N7PD Declarative Programming

Logic Programming – Prolog

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What is logic programming?

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Points 1 and 2 implies that we will manipulate **predicates**, i.e. expressions that have a truth value.

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$Integer(one)$	*one* is an integer
$Plays(john, soccer, isae)$	John plays soccer for the ISAE team

Point 3 implies that we will **ask questions to programs**, e.g. "who plays for the ISAE soccer team?"

Predicates, terms and functions

Predicates represent **properties** verified by **objects**.

For instance, *Plays*$/3$ is a predicate of arity 3 representing the fact that somebody (the first argument of the predicate) plays a sport (the second argument of the predicate) in a team (the third argument).

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In first-order logic, **objects** are represented by **terms**.

Terms can be:

- **constants**, e.g. $\mathit{Plays}(\mathit{john}, \mathit{soccer}, \mathit{isae})$
- **functions applied to terms**, e.g. $\mathit{Plays}(\mathit{father}(\mathit{john}), \mathit{soccer}, \mathit{isae})$
- **variables**, e.g. $\forall x \; \mathit{Plays}(x, \mathit{soccer}, \mathit{isae})$

Going back to natural numbers (again)

For instance, let us consider natural numbers. We will define them by:

- a **constant term** z
- an **unary function** s (successor)
- a **predicate** $=$ (in infix notation)

$s(s(z))$ represents the successor of the successor of z (2 in real life) and $s(z) = z$ should be false.

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Exercise

We want to define addition. What will you choose, a function, a constant, a predicate?

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We want now to define a **predicate** *add* representing addition. What is its arity?

Addition of two natural numbers

We can define addition with a predicate *add/3* and the following fact and rule:

1. $\forall x \; add(z, x, x)$
2. $\forall x \forall y \forall r \; add(x, y, r) \rightarrow add(s(x), y, s(r))$

Using this theory, we can exhibit a **proof** of $add(s(s(z)), s(s(z)), s(s(s(s(z)))))$ in a particular **formal system**.

$$\dfrac{\dfrac{add(z, s(s(z)), s(s(z))) \qquad add(x, y, r) \rightarrow add(s(x), y, s(r))}{add(s(z), s(s(z)), s(s(s(z))))} \qquad add(x, y, r) \rightarrow add(s(x), y, s(r))}{add(s(s(z)), s(s(z)), s(s(s(s(z)))))}$$

Yes, $2 + 2 = 4$ ☺

What is Prolog?

Prolog (*Programmation en logique*) is a logic programming language initially developed in the 70s for natural language processing.

In Prolog, rules can only have the following form (they are called Horn clauses)

$$B_1 \wedge \ldots \wedge B_n \rightarrow A$$

i.e. "B_1 and …and B_n implies A".

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Prolog is both

- **declarative**: you describe the problem to solve
- **procedural**: there is a procedure that explains how Prolog answers questions

Prolog basic syntax and principles

Instructions in a Prolog program can be viewed as the **premises** of an argument.

A request can be viewed as the **conclusion** of an argument from the previous premises.

The fact that the conclusion can be deduced from the premises is proved by using the **Resolution** formal system for First-Order Logic (FOL).
Variables are used and instanciated.

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Syntax (Prolog)

- representing data: using **terms**
- identifier beginning by a **lowercase** letter: function or predicate symbol
- identifier beginning by a **uppercase** letter: variable symbol

Prolog: back to addition

The Prolog program corresponding to our “definition” of addition:

add.pl

```
add(z, X, X).
add(s(X), Y, s(R)) :- add(X, Y, R).
```

Prolog: back to addition

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```

We can "ask" Prolog questions: `add(s(s(z)),s(s(s(z))),W)`?

- ➥ meaning: is there a `W` such that `add(s(s(z)),s(s(s(z))),W)` holds, i.e. `W = 2 + 3`?

In this case, Prolog answers `W = s(s(s(s(s(z)))))`.

Cool. Prolog knows how to add two natural numbers.

Prolog applications

Is Prolog really used in real life?

- NASA Clarissa
  - voice-operated procedure browser in International Space Station
  - help astronauts with complex procedures
- IBM Watson
  - a question-answering computer system
  - won the quiz show Jeopardy in 2011

Lally, Adam and Paul Fodor (Mar. 2011).
Natural Language Processing With Prolog in the IBM Watson System.
https://www.cs.nmsu.edu/ALP/2011/03/natural-language-processing-with-prolog-in-the-ibm-watson-system/.

Outline

Program clause

Definition (Prolog program)

A Prolog program is a **sequence** of clauses.

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Syntax (Program clause)

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If $n = 0$, then the clause is a **fact** and is simply denoted by A**.**

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beware of us!

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Query clause

Syntax (Query clause)

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$$:\!-B_1, \ldots, B_n.$$

Intuition: are there values for variables such that $B_1, \ldots, B_n$ are **all** true?

Beware

For the program clause $B(X)$:- $C(Y, X)$.

- X is **universally** quantified
- Y is **existentially** quantified

Terminology: Prolog resolvent

Definition (Most general unifier)

If $p(t_1, \ldots, t_n)$ and $p(s_1, \ldots, s_n)$ are two atoms such thath the subsitution σ is a most general unifier of those atoms, then $p(t_1, \ldots, t_n)$ and $p(s_1, \ldots, s_n)$ are unifiable by **mgu** σ.

Definition (Prolog resolvent)

Let $R = \texttt{:-}\ A_1, \ldots, A_m$ be a query clause and $C = A'_1\ \texttt{:-}\ B_1, \ldots, B_p$ be a program clause with $m > 0$ and $p \geq 0$. If A_1 and A'_1 are unifiable by σ, then the new query clause $R' = \texttt{:-}\ \sigma(B_1, \ldots, B_p, A_2, \ldots, A_m)$ is called **Prolog resolvent** of R and C.

Idea behind Prolog resolvent

The idea behind Prolog resolvent is the same as behind Resolution in First-Order Logic (cf. last slides in this part).

More intuitively, consider a program clause $A' \texttt{:-} B_1, \ldots, B_n$. It can be read as “if $B_1, \ldots, B_n$ hold, then A' holds”...

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...but you can also understand the clause as “to prove A', it is sufficient to prove $B_1, \ldots, B_n$”.

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...but you can also understand the clause as "to prove A', it is sufficient to prove $B_1, \ldots, B_n$".

Thus, when you have a request involving A such that A and A' are unifiable, you can replace the A part of the request by $B_1, \ldots, B_n$ (given the substitution).

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...but you can also understand the clause as "to prove A', it is sufficient to prove $B_1, \ldots, B_n$".

Thus, when you have a request involving A such that A and A' are unifiable, you can replace the A part of the request by $B_1, \ldots, B_n$ (given the substitution).

Does it end? It depends (cf. next slides), but facts can be used, e.g.:

clause	`add(X, z, X).`
request	`add(s(s(z)), z, W).`
resolvent	empty clause with substitution {`X/s(s(z))`, `W/X`}

Evaluation algorithm

There is a **non-determinist** evaluation algorithm for Prolog.

Inputs: a Prolog program P and a query clause R

Output: two possibilities

- a substitution σ for the variables appearing in R (if R does not contain variables, the output is `YES`);
- `NO`

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Output: two possibilities

- a substitution σ for the variables appearing in R (if R does not contain variables, the output is `YES`);
- `NO`

Given a Prolog program P and a query R, an evaluation of R can have three issues:

- ending with success
- ending with `NO`
- **no ending!**

Algorithm

Algorithm 1.1: Prolog evaluator

Input: a Prolog program P and a query R
Output: a substitution σ for variables appearing in R, else NO

$R_c \leftarrow R$;
$mgu_c \leftarrow \emptyset$;
while $R_c =$:- $G_1, \ldots, G_k \neq \emptyset$ **do**
    choose $C = G'_1$:- $D_1, \ldots D_t \in P$ st G_1 et G'_1 unifiable by σ ;
    if C does not exist **then**
        break ;
    end
    $R_c \leftarrow$ Prolog resolvent of G_1 and C (replace G_1) ;
    $mgu_c \leftarrow mgu_c \circ \sigma$;
end
if $R_c = \emptyset$ **then**
    compute restriction σ' of mgu_c to R variables ;
    if $\sigma' = \emptyset$ **then**
        return YES ;
    else
        return σ' ;
    end
else
    return No ;
end

Outline

Example

Program

```
1 parent(jack,mary).
2 parent(louise,jack).
3 parent(franck,john).
4 ancestor(X,Y) :- parent(X,Y).
5 ancestor(X,Y) :- ancestor(X,Z), parent(Z,Y).
```

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Query

```
:- ancestor(W,mary)
```

Example

Program

1. `parent(jack,mary).`
2. `parent(louise,jack).`
3. `parent(franck,john).`
4. `ancestor(X,Y) :- parent(X,Y).`
5. `ancestor(X,Y) :- ancestor(X,Z), parent(Z,Y).`

Query

```
:- ancestor(W,mary)
```

Some evaluation **examples** are presented in the next slides.

Example 1: evaluation with success

$\sigma = \emptyset$

```
ancestor(W,mary)
```

```
parent(jack,mary).
parent(louise,jack).
parent(franck,john).
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- ancestor(X,Z),
                 parent(Z,Y).
```

Example 1: evaluation with success

$\sigma = \{W/X1, Y1/mary\}$

```
ancestor(W,mary)
       |
parent(X1,mary)
```

```
parent(jack,mary).
parent(louise,jack).
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ancestor(X,Y) :- parent(X,Y).
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```

Example 1: evaluation with success

$$\sigma = \{W/X1, Y1/mary, X1/jack\}$$

```
ancestor(W,mary)
       |
parent(X1,mary)
       |
       ∅
```

```
parent(jack,mary).
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ancestor(X,Y) :- parent(X,Y).
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ancestor(W,mary)
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parent(X1,mary)
       |
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```

```
parent(jack,mary).
parent(louise,jack).
parent(franck,john).
ancestor(X,Y) :- parent(X,Y).
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```

Answer: $\{W/jack\}$

Example 2: evaluation with success

$\sigma = \emptyset$

```
ancestor(W,mary)
```

```
parent(jack,mary).
parent(louise,jack).
parent(franck,john).
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- ancestor(X,Z),
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```

Example 2: evaluation with success

$$\sigma = \{W/X1, Y1/mary\}$$

```
      ancestor(W,mary)
              |
ancestor(X1,Z1),parent(Z1,mary)
```

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parent(jack,mary).
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Example 2: evaluation with success

$$\sigma = \{W/X1, Y1/mary, X1/X2, Z1/Y2\}$$

```
      ancestor(W,mary)
              |
ancestor(X1,Z1),parent(Z1,mary)
              |
 parent(X2,Y2),parent(Y2,mary)
```

```
parent(jack,mary).
parent(louise,jack).
parent(franck,john).
```
`ancestor(X,Y) :- parent(X,Y).`
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```

Example 2: evaluation with success

$$\sigma = \{W/X1, Y1/mary, X1/X2, Z1/Y2, X2/louise, Y2/jack\}$$

```
ancestor(W,mary)
        |
ancestor(X1,Z1),parent(Z1,mary)
        |
parent(X2,Y2),parent(Y2,mary)
        |
parent(jack,mary)
```

```
parent(jack,mary).
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Answer: $\{W/louise\}$

Example 3: evaluation with failure

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Example 3: evaluation with failure

$$\sigma = \{W/X1, Y1/mary\}$$

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ancestor(W,mary)
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ancestor(X1,Z1),parent(Z1,mary)
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Answer: `NO`

Example: evaluation with no ending

Using only clause `ancestor(X,Y) :- ancestor(X,Z),parent(Z,Y).`

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```
ancestor(W,mary)
```

Example: evaluation with no ending

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                   |
ancestor(X1,Z1),parent(Z1,mary)
```

Example: evaluation with no ending

Using only clause `ancestor(X,Y) :- ancestor(X,Z),parent(Z,Y).`

```
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                        |
        ancestor(X1,Z1),parent(Z1,mary)
                        |
ancestor(X2,Z2),parent(Z2,Y2),parent(Y2,mary)
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Example: evaluation with no ending

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                     |
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ancestor(X2,Z2),parent(Z2,Y2),parent(Y2,mary)
                     |
                     ⋮
```

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Using only clause `ancestor(X,Y) :- ancestor(X,Z),parent(Z,Y).`

```
ancestor(W,mary)

ancestor(X1,Z1),parent(Z1,mary)

ancestor(X2,Z2),parent(Z2,Y2),parent(Y2,mary)
⋮
⋮
```

Example: evaluation with no ending

Using only clause `ancestor(X,Y) :- ancestor(X,Z),parent(Z,Y).`

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                        |
        ancestor(X1,Z1),parent(Z1,mary)
                        |
ancestor(X2,Z2),parent(Z2,Y2),parent(Y2,mary)
                        |
                        ⋮
                        |
                        ⋮
```

No ending!

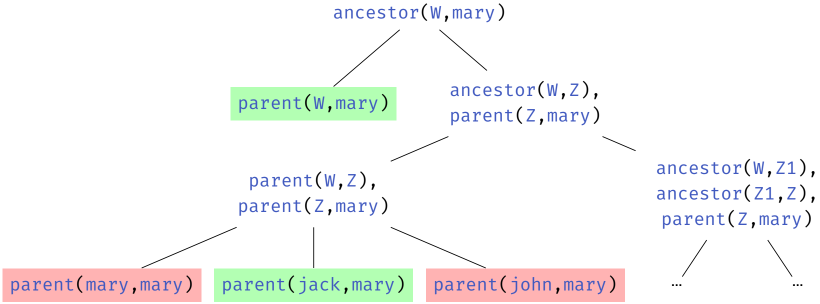
Outline

Definition (Prolog search tree)

A Prolog search tree for a program P and a query R is a tree whose nodes are query clauses and such that:

- its **root** is R;
- if a node is a non-empty query `:-` $A_1, \ldots, A_n$ (where $n > 0$) and $C_1, \ldots, C_k$ (where $k > 0$) are the program clauses (**appearing in this order** in P) whose heads are unifiable with A_1, then this node has k children $Res_1, \ldots, Res_k$ where for $i \in \{1, \ldots, k\}$, Res_i **is the Prolog resolvent** of A_1 with the clause C_i;
- if a node is a non-empty query `:-` $A_1, \ldots, A_n$ (where $n > 0$) and if there is no program clause whose head is unifiable with A_1, then this node is a **failure leaf**;
- if a node is the empty query, then this node is a **success leaf**.

Prolog search tree for our example (simplified)



The Prolog tree on our example is **infinite on the right**.

Depth-first exploration

- if the current branch ends with success, the evaluation stops and give the corresponding answer;
- if the current branch ends with failure, the next branch is considered. The next clause usable for the query represented by the parent node of the current node is chosen (**backtracking**);
- if after having given the result of a success branch the user send a “continue” instruction, the next branch is considered as if the current branch had failed (**backtracking**);
- if the branch does not end, the evaluator does not stop.

Thus...

- the answers to the query R are given into an order that depends of the writing order of clauses and atoms in P;
- infinite branch $\Rightarrow$ the interpreter does not stop;
- the answer `NO` corresponds to the case where the tree is **finite** and where every branch is a failure branch.

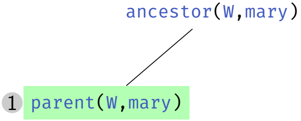
Prolog search tree building on our example (simplified)

`ancestor(W,mary)`

- - - - - - - - - → user backtracking after success

- - - - - - - - - → Prolog backtracking after failure

Prolog search tree building on our example (simplified)



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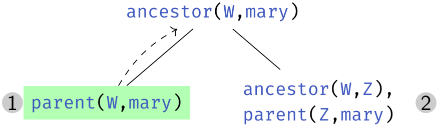

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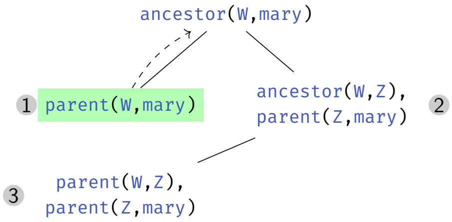

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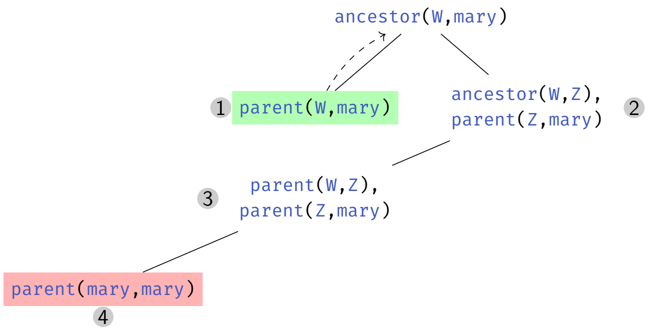

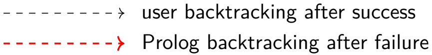

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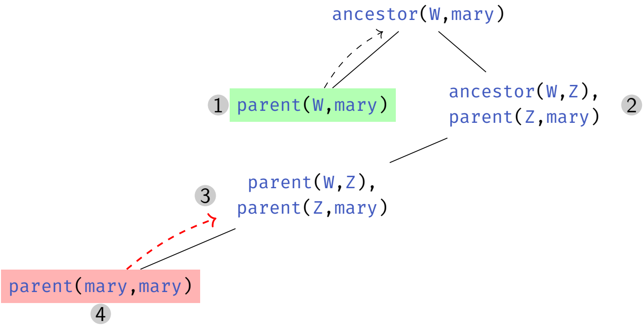

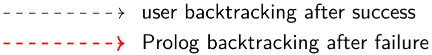

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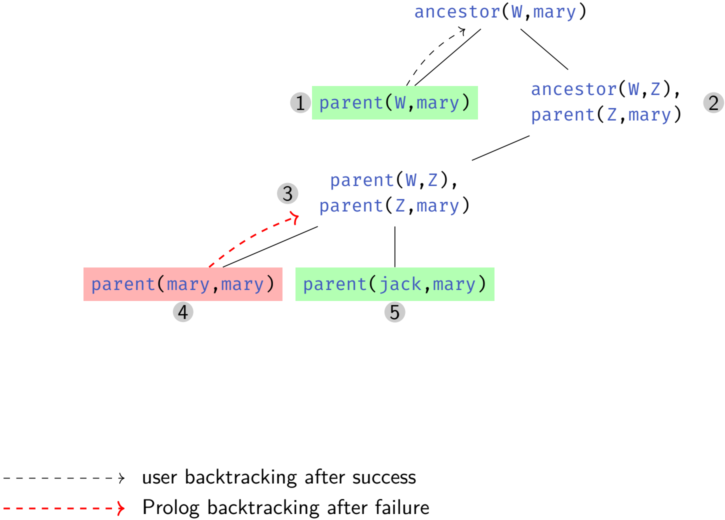

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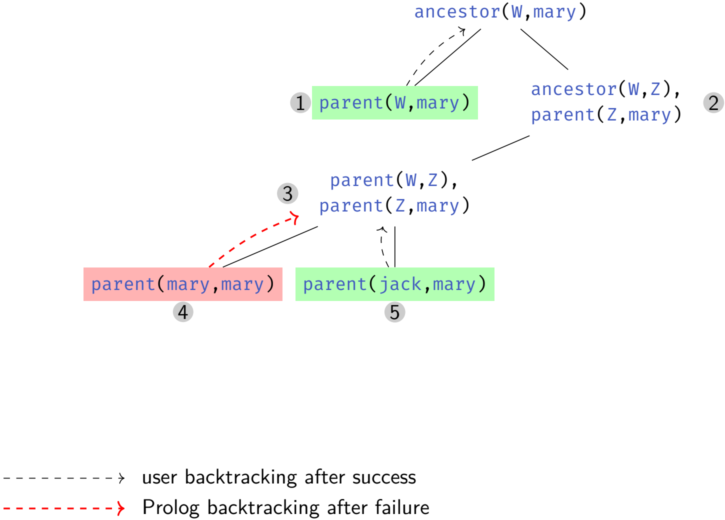

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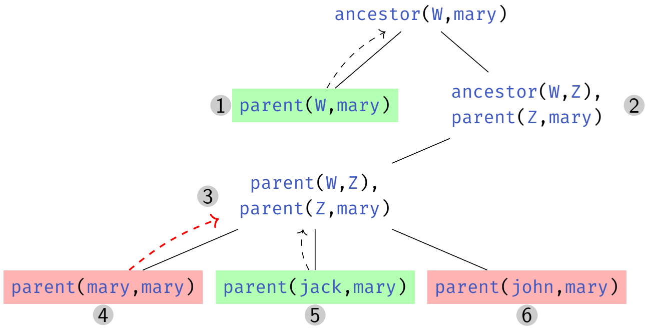

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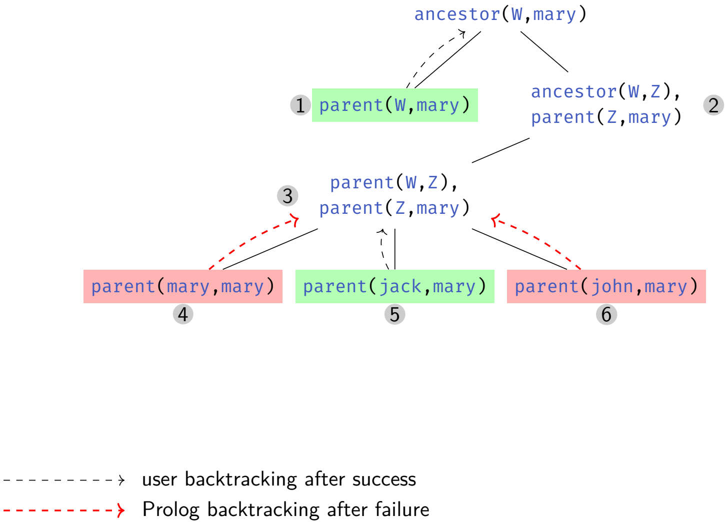

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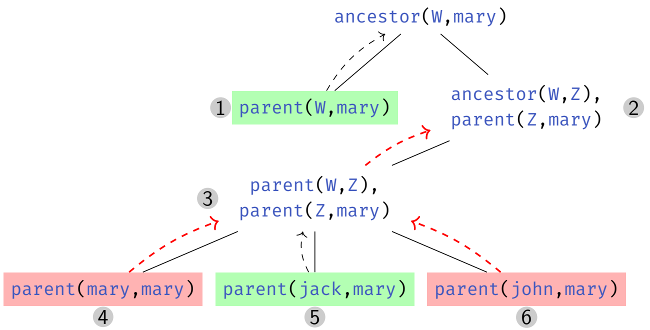

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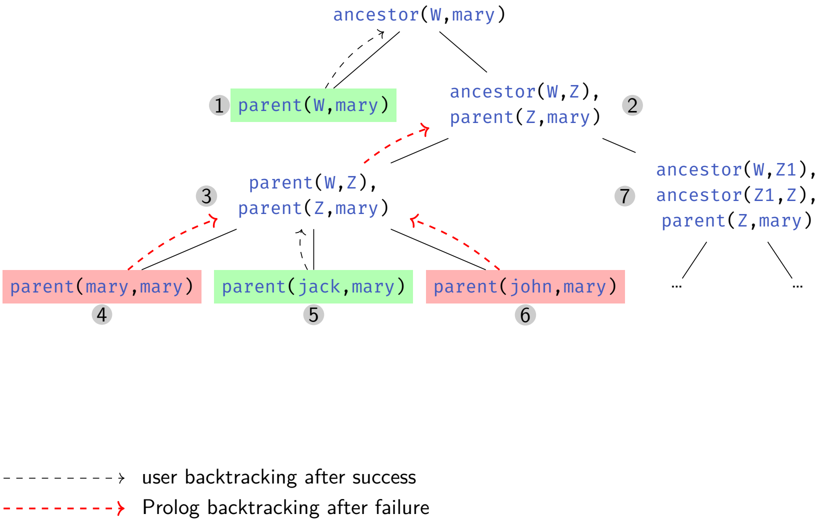

Atoms order is important!

Theorem

*The order of **atoms** in the clauses bodies determine the structure of the search tree.*

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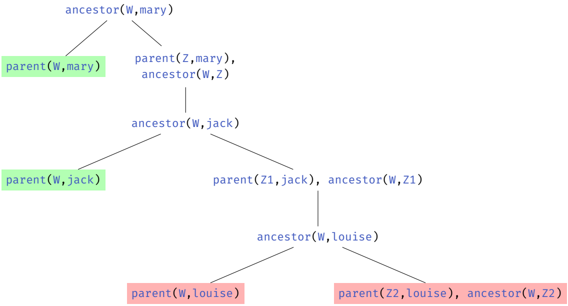
For instance, replace in the previous program the clause 5 by
`ancestor(X,Y) :- parent(Z,Y),ancestor(X,Z).`



Clauses order is important!

Theorem

*The order of the **clauses** in the program is important:*

- *the answers order can change*
- *the Prolog interpretor can loop*

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- *the answers order can change*
- *the Prolog interpreter can loop*

For instance, swap clauses 4 and 5...

```
ancestor(W,mary)
        |
ancestor(W,Z1), parent(Z1,mary)
        |
ancestor(W,Z2), parent(Z2,Z1), parent(Z1,mary)
        |
       ...
```

Outline

Using negation

Let us suppose that we want to express the following fact: "Christophe likes all programming languages except ugly languages like MATLAB or Visual Basic".

We can write

```
likes(christophe, X) :- prog_language(X).
```

but we must exclude ugly languages...

Negation as failure

The previous situation is so common that a predicate **`not`** has been defined.
The **`not`** operator is also written `\+`. It is called **negation as failure**.

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We could have rewritten the previous program as

```
likes(christophe, X) :- \+ ugly_language(X),
                        prog_language(X).
```

The problem with negation as failure

Consider the following program

```
smart(albert).
smart(edsger).
```

and ask the following question: `\+ smart(christophe).`

Does it mean that Christophe is not smart (maybe)?

The problem with negation as failure

Consider the following program

```
smart(albert).
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```

and ask the following question: `\+ smart(christophe).`

Does it mean that Christophe is not smart (maybe)?

The meaning of Prolog answer to the question is the following: given the program, there is not enough information to prove that Christophe is smart.

This is called the **closed world assumption** (CWA): everything that is true can be derived from the program.

Negation as failure: example

Let us consider the following program to choose the university you want to attend a MSc in:

```
good_standard(mit).
good_standard(berlin).

expensive(mit).

reasonable(X) :- \+ expensive(X).
```

Ask the following questions. What happens?

- `good_standard(X), reasonable(X).`
- `reasonable(X), good_standard(X).`

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Ask the following questions. What happens?

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- `reasonable(X), good_standard(X).`

Important

`\+ expensive(X)` in the request means "**for all X**, not `expensive(X)`".

Outline

Prolog and Resolution?

The formula associated to $A_1 :- B_1, \ldots, B_m$ with variables $X_1, \ldots, X_n$ is $\forall x_1 \ldots \forall x_n \; ((B_1 \land \ldots \land B_m) \rightarrow A_1)$, thus $\neg B_1 \lor \ldots \lor \neg B_m \lor A_1$.

NB: there is only one positive literal in the clause.

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The formula associated to the query $:- B_1, \ldots, B_m$ is $\forall x_1 \ldots \forall x_n \; (B_1 \wedge \ldots \wedge B_m) \rightarrow \bot$, thus $\neg B_1 \vee \ldots \vee \neg B_m$.

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Intuition: find a refutation with Resolution!

Definition (Horn clause)

A clause is **defined** if it contains one and only one positive literal. A clause is **negative** if it does not contain positive literal.
A **Horn clause** is either a defined clause, either a negative clause.

Link between Prolog and Resolution

For instance: the first successful branch.

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A successful branch **is just a refutation using Resolution** from the set of Horn clauses.

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A **constructive** proof is found (the variables are assigned).

Prolog uses the Selective Linear Definite clause Resolution: only one literal can be **selected** for Resolution application, namely the most recent one in the clause.