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Effect of Swing Legs on Turning Motion of a Free-Falling Cat Robot

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***Abstract* - Based on self-righting principle of a free-falling cat, a multi-rigid-body dynamic model of a cat robot is built with the consideration of swing legs and the relation between turning rate and swing angle of legs is got. Mechanical structure of the cat robot is designed and dynamics simulation is carried out to validate the theoretical analysis. Simulation results indicate that flip angle of the cat robot becomes larger with the increase of angle or angular velocity of swing legs. By contrast, the change of swing angle has more influence on flip angle of the cat robot. Adjusting angle and angular velocity of swing legs to suitable numerical values, energy consumption of turning motion can be reduced effectively.**

Index Terms - Cat Robot, Swing Legs, Flip Angle, Energy Consumption, Modeling and Simulation.

I. INTRODUCTION

It is well known that a cat can manage to right itself and land safely on its feet after being released upside-down from rest. This phenomenon is widely researched by scholars and provides an idea to solve the problem of safe landing of legged robot which falls from a height.

Explanation for the phenomenon can be traced back to the end of the 19th century. Guyou [1] researched the turning motion of a falling cat according to the law of angular momentum conservation. Loitsyansky [2] thought that the cat rotates tail rapidly and its body turns in reaction, but the experiment proved that a cat without tail can still restore its posture and perform safe landing. Kane and Scher [3] explained that the cat bends its torso but does not twist after falling from midair. They used two identical axial-symmetric rigid bodies to represent the front and back halves of the cat and established equations of motion. The numerical analysis result showed that motion of the model matched the photographic record of a falling cat very well.

In recent years, research on the problem of falling cats mainly includes the following aspects: motion control, mechanism analysis and prototype development. Takahashi [4] designed a quantum neural controller to solve the control problem that brings the falling cat from the initial to desired position. Nakano et al. [5] proposed a reinforcement learning approach to the control problem of a free-falling cat. Shields et al. [6] created a simplified model of a flipping cat robot and designed a simulation to demonstrate the mechanism of a prototype. Kawamura [7] developed a robotic cat with two rectangular columns which are connected by a flexible spine and the robotic cat can right itself in midair. Liu [8] obtained the general characteristics of the turning motion of a free-falling cat. Zhong [9] explained that the free-falling cat can land on its feet from arbitrary posture. Ge and Guo [10] implemented the attitude planning of a free-falling cat and the optimal of control input by using spline approximation. Zhen et al. [11] applied the Udwadia–Kalaba approach to study the falling cat's movements. Yi and Ge [12] presented a hybrid optimal strategy to solve the attitude optimal control problem of a free-falling cat. Through observing a large number of high-speed photographs of the falling cat, we know that its legs swing regularly. However, most of researches about the falling cat do not consider the swing legs.

This paper mainly studies the influence of swing legs on the turning motion of a free-falling cat. First, we establish a simplified model of a robotic cat with the consideration of swing legs and analyses the relation between swing angle and turning rate. Next, we design the mechanical structure of the cat robot and a simulation environment has been constructed using ADAMS. Finally, we develop a cat robot with five servo actuators and then use a high-speed camera to shoot its turning motion.

II. MODELING OF SYSTEM DYNAMICS

This section introduces characteristic of swing legs of the falling cat and proposes some assumptions in order to make the research convenient. Then we build a simplified model of a cat robot. According to the law of angular momentum conservation, we establish motion equations to analyse the effect of swing angle on moment of inertia and turning rate of the cat robot.

A. Characteristic of Swing Legs

Photographs of a falling cat are shown in Fig. 1. Observing the pictures, we can know that when the cat is released from complete rest while upside-down, it first retracts forelegs and hindlegs close to the head and tail respectively and keep contractive posture for a while. Then the cat extends its forelegs and hindlegs to right itself completely. In order to simplify the inertia model of the cat robot, some assumptions are made as follows: 1) The swing of forelegs and hindlegs is axial-symmetric; 2) Two legs on each half body are assumed

to move as one; 3) The processes of leg retraction and extention cost the same time.

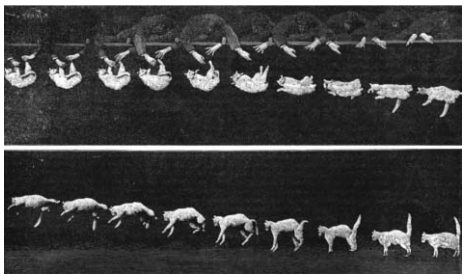


Fig. 1 Photographs of a falling cat [13].

B. Analysis of Moments of Inertia

We use two kinds of circular columns whose size and mass distribution are identical to respectively represent torso and leg of the robotic cat model, as shown in Fig. 2. The front and back halves of its body are symmetric and the related parameters are marked in the multi-rigid-body dynamic model, as shown in Fig. 3. Torso is fixed with a constant angle 2θ between X_1 and X_2 axes. θ is the bending angle of waist and its range is from 0° to 90°. Forelegs and hindlegs respectively swing around Y_1 and Y_2 axes. β is the swing angle of legs and its range is from 0° to 90°.

Since the robotic cat is released from complete rest, the model has no initial angular momentum and no external force is applied to it. The front and back halves of its body respectively rotate around X_1 and X_2 axes in the same direction with angular velocity w_s. According to the law of angular momentum conservation, the entirety of cat robot will rotate around the X_a axis which through two gravity centers, G_1 and G_2, of the front and back halves of the cat robot in the opposite direction with the angular velocity w.

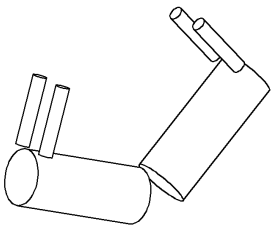


Fig. 2 Robotic cat model.

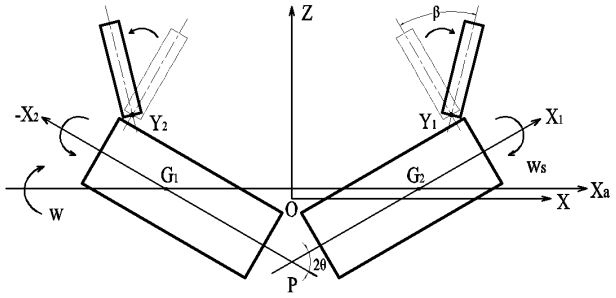


Fig. 3 The multi-rigid-body dynamic model.

Based on the above analysis, we can get the following equation:

$$2Iw + 2I_s w_s \cos\theta = 0 \tag{1}$$

where I is moment of inertia of each half body related to X_a axis, I_s is moment of inertia of each half body respectively related to X_1 and X_2 axes. From (1), we can get

$$w_s = -\frac{I}{I_s \cos\theta} w \tag{2}$$

Turning rate of the cat robot is defined by (3) [14]:

$$R_t = 1 + \frac{w}{w_s} = 1 - \frac{I_s}{I}\cos\theta \tag{3}$$

From (3), we know that the next step is to solve the expression of I_s and I. I_s is expressed in (4),

$$I_s = I_{T1} + I_{L1} \tag{4}$$

where I_{T1} is moment of inertia of front half of torso related to X_1 axis, I_{L1} is moment of inertia of forelegs related to X_1 axis. The expressions of I_{T1} and I_{L1} are as follows:

$$I_{T1} = \frac{1}{2}MR^2 \tag{5}$$

where M is the mass of the front half of torso, R is the radius of the front half of torso . I_{L1} is expressed in (6) [15],

$$I_{L1} = \frac{1}{2}mr^2 + \frac{1}{12}m\sin^2(\frac{\pi}{2} - \beta)(h^2 - 3r^2) + md_1^2 \tag{6}$$

where m is the mass of forelegs, r is the radius of the leg, h is the leg length, d_1 is the distance between the centre of gravity of the leg and X_1 axis. The expression of d_1 is as follows:

$$d_1 = \sqrt{[R + r + \frac{1}{2}h\sin(\frac{\pi}{2} - \beta)]^2 + (R - r)^2} \tag{7}$$

I is expressed in (8),

$$I = I_T + I_L \tag{8}$$

where I_T is moment of inertia of the front half of torso related to X_a axis, I_L is moment of inertia of forelegs related to X_a axis. The expressions of I_T and I_L are as follows [15] :

$$I_T = \frac{1}{2}MR^2 + \frac{1}{12}M\sin^2\theta(H^2 - 3R^2) + M\left(\frac{mL_z}{M+m}\right)^2 \tag{9}$$

where H is the length of the front half of torso, L_z is the center of gravity of the leg corresponds to the coordinate of the Z axis.

$$L_z = c + \frac{1}{2}h\sin(\frac{\pi}{2} - \beta + \theta) \tag{10}$$

where c is the distance between Y_1 axis and X axis. Its expression is as follows:

$$c = \frac{r + 2R}{\cos\theta} + [\frac{1}{2}H - \frac{R}{\tan\theta} - (r + 2R)\tan\theta - r]\sin\theta \tag{11}$$

$$I_L = \frac{1}{2}mr^2 + \frac{1}{12}m\sin^2(\frac{\pi}{2} - \beta + \theta)(h^2 - 3r^2) + md_a{}^2 \tag{12}$$

where d_a is the distance between the centre of gravity of the leg and X_a axis. Its expression is as follows:

$$d_a = \sqrt{(L_z - \frac{mL_z}{M+m})^2 + (R - r)^2} \tag{13}$$

We assign numerical values to parameters except β in order to simplify expression of I_s and I. Where M=2kg, m=0.5kg, R=40mm, r=15mm, H=150mm, h=100mm, θ=30°. The numerical values given for the parameters are based on the current model of the cat robot, then we can obtain:

$$I_s = (1.6\cos^2\beta + 2.8\cos\beta + 3.5)\times 10^{-3} \quad (14)$$

$$I = [1.4\cos^2(\frac{\pi}{6}-\beta) + 3.1\cos(\frac{\pi}{6}-\beta) + 5.1]\times 10^{-3} \quad (15)$$

Plugging (13) and (14) into (3), we get the relation between turning rate and swing angle of legs:

$$R_t = 1 - \frac{1.6\cos^2\beta + 2.8\cos\beta + 3.5}{1.4\cos^2(\frac{\pi}{6}-\beta) + 3.1\cos(\frac{\pi}{6}-\beta) + 5.1} \times \frac{\sqrt{3}}{2} \quad (16)$$

Fig. 4 shows the relation between turning rate and swing angle. Fig. 5 shows the effect of swing angle on *I and* I_s. From Fig. 4, we can see that the turning rate becomes larger with the increase of swing angle. From Fig.5, we know that I_s becomes smaller with the increase of swing angle, but I increases first and then decreases as a result of the bending angle of waist. Since the ratio of I_s to I becomes smaller with the increase of swing angle, so according to (3) we can know that the turning rate becomes larger when the bending angle is constant.

III. Dynamics Simulation

In this section, we design a 3D modeling of a cat robot and then use ADAMS to analyse the influence of angle and angular velocity of swing legs on flip angle and energy consumption of the turning motion of the cat robot. The simulation results also provide reference data for the final prototype experiment.

A. Structure Design of a Cat Robot

We design the structure of a cat robot which consists of the waist, torso and legs, as shown in Fig. 6. The material of each part is resin whose density is 1.1 g/cm^3. The motion of each part is driven by a servo actuator whose mass is 52g. Servo actuator 5 and 9 separately control the rotation of the front and back halves of the cat robot. Servo actuator 7 adjusts the bending angle of the waist. Servo actuator 2 and 3 separately control the swing of forelegs and hindlegs which are fixed on the gear shafts. The transmission ratio is 0.607.

B. Simulation Analysis

In order to simplify the analysis, we assume that the motion of each part is uniform. We define the drop height of the cat robot is 2m and its bending angle of waist is 30°, so the fall time is 0.64s. We set the time of turning motion is the initial 0.5s, so the angular velocity of servo actuator 5 and 9 is 12.56rad/s. Then the cat robot flattens its waist in the remaining 0.14s, so the angular velocity of servo actuator 7 is 7.48rad/s. In the simulation environment, we set the end time is 0.64s and steps are 500.

1) Effect of Swing Angle of Legs on Turning Motion

According to the principle of single variable, we set the angular velocity of swing legs is constant and its numerical value is 8rad/s, we can obtain the angular velocity of servo actuator 2 and 3 is 4.86rad/s. The range of swing angle of simulation input is from 0° to 90° and the interval angle is 10°. Fig. 7 shows the relation between flip angle and swing angle. It can be seen from the figure that swing angle goes from 0° to 90°, flip angle of the cat robot goes from 102° to 210°.

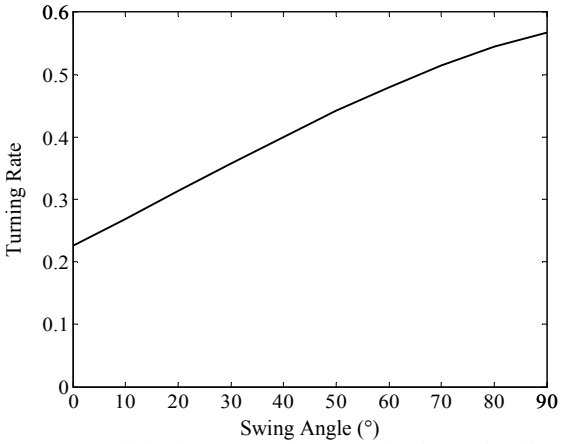


Fig. 4 Relation between turning rate and swing angle of legs.

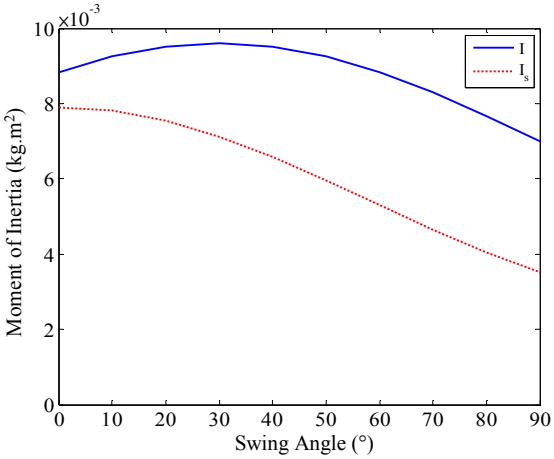
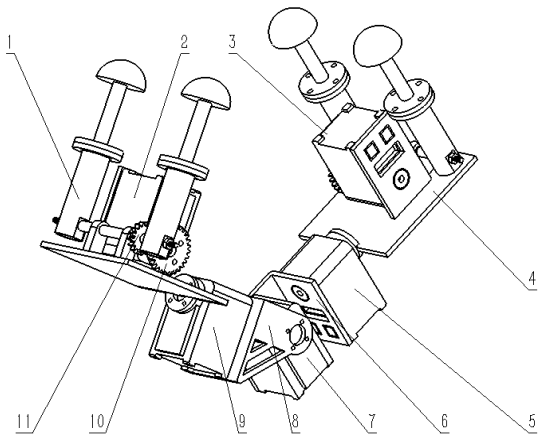


Fig. 5 The effect of swing angle of legs on moment of inertia I and I_s.



1-Leg; 2, 3, 5, 7, 9- Servo Actuator; 4, 6, 8- Connecting Plate; 10- Gear; 11- Gear Shaft.

Fig. 6 3D modeling of the cat robot.

So the simulation result is consistent with the conclusion from previous theoretic analysis. In all angles of simulation input, the cat robot has the minimum turning error when the swing angle of legs is 60°. The corresponding flip angle of the cat robot is 184° and the righting process is shown in Fig. 8.

The total energy consumption W of turning motion of the cat robot is composed of two parts: the energy consumption W_1 of the rotation of the front and back halves of the cat robot and the energy consumption W_2 of the swing of forelegs and hindlegs. The expressions of W_1 and W_2 are as follows:

$$W_1 = 2w_1\int_0^{0.5} T_1 dt \tag{17}$$

where w_1 and T_1 are angular velocity and torque of servo actuator 5 and 9, respectively.

$$W_2 = 2w_2\int_0^{0.5} T_2 dt \tag{18}$$

where w_2 and T_2 are angular velocity and torque of servo actuator 2 and 3, respectively. The variation curves of T_1 and T_2 with time under different swing angles are shown in Fig. 9 and Fig. 10, respectively.

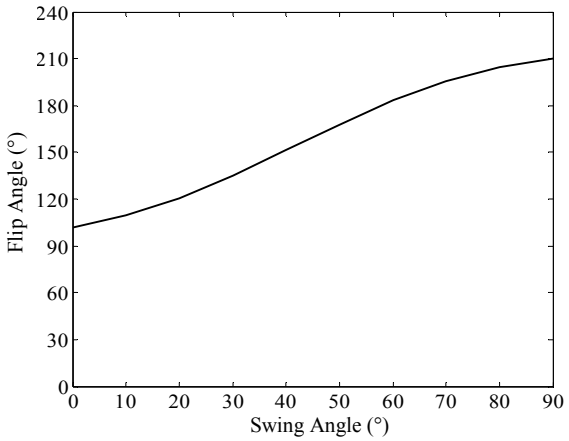


Fig. 7 Relation between flip angle of the cat robot and swing angle of legs.

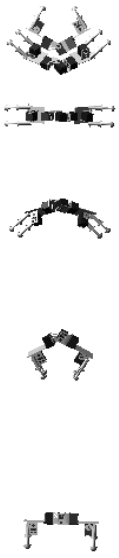


Fig. 8 Righting process of the cat robot.

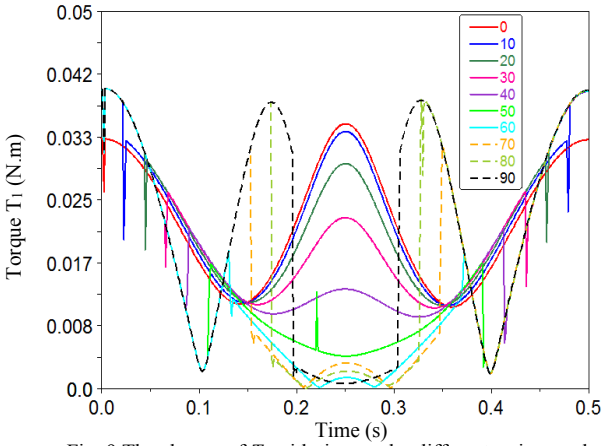


Fig. 9 The change of T_1 with time under different swing angles.

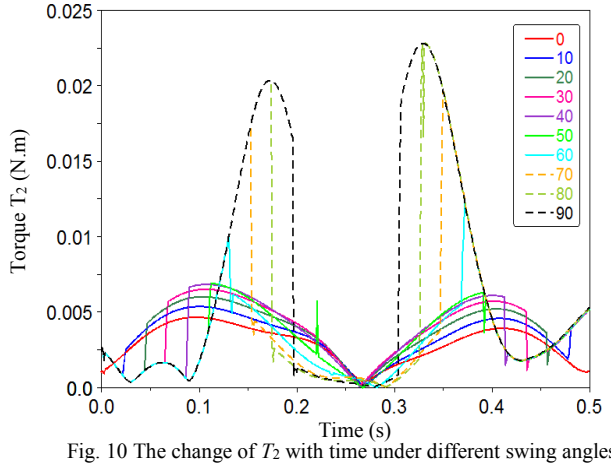


Fig. 10 The change of T_2 with time under different swing angles.

The integral value of the torque curve is multiplied with the corresponding angular velocity, then the each part and total energy consumption can be obtained, as shown in Fig. 11.

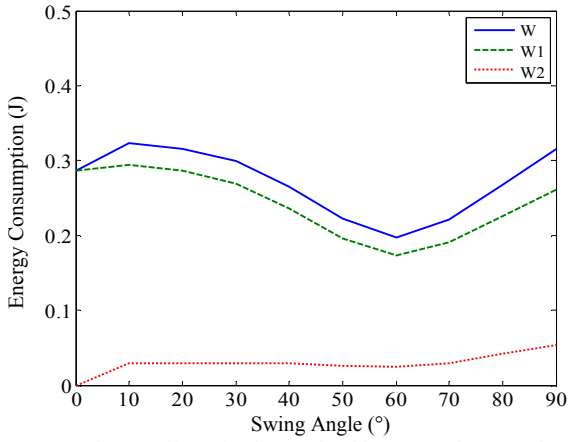


Fig. 11 Effect of swing angle of legs on each part and total energy consumption.

It's easy to see that the swing legs can generate energy consumption, but they are beneficial to reduce the energy consumption of rotation of the front and back halves of the cat robot. In all angles of simulation input, the optimal swing

angle value corresponding to the lowest energy consumption is 60°. The total energy consumption of the cat robot under this swing angle is reduced by 31.1% compared with the cat robot without legs swinging.

2) Effect of Angular Velocity of Swing legs on Turning Motion

According to the principle of the single variable, we set the swing angle of legs is constant and angle value is 60°. The numerical values of angular velocity are as follows: 8rad/s, 12rad/s, 16rad/s, 20rad/s. Fig. 12 shows the relation between flip angle and angular velocity. When the angular velocity goes from 8rad/s to 20rad/s, flip angle goes from 183° to 184.2°. So increasing the angular velocity can also improve the turning rate, but the influence is very small. The variation curves of T_1 and T_2 with time under different angular velocities are shown in Fig. 13 and Fig. 14, respectively.

Similarly, we can obtain the variation curves of each part and total energy consumption, as shown in Fig. 15. It can be seen from the figure that each part and total energy consumption become larger with the increase of the angular velocity. The increase of energy consumption of swing legs is particularly evident. So in order to keep the total energy consumption of turning motion of the cat robot at a lower level, the angular velocity of swing legs should be small under the premise that the legs can finish the swing within the stipulated time.

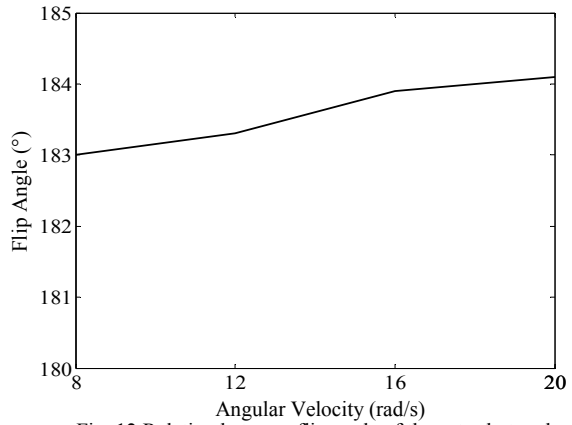


Fig. 12 Relation between flip angle of the cat robot and angular velocity of swing legs.

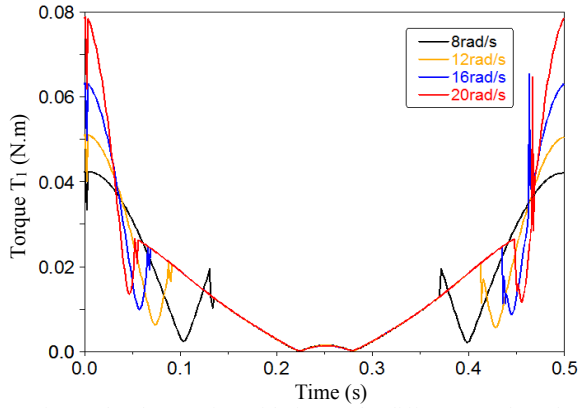


Fig.13 The change of T_1 with time under different angular velocities.

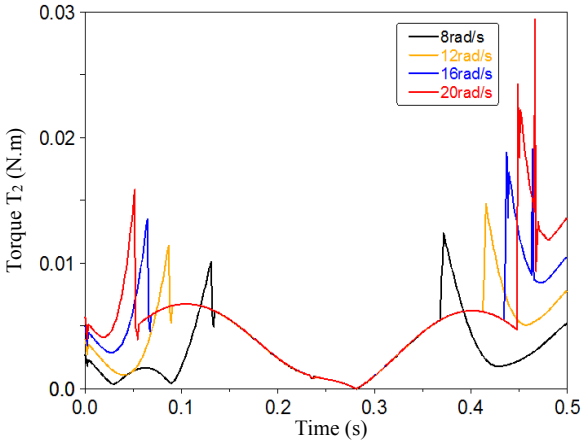


Fig. 14 The change of T_2 with time under different angular velocities.

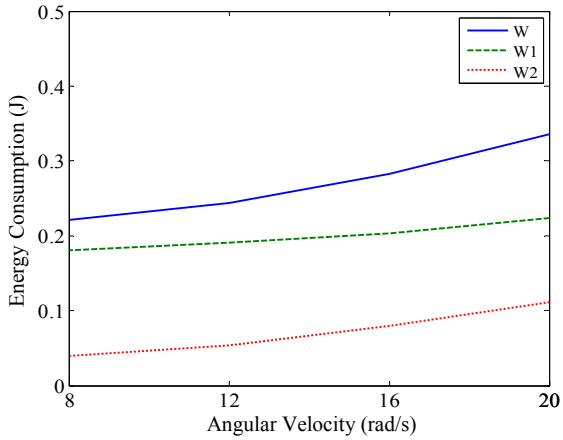


Fig. 15 Effect of angular velocity of swing legs on each part and total energy consumption.

Fig. 16 and Fig. 17 respectively show the variations of flip angle and energy consumption when angle and angular velocity of swing legs change together. We can see from the two figures that adjusting angle and angular velocity of swing legs to suitable numerical values, not only can the flip angle be improved, but also the energy consumption can be reduced.

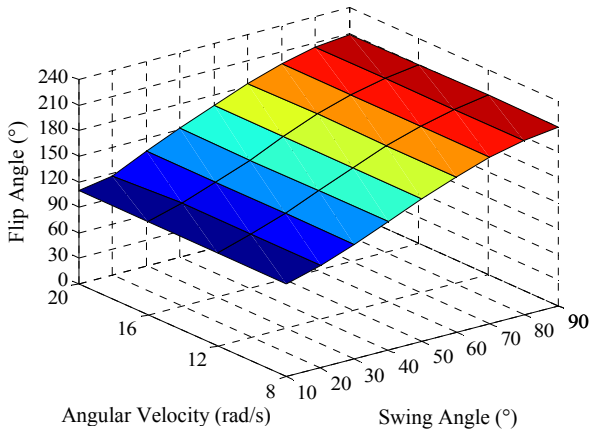


Fig. 16 Effect of angle and angular velocity of swing legs on flip angle of the cat robot.

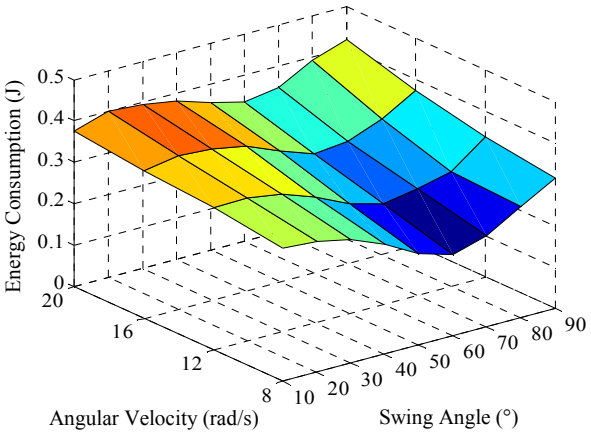


Fig. 17 Effect of angle and angular velocity of swing legs on total energy consumption.

In order to indicate whether the time of turning motion will change the influence of swing legs on flip angle and energy consumption of the cat robot, we do the same simulation when the time of turning motion is 0.4s, 0.6s, 0.8s and 1s, respectively. The results show that the change laws of flip angle and energy consumption with different swing angles and angular velocities are the same under different time of turning motion. In all angles of simulation input, the optimal swing angle value remains unchanged, but the energy of consumption of the cat robot will be reduced with the increase of time of turning motion. The longer time of turning motion means that the cat robot will fall from a higher height and it needs to withstand a greater impact from contact surface.

IV. Experiment

Experimental prototype of the cat robot is shown in Fig. 18 and the control of each servo actuator is based on STM32 MCU. The experiment of turning motion of the cat robot is carried out by suspended method. The experimental process is recorded by a high-speed camera.

The values of motion parameters of the prototype are set as follows: bending angle of waist θ is 30°, swing angle of legs β is 60°, angular velocity of swing legs is 8rad/s, the total time is 0.64s, where the time of turning motion is the initial 0.5s, then the cat robot unbends its waist in the remaining 0.14s. The angular velocity of servo actuator 5 and 9 is 12.56rad/s, the angular velocity of servo actuator 7 is 7.48rad/s, the angular velocity of servo actuator 2 and 3 is 4.86rad/s.

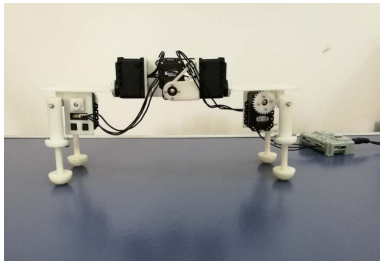


Fig. 18 Experimental prototype of the cat robot.

The comparison between simulation result and the actual righting process of the cat robot prototype is shown in Fig. 19. In order to facilitate the comparison, the experimental photos are rotated 90 degrees clockwise. It can be seen from the pictures that the actual righting process of the prototype is in good agreement with simulation result.

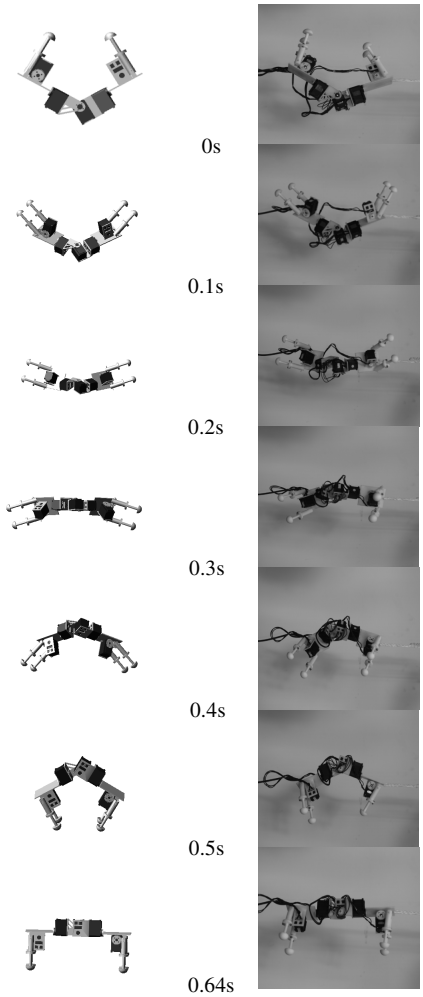


Fig. 19 The comparison between simulation result and actual righting process of the cat robot prototype.

V. Conclusions and Future Work

In this paper, a multi-rigid-body dynamic model of a free-falling cat robot is built with the consideration of swing legs. Through theoretical analysis and calculation, we come to the conclusion that turning rate of the cat robot will increase when swing angle of legs becomes large. We design a 3D modeling of a cat robot and then use ADAMS to carry out dynamic simulation. The simulation results show that adjusting the angle and angular velocity of the swing legs to suitable numerical values, not only can the flip angle be improved, but also the energy consumption can be reduced effectively. A cat robot prototype is developed and the turning motion of the prototype is implemented successfully in the experiment.

For future work, we will add gyroscopes and pressure sensors to the cat robot prototype in order to get the accurate data of turning motion and indicate the influence of landing attitude, elastic leg and flexible waist on landing stability, finally present mechanisms and implementation methods of safe landing of quadruped robots.

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