Pyramidal Lucas Kanade Optical Flow

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1 Pyramidal Lucas-Kanade Optical Flow Project

1.0.1 Data Set Visualization

[12]:
```
import matplotlib.pyplot as plt
import cv2


img0 = cv2.imread('im0.png')
img1 = cv2.imread('im1.png')


img0_rgb = cv2.cvtColor(img0, cv2.COLOR_BGR2RGB)
img1_rgb = cv2.cvtColor(img1, cv2.COLOR_BGR2RGB)

fig, axes = plt.subplots(1, 2, figsize=(12, 6))

# == Visualization ===
axes[0].imshow(img0_rgb)
axes[0].set_title('img0')
axes[0].axis('off')

axes[1].imshow(img1_rgb)
axes[1].set_title('img1')
axes[1].axis('off')

# Show the plot
plt.show()
```

img0

img1

[13]:
```
import numpy as np
import cv2
import matplotlib.pyplot as plt
```

The Sobel operator computes the image gradients in the x and y directions using the convolution of the image with two kernels. The horizontal Sobel kernel G_x is given by:

$$G_x = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

and the vertical Sobel kernel G_y is:

$$G_y = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

The gradients are computed by convolving the image $I(i, j)$ with the kernels, where:

$S_x(i, j) = \sum_{m=-1}^{1} \sum_{n=-1}^{1} G_x(m, n) \cdot I(i + m, j + n)$

and

$S_y(i, j) = \sum_{m=-1}^{1} \sum_{n=-1}^{1} G_y(m, n) \cdot I(i + m, j + n)$

These give the gradients in the x-direction I_x and y-direction I_y. The magnitude of the gradient is computed as:

$\text{Magnitude} = \sqrt{I_x^2 + I_y^2}$

indicating edge strength, while the direction of the gradient is:

$\text{Direction} = \arctan\left(\frac{I_y}{I_x}\right)$

representing edge orientation. These gradients provide crucial information about edges and structure in the image.

[14]:
```
def compute_gradients(img):
    Ix = cv2.Sobel(img, cv2.CV_32F, 1, 0, ksize=3)  # Positive Ix = intensity␣
↪increases rightward
    Iy = cv2.Sobel(img, cv2.CV_32F, 0, 1, ksize=3)  # Positive Iy = intensity␣
↪increases downward
    return Ix, Iy
```

The function `warp_image(img,u,v)` performs image warping by shifting the image according to displacement The displacement vectors `u` and `v` are added to the respective coordinates, giving the new coordinates:

$$\text{map}_x = \text{grid}_x + \texttt{u}, \tag{1}$$

$$\text{map}_y = \text{grid}_y + \texttt{v}. \tag{2}$$

These new coordinates are then used in the function **`cv2.remap`**, which maps the image pixels to their new locations. The remapping uses linear interpolation with border reflection (`cv2.BORDER_REFLECT`) to handle pixels at the image boundaries.

[15]:
```
def warp_image(img, u, v):
    h, w = img.shape
    grid_x, grid_y = np.meshgrid(np.arange(w), np.arange(h))
    map_x = (grid_x + u).astype(np.float32)  # Positive u shifts I2_l rightward␣
↪to match I1_l
    map_y = (grid_y + v).astype(np.float32)  # Positive v shifts I2_l downward
    return cv2.remap(img, map_x, map_y, interpolation=cv2.INTER_LINEAR,␣
↪borderMode=cv2.BORDER_REFLECT)
```

The function `build_pyramid(img, levels)` creates an image pyramid, which is a series of images with progressively lower resolutions. The function starts by initializing the pyramid with the original image. Then, for each level from 1 to `levels-1`, the image is downsampled using the `cv2.pyrDown` function, which reduces the image size by a factor of 2. Each downsampled image is appended to the pyramid list:

$$\text{pyramid} = [\text{img}]$$

For each subsequent level, the image is downsampled:

$$\text{img} = \text{cv2.pyrDown(img)}$$

After all levels are generated, the pyramid is reversed to return the images from the smallest to the largest resolution. The final pyramid is:

$$\text{pyramid[::-1]}$$

[16]:
```
def build_pyramid(img, levels):
    pyramid = [img]
    for _ in range(1, levels):
        img = cv2.pyrDown(img)
        pyramid.append(img)
    return pyramid[::-1]
```

The function `pyramidal_lucas_kanade(I1, I2, levels=4, window_size=15, iterations=3, eigen_thresh=1e-4)` implements the Lucas-Kanade optical flow method using a pyramidal approach. The function starts by checking that the input images I_1 and I_2 have the same shape, are grayscale, and that the `window_size` is odd. It also checks that the pyramid level is not too small for the chosen window size. The images are normalized by dividing them by 255:

$$I_1 = \frac{I_1}{255.0}, \quad I_2 = \frac{I_2}{255.0}$$

Next, image pyramids for both images are generated using the `build_pyramid` function. The initial optical flow is set to zero:

$$u = v = 0$$

For each pyramid level, the flow is refined using the Lucas-Kanade method. The optical flow is upsampled from the previous level using `cv2.pyrUp`, and resized to the current level's dimensions. The gradients I_x and I_y are computed using the `compute_gradients` function, and the flow is updated using the equation:

$$A = \begin{pmatrix} I_x & I_y \end{pmatrix}, \quad b = -I_t$$

where I_t is the temporal difference between the warped image and the original image at the current level. The least squares solution is computed using:

$$\mathbf{A}^T\mathbf{A}\nu = \mathbf{A}^T\mathbf{b}$$

where $\nu = \begin{pmatrix} u \\ v \end{pmatrix}$ is the optical flow at the pixel. The flow is updated at each pixel, and the process is repeated for each level of the pyramid. The final flow vectors u and v are returned.

[17]:
```
def pyramidal_lucas_kanade(I1, I2, levels=4, window_size=15, iterations=3,␣
↪eigen_thresh=1e-4):
    if I1.shape != I2.shape:
        raise ValueError("Input images must have the same shape")
    if len(I1.shape) != 2:
        raise ValueError("Input images must be grayscale")
    if window_size % 2 == 0:
        raise ValueError("window_size must be odd")
    min_size = min(I1.shape) / (2 ** (levels - 1))
    if min_size < window_size:
```

```
        raise ValueError("Pyramid level too small for window_size")

    I1 = I1.astype(np.float32) / 255.0
    I2 = I2.astype(np.float32) / 255.0

    pyr1 = build_pyramid(I1, levels)
    pyr2 = build_pyramid(I2, levels)

    u = np.zeros_like(pyr1[0])
    v = np.zeros_like(pyr1[0])

    hw = window_size // 2

    for level in range(levels):
        I1_l = pyr1[level]
        I2_l = pyr2[level]

        if level != 0:
            u = cv2.pyrUp(u) * 2
            v = cv2.pyrUp(v) * 2
            h, w = I1_l.shape
            u = cv2.resize(u, (w, h), interpolation=cv2.INTER_LINEAR)
            v = cv2.resize(v, (w, h), interpolation=cv2.INTER_LINEAR)

        Ix, Iy = compute_gradients(I1_l)

        for _ in range(iterations):
            I2_warped = warp_image(I2_l, u, v)
            It = I2_warped - I1_l  # Follow pseudocode

            for y in range(hw, I1_l.shape[0] - hw):
                for x in range(hw, I1_l.shape[1] - hw):
                    Ix_win = Ix[y - hw:y + hw + 1, x - hw:x + hw + 1].flatten()
                    Iy_win = Iy[y - hw:y + hw + 1, x - hw:x + hw + 1].flatten()
                    It_win = It[y - hw:y + hw + 1, x - hw:x + hw + 1].flatten()

                    A = np.vstack((Ix_win, Iy_win)).T
                    b = -It_win.reshape(-1, 1)

                    ATA = A.T @ A
                    if np.min(np.linalg.eigvals(ATA)) < eigen_thresh:
                        continue

                    try:
                        nu = np.linalg.lstsq(A, b, rcond=None)[0]
                        u[y, x] += nu[0, 0]
                        v[y, x] += nu[1, 0]
```

```
                    except np.linalg.LinAlgError:
                        continue

    return u, v
```

Visualization

[18]:
```
# === Visualization ===

def draw_quiver(u, v, step=10, title="Optical Flow", scale=5,␣
↪exaggeration_factor=50):
    h, w = u.shape
    y, x = np.mgrid[0:h:step, 0:w:step]
    u_s = u[::step, ::step] * exaggeration_factor  # Apply exaggeration factor
    v_s = v[::step, ::step] * exaggeration_factor  # Apply exaggeration factor

    magnitude = np.sqrt(u_s**2 + v_s**2)

    plt.figure(figsize=(12, 8))
    plt.quiver(x, y, u_s, -v_s, magnitude, angles='xy', scale_units='xy',␣
↪scale=scale, cmap='jet')
    plt.colorbar(label="Flow Magnitude")
    plt.gca().invert_yaxis()
    plt.title(title)
    plt.show()
```

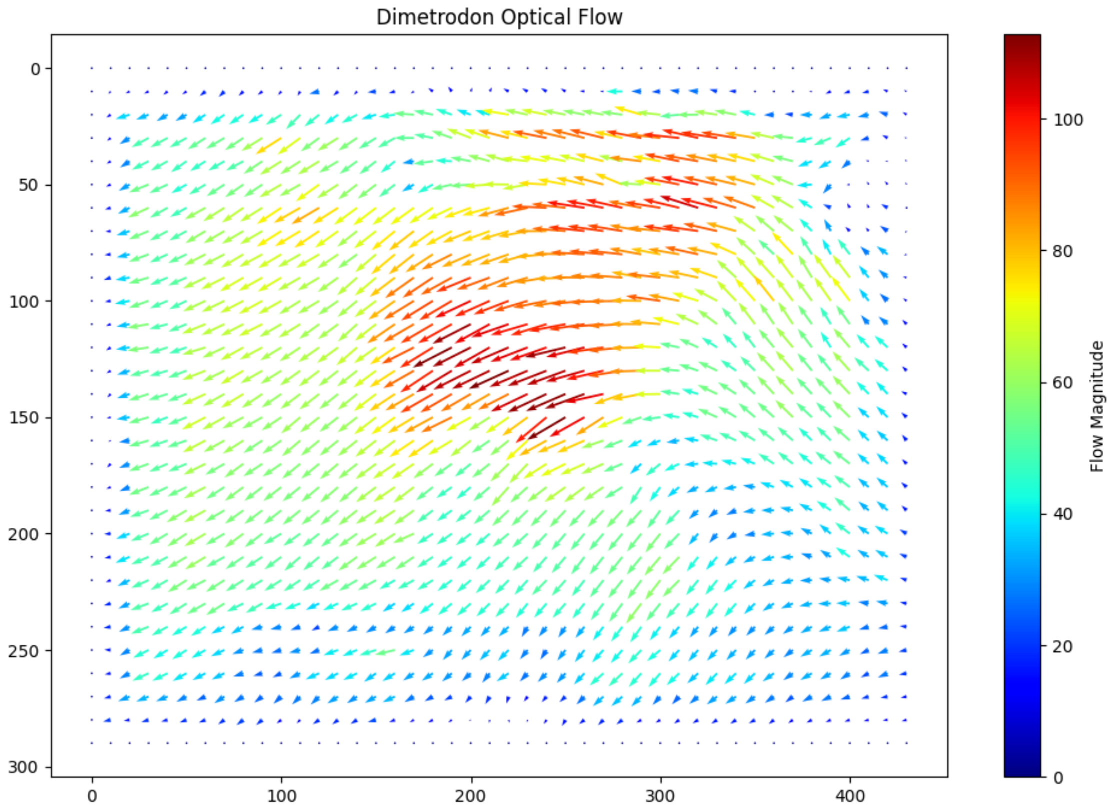
[20]:
```
# === Main Script ===

if __name__ == "__main__":
    # Load the images from Dimetrodon dataset
    I1 = cv2.imread('/content/im0.png', cv2.IMREAD_GRAYSCALE)
    I2 = cv2.imread('/content/im1.png', cv2.IMREAD_GRAYSCALE)

    # Safety check
    if I1 is None or I2 is None:
        raise FileNotFoundError("One or both input images could not be loaded.")

    # Run pyramidal Lucas-Kanade
    u, v = pyramidal_lucas_kanade(I1, I2, levels=4, window_size=15, iterations=3)

    # Visualize the optical flow using quiver plot with exaggerated flow
    draw_quiver(u, v, step=10, title="Dimetrodon Optical Flow",␣
↪exaggeration_factor=50)
```



Dimetrodon Optical Flow
0
50
100
150
200
250
300
0
100
200
300
400
100
80
60
40
20
0
Flow Magnitude