Brac University
MAT 110
Assignment-04
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1. Calculate the following limits. If a limit is ∞ or -∞, please say so. Make sure you show all your work and justify all your answers.

(a) $\lim_{x\to\infty}\sqrt{\frac{-2x+1}{3-7x}}$

1 Answer:

$$= \lim_{x\to\infty}\left(\sqrt{\frac{-2+\frac{1}{x}}{\frac{3}{x}-7}}\right)$$

$$= \sqrt{\frac{\lim_{x\to\infty}(-2+\frac{1}{x})}{\lim_{x\to\infty}(\frac{3}{x}-7)}}$$

$$= \sqrt{\frac{-2}{-7}}$$

$$= \sqrt{\frac{2}{7}} \qquad \text{(Ans.)}$$

(b) $\lim_{x\to\infty}$x ln x with L'Hopital's rule

Answer: Given,

$\lim_{x\to 0}$x ln x
$=\lim_{x\to 0}$x $\frac{1}{x}$+ lnx [By using L'hospital's rule]

$= \lim_{x\to 0}1+$ lnx
$=$ 0 (Ans.)

(c) $\lim_{x\to\infty}\frac{1-cos(4x)}{8x^2}$

Answer: $\lim_{x\to\infty}\frac{1-cos(4x)}{8x^2}$

$= \frac{1}{8}.\lim_{x\to 0}\left(\frac{1-cos(4x)}{x^2}\right)$

$= \frac{1}{8}.\lim_{x\to 0}\left(\frac{4sin(4x)}{2x}\right)$

$= \frac{1}{8}.\lim_{x\to 0}\left(\frac{16cos(4x)}{2}\right)$

$= \frac{1}{8}.\frac{16cos(4.0)}{2}$ [$\because$ x =0]

$=$ 1

(Ans.)

2. Find the derivative of the function f(x) = ln $(\tan^{-1}(\sqrt{\frac{1-x}{1+x}})$. Simplify your answers as much as possible. Show all your work.

Answer: Given,

$$f(x) = \ln\left(\tan^{-1}(\sqrt{\tfrac{1-x}{1+x}}\right)$$

let,
x= $\cos\theta$
$\Rightarrow \theta = cosx^{-1}$

Here, f(x) $= \ln\left(\tan^{-1}(\sqrt{\frac{1-x}{1+x}}\right)$

$= \ln\left(\tan^{-1}(\sqrt{\frac{1-cos\theta}{1+cos\theta}}\right)$

$= \ln\left(\tan^{-1}(\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right)$

$= \ln\left(\tan^{-1}(\tan\frac{\theta}{2}\right)$
$= \ln\frac{\theta}{2}$

$= \ln\theta - ln2$

Now , y= f(x)= $\ln(\cos^{-1} x) - ln2$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\Big\{\ln\Big(\cos^{-1} x) - ln2\Big\}$

$= \frac{1}{\cos^{-1} x}.$ - $\frac{1}{\sqrt{1-x^2}}$

$=$ - $\frac{1}{\cos^{-1} x\sqrt{1-x^2}}$

(Ans.)

3. Test the differentiability of the function f(x) = |x - 3| at x = 3.

Answer:

The given function,

f(x)=x-3
⇒f(x)=x-3 if x≥3
=3-x (if x < 3)

Now, checking the differentiability at x = 3

L.H.S. = $\lim_{x\to3} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x\to3} \frac{f(3-x)-f(0)}{x-3}$

= $\lim_{x\to3}$(-1)
= -1

R.H.S. = $\lim_{x\to3} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x\to3} \frac{f(3-x)-f(0)}{x-3}$

= 1

∴ L.H. S.≠R.H.S.
Therefore, f(x) is not differentiable.

(Ans.)

4. Validate the continuity of the function f(x) = $\frac{x^2-9}{x-3}$ at x = 3.

Answer:

Given,
f(x) = $\frac{x^2-9}{x-3}$

At x=3, the limiting value of f(x) is,

L.H.L. = $\lim_{x\to 3^-} \frac{x^2-9}{x-3}$
= 6

R.H.L. = $\lim_{x\to 3^+} \frac{x^2-9}{x-3}$
= 6

Functional value, f(x) = $\frac{x^2-9}{x-3}$
$\therefore$ f(0) = $\frac{0}{0}$
= undefined
$\therefore$ limiting value$\neq$functional value
$\therefore$f(x)= $\frac{x^2-9}{x-3}$is not continuous at x=3 . (Ans.)

5. Find the relative maxima and minima from the function f(x) = 4x^3- 3x - 1. Locate all the extrema as x_0, x_1, $\cdots$, x_n

Answer:

Given,
f(x) = 4x^3- 3x - 1

1st derivative , f'(x) = $12x^2 - 3$

Again, 2nd derivative, f"(x) = 24x

Let, f'(x)=0
$\Rightarrow 12x^2 - 3 = 0$
$\Rightarrow 3(2x+1)(2x-1) = 0$
$\Rightarrow (2x+1)(2x-1) = 0$

so, x = $\frac{1}{2}$, $-\frac{1}{2}$
$\therefore$ the limiting points are x =$\frac{1}{2}$, and x = -$\frac{1}{2}$

Now, implementing the values of x into f"(x),

f"(x) = f"(0.5) = 24 $\times\frac{1}{2} = 12$

f"(x) = f"(-0.5) = 24 $\times(-\frac{1}{2}) = -12$

Interval	Test x value	f'(x)	conclusion
(-∞, - $\frac{1}{2}$)	-1	$9 > 0$	increasing
(- $\frac{1}{2}$, + $\frac{1}{2}$)	0	$-3 < 0$	decreasing
(+ $\frac{1}{2}, -\infty$)	1	$9 > 0$	increasing

It can be seen from the table that ,
f (x) is increasing before x = - $\frac{1}{2}$, After decreasing and defined at x = - $\frac{1}{2}$.
So, f (x) has a minimum relative point at x= - $\frac{1}{2}$.

Again, f(x) is decreasing before x = + $\frac{1}{2}$. increasing and defined at x = + $\frac{1}{2}$.
So, f (x) has a maximum relative point at x= + $\frac{1}{2}$.

$\therefore$ relative maxima at x= + $\frac{1}{2}$= - 12

$\therefore$ relative minima at x= - $\frac{1}{2}$= + 12

(Ans.)