Answer to the Q. NO- 01(a)

Given,

$$f(x) = x^3+x^2-4x-4$$

$$\therefore x^3+x^2-4x-4=0$$

$$\Rightarrow x^2(x+1)-4(x+1)=0$$

$$\Rightarrow (x+1)(x^2-4)=0$$

$$\Rightarrow (x+1)(x+2)(x-2)=0$$

Here,

$x+1=0$

$\Rightarrow x=-1$

And,

$x+2=0$

$\Rightarrow x=-2$

Again,

$x-2=0$

$\Rightarrow x=2$

Now,

$$x^3+x^2-4x-4=0$$

$$\Rightarrow 4x = x^3+x^2-4$$

$$\Rightarrow x = \frac{x^3+x^2-4}{4}$$

$$\therefore g_1(x) = \frac{x^3+x^2-4}{4}$$

Again,

$$x^3+x^2-4x-4=0$$

$$\Rightarrow x^2 = -x^3+4x+4$$

$$\Rightarrow x = \sqrt{-x^3+4x+4}$$

$$\therefore g_2(x) = \sqrt{4x-x^3+4}$$

(Ans:)

[P.T.O.]

Ans. to the Q. NO - 01 (b)

From previous part (a),

We get,

$$g_1(x) = \frac{x^3+x^2-4}{4}$$

$$\therefore g_1'(x) = \frac{1}{4}(3x^2+2x)$$

And,

$$g_2(x) = \sqrt{4x-x^3+4}$$

$$\Rightarrow g_2'(x) = \frac{4-3x^2}{2(4x-x^3+4)^{1/2}}$$

Now, for $g_1'(x)$:

$$\lambda = |g_1'(x)|$$

$$= \left|\frac{3x^2+2x}{4}\right|$$

$x_* = -1,\ \lambda = 0.25 \Rightarrow$ linear convergence

$x_* = -2,\ \lambda = 2 \Rightarrow$ Divergence

$x_* = 2,\ \lambda = 4 \Rightarrow$ Divergence

$\therefore g_1(x)$ is linearly converging to $x_* = -1$

[P.T.O.]

And, for $g_2(x)$:

$$\lambda = \left| g_2'(x) \right|$$

$$= \frac{(4-3x^2)}{2(4x-x^3+4)^{1/2}}$$

$x_* = -1$, $\lambda = 0.5 \Rightarrow$ linear convergence

$x_* = -2$, $\lambda = 2 \Rightarrow$ Divergence

$x_* = 2$, $\lambda = 2 \Rightarrow$ Divergence

$\therefore$ $g_2(x)$ is linearly converging $\Rightarrow x = -1$.

(Ans:)

Answer to the Q. NO - 02 (a)

Given,

$$f(x) = xe^{x} - 1$$

$$\Rightarrow f'(x) = xe^{x} + e^{x}$$

K	x_K	$f(x_K)$
0	1.500	5.722
1	0.9893	1.660
2	0.6789	0.3385
3	0.5766	0.02633
4	0.5672	1.567×10^{-4}
5	0.5671	-1.196×10^{-4}

$$x_{K+1} = x_K - \frac{f(x_K)}{f'(x_K)}$$

$\therefore$ Solution of $f(x) = 0$ is 0.5671

(Ans:)

[P.T.O]

Ans. to the Q. NO- 02(b)

Given,

$$g(x) = \frac{2x+1}{\sqrt{x+1}}$$

We need to show that,

to be superlinearly convergent,

$x^* = -3/2$;

$\therefore \quad g'(x^*) = 0$

$\therefore \quad g'(-3/2) = 0$

Now,

$$g(x) = \frac{2x+1}{\sqrt{x+1}}$$

$$= \frac{2x+1}{(x+1)^{1/2}}$$

$$= (2x+1)(x+1)^{-1/2}$$

$$\Rightarrow g'(x) = \frac{d}{dx}\left[(2x+1)(x+1)^{-1/2}\right]$$

let,

$u = 2x+1$

$\Rightarrow u' = 2$

and, $v = (x+1)^{-1/2}$

$\Rightarrow v' = -\frac{1}{2}(x+1)^{-3/2}$

[P.T.O.]

Now,

$$g'(x) = u'v + uv'$$

$$= 2(x+1)^{-1/2} + (2x+1)\left(-\tfrac{1}{2}(x+1)^{-3/2}\right)$$

$$= \frac{2}{\sqrt{x+1}} - \frac{2x+1}{2(x+1)^{3/2}}$$

Now,

finding a common denominator:

$$g'(x) = \frac{2 \cdot 2(x+1)^{3/2}}{\sqrt{x+1} \cdot 2(x+1)^{3/2}} - \frac{2x+1}{2(x+1)^{3/2}}$$

$$= \frac{4(x+1)^{3/2}}{2(x+1)^{3/2}} - \frac{2x+1}{2(x+1)^{3/2}}$$

$$= \frac{4(x+1) - (2x+1)}{2(x+1)^{3/2}}$$

$$= \frac{4x+4-2x-1}{2(x+1)^{3/2}}$$

$$= \frac{2x+3}{2(x+1)^{3/2}}$$

[P.T.O.]

Now,

evaluating $g'(x)$ at $x^* = -3/2$:

$$g'(-3/2) = \frac{2\cdot(-3/2)+3}{2(-3/2+1)^{3/2}}$$

$$= \frac{-3+3}{2\cdot(-1/2)^{3/2}}$$

$$= \frac{0}{2(-1/2)^{3/2}}$$

$$= 0$$

$\therefore g'(-3/2) = 0$

$\therefore$ the fixed point iteration using $g(x) = \frac{2x+1}{\sqrt{x+1}}$ will be superlinearly convergent to $x^* = -3/2$.

[showed]

Ans. to the Q. NO - 03 (a)

Given function,

$$f(x) = x^3 - x^2 - 3x + 2$$

We Know,

Newton-Raphson method's formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

[when close to root and where $f'(x) = 0$; (not near a turning point)]

Roots at [-4,4]:

$x = -1.618$

$x = 0.6180$

$x = 2.000$

Turning points:

$x = -0.721$

$x = 1.387$

Now, avoiding the turning points,

for root : (1) : $x = -1.618$

avoiding turning point at $x = -0.721$,

choosing an interval from left,

[-2, 1]

It contains the root, $x = -1.618$; away from the turning point.

[P.T.O.]

for root : (2) : $x = 0.6180$

here,
avoiding both turning point $x = -0.721$ and $x = 1.387$;
but choosing an interval between them,

$$[0, 1]$$

It contains $x = 0.6180$; away from the given turning points.

for root : (3) : $x = 2.000$

here,
avoiding the turning point at $x = 1.387$, choosing an interval in its right:

$$[1.5, 2.5]$$

It contains $x = 2.000$ and away from the turning point, $x = 1.387$.

∴ The correct intervals, including the roots it contains, avoiding the turning points are :

$$[-2, -1],\ [0, 1],\ [1.5, 2.5]$$

(Ans:)

[P.T.O]

Ans. to the Q. NO- 03(b)

Yes,
this can be solved using the Quasi Newton method.

It is a numerical method as like Newton-Raphson method; but this Quasi Newton method has some different characteristics. This Quasi-Newton method prevents direct computation of the derivative; most importantly the turning points; where $f'(x) = 0$; and where we are unable to calculate the next point; or in cases of "Math error".

Hence,
it can be said that, this can be solved using the Quasi-Newton method.