MPCS 58020 Homework 1

Each problem is worth 10 points. Along with your code, plots, and written solutions, please submit a README with instructions on how to run your code, a list of any references you used, and a description of any shortcomings.

For each of the first five problems, write a short program (numerical experiment) and execute it on the specified set of parameters. You may use the language of your choice provided that you do not use high-level functionality that substitutes for an implementation of the details of the experiment. Submit your source code together with the specified output.

Note: For this assignment, I'd like the random selection part of the code to be done explicitly, not as part of a higher-level library. That is, I would like to see the call to a uniform random number generator directly in your code. I will relax this in the future. If you are unsure about the use of a particular high-level built-in function, please ask on Slack.

1. Estimate the probability that at least three people have the same birthday for a group of m people. Run the simulation for values of m from 3 to 300 and plot the probability as a function of m. You do not have to account for leap years.

2. Estimate the probability that at least two people in a group of m people have a birthday within one day of each other (either the same day, one day before, or one day after). Run the simulation for values of m from 2 to 100 and plot the probability as a function of m. You do not have to account for leap years.

3. Estimate the probability that more than 50% of a group of m people have a summer birthday in a non-leap year, where summer = June, July, or August for simplicity. Use the exact total number of days across these months in these calculations; do not make the simplifying assumption that summer spans $\frac{1}{4}$ of the days in a year. Run the simulation for values of m from 2 to 100 and plot the probability as a function of m.

4. If you draw five cards at random from a standard deck, estimate the probability of getting a straight with all four suits represented.

5. Given three bags

   - Bag A: 700 red marbles; 300 blue marbles
   - Bag B: 300 red marbles; 700 blue marbles
   - Bag C: 500 red marbles; 500 blue marbles

   Pick a bag randomly. Then choose 10 marbles *without* replacement. If the sequence B B B B B R B B B R is obtained, estimate the probabilities that it came from Bag A, Bag B, and Bag C?

Problems 6-10 involve derivations and a few simple calculations. You must show your work by submitting either typeset solution (in LaTex) or a scanned, neatly handwritten version.

6. An airplane can safely fly if at least half of its engines are functional. If each engine independently functions with probability p, for what values of p is a three-engine plane safer than a five-engine plane?

7. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what should it charge the customer so that its expected profit will be 10% of A?

8. Two design teams, one named C and the other N, are asked to separately design a new product within a month. From experience we know that:

   (a) The probability that team C is successful is 2/3.
   (b) The probability that team N is successful is 1/2.
   (c) The probability that at least one team is successful is 3/4.

   Assuming that exactly one successful design is produced, what is the probability that it was designed by team N?

9. Using the basic properties of set theory, show that

$$A \cup B \cup C = A \cup (A^c \cap B) \cup (A^c \cap B^c \cap C)$$

10. Amanda and Kyle are scheduled to meet at Hallowed Grounds at 1pm to grade the MPCS 58020 homework. After a year-long quarantine, they both struggle with time-management and the commute, and will each arrive separately with independent delays uniformly distributed between 0 and 60 minutes. If Amanda arrives and Kyle does not show up within 15 minutes, she will leave (and vice versea). What is the probability that your homework gets graded on time?