Exercise

June 14, 2016

1 Introduction

A linear least square problem has the form

$$\text{minimize} \quad ||Fx - f|| \tag{1}$$

where

$$F \in \mathcal{R}^{m \times n} \text{ and } f \in \mathcal{R}^m$$

Typically, then

$$m \gg n.$$

Sometimes the solution to (1) has very large norm i.e. $||x||$ is large. This may be undesirable and therefore one may solve

$$\text{minimize} \quad ||Fx - f|| + \sum_{j=1}^{n} \lambda_j |x_j| \tag{2}$$

instead where $\lambda_j > 0$ is positive parameters. Clearly, the additional term has a regularizing effect. Think about what happens as λ goes to infinity.

See `http://www.swissquant.com/data/docs/en/1901/Mathematical-Challenge-March-2014.pdf` for a motivation.

Data for the problem can be obtained as follows

```
import lassodata
(F,f) = lassodata.get(1000,100,279)
```

where `lassodata.py` is on github page. Use $\lambda = 0.1$.

2 Questions

1. Formulate (2) as a conic quadratic optimization problem.
   (a) Is the problem feasible?
   (b) Is the interior of the feasible solutions nonempty.
   (c) How many nonzeros are there approximately in the most dense row and column of the constraint matrix.

2. Implement the formulated problem in MOSEK Fusion.
3. State the dual problem to (2).
   (a) Is the dual problem feasible?
   (b) Is the interior of the feasible solutions to the dual nonempty.
   (c) How many nonzeros are there approximately in the most dense row and column of the constraint matrix of the dual.
4. Implement the dual problem of (2) in MOSEK Fusion.