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Pages Sans nom FR-FR Exercices 1A En Optimisation

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TD4 ex. 1

$$f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$$
$$(w, b) \longmapsto \|w\|^2$$

- $\exists (w_0, b_0) \in C$.

$$(P) \iff \begin{cases} \text{Min } \|w\|^2 \\ \langle w, x\rangle + b \geq 1 \quad \forall x \in \mathcal{X}^1 \\ \langle w, x\rangle + b \leq -1 \quad \forall x \in \mathcal{X}^2 \\ \|w\| \leq \|w_0\| = M \qquad w \text{ borné} \\ (w, b) \in \mathbb{R}^{n+1} \end{cases}$$

Soit $x_i^1 \in \mathcal{X}^1$ $\quad b \geq 1 - \langle w, x_i^1\rangle \geq 1 - M\|x_i^1\|$

Soit $x_j^2 \in \mathcal{X}^2$ $\quad b \leq -1 - \langle w, x_j^2\rangle \leq -1 + \|M\| \, \|x_j^2\|$

$\Rightarrow$ b est borné.

$\Rightarrow$ C borné.

$C \neq \emptyset$, C fermé, borné (donc compact)
f est continue
$\Big\} \Rightarrow$ (P) admet 1 solution.

1.2) $$\mathcal{L}(w, b, \mu^1, \mu^2) = \|w\|^2 - \sum_{i=1}^{n_1} \mu_i^1 (\langle w, x_i^1\rangle + b - 1) + \sum_{i=1}^{n_2} \mu_i^2 (\langle w, x_i^2\rangle + b + 1)$$

$\mu^1 \in \mathbb{R}^{n_1}$, $\mu^2 \in \mathbb{R}^{n_2}$

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$$(P) \iff \begin{cases} \text{Min } \|w\|^2 \\ \langle w,x\rangle + b \geq 1 \quad \forall x \in \mathcal{X}^1 \\ \langle w,x\rangle + b \leq -1 \quad \forall x \in \mathcal{X}^2 \\ \|w\| \leq \|w_0\| = M \quad w \text{ borné} \\ (w,b) \in \mathbb{R}^{n+1} \end{cases}$$

Soit $x_i^1 \in \mathcal{X}^1$ $\quad b \geq 1 - \langle w, x_i^1\rangle \geq 1 - M\|x_i^1\|$

Soit $x_j^2 \in \mathcal{X}^2$ $\quad b \leq -1 - \langle w, x_j^2\rangle \leq -1 + \|M\|\,\|x_j^2\|$

$\implies$ b est borné.

$\implies$ C borné.

$$\left.\begin{array}{l} C \neq \emptyset, \ C \text{ fermé, borné (donc compact)} \\ f \text{ est continue} \end{array}\right\} \implies (P) \text{ admet 1 solution.}$$

1.2) $\mathcal{L}(w,b,\mu^1,\mu^2) = \|w\|^2 - \sum_{i=1}^{n_1} \mu_i^1(\langle w, x_i^1\rangle + b - 1) + \sum_{i=1}^{n_2} \mu_i^2(\langle w, x_i^2\rangle + b + 1)$

$\mu^1 \in \mathbb{R}^{n_1}$, $\mu^2 \in \mathbb{R}^{n_2}$

$\langle w,x\rangle + b \geq 1 \quad \forall x \in \mathcal{X}^1$ $\quad \begin{cases} \langle w, x_1^1\rangle + b - 1 \geq 0 \\ \vdots \\ \langle w, x_{n_1}^1\rangle + b - 1 \geq 0 \end{cases}$

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$$\begin{cases} \mu^1_i(\langle w, x^1_i\rangle + b - 1) = 0 & \forall i = 1,\dots,n_1 \\ \mu^2_i(\langle w, x^2_i\rangle + b + 1) = 0 & \forall i = 1,\dots,n_2 \end{cases}$$

les contraintes sont affines $\Longrightarrow$ HQC.

P est convexe

C est convexe.

f est convexe

les fct sont $C^1$

$\Longrightarrow$ KKT est une CNS

$\Longrightarrow$ $\mathcal{L}$ est convexe en $(w, b)$

$$\nabla_{(w,b)}\mathcal{L}(w, b, \mu_1, \mu_2) = 0$$

$$\Longleftrightarrow \min_{(w,b)} \mathcal{L}(w, b, \mu_1, \dots$$

$$\nabla_{(w,b)}\mathcal{L}(w, b, \mu_1, \mu_2) = \begin{pmatrix} 2w - \sum_{i=1}^{n_1} \mu^1_i x^1_i + \sum_{i=1}^{n_2} \mu^2_i x^2_i \\ -\sum_{i=1}^{n_1} \mu^1_i + \sum_{i=1}^{n_2} \mu^2_i \end{pmatrix} \begin{matrix} \} \; n \\ \; 1 \end{matrix} \Big\} (n+1)$$

1.4) $\mathcal{L}(w, b, \mu) = \|w\|^2 + \langle \mu, DXw + bDe + e\rangle$ convexe en $(w, b)$

$$\nabla_{(w,b)}\mathcal{L}(w, b, \mu) = \begin{pmatrix} 2w + (DX)^T\mu \\ \langle \mu, De\rangle \end{pmatrix} = 0 \Longleftrightarrow \min_{(w,b)} \mathcal{L}(w, b, \mu)$$

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$\mu, DXw>$

$(DX)^T\mu, -(DX)^T\mu/2>$

$(DX)^T\mu, \quad <(DX)^T\mu,(DX)^T\mu>$

$-\frac{1}{2}<(DX)^T\mu,(DX)^T\mu>$

1.4) $\mathcal{L}(w,b,\mu) = \|w\|^2 + <\mu, DXw + bDe + e>$ convexe en $(w,b)$

$\nabla_{(w,b)}\mathcal{L}(w,b,\mu) = \begin{pmatrix} 2w + (DX)^T\mu \\ <\mu, De> \end{pmatrix} = 0 \iff \underset{(w,b)}{\mathrm{Min}}\, \mathcal{L}(w,b,\mu)$

Si $<\mu, De> = 0$ alors $w = -(DX)^T\mu/2$

$\psi(\mu) = \underset{(w,b)}{\mathrm{Min}}\, \mathcal{L}(w,b,\mu) = -\frac{1}{4}\|(DX)^T\mu\|^2 + b\underbrace{\cancel{<\mu, De>}}_{=0} + <\mu, e>$

Si $<\mu, De> \neq 0$, $\mathcal{L}(w,b,\mu) = \|w\|^2 + <\mu, DXw> + <\mu, e> + b<\mu, De>$

Si $b = -\alpha<\mu, De>$

$\mathcal{L}(w,b,\mu) \to -\infty$

$\alpha \to +\infty$.

$\psi(\mu) = -\infty$

$(D) \begin{cases} \mathrm{Max}\ \psi(\mu) = -\frac{1}{4}\|(DX)^T\mu\|^2 + <\mu, e> \\ \mu \geq 0 \\ \boxed{(De)^T\mu = 0} \end{cases}$

$-\frac{1}{2}<(DX)^T\mu,(DX)^T\ldots$ $\quad <\mu, De>$ $\quad (\omega, b)$

Si $<\mu, De> = 0$ alors $\boxed{\omega = -(DX)^T\mu/2}$

$$\psi(\mu) = \min_{(\omega,b)} \mathcal{L}(\omega,b,\mu) = -\frac{1}{4}\|(DX)^T\mu\|^2 + b\underbrace{\cancel{<\mu, De>}}_{=0} + <\mu, e>$$

Si $<\mu, De> \neq 0$, $\mathcal{L}(\omega,b,\mu) = \|\omega\|^2 + <\mu, DX\omega> + <\mu, e> + b<\mu, De>$

Si $b = -\alpha<\mu, De>$

$\mathcal{L}(\omega,b,\mu) \longrightarrow -\infty$

$\alpha \to +\infty$

$\psi(\mu) = -\infty$

$$(D)\begin{cases} \operatorname{Max} \psi(\mu) = -\frac{1}{4}\|(DX)^T\mu\|^2 + <\mu, e> \\ \mu \geq 0 \\ \boxed{(De)^T\mu = 0} \end{cases}$$

$$\begin{cases} \operatorname{Min} f(x) \\ g(x) \leq 0 \end{cases}$$

$\mathcal{L}(x,\mu) = f(x) + <\mu, g(x)>$ $\qquad \mu \geq 0.$