MAT216 (Linear Algebra and Fourier Analysis)
Assignment 02 (07 July 2024)

Deadline: 25 July, Thursday, during class hour

*each question equals 4 marks, total marks (4 × 5) = 20

1. (a) Vector space: A set V equipped with two binary operations addition and scalar mul tiplication is called a vector space over the field F, if V satisfies the following 10 axioms: i. Closed under addition: u +v ∈ V , for all u, v ∈ V

   ii. Commutativity: u + v = v + u, for all u, v ∈ V

   iii. Associativity: u + (v + w) = (u + v) + w, for all u, v, w ∈ V

   iv. Existence of additive identity: For each u ∈ V , there exists 0 ∈ V such that u + 0 = u = 0 + u

   v. Existence of additive inverse: For each u ∈ V , there exists −u ∈ V such that u + (−u) = 0 = (−u) + u

   vi. Closed under scalar multiplication: ku ∈ V for all u ∈ V and k ∈ F vii. Associativity of scalar multiplication: (ab)u = a(bu), for all u ∈ V and a, b ∈ F viii. Distributive law: a(u + v) = au + av, for all a ∈ F and u ∈ V

   ix. Distributive law: (a + b)u = au + bu, for all a, b ∈ F and u ∈ V

   x. Unite of scalar multiplication: 1u = u, where, 1 ∈ F and for all u ∈ V . Let V be the set of all pairs of real numbers of the form (1, x) where addition and scalar multiplication is defined as follows

$$(1, x_1) + (1, x_2) = (1, x_1 + x_2)$$
$$k(1, x) = (1, kx)$$

   Determine whether V is a vector space or not.

   (b) Let W be the set of all vectors of the form (a, b, c), where $b = a+c+ 1$. Is W a subspace of R^3? If yes, verify all the three conditions of subspace and if no, give a counter example.

2. Determine whether the vectors span R^3 or not ?

   (a) $v_1 = (2, 2, 2)$, $v_2 = (0, 0, 3)$, $v_3 = (0, 1, 1)$

   (b) $v_1 = (2, -1, 3)$, $v_2 = (4, 1, 2)$, $v_3 = (8, -1, 8)$

3. Determine whether the vectors are linearly independent or are linearly dependent in R^3?

   (a) $v_1 = (-3, 0, 4)$, $v_2 = (5, -1, 2)$, $v_3 = (1, 1, 3)$

   (b) $v_1 = (-2, 0, 1)$, $v_2 = (3, 2, 5)$, $v_3 = (6, -1, 1)$, $v_4 = (7, 0, -2)$

4. Find the rank and nullity of the matrix

□ □ −3 6 −4 5 8
−1 1 −4
A = 1 −2 2 −7 2 □ □
3 −1

5. Let $T : R^4 \rightarrow R^3$ be the linear transformation defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 - x_2 + x_3 + x_4,\ x_1 + 2x_3 - x_4,\ x_1 + x_2 + 3x_3 - 3x_4)$$ Find

rank of T.