}{a+b} = 8\ 8{,}79\ kN$$

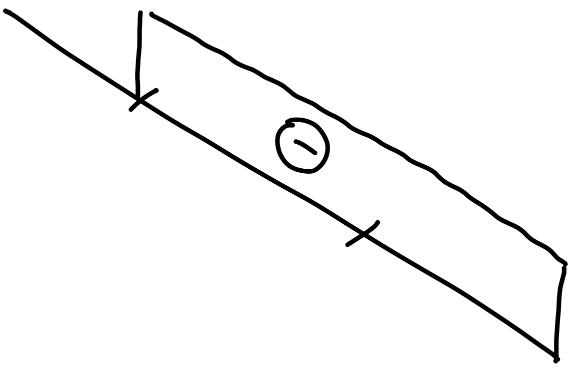
$$\sum F_z = 0 = \frac{-Ta}{a+b} + T + C \Rightarrow C = 82{,}21\ kN$$

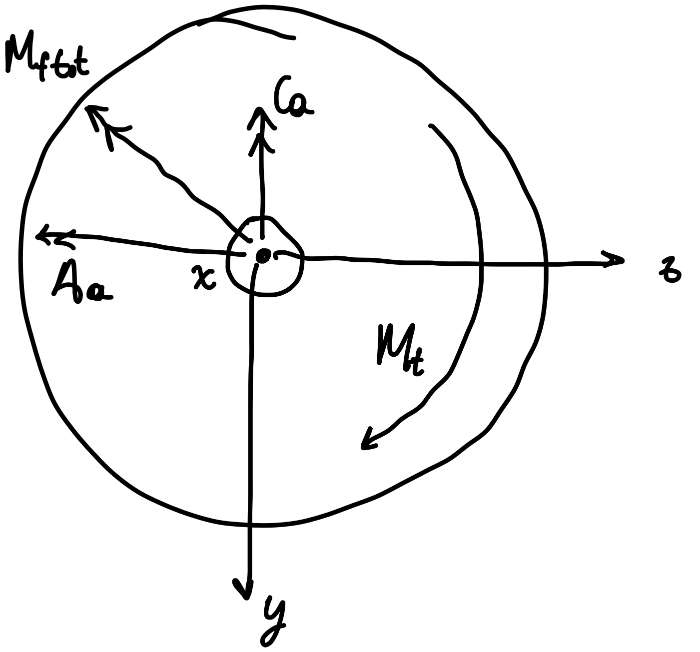
$$\sum M_z = 0 = D(a+b) - Ra \Rightarrow D = \frac{Ra}{a+b} = 32{,}19\ kN$$

$$\sum F_y = 0 = D - R + A \Rightarrow A = R - D = 29{,}81\ kN$$



$\underline{M_t}$





$$M_f^{tot} = \sqrt{(Ca)^2+(Aa)^2} = 23,61\,kNm$$

$$\sigma_n = \frac{32 M_f}{\pi d^3} = 30,06\,MPa$$

$$\tau_n = \frac{16 M_t}{\pi d^3} \simeq 49\,MPa$$

<u>PP</u>

$$\sigma_{MAX} = k_t\,\sigma_n = 66,13\,MPa$$

$$\tau_{MAX} = k_t\,\tau_n = 107,8\,MPa$$

$$\sigma^*_{GT} = \sqrt{\sigma_{MAX}^2 + 4\tau_{MAX}^2} = 225,5\,MPa$$

$$\eta = \frac{R_s}{\sigma^*_{GT}} = 1,83 > 1,5 \checkmark$$

Tutti gli altri carichi sono statici $\rightarrow$

$$\left.\begin{aligned} \sigma_a &= \sigma_n \\ \sigma_m &= 0 \end{aligned}\right\} R = -1$$

$$\tau_M = \tau_n$$

$$\sigma'_{FAf} = \frac{\sigma_{FAf}\, b_2 b_3}{1+q(k_t-1)} = 105,94\,MPa = \sigma_{lim}$$

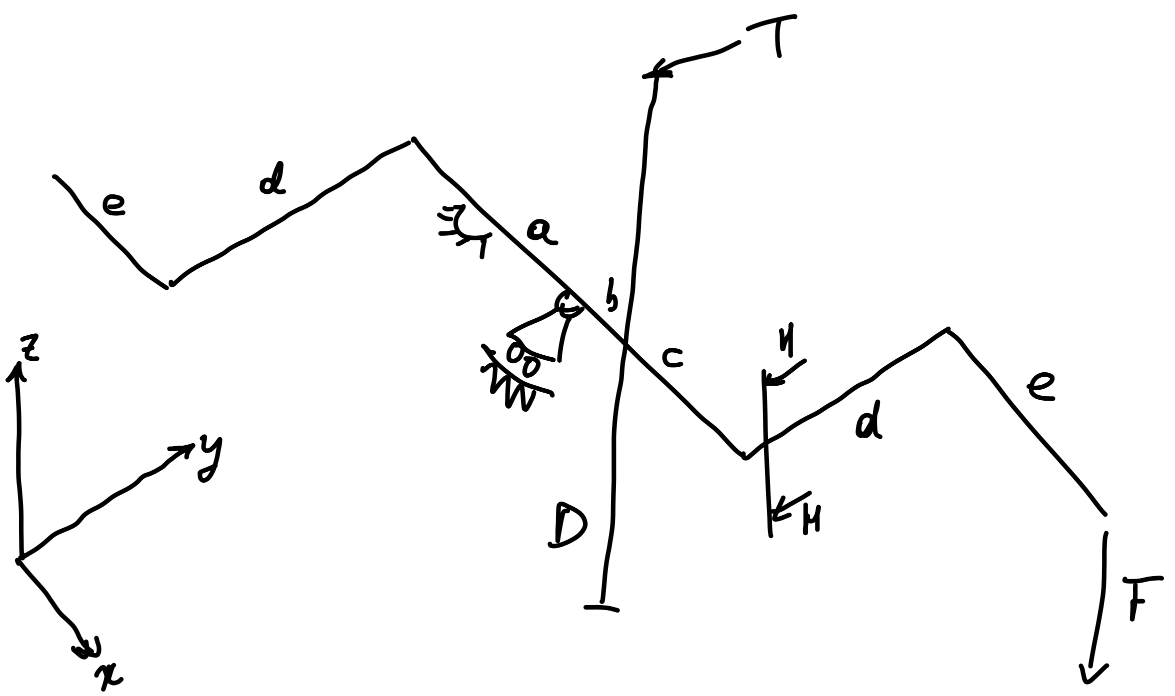
perché no σ_m

$$\sigma_{GP}^{*} = \sqrt{\sigma_a^2 + H^2 \tau^2} = 31,75\ MPa$$

$$H^2 = \left(\frac{\sigma_{lim}}{0,77\ R_m}\right)^2 = 0,0435$$

$$\eta = \frac{\sigma_{lim}}{\sigma_{GP}^{*}} = 3,34 > 2\ \checkmark$$

Esercizio 5 (Ultimo esercizio)



$F = 50N$

$a = 80\ mm$

$b = 25\ mm$

$c = 20\ mm$

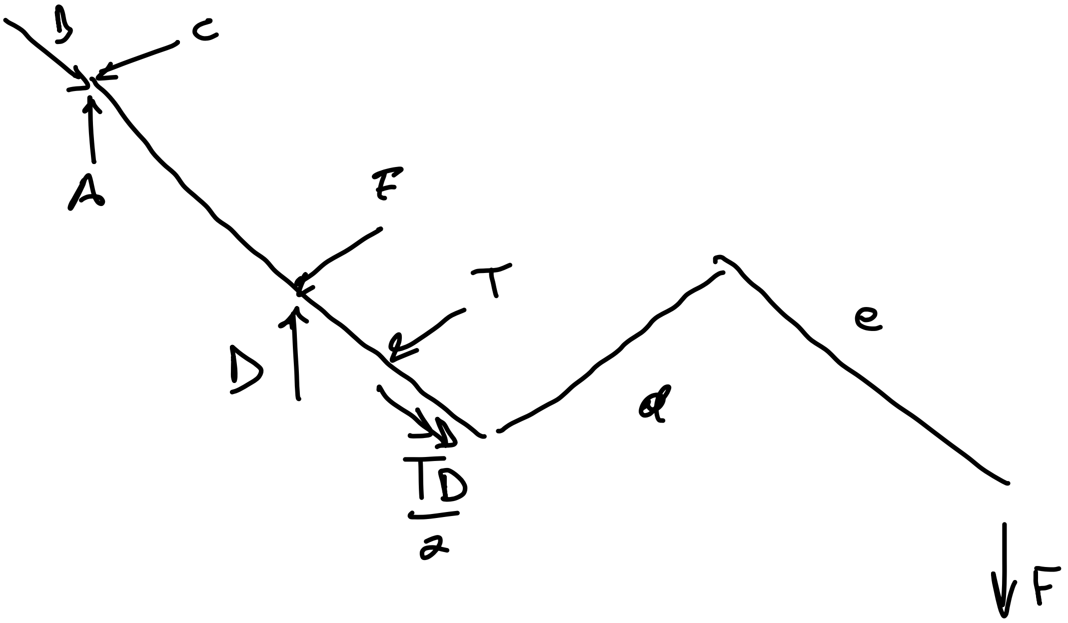
$d = 170\ mm$

$e = 100\ mm$

$b_2 = 0,8$

$b_3 = 0,9$

$R_m = 900\ MPa$



$$\sum M_x = 0 = \frac{TD}{2} - Fd \Rightarrow T = \frac{2Fd}{D} = 680\,N$$

$$\sum M_z = 0 = -Ea - T(a+b) \Rightarrow E = \frac{-T(a+b)}{a} = -892{,}5\,N$$

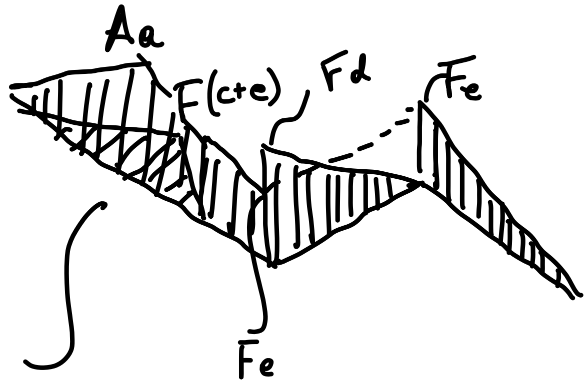
$$\sum F_y = 0 = C + E + T \Rightarrow C = -212{,}5\,N$$

$$\sum M_y = 0 = -Da + F(a+b+c+e) \Rightarrow D = F\,\frac{a+b+c+e}{a} = 1406{,}3\,N$$

$$\sum F_z = 0 = A + D - F \Rightarrow A = -906{,}25\,N$$

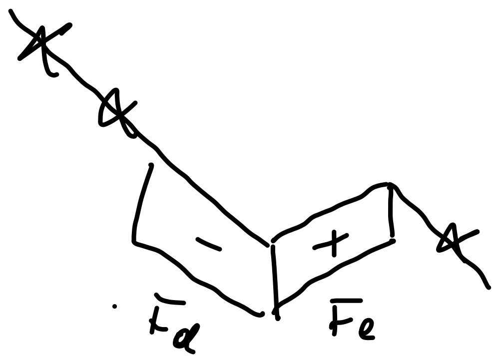
$\underline{M_f}$

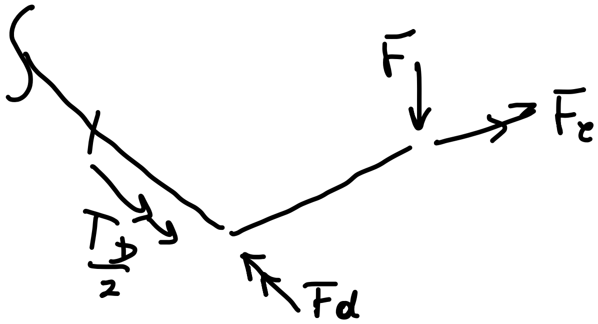
su cerniera M=0 perchè è ultimo vincolo

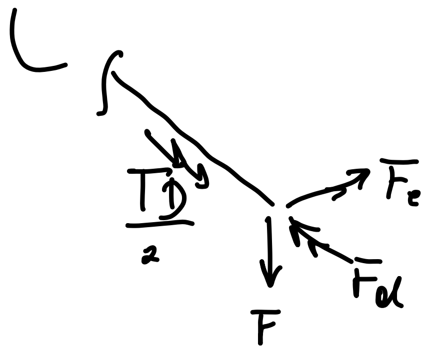


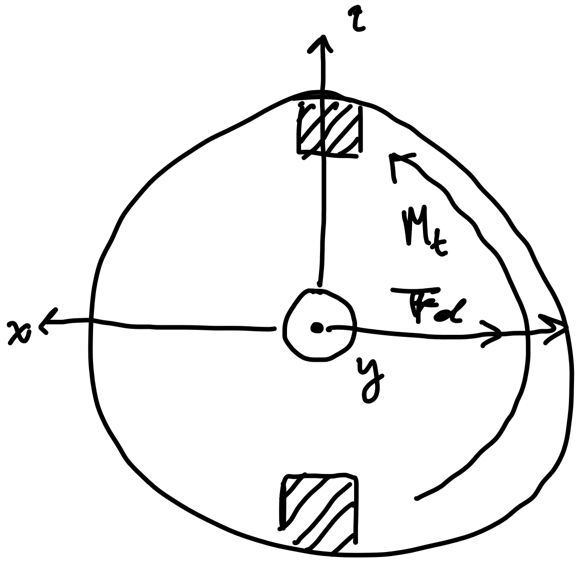
Carrello non rompe l'albero, quindi può esercitare $M \neq 0$

M_t









$$F = F_0 \sin \omega t$$

$$T = T_0 \sin \omega t$$

$$\sigma_a = \frac{32 M_f}{\pi d^3}$$

↳ perché non c'è σ_m

$$\tau_a = \frac{16 M_t}{\pi d^3}$$

↳ perché non c'è τ_a

$$\sigma_{lim} = \sigma'_{FAf} = \frac{\overbrace{0{,}5\, Rm}^{\sigma'_{FAf}}\, b_2 b_3}{K_{f,f}} = 324 \text{ MPa}$$

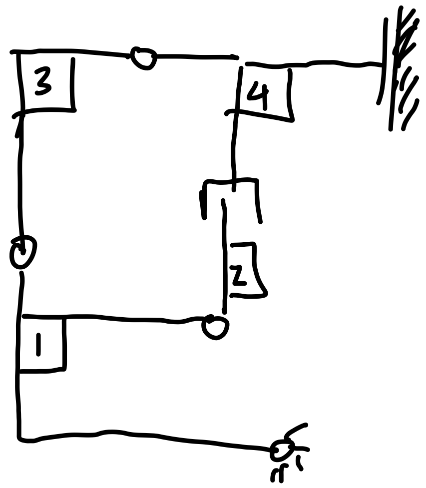
$$\tau_{lim} = \tau_{FAt} = \frac{0{,}25\, Rm\, b_2 b_3}{K_{f,t}} = 162 \text{ MPa}$$

$$H^2 = \left(\frac{\sigma_{lim}}{\tau_{lim}}\right)^2 = 4$$

$$\eta = \frac{\sigma_{lim}}{\sigma^*_{GP}} = 2$$

$$d_{min} : \sigma^*_{GP} = \frac{\sigma_{lim}}{2} = \sqrt{\left(\frac{32\,Fd}{\pi d^3}\right)^2 + H^2\left(\frac{16\,Fe}{\pi d^3}\right)^2} \Rightarrow d_{min} = 20{,}2\,mm$$

Esercizio 6



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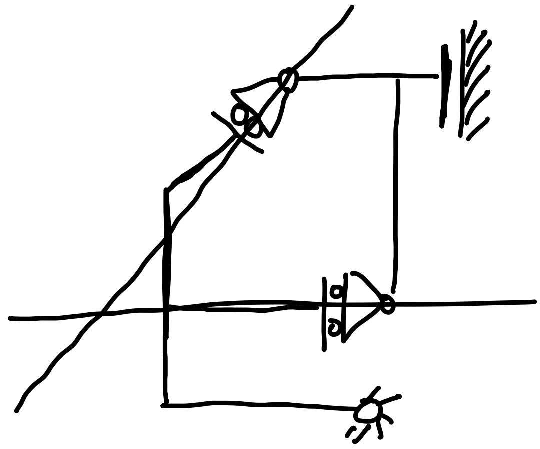

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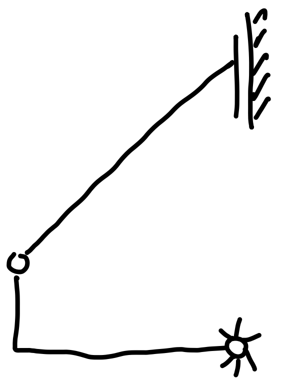
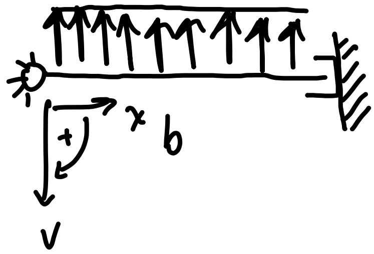
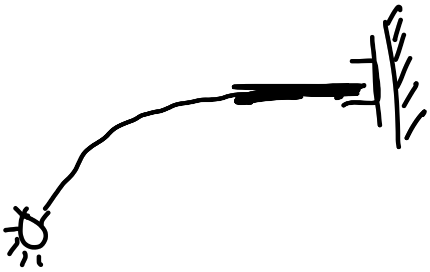
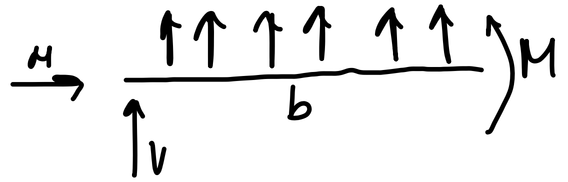


A3C non allineato

Esercizio 7





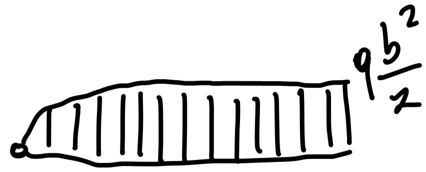


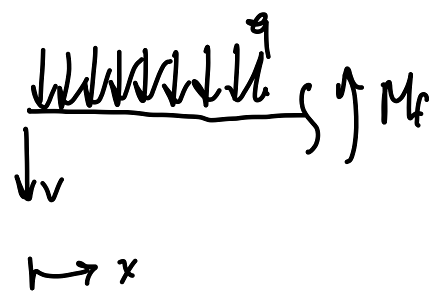
$\sum F_x = H = 0$

$\sum F_y = 0 = V - qb \Rightarrow V = qb$

$\sum M = 0 = M + q\frac{b^2}{2} \Rightarrow M = -\frac{qb^2}{2}$

M_f





Linea Elastica

$EJ\, v''(x) = -M_f = \frac{qx^2}{2} - qbx$

$\sum M = 0 = qbx - \frac{qx^2}{2}$

$$EJ\, v'(x) = \frac{qx^3}{6} - \frac{qbx^2}{2} + A$$

$$EJ\, v(x) = \frac{qx^4}{24} - \frac{qbx^3}{6} + Ax + B$$

$v(0) = 0 \Rightarrow B = 0$

$v'(b) = 0 \Rightarrow v'(b) = \frac{1}{EJ}\left(\frac{qb^3}{6} - \frac{qb^3}{2} + A\right) =$

$\Rightarrow A = \frac{1}{3} qb^3$

$$v(x) = \frac{1}{EJ}\left(-q\frac{b}{6}x^3 + \frac{qx^4}{24} + \frac{qb^3}{3}x\right)$$

$$v(b) = -\frac{1}{EJ}\, qb^4 \frac{5}{24}$$