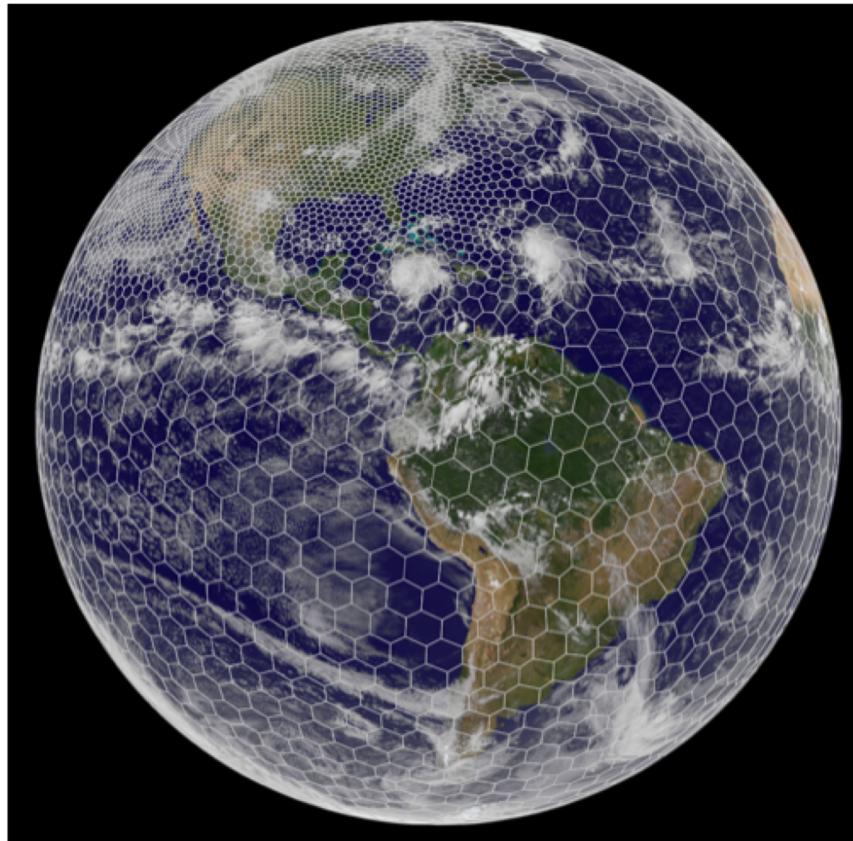


# An Overview of the Structure of MPAS Meshes

A defining feature of MPAS models is their use of **centroidal Voronoi tessellations** (CVTs) with a C-grid staggering

- When constrained to lie on the surface of a sphere, we often call them spherical centroidal Voronoi tessellations (SCVTs)



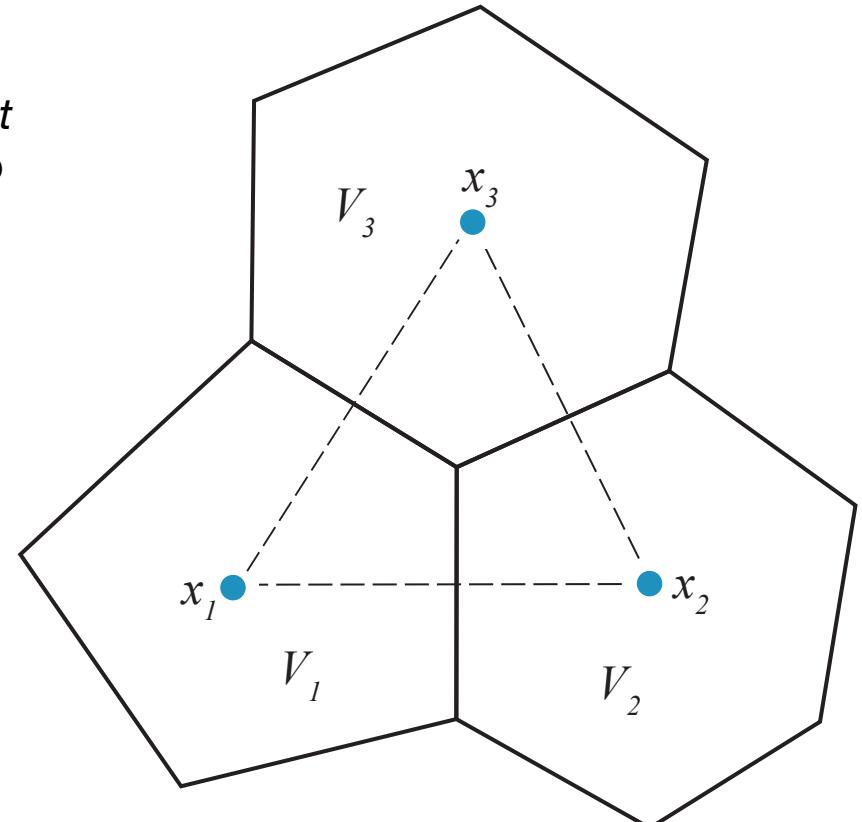
A defining feature of MPAS models is their use of **centroidal Voronoi tessellations** (CVTs) with a C-grid staggering

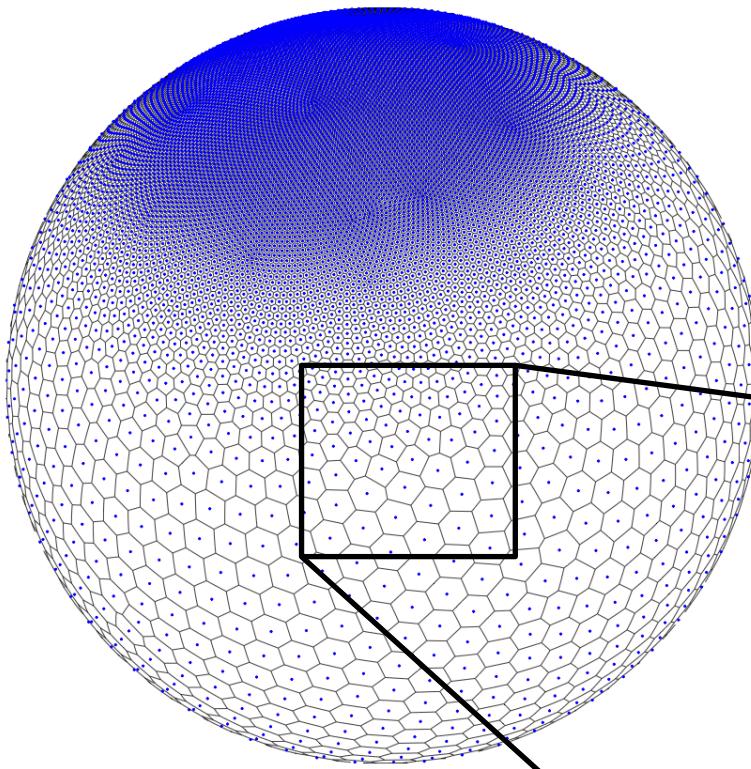
- When constrained to lie on the surface of a sphere, we often call them spherical centroidal Voronoi tessellations (SCVTs)

**Voronoi** = each grid volume (cell)  $V_i$  is uniquely associated with a *generating point*  $x_i$  such that all points within  $V_i$  are closer to  $x_i$  than to any other  $x_j$

- Lines joining generating points of adjacent cells are
  1. bisected by the shared cell face; and
  2. intersect the shared cell face at a right angle.

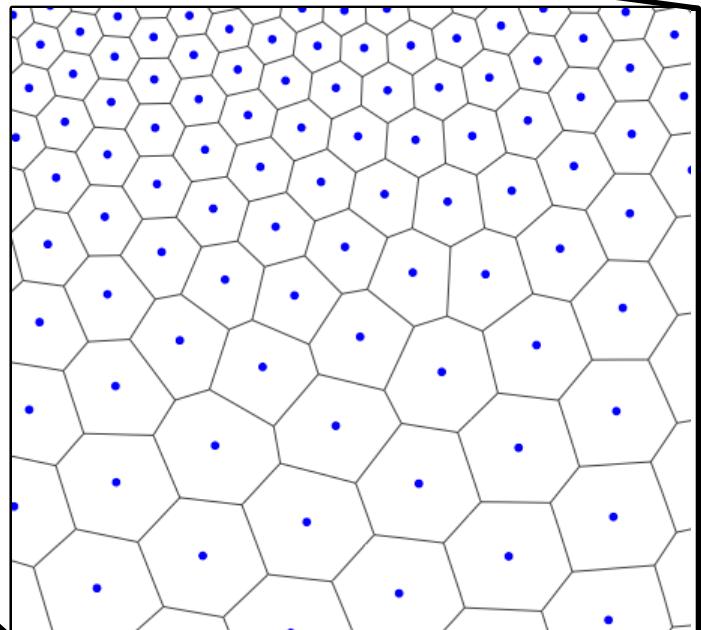
**Centroidal** = the generating point for each Voronoi cell is also the mass centroid of that cell (**w.r.t. some density function**)



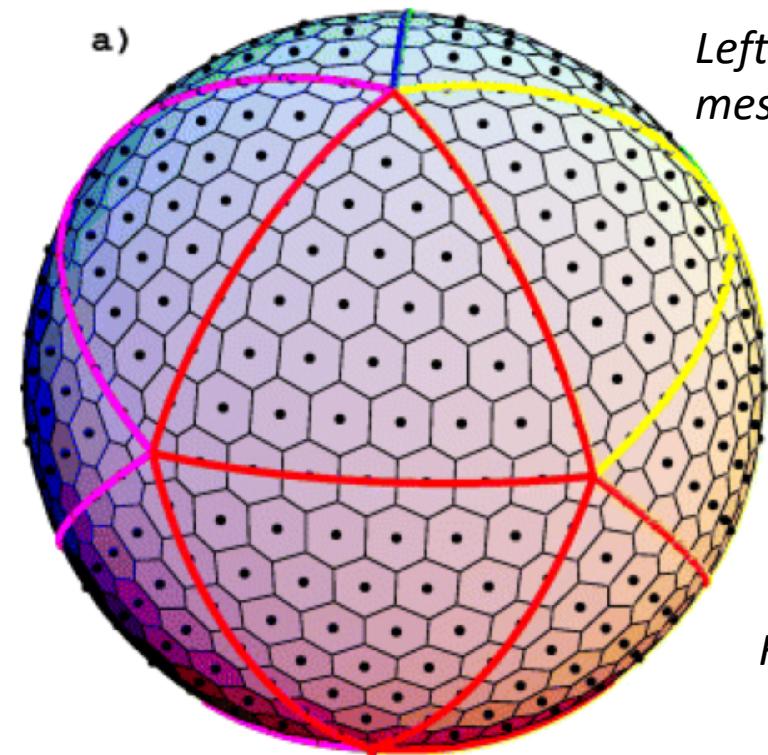


*Observe that we have a fully unstructured horizontal mesh, not just a deformation of the icosahedral mesh! **Cells may have 5, 6, 7, or more sides.***

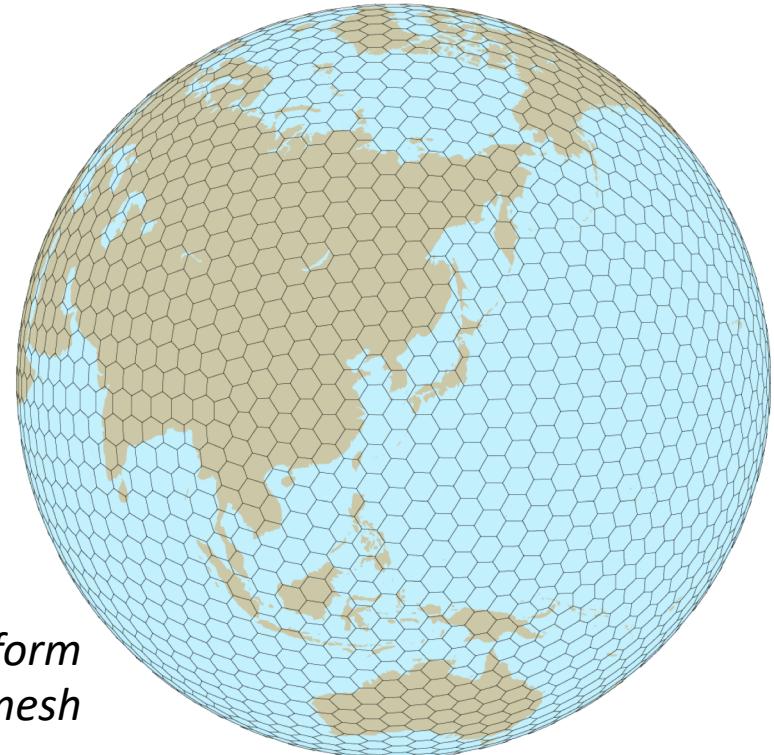
- Given a set of generating points, the primal mesh (finite volume mesh) in MPAS is defined by the Voronoi tessellation induced by this set
- The centroidal aspect of CVTs is used to produce meshes with smoothly-varying resolution



Quasi-uniform MPAS meshes look just like icosahedral meshes...



*Right: A quasi-uniform MPAS mesh*

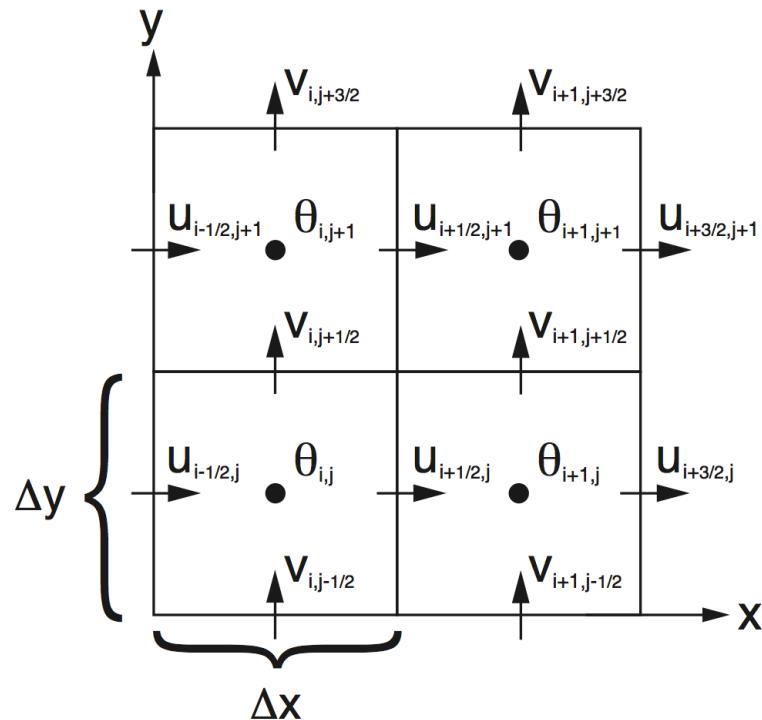


... but the MPAS solver considers every mesh as a completely general, unstructured mesh: there are no special cases!

## How do MPAS, *and you*, keep track of this unstructured Voronoi mesh?

One can start to imagine way to identify neighboring cells implicitly based on the index or location of each cell

- In a rectangular mesh, our neighbors are at  $(i+1, j)$ ,  $(i-1, j)$ ,  $(i, j+1)$ ,  $(i, j-1)$

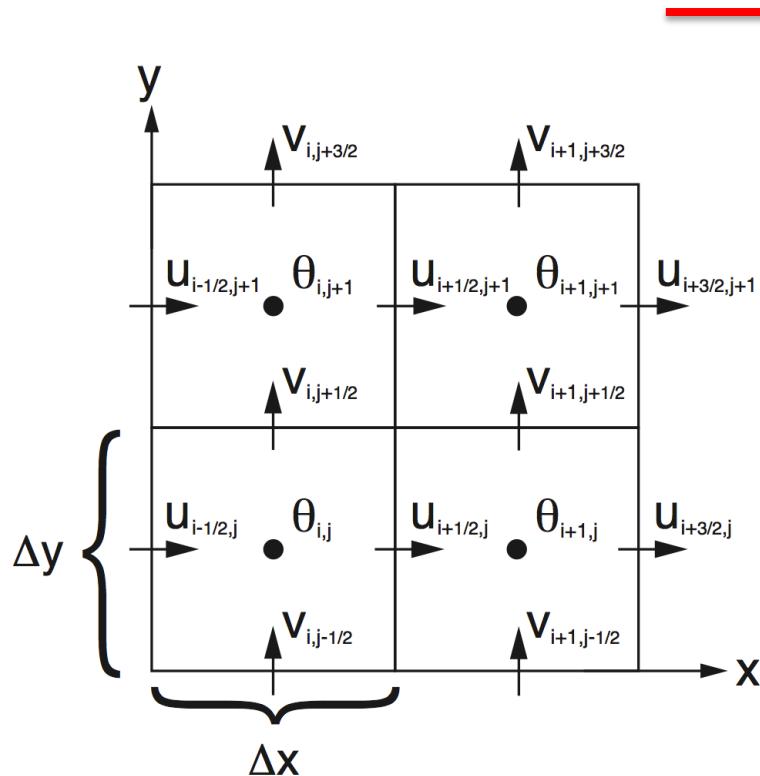


Above: A region from the ARW C-staggered grid, stored in a 2-d array.

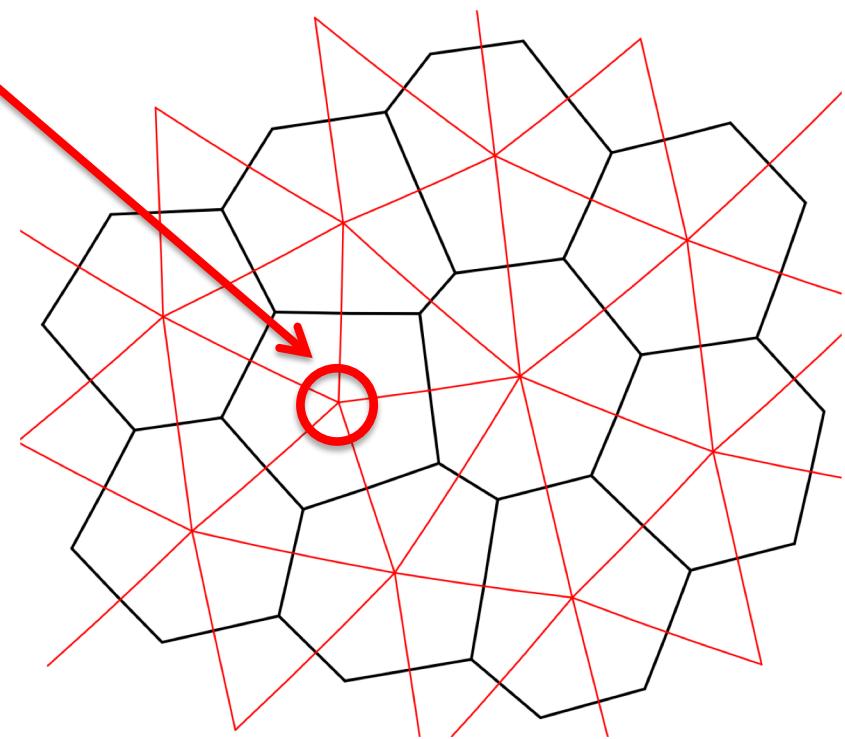
# How do MPAS, *and you*, keep track of this unstructured Voronoi mesh?

One can start to imagine way to identify neighboring cells implicitly based on the index or location of each cell

- In a rectangular mesh, our neighbors are at  $(i+1, j)$ ,  $(i-1, j)$ ,  $(i, j+1)$ ,  $(i, j-1)$
- Who is the “next” cell after this one in any direction?



*Above:* A region from the ARW C-staggered grid, stored in a 2-d array.

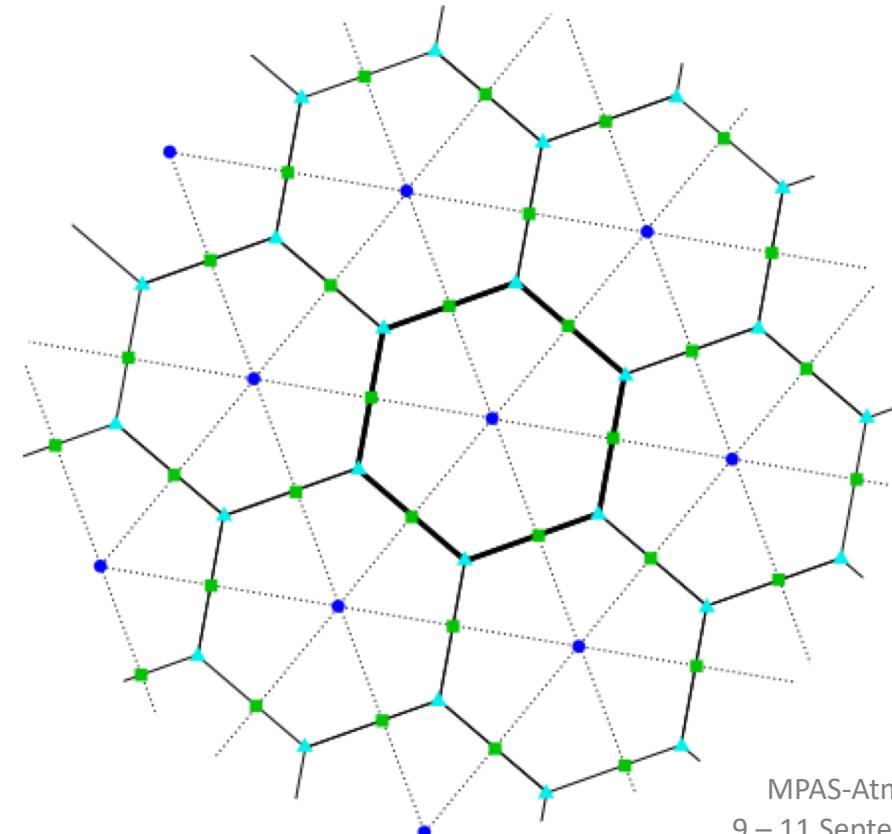


*Above:* A region from an MPAS mesh showing Voronoi regions (black) and Delaunay triangles (red).

# How do MPAS, *and you*, keep track of this unstructured Voronoi mesh?

Schemes for implicitly finding the indices/identities (the “IDs”) of neighboring mesh elements (i.e., cells, edges, vertices) are bound to fail...

... so we must find them explicitly through connectivity fields that are the foundation of the MPAS mesh representation.

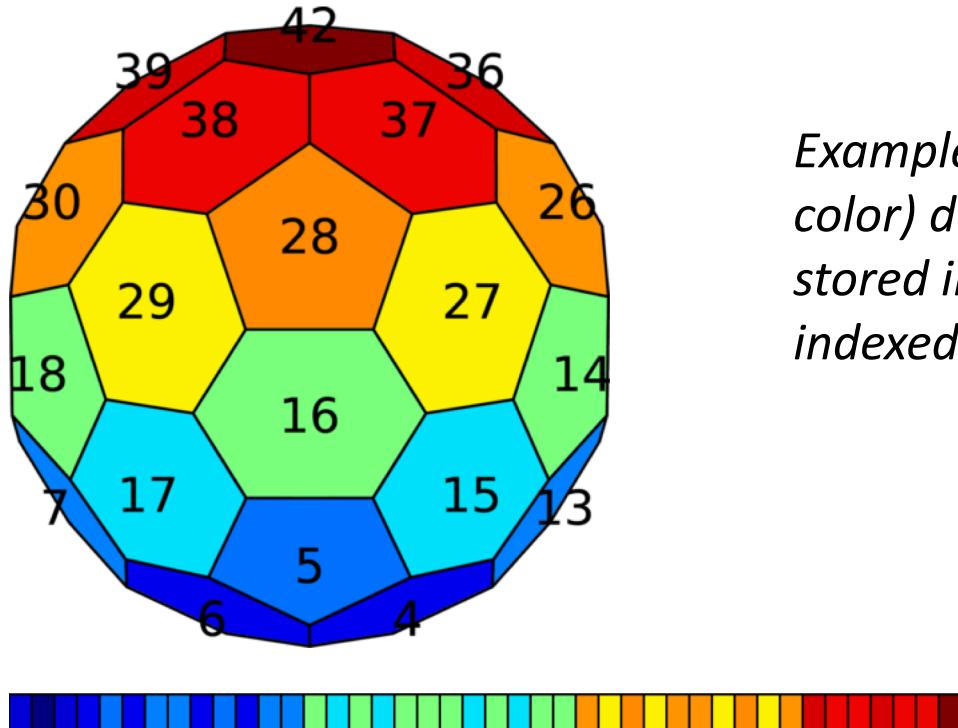


Three types of mesh elements are tracked in the mesh representation:

- **Cell** locations (blue circles) - the generating points of the Voronoi mesh
- **Vertex** locations (cyan triangles) - the corners of primal mesh cells
- **Edge** locations (green squares) - the points where the dual mesh edges intersect the primal mesh edges

How do MPAS, *and you*, keep track of this unstructured Voronoi mesh?

*For the unstructured, horizontal dimension there is nothing to be gained from using 2-d arrays...  
...hence, the horizontal dimension is collapsed into a single array dimension: we then have a simple list of elements*

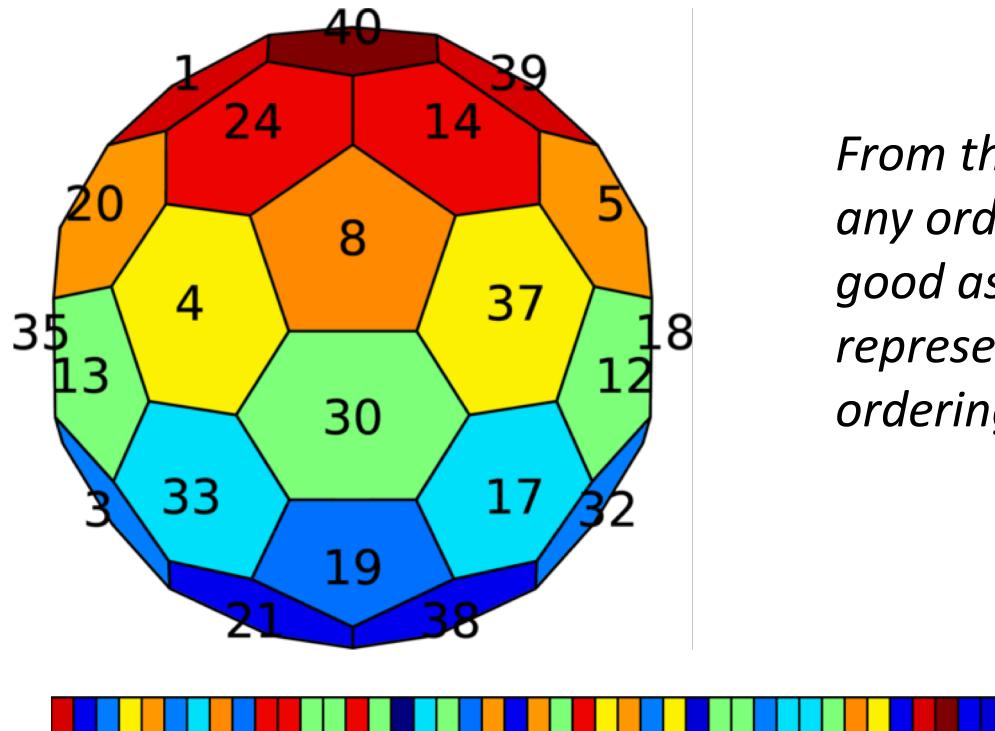


*Example: For some 2-d field (shown in color) defined on mesh cells, that field is stored in a 1-d array (bottom) that is indexed by cell number (labeled in black).*

## How do MPAS, *and you*, keep track of this unstructured Voronoi mesh?

*For the unstructured, horizontal dimension there is nothing to be gained from using 2-d arrays...*

*...hence, the horizontal dimension is collapsed into a single array dimension: we then have a simple list of elements*



*From the perspective of the MPAS solver, any ordering of cells in the mesh is as good as any other<sup>1</sup>, as long as the mesh representation is consistent with this ordering.*

<sup>1</sup>*Though some orderings may give better performance, e.g., due to better cache reuse.*

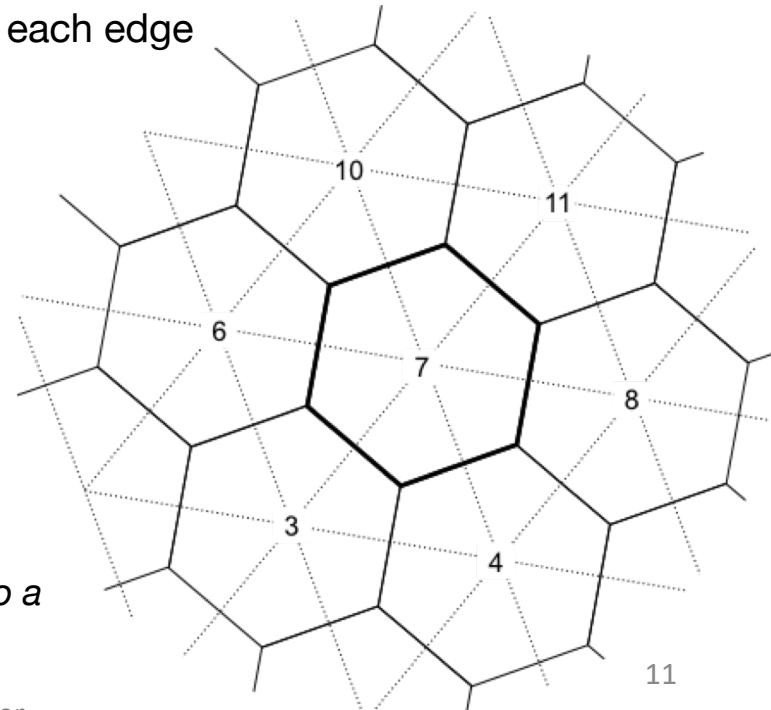
## Explicit connectivity fields describe the structure of a mesh

- **nEdgesOnCell(nCells)** – the number of neighbors for each cell
- **cellsOnCell(maxEdges, nCells)** – the indices of neighboring cells for each cell
- **edgesOnCell(maxEdges, nCells)** – the indices of bounding edges for each cell
- **verticesOnCell(maxEdges, nCells)** – the indices of corner vertices for each cell
- **edgesOnVertex(vertexDegree,nVertices)** – the indices of edges incident with each vertex
- **verticesOnEdge(2,nEdges)** – the indices of endpoint vertices for each edge
- **cellsOnVertex(vertexDegree,nVertices)** – the indices of cells meeting at each vertex
- **cellsOnEdge(2,nEdges)** – the indices of cells separated by each edge

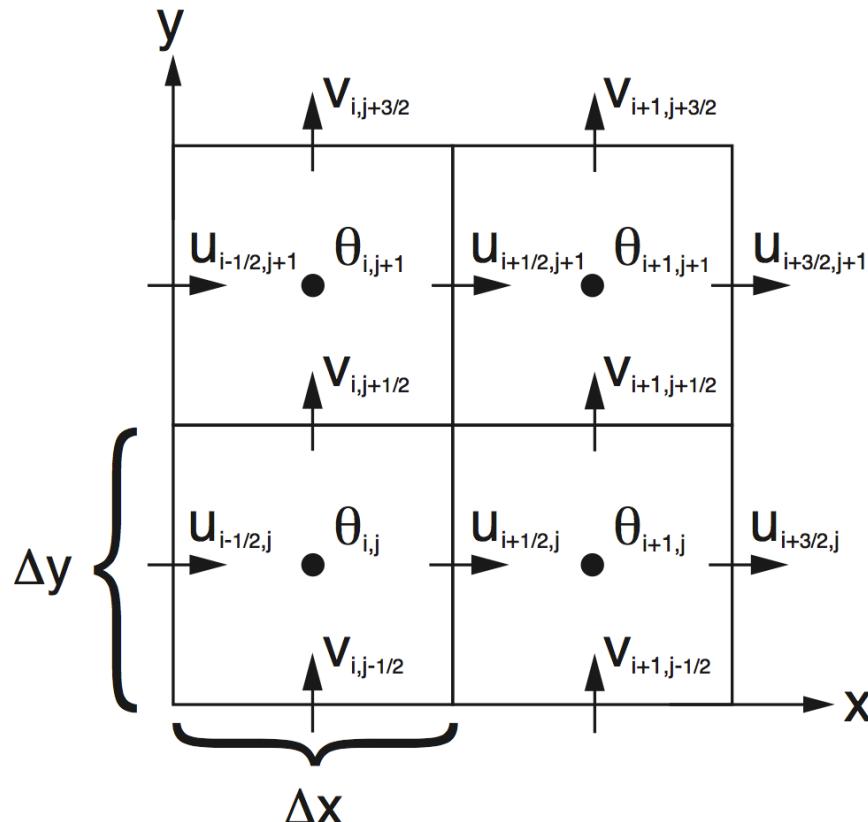
```

nEdgesOnCell(7)=6   cellsOnCell(1,7)=8
                     cellsOnCell(2,7)=11
                     cellsOnCell(3,7)=10
                     cellsOnCell(4,7)=6
                     cellsOnCell(5,7)=3
                     cellsOnCell(6,7)=4
    
```

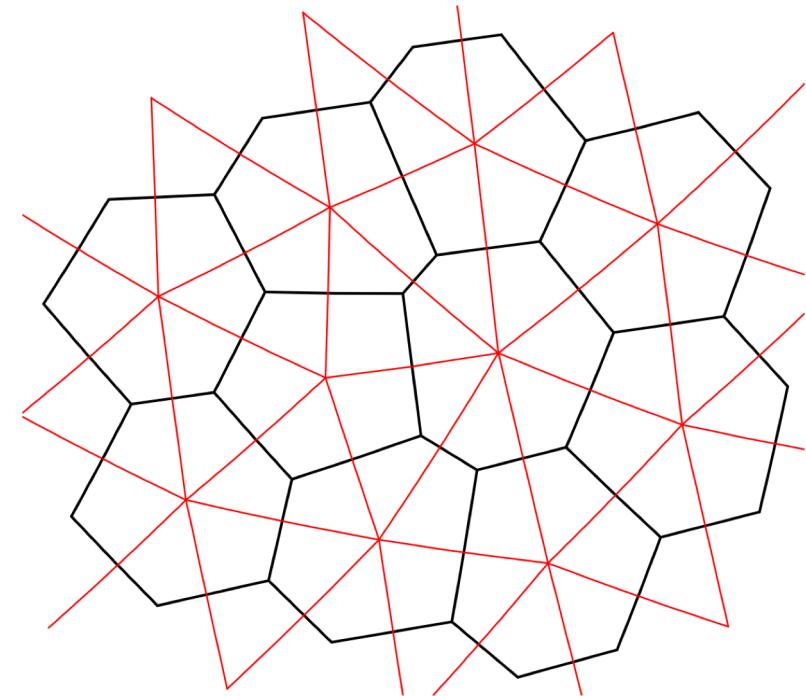
*At model start-up, all indices in these arrays are re-numbered to a local indexing scheme.*



In a WRF grid, which directions represent positive U and positive V?

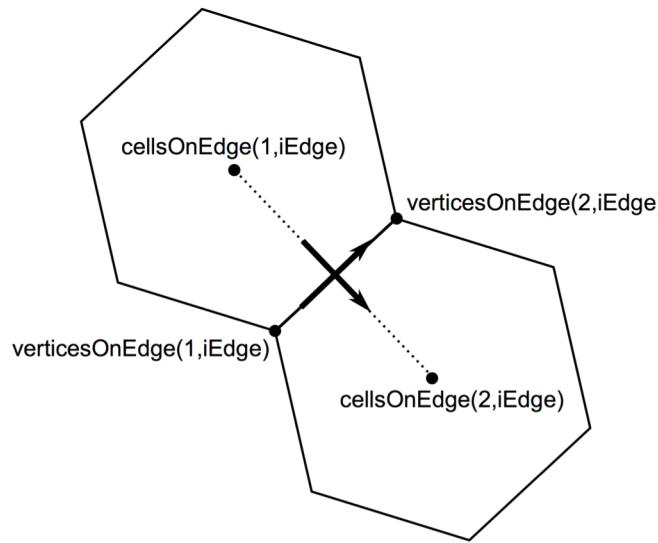


On a rectangular grid, one might say that positive U flows from left to right, and positive V flows from bottom to top when looking down on the xy-plane.



On a CVT mesh, one could introduce a similar definition, but we have only U, not V, so such a definition becomes more complicated...

## Horizontal wind vectors: how are these defined?



Earth-relative horizontal winds,  $u_{\text{zonal}}$  and  $u_{\text{meridional}}$ , can be calculated using  $u$  and  $v$ :

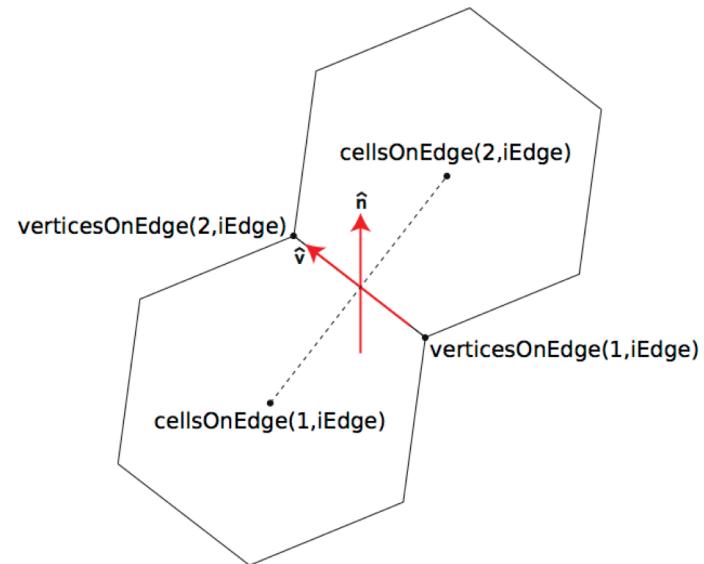
$$\begin{bmatrix} u_\lambda \\ u_\phi \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $\alpha$  is angleEdge.

Positive  $u$  (normal) velocity is always defined as flow from `cellsOnEdge(1,iEdge)` to `cellsOnEdge(2,iEdge)` for edge `iEdge`

Positive  $v$  (tangential) velocity is always defined as flow from `verticesOnEdge(1,iEdge)` to `verticesOnEdge(2,iEdge)` for edge `iEdge`

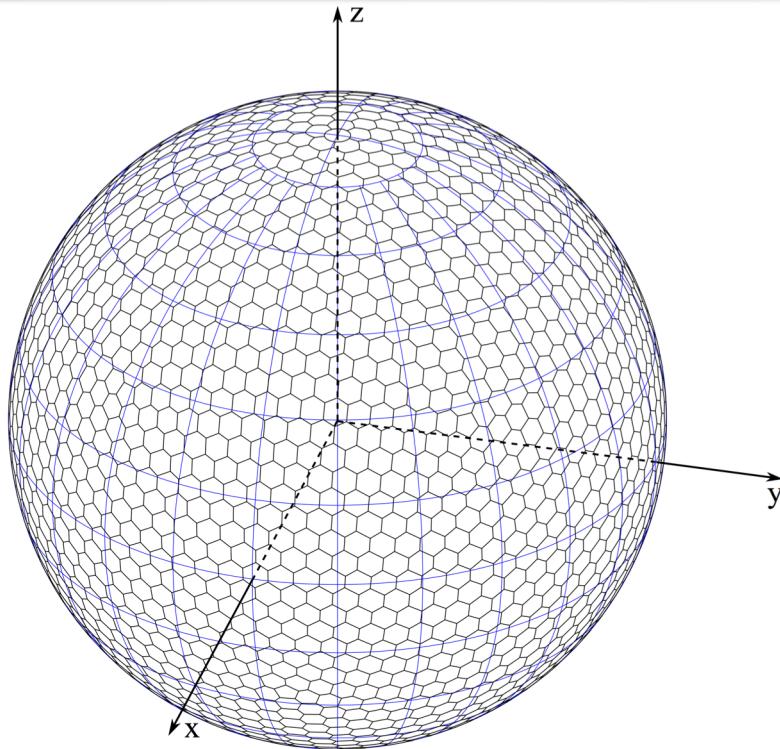
The cross product of the positive  $u$  and  $v$  vectors always points upward (out of the plane)



`angleEdge(nEdges)` – angle between east and positive  $u$

$$\text{angleEdge} = \arcsin \|\hat{n} \times \hat{v}\|$$

## Which mesh geometries does MPAS support?



On the surface of the sphere: all distances and areas are computed in spherical geometry.

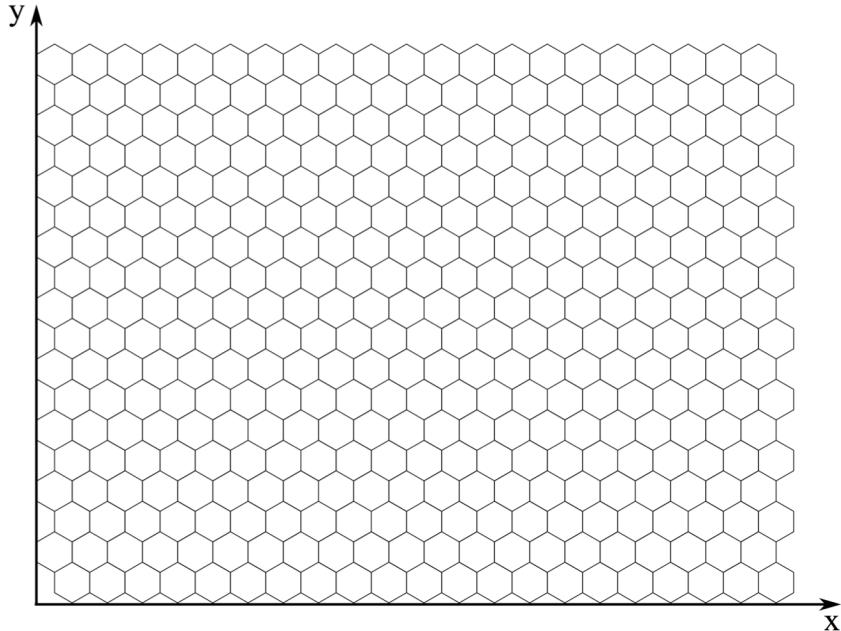
$$x = r \cos(\lambda) \cos(\phi)$$

$$y = r \sin(\lambda) \cos(\phi)$$

$$z = r \sin(\phi)$$

$$\phi = \arcsin\left(\frac{z}{r}\right)$$

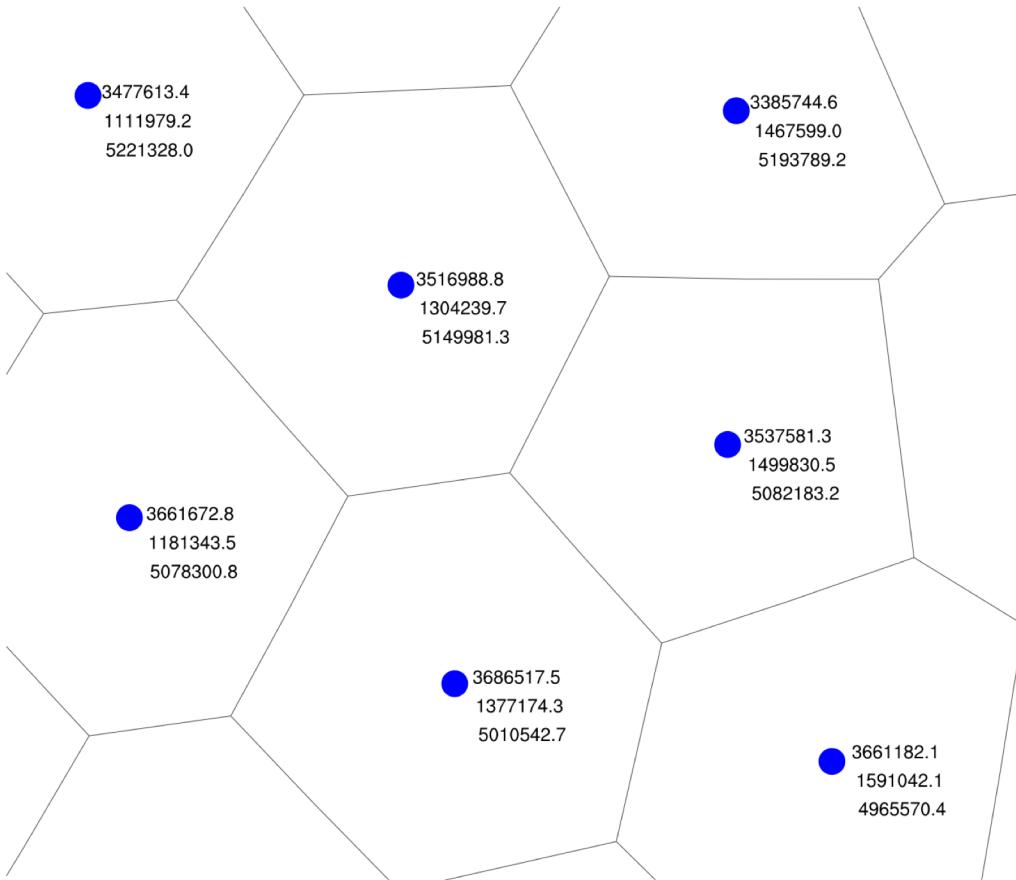
$$\lambda = \arctan\left(\left|\frac{y}{x}\right|\right)$$



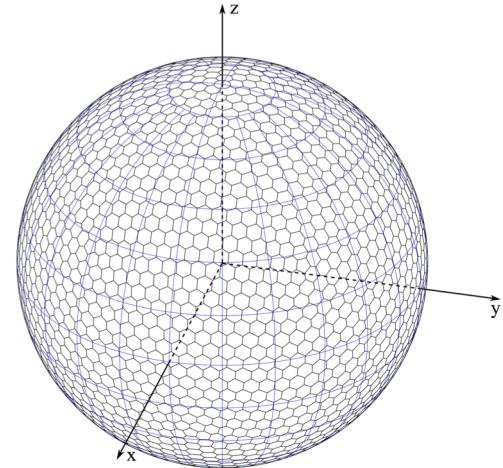
In the Cartesian plane: all distances and areas are computed in Euclidean geometry.

*In the plane, only doubly periodic boundaries are currently supported.*

## Notes about MPAS mesh geometry



Above: Cartesian coordinates for cell locations near (52.9°N lat, 20.8°E lon) in a variable-resolution spherical mesh with radius 6371229 m.



Global Cartesian coordinates are computed for each element

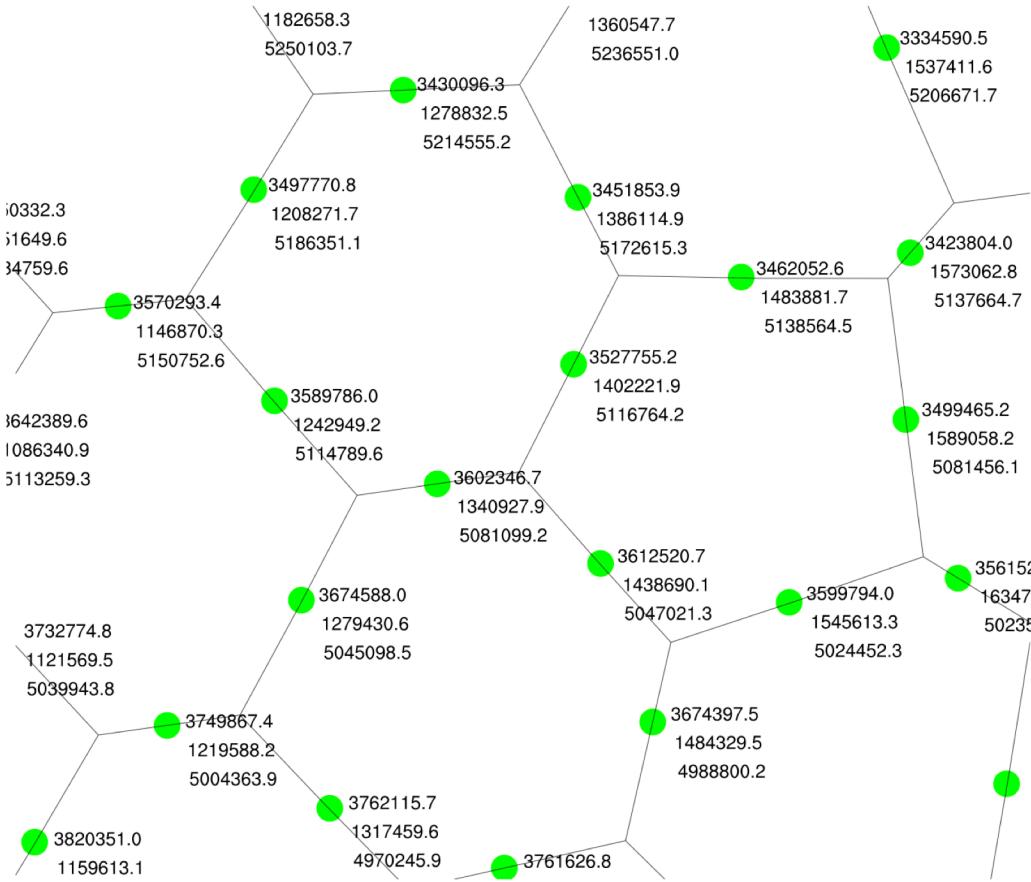
- For planar meshes, coordinates lie in the plane  $z=0$
- For spherical meshes, coordinates lie on the surface of the sphere

For cells: **xCell**, **yCell**, **zCell**

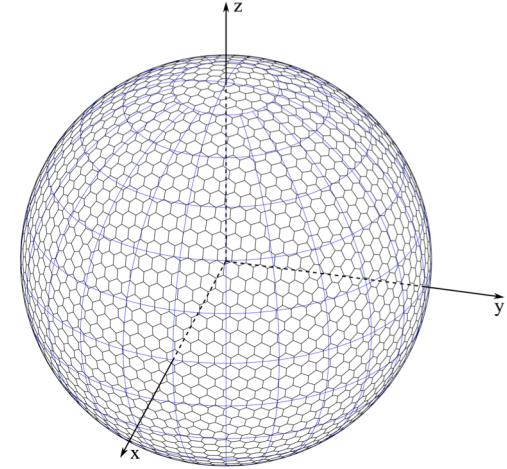
Latitudes and longitudes are computed from Cartesian coordinates as described earlier

- positive x-axis through 0° longitude
- positive y-axis through 90° longitude
- positive z-axis through 90° latitude

## Notes about MPAS mesh geometry



Above: Cartesian coordinates for cell locations near (52.9°N lat, 20.8°E lon) in a variable-resolution spherical mesh with radius 6371229 m.



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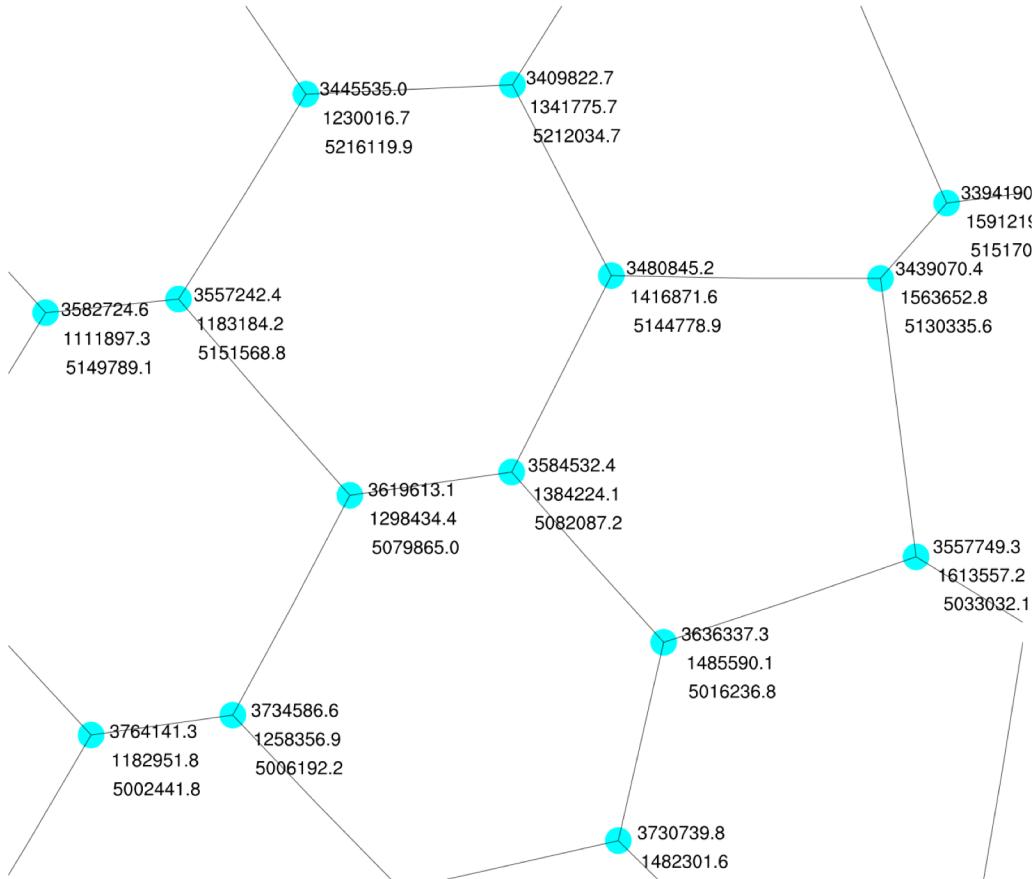
- For planar meshes, coordinates lie in the plane  $z=0$
- For spherical meshes, coordinates lie on the surface of the sphere

For cells: **xEdge**, **yEdge**, **zEdge**

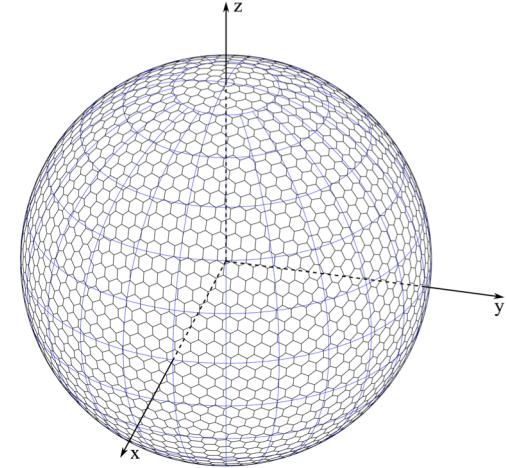
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# Notes about MPAS mesh geometry



Above: Cartesian coordinates for cell locations near (52.9°N lat, 20.8°E lon) in a variable-resolution spherical mesh with radius 6371229 m.



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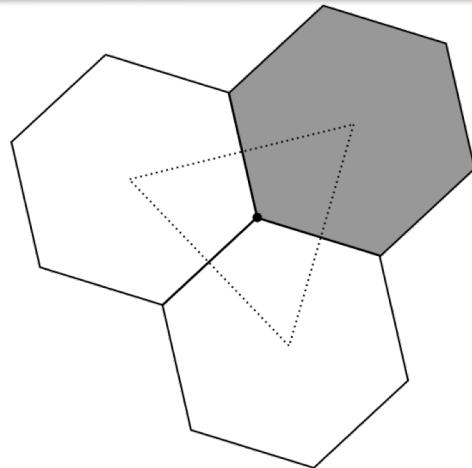
- For planar meshes, coordinates lie in the plane  $z=0$
- For spherical meshes, coordinates lie on the surface of the sphere

For cells: **xVertex**, **yVertex**, **zVertex**

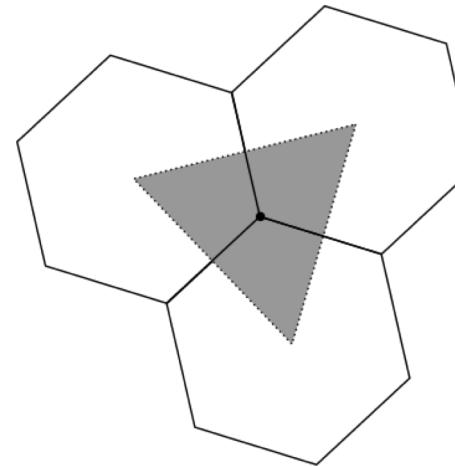
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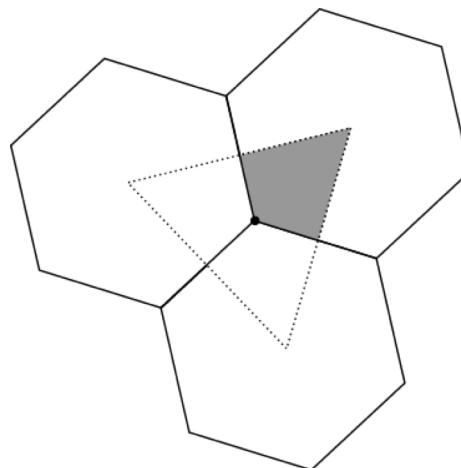
## Mesh geometry fields in MPAS



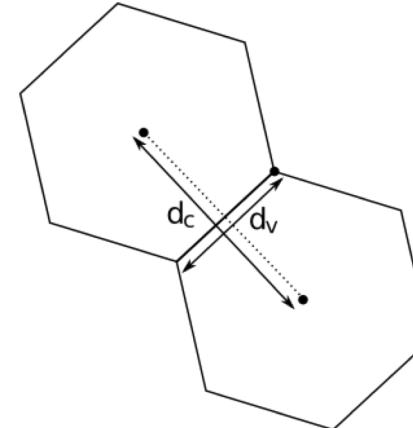
`areaCell(nCells)` – area of each cell



`areaTriangle(nVertices)` – area of each dual-grid cell

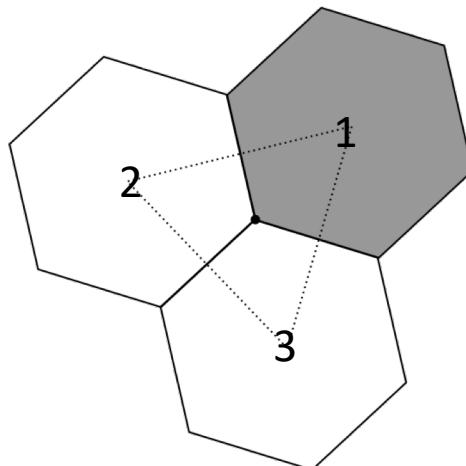


`kiteAreasOnVertex(vertexDegree,nVertices)` –  
area of intersection between dual- and primal-mesh cells

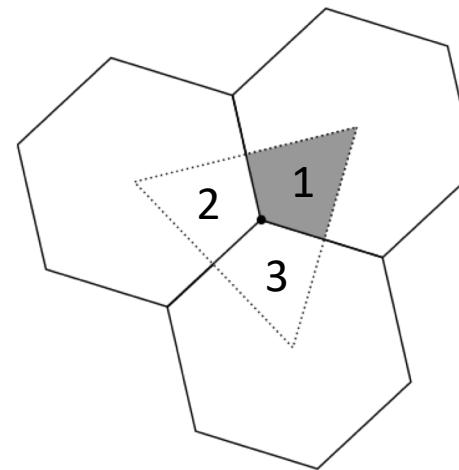


`dcEdge(nEdges)` – distances between cell centers  
`dvEdge(nEdges)` – length of each edge

- For **\*On** arrays (e.g., `cellsOnCell`), elements are listed in anti-clockwise order
  - Whenever possible, starting points are consistent between indexing arrays (e.g., `cellsOnVertex` and `kiteAreasOnVertex`)
  - E.g., the first `edgeOnCell` separates a given cell from the first `cellOnCell`
- All indices are 1-based
  - (MPAS is written in Fortran, after all...)



`cellsOnVertex(:,V)`



`kiteAreasOnVertex(:,V)`

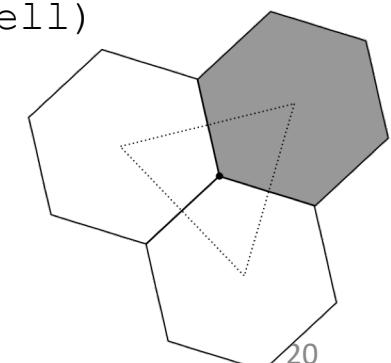
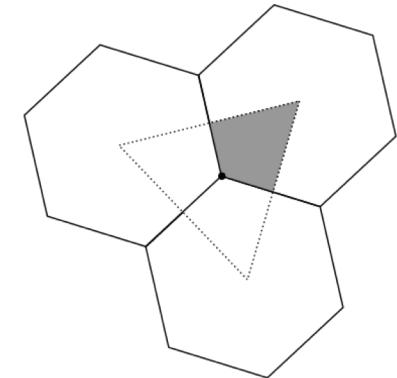
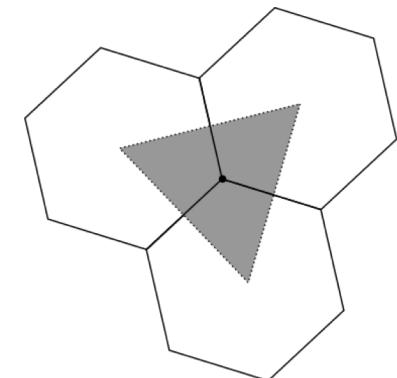
An example of using mesh fields: Averaging a vertex-based field, `vorticity(nVertLevels, nVertices)`, to cells as `vortcell(nVertLevels, nCells)`:

```

vortcell(:,:) = 0.0
do iVtx = 1, nVertices
do j = 1, vertexDegree
    iCell = cellsOnVertex(j,iVtx)
    vortcell(:,j) = vortcell(:,j) +
                    kiteAreasOnVertex(j,iVtx) * vorticity(:,iVtx)
end do
end do

do iCell = 1, nCells
    vortcell(:,iCell) = vortcell(:,iCell) / areaCell(iCell)
end do

```



## What do MPAS meshes look like in netCDF files?

When stored in netCDF files (“*grid.nc*”), MPAS meshes have at least the following dimensions:

dimensions:

```
nCells = 40962 ;  
nEdges = 122880 ;  
nVertices = 81920 ;  
  
maxEdges = 10 ;  
maxEdges2 = 20 ;  
TWO = 2 ;  
vertexDegree = 3 ;
```

The number of cells, edges, and vertices in the mesh.

For global, spherical meshes:  
 $nVertices = 2 * (nCells - 2)$   
 $nEdges = 3 * (nCells - 2)$

For *doubly-periodic* planar meshes:  
 $nEdges = nCells + nVertices$

For *limited-area* meshes:  
 $nEdges + 1 = nCells + nVertices$

When stored in netCDF files (“*grid.nc*”), MPAS meshes have at least the following dimensions:

dimensions:

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```

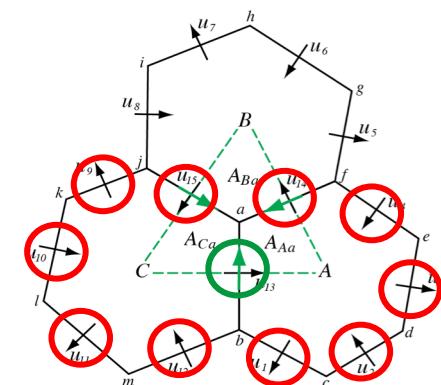
The maximum number of faces (edges) any cell can have; equivalent to the maximum number of cell neighbors or vertices that a cell can have.

When stored in netCDF files (“*grid.nc*”), MPAS meshes have at least the following dimensions:

dimensions:

```
nCells = 40962 ;
nEdges = 122880 ;
nVertices = 81920 ;
maxEdges = 10 ;
maxEdges2 = 20 ;
TWO = 2 ;
vertexDegree = 3 ;
```

The maximum number of edges that participate in the reconstruction of tangential velocities at cell faces (edges).



When stored in netCDF files (“*grid.nc*”), MPAS meshes have at least the following dimensions:

dimensions:

```
nCells = 40962 ;  
nEdges = 122880 ;  
nVertices = 81920 ;  
maxEdges = 10 ;  
maxEdges2 = 20 ;  
TWO = 2 ;  
vertexDegree = 3 ;
```

Always 2 (every dimension must have a name in netCDF). Used for, e.g., the number of vertices forming the endpoints of edges and the number of cells separated by an edge.

## What do MPAS meshes look like in netCDF files?

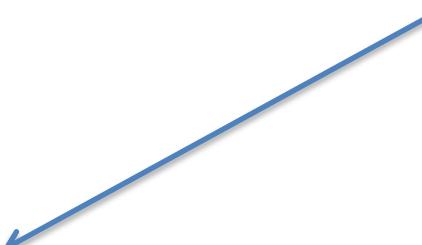
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dimensions:

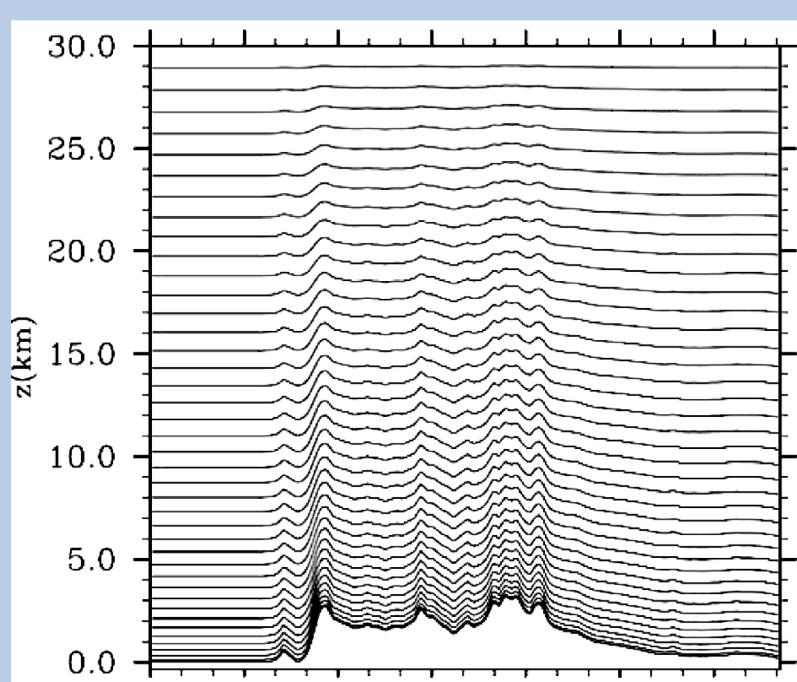
```
nCells = 40962 ;  
nEdges = 122880 ;  
nVertices = 81920 ;  
maxEdges = 10 ;  
maxEdges2 = 20 ;  
TWO = 2 ;  
vertexDegree = 3 ;
```

The number of cells/edges that meet at each vertex.

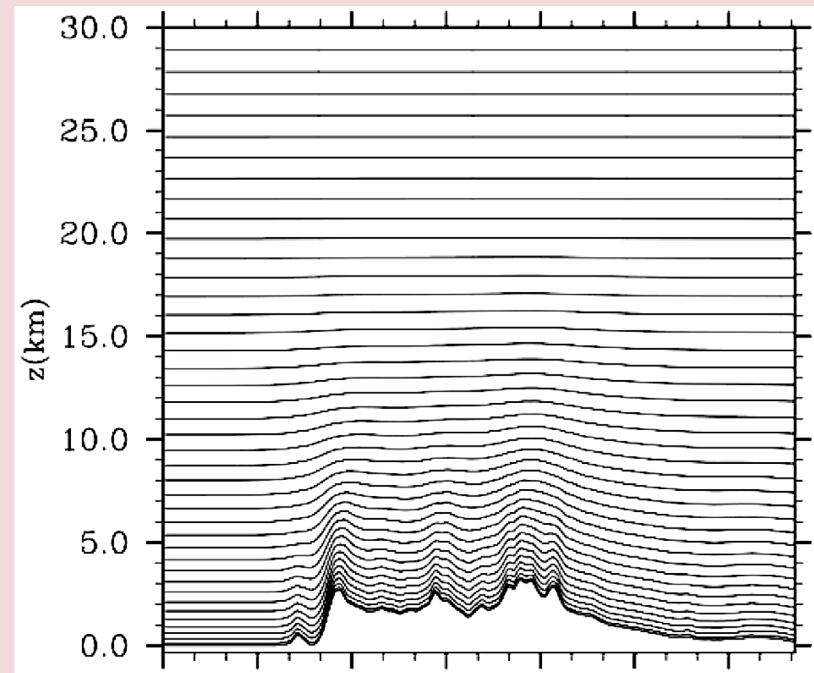
*In principle, quadrilateral meshes could be represented by setting vertexDegree = 4*



## What about the vertical grid?



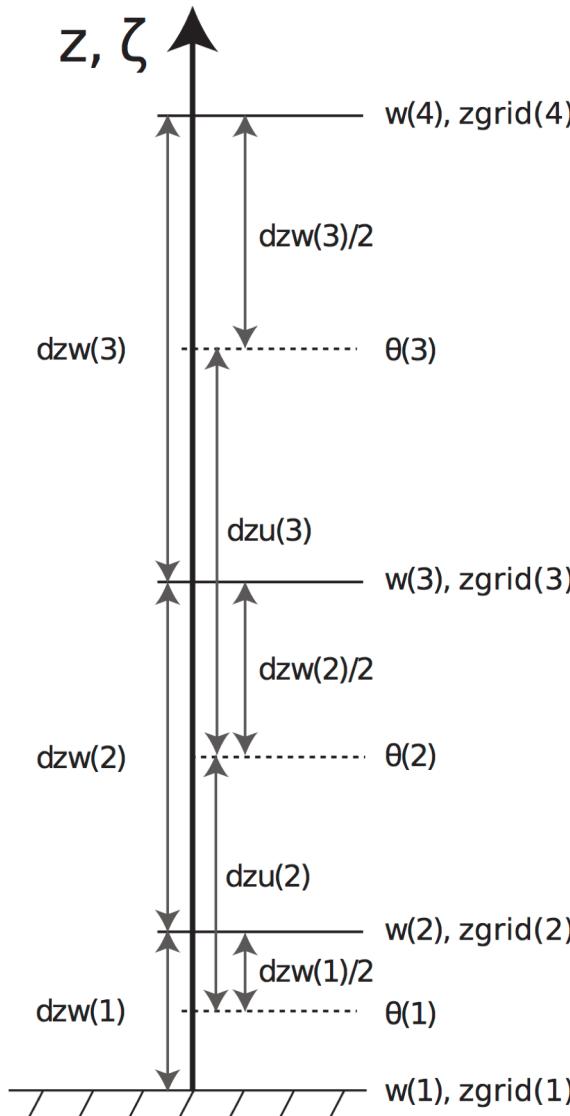
**WRF**  
Pressure-based  
terrain-following sigma  
vertical coordinate



**MPAS**  
Height-based hybrid smoothed  
terrain-following vertical  
coordinate

- Improved numerical accuracy

# Vertical grid



The MPAS-Atmosphere vertical grid is also staggered:

- vertical velocities on  $w$  levels
- all other fields on  $\Theta$  levels

$zgrid$  gives geometric height at  $w$  levels

$\Theta$  levels lie at the midpoints of bracketing  $w$  levels

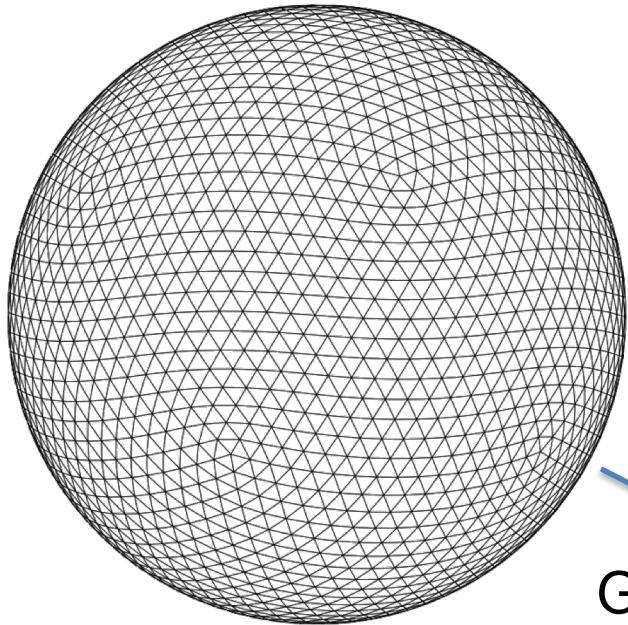
To vertically interpolate field  $F$  from theta levels to  $w$  levels:

$$fzp(k) = 0.5 * dzw(k) / zu(k)$$

$$fzm(k) = 0.5 * dzw(k-1) / zu(k)$$

$$F_w(k) = fzm(k) * F_\Theta(k) + fzp(k) * F_\Theta(k-1)$$

## How are meshes partitioned into blocks for parallelization?

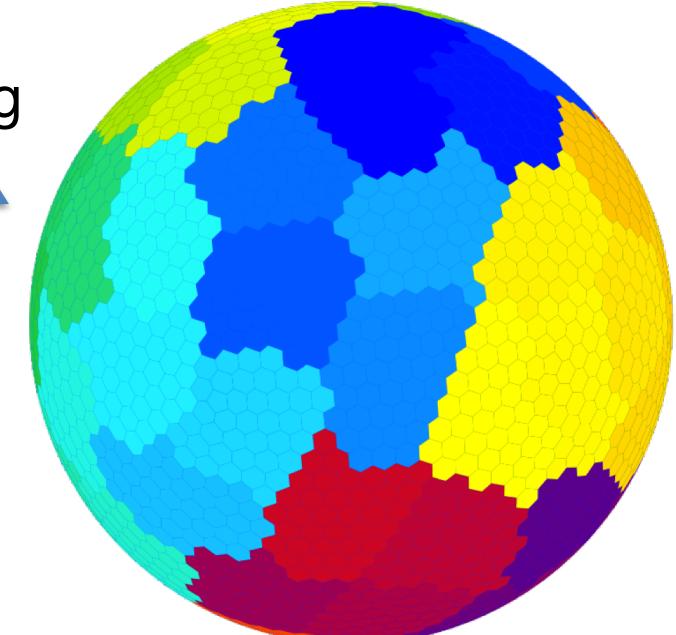


Graph partitioning

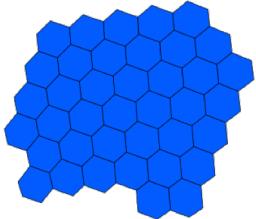
- The *dual* mesh of a Voronoi tessellation is a Delaunay triangulation – essentially the connectivity graph of the cells
- Parallel decomposition of an MPAS mesh then becomes a graph partitioning problem: ***equally distribute nodes among partitions (give each process equal work) while minimizing the edge cut (minimizing parallel communication)***

We use the Metis package for parallel graph decomposition

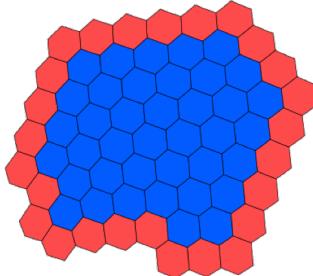
- Currently done as a pre-processing step, but could be done “on-line”
- Fortunately, Metis runs quickly, and a partitioning into  $n$  pieces only needs to be done once for a given mesh



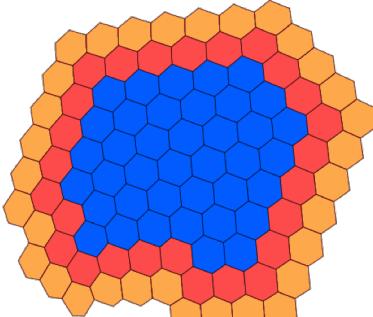
## How are meshes partitioned into blocks for parallelization?



Block of cells owned by a process

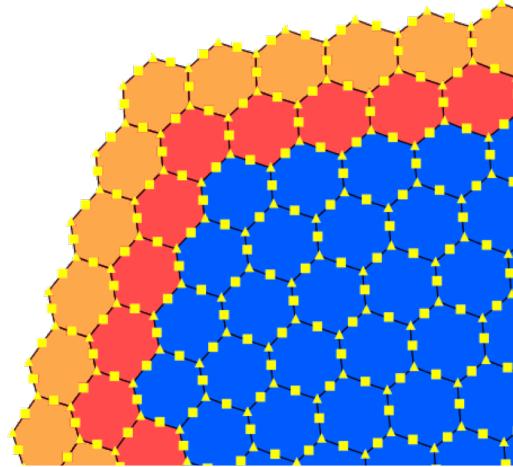


Block plus one layer of halo/ghost cells

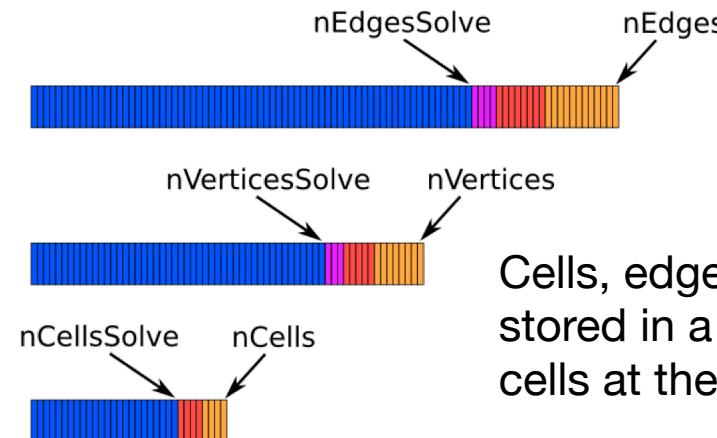


Block plus two layers of halo/ghost cells

Given an assignment of cells to a process, any number of layers of halo (ghost) cells may be added



With a complete list of cells stored in a block, adjacent edge and vertex locations can be found; we apply a simple rule to determine ownership of edges and vertices adjacent to real cells in different blocks

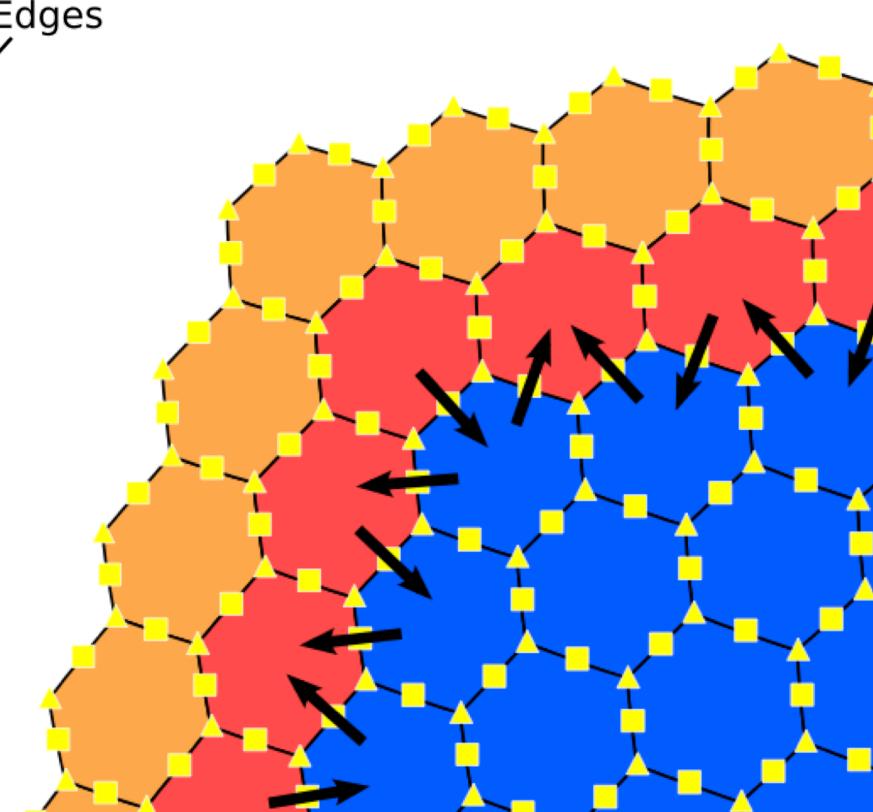
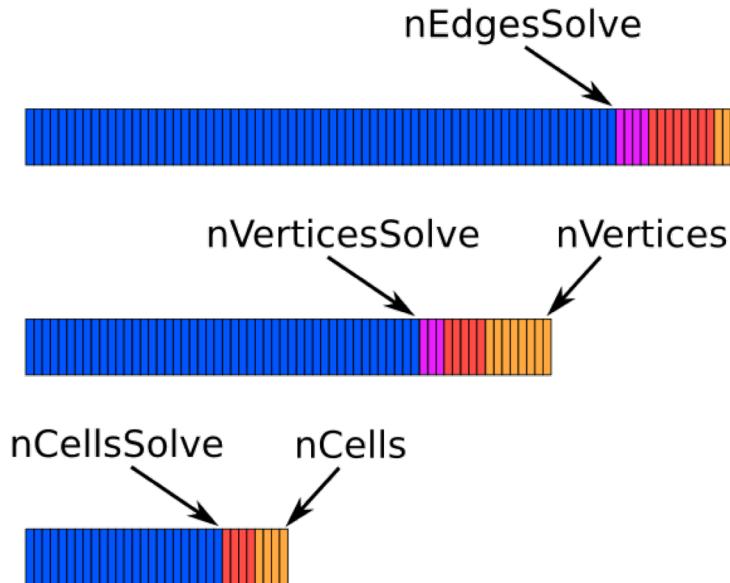


Cells, edges, and vertices are stored in a 1-d array, with halo cells at the end of the array

## How are meshes partitioned into blocks for parallelization?

An edge  $E$  is an owned edge **iff**  $\text{cellsOnEdge}(1, E)$  is an owned cell

A vertex  $V$  is an owned vertex **iff**  $\text{cellsOnVertex}(1, V)$  is an owned cell



For  $n$  layers of ghost cells, we have  $n+1$  layers of ghost edges and ghost vertices.