Assignment - 03

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CSE331
Automata and Computability

ASSIGNMENT 3
TOTAL MARKS: 30
DEADLINE: 09 JANUARY, 2025

09

There are a total of three problems. You have to solve all of them!

Problem 1 (CO3): Context-Free Grammars (10 points)

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Consider the following languages over Σ. Recall that for a string w, |w| denotes the length of w.

$$L_1 = \{w \text{ is an even length palindrome}\}$$

$$L_2 = \{\text{Length of } w \text{ is even}\}$$

$$L_3 = \{x11y \mid x, y \in L_2, \mid x \mid = \mid y \mid\}$$

$$L_4 = L_1 \cap L_3$$

(a) **Give** a context free grammar for L_1. (3 points)

(b) **Give** a context free grammar for L_3. (4 points)

(c) **Give** a context free grammar for L_4. (3 points)

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \to \mathtt{1}A \mid \mathtt{1}C \mid \mathtt{0}B \mid \mathtt{00}A$$
$$B \to \mathtt{0}A \mid \mathtt{1}B \mid \mathtt{00}B$$
$$C \to \mathtt{0}C\mathtt{0} \mid \mathtt{0}C\mathtt{1} \mid \mathtt{1}C\mathtt{0} \mid \mathtt{1}C\mathtt{1} \mid \varepsilon$$

(a) **Give** a leftmost derivation for the string $\mathtt{01011001}$. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)

(d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (2 point)

Problem 3 (CO4): Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages.

$$L_1 = \{w \mid w \text{ starts and ends with the same character}\}$$

$$L_2 = \{w \mid \text{the number of 0s in w is not the same as the number of 1s}\}$$

(a) **Give** the state diagram of a pushdown automaton that recognizes L_1. (4 points)

(b) **Give** the state diagram of a pushdown automaton that recognizes L_2. (6 points)

Context - Free Grammars :

Answer to the Q. NO - 01 (a)

Given,

$L_1 = \{ \omega \text{ is an even length palindrome} \}$

$L_2 = \{ \text{Length of } \omega \text{ is even} \}$

$L_3 = \{ x\,11\,y \mid x, y \in L_2,\ |x| = |y| \}$

$L_4 = L_1 \cap L_3$

for L_1,

an even length palindrome :-

for $\omega = \epsilon : \epsilon \in L_1$; generated by $S \to \epsilon$

for $\omega = aa : S \to aSa \to aa$

for $\omega = abba : S \to aSa \to abSba \to abba$

∴ context - free grammar for L_1 ;

$$S \to aSa \mid bSb \mid \epsilon$$

here,
ϵ represents empty string, which ensures base case.

(Ans:)

[P.T.O.]

Answer to the Q. NO – 01 (b)

Given,

$$L_3 = \{ x\,11\,y \mid x, y \in L_2, |x| = |y| \}$$

$$L_2 = \{ \text{Length of } \omega \text{ is even} \}$$

Let us define,

S_3 for L_3

and S_2 for L_2

context free grammar formation :-

for $S_3 \rightarrow S_2\,11\,S_2$

for $S_2 \rightarrow a\,S_2\,a \mid b\,S_2\,b \mid \epsilon$

(Ans:)

here,
$|x\,11\,y|$
This '11' portion seperates x and y.

$|x| = |y|$
The length of x and y must match the same even length

Which verifies :-

$S_2 \rightarrow aa$

$S_2 \rightarrow abba$, etc.

$S_3 \rightarrow S_2\,11\,S_2$ [This ensures x, y are generated with equal lengths]

for $x = \epsilon$,

$y = \epsilon$,

$S_3 \rightarrow \epsilon\,11\,\epsilon = 11$

for $x = aa$,

$y = bb$

$S_3 \Longrightarrow S_2\,11\,S_2 \rightarrow aa\,11\,bb$ (Ans:)

[P.T.O.]

Answer to the Q. NO- 01(c)

Given,

$L_1 = \{ \omega \text{ is even length palindrome} \}$

$L_3 = \{ x11y \mid x, y \in L_2, |x| = |y| \}$

$L_4 = L_1 \cap L_3$

Context free Grammar for L_4 :-

$S \rightarrow aSa \mid bSb$ [Generates even length palindrome]

$M \rightarrow aM \mid bM \mid \epsilon$ [Generates even length strings]

$S' \rightarrow M11M$ [Generates strings of the form $x11y$; that ensures x and y are even length]

∴ The context free Grammar for L_4 :

$S \rightarrow aSa \mid bSb \mid M11M$

$S' \rightarrow aM \mid bM \mid \epsilon$

(Ans:)

[P.T.O.]

Answer to the Q. NO - 02 (a)

Derivations, Parse Trees and Ambiguity:

Given,

$$A \rightarrow 1A \mid 1C \mid 0B \mid 00A$$
$$B \rightarrow 0A \mid 1B \mid 00B$$
$$C \rightarrow 0C0 \mid 0C1 \mid 1C1 \mid \epsilon$$

String → 01011001.

Now,

leftmost derivation :-

$$\begin{aligned} A &\\ &\Rightarrow 0B \\ &\Rightarrow 00A \\ &\Rightarrow 001C \\ &\Rightarrow 0011C0 \\ &\Rightarrow 00110 01 \end{aligned}$$

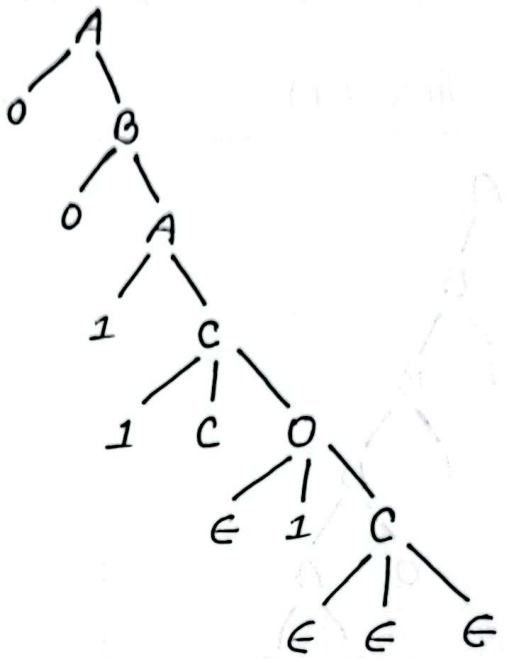
(Ans:)

[P.T.O.]

Answer to the Q. NO - 02(b)

for the leftmost derivation A got in answer of 02(a),

The Parse Tree is drawn below :-



(Ans:)

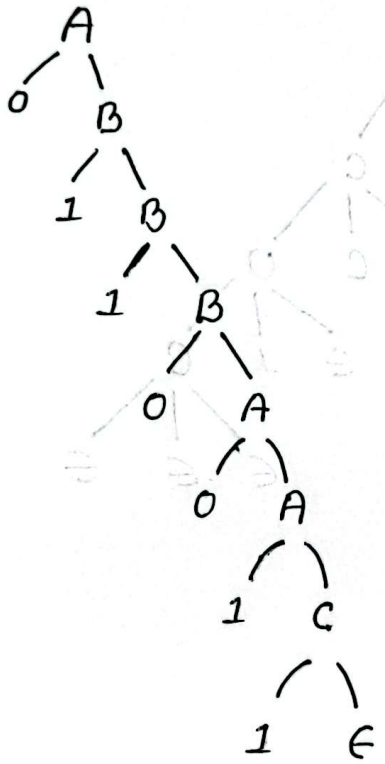
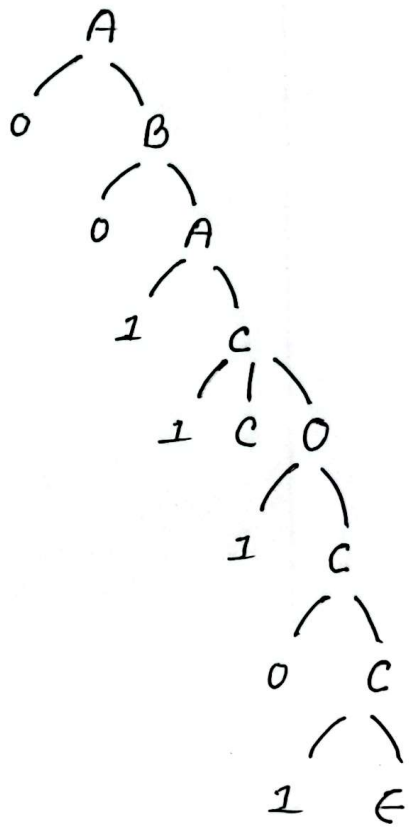
[p.T.O.]

Answer to the Q. NO - 02(c)

Ambiguity demonstration:

except 0101101, [found in 2(b)]

Two more parse trees are drawen below:

Parse Tree (I)	Parse Tree (II)
	

(Ans:)

[P.T.O.]

Answer to the Q. NO - 02 (d)

Given grammar,

$$A \rightarrow 1A \mid 1C \mid 0B \mid 00A$$
$$B \rightarrow 0A \mid 1B \mid 00B$$
$$C \rightarrow 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid \epsilon$$

To find,

a string of length six such that ω has exactly one parse tree in the given grammar:

Let,

The string of length six = 000000

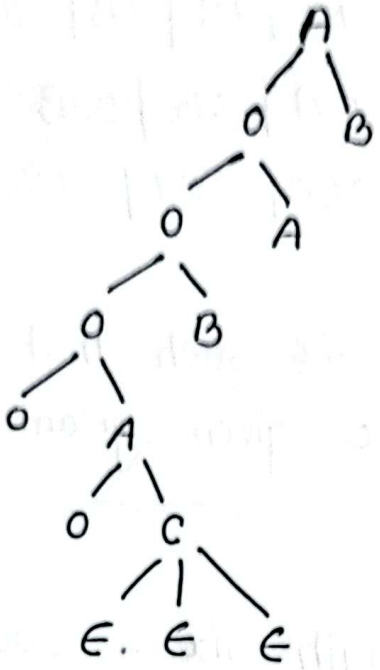
∴ leftmost derivation:

$$A$$
$$\Rightarrow 00A$$
$$\Rightarrow 0000A$$
$$\Rightarrow 00000A$$
$$\Rightarrow 000000 \quad [\text{using rule, } A \rightarrow \epsilon]$$

This string '000000' of six length has only one possible parse tree.

[The parse tree is drawn in next page.] [P.T.O.]

parse tree for string : 000 000



(Ans)

[P.T.O.]

Pushdown Automata :

Answer to the Q. NO - 03 (a)

Given,

let

$$\Sigma = \{0, 1\}$$

$L_1 = \{\omega \mid \omega$ starts and ends with same character$\}$

here,

symbol, pop ⟶ push

State diagram:-

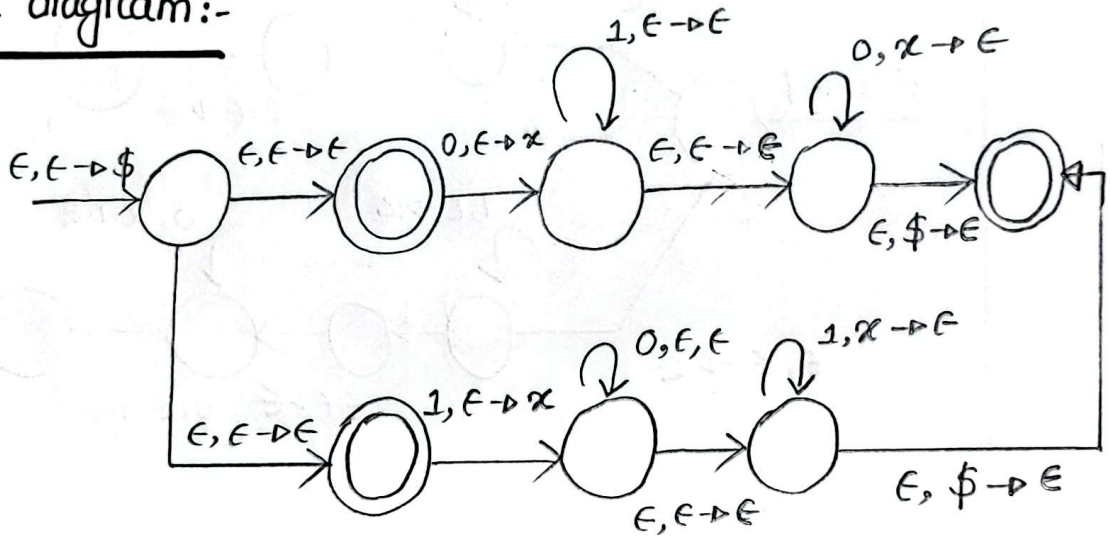


Figure: PDA State diagram for L1.

(Ans:)

[P.T.O.]

Answer to the Q. NO - 03(b)

Given,

Let, $\Sigma = \{0, 1\}$

$L_2 = \{\omega \mid$ the number of 0s in ω is not the same as the number of 1s$\}$

State diagram:

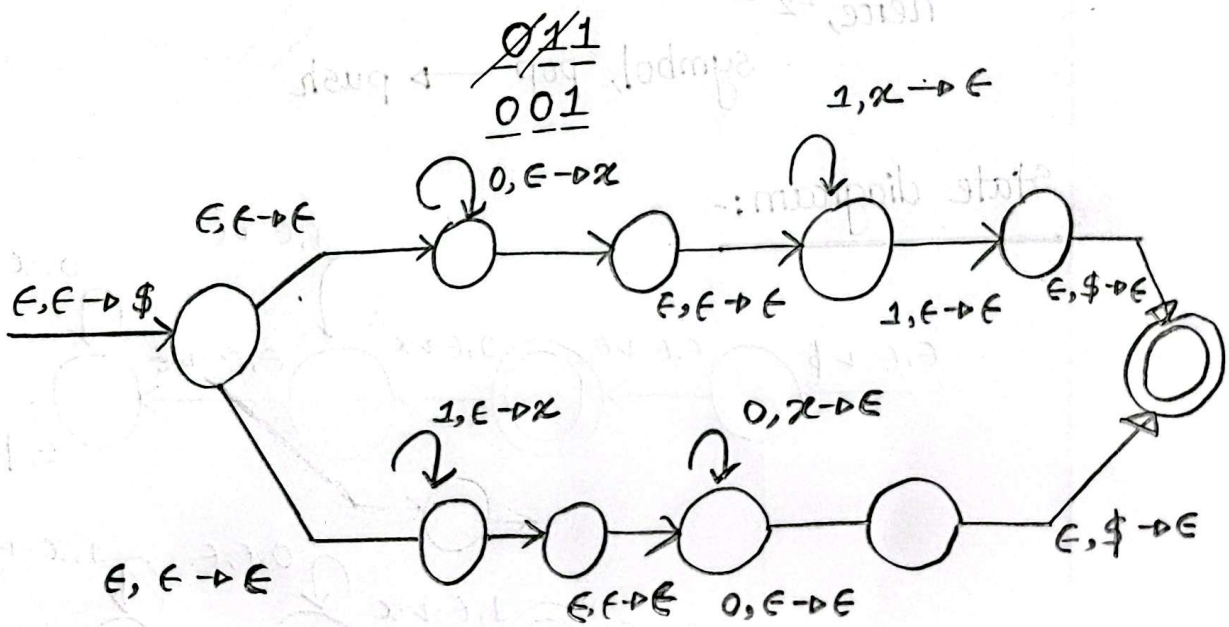


Figure: PDA State Diagram for L_2.

(Ans:)