Assignment- 05

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Answer to the Q. NO - 01 (a)

Given,

$$x_1 + 6x_2 + 2x_3 = 10$$

$$3x_1 + 2x_2 + x_3 = 6$$

$$4x_1 + 5x_2 + 2x_3 = 9$$

∴ The matrix equation will be :

$$\underbrace{\begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}}_{b}$$

(Ans:)

Answer to the Q.NO-01 (b)

Here,

$$\begin{bmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

F' $\quad$ A'

$; m_{21} = \frac{A_{21}}{A_{11}} = \frac{3}{1} = 3$

$; m_{31} = \frac{A_{31}}{A_{11}} = \frac{4}{1} = 4$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

F' $\quad$ A'

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix}$$

F^2 $\quad$ A^2

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1.1875 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix}$$

$m_{32} = \frac{A_{32}}{A_{22}} = \frac{-19}{16} = 1.1875$

$$\begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.062 \end{bmatrix}$$

A^3

$$\therefore F' = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$$F^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1.1875 & 1 \end{bmatrix}$$

(Ans:)

Answer to the Q. NO- 01 (c)

For Lower triangular unit matrix,

$$L = (F^1)^{-1} (F^2)^{-1};$$

$$\text{or } L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 1.1875 & 1 \end{bmatrix}$$

(Ans:)

Answer to the Q. NO - 01 (d)

From 'b' and 'c',

We know,

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 1.1875 & 1 \end{bmatrix} \; ; \; U = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.062 \end{bmatrix}$$

To solve these,

(i) $Ly = b$

(ii) $Ux = y$

for (i):

$$Ly = b$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 1.1875 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$\Rightarrow y_1 = 10$

$\Rightarrow 3y_1 + y_2 = 6$

$\Rightarrow y_2 \Rightarrow -24$

$\therefore$ Now,

$$4y_1 + 1.1875y_2 + y_3 = 9$$

$$\Rightarrow 40 - 28.5 + y_3 = 9$$

$$\Rightarrow y_3 = -2.5$$

$\therefore$ Solving $\Rightarrow$

we get,

$$y_1 = 10$$

$$y_2 = -24$$

$$y_3 = -2.5$$

For (11) :

$$Ux = y$$

$$\Rightarrow \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.062 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -24 \\ -2.5 \end{bmatrix}$$

$$\Rightarrow -0.062\, x_3 = -2.5$$

$$\Rightarrow x_3 = 40.322$$

[P.T.O.]

$\therefore\ -16x_2 - 5x_3 = -24$

$\Rightarrow -16x_2 - 201 \cdot 61 = -24$

$\Rightarrow x_2 = -11 \cdot 1$

And,

$x_1 + 6x_2 + 2x_3 = 10$

$\Rightarrow x_1 - 66 \cdot 6 + 80 \cdot 644 = 10$

$\Rightarrow x_1 = -4 \cdot 044$

$\therefore\ x_1 = -4 \cdot 044$

$x_2 = -11 \cdot 1$

$x_3 = 40 \cdot 322$

(Ans:)

Answer to the Q. NO- 02(a)

Given,

$$6x_2 + 3x_3 = 10$$

$$3x_1 + 2x_2 + x_3 = 6$$

$$4x_1 + 5x_2 + 2x_3 = 9$$

Identifying the matrices A, x, b.

We know,

$$Ax = b$$ [matrix equation form]

∴ Now,

rewriting the equations in a standard form,

$$0x_1 + 6x_2 + 3x_3 = 10$$

$$3x_1 + 2x_2 + x_3 = 6$$

$$4x_1 + 5x_2 + 2x_3 = 9$$

∴ the matrix will be :

$$\underbrace{\begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}}_{b}$$

(Ans:)

Ans. to the Q. NO - 02 (b)

A pivoting problem occurs when :-

⇒ a pivot element (the leading element used for elimination in a row) = 0.

⇒ using a pivot (0) to eliminate elements below it, we would end up dividing by 0; ≈ undefined.

from 'a',

we get,

$$A = \begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

here,

$a_{11} = 0$

∴ A has pivoting problem.

Therefore, to use Gaussian elimination without encountering division by 0; I need to use row interchange to determine a non-zero element to the pivot position. (Ans:)

Ans. to the Q. NO - 02(c)

from 'a',

the Augmented matrix, $Aug(A) = \left[\begin{array}{ccc|c} 0 & 6 & 2 & 10 \\ 3 & 2 & 1 & 6 \\ 4 & 5 & 2 & 9 \end{array}\right]$

Now,

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 4 & 5 & 2 & 9 \end{array}\right]$$

[$\because$ for 1^{st} pivot, swapping R_1 & R_2]

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 4-\frac{4}{3}(3) & 5-\frac{4}{3}(2) & 2-4/3(1) & 9-4/3(6) \end{array}\right]$$

[$\because m_{31} = \frac{a_{31}}{a_{11}} = 4/3$]

[$\because R_3 = R_3 - m_{31}R_1 = R_3 - 4/3 R_1$]

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 7/3 & 2/3 & 1 \end{array}\right]$$

[P.T.O.]

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 7/3 - 7/18(6) & 2/3 - 7/18(2) & 1 - 7/18(10) \end{array}\right]$$

$$\left[\because m_{32} = \frac{a_{32}}{a_{22}} = \frac{7/3}{6} = 7/18\right]$$

$$\left[\because R_3 = R_3 - m_{32}R_2 = R_3 - 7/18\, R_2\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 0 & 5/9 & -\frac{26}{9} \end{array}\right]$$

$\therefore$ the upper triangular matrix U, (from the coefficients is) :

$$U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & 5/9 \end{bmatrix}$$

(Ans)

Answer to the Q. NO-02 (d)

from '(c)',

We get,

the upper triangular matrix,

$$U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & 5/9 \end{bmatrix}$$

from this matrix,

system of equations :

$$3x_1 + 2x_2 + x_3 = 6 \quad \text{---} \quad (i)$$

$$6x_2 + 2x_3 = 10 \quad \text{---} \quad (ii)$$

$$(5/9) \cdot x_3 = -26/9 \quad \text{---} \quad (iii)$$

from (iii) $\Rightarrow$,

$$(5/9) \cdot x_3 = -26/9$$

$$\Rightarrow 5x_3 = -26$$

$$\Rightarrow x_3 = -26/5$$

[P.T.O.]

Substituting the value of x_3 in eqn (ii) $\Rightarrow$

$$6x_2 + 2\left(-\frac{26}{5}\right) = 10$$

$$\Rightarrow 6x_2 + 2\left(-26/5\right) = 10$$

$$\Rightarrow 6x_2 + \left(-52/5\right) = 10$$

$$\Rightarrow 6x_2 = 10 + 52/5$$

$$\Rightarrow 6x_2 = \frac{102}{5}$$

$$\Rightarrow x_2 = 17/5$$

Now,

Substituting the value of x_2 & x_3 in eqn (i) $\Rightarrow$

$$3x_1 + 2\cdot\left(17/5\right) + \left(-26/5\right) = 6$$

$$\Rightarrow 3x_1 + 34/5 - 26/5 = 6$$

$$\Rightarrow 3x_1 + 8/5 = 6$$

$$\Rightarrow 3x_1 = 22/5$$

$$\Rightarrow x_1 = 22/15$$

∴ the solution of the given linear system by Gaussian elimination method :

[using the upper triangular matrix found in question's answer : ("d")]

$$x_1 = 22/15$$

$$x_2 = 17/5$$

$$x_3 = -26/5$$

(Ans :)