## Ans. to the Q. NO - 01(a)

Given,

| Time, t(sec) | Velocity, v($ms^{-1}$) |
|---|---|
| 2 | 10 |
| 4 | 20 |
| 6 | 25 |

From the given Table,

$x_0 = 2$

$x_1 = 4$

$x_2 = 6$

And,

$f(x_0) = 10$

$f(x_1) = 20$

$f(x_2) = 25$

Here, number of nodes = 3

∴ the degree of polynomial, = 3 - 1 = 2

We know,

$$P_n(x) = a_0x^0 + a_1x^1 + a_2x^2 + \cdots + a_nx^n$$

$$= a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

$$\therefore P_2(x) = a_0x^0 + a_1 \cdot x^1 + a_2x^2$$

$$\therefore P_2(x_0) = a_0(x_0)^0 + a_1(x_0)^1 + a_2(x_0)^2 \Rightarrow a_0 + 2a_1 + 4a_2 = 10$$

$$\therefore P_2(x_1) = a_0(x_1)^0 + a_1(x_1)^1 + a_2 \cdot (x_2)^2 \Rightarrow a_0 + 4a_1 + 16(a_2) = 20$$

$$\therefore P_2(x_2) = a_0(x_2)^0 + a_1(x_2)^2 + a_2(x_2)^2 \Rightarrow a_0 + 6a_1 + 36a_2 = 25$$

[P.T.O.]

Now,

This can be written in matrix form:

$$\begin{bmatrix} (x_0)^0 & (x_0)^1 & (x_0)^2 \\ (x_1)^0 & (x_1)^1 & (x_1)^2 \\ (x_2)^0 & (x_2)^1 & (x_2)^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & (x_0)^1 & (x_0)^2 \\ 1 & (x_1)^1 & (x_1)^2 \\ 1 & (x_2)^1 & (x_2)^2 \end{bmatrix}^{-1} \times \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix}^{-1} \times \begin{bmatrix} 10 \\ 20 \\ 25 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 8.75 \\ -0.625 \end{bmatrix}$$

Now,

$$P_2(x) = a_0x^0 + a_1x^1 + a_2x^2$$

$$= -5 + 8.75x + (-0.625).x^2$$

$$\therefore P_2(7) = -5 + 8.75 \times (7) + (-0.625) \times (7)^2$$

$$= 25.625$$

## Ans. to the Q. NO - 01 (b)

From given info,

$$x_0 = 2$$
$$x_1 = 4$$
$$x_2 = 6$$

And,

$$f(x_0) = 10$$
$$f(x_1) = 20$$
$$f(x_2) = 25$$

Here,

number of nodes = 3

$\therefore$ the degree of polynomial, $n = 3-1 = 2$

We Know,

Lagrance method,

$$P_2(x) = l_0(x) \cdot f(x_0) + l_1(x) \cdot f(x_2) \cdot + l_2(x) \cdot f(x_2)$$

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right) \times \left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$= \left(\frac{x-4}{2-4}\right) \times \left(\frac{x-6}{2-6}\right)$$

$$= \left(\frac{x-4}{-2}\right) \times \left(\frac{x-6}{-4}\right)$$

$$= \frac{1}{8} \cdot (x-4)(x-6)$$

[P.T.O.]

And,

$$l_1(x) = \frac{x_1 - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_2}$$

$$= \frac{x-2}{4-2} \times \frac{x-6}{4-6}$$

$$= \frac{x-2}{2} \times \frac{x-6}{-2}$$

$$= \frac{1}{4}(x-2)(x-6)$$

Again,

$$l_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_2 - x_1}$$

$$= \frac{x-2}{6-2} \times \frac{x-4}{6-4}$$

$$= \frac{x-2}{4} \times \frac{x-4}{2}$$

$$= \frac{1}{8}(x-2).(x-4)$$

Now, substituting these values into formula of Lagrange method:

$$P_2(x) = l_0(x).f(x_0) + l_1(x).f(x_2) + l_2(x).f(x_2)$$

$$= 10 \times \left(\frac{1}{8}\right) \times (x-4) \times (x-6) + 20 \times \left(-\frac{1}{4}\right)(x-2)(x-6) + 25 \times \left(\frac{1}{8}\right)(x-2)(x-4)$$

$$= \frac{5(x-4)(x-6)}{4} + -5.(x-2)(x-6) + \frac{25}{8}(x-2)(x-4)$$

(Ans) [P.T.O.]

## Ans. to the Q. NO- 01 (C)

According to the question,

if a new data point is added in the given scenario,

I would use Newton's Divided Difference method ; which would be more effective to find new interpolating polynomial.

Because,

Newton's Divided Difference method can help to determine the required update with recomputing the whole polynomial.

Whereas, Using Lagrange interpolation method and Vandermonde matrix method — are time consuming as these methods need recomputation of the whole new system from the beginning.

[P.T.O.]

## Degree of the new polynomial :

The given original polynomial,

→ 3 nodes

∴ degree, $n = 3-1 = 2$

Now, If a new data point is added,

→ no. of nodes = 4

∴ degree, $n = 4-1$
$= 3$

∴ new polynomial with 1 new data point will have degree = 3.

(Ans:)

## Ans. to the Q. NO - 02(a)

Given,

$$f(x) = x\sin(x)$$

nodes $\Rightarrow$ $x_0 = -\pi/2$,

$x_1 = 0$

$x_2 = -\pi/2$

Here,

$$f(-\pi/2) = -\pi/2 \sin(-\pi/2)$$

$$= -\pi/2(-1)$$

$$= \pi/2$$

$$f(0) = 0 \sin\cdot 0$$

$$= 0$$

$$f(\pi/2) = \pi/2 \sin\cdot \pi/2$$

$$= \pi/2\cdot(1)$$

$$= \pi/2.$$

We know, According to Newton's divided difference,

The first divided difference, $f[x_0 - x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$$= \frac{f(0) - f(-\pi/2)}{0 - (-\pi/2)}$$

[P.T.O.]

$$= \frac{0 - \pi/2}{\pi/2}$$

$= 1.$

Again, $$f[x_1, x_2] = \frac{f(\pi/2) - f(0)}{\pi/2 - 0}$$

$$= \frac{\pi/2 - 0}{\pi/2}$$

$= 1.$

Now,

The second divided difference,

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$= \frac{1 - (-1)}{\pi/2 - (-\pi/2)}$$

$$= \frac{1 - (-1)}{\pi}$$

$$= \frac{2}{\pi}$$

[P.T.O.]

Now,

We know,

Newton's interpolation formula for polynomial:

$$P_2(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0) \cdot (x - x_1)$$

here,

substituting the values:

$$P_2(x) = \pi/2 - (x + \pi/2) + \frac{2}{\pi}(x + \pi/2) \cdot x$$

$$= \pi/2 - x - \pi/2 + \frac{2}{\pi} x^2 + \frac{2}{\pi} \cdot \frac{\pi}{2} x$$

$$= -x + \frac{2}{\pi} x^2 + x$$

$$= \frac{2}{\pi} x^2$$

∴ The interpolating polynomial for the given function,

$$P_2(x) = \frac{2}{\pi} x^2.$$

(Ans:)

[P.T.O.]

## Ans. to the Q. NO - 02(b)

from answer of '02(a)',

we get,

The interpolating polynomial,

$$P_2(x) = \frac{2}{\pi}x^2$$

Given point,

$$x = \pi/4$$

Now,

$$P_2(\pi/4) = \frac{2}{\pi}\left(\frac{\pi}{4}\right)^2$$

$$= \frac{2}{\pi} \times \frac{\pi^2}{16}$$

$$= \frac{2\pi^2}{16\pi}$$

$$= \frac{\pi}{8}$$

$\because$ Given,

$$\left[f(x) = x\sin(x)\right]$$

$$\therefore f(\pi/4) = \pi/4 \sin \pi/4$$

$$= \pi/4 \times \frac{\sqrt{2}}{2}$$

$$= \frac{\pi\sqrt{2}}{8}$$

[P.T.O.]

Now,

percentage relative error,

$$= \frac{|\text{Actual Value} - \text{Approximate value}|}{|\text{Actual value}|} \times 100\%$$

$$= \left| \frac{\frac{\pi\sqrt{2}}{8} - \frac{\pi}{8}}{\frac{\pi\sqrt{2}}{8}} \right| \times 100\%$$

$$= \left| \frac{\frac{\pi}{8}(\sqrt{2}-1)}{\frac{\pi\sqrt{2}}{8}} \right| \times 100\%$$

$$= \frac{(\sqrt{2}-1)}{\sqrt{2}} \times 100\%$$

$$= \left| \frac{(1.414-1)}{(1.414)} \right| \times 100\%$$

$$= 29.2786\ \%$$

$\therefore$ The percentage relative error is $29.2786\%$. (approximately), at $\pi/4$.

(Ans:)

[P.T.O.]

## Ans. to the Q. NO- 02 (c)

Given nodes,

$$-\pi/2, 0, \pi/2$$

The new node, $\pi$

$\therefore$ $x_0 = -\pi/2$

$x_1 = 0$

$x_2 = \pi/2$

$x_3 = \pi$

for given function, $f(x) = x \sin x$,

adding $x_3 = \pi$ in the given function:

$$f(\pi) = \pi \cdot \sin\pi$$
$$= \pi \times 0$$
$$= 0$$

$\because \sin(n\pi) = 0$
[for any integer n]

from the answer of '(2)(a)' and '2(b)',

we get,

$$f[x_0, x_1] = -1$$
$$f[x_1, x_2] = 1$$
$$f[x_0, x_1, x_2] = \frac{2}{\pi}$$

[P.T.O]

Now, for $x_3 = \pi$,

$$f[x_2, x_3] = \frac{f(\pi) - f(\pi/2)}{\pi - \pi/2}$$

$$= \frac{0 - \pi/2}{\pi - \pi/2}$$

$$= \frac{-\pi/2}{\pi/2}$$

$$= -1$$

And,

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{\pi - 0}$$

$$= \frac{-1 - 1}{\pi}$$

$$= \frac{-2}{\pi}$$

And,

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{\pi - (-\pi/2)}$$

$$= \frac{-2/\pi - 2/\pi}{\frac{3\pi}{2}}$$

$$= \frac{-4/\pi}{\frac{3\pi}{2}}$$

$$= -\frac{8}{3\pi^2}$$

[P.T.O.]

Now,

We Know,

Newton's form of the interpolating polynomial:

$$P_3(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0) \cdot (x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

here,

substituting the values:

$$P_3(x) = \pi/2 + (-1)\,(x + \pi/2) + \frac{2}{\pi}(x + \pi/2) \cdot x + \left(-\frac{8}{3\pi^2}\right) \cdot (x + \pi/2) \cdot x \cdot (x - \pi/2)$$

$$= \pi/2 - x - \pi/2 + \frac{2}{\pi}(x^2 + \pi/2\, x) - \frac{8}{3\pi^2}\left(x^3 - \pi/2\, x^2 + \pi/2\, x^2 - \pi^2/4\, x\right)$$

$$= -x + 2/\pi\, x^2 - 8/3\pi^2 \cdot x^3 + 8/3\pi^2 \cdot \pi^2/4\, x$$

$$= -x + \frac{2}{\pi} x^2 - \frac{8}{3\pi^2} x^3 + \frac{8}{12} x$$

$$= -x + \frac{2}{\pi} x^2 - \frac{8}{3\pi^2} x^3 + \frac{2}{3} x$$

$$= \frac{2}{3} x - x + \frac{2}{\pi} x^2 - \frac{8}{3\pi^2} \cdot x^3$$

$$= -1/3\, x + \frac{2}{\pi} x^2 - \frac{8}{3\pi^2} \cdot x^3$$

[P.T.O.]

$$\therefore P_3(x) = -\frac{1}{3}x + \frac{2}{\pi}x^2 - \frac{8}{3\pi^2}x^3$$

[for degree 3]

[$\because$ n $\Rightarrow$ 4 − 1 = 3
points]

(Ans:)