

Lee: 06 (22-06-24)

Q: find inverse of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$$

$$\begin{bmatrix} [A|I] \\ [I|A^{-1}] \end{bmatrix} \sim$$

Sol:

$$[A|I]$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array} \right]$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array} \right]$$

$$R_2' = R_2 - 2R_1$$

$$R_3' = R_3 - R_1$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array} \right]$$

$$R_3' = R_3 + 2R_2$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right]$$

$$R_3' = (-1) R_3$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right]$$

$$R_2' = R_2 + 3R_3$$

$$R_1' = R_1 - 3R_3$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right] \quad \begin{array}{l} R_1' = R_1 - 2R_2 \\ R_2' = R_2 - R_1 \end{array}$$

$$= [I | A^{-1}]$$

$$\therefore A^{-1} = \begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$$

(Ans)

\* ~~show~~ find  $A^{-1}$  if  $A$  is invertible -

$$A = \begin{pmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{pmatrix}$$

$$[A | I]$$

$$\begin{pmatrix} 1 & 6 & 4 & | & 1 & 0 & 0 \\ 2 & 4 & -1 & | & 0 & 1 & 0 \\ -1 & 2 & 5 & | & 0 & 0 & 1 \end{pmatrix}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array} \right) \begin{array}{l} \cdot R_2' = R_2 - 2R_1 \\ R_3' = R_3 + R_1 \end{array}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array} \right) : R_3' = R_3 + R_2$$

Since, we have obtained a row of zeros on the left side,  $A$  is not invertible.  
(Ans)