

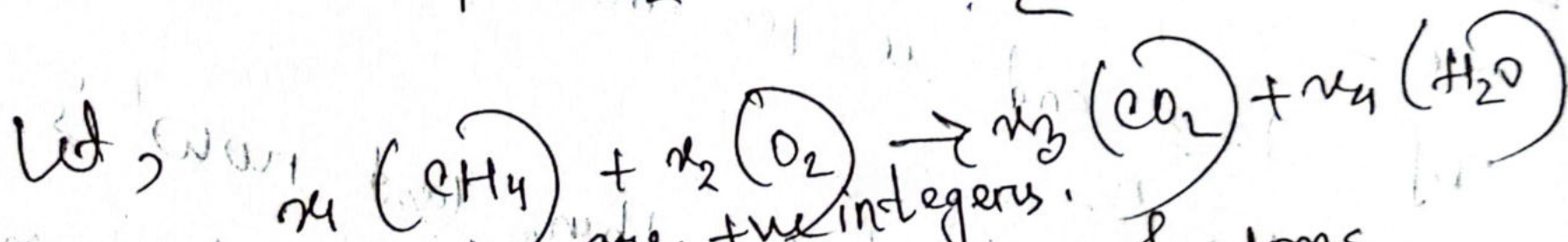
Application of System of Linear Eqⁿs

1. Balancing chemical Equations

Q: Balance the chemical eqⁿ using system of linear eqⁿs.



Solⁿ



where, x_1, x_2, x_3, x_4 are the integers.

Sol: To balance the eqⁿ, the number of atoms on both left & right sides must be equal i.e.

	left side	=	right side
# of carbon	x_1	=	x_3

# of Hydrogen	$4x_1$	=	$2x_4$
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# of oxygen	$2x_2$	=	$2x_3 + x_4$
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So, the corresponding homogeneous linear system \Rightarrow RHS = 0

$$x_1 - x_3 = 0$$

$$4x_1 - 2x_4 = 0$$

$$2x_2 - 2x_3 - x_4 = 0$$

- i The augmented matrix \Rightarrow

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 4 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array} \right); R_2' = R_2 - 4R_1$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 2 & -1 & -\frac{1}{2} & 0 \end{array} \right); R_3' = \frac{R_3}{2}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array} \right); R_2 \leftrightarrow R_3$$

This is in row echelon form. The corresponding system \Rightarrow

$$x_1 - x_3 = 0$$

$$x_2 - x_3 - \frac{1}{2}x_4 = 0$$

$$x_3 - \frac{1}{2}x_4 = 0$$

There are 3 eqs in 4 unknowns. So, $(4-3)=1$ free variable.

Here, x_4 is the free variable.

$$\text{Let, } x_4 = t$$

$$\therefore x_3 = \frac{t}{2}$$

$$x_2 = x_3 + \frac{x_4}{2} = \frac{t}{2} + \frac{t}{2} = t$$

$$x_1 = x_3 = \frac{t}{2}; t \in \mathbb{R}$$

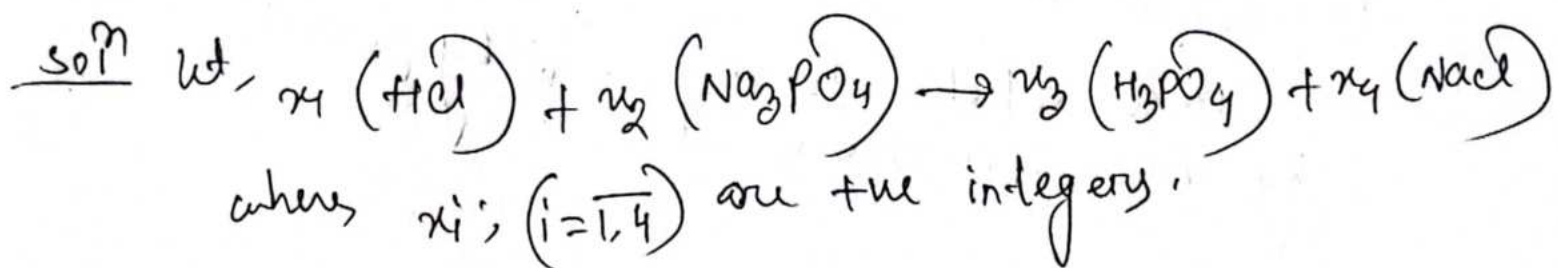
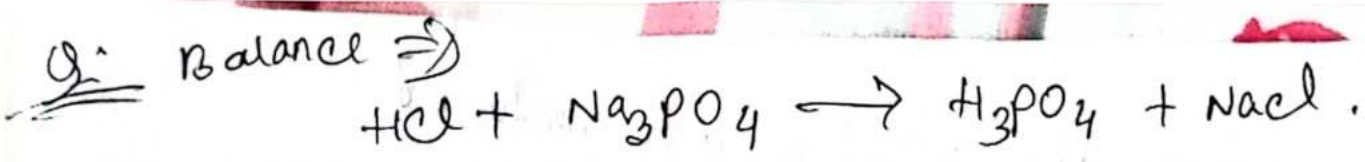
The smallest value of t for which x_1, x_2, x_3, x_4 are integers is $t=2$.

$$\text{So, } t=2 \Rightarrow x_1=1, x_2=2, x_3=1, x_4=2$$

\therefore The balanced eqn \Rightarrow



Ans



	Left	Right
H :	x_1	$= 3x_3$
Cl :	x_1	$= x_4$
Na :	$3x_2$	$= x_4$
P :	x_2	$= x_3$
O :	$4x_2$	$= 4x_3$

\therefore

$$\begin{aligned} x_1 - 3x_3 &= 0 \\ x_1 - x_4 &= 0 \\ 3x_2 - x_4 &= 0 \\ x_2 - x_3 &= 0 \\ 4x_2 - 4x_3 &= 0 \end{aligned}$$

Augmented matrix \Rightarrow

$$\left(\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 4 & -4 & 0 & 0 \end{array} \right)$$

[transform the system into reduced row echelon form] \rightarrow do by yourself.
by row operations

The reduced row echelon form is

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

\therefore The corresponding system is

$$x_1 - x_4 = 0$$

$$x_2 - \frac{1}{3}x_4 = 0$$

$$x_3 - \frac{1}{3}x_4 = 0$$

3 eq's in 4 unknowns. So, $(4-3) = 1$ free variable.

Here, x_4 is free variable.

So, let, $x_4 = t$; $t \in \mathbb{R}$.

$$\therefore x_3 = \frac{t}{3}, x_2 = \frac{t}{3}, x_1 = t.$$

The smallest value of t for which all x_i 's are +ve integers is $t = 3$.

$t=3 \Rightarrow x_1=3, x_2=1, x_3=1, x_4=3$
 \therefore The balanced eqn \Rightarrow



(Ans)

2. Polynomial Interpolation

**NB. If n points are given then we'll get a polynomial of degree $(n-1)$ or less whose graph passes through those n points.

Q. find a polyⁿ whose graph passes through the pts \Rightarrow
 $(1, 3), (2, -2), (3, -5), (4, 0).$

Solⁿ. Since there are 4 pts, we'll ~~assume~~ use a polynomial of degree $(4-1) = 3$.

Let the required polyⁿ be \Rightarrow

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \quad \text{--- (*)}$$

Since the polyⁿ (*) passes through $(1, 3)$ i.e.

when $x=1, P(x) = 3$.

$$\text{So, (i)} \Rightarrow a_0 + a_1 + a_2 + a_3 = 3$$

Similarly,

$$a_0 + 2a_1 + 2^2a_2 + 2^3a_3 = -2$$

$$a_0 + 3a_1 + 3^2a_2 + 3^3a_3 = -5$$

$$a_0 + 4a_1 + 4^2a_2 + 4^3a_3 = 0$$

So, the system \Rightarrow Augmented matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 3 \\ 1 & 2 & 4 & 8 & -2 \\ 1 & 3 & 9 & 27 & -5 \\ 1 & 4 & 16 & 64 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & 7 & -5 \\ 0 & 2 & 8 & 26 & -8 \\ 0 & 3 & 15 & 63 & -3 \end{pmatrix}$$

$$; r_2' = r_2 - r_1$$

$$r_3' = r_3 - r_1$$

$$r_4' = r_4 - r_1$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & 7 & -5 \\ 0 & 0 & 1 & 4 & -4 \\ 0 & 0 & 1 & 5 & -1 \end{pmatrix}$$

$$r_3' = \frac{r_3}{2}$$

$$r_4' = \frac{r_4}{3}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & 7 & -5 \\ 0 & 0 & 1 & 6 & 1 \\ 0 & 0 & 2 & 14 & 4 \end{pmatrix}$$

$$; r_3' = r_3 - r_2$$

$$r_4' = r_4 - r_2$$

$$2 \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 6 & 1 & 3 & -5 \\ 0 & 0 & 1 & 1 \\ 6 & 6 & 0 & 2 \end{array} \right) ; R_4' = R_4 - 2R_3$$

$$2 \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right) ; R_4' = \frac{R_4}{2}$$

, This is in row echelon form.

The corresponding system \Rightarrow

$$a_0 + a_1 + a_2 + a_3 = 3 \quad \text{--- (1)}$$

$$a_1 + 3a_2 + 7a_3 = -5 \quad \text{--- (2)}$$

$$a_2 + 6a_3 = 1 \quad \text{--- (3)}$$

$$a_3 = 1 \quad \text{--- (4)}$$

By backward substitution \Rightarrow

$$(4) \Rightarrow a_3 = 1$$

$$(3) \Rightarrow a_2 = 1 - 6a_3 = 1 - 6 = -5$$

$$(2) \Rightarrow a_1 = -5 - 3(-5) - 7(1) = 3$$

$$(1) \Rightarrow a_0 = 3 - 3 + 5 - 1 = 4$$

$$(*) \Rightarrow p(n) = 4 + 3n - 5n^2 + n^3$$

(Ans)

Practice prob^m \Rightarrow

page - 95 (Howard Anton)

Exercise 9-12, 13, 15,

T1
