Assignment ~ 03

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Answer to the Q. NO - 01

Given,

$$F(t) = (t+2)^3 e^{2t}$$

$$= \left(t^3 + 6t^2 + 12t + 8\right) e^{2t}$$

$$= e^{2t} t^3 + e^{2t} \cdot 6t^2 + 12 e^{2t} t + 8 e^{2t}$$

Now,

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{e^{2t} t^3\} + 6\mathcal{L}\{e^{2t} t^2\} + 12\mathcal{L}\{e^{2t} t\} + 8\mathcal{L}\{e^{2t}\}$$

$$= \frac{3!}{(s-2)^4} + 6 \cdot \frac{2!}{(s-2)^3} + 12 \cdot \frac{1!}{(s-2)^2} + 8 \cdot \frac{1}{(s-2)}$$

$$= \frac{6}{(s-2)^4} + \frac{12}{(s-2)^3} + \frac{12}{(s-2)^2} + \frac{8}{(s-2)}$$

$$= F(s)$$

$\therefore$ Laplace Transform of : $F(t) = (t+2)^3 e^{2t}$

is,

$$\mathcal{L}\{(F(t)\} = \frac{6}{(s-2)^4} + \frac{12}{(s-2)^3} + \frac{12}{(s-2)^2} + \frac{8}{(s-2)}$$

(Ans:)

[P.T.O]

Answer to the Q. NO - 02

Given,

$$F(t) = (t^2 - 3t + 2)\sin 3t$$

$$= t^2 \sin 3t - 3t.\sin 3t + 2 \sin 3t$$

Now,

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{t^2 \sin 3t\} - 3\mathcal{L}\{t.\sin 3t\} + 2\mathcal{L}\{\sin 3t\}$$

$$= (-1)^2 \frac{d^2}{ds^2}\left[\mathcal{L}\{\sin 3t\}\right] - 3.\frac{6s}{(s^2+9)^2} + 2.\frac{3}{s^2+9}$$

$$= \frac{d^2}{ds^2}\left[\frac{3}{s^2+9}\right] - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

$$= \frac{d}{ds}\left(-3(s^2+9)^{-2}.2s\right) - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

$$= \frac{d}{ds}\left[-\frac{6s}{(s^2+9)^2}\right] - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

$$= \frac{-6\left[(s^2+9)^2 - s\{2(s^2+9).2s\}\right]}{(s^2+9)^4} - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

$$= \frac{-6\left[(s^2+9)^2 - 4s^2(s^2+9)\right]}{(s^2+9)^4} - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

[P.T.O.]

$$= \frac{24s^2 - 6(s^2+9)}{(s^2+9)^3} - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

$$= \frac{18s^2 - 54}{(s^2+9)^3} - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

$$= f(s)$$

$\therefore$ Laplace Transform of : $F(t) = (t^2 - 3t + 2)\sin 3t$

is :

$$\mathcal{L}\{F(t)\} = \frac{18s^2 - 54}{(s^2+9)^3} - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$$

(Ans:)

Answer to the Q. NO - 03

Given,

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$$

Here,

$$f(s) = \frac{s}{(s^2+a^2)^2}$$

$$\left[\because F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)\, e^{-st} \cdot dt\right]$$

$$= \frac{1}{2a} \cdot \frac{2as}{(s^2+a^2)^2}$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{1}{2a} \cdot \mathcal{L}^{-1}\left\{\frac{2as}{(s^2+a^2)^2}\right\}$$

$$= \frac{1}{2a} \cdot t \sin at$$

$$= \frac{t \cdot \sin a \cdot t}{2a}$$

$$= F(t)$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{t \cdot \sin a \cdot t}{2a}$$

(Ans:)

[P.T.O.]

Answer to the Q. NO - 04

Given,

$$\mathcal{L}^{-1}\left\{\frac{s^2-3}{(s+2)(s-3)(s^2+2s+5)}\right\}$$

Here,

$$\frac{s^2-3}{(s+2)(s-3)(s^2+2s+5)} \equiv \frac{A}{(s+2)} + \frac{B}{(s-3)} + \frac{Cs+D}{s^2+2s+5}$$

$$\Rightarrow s^2-3 = A(s-3)(s^2+2s+5) + B(s+2)(s^2+2s+5) + (Cs+D)(s+2)(s-3)$$

$$\Rightarrow s^2-3 = As^3 + 2As^2 + 5As - 3As^2 - 6As - 15A + Bs^3 + 2Bs^2 + 5Bs + 2Bs^2 + 4Bs + 10B + Cs^3 - 6s^2 - 6Cs + Ds^2 - Ds - 6D$$

$$\Rightarrow s^2-3 = (A+B+C)s^3 + (-A+4B-C+D)s^2 + (-A+9B-6C-D)s + (-15A+10B-6D)$$

Now,

Assuming, $s=-2$, $A = -\frac{1}{25}$

Assuming, $s=3$, $B = +\frac{6}{100} = \frac{3}{50}$

[P.T.O.]

From coefficient of s^2:

$$-A + 4B - 6D = 1$$

And,

From s^3 & $\to$ coefficient:

$$A+B+C = 0$$

$$\therefore C = -\frac{1}{50}$$

$$\therefore D = \frac{7}{10}$$

Now,

$$f(s) = \frac{s^2-3}{(s+2)(s-3)(s^2+2s+5)}$$

$$= \frac{-\frac{1}{25}}{(s+2)} + \frac{\frac{3}{50}}{(s-3)} + \frac{-\frac{s}{50}+\frac{7}{10}}{(s^2+2s+5)}$$

$$= \frac{-\frac{1}{25}}{(s+2)} + \frac{\frac{3}{50}}{(s-3)} + \frac{-\frac{s}{50}-\frac{1}{50}+\frac{1}{50}+\frac{7}{10}}{(s+1)^2+(2)^2}$$

$$= \frac{-\frac{1}{25}}{(s+2)} + \frac{\frac{3}{50}}{(s-3)} - \frac{1}{50}\cdot\frac{s+1}{(s+1)^2\cdot(2)^2} + \frac{\frac{18}{25}}{(s+1)^2+(2)^2}$$

Now,

$$\mathcal{L}^{-1}\left\{F(s)\right\} = -\frac{1}{25}\cdot\mathcal{L}^{-1}\left\{\frac{1}{(s+2)}\right\} + \frac{3}{50}\cdot\mathcal{L}^{-1}\left\{\frac{1}{(s-3)}\right\}$$

$$-\frac{1}{50}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+2^2}\right\}$$

$$+\frac{18}{(25\times 2)}\cdot L^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}$$

$$= -\frac{1}{25}e^{-2t} + \frac{3}{50}e^{3t} - \frac{1}{50}\cdot e^{-t}\cdot\cos 2t$$

$$+\frac{18}{50}\cdot e^{-t}\sin 2t$$

$$= F(t).$$

$$\therefore \mathcal{L}^{-1}\left\{F(s)\right\} = -\frac{1}{25}e^{-2t} + \frac{3}{50}\cdot e^{3t} - \frac{1}{50}\cdot e^{-t}\cdot\cos 2t$$

$$+\frac{18}{50}\cdot e^{-t}\cdot\sin 2t$$

(Ans:)

[P.T.O.]

Answer to the Q. NO-05

Given,

$$\int_0^\infty t^2 \cdot e^{-2t} \cdot \cos t \cdot dt$$

$$= \int_0^\infty t^2 \cdot \cos t \cdot e^{-2t} \cdot dt$$

$$= \mathcal{L}\{t^2 \cdot \cos t\}_{(s=2)}$$

$$= \mathcal{L}\{t^2 \cdot F(t)\}_{(s=2)}$$

$$= (-1)^2 \cdot \frac{d^2}{ds^2}\left[\mathcal{L}\{F(t)\}\right]$$

$$= 1 \cdot \frac{d^2}{ds^2}\left[\mathcal{L}\{F(t)\}\right]$$

$$= 1 \cdot \frac{d^2}{ds^2}\left(\frac{s}{(s^2+1)}\right)$$

$$= \frac{d}{ds}\left[\frac{(s^2+1) - s \cdot 2s}{(s^2+1)^2}\right]$$

$$= \frac{d}{ds}\left(\frac{-s^2+1}{(s^2+1)^2}\right)$$

$$= \left[\frac{(s^2+1)^2 \cdot (-2s) - (1-s^2) \cdot 2(s^2+1) \cdot 2s}{(s^2+1)^4}\right]_{s=2}$$

[P.T.O.]

$$= \frac{4}{125}$$

$$\therefore \int_0^\infty t^2 e^{-2t} \cos t \, dt = \frac{4}{125}$$

(Ans:)