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FR-FR Exercices 1A En Optimisation

Talking: Joseph Gergaud

$\Longrightarrow$ Si s^* une sol de (P^{rc}) alors

<u>Cas 1</u> si $\|s^*\| < \delta^{(k)}$ donc s^* est un minimum local de q.

$$\left.\begin{array}{l}\Rightarrow \nabla^2 q(s^*) \text{ est semi-déf.} > 0 \quad (CN2) \\ \nabla^2 q(s) = H \text{ est semi-déf} \geq 0 \;\; \forall s\end{array}\right\} \Longrightarrow q \text{ est convexe.}$$

$\Rightarrow s^*$ est un min. global de q.

lemme $\Longrightarrow$
$$\begin{cases} Hs^* = -g \\ \mu^*(\|s^*\| - \delta) = 0 \\ H + 2\mu^* I \text{ semi déf} \geq 0 \end{cases} \qquad \mu^* = 0.$$

<u>Cas 2</u> Si $\|s^*\| = \delta \Rightarrow$ on a l'HQC.

$c(s) = \|s\|^2 - \delta \qquad c(s) \leq 0 \qquad \nabla c(s) = 2s \neq \vec{0}$ si $\|s\| = \delta$.

(KKT) $\qquad \mathcal{L}(s,\mu) = g^T s + \frac{1}{2} s^T H s + \mu(\|s\|^2 - \delta)$

$$\begin{cases} \nabla_s \mathcal{L}(s,\mu) = \underline{g + Hs + 2\mu s} = 0. \\ \qquad = \underline{(H + 2\mu I)s + g} \quad 1. \\ \mu(\|s\|^2 - \delta) = 0 \quad 2. \\ \|s\|^2 \leq \delta \end{cases}$$

$$c(s) = \|s\|^2 - \delta \qquad c(s) \leq 0 \qquad \nabla c(s) = 2s \neq \vec{0}$$

$s : \|s\| = \delta.$

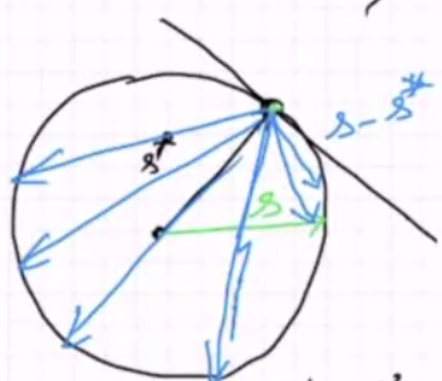
(KKT) $\quad \mathcal{L}(s,\mu) = g^T s + \frac{1}{2} s^T H s + \mu(\|s\|^2 - \delta)$

$$\begin{cases} \nabla_s \mathcal{L}(s,\mu) = \underline{g + Hs + 2\mu s} = 0. \quad = \underline{(H + 2\mu I)s + g} \quad 1. \\ \mu(\|s\|^2 - \delta) = 0 \quad 2. \\ \|s\|^2 \leq \delta \end{cases}$$

CN2. $\quad \forall d \in \operatorname{Ker} c'(s) \qquad d^T \underline{\nabla_{ss} \mathcal{L}(s,\mu)} d \geq 0$

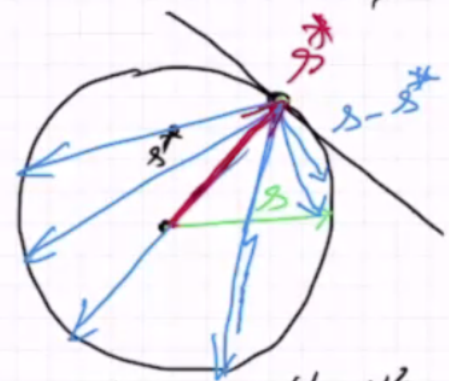
$\nabla c(s) = 2s.$

$$\left.\begin{array}{l} \forall d \ \langle d, s \rangle = 0 \\ \quad d^T (H + 2\mu I) d \geq 0. \end{array}\right\}$$



Montrons que $\forall s \quad \|s\|^2 = \delta$ ...

$$\left.\begin{array}{l}\forall d \quad \langle d, s\rangle = 0 \\ d^T(H+2\mu I)\,d \geq 0.\end{array}\right\}$$



Montrons que $\forall s \quad \|s\|^2 = \delta$ on a $\Big\{$ (1)

- $(s-s^*)^T(H+2\mu I)(s-s^*) \geq 0$

$\to \{s - s^*, \ \|s\|^2 = \delta\}$ prend toutes les directions possible dans le demi-hyperplan ouvert d'equation D
$\langle d, s^*\rangle < 0.$

Alors si d, $\langle d, s\rangle \neq 0$ alors $d \in D$ ou $-d \in D$.

$\exists \alpha > 0, \quad d = \alpha(s - s^*)$

$$d^T(H+2\mu I)\,d = \alpha^2 (s-s^*)^T(H+2\mu I)(s-s^*) \geq 0.$$

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Montrons (1).

s^* est sol de (P^{rc}) donc $q(s^*) \leq q(s)$ $\forall s$ $\|s\|^2 = \delta$

$$g^T s^* + \frac{1}{2} s^{*T} H s^* \leq g^T s + \frac{1}{2} s^T H s \quad \forall s \; \|s\|^2 = \delta = \|s^*\|^2$$

$$\Longrightarrow \quad g^T s^* + \frac{1}{2} s^{*T} H s^* + \mu(\|s^*\|^2 - \delta) \leq g^T s + \frac{1}{2} s^T H s + \mu(\|s\|^2 - \delta)$$

$$(H + 2\mu I) s^* = -g$$

$$-s^{*T}(H + 2\mu I)^T s^* + \frac{1}{2} s^{*T} H s^* + \mu(\|s^*\|^2 - \delta)$$

$$\leq -s^{*T}(H + 2\mu I)^T s + \frac{1}{2} s^T H s + \mu(\|s\|^2 - \delta)$$

$$\Longrightarrow \quad \frac{1}{2} s^{*T} H s^* + \mu \|s^*\|^2 - s^{*T} H^T s - 2\mu s^{*T} s + \frac{1}{2} s^T H s + \mu \|s\|^2 \geq$$

$$\frac{1}{2}(s - s^*)^T (H + 2\mu I)(s - s^*)$$

$$= \frac{1}{2} s^T H s - s^T H s^* + \frac{1}{2} s^{*T} H s^* + \mu \|s\|^2 + \mu \|s^*\|^2 - 2\mu s^{*T} s$$

(s^*, μ^*) vérifie 1., 2. et 3

$= \frac{1}{2} s^T H s - s^T H s^* + \frac{1}{2} s^{*T} H s^* + \mu \|s\|^2 + \mu \|s^*\|^2 - 2\mu s^{*T} s$

≥ 0

$\Leftarrow$ (s^*, μ^*) vérifie 1., 2. et 3

Si $\mu^* = 0$. $\begin{cases} \text{alors } H \text{ est semi déf.} \geq 0 \\ H s^* = -g \end{cases}$

lemme $\Rightarrow s^*$ est un min. global de q.

Sinon $\mu^* \neq 0$ 1.2 $\Rightarrow$ $\|s^*\|^2 = \delta$ $\Rightarrow$ HQC.

$\mathcal{L}(s,\mu) = q(s) + \mu(\|s\|^2 - \delta)$

$\nabla_s \mathcal{L}(s,\mu) = (H + 2\mu I) s + g$

$\nabla_{ss} \mathcal{L}(s,\mu) = H + 2\mu I$ semi. déf. ≥ 0 (1.3)

donc $\mathcal{L}$ est convexe en s.

Donc $\nabla_s \mathcal{L}(s,\mu) = 0 \iff \underset{s}{\mathrm{Min}}\, \mathcal{L}(s,\mu)$.

$\begin{cases} \underset{s}{\mathrm{Min}}\, \mathcal{L}(s,\mu) \\ \mu^*(\|s^*\|^2 - \delta) = 0 \end{cases}$ CS. pt col.

Ex. 3

$$f(s) = \frac{1}{2} < r(\beta^{(k)}), r(\beta^{(k)}) > \quad c$$

$$+ < r(\beta^{(k)}), J(\beta^{(k)}).s >$$

$$+ \frac{1}{2} < J(\beta^{(k)}).s, J(\beta^{(k)})s >$$

$$= q(s) + c^{te} \qquad = c^{te} + g^T s + \frac{1}{2} s^T H s.$$

$$g = J(\beta^{(k)})^T r(\beta^{(k)})$$

$$H = \underbrace{J(\beta^{(k)})^T J(\beta^{(k)})}_{} = X^T X$$

$$\begin{cases} (H + 2\mu^* I) s^* = -g & \mu^* \geq 0 \\ \mu^* (\|s^*\|^2 - \delta) = 0 \\ H + 2\mu^* I \text{ semi déf. } \geq 0 & (*) \end{cases}$$

ici H est semi-déf. ≥ 0.

$$s^T(X^T X)s = (s^T X^T)(Xs) = (Xs)^T Xs = <Xs, Xs> \geq 0.$$

Soit $\mu^* = 0$. et alors

$$Hs^* = -g \quad \text{donc} \quad \underline{s^* = -H^{-1} g} \qquad rg(\overbrace{J(\beta^{(k)})}^{X}) = p.$$

$$X^T X x = 0 \Rightarrow x^T (X^T X) x = 0 \Rightarrow <Xx, Xx> = 0 \Rightarrow Xx = 0 \Big/ \Rightarrow x = 0$$

$rg\, X = p$