Assignment - 01

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communer to the g. No. - 01

Given Linear system,

$$2x_{1} - x_{2} + 3x_{3} + 4x_{4} = 9$$

$$x_{4} - 2x_{3} + 7x_{4} = 11$$

$$3x_{1} - 3x_{2} + x_{3} + 5x_{4} = 8$$

$$2x_{1} + x_{2} + 4x_{3} + 4x_{4} = 10$$

The corrresponding augmented matrix =>

$$\begin{pmatrix}
2 & -1 & 3 & 4 & 9 \\
1 & 0 & -2 & 7 & 11 \\
3 & -3 & 1 & 5 & 8 \\
2 & 1 & 4 & 4 & 10
\end{pmatrix}$$

$$R_2' = 2R_2 - R_1$$

 $R_3' = 2R_3 - 3R_1$
 $R_4' = R_4 - R_1$

$$R_3' = R_3 + 3R_2$$

$$R_4' = R_4 - 2R_2$$

P. T. O.



$$\sim \begin{pmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & 2 & \frac{9}{2} \\ 0 & 1 & -7 & 10 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}; R_{1}' = \frac{1}{2} R_{1}$$

$$R_{2}' = \frac{1}{28} R_{3}$$

$$R_{4}' = -\frac{1}{140} R_{4}$$

Herre, this is in the row echelon form.

Now,
The corresponding system
$$\Rightarrow$$

$$\chi_{1} - \frac{1}{2} \chi_{2} + \frac{3}{2} \chi_{3} + 2\chi_{4} = \frac{9}{2} - 0$$

$$\chi_{2} - 7\chi_{3} + 10\chi_{4} = 13 - 0$$

$$\chi_{3} - \chi_{4} = -1 - 0$$

$$\chi_{4} = 2 - 0$$

$$\chi_{4} = 2 - 0$$

from (1)
$$\Rightarrow \chi_2 = 13 + 7 \cdot (1) - 10 \cdot (2) = 0$$

from (1) $\Rightarrow \chi_1 = \frac{9}{2} + \frac{1}{2}(0) - \frac{3}{2}(1) - 2(2)$
 $= -1$

$$\therefore \left(\chi_1, \chi_2, \chi_3, \chi_4 \right) = \left(-1, 0, 1, 2 \right)$$

communer to the g. NO - 02

$$-2q + 3n = 1$$

$$3p + 6q - 3n = -2$$

$$6p + 6q + 3p = 5$$

using Gauss-Jordan elimination method:

The augmented matrix is =>

$$\begin{pmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 3 & 6 & -3 & | & -2 \\ 0 & -2 & 3 & | & 1 \\ 0 & 0 & 0 & | & 6 \end{pmatrix}; \quad R_3' = R_3 - 3R_2$$

$$\begin{bmatrix}
0 & 0 & 0 & | & 6 \\
0 & 0 & 0 & | & 6
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 2 & | & \frac{1}{3} \\
0 & 1 & -\frac{3}{2} & | & -\frac{1}{2} \\
0 & 0 & 0 & | & 6
\end{bmatrix}$$

$$\begin{bmatrix}
R_1' = \frac{1}{3} R_1 \\
R_2' = -\frac{1}{2} R_2$$

P.T.0.



Now,

$$p + 2p = \frac{1}{3}$$

$$9 - \frac{3}{2}p = -\frac{1}{2}$$

$$0.p + 0.q + 0.p = 6$$
 Got from the last

$$\Rightarrow b \circ d \Rightarrow 0 = 6 \text{ partially appears}$$

- : the given system has no solution.
- : the system is in combitent.

com. to the g. NO-3(a)(1)

Given system,

$$x+y+\lambda z = 1$$

$$x+\lambda y+z=\lambda^2$$

$$\lambda x+y+z=\lambda^2$$

 $x + \lambda y + z = \lambda$ $\lambda x + y + z = \lambda^{2}$ The corresponding augmented system =>

$$\left(\begin{array}{ccc|c}
1 & 1 & \lambda & 1 \\
1 & \lambda & 1 & \lambda \\
\lambda & 1 & 1 & \lambda^2
\end{array}\right)$$

$$\sim \begin{pmatrix} 1 & 1 & \gamma & 1 \\ 0 & \lambda - 1 & 1 - \lambda & \lambda - 1 \\ 0 & 0 & -\lambda^2 - \lambda + 2 & \gamma^2 - 1 \end{pmatrix}; R_3 = R_3 + R_2$$

P. T. O.

O unique solution :-

Herre, if $n \neq 1$ and $\lambda \neq -2$

then it can be defined as eqn - 1 as:-

$$\begin{pmatrix} 1 & 1 & \lambda & 1 & 1 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ \hline & & & & & & \\ & & & & & \\ \end{pmatrix}, \quad R_{2}' = \frac{1}{\lambda - 1} \cdot R_{2}$$

$$R_{3}' = \frac{1}{-(\lambda + 2)(\lambda - 1)} \cdot R_{3}$$

as it is in row echelon forem and all three columns has a leading 1.

: the given system has a unique solution.

O No Solution :
Here, if $\lambda = -2$,

-then egn ---- (1) ⇒

$$\begin{pmatrix}
1 & 1 & -2 & | & 1 \\
0 & -3 & 3 & | & -3 \\
0 & 0 & 0 & | & 3
\end{pmatrix}$$

from the last row, 0.2 + 0.9 + 0.7 = 3 $\Rightarrow 0 = 3$ This is not possible.

: It means the system has no solution. and the system is incomistent.

when
$$\lambda = 1$$
,

the eqn ---- $0 \Rightarrow$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

when $\lambda = 1$;

all the values of 2nd and 3nd now becomes zero and there will be infinite amount of many solutions.

Amwers: ① Unique solution \Rightarrow when $\lambda \neq 1$ & $\lambda \neq -2$ for all the values

- ① No solution \Rightarrow when $\beta = -2$
- (ii) Many solution \Rightarrow infinitely many solution when $\lambda = 1$.

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Amwere to the g. NO-03(b)

Given Lineare System,

$$x+2y-3z=0$$

$$3x-y+2z=\beta$$

The corrresponding augmented matrix >

$$\begin{pmatrix}
1 & 2 & -3 & \alpha \\
3 & -1 & 2 & \beta \\
2 & -10 & 16 & 28
\end{pmatrix}$$

$$\begin{pmatrix}
1 & 2 & -3 & \alpha \\
0 & -7 & 11 & \beta - 3\alpha \\
0 & -14 & 22 & 2\gamma - 2\alpha
\end{pmatrix}; R_{2}' = R_{2} - 3R_{1}$$

Herce, the given system will be comsistent if >>

$$4\alpha + 2\gamma - 2\beta = 0$$

$$\Rightarrow 2\left(2\alpha + \gamma - \beta\right) = 0$$

$$\Rightarrow 2\alpha + \gamma - \beta = 0$$

(Ams)

communer to the g. NO - 04(a)

Given system,

$$x_{1} + x_{2} + x_{3} = 1$$
 $2x_{1} + 2x_{2} + 2x_{3} = 1$
 $3x_{1} + 3x_{2} + 3x_{3} = 2$

The augmented matrix >

$$\left(\begin{array}{ccc|cccc}
1 & 1 & 1 & 1 \\
2 & 2 & 2 & 1 \\
3 & 3 & 3 & 2
\end{array}\right)$$

Now, $x_1 + x_2 + x_3 = 1$

$$\Rightarrow 0. \alpha_1 + 0. \alpha_2 + 0. \alpha_3 = 1 \quad \text{[from 2nd & 3rd row]}$$

$$\Rightarrow$$
 0 = -1

which is not possible.

- : The system has no solution.
- .. The system is incomistent.

(Am.)



communer to the g. NO - 04 (b)

Gilven system,

$$x_1 + 2x_2 + x_3 + x_4 = 6$$

$$x_1 - x_2 - x_4 = -2$$

$$\alpha_{1} + 8\alpha_{2} + \alpha_{3} + 5\alpha_{4} = 22$$

$$2x_{1} + 7x_{2} + 2x_{3} + 4x_{4} = 20$$

The augmented corrusponding matrix =>

$$\begin{pmatrix}
1 & 2 & 1 & 1 & 6 \\
1 & -1 & 1 & -1 & 6 \\
1 & 8 & 1 & 5 & 22 \\
2 & 7 & 2 & 4 & 20
\end{pmatrix}$$

$$\sim \begin{pmatrix}
1 & 2 & 1 & 1 & | & 6 \\
0 & -3 & 0 & -2 & | & -8 \\
1 & 8 & 1 & 5 & | & 22 \\
2 & 7 & 2 & 4 & | & 20
\end{pmatrix}; R_{2} = R_{2} - R_{1}$$

$$\sim \begin{pmatrix}
1 & 2 & 1 & 1 & | & 6 \\
0 & -3 & 0 & -2 & | & -8 \\
0 & 6 & 0 & 4 & | & 16 \\
2 & 7 & 2 & 4 & | & 20
\end{pmatrix}; R_3' = R_3 - R_1$$

$$\sim \begin{pmatrix}
1 & 2 & 1 & 1 & | & 6 \\
0 & 1 & 0 & \frac{2}{3} & | & \frac{8}{3} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}; R_3' = R_3 - 6R_2 \\
R_4' = R_4 - 3R_2$$

Herre,

The system is consistent.

The corresponding system \Rightarrow

$$\chi_{1} + \chi_{3} - \frac{1}{3} \chi_{4} = \frac{2}{3}$$

$$\chi_{2} + \frac{2}{3} \chi_{4} = \frac{8}{3}$$

There are 2 equations, 4 unknowns.

 \therefore we have (4-2)=2 free variables.

3rd & 4th column have no leading 1.

: 23 and 24 are free variables. [P.T.O.]

let,

$$x_3 = \pm_1$$
; $x_4 = \pm_2$

$$\therefore x_1 = \frac{2}{3} + \frac{1}{3} + \frac{1}{2} - + \frac{1}{2}$$

$$\chi_2 = \frac{8}{3} - \frac{2}{3} t_2$$

$$\therefore \left(x_{1}, x_{2}, x_{3}, x_{4} \right) = \left(\frac{2}{3} + \frac{1}{3} t_{2} - t_{1}, \frac{8}{3} - \frac{2}{3} t_{2}, t_{1}, t_{2} \right)$$

Now,

where
$$t_1, t_2 \in \mathbb{R}$$

the system is promised of

nic eciciospending splen

putting values to to & t2, >

$$t_1 = 1$$
 and $t_2 = 1$, we get \Rightarrow

$$x_1 = 0$$
,

$$22 = 2$$

$$x_4 = 1$$

Again,
$$t_1 = 10$$
 and $t_2 = 10$, we get \Rightarrow

$$\varkappa_1 = -6$$

$$\chi_2 = -4$$

communer to the g. NO-04(c)

Given system;

$$x_{1} + 2x_{2} + 3x_{3} + 4x_{4} = 0$$
 $2x_{1} + 3x_{2} + 4x_{3} = 1$
 $3x_{1} + 4x_{2} + x_{4} = 2$
 $4x_{1} + x_{3} + 2x_{4} = 3$

The corresponding augmented matrixe
$$\Rightarrow$$

$$\begin{pmatrix}
1 & 2 & 3 & 4 & 0 \\
2 & 3 & 4 & 0 & 1 \\
3 & 4 & 0 & 1 & 2 \\
4 & 0 & 1 & 2 & 3
\end{pmatrix}$$

$$\sim \begin{pmatrix}
1 & 2 & 3 & 4 & 0 \\
0 & -1 & -2 & -8 & 1 \\
0 & -2 & -9 & -11 & 2 \\
0 & -8 & -11 & -14 & 3
\end{pmatrix}; R_{2}' = R_{2} - 2R_{1}$$

$$R_{3}' = R_{3} - 3R_{1}$$

$$R_{4}' = R_{4} - 4R_{1}$$

$$\sim \begin{pmatrix}
1 & 2 & 3 & 4 & 0 \\
0 & 1 & 2 & 8 & -1 \\
0 & -2 & -9 & -11 & 2 \\
0 & -8 & -11 & -14 & 3
\end{pmatrix}; R_{2} = (-1) \cdot R_{2}$$

$$\sim \begin{pmatrix}
1 & 2 & 3 & 4 & 0 \\
0 & 1 & 2 & 8 & -1 \\
0 & 0 & -5 & 5 & 0 \\
0 & 8 & -11 & -14 & 3
\end{pmatrix}; R_3' = R_3 + 2R_2$$

p.1.0.



This is in Reduced Row echelon form. and the system is comsistent.

$$24 = -\frac{1}{11}$$

$$(x_1, x_2, x_3, x_4) = \left(\frac{9}{11}, -\frac{1}{11}, -\frac{1}{11}, -\frac{1}{11}\right)$$

(com:)

cam. to the g. NO - 05

Given chemical reaction,

$$\dot{\varkappa}_{1}\left(co_{2}\right) + \varkappa_{2}\left(H_{2}O\right) \longrightarrow \varkappa_{3}\left(c_{6}H_{12}O_{6}\right) + \varkappa_{4}\left(o_{2}\right)$$

where $x_1, x_2, x_3, x_4 \Rightarrow \text{integers}$.

To balance the given equation,

the number of atoms on both sides of the equation must be equal.

Now, Left side right side Number of Carebon (C):
$$x_1 = 6x_3$$
Number of Hydrogen (H): $2x_2 = 12x_3$

Numbers of Oxygen (0):
$$2x_1+x_2 = 6x_3+2x_4$$

Now, the corresponding homogenous linear system=>

$$x_1$$
 $-6x_3 = 0$
 $2x_2$ $-12x_3 = 0$
 $2x_1 + x_2 - 6x_3 - 2x_4 = 0$

The corrresponding augmented matrize >

$$\begin{pmatrix}
1 & 0 & -6 & 0 & 0 \\
0 & 2 & -12 & 0 & 0 \\
2 & 1 & -6 & -2 & 0
\end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 2 & -12 & 0 & 0 \\ 0 & 1 & 6 & -2 & 0 \end{pmatrix}; R_3 = R_3 - 2R_1$$

$$\sim \begin{pmatrix}
1 & 0 & -6 & 0 & 0 \\
0 & 2 & -12 & 0 & 0 \\
0 & 0 & 12 & -2 & 0
\end{pmatrix}; R_3' = R_3 - \frac{1}{2} R_2$$

$$\sim \begin{pmatrix}
1 & 0 & 0 & -1 & 0 \\
0 & 2 & 0 & -2 & 0 \\
0 & 0 & 12 & -2 & 0
\end{pmatrix}; R_{2}' = R_{2} + R_{3} \\
R_{1}' = R_{1} + \frac{1}{2}R_{3}$$

Now, The corresponding system >

$$x_{1} - x_{4} = 0 - 0$$

$$x_{2} - x_{4} = 0 - 0$$

$$x_{3} - \frac{1}{6}x_{4} = 0 - 0$$

There are 3 equations and 4 unknowns;

.: we have (4-3) = 1 free variable.

Herce, 4th column has no leading 1.

: 24 is free varciable.

$$\alpha_3 = \frac{1}{6} \pm$$

$$\therefore \left(\varkappa_1, \, \varkappa_2, \, \varkappa_3, \, \varkappa_4 \right) = \left(t, \, t, \, \frac{1}{6} t, \, t \right)$$

[where I ER]

The smallest value of t for which, all the values of x_i are positive integers is t=6.

$$\chi_1 = 6$$

$$\chi_2 = 6$$

$$23 = 1,$$

$$\chi_4 = 6$$

: The balanced equation >