MAT120

Assignment ~ 01

Submitted by:

Name : Tasnim Rahman Moumita

ID : 22301689

Course Title : Integral Calculas & Differential Equations

Course Code : MAT120

Section : 17

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Answer to the Q. NO - 01

Date : ....

(a)

$$\int \frac{\sec^4(2t)}{\tan^9(2t)}\, dt$$

$$= \int \frac{\sec^4(u)}{2\tan^9(u)}\, du \quad [u\text{-substitution}]$$

$$= \frac{1}{2}\int \frac{\sec^4(u)}{\tan^9(u)}\, du \quad [\because \int a\cdot f(x)\cdot dx = a\cdot f(x)\cdot dx]$$

$$= \frac{1}{2}\int (1+\tan^2(u))\sec^2(u) \quad \frac{1}{\tan^9(u)}\cdot du$$

$$= \frac{1}{2}\int \frac{1+v^2}{v^9}\cdot dv$$

u-substitution, again,
let,
$\tan(u) = v$

$$= \frac{1}{2}\int \frac{1}{v^9} + \frac{1}{v^7} \quad dv$$

$$= \frac{1}{2}\left(\int \frac{1}{v^9}\, dv + \int \frac{1}{v^7}\, dv\right)$$

$$[\because \int f(x) \pm g(x)\cdot dx = \int f(x)\, dx \pm \int g(x)\cdot dx$$

$$= \frac{1}{2}\left(-\frac{1}{8v^8} - \frac{1}{6v^6}\right)$$

$$= \frac{1}{2}\left(-\frac{1}{8\tan^8(2t)} - \frac{1}{6\tan^6(2t)}\right.$$

$$= \frac{1}{2}\left(-\frac{1}{8}\cot^8(2t) - \frac{1}{6}\cot^6(2t)\right) + c$$

$$= \frac{1}{2}\left(-\frac{1}{8}\cot^8(2t) - \frac{1}{6}\cot^6(2t)\right) + c$$

(Ans:)

Answer to the Q. NO-1(b)

$$I_n = \int \csc^n x \, dx$$

let,
$u = \csc^{n-2} x$
$v = \csc^2 x$

$$= \int \csc^{n-2} x \cdot \csc^2 x \cdot dx$$

$$= \csc^{n-2} x \int \csc^2 x \cdot dx - \int \left\{ \frac{d}{dx}\left(\csc^{n-2} x\right) \cdot dx \cdot \int\left(\csc^2 x \cdot dx\right) \right\} \cdot dx$$

$$= -\csc^{n-2} x \cdot \cot x - \int \left\{ (n-2) \cdot \csc^{n-3} x \left(-\csc x \cdot \cot x\right)\left(-\cot x\right) \right\} dx$$

$$= -\csc^{n-2} x \cot x - \int (n-2) \cdot \csc^{n-2} x \cdot \cot^2 x \cdot dx$$

$$= -\csc^{n-2} x \cdot \cot x - (n-2) \int \csc^{n-2} x \left(\csc^2 x - 1\right) dx$$

$$= -\csc^{n-2} x \cdot \cot x - (n-2) \int \csc^n x \, dx + (n-2) \int \csc^{n-2} x \cdot dx$$

$$= -\csc^{n-2} x \cdot \cot x - (n-2) I_n + (n-2) I_{n-2}$$

$$\therefore I_n = -\csc^{n-2} x \cdot \cot x - (n-2) I_n + (n-2) I_{n-2}$$

$$\Rightarrow I_n + (n-2) I_n = -\csc^{n-2} x \cdot \cot x + (n-2) I_{n-2}$$

$$\Rightarrow I_n \cdot (n-1) = -\csc^{n-2} x \cdot \cot x + (n-2) I_{n-2}$$

$$\Rightarrow I_n = -\frac{1}{n-1} \cdot \csc^{n-2} x \cdot \cot x + (n-2) \cdot I_{n-2}$$

$$\therefore \int \csc^n x \, dx = -\frac{1}{n-1} \csc^{n-2} x \cdot \cot x + \frac{n-2}{n-1} \int \csc^{n-2} x + C$$

(Ans:)

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Answer to the Q. NO - 1 (c)

$$\int_{-4}^{6} |t^3 - 4t^2 - 4t + 16| \, dt$$

$$= \int_{-4}^{-2} -(t^3 - 4t^2 - 4t + 16)\, dt + \int_{-2}^{3} (t^3 - 4t^2 - 4t + 16)\, dt$$

$$= -\left[\frac{x^4}{4} - \frac{4t^3}{3} - 2t^2 + 16t\right]_{-4}^{-2} + \left[\frac{t^4}{4} - \frac{4t^3}{3} - 2t^2 + 16t\right]_{-2}^{3} - \left[\frac{t^4}{4} - \frac{4t^3}{3} - 2t^2 + 16t\right]_{3}^{4} + \left[\frac{t^4}{4} - \frac{4t^3}{3} - 2t^2 + 16t\right]_{4}^{6}$$

$$= \frac{1027}{6}$$

(Ans:)

$|t^3 - 4t^2 - 4t + 16|$

$= (-t^3 - 4t^2 - 4t + 16),\quad -4 \le t < -2$

$(t^3 - 4t^2 - 4t + 16),\quad [-2 < t < 3]$

$(t^3 - 4t^2 - 4t + 16),\quad 3 \le t < 4$

$t^3 - 4t^2 - 4t + 16,\quad 4 \le t \le 6$

Answer to the Q. NO - 1(D)

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$$\int 9t^{11} \cos(1-t^6)\, dt$$

Let,

$u = 1 - t^6$

$\Rightarrow du = -6t^5\, dt$

$\Rightarrow -\frac{du}{6} = t^5\, dt$

$\Rightarrow [\because u = 1 - t^6]$

$\Rightarrow t^6 = 1 - u$

$$= -9\int \frac{(1-u)\cdot\cos(u)}{6}\, du$$

$$= -\frac{3}{2}\int (1-u)\cdot\cos(u)\cdot du$$

$$= -\frac{3}{2}\left[(1-u)\int\cos(u)\cdot du - \int\left\{\frac{d}{du}(1-u)\int\cos(u)\cdot du\right\}du\right]$$

$$= -\frac{3}{2}\left[(1-u)\sin(u) + \int\sin(u)\cdot du\right]$$

$$= -3/2\left[(1-u)\sin(u) - \cos(u)\right] + c$$

$$= -3/2\left[(1-1+t^6)\cdot\sin(1-t^6) - \cos(1-t^6)\right] + c$$

$$= -3/2\left[t^6\sin(1-t^6) - \cos(1-t^6)\right] + c$$

(Ans:)

Answer to the Q. NO- 1 (e)

$$\int \left[9\sin^5(3x) - 2\cos^2(3x)\right] \operatorname{cosec}^4(3x)\,dx$$

$$= \int \left[9\sin^5(3x) - 2\cos^2(3x)\right] \frac{1}{\sin^4(3x)} \cdot dx$$

$$= 9\int \sin(3x)\cdot dx - 2\int \frac{\cos^2(3x)}{\sin^2(3x)} \cdot \frac{1}{\sin^2(3x)} \cdot dx$$

$$= 9\left(-\frac{\cos(3x)}{3}\right) - 2\int \cot^2(3x)\cdot \operatorname{cosec}^2(3x)\cdot dx$$

$$= -3\cos(3x) + \frac{2}{3}\int u^2\cdot du$$

$$= \frac{2}{9}u^3 - 3\cos(3x)$$

$$= \frac{2}{9}\cot^3 x - 3\cos(3x) + C$$

Let,

$u = \cot 3x$

$\Rightarrow d = -3\operatorname{cosec}^2 3x\cdot(dx)$

$\Rightarrow -\frac{du}{3} = \operatorname{cosec}^2(3x)\,dx$

(Ans:)

Answer to the Q. NO-1(f)

Date :

$$\int \left[17\left(xe^{x}+e^{x}\right)\sin\left(x.e^{x}\right)-14\sin(x)\right]\cdot dx$$

$$=17\int\left(xe^{x}+e^{x}\right)\sin\left(xe^{x}\right)dx-14\int\sin(x)\cdot dx$$

$$=17\int\sin u\cdot du+14\cos x$$

$$=-17\cos u+14\cos x$$

$$=14\cos x-17\cos\left(xe^{x}\right)+c$$

(Ans.)

let,

$x = xe^{x}$

$\Rightarrow du = \left(xe^{x}+e^{x}\right)\cdot dx$

Answer to the Q. NO-2

$f(x) = x^2 + 5x + 6$

interval $[1, 3]$

(a) Left endpoint:-

$n = 2$, width, $\Delta x = \frac{3-1}{2} = 1$

$x_0 = 1$,

$x_1 = 2$

$\therefore L_x = f(x_0)\Delta x + f(x_1)\Delta x$

$= f(1)\cdot 1 + f(2)\cdot 1$

$= ((1)^2 + 5\cdot 1 + 6) + (2^2 + 5\cdot 2 + 6)\cdot 1$

$= 32$ (Ans:)

(b) Right endpoint:-

intervals $\Rightarrow$ $x_1 = 2$,

$x_2 = 3$

$\therefore R_x = f(x_1)\cdot \Delta x + f(x_2)\cdot \Delta x$

$= f(2)\cdot 1 + f(3)\cdot 1$

$= 50$ (Ans:)

(c) midpoints:

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intervals,

$$x_{0\cdot5} = 1\cdot5$$

$$x_{1\cdot5} = 2\cdot5$$

$$\therefore M_x = f(x_{0\cdot5})\Delta x + f(x_{1\cdot5})\cdot\Delta x$$

$$= f(1\cdot5)\cdot1 + f(2\cdot5)\cdot1$$

$$= ((1\cdot5)^2 + 5\cdot(1\cdot5) + 6) + ((2\cdot5)^2 + 5\cdot(2\cdot5) + 6)\cdot1$$

$$= 40\cdot5$$

(Ans:)