3

Robot Attitude

3.1 Introduction

This chapter is devoted to the introduction of coordinate frames and rotations. This is important in the study of the motion of any type of vehicle such as airplanes, ships, and automobiles as well as mobile robots. It will be seen that frames provide an efficient means of keeping track of vehicle orientation and also enable simple conversion of displacement with respect to an intermediate frame to displacement with respect to a fixed frame.

3.2 Definition of Yaw, Pitch, and Roll

Shown in Figure 3.1 is a mobile robot with a coordinate frame attached. This frame moves with the robot and is called the robot frame. The y axis is aligned with the longitudinal axis of the robot, and the x axis points out the right side. The z axis points upward to form a right-handed system. This type of frame definition is commonly used in the field of robotics. It differs from the convention used by those in aerospace where the x axis is aligned with the longitudinal axis, the y axis is to the right, and the z axis points down, still a right-handed system.

As the robot moves about, it experiences translation or change in position. In addition to this, it may also experience rotation or change in attitude. The various rotations of the robot are now defined. Yaw is rotation about the z axis in the counter-clockwise direction as viewed looking into the z axis. Pitch is rotation about the new (after the yaw motion) x axis, in the counter-clockwise direction as viewed looking into the x axis, i.e., front end up is positive pitch. Roll is rotation about the new (after both yaw and pitch) y axis in the counter-clockwise direction as viewed looking into the y axis, i.e., left side of vehicle up is positive. In the system used by those in the aerospace field, pitch is

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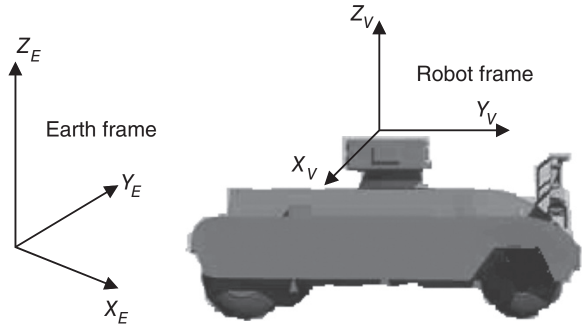

Figure 3.1 Mobile robot with earth and robot coordinate frames.

counter-clockwise rotation about the y axis while roll is counter-clockwise rotation about the x axis, i.e., the roles of the x and y axes are reversed with respect to these two rotations.

3.3 Rotation Matrix for Yaw

The rotation matrices for basic rotations are now derived. For yaw we have the diagram shown in Figure 3.2. Axes 1 represent the robot coordinate frame before rotation and axes 2 represent the robot coordinate frame after positive yaw rotation by the amount ψ. The z axes come out of the paper. It bears repeating that counter-clockwise rotation about the z axis is taken as positive yaw.

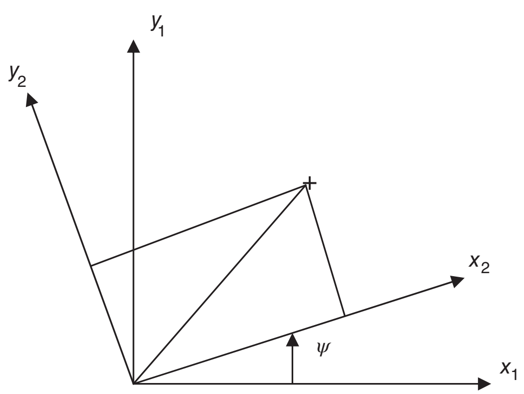

Figure 3.2 Frame 2 yawed with respect to frame 1.

We wish to express in the original coordinate frame 1 the location of a point whose coordinates are given in the new frame 2. For x and y we have

$$x_1 = x_2 \cos\psi - y_2 \sin\psi$$
$$y_1 = x_2 \sin\psi + y_2 \cos\psi$$

and for z

$$z_1 = z_2$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

Thus the rotation matrix for yaw is

$$R_{yaw}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.1}$$

Example 1 *A vector expressed in the rotated coordinate system with ψ of $\pi/2$ is given by*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Express this vector in the original coordinate system.

Solution 1

The expression of this vector in the original coordinate system becomes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\pi/2 & -\sin\pi/2 & 0 \\ \sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Note that the Euclidean norm of each column of the rotation matrix is one and that each column is orthogonal to each of the others. This is the definition of an orthonormal matrix. A convenient property of such matrices is that the inverse is simply the transpose, i.e.,

$$R_{yaw}(\psi)^{-1} = R_{yaw}(\psi)^T \tag{3.2}$$

or

$$R_{yaw}(\psi)^T R_{yaw}(\psi) = I$$

This property can be proved by premultiplying an orthonormal matrix by its transpose and then using the properties which it possesses, i.e.,

$$\begin{aligned} \langle col_i, col_j \rangle &= 1, \quad i = j \\ &= 0 \quad i \neq j \end{aligned}$$

3.4 Rotation Matrix for Pitch

For pitch, we have the situation depicted in Figure 3.3. The x axes come out of the paper. Note again that front end up corresponds to positive pitch.

Again we wish to express in the original coordinate frame the location of a point whose coordinates have been given in the new frame. For x and z we have

$$\begin{aligned} y_1 &= y_2 \cos\theta - z_2 \sin\theta \\ z_1 &= y_2 \sin\theta + z_2 \cos\theta \end{aligned}$$

and for x

$$x_1 = x_2$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

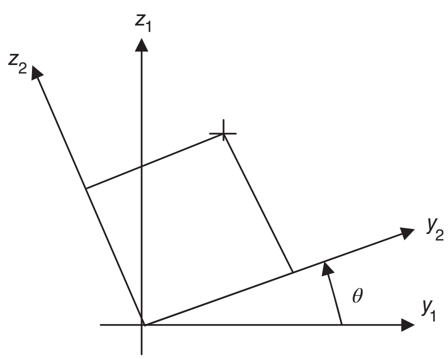

Figure 3.3 Frame 2 pitched with respect to frame 1.

Thus the rotation matrix for pitch is

$$R_{pitch}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{3.3}$$

Example 2 *A vector expressed in the rotated coordinate system with θ of $\pi/2$ (i.e., pitched up by the angle $\pi/2$) is given by*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Express this vector in the original coordinate system.

Solution 2

The expression of this vector in the original coordinate system becomes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 \\ 0 & \sin\pi/2 & \cos\pi/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

One may easily verify that the rotation matrix for pitch is also orthonormal.

3.5 Rotation Matrix for Roll

Finally we treat roll. This is counter-clockwise rotation about the y axis which results in left side up being defined as positive roll. The y axes come out of the paper as is shown in Figure 3.4.

Once more we wish to express in the original coordinate frame the location of a point whose coordinates are given in the new frame. For x and z we have

$$x_1 = x_2 \cos\phi + z_2 \sin\phi$$
$$z_1 = -x_2 \sin\phi + z_2 \cos\phi$$

and for y

$$y_1 = y_2$$

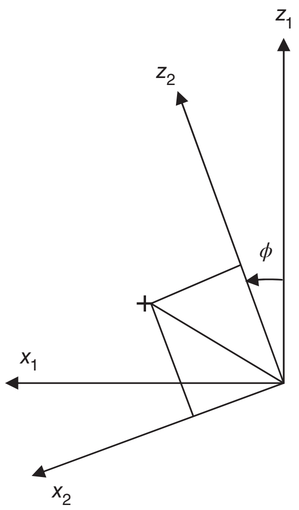

Figure 3.4 Frame 2 rolled with respect to frame 1.

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

Thus the rotation matrix for roll is

$$R_{roll}(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \tag{3.4}$$

which is also orthonormal.

Example 3 *A vector expressed in the rotated coordinate system with ϕ of $\pi/2$ is given by*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Express this vector in the original coordinate system.

Solution 3

In the original coordinate system the expression of this vector becomes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\pi/2 & 0 & \sin\pi/2 \\ 0 & 1 & 0 \\ -\sin\pi/2 & 0 & \cos\pi/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Another way to think about the definitions of these different rotations is to reference them to the longitudinal axis of the vehicle, starting with the vehicle level and pointing along the y axis of the reference frame. Yaw is the rotation of the longitudinal axis of the robot in the horizontal plane. CCW rotation as viewed from above is taken as positive. Pitch is the rotation of the longitudinal axis of the robot in a plane perpendicular to the horizontal plane. Front end up is taken as positive. Roll is the rotation of the robot about its longitudinal axis. Left side up is taken as positive.

It is worth reiterating that each of these rotation matrices is orthonormal, i.e., the columns are all orthogonal to each other, and each column has Euclidean norm of one, making the inverse equal to the transpose.

3.6 General Rotation Matrix

We now define the general rotation matrix. After a frame has been yawed, pitched, and rolled, in this specific order, a point with coordinates given in this new frame may be converted into its coordinates in the original frame by the following operation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = R_{yaw}(\psi)R_{pitch}(\theta)R_{roll}(\phi)\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 \tag{3.5}$$

Note that the conversion back into the original coordinates is in the reverse order of the rotations; i.e., roll was the last rotation. Therefore, it is the first matrix to operate on the coordinates of the point in question. Yaw was the first rotation; therefore, it is the last matrix to operate on the point in question. By multiplying these three rotation matrices together in the order shown above, we have the general rotation matrix:

$$R(\psi,\theta,\phi) = \begin{bmatrix} \cos\psi\cos\phi - \sin\psi\sin\theta\sin\phi & -\sin\psi\cos\theta & \cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ \sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\theta & \sin\psi\sin\phi - \cos\psi\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \sin\theta & \cos\theta\cos\phi \end{bmatrix} \tag{3.6}$$

It is easy to show that this product of orthonormal matrices is also orthonormal. Thus the general rotation matrix is also orthonormal.

As was the case for the individual rotation matrices, this general rotation matrix can be used to express a vector in an original coordinate frame when it has first been expressed in a frame that has been rotated with respect to the original frame. No matter what the attitude of a vehicle or how it arrived at this attitude, there exists a set of rotations in the order prescribed, yaw, pitch, and roll, which will yield this very same attitude.

A more generic expression of attitude that does not depend on one's choice of rotation order is the matrix comprised of direction cosines of the axes of frame 2 with the axes of frame 1. The components of the first column are successively the inner product of the x unit vector of frame 2 with the x unit vector of frame 1, the inner product of the x unit vector of frame 2 with the y unit vector of frame 1 and the inner produce of the x unit vector of frame 2 with the z unit vector of frame 1. Likewise, the components of the second column are successively the inner product of the y unit vector of frame 2 with the x unit vector of frame 1, the inner product of the y unit vector of frame 2 with the y unit vector of frame 1 and the inner produce of the y unit vector of frame 2 with the z unit vector of frame 1. Finally the components of the third column are successively the inner product of the z unit vector of frame 2 with the x unit vector of frame 1, the inner product of the z unit vector of frame 2 with the y unit vector of frame 1, and the inner produce of the z unit vector of frame 2 with the z unit vector of frame 1. In other words:

$$R_{21} = \begin{bmatrix} U_{x2}^T U_{x1} & U_{y2}^T U_{x1} & U_{z2}^T U_{x1} \\ U_{x2}^T U_{y1} & U_{y2}^T U_{y1} & U_{z2}^T U_{y1} \\ U_{x2}^T U_{z1} & U_{y2}^T U_{z1} & U_{z2}^T U_{z1} \end{bmatrix} \tag{3.7}$$

The entries of the matrix $R(\psi, \theta, \phi)$ given in Eq. (3.6) may be equated to this matrix yielding the values for yaw, pitch, and roll which when executed in that order would yield the given orientation. Equating terms it may be readily seen that

$$\tan\psi = -U_{y2}^T U_{x1} / U_{y2}^T U_{y1}$$

$$\sin\theta = U_{y2}^T U_{z1}$$

and

$$\tan\phi = -U_{x2}^T U_{z1} / U_{z2}^T U_{z1}$$

3.7 Homogeneous Transformation

There are situations where one frame is not only rotated with respect to another, but is also displaced. Suppose frame 2 is both rotated and displaced with respect to frame 1. Then a vector initially expressed with respect to frame 2 can be expressed with respect to frame 1 as below.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = R(\psi,\theta,\phi) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 + \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\textit{origin of frame 2 expressed in frame 1 coords}}$$

or in shorthand notation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = R_{21} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 + \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}_{21} \tag{3.8}$$

If one goes through a series of transformations, the operations become even more cumbersome. For the case of two transformations the equations are

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = R_{32} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_3 + \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}_{32}$$

and

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = R_{21} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 + \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}_{21}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = R_{21}R_{32} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_3 + R_{21} \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}_{32} + \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}_{21}$$

This can be written more concisely as a single operation using the *homogeneous transformation*. For a single transformation containing translation and rotation

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_1 = A_{21} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_2 \tag{3.9}$$

where for A_{21} we have

$$A_{21} = \left[\begin{array}{ccc|c} & & & x_o \\ & R_{21} & & y_o \\ & & & z_o \\ 0 & 0 & 0 & 1 \end{array}\right] \tag{3.10}$$

Note that the upper left three-by-three matrix is the rotation matrix while the upper portion of the right column is comprised of the origin of frame 2 in frame 1 coordinates. Here x_o, y_o, and z_o could represent, for example, the origin of the sensor frame in vehicle coordinates. One can use this transformation to convert a vector specified in one set of coordinates to its expression in another set of coordinates in a single operation. When using this homogeneous transformation, the position vectors are converted to dimension four by appending a 1 as the fourth entry. This is necessary not only to make the matrix operations conformal, but also to couple in the location of the origin of the second coordinate system with respect to the original coordinate system.

Example 4 *Let frame 2 be both rotated and displaced with respect to frame 1. The rotation is a yaw of 90°*

$$R_{21} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the displacement of the origin of frame 2 with respect to frame 1 is

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

Now let the point of interest be given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\textit{expressed in frame 2}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Express this vector in frame 1.

Solution 4

In frame 1 the expression of this vector becomes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\textit{expressed in frame 1}} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\textit{expressed in frame 1}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 0 \end{bmatrix}$$

Now solving this problem by using the homogeneous transformation matrix we have

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\textit{expressed in frame 1}} = \begin{bmatrix} 0 & -1 & 0 & 10 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{\textit{expressed in frame 1}} = \begin{bmatrix} 10 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$

The homogeneous transformation matrices can be multiplied just as the rotation matrices can. Thus the homogeneous transformation to take a vector from its expression in frame 3 coordinates to its expression in frame 2 coordinates and finally to its expression in frame 1 coordinates can be written

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_1 = [A_{21}][A_{32}] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_3 \tag{3.11}$$

or

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_1 = [A_{31}] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_3$$

where

$$[A_{31}] = [A_{21}][A_{32}]$$

Another interesting and useful property of the homogeneous transformation matrix is that its inverse can be expressed as

$$\begin{bmatrix} & R_{21} & & \begin{matrix} x_o \\ y_o \\ z_o \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} & R_{21}^T & & \begin{matrix} a \\ b \\ c \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.12}$$

where the entries in the upper part of the last column are defined by

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -R_{21}^{T} \begin{bmatrix} x_o \\ y_0 \\ z_o \end{bmatrix} \tag{3.13}$$

In all of these, use has been made of the fact that for a rotation matrix

$$R^{-1} = R^{T}$$

since the rotation matrix is orthonormal.

This homogeneous transformation provides a concise means of expressing a vector in an original frame when the second frame is both rotated and translated with respect to the original frame. Its convenience becomes even more pronounced when there is a series of transformations, e.g., sensor frame to vehicle frame and then vehicle frame to earth frame.

3.8 Rotating a Vector

Another important application of the rotation matrix is to express the new coordinates of a vector after the vector itself has been yawed, pitched, and rolled. Here the same coordinate frame is used before and after the rotation. To illustrate, consider an initial vector expressed in frame 1. This vector is now rotated about the z axis. The expression for this rotated vector, again in frame 1, is given by the following

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{after\ rotation} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{before\ rotation}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{after\ rotation} = R_{yaw}(\psi) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{before\ rotation} \tag{3.14}$$

This same process holds for each of the rotations. Thus, if a vector is rotated first about the y axis, then about the x axis, and finally about the z axis, the new vector in the original frame is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{after\ rot} = \begin{bmatrix} \cos\psi\cos\phi - \sin\psi\sin\theta\sin\phi & -\sin\psi\cos\theta & \cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ \sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\theta & \sin\psi\sin\phi - \cos\psi\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \sin\theta & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{before\ rot}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{after\ \ rotation} = R(\psi, \theta, \phi) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{before\ \ rotation} \tag{3.15}$$

To reiterate, in this second application of rotation matrices, the vector on the right-hand side of the equation is the vector before its rotation and the result on the left-hand side is the vector after its rotation. Both vectors are expressed in the same frame.

We shall find important uses for these rotation matrices and homogeneous transformation matrices in the chapters that follow.

Exercises

1 Evaluate the rotation matrix for the case where $\psi = \pi/2,\ \ \theta = -\pi/2$, and $\phi = 0$.

2 Evaluate the rotation matrix for the case where $\psi = 0,\ \ \theta = \pi/2$, and $\phi = \pi/2$.

3 Evaluate the homogeneous transformation for the case where the second frame has orientation with respect to the first frame of $\psi = \pi/2,\ \theta = -\pi/2$, and $\phi = 0$ and location with respect to the first frame of $x = 3$, $y = 2$, and $z = 1$.

4 Evaluate the homogeneous transformation for the case where the second frame has orientation with respect to the first frame of $\psi = 0,\ \ \theta = \pi$, and $\phi = \pi/2$ and location with respect to the first frame of $x = 1, y = 3$, and $z = 2$.

5 Given a target whose location is expressed in frame 2 as $x = 1, y = 2$, and $z = 0$, find its location with respect to frame 1. The origin of frame 2 is at $x = 20$, $y = 10$, and $z = 1$ with respect to frame 1. The orientation of frame 2 with respect to frame 1 is $\psi = \pi/2,\ \ \theta = \pi/4$, and $\phi = 0$. Solve using a rotation plus a translation and also solve using the homogeneous transformation.

6 A target is located in frame 3 at coordinates $x = 3, y = 2$, and $z = 1$. The origin of frame 3 is at coordinates $x = 10, y = 0$, and $z = 0$ with respect to frame 2. The orientation of frame 3 with respect to frame 2 is $\psi = \pi/4,\ \ \theta = -\pi/4$, and $\phi = 0$. The origin of frame 2 is at coordinates $x = 0, y = 5$, and $z = 0$ with respect to frame 1. The orientation of frame 2 with respect to frame 1 is $\psi = \pi,\ \ \theta = 0$, and $\phi = \pi/2$.

A Compute the homogeneous transformation describing frame 3 with respect to frame 2 and determine the location of the target in frame 2.
B Next compute the homogeneous transformation describing frame 2 with respect to frame 1 and determine the location of the target in frame 1.
C Finally multiply the homogeneous transformations together (in the proper order) and determine the location of the target in frame 1 in a single step.

A vector has coordinates $[1\ \ 0\ \ 0]'$. This vector is to be rotated about the y axis by the amount $\pi/2$. Use the rotation matrix to determine the resulting vector.

7 A vector has coordinates $[1\ \ 0\ \ 0]'$. This vector is to be rotated about the y axis by the amount $\pi/2$ and then about the z axis by the amount $-\pi/2$. Use the rotation matrix to determine the resulting vector.

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