Assignment - 03

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Course code : MAT120

Course Title : Integral Calculus & Differential Equations

Section : 17

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Answer to the Q. NO - 01

Given,

$\iint_R (x+y)\,dA$ where the region R is defined as $\{(x,y) \mid 1 \le x^2+y^2 \le 4,\ x \le 0\}$

We know,

$x = r\cos\theta$

$y = r\sin\theta$

Here,

$1 \le x^2+y^2 \le 4$

$\Rightarrow 1 \le r^2 \le 4$

$\Rightarrow 1 \le r \le 2$

Now,

$$\iint_R (x+y)\,dA$$

Here, $dA = r\,dr\,.\,d\theta$

Now Again,

$$\int_{\pi}^{2\pi}\int_{1}^{2} (r\cos\theta + r\sin\theta)\, r\,.\,dr\,.\,d\theta$$

$$= \int_{\pi}^{2\pi}\int_{1}^{2} \{r^2(\cos\theta) + r^2\sin\theta\}\,.\,dr\,.\,d\theta$$

$$= \int_{\pi}^{2\pi} \left[\frac{r^3}{3}\cos\theta + \frac{r^3}{3}\sin\theta\right]_1^2 .\,d\theta$$

$$= \int_{\pi}^{2\pi} \left[\frac{8}{3}\cos\theta + \frac{8}{3}\sin\theta - \frac{1}{3}\cos\theta - \frac{1}{3}\sin\theta\right].\,d\theta$$

(P.T.O.)

$$\int_{\pi}^{2\pi} \frac{7}{3}\left(\cos\theta + \sin\theta\right). d\theta$$

$$= \frac{7}{3}\left[\sin\theta - \cos\theta\right]_{\pi}^{2\pi}$$

$$= 7/3 \left[\sin(2\pi) - \cos(2\pi) - \sin(\pi) + \cos(\pi)\right]$$

$$= \frac{7}{3} \times (-2)$$

$$= -\frac{14}{3}$$

(Ans:)

Answer to the Q. NO- 02 (a)

Given,

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$$

$$=\int_0^{2\pi}\int_0^{3}\int_0^{9-r^2} z\sqrt{r^2+z^2}\; r\cdot dz\cdot dr\cdot d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\int_0^{3}\int_0^{\sqrt{9-r^2}} 2\cdot z\cdot r\sqrt{r^2+z^2}\,dz\,dr\,d\theta$$

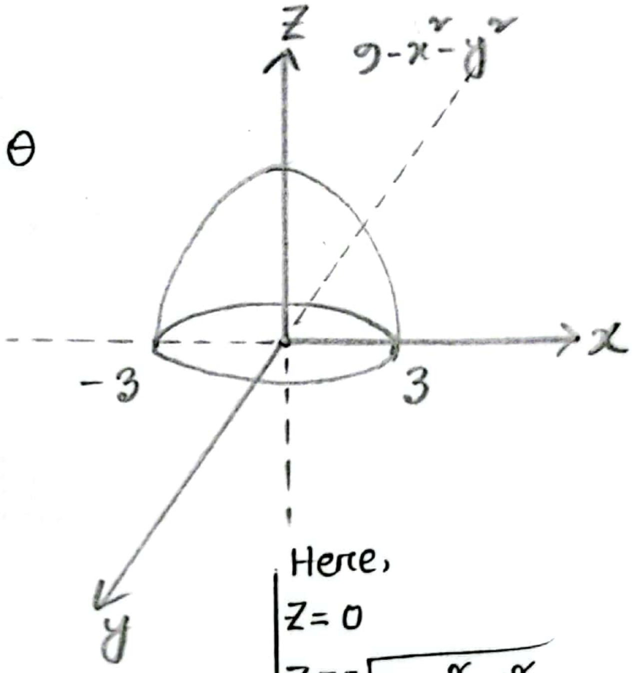
$$=\frac{1}{2}\int_0^{2\pi}\int_0^{3}\int_9^{r^2} r\sqrt{u}\cdot du\cdot dr\cdot d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\int_0^{3}\left[r\cdot\frac{2}{3}u^{3/2}\right]_9^{r^2} dr\cdot d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\int_0^{3}\left[\frac{2r^{5/2}}{3}-18r\right]dr\cdot d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\left[\frac{2\times 2}{7\times 3}r^{7/2}-\frac{18r^2}{2}\right]_0^3 d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\left[\frac{4}{21}(3)^{7/2}-\frac{18\times(3)^2}{2}\right].d\theta$$



Here,

$z=0$

$z=\sqrt{9-x^2-y^2}$

$=\sqrt{9-(x^2+y^2)}$

$=\sqrt{9-r^2}$

let,

$u=r^2+z^2$

$\Rightarrow du=2z\cdot dz$

when,

$z=0\ ,\ u=r^2$

$z=\sqrt{9-r^2}$

$u=(9-r^2+r^2)$

$u=9$

(P.T.O.)

$$\frac{1}{2}\int_0^{2\pi}\left[\frac{4}{21}\times 3^{7/2} - 81\right]\cdot d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}\left[\frac{4}{21}\times 3^{7/2} - 81\right]\cdot d\theta$$

$$= \frac{1}{2}\left[\frac{4}{21}\times 3^{(7/2)} - 81\right]\cdot \left[\theta\right]_0^{2\pi}$$

$$= \left(\frac{1}{2}\times\frac{4}{21}\times 3^{(7/2)} - \frac{1}{2}\times 81\right)\times 2\pi$$

$$= \left(\frac{4}{21}\cdot 3^{7/2} - 81\right)\pi$$

(Ans:)

Answer to the Q. NO - 02(b)

Given,

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z.\sqrt{x^2+y^2+z^2}\,dz.\,dy.\,dx$$

$$=\int_0^{2\pi}\int_0^{\pi/2}\int_0^{3} \rho\cos\varphi\sqrt{\rho^2}.\ \rho^2\sin\varphi.\,d\rho.\,d\varphi.\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\pi/2}\int_0^{3} \rho^4\cos\varphi.\sin\varphi\,d\rho.\,d\varphi.\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\pi/2}\left[\frac{\rho^5}{5}\cos\varphi.\sin\varphi\right]_0^3 d\varphi.\,d\theta$$

$$=\frac{243}{5}\int_0^{2\pi}\int_0^{1} u.\,du.\,d\theta$$

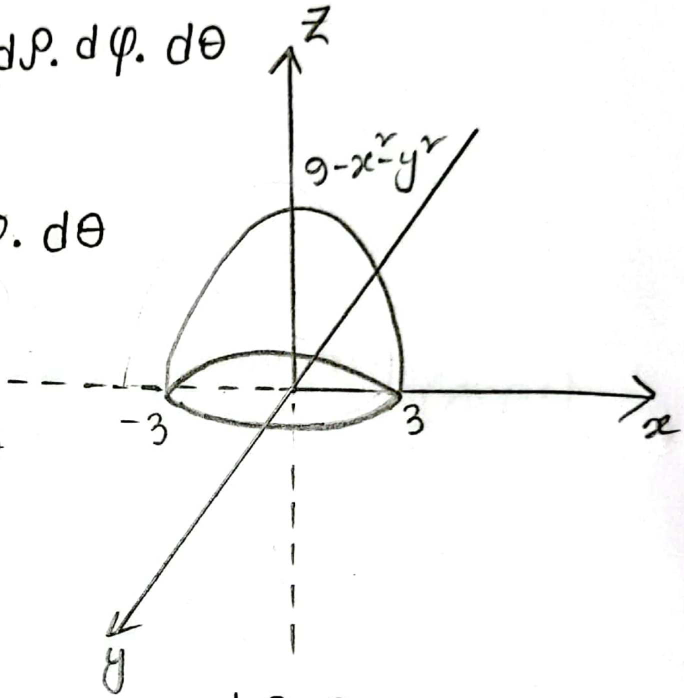
$$=\frac{243}{5}\int_0^{2\pi}\left[\frac{u^2}{2}\right]_0^1.\,d\theta$$

$$=\frac{243}{5}\int_0^{2\pi}\frac{1}{2}.\,d\theta$$

$$=\frac{243}{5}\times\frac{1}{2}\left[\theta\right]_0^{2\pi}$$

$$=\frac{243}{10}\times 2\pi$$

$$=\frac{243\pi}{5}\ \text{(Ans:)}$$



$z=0$

$z=\sqrt{9-x^2-y^2}$

$\Rightarrow x^2+y^2+z^2=9$

$\therefore\ \rho=3$

let, $u=\sin\varphi$

$\Rightarrow du=\cos\varphi.\,d\varphi$

when,

$\varphi=0,\ u=0$

$\varphi=\pi/2,\ u=1$

(P.T.O.)

Answer to the Q. NO - 03

given,

$$\cos x\, dx + \left(1 + \frac{2}{y}\right) \sin x \cdot dy = 0$$

We can see that, it is in the format :-

$$M(x,y)\, dx + N(x,y)\, dy = 0$$

Here,

$$M(x,y) = \cos x$$

$$N(x,y) = \left(1 + \frac{2}{y}\right) \sin x$$

We know,

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \rightarrow \boxed{\text{Non-exact equation.}}$$ (Ans:)

Now,

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y}}{M} = \frac{\left(1+\frac{2}{y}\right)\cos x - 0}{\cos x}$$

$$= 1 + \frac{2}{y}$$

$$\therefore e^{\int \left(1+\frac{2}{y}\right)} = e^{y + 2\ln y}$$

$$= y^2 e^y$$

Now, multiplying the given ODE with $e^{\int\left(1+\frac{2}{y}\right)} = y^2 e^y$; $\Rightarrow$

$$y^2 e^y (\cos x)\, dx + y^2 e^y \left(1 + \frac{2}{y}\right) \sin x\, dy = 0$$

$$\Rightarrow y^2 e^y \cos x\, dx + y^2 e^y \sin x\, dy + 2y e^y \sin x \cdot dy = 0$$

$$\Rightarrow y^2 e^y (\sin x)\, dx \cdot y^2 \sin x \; (y^y)\, dx + e^y \sin x\, (y^2)\, dx = 0$$

(P.T.O.)

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$(y^2 e^y \sin x)\, dx = 0$

Now,

taking both sides $\Rightarrow$

$y^2 e^y \sin x + c = 0$

(Ans:)

Answer to the Q. NO-04

At $t = 0$, $P = 500$

At $t = 10$, $P = 575$ [$\because$ 15% of 500 increased in 10 years]

According to the given question,

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = K \cdot P$$ [Here, K = proportionality constant]

$$\Rightarrow \int \frac{dP}{P} = \int K \cdot dt$$

$$\Rightarrow \ln|P| = Kt + C \Rightarrow P = e^{Kt} \cdot e^{C} \Rightarrow P = C_1 \cdot e^{Kt}$$

Now, $\ln(575) = K(10) + \ln(500)$

$$\Rightarrow K = \frac{1}{10} \ln\left(\frac{575}{500}\right)$$

$$\Rightarrow K = 0.0139762$$

(P.T.O.)

$\therefore P = 500e^{0.0139762t}$

$\therefore$ The population in 30 years will be,

$P = 500e^{0.0139762 * 30} = 760.437$

≈ 761 people

(Ans:)

And,

the rate of population growth will be,

$$\frac{dP}{dt} = \frac{d\left(500e^{0.0139762t}\right)}{dt}$$

$$= (500 \times 0.0139762)e^{0.0139762t}$$

$$= (500 \times 0.0139762)e^{0.0139762 \times 30} \quad [\because t = 30]$$

$$= (6.9881)e^{0.0139762 \times 30}$$

$$= 10.6280$$

≈ 11 people / year. (approximately)

$\therefore$ The population in 30 years will be 761 people. (Ans:)

$\therefore$ The rate of the population growth at $t = 30$ years will be $= 11$ people / year (approximately)

(Ans:) (P.T.O.)

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Answer to the Q. NO - 05

Given,

$$\frac{dr}{d\theta} + r\sec\theta = \cos\theta$$

The intigrating factors,

$$e^{\int \sec\theta . d\theta}$$

$$e^{\ln|\sec\theta + \tan\theta|}$$

Now,

$\sec\theta + \tan\theta$

$$(\sec\theta + \tan\theta)\frac{dr}{d\theta} + r\sec\theta(\sec\theta + \tan\theta) = \cos\theta(\sec\theta + \tan\theta)$$

$$\Rightarrow \frac{d}{d\theta}\{r(\sec\theta + \tan\theta)\} = (\sec\theta + \tan\theta).\cos\theta$$

$$\Rightarrow \int \frac{d}{d\theta}\{r(\sec\theta + \tan\theta)\} d\theta = \int\{(\sec\theta + \tan\theta).\cos\theta\}.d\theta$$

$$\Rightarrow \int \frac{d}{d\theta}\{r(\sec\theta + \tan\theta) d\theta = \int 1 + \sin\theta . d\theta$$

$$\Rightarrow r(\sec\theta + \tan\theta) = \theta + \cos\theta + C$$

$$\therefore r = \frac{\theta}{\sec\theta + \tan\theta} + \frac{\cos\theta}{\sec\theta + \tan\theta} + \frac{C}{\sec\theta + \tan\theta}$$

(Ans:)