MAT216 (Linear Algebra and Fourier Analysis)
Assignment 03 (22 September 2024)

Deadline: 28 September, Saturday.
***You MUST submit your assignment in the Google Classroom assignment section. Otherwise, your assignment will not be counted

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* each question equals 4 marks, total marks $(4 \times 5) = 20$

1. Consider the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

   (a) Calculate all eigenvalues of A

   (b) Calculate the eigenvectors corresponding to each eigenvalues obtained in (a).

2. Find a matrix P that diagonalizes, $A = \begin{bmatrix} 5 & 1 \\ 0 & 6 \end{bmatrix}$. Also, calculate $P^{-1}AP$

3. Consider the function $f(x) = |x|$; $-\pi < x < \pi$, which is defined as follows:

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ -x, & -\pi < x < 0 \end{cases}$$

   (a) Sketch the graph of $f(x)$ and find out if the function $f(x)$ is an odd or an even function.

   (b) Find the Fourier series expansion of $f(x)$.

4. Find the Fourier sine series for the function $f(x) = x^3$; $(0, \pi)$

5. (a) Find Fourier cosine transform of $f(x) = e^{-2x}$, $x \geq 0$.

   (b) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$