

```
using Vaje09
using Plots
```

1 Numericna integracija

Splosna oblika numericne interacije je:

$$\int_a^b f(x)dx \approx \sum_{i=1}^n w_i f(x_i)$$

kjer so x_i vozlišča in w_i uteži.

Integracijsko pravilo zahteva da določimo utezi ter vozlišca.

Uporabili bomo metode:

- trapezno pravilo
- Simpsonovo pravilo
- Gauss-Legendrove kvadrature

Ocenili bomo napako in primerjali rezultate.

1.1 Sestavljeno trapezno pravilo

Osnovno trapzeno pravilo:

$$\int_a^b f(x)dx \approx \frac{b-a}{2}(f(a) + f(b)) + R_f$$

kjer je R_f napaka, odvisna od f .

Sestavljeno trapzeno pravilo z n podintervali:

$$\int_a^b f(x)dx \approx \sum_{i=1}^n \frac{h}{2}(f(x_i) + f(x_{i+1})) + R_f = \frac{h}{2}(f(a) + 2f(a+h) + \dots + 2f(b-h) + f(b)) + R_f$$

1.1.1 Izračunajmo integral sin na $[0, 1]$

$$\int_0^1 \sin(x)dx = -\cos(1) + \cos(0) = 1 - \cos(1)$$

```
Ip = 1 - cos(1) # prava vrednost
```

```
| 0.45969769413186023
```

Izračunajmo integral s trapezno metodo z $n = 5$ koraki.

```
It5 = integral(sin, trapez, (0, 1, 5))
```

```
| 0.4581643459604437
```

```
relative_err5 = (It5 - Ip) / Ip
```

```
| -0.0033355576740759334
```

Izračunajmo integral s trapezno metodo z $n = 10$ koraki.

```
It10 = integral(sin, trapez, (0, 1, 10))
```

```
| 0.4593145488579764
```

```
relative_err10 = (It10 - Ip) / Ip
```

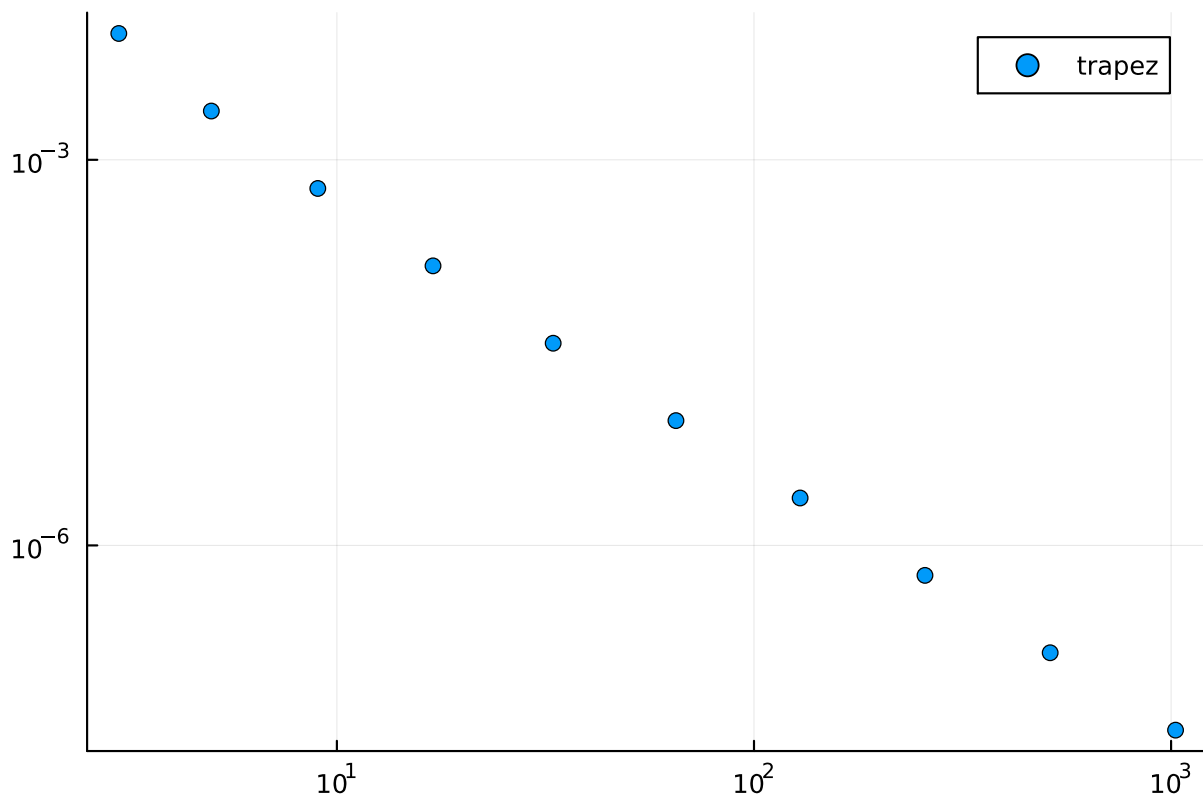
```
| -0.000833472255299005
```

Graf napake v odvisnosti od števila korakov trapezne metode.

```
napaka_t(n) = Ip - integral(sin, trapez, (0, 1, n))
```

```
tabn = 2 .^ (1:10)
```

```
scatter(tabn .+ 1, abs.(napaka_t.(tabn)), xaxis=:log10, yaxis=:log10, label="trapez")
```



```
# Približen red metode
red_trapez = hcat(ones(length(tabn)), log10.(tabn)) \ log10.(abs.(napaka_t.(tabn)))

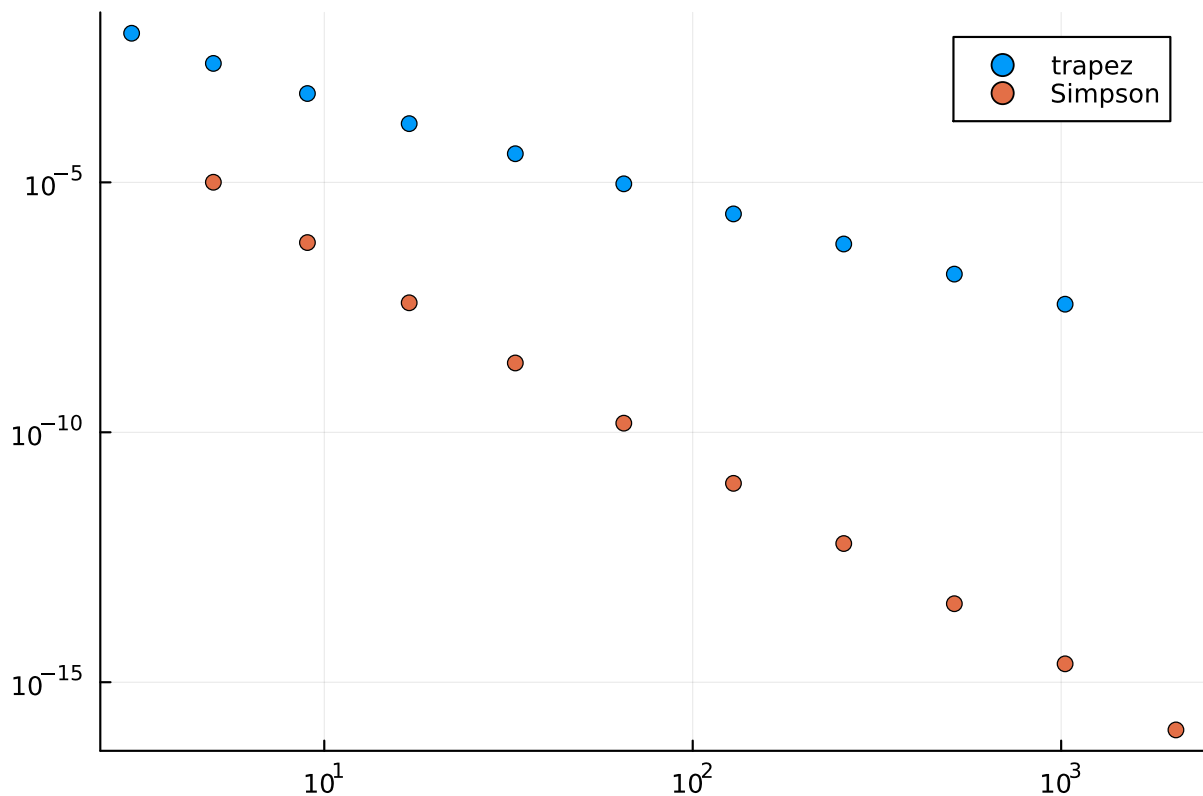
2-element Vector{Float64}:
-1.41579447032699
-2.00040613639406
```

1.2 Simpsonovo pravilo

Graf napake za Simpsonovo metodo

```
napaka_s(n) = Ip - integral(sin, simpson, (0, 1, n))

scatter!(2 * tabn .+ 1, abs.(napaka_s.(tabn)), xaxis=:log10, yaxis=:log10, label="Simpson")
```



| 1.8818280267396403e-13

Graf napake za Gauss-Legendrove kvadrature

```
napaka_gl(n) = Ip - integral(sin, gl, (0, 1, n))
```

```
scatter!(tabn, abs.(napaka_gl.(tabn)) .+ 5e-16, xaxis=:log10, yaxis=:log10,  
label="Gauss-Legendre")
```

