MAT216

class - 07
27·06·2024

Chapter - 04 : (Vector Space) :-

* **Definition:** $V \neq 0$; is called vector space if →

properties :

1. If $\underline{u}, \underline{v} \in V$, then, $\underline{u} + \underline{v} \in V$
2. $\underline{u} + \underline{v} = \underline{v} + \underline{u}$
3. $\underline{u} + (\underline{v} + \underline{w}) = (\underline{u} + \underline{v}) + \underline{w}$
4. $\exists \in V$, called a zero vector such that $\underline{0} + \underline{u} = \underline{u} + \underline{0} =$
   $\forall \underline{u} \in V$

$\exists$ → There exist

$\forall$ → for all

(Addition)

Paricel

CamScanner

5. $\forall \underline{u} \in V, \exists -\underline{u} \in V$, called a negative of $\underline{u}$ such that $\underline{u} + (-\underline{u}) = \underline{0} = (-\underline{u}) + \underline{u}$ (Addition)

6. If K is any scalar, & $\underline{u} \in V$

then, $K\underline{u} \in V$

7. $K(\underline{u}+\underline{v}) = K\underline{u} + K\underline{v}$

8. $(K+m)\underline{u} = K\underline{u} + m\underline{u}$

9. $K(m\underline{u}) = (Km)\underline{u}$

10. $1 . \underline{u} = \underline{u}$

(multiplication.)

Example:

Show that the set of all 2×2 matrixes with real entries form a vector space under user a matr addition & scalar multiplication.

Solution: let,

$$V = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \middle| \; a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R} \right\}$$

(1) let, $\underline{u}, \underline{v} \in V$

$$\underline{u} + \underline{v} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} + \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}$$

CamScanner

$$= \begin{pmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{pmatrix} \in V$$

since it is a 2×2 real matrise.

(2) $$\underline{u}+\underline{v} = \begin{pmatrix} u_{11} & u_{12} \\ v_{21} & v_{22} \end{pmatrix} + \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}$$

$$= \begin{pmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{pmatrix}$$

$$= \begin{pmatrix} v_{11}+u_{11} & v_{12}+u_{12} \\ v_{21}+u_{21} & v_{22}+u_{22} \end{pmatrix}$$

$$= \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix} + \begin{pmatrix} u_{11} & +u_{12} \\ \cancel{v}u_{21} & +u_{22} \end{pmatrix}$$

(3) $$\underline{u}+(\underline{v}+\underline{w}) = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} + \begin{pmatrix} v_{11}+w_{11} & v_{12}+w_{12} \\ v_{21}+w_{21} & v_{22}+w_{22} \end{pmatrix}$$

$$= \begin{pmatrix} u_{11}+(v_{11}+w_{11}) \dots \\ u_{21}+(v_{21}+w_{21}) \dots \end{pmatrix}$$

Paricel

CS CamScanner

$$= \begin{pmatrix} (u_{11}+v_{11})+w_{11} & \cdots \\ (u_{21}+v_{21})+w_{21} & \cdots \end{pmatrix}$$

----

4) $$0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in V$$

$$\forall u,\ \underline{u}+\underline{0} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} = \underline{u}$$

(5) $$\forall \underline{u} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \in V \ \exists -\underline{u}$$

$$= \begin{pmatrix} -u_{11} & -u_{12} \\ -u_{21} & -u_{22} \end{pmatrix} \in V$$

$$\Rightarrow u + (-\underline{u}) = 0$$

(6) $$K \cdot \underline{u} = K \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} = \begin{pmatrix} Ku_{11} & Ku_{12} \\ Ku_{21} & Ku_{22} \end{pmatrix} \in V$$

since it is a real matrice.

(7) $K(\underline{u}+\underline{v}) = K\begin{pmatrix} u_{11}+v_{11} & u_{12}+v_{12} \\ u_{21}+v_{21} & u_{22}+v_{22} \end{pmatrix}$

$= \begin{pmatrix} K(u_{11}+v_{11}) \cdots\cdots & \\ & \end{pmatrix}$

$= \begin{pmatrix} Ku_{11} & Ku_{12} \\ Ku_{21} & Ku_{22} \end{pmatrix}$ --------

(8) $(K+m)\,\underline{u} = K\underline{u} + m\underline{u}$

(9) $K(m\underline{u}) = K\begin{pmatrix} mu_{11} & mu_{12} \\ mu_{21} & mu_{22} \end{pmatrix} = \begin{pmatrix} K(mu_{11}) & K(mu_{12}) \\ K(mu_{21}) & K(m \cdot u_{22}) \end{pmatrix}$

$= \begin{pmatrix} (Km)u_{11} & (Km)u_{12} \\ (Km)u_{21} & (Km)u_{22} \end{pmatrix}$

$= Km(\underline{u})$

$* \; V = \mathbb{R}^2 = \{(x,y) \mid x, y \in \mathbb{R}\}$

$\underline{u} + \underline{v} = (u_1+v_1,\ u_2+v_2)$

$K\underline{u} = (Ku_1, 0)$

Is V a vector space?

Paricel

CS CamScanner

* V is the set of positive real numbers

Define,

$$\underline{u}+\underline{v} = u \cdot v$$

$$k \cdot u = u^k$$

Is V a vector space?

Solution:-

$$V = \left\{ x \;\middle|\; \begin{array}{l} x \in \mathbb{R}^+ \\ x \in \mathbb{R},\ x>0 \end{array} \right\}$$

(9) $k(m\underline{u}) = k(u^m)$

$= (u^m)^k$

$= u^{mk}$

$= u^{km}$

$= (km)u$

(2) $u+v = u \cdot v$

$= v \cdot u$

$= \underline{v}+u$

(3) $\underline{u}+(\underline{v}+\underline{w}) = \underline{u}+(\underline{v}w)$

$= u \cdot (vw)$

$= (uv) \cdot w$

$= (u+v) \cdot w$

$= (u+v)+w$

(4)

$u+1 = u \cdot 1 = u$

$1+u = 1 \cdot u = u$

CamScanner

(5). $u + \frac{1}{u} = u \cdot \frac{1}{u} = 1.$

L.H.S. =

(7) $= K(\underline{u} + \underline{v}) = K(uv) = (uv)^K = u^K v^K = (Ku) \cdot (Kv)$

$= Ku + Kv$

R.H.S. $= K\underline{u} + K\underline{v} = u^K + v^K = u^K \cdot v^K$

MAT216

class - 08

29 · 06 · 2024

* Quiz - 1 ✓.

* let, V be the set of all functions where + and $\cdot$ is defined as —

$$(f+g)(x) = f(x) + g(x) \quad \Big| \quad f, g \in V$$
$$\& \ (\alpha f)(x) = \alpha f(x) \quad \Big| \quad \alpha = \text{scalar}$$

show V is a V.S. (Vector Space)

Solution:-

(i) let, $f, g \in V$

(ii) $(f+g)x = f(x) + g(x) = g(x) + f(x) = (g+f)(x)$

$\Rightarrow f+g = g+f$

Paricel

CS CamScanner

(iii) $\left(f+(g+h)\right)x = f(x)+(g+h)(x)$

$= f(x)+\left(g(x)+h(x)\right)$

$=\left(f(x)+g(x)\right)+h(x) = (f+g)\,(x)+h(x)$

$= \left((f+g)+h\right)(x)$

$\therefore\ f+(g+h) = (f+g)+h.$

(iv) let, $\underline{0}$ be the zero function.

$\underline{0}(x)=0 \ . \ \forall x$

$(f+\underline{0})(x) = f(x)+\underline{0}(x) = f(x)+0 = f(x)$

$(\underline{0}+f)(x) = f(x).$

(v) let,

$(-f)(x) = -f(x)$

$\left(f+(-f)\right)(x) = f(x)+(-f)(x) = f(x)-f(x) = 0 = \underline{0}(x)$

$\therefore\ f+(-f) = \underline{0}$

(vii) let, $\alpha \in \mathbb{F},\ f g \in V$

$\left(\alpha(f+g)\right)(x) = \alpha\,(f+g)(x)$

$= \alpha\,[f(x)+g(x)]$

$= \alpha f(x)+\alpha g(x)$

$= (\alpha f+\alpha g)\,(x)$

$\therefore\ \alpha(f+g) = \alpha f+\alpha g.$

CamScanner

(viii) $((k+m)f)(x) = (k+m)f(x)$

$= kf(x) + mf(x)$

$= (kf + mf)(x)$

$\therefore (k+m)f = kf + mf.$

(ix) $k(mf)(x) = (km)f(x)$

(x) $(1 \cdot f)(x) = 1 \cdot f(x)$

$= f(x)$

$\therefore 1 \cdot f = f.$

$\therefore$ V is vector space. (Ans)

(showed)

Paricel

CS CamScanner