

There are a total of three problems. You have to solve all of them!

Problem 1 (CO3): Context-Free Grammars (10 points)

Let $\Sigma = \{a, b\}$. Consider the following languages over Σ . Recall that for a string w , $|w|$ denotes the length of w .

$$L_1 = \{w \mid w \text{ is an even length palindrome}\}$$

$$L_2 = \{w \mid \text{Length of } w \text{ is even}\}$$

$$L_3 = \{x11y \mid x, y \in L_2, |x| = |y|\}$$

$$L_4 = L_1 \cap L_3$$

- (a) **Give** a context free grammar for L_1 . (3 points)
- (b) **Give** a context free grammar for L_3 . (4 points)
- (c) **Give** a context free grammar for L_4 . (3 points)

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \rightarrow 1A \mid 1C \mid 0B \mid 00A$$

$$B \rightarrow 0A \mid 1B \mid 00B$$

$$C \rightarrow 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid \epsilon$$

- (a) **Give** a leftmost derivation for the string 01011001. (3 points)
- (b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)
- (c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)
- (d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (2 point)

Problem 3 (CO4): Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages.

$$L_1 = \{w \mid w \text{ starts and ends with the same character}\}$$

$$L_2 = \{w \mid \text{the number of 0s in } w \text{ is not the same as the number of 1s}\}$$

- (a) **Give** the state diagram of a pushdown automaton that recognizes L_1 . (4 points)
- (b) **Give** the state diagram of a pushdown automaton that recognizes L_2 . (6 points)