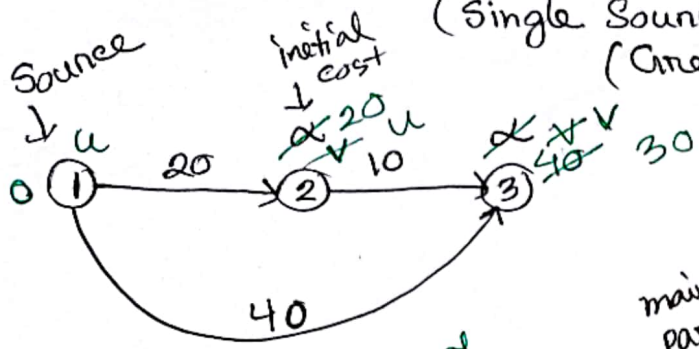


1

# Dijkstra's Algorithm. (Single Source Shortest Path) (Greedy Approach)



\* Relaxation

main part: 
$$\begin{cases} \text{if } d(u) + c(u, v) < d(v) \\ d(v) = d(u) + c(u, v) \end{cases}$$

$$\begin{aligned} d(u) + c(u, v) &< d(v) \\ 0 + 20 &< \infty \Rightarrow \text{Yes} \\ d(v) &= d(u) + c(u, v) \\ &= 20 \end{aligned}$$

1 to 3:

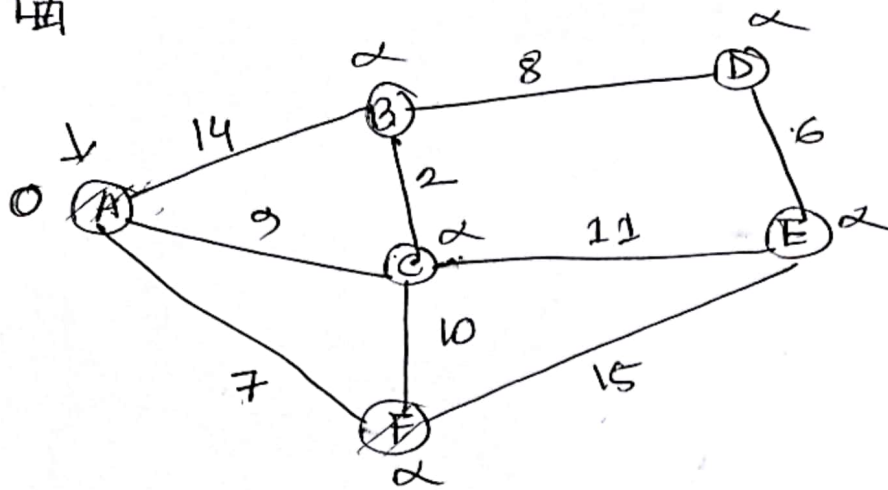
$$0 + 40 < \infty \Rightarrow \text{Yes}$$

2 to 3

$$\begin{aligned} d(u) + c(u, v) &< d(v) \\ 20 + 10 &< 40 \\ 30 &< 40 \rightarrow \text{Yes} \end{aligned}$$

As we have to find Shortest Path is a minimization problem. Minimization is a optimization problem. So Optimization problem can be solved using Greedy Approach.

Greedy Method say that: A problem should be solved by stages taking one step at a time and considering one input at a time to get a optimal solution.



Visited Vertices	A	B	C	D	E	F
A	0	∞	∞	∞	∞	∞
F		14	9	∞	∞	7
C		14	9	∞	22	
B		11		∞	20	
D				19	20	
					20	

A to E → Shortest Path 000009 E, C, A

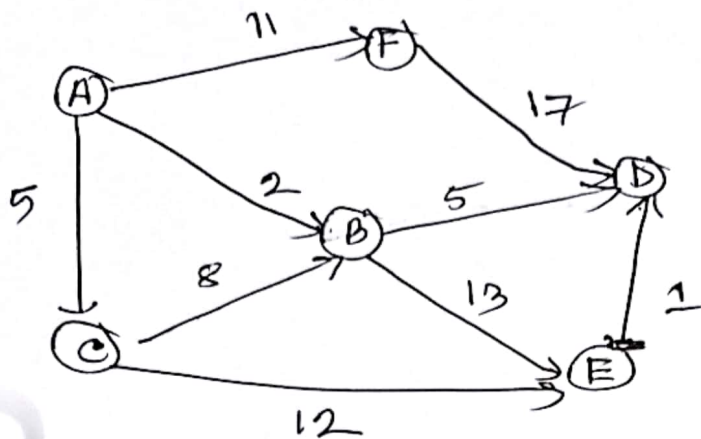
$$A, C, E \rightarrow 9 + 11 = 20$$

A → D

D, B, C, A

A, C, B, D

$$9 + 2 + 8 = 19$$



	A	B	C	D	E	F
A	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
B		2	5	$\infty$	$\infty$	11
C			15	7	15	11
D				7	15	11
F					<del>8</del> 15	11
E					15	<del>11</del>

$A \rightarrow E$

E, B, A

A, B, E

$0 + 2, 13 \rightarrow 15$



Dijkstra (Graph, Source)

Create vertex set  $Q \rightarrow$  Data Structure Heap  
 min heap use  $\text{priority}$  with minimum time.

for each vertex  $v$  in graph

$$\text{dist}[v] = \infty$$

add  $v$  to  $Q \rightarrow$  Build Heap  
 $O(V)$

$$\text{dist}[\text{Source}] = 0$$

while  $Q$  is not empty

$$u = \text{Extract-min}[Q]$$

for each neighbour  $v$  of  $u$

$$\text{if } d(u) + c(u, v) < d(v)$$

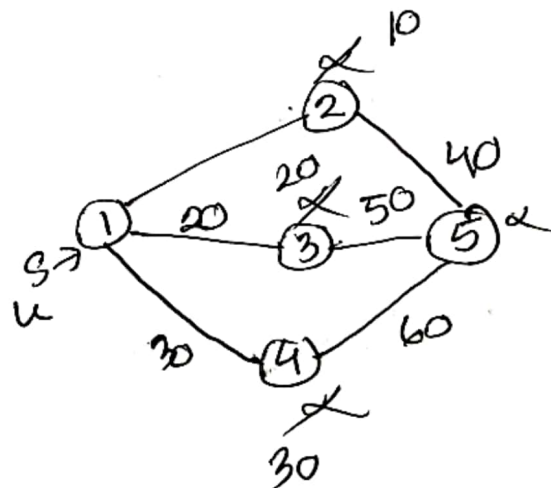
$$d(v) = d(u) + c(u, v)$$

min heap  
 Remove

$$O(\log V)$$

$\uparrow$  total element  
 $\uparrow$  total vertex  
 $V$  times

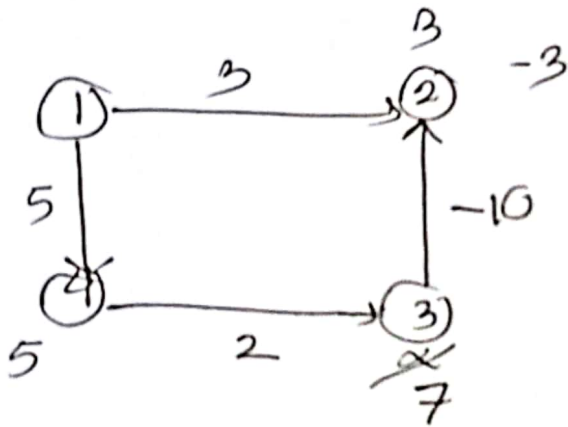
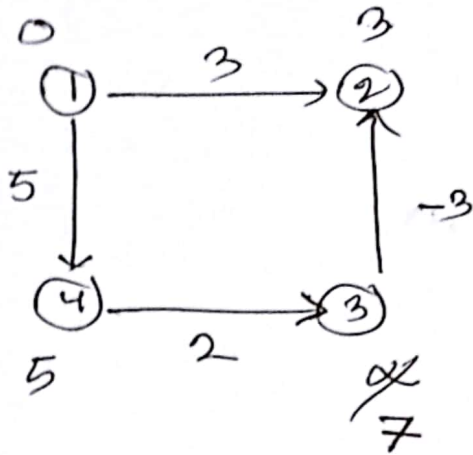
$$O(V \log V)$$



$$O(V) + O(V) + O(V \log V) + O(E \log V)$$

$$= O(E \log V)$$

Draw back:



may work or may not work in negative weight.