Lecture-01

29 May 2024

Topic: Systems of Linear Equations $\rightarrow$ Eqn (Howard Anton sec: 1.1)

- Example of a linear eqn in 2 variables/unknowns –

$$ax + by = c$$

- Linear eqn in n variables –

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

if $b = 0$, then we've –

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0 \longrightarrow$$ this is called a homogeneous linear equation in the variables $x_1, x_2, \cdots x_n$.

- Example of system of linear Eqns :=>

$$\left.\begin{aligned} 5x + y &= 3 \\ 2x - y &= 4 \end{aligned}\right\} \text{ — (1)}$$

$\rightarrow$ this is a system of 2 linear eqns in 2 unknowns

Augmented matrix — Augmented matrix of a system of linear eqns is formed using the co-efficients of the unknowns & the right hand side constants.

For example, for the above system (1), the augmented matrix is $\Rightarrow$

right hand side constants

$$A = \left(\begin{array}{cc|c} 5 & 1 & 3 \\ 2 & -1 & 4 \end{array}\right)$$

Or, we can avoid the vertical bar to write -

$$A = \begin{pmatrix} 5 & 1 & 3 \\ 2 & -1 & 4 \end{pmatrix}$$

[dimension of $A = 2\times3$.]

* A general ~~line~~ system of m equations in n unknowns is represented by $\Rightarrow$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

So, the corresponding augmented matrix is for the above system is given by $\Rightarrow$

$$A = \left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right)$$

Since, A has m rows and $(n+1)$ columns, so dimension of the matrix A is $m \times (n+1)$.

* Elementary row operations: There are 3 main row operations:-

(1.) multiply a row ~~through~~ by a non-zero constant

Ex:
$$\begin{pmatrix} 1 & 2 & 4 \\ 5 & 3 & 9 \\ -2 & 0 & 6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 4 \\ 5\times2 & 3\times2 & 9\times2 \\ -2 & 0 & 6 \end{pmatrix} ; \; r_2' = 2r_2$$

$$= \begin{pmatrix} 1 & 2 & 4 \\ 10 & 6 & 18 \\ -2 & 0 & 6 \end{pmatrix}.$$

(02) Interchange two ~~no~~ rows.

Ex:

$$\begin{pmatrix} 1 & 2 & 3 \\ -9 & 0 & 5 \\ 4 & 16 & 11 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 3 \\ 4 & 16 & 11 \\ -9 & 0 & 5 \end{pmatrix} \quad r_2 \longleftrightarrow r_3$$

(03) Add a constant times one row to another.

Ex.

$$\begin{pmatrix} 1 & 4 & 9 \\ 0 & 2 & 3 \\ -1 & 3 & 4 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 4 & 9 \\ 0+(2\times 1) & 2+(2\times 4) & 3+(2\times 9) \\ -1 & 3 & 4 \end{pmatrix} ; \; r_2' = r_2 + 2r_1$$

$$= \begin{pmatrix} 1 & 4 & 9 \\ 2 & 10 & 21 \\ -1 & 3 & 4 \end{pmatrix}$$