Lecture: 04

Exercise 1.2

* KR

(25) Determine the values of 'a' for which the system has no solⁿ, exactly one solⁿ, or infinitely many solⁿs.

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

solⁿ.

Augmented matrix =>

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right] ; \; r_2' = r_2 - 3r_1, \; r_3' = r_3 - 4r_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & -7 & a^2-2 & a-14 \end{array}\right] ; \; r_2' = \left(-\frac{1}{7}\right) r_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right] ; \; r_3' = r_3 + 7r_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & (a+4)(a-4) & a-4 \end{array}\right] \quad \text{———} \ (*)$$

Now we see that $\Rightarrow$

(1.) when $a = 4$, then ~~the~~ all the entries of the last row becomes zero, and hence, there will be infinitely many solutions.

(2) when $a = -4$ then ~~(*)~~ $(*) \Rightarrow$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & 0 & -8 \end{array}\right]$$

The last row implies $\Rightarrow$

$$0.x + 0.y + 0.z = -8$$

~~which is inconsistent~~.

which is impossible for any real values of x, y, z.

So, the system is inconsistent & hence No sol^n exists.

(3) If $a \neq 4$ and $a \neq -4$, then, we can write (*) as -

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 1 & \frac{1}{a+4} \end{array}\right] ; \quad r_3' = \frac{r_3}{(a+4)(a-4)}$$

Now this is in row echelon form & all the columns has a leading 1. So, there exists a unique solution.

__Ans:__ Infinitely many sol^n when $a = 4$

No sol^n when $a = -4$

Unique sol^n when ~~ex~~ for all values of ~~except~~ $a \neq 4$ and $a \neq -4$.

(27) What condition a, b, & c must satisfy for the linear system to be consistent?

$\Rightarrow$
$$x+3y-z=a$$
$$x+y+2z=b$$
$$2y-3z=c$$

Sol^n. Augmented matrix $\Rightarrow$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 1 & 1 & 2 & b \\ 0 & 2 & -3 & c \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & -2 & 3 & b-a \\ 0 & 2 & -3 & c \end{array}\right] ; r_2' = r_2 - r_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & 1 & -\frac{3}{2} & \frac{a-b}{2} \\ 0 & 1 & -\frac{3}{2} & \frac{c}{2} \end{array}\right] ; r_2' = \left(-\frac{1}{2}\right) r_2, \quad r_3' = \left(\frac{1}{2}\right) r_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & 1 & -3/2 & \frac{a-b}{2} \\ 0 & 0 & 0 & \frac{c}{2}-\frac{a-b}{2} \end{array}\right] ; r_3' = r_3 - r_2$$

$$= \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & 1 & -\frac{3}{2} & \frac{a-b}{2} \\ 0 & 0 & 0 & \frac{c-a+b}{2} \end{array}\right]$$

The system will be consistent if

$$\frac{c-a+b}{2} = 0$$

$$\Rightarrow c-a+b = 0$$

$\therefore -a+b+c = 0$ is the required condition.

(Ans)

*AP VIMS

Determine the value of λ so that the system has

i) unique sol^n ii) many sol^n iii) no sol^n

$$x+y-z = 1$$
$$2x+3y+\lambda z = 3$$
$$x+\lambda y+3z = 2$$

Sol^n.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & 3 & \lambda & 3 \\ 1 & \lambda & 3 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & \lambda-1 & 4 & 1 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 - 2r_1 \\ r_3' = r_3 - r_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & 4-(\lambda-1)(\lambda+2) & 2-\lambda \end{array}\right] \quad ; r_3' = r_3 - (\lambda-1) r_2$$

$$4 - (\lambda-1)(\lambda+2)$$
$$= 4 - \{\lambda^2+\lambda-2\}$$
$$= -\lambda^2-\lambda+6$$
$$= -\lambda^2-3\lambda+2\lambda+6 = -\lambda(\lambda+3) + 2(\lambda+3)$$
$$= (\lambda+3)(2-\lambda)$$

$$\approx \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & (\lambda+3)(2-\lambda) & 2-\lambda \end{array}\right] \quad \text{---} (*)$$

We see —

① if $\lambda = 2$, then all the entries in the last row are 0. i.e. we have $0 = 0$ which is true. So, the system has infinitely many sol^n.

② If $\lambda = -3$, then last row becomes

$$0 \cdot x + 0 \cdot y + 0 \cdot z = 5$$

$$\Rightarrow 0 = 5 \text{; which is not true}$$

i.e the system becomes inconsistent. So, no sol^n.

③ If $\lambda \neq 2$ and $\lambda \neq -3$, then $(*) \Rightarrow$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & 1 & \frac{1}{\lambda+3} \end{array}\right] ; \; r_3' = \frac{r_3}{(\lambda+3)(2-\lambda)}$$

i.e. all the column's have a leading 1. So, the system has unique sol^n.

<u>Ans:</u> many sol^n if $\lambda = 2$, no sol^n if $\lambda = -3$, unique sol^n for all $\lambda \neq 2$ and $\lambda \neq -3$.

*** V.V.S

Q. For what values of λ & μ, the system

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has - i) no sol^n. ii) many sol^n, iii) unique sol^n?

Sol^n.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right] ; \begin{array}{l} r_2' = r_2 - r_1 \\ r_3' = r_3 - r_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right] ; r_3' = r_3 - r_2$$

we see -

① If $\lambda=3$ and $\mu=10$, the last row is $0=0$.

So many sol^n.

② If $\lambda=3$, & $\mu \neq 10$, then, last row is

$0 = \mu-10 \neq 0$; which is not true

So, no sol^n.

(3) for unique sol^n co-efficient of z in the 3rd row must be non-zero, i.e. $\lambda - 3 \neq 0$

$\Rightarrow \lambda \neq 3$

~~but $\mu - 10$ can be any~~

but the right hand constant in the 3rd eq^n can be anything. so, μ can be any real number i.e. $\mu \in \mathbb{R}$.

Ans: many sol if $\lambda = 3, \& \mu = 10$.

no sol^n if $\lambda = 3, \& \mu \neq 10$

unique sol^n if $\lambda \neq 3, \& \mu \in \mathbb{R}$.

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