Pool Models

A classification by required and obtainable information

Markus Müller, Holger Metzler, Carlos Sierra

August 7, 2019

- What are pool models?
- Why do we need them?
- What can they be used for?
    - What is needed?
    - What can we learn from them?

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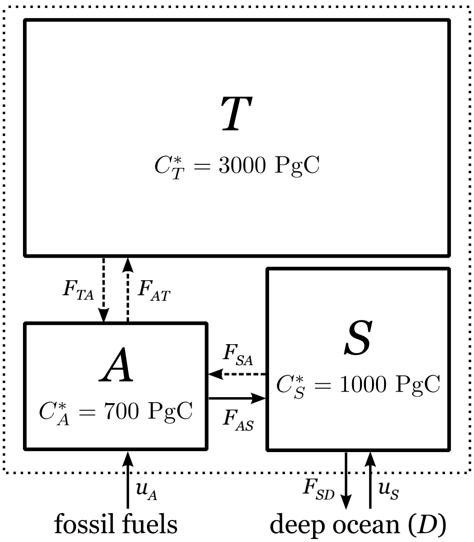
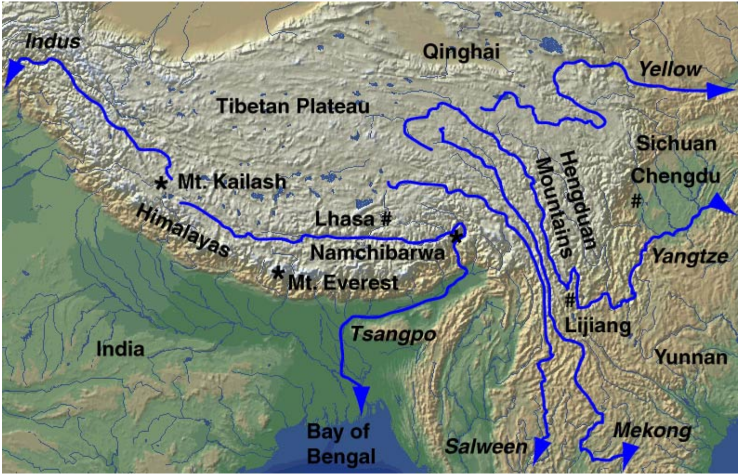
Outline

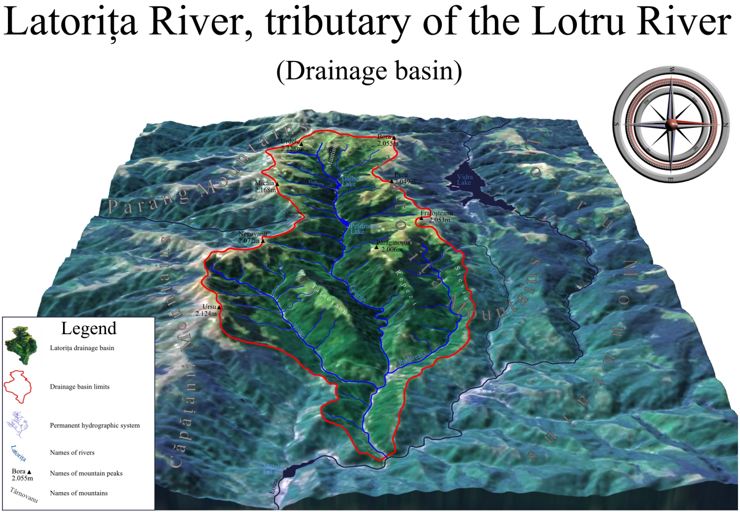
Outline


T
$C_T^* = 3000$ PgC
F_{TA}
F_{AT}
S
A
F_{SA}
$C_S^* = 1000$ PgC
$C_A^* = 700$ PgC
F_{AS}
u_A
F_{SD}
u_S
fossil fuels
deep ocean (D)

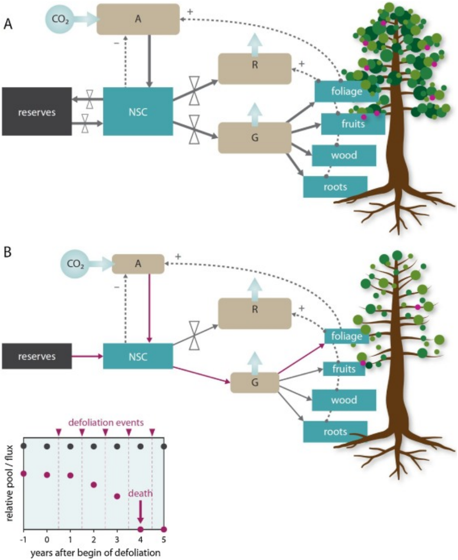
Hydrology Watersheds



Hydrology Catchments

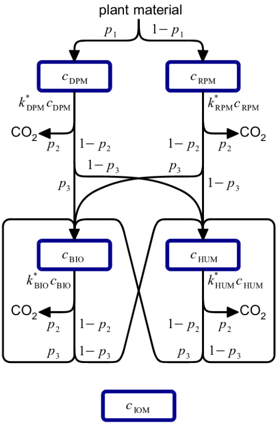


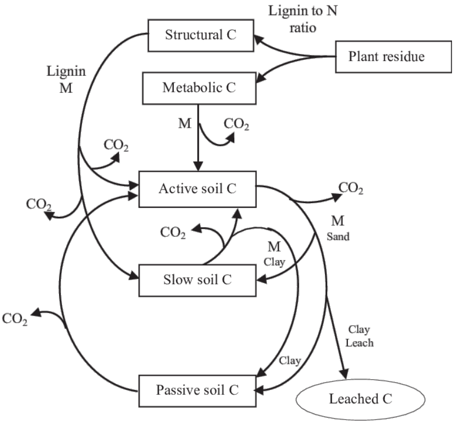
Plant Physiology/ Carbon allocation



?

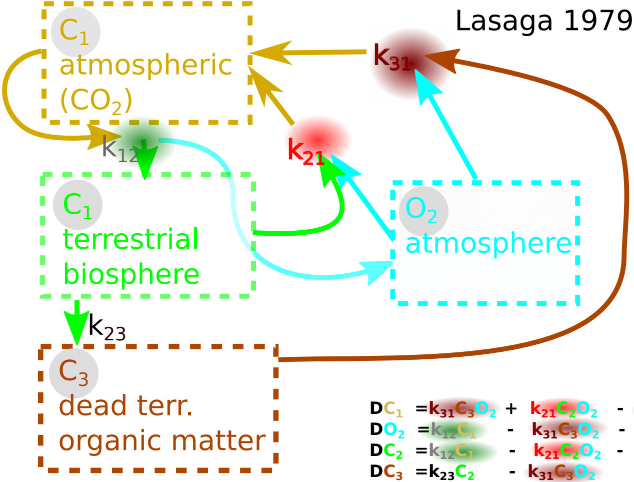
Organic Matter Decomposition / Soil Models





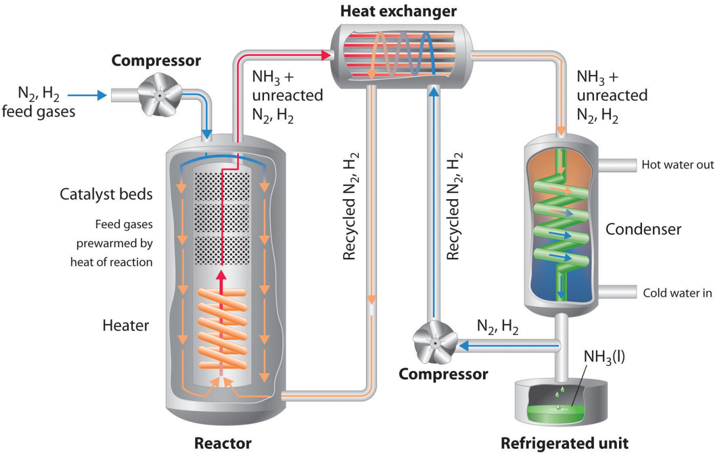
Ecosystem Models

Lasaga 1979

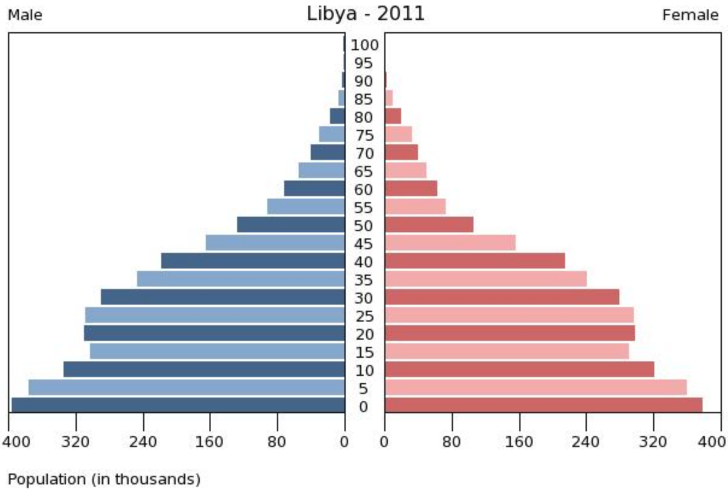


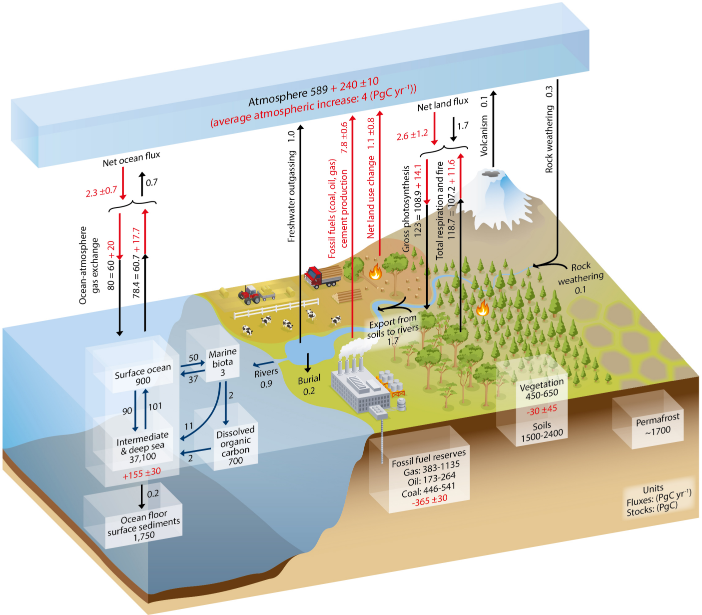
$$
\begin{aligned}
DC_1 &= k_{31}C_3O_2 + k_{21}C_2O_2 - k_{12}C_1\\
DO_2 &= k_{12}C_1 - k_{31}C_3O_2 - k_{21}C_2O_2\\
DC_2 &= k_{12}C_1 - k_{21}C_2O_2 - k_{23}C_2\\
DC_3 &= k_{23}C_2 - k_{31}C_3O_2
\end{aligned}
$$

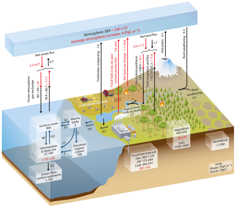
Chemical Reactors

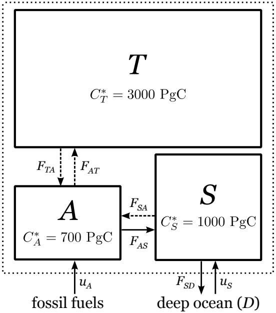


Population Dynamics




Atmosphere 589 + 240 ±10
(average atmospheric increase: 4 (PgC yr-1))
Net ocean flux
2.3 ±0.7
0.7
Ocean-atmosphere gas exchange
80 = 60 + 20
78.4 = 60.7 + 17.7
Freshwater outgassing 1.0
Fossil fuels (coal, oil, gas) cement production 7.8 ±0.6
Net land use change 1.1 ±0.8
Net land flux
2.6 ±1.2
1.7
Gross photosynthesis 123 = 108.9 + 14.1
Total respiration and fire 118.7 = 107.2 + 11.6
Volcanism 0.1
Rock weathering 0.3
Rock weathering 0.1
Export from soils to rivers 1.7
Rivers 0.9
Burial 0.2
Surface ocean 900
50
37
Marine biota 3
90
101
2
11
2
Intermediate & deep sea 37,100
+155 ±30
Dissolved organic carbon 700
0.2
Ocean floor surface sediments 1,750
Vegetation 450-650
-30 ±45
Soils 1500-2400
Permafrost ~1700
Fossil fuel reserves
Gas: 383-1135
Oil: 173-264
Coal: 446-541
-365 ±30
Units
Fluxes: (PgC yr-1)
Stocks: (PgC)

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F_{AT}
A
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S
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u_A
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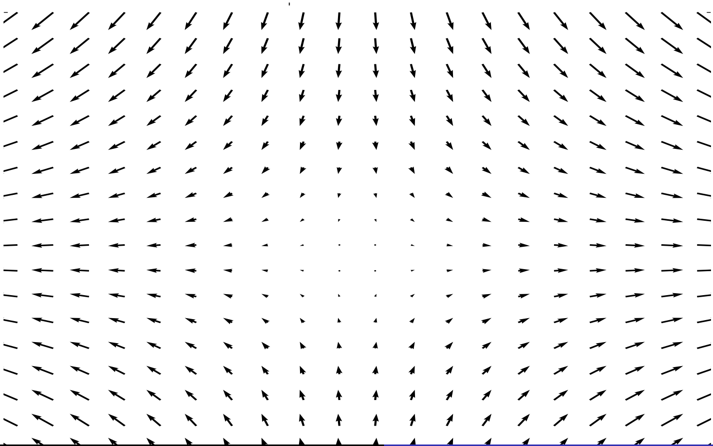
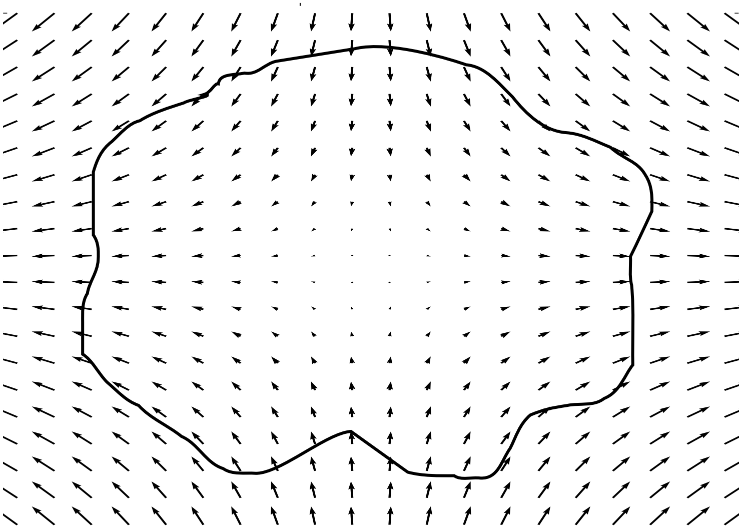
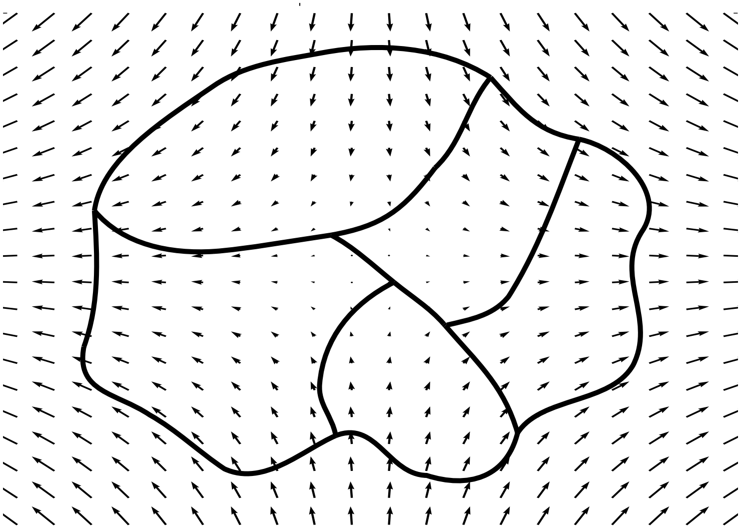
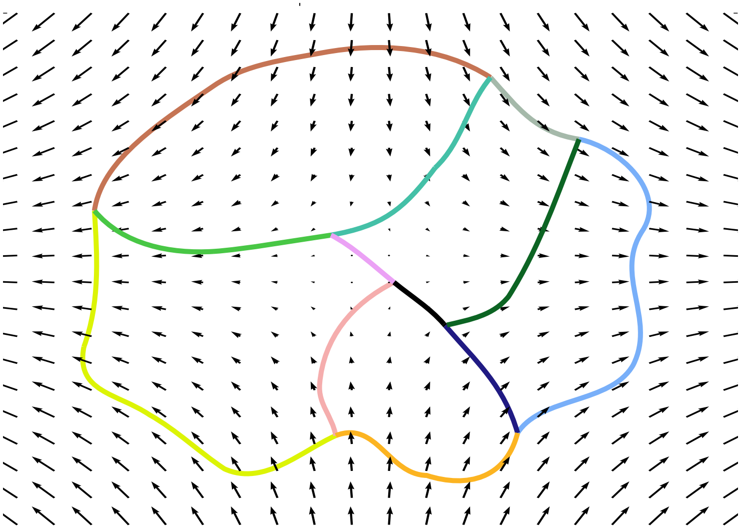
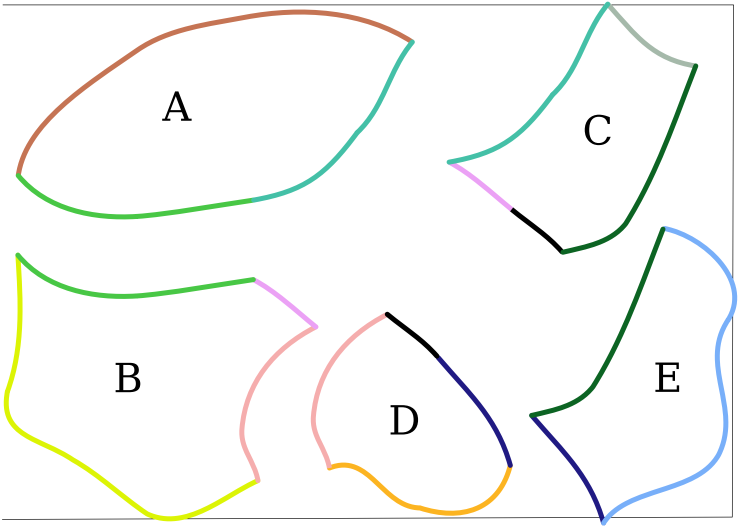
deep ocean (D)

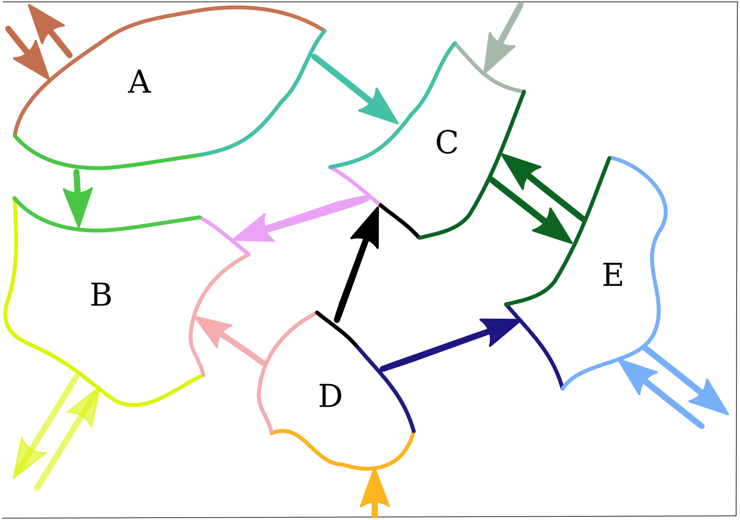


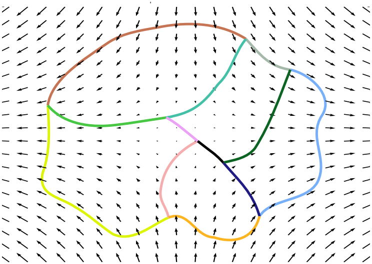


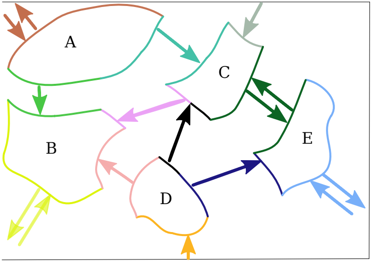


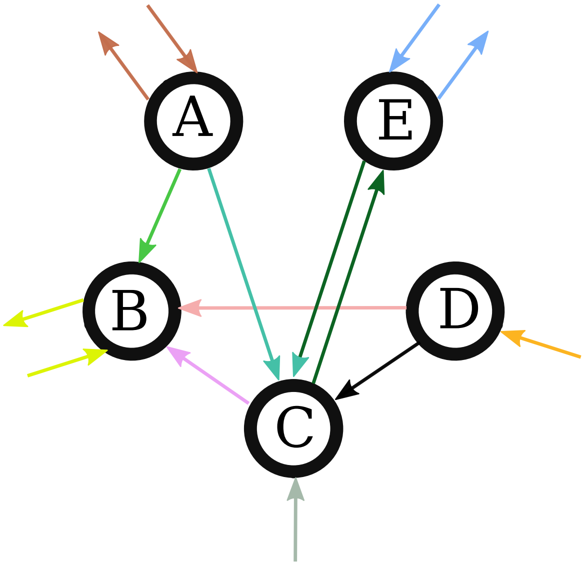


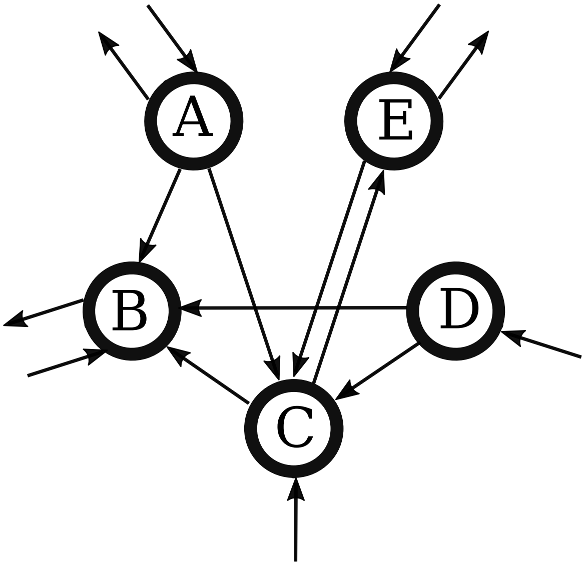

A
B
C
D
E


A
B
C
D
E

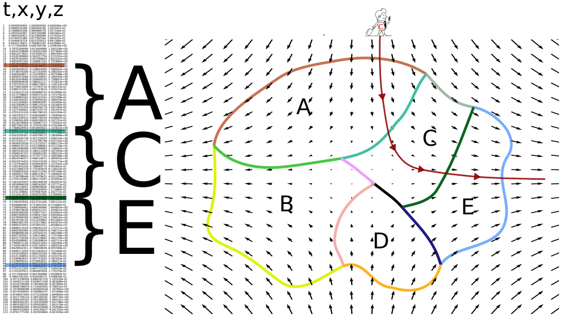



A
B
C
D
E


A
E
B
D
C

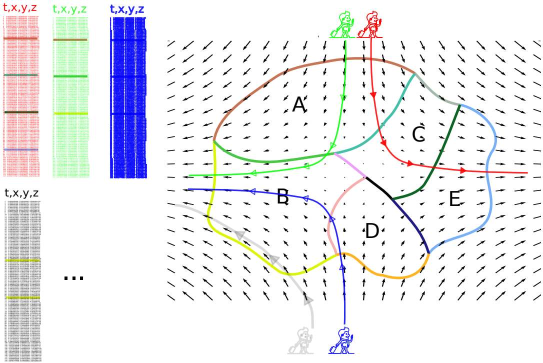

A
E
B
D
C


C
PASSPORT
PASSPORT
HONG KONG
-4 JUN 2011
27 07 16 7
MALAYSIA IMMIGRATION
DEPARTURE
ČR České Velenice
AEROPORT POLE CARAIBES - ABYMES
26 MARS 2016
GUADELOUPE
UGANDA
12 JUN 2010
ENTEBBE
APR -3 2015
EGYPT
PORT DE FORT DE FRANCE
21 MARS 2016
MARTINIQUE
14 AVR. 2011
H 124 MARTINIQUE



Pools arrival

1	A	16.45
2	C	42.45
3	E	68.56
4	Ext	94.23
...		



Pools arrival		
1	A	16.45
2	C	42.45
3	E	68.56
4	Ext	94.23

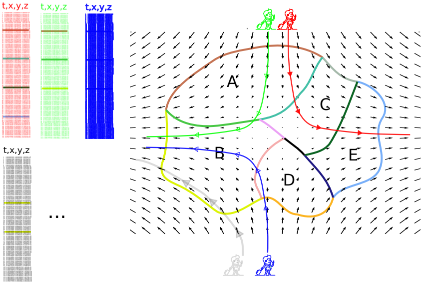
Pools arrival		
1	D	14.45
2	B	52.45
3	Ext	94.23

Pools arrival		
1	D	12.45
2	B	32.45
3	Ext	194.23

Pools arrival		
1	B	16.45
2	Ext	194.23

...

Possible Descriptive Statistics



Pools		arrival
1	A	16.45
2	C	42.45
3	E	68.56
4	Ext	94.23

Pools		arrival
1	D	14.45
2	B	52.45
3	Ext	94.23

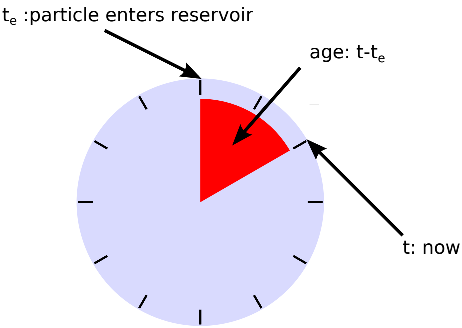
Pools		arrival
1	D	12.45
2	B	32.45
3	Ext	194.23

Pools		arrival
1	B	16.45
2	Ext	194.23

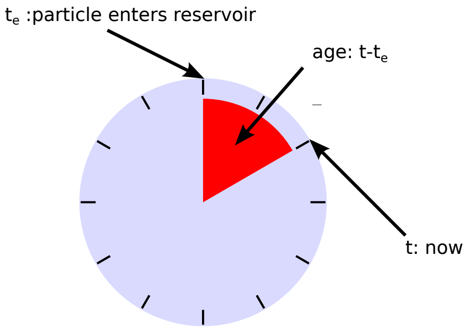
...

- average time spent in a pool A ,B...
- average time spent in the whole system
- average time of particles spent between pool C and E under the assumption of having entered by pool D (weird but possible...)
- number / mass of particles in pool A, B,
- Deathrate of pool A.
-

Age of a Particle

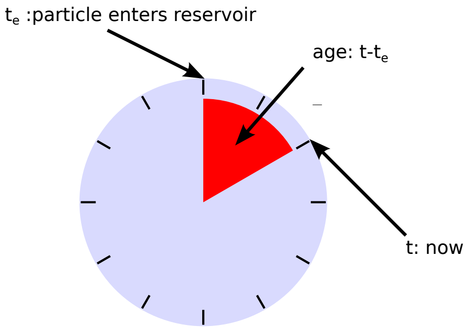


Age of a Particle



- The "age " is always defined in *context* of the reservoir

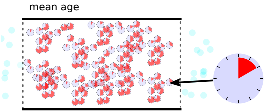
Age of a Particle



- The “age ” can not be negative!

Mean Age

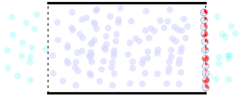
- Which set of particles to use for the average?
  proposition: *all* particles that are in the reservoir at the given time.
  $\rightarrow$ usually depends on input rates as well as the dynamics of the system.



$$\bar{a}(t) = \frac{a_1 + a_2 + \cdots + a_N}{N}$$

With $N = N(t)$ the number of all particles in the reservoir at time t.

Mean Transit Time



$$\bar{t}_r(t) = \frac{a_1 + a_2 + \cdots + a_{n_o}}{n_o}$$

With $n_o = n_o(t)$ the number of particles just leaving at time t

- Can be time dependent as well

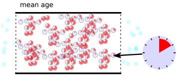
Mean Transit Time

$$\bar{t}_r(t) = \frac{a_1 + a_2 + \cdots + a_{n_o}}{n_o}$$

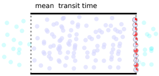
With $n_o = n_o(t)$ the number of particles just leaving at time t

- Can be time dependent as well
- Includes only the subset of particles that are just leaving at the given time. (Can only be computed when there is an output stream)

Differences between mean age and mean transit time



- Includes all particles that are in the reservoir at the given time.
- Directly coupled to input rates



- Includes only the subset of particles that are just leaving at the given time.
- Indirectly coupled to inputs

Iteration over all particles

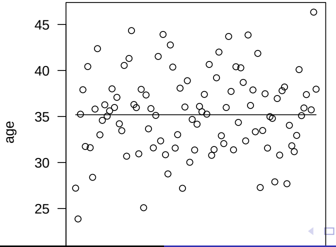
1. To compute the mean transit time we have to identify the particles just leaving.
2. Ask every leaving particle when it entered and compute its age.
3. Iterate over all particle and compute the average of their ages.

Iteration over all particles

1. To compute the mean transit time we have to identify the particles just leaving.
2. Ask every leaving particle when it entered and compute its age.
3. Iterate over all particle and compute the average of their ages.

$$\bar{t}_r(t) = \frac{a_1 + a_2 + \cdots + a_{n_o}}{n_o}$$

With $n_o = n_o(t)$ the number of particles just leaving at time t



Iteration over all ages

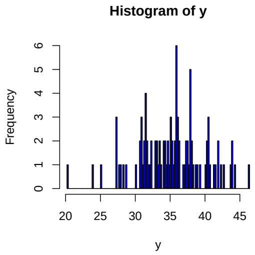
1. as above
2. as above + make a histogram of all ages
3. iterate over all ages and compute their weighted average

Iteration over all ages

1. as above
2. as above + make a histogram of all ages
3. iterate over all ages and compute their weighted average

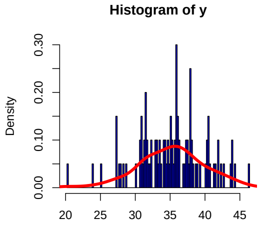
$$\bar{t}_r(t) = \frac{a_1 n_{a_1} + a_2 n_{a_2} + \cdots + a_n n_{a_n}}{n_o}$$

With $n_o = n_o(t) = n_{a_1} + n_{a_2} + \cdots + n_{a_n}$



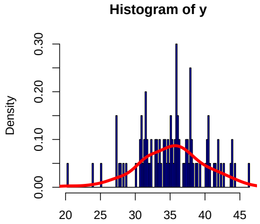
Integration over a density

$$
\begin{aligned}
\bar{t}_r(t) &= \lim_{n\to\infty} \frac{a_1 n_{a_1} + a_2 n_{a_2} + \cdots + a_n n_{a_n}}{n_o} \\
&= \lim_{n\to\infty} \sum_{minage}^{maxage} a \frac{n(a)}{n_o} da \\
&= \int_{minage}^{maxage} a\psi(a) da
\end{aligned}
$$

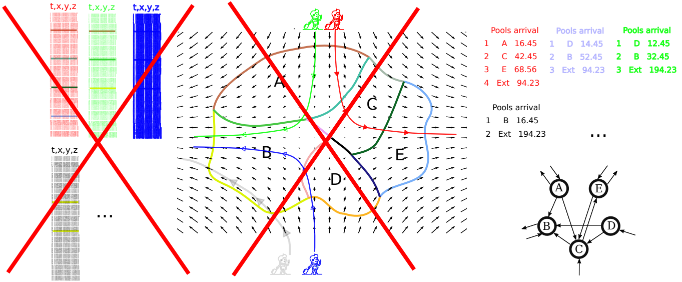


Same procedure for age density

$$\begin{aligned}\bar{a}(t) &= \lim_{n\to\infty} \frac{a_1 n_{a_1} + a_2 n_{a_2} + \cdots + a_n n_{a_n}}{n_p} \\ &= \lim_{n\to\infty} \sum_{minage}^{maxage} a \frac{n(a)}{n_p} da \\ &= \int_{minage}^{maxage} a\phi(a) da\end{aligned}$$

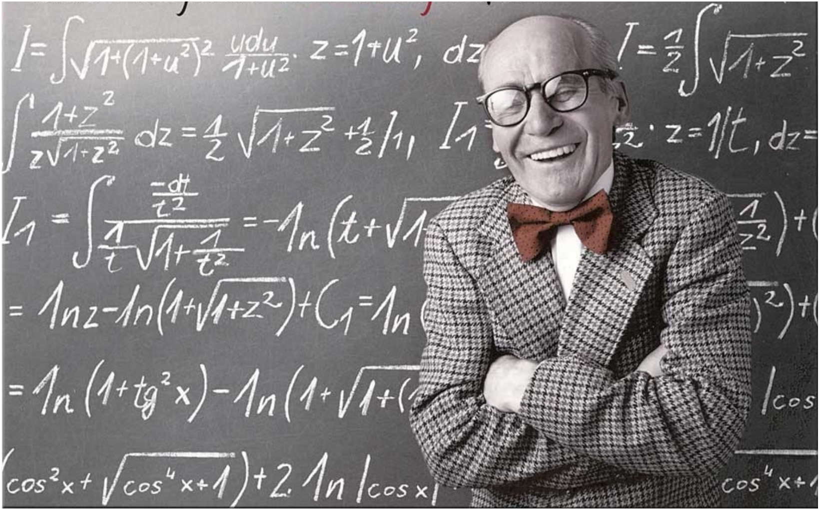


Possible Predictive Statistics ?



- Could we make a rule to predict the number of particles exiting from Pool E at time using only the particle passports? (Assuming that the exit from E is not recorded.)
- Could we make a rule to predict the age distribution of particles exiting from Pool E at time using only the particle passports? (Assuming again that the exit from E is not recorded.)

Thank you for your attention



bibliography