Assignment-06

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Answer to the Q. NO - 01 (a)

Forming an overdetermined system :-

$$x+y+z=6$$
$$2x-y+3z=14$$
$$4x+y+z=13$$
$$3x+2y+2z=15$$

$\therefore$ Matrix Form will be : $[Ax=b]$

$$A = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 6 \\ 14 \\ 13 \\ 15 \end{bmatrix}}_{b}$$

Ans. to the Q. NO- 01 (b)

Solving the system [from ('a')] using the QR decomposition method:

Here,

$$Q = \begin{bmatrix} 0\cdot1826 & 0\cdot3652 & 0\cdot9129 \\ 0\cdot3652 & -0\cdot8765 & 0\cdot3033 \\ 0\cdot7303 & 0\cdot1096 & -0\cdot2531 \\ 0\cdot5478 & 0\cdot2929 & -0\cdot0785 \end{bmatrix}$$

[P.T.O.]

$$R = \begin{bmatrix} 5.4772 & 1.0954 & 2.7381 \\ 0 & 2.7386 & 0.9123 \\ 0 & 0 & 0.5477 \end{bmatrix}$$

We know,

$$QRx = b = Rx = Q^T b$$

Now,

$$Q^T b = \begin{bmatrix} 25.64 \\ 4.11 \\ 2.19 \end{bmatrix}$$

Now,

Solving the upper triangular system:-

$$Rx = Q^T b = \begin{bmatrix} 5.4772 & 1.0954 & 2.7381 \\ 0 & 2.7386 & 0.9123 \\ 0 & 0 & 0.5477 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 25.64 \\ 4.11 \\ 2.19 \end{bmatrix}$$

[P.T.O]

Using back substitution :-

for 3rd row :-

$$0\cdot548\, x_3 = 2\cdot19$$

$$\Rightarrow x_3 = \frac{2\cdot19}{5\cdot48}$$

$$\boxed{\Rightarrow x_3 \approx 4\cdot00}$$

for 2nd row :-

$$2\cdot739 x_2 + 0\cdot912\cdot4 = 4\cdot11$$

$$\Rightarrow x_2 = \frac{4\cdot11 - 3\cdot648}{2\cdot739}$$

$$\Rightarrow \boxed{x_2 \approx 0\cdot17}$$

for 1st row :-

$$5\cdot477 x_1 + 1\cdot095\cdot0\cdot17 + 2\cdot738\cdot4 = 25\cdot64$$

$$\Rightarrow \boxed{x_1 \approx 2\cdot00}$$

$\therefore x_1 = 2\cdot00$

$x_2 = 0\cdot17$

$x_3 = 4\cdot00$

(Ans:)

[P.T.O.]

Ans. to the Q. NO- 02(a)

Given,

$$f(x) = \int_0^2 e^{0.5x} + \sin(x)$$

$$= \int_0^2 e^{0.5x} dx + \int_0^2 \sin x . dx$$

$$= \frac{1}{0.5}\left[e^{0.5x}\right]_0^2 + \left[-\cos x\right]_0^2$$

$$= 2\left(e^{0.5\times 2} - e^{0}\right) + \left[-\cos(2) + \cos(0)\right]$$

$$= 2e - 2 - \cos(2) + 1$$

$$= 2 \times 2.718 - 2 - 0.416 + 1$$

$$= 4.02$$

$$\therefore f(x) = \int_0^2 e^{0.5x} + \sin(x)$$

(Ans:)

[P.T.O]

Answer to the Q. NO - 02(b)

Computing the numerical integral by using the Newton - Cotes formula with $n=1$:-

We Know,

$$\int_0^2 f(x) \cdot dx \approx \frac{h}{2} \left[f(x_0) + f(x_1) \right]$$

Where,

$h = 2$

$x_0 = 0$

$x_1 = 2$

Now,

$$f(0) = 1+0$$
$$= 1$$

$$f(2) = e^1 + \sin(2)$$
$$= 2 \cdot 718 + 0 \cdot 909$$
$$= 3 \cdot 627$$

$$\therefore I = \frac{2}{2} \left(1 + 3 \cdot 627 \right)$$
$$= 4 \cdot 627$$
$$\approx 4 \cdot 63$$

(Ans:)