Assignment - 02

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There are a total of five problems. You have to solve all of them.

Problem 1 (CO5): Nonregular Language (25 points)

Use the pumping lemma to **demonstrate** that L_1, L_2, L_3, L_4 and L_5 is not regular.

(a) $L_1 = \{ww \mid w \in \{0,1\}^*\}$ (5 points)

(b) $L_2 = \{w \in \{0,1\}^* : 0^x 1^y 0^z$ where $z > x + y$ and $x, y \geq 0$ $\}$ (5 points)

(c) $L_3 = \{w \in \{0,1\}^* : w$ is a palindrome$\}$ (5 points)

(d) $L_4 = \{w \in \{a,b\}^* :$ numbers of a in w is a prime number$\}$ (5 points)

(e) $L_5 = \{w \in \{0\}^* : 0^{3^n}$ where $n \geq 0\}$ (5 points)

Answer to the Q. NO - 01(a)

Given,

$$L_1 = \{ww \mid w \in \{0,1\}^*\}$$

Step-1: Assumption: let us assume,
L_1 is regular.

Step-2: Pumping Length: let,
p = pumping length

Step-3: Choose:

for string, $S_1 = 0101 \in L_1$

here, $S_1 = 0^p 1^p 0^p 1^p \in L_1$

here,
$S_1 = ww$,
$w = 0^p 1^p$

Dividing S_1 into xyz;
such that $|xy| \leq p$ &
$|y| > 0$

here, $|xy| \leq p$

$xy \rightarrow$ consists of only for 0's.

here,
let's assume,
$y = 0^k$;
$k > 0$

Now, pumping y,

$xyy'z = 0^{p+k} 1^p 0^p 1^p$

It is not in L_1, because it is not in form ww.

$\therefore$ " There is a contradiction.

Step-4: Conclusion: Therefore, L_1 is non-regular.

[proved] [P.T.O.]

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Answer to the Q. NO - 01(b)

Given,

$$L_2 = \{ \omega \in \{0,1\}^* : 0^x 1^y 0^z \text{ where } z > x+y \text{ and } x,y \geq 0 \}$$

Step-1: Assumption:

Let us assume,

L_2 is regular.

Step-2: Pumping length: Let, p = pumping length.

for a string, $S_2 = 0^p 1^p 0^{2p+1}$

Here,
$x = p$
$y = p$
$\therefore z = 2p+1;$
It satisfies $\Rightarrow$
$z > x+y$

Step-3: Choose:

Here, $S_2 \Rightarrow$ spliting into xyz,

Now,

$S_2' = xyy'z$

$\therefore S_2' = 0^{p+|y|} 1^p 0^{2p+1}$.

Where,
$|xy| \leq p$
$|y| > 0$
$\therefore y$ consists of only 0's.

Here,
x increases,

but, $z > x+y$; [no longer holds; as $x+y$ increases beyond z.]

Hence, There is a contradiction.

Step-4: Conclusion: Therefore, L_2 is non-regular.

[proved]

[P.T.O.]

Answer to the Q. NO - 01 (c)

Given,

$$L_3 = \{ w \in \{0,1\}^* : w \text{ is a palindrome} \}$$

Step-1: Assumption: Let us assume,
L_3 is regular.

Step-2: Pumping length: Let,
p = pumping length.

Step-3: Choose:
for string, $S_3 = 0^p . 10^p \in L_3$

Spliting S_3 into xyz,

$$S_3 = 0^x 10^z ;$$

S_3 consists of p 0s, followed by a single 1 the another p 0s. Which creates symmetry.

And,

where, $x = p - K$
$z = p$
$0 < K \leq p$

pumping y,

$$xyyz = 0^{(p+K)} 10^p$$

Here,
Increases number 0 s before the 1, which breaks the symmetry of palindrom.

∴ There is a contradiction.

Step-4: Conclusion: Therefore, L_3 is non-regular.
[proved]

[P.T.O.]

Answer to the Q. NO- 01(d)

Given,

$L_4 = \{ \omega \in \{a,b\}^* :$ numbers of a in ω is a prime number $\}$

Step-1: Assumption:

let us assume,

L_4 is regular.

Step-2: Pumping Length:

let, $p =$ pumping length.

Step-3: Choose:

for string, $S_4 = a^p \in L_4$

Now,

Spliting S_4 into xyz,

here, $p =$ prime.

let,

$y = a^m$; $m > 0$

Where,
$|xy| \leq p$ &
$|y| > 0$

$|xy| \leq p \rightarrow xy$ consists of only for a's.

Now, pumping y,

$xyy'z = a^{(p+m)}$

Here, it is not in L_4;
as the number of a's migh not be a prime number here; for every case.

$\therefore$ There is a contradiction.

Step-4: Conclusion: Therefore, L_4 is non-regular.

[proved] [P.T.O]

Answer to the Q. NO - 01(e)

Given,

$$L_5 = \{ \omega \in \{0\}^* : 0^{3^n} \text{ where } n \geq 0 \}$$

Step-1: Assumption: let us assume, L_5 is regular.

Step-2: Pumping Length: let, $p =$ pumping length.

Step-3: Choose:

for $S_5 = 0^{3^n} \in L_5$

$= 0^{3p} \in L_5$

where, $3^n \geq p$; $n \geq 0$

it satisfies; $|s| = 3^n$.

Now, Spliting S_5 into xyz;

$S_5' = 0^x 0^y 0^z$;

here,

$x = p - k$

$y = k$

$z = 2p$; &

$0 < k \leq p$

where,

$|xy| \leq p$

$|y| > 0$

$y = 0^k$

(since y consist only of 0s)

Now, pumping y,

$xyy'z = 0^{(p+k)} 0^z$

$= 0^{(p+k)} 0^{(2p)}$

$= 0^{(3p+k)}$

Here, this pumping adds $k \rightarrow$ 0s into the

[P.T.O.]

string ;

$\because k>0$ & p is constant;

Hence, the length of the ~~r~~ new pumped string will not always be a multiple of 3.

$\therefore$ There is a contradiction.

Step-4: Conclusion :

Therefore, L_5 is non-regular.

[proved]