Lecture-11

Null space: Let, A be a matrix. Then the nullspace of A is the solution space of :-

$$A\underline{x} = \underline{0}.$$

Nullity. The # of vectors in the basis of the null space of A is called nullity of A. It is denoted by nullity (A).

Ex. find ~~nullity of~~ nullspace & hence nullity of

$$A = \begin{pmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{pmatrix}$$

Sol^n Let, $A\underline{x} = 0$.

$$\Rightarrow \begin{pmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Augmented matrix $\Rightarrow (A \mid 0)$

$\sim$ [perform appropriate row operations to transform it into row echelon form]

$$\sim \left(\begin{array}{cccccc|c} 1 & 0 & -4 & -28 & -37 & 13 & 0 \\ 0 & 1 & -2 & -12 & -16 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

This is in row echelon form. The corresponding system $\Rightarrow$

$$x_1 - 4x_3 - 28x_4 - 37x_5 + 13x_6 = 0$$
$$x_2 - 2x_3 - 12x_4 - 16x_5 + 5x_6 = 0$$

There are 2 eq^{n}s in 6 unknowns

$\therefore (6-2) = 4$ free variables

Here, x_3, x_4, x_5, x_6 are free variables

Let, $x_3 = r$, $x_4 = s$, $x_5 = t$, $x_6 = u$.

$\therefore x_2 = 2r + 12s + 16t - 5u$

$x_1 = 4r + 28s + 37t - 13u$

So the null space ~~(solution~~ is the solution space of this homogeneous system.

which is -

$$\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 4r + 28s + 37t - 13u \\ 2r + 12s + 16t - 5u \\ r \\ s \\ t \\ u \end{pmatrix} ; r, s, t, u \in \mathbb{R} \right\}$$

(Ans)

Now, we find the basis of this nullspace.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 4r \\ 2r \\ r \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 28s \\ 12s \\ 0 \\ s \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 37t \\ 16t \\ 0 \\ 0 \\ t \\ 0 \end{pmatrix} + \begin{pmatrix} -13u \\ -5u \\ 0 \\ 0 \\ 0 \\ u \end{pmatrix}$$

$$= r\begin{pmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 28 \\ 12 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 37 \\ 16 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + u\begin{pmatrix} -13 \\ -5 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Basis $= \{ (4,2,1,0,0,0), (28,12,0,1,0,0), (37,16,0,0,1,0), (-13,-5,0,0,0,1) \}$

So, nullity $= 4$.

Imp * $\underline{\text{Th}^m}$: If A is a matrix with n columns, then:

$$\text{rank} + \text{nullity}(A) = n.$$

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