Assignment - 02

"CFG"

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Course Title : Automata & Computability

Course Code : CSE331

Section : 20

Group : Solo

Date of submission : 01.05.2025

Answer to the Q. NO - 03 (a)

Given,

$$L = \{ w \in \{a, b, c, p, q, r, \#\}^* : a^i \# c^n p^{2x} q^y r^z b^j$$

Where, $i = j + k$,
$y = 3x + z$,
n is odd and
$i, j, k, n, x, y, z \geq 0 \}$

$S \rightarrow A \# C P Q B$

$A \rightarrow a\ A1$ [a^i ; such that $i = j + k$]

$A_1 \rightarrow a\ A1\ B \mid \varepsilon$

$B \rightarrow bB \mid \varepsilon$ [$b^{(j)}$]

$C \rightarrow C\ C1$ [c^n]; [n is odd]

$C_1 \rightarrow C\ C_2$

$C_2 \rightarrow C\ C1 \mid \varepsilon$

$P \rightarrow P^P \mid \varepsilon$

$Q \rightarrow qqR$ [$q^{(2x)} \rightarrow Q$, $r^{(3x+z)} \rightarrow R$]

$R \rightarrow rrrQ \mid R_Z$

$R_Z \rightarrow r\ R_Z \mid \varepsilon$

Ans. to the Q. NO - 03(b)(ii)

Given,

$L = \{ w \in \{0,1,2\}^* : w = 0^i 2^j 1^k,$

(ii) $i = 3k$, j is odd and $i, j, k \geq 0$

$S \rightarrow ABC$

$A \rightarrow 000\ 111 \mid \varepsilon$

$B \rightarrow 2B2 \mid 2$

$C \rightarrow 1C \mid \varepsilon$

Ans. to the Q. NO - 03(b)(iii)

(iii) i is a multiple of two, k is two more than a multiple of 3, $j = k + i$, and $i, j, k \geq 0$

$S \rightarrow A\ B\ C$

$A \rightarrow 0\ 0\ A \mid \varepsilon$

$B \rightarrow 111\ B \mid 11$

$C \rightarrow 2C \mid \varepsilon$

here,
$i = 2n$
$k = 3m + 2$
$j = i + k$

Answer to the Q. NO- 03 (iv)(b)

Given, $i+j > k$ and $i, j, k \geq 0$

$S \rightarrow A, B$

$A \rightarrow 0S1 \mid 0A \mid B$

$B \rightarrow 2S1 \mid 2B \mid \varepsilon$

Answer to the Q. NO- 03(v)(b)

Given, $i+k$ is even, $j = i+k$, $j >= 1$

$S \rightarrow 0S12 \mid A$ [each 0 & 1 pair increases i, k by 1 each; total +2 => even sum]

$A \rightarrow 0A12 \mid \varepsilon$ [ε → possible empty case]

Answer to the Q. NO - 03(c)

Given,

$L = \{w \in \{0,1\}^* :$ the parity of 0s and 1s is different in $w\}$

$A \rightarrow 0B \mid 1C$ → if even→0, odd→1

$B \rightarrow 0A \mid 1D \mid \varepsilon$ → if even→1, odd,→0

$C \rightarrow 0D \mid 1A \mid \varepsilon$

$D \rightarrow 0C \mid 1B$

Answer to the Q. NO - 03(d)

Given,

$L = \{w \in \{0,1\}^* :$ the number of 0s and 1s are different in $w\}$

$S \rightarrow A \mid B$

$A \rightarrow 0A1 \mid 1A0 \mid 0A \mid 0$

$B \rightarrow 0B1 \mid 1B0 \mid 1B \mid 1$

Answer to the Q. NO - 03(e)

Given,

$L = \{ 1^i 0 2^j 1^k \mid i, j, k \geq 0, 3i \geq 4k+2, j \text{ is not divisible by three} \}$

$S \rightarrow 1S1 \mid 1S \mid \varepsilon$

$A \rightarrow 00A \mid 00$

$B \rightarrow 1B1 \mid 1$

$S \rightarrow A \mid B$

Answer to the Q. NO- 03 (f)

Given,

$$L_1 = \{ w \in \Sigma^* : w \text{ contains exactly two 1s} \}$$

$$L_2 = \{ x \# y : x \in \Sigma^*, y \in L_1, |x| = |y| \}$$

$S \rightarrow X \# Y$

$X \rightarrow 0X0 \mid 0X1 \mid 1X0 \mid 1X1 \mid \#$

$Y \rightarrow 0Y \mid 1M$

$M \rightarrow 0M \mid 1N$

$N \rightarrow 0N \mid \varepsilon$

Answer to the Q. NO - 03 (a)

Given,

$L_1 = \{\omega \in \Sigma^* : \omega \text{ contains at least three 1s}\}$

$L_2 = \{x \# y : x \in (\Sigma\Sigma)^*, y \in L_1, |x| = |y|\}$

$S \rightarrow X \# Y$

$X \rightarrow \varepsilon \mid 00X \mid 01X \mid 10X \mid 11X$

$Y \rightarrow 1M1N1P \mid 1M1N1 \mid 1M11 \mid 111$

$M \rightarrow 0M \mid \varepsilon$

$N \rightarrow 0N \mid \varepsilon$

$P \rightarrow 0P \mid \varepsilon$