

3) Design a Context Free Grammar for the Language:

a) $L = \{w \in \{a,b,c,p,q,r,\#\}^* : a^i \#^n c^k p^{2x} q^y r^z b^j \text{ where } i=j+k, y=3x+z, n \text{ is odd and } i,j,k,n,x,y,z \geq 0\}$

b) $L = \{w \in \{0,1,2\}^* : w = 0^i 2^j 1^k, [\text{whereconditions.....}] \}$

where...

~~i) $i = k, i, k \geq 1 \text{ and } j \geq 2$~~

ii) $i = 3k, j \text{ is odd and } i,j,k \geq 0$

iii) $i \text{ is a multiple of two, } k \text{ is two more than a multiple of 3, } j = k+i, \text{ and } i,j,k \geq 0$

iv) $i+j > k \text{ and } i,j,k \geq 0$

v) $i+k \text{ is even, } j = i+k \text{ and } j \geq 1$

c) $L = \{w \in \{0,1\}^* : \text{the parity of 0s and 1s is different in } w\}$

d) $L = \{w \in \{0,1\}^* : \text{the number of 0s and 1s are different in } w\}$
[Hint: First, try to solve for an equal number of 0s and 1s in w]

e) $L = \{1^i 0 2^j 1^k \mid i, j, k \geq 0, 3i \geq 4k + 2, j \text{ is not divisible by three}\}$

f) Recall that for a string w , $|w|$ denotes the length of w . $\Sigma = \{0,1\}$

$$L1 = \{w \in \Sigma^* : w \text{ contains exactly two 1s}\}$$

$$L2 = \{x\#y : x \in \Sigma^*, y \in L1, |x| = |y|\}$$

Construct a CFG for $L2$.

g) Recall that for a string w , $|w|$ denotes the length of w . $\Sigma = \{0,1\}$

$$L1 = \{w \in \Sigma^* : w \text{ contains at least three 1s}\}$$

$$L2 = \{x\#y : x \in (\Sigma\Sigma)^*, y \in L1, |x| = |y|\}$$

Construct a CFG for $L2$.