1 Interpolacija

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

f je interpolant na (x_i, y_i), ce $f(x_i) = y_i$ za vse i.

1.1 Cebiševe tocke

Za njih velja da je interpolacija numericno stabilna.

1.2 Zlepki

Funkcije z vec predpisi

2 Hermitov kubicni zlepek

Poleg vrednosti funkcije, je na voljo tudi vrednost odvoda.

$$(x_1, y_1, dy_1), (x_2, y_2, dy_2), \ldots, (x_n, y_n, dy_n)$$

So zlepki C1, saj drugi odvodi niso zvezni.

Podatkov: $2n$, $n - 1$ intervalov.

Na vsakem intervalu imamo 4 podatke.

Enacbe so med intervali med seboj **neodvisne**.

Ko resujemo sistem enacb s 4 neznankami: y_0, y_1, dy_0, dy_1 racunamo naslednji interpolacijski polinom:

$$p_3(x) = a + bx + cx^2 + dx^3$$

Najprej resimo problem na $[0, 1]$.

Imamo naslednjo Hermitovo bazo polinomov.

$$h_{00}(t)(p(0)) = 1$$
$$h_{00}(t)(p(1)) = 0$$
$$h_{00}(t)(p'(0)) = 0$$
$$h_{00}(t)(p'(1)) = 0$$

$$h_{01}(t)(p(0)) = 0$$
$$h_{01}(t)(p(1)) = 1$$
$$h_{01}(t)(p'(0)) = 0$$

$$h_{01}(t)(p'(1)) = 0$$

$$h_{10}(t)(p(0)) = 0$$
$$h_{10}(t)(p(1)) = 0$$
$$h_{10}(t)(p'(0)) = 1$$
$$h_{10}(t)(p'(1)) = 0$$

$$h_{11}(t)(p(0)) = 0$$
$$h_{11}(t)(p(1)) = 0$$
$$h_{11}(t)(p'(0)) = 0$$
$$h_{11}(t)(p'(1)) = 1$$

$$h_{00}:$$

$$h_{ij}(0) = a_{ij}$$
$$h_{ij}(1) = a_{ij} + b_{ij} + c_{ij} + d_{ij}$$
$$h'_{ij}(0) = b_{ij}$$
$$h'_{ij}(1) = b_{ij} + 2c_{ij} + 3d_{ij}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$

Preslikati moramo interval $[0, 1] \leftrightarrow [x_i, x_{i+1}]$

$$t : 0$$
$$f(x(t)) = y_i$$
$$\frac{d}{dt} = dy_i$$

$$t : 1$$
$$f(x(t)) = y_{i+1}$$
$$\frac{d}{dt} = (x_{i+1} - x_i)$$

$$x : x_i$$
$$f(x) = y_i$$
$$f'(x) = dy_i$$

$$x : x_{i+1}$$
$$f(x) = y_{i+1}$$
$$f'(x) = dy_{i+1}$$

Ce t definiramo kot: $t = \frac{x-x_i}{x_{i+1}-x_i}$

Potem so vrednosti v robnih tockah:

$$t(x_i) = 0$$
$$t(x_{i+1}) = 1$$

Odvodi so naslednji: v t odvajamo po x in dobimo:

$$\frac{dt}{dx} = \frac{1}{x_{i+1} - x_i}$$

Inverz je reciprocna vrednost:

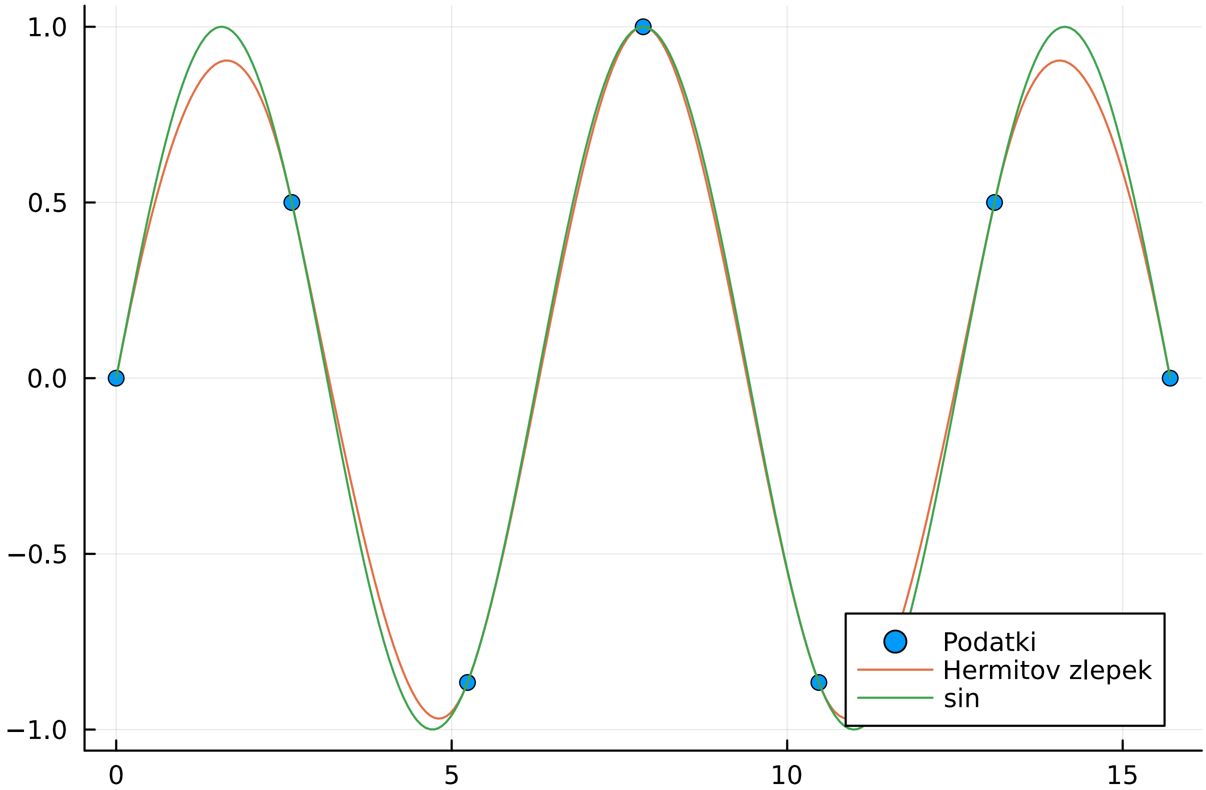
$$\frac{dx}{dt} = \frac{x_{i+1} - x_i}{1}$$

$$\frac{d}{dt} f(x(t)) = f'(x) * x'(t) = f'(x) * (x_{i+1} - x_i)$$

```
using Vaje08
using Plots

x = range(0, 5pi, 7)
y = sin.(x)
dy = cos.(x)

scatter(x, y, label="Podatki")
Z = HermitovZlepek(x, y, dy)
plot!(x -> Z(x), 0, 5pi, label="Hermitov zlepek")
plot!(sin, 0, 5pi, label="sin")
```


1.0
0.5
0.0
−0.5
−1.0
0
5
10
15
Podatki
Hermitov zlepek
sin