Bonus Assignment

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Given function,

$$f(x) = x - x^2$$

Answer to the Q. NO - 01(a)

we know,

for even function, $f(-x) = f(x)$

for odd function, $f(-x) = -f(x)$

checking → (for even function)

$$f(-x) = -x - (-x)^2$$
$$= -x - x^2$$

checking → (for odd function)

$$f(x) = -x - (-x)^2$$
$$= -x - (x)^2$$
$$= -(x + x^2)$$

∵ The given function does not fulfill the condition for being odd or even,

∴ $f(x) = x - x^2$; is neither odd, nor even.

Answer to the Q. NO - 01(b)

Here,

$$L = \pi$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right)$$

Now,

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2).\, dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi^2}{2} - \frac{\pi^3}{3}\right) - \left(\frac{(-\pi)^2}{2} - \frac{(-\pi)^3}{3}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^3}{3} - \frac{\pi^2}{2} - \frac{\pi^3}{3}\right]$$

$$= \frac{1}{\pi}\left(-\frac{2\pi^3}{3}\right)$$

$$= \frac{-2\pi^2}{3}$$

$$\therefore \boxed{a_0 = \frac{-2\pi^2}{3}}$$

Now,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left[(x-x^2)(\cos nx)\right]$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(\frac{\sin nx}{n}\right)+(1-2x)\left(\frac{\cos nx}{n^2}\right) + \frac{2\sin nx}{n^3}\right]_{-\pi}^{\pi}$$

D	I
$\oplus\ x+x^2$	$\cos nx$
$\ominus\ 1-2x$	$\frac{\sin nx}{n}$
$\oplus\ -2$	$-\frac{\cos nx}{n^2}$
0	$-\frac{\sin nx}{n^3}$

$$= \frac{1}{\pi}\left[\left(\frac{(1-2\pi)(-1)^n}{n^2}\right)-\left(\frac{1+2\pi(-1)^n}{n^2}\right)\right]$$

$$\therefore a_n = \frac{1}{\pi}\left[\frac{(1-2\pi)(-1)^n-(1+2\pi)(-1)^n}{n^2}\right]$$

$$\boxed{a_n = \frac{-4(-1)^n}{n^2}}$$

Again,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left[(x-x^2)\sin(nx)\right]$$

$$= \frac{1}{\pi}\left[-(x-x^2)\left(\frac{\cos nx}{n}\right)+(1+2x)\left(\frac{\sin nx}{n^2}\right) - \frac{2\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

D	I
$\oplus\ x-x^2$	$\sin nx$
$\ominus\ 1-2x$	$-\frac{\cos nx}{n}$
$\oplus\ -2$	$-\frac{\sin nx}{n^2}$
0	$\frac{\cos nx}{n^3}$

$$= \frac{1}{\pi}\left[\left(-\pi-\pi^2\right)\frac{(-1)^n}{n} - \frac{2(-1)^n}{n^3}\right) - \left(-\pi-\pi^2\right)\left(\frac{-1}{n}\right)^n - \frac{2(-1)^n}{n^3}\right]$$

$$= \frac{1}{\pi}\left[\left(\pi+\pi^2\right)\frac{(-1)^n}{n} - \frac{\left(\pi-\pi^2\right)(-1)^n}{n}\right]$$

$$= \frac{1}{\pi}\left(\frac{\left(\pi+\pi^2\right)(-1)^n - \left(\pi-\pi^2\right)(-1)^n}{n}\right)$$

$$\therefore \boxed{bn = \frac{2\pi(-1)^n}{n}}$$

Now, The final fourier series will be:

$$f(x-x^2) = -\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi}\cdot\frac{(1-2\pi)(-1)^n - (1+2\pi)(-1)^n}{n^2}\cos nx + \frac{1}{\pi}\frac{\left(\pi+\pi^2\right)(-1)^n - \left(\pi-\pi^2\right)(-1)^n}{n}\sin nx\right]$$

$$\therefore f(x-x^2) = -\frac{\pi^2}{3} + 4\left[\frac{\cos x}{(1)^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \frac{\cos 4x}{4^2} + \cdots\right] + 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \cdots\right]$$

(Ans:)