A Classical Macroeconometric Model

For the United States

by

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Abstract

A statistical definition of the natural unemployment rate
hypothesis is advanced and tested. A particular illustrative structural
macroeconomic model satisfying the definition is set forth and estimated.
The model has "classical" policy implications, implying a number of
neutrality propositions asserting the invariance of the conditional
means of real variables with respect to the feedback rule for the money
supply. The aim is to test how emphatically the data reject a model
incorporating rather severe "classical" hypotheses.

This paper estimates a small, linear, classical macroeconometric model for the postwar U.S. One reason for estimating the model is to produce a simple device capable of generating unconditional forecasts of key economic aggregates such as the unemployment rate, the price level, and the interest rate. But a more important reason is that as part of the estimation process, the hypotheses underlying the model are subjected to empirical testing. Since these hypotheses are very "classical" and sharply at variance with Keynesian macroeconomics, it would be useful to know at what confidence levels the data reject them.

The present model is considerably more monetarist than is the St. Louis model.[1] Indeed, as interpreted and manipulated by its builders, the St. Louis model is incapable of rationalizing prominent monetarist positions. In particular, it implies that simple x-percent growth rules for money can generally be improved upon by adopting rules with feedback from past endogenous variables to current money.[2] By way of contrast, the present model is one in which an x-percent growth rule for the money supply seems not to be dominated by any rule with feedback.[3]

The deterministic (nonrandom) classical model, the static analysis of which is enshrined in macroeconomics textbooks, has never been taken seriously because its predictions seem so terribly at variance with the data. In particular, it is hard to explain the observed

persistent movements in employment and unemployment with the textbook classical model. How meaningfully integrating random disturbances into the classical model would affect the analysis is a matter about which there is presently little agreement. On the one hand, in his AEA Presidential address, James Tobin [20] seemed to assert that the presence of random disturbances in demand and supply schedules so alters the character of the general system that it sets up an exploitable trade-off between unemployment and inflation even in a system where all agents optimize. On the other hand, Robert Lucas [9] has analyzed a general equilibrium system in which agents cope optimally with the existence of uncertainty. While there exist "nonneutralities" in that system, there aren't any nonneutralities that the government can either exploit or offset by way of countercyclical policy.

This paper formulates, tests and estimates a version of the classical model that has its origin in hypotheses that place severe restrictions on the random behavior of unemployment, output, and the interest rate. The model implies that those three "real" variables are econometrically exogenous with respect to variables measuring monetary and fiscal policies. As a consequence, government manipulations of monetary and fiscal policy variables have no predictable effects on unemployment, output, or the interest rate, and hence are useless for pursuing countercyclical policy. Such implications are in the nature of neutrality results, albeit ones that require drawing some fairly fine econometric distinctions. The key elements of the model that provide the sources of the restrictions on the stochastic nature of output, unemployment, and interest are: (a) a drastic version of the natural unemployment rate hypothesis; (b) the expectations theory of the term

structure of interest rates; and (c) the assumption that the public's expectations are "rational."

The chief novelty of this paper is its formulation of a drastic, statistical definition of the natural unemployment rate hypothesis. That definition is not dependent on any particular macroeconomic structural model, being compatible with a variety of structures one could imagine. The particular structural model presented in this paper is intended only as an illustrative example that satisfies this definition of the natural rate hypothesis. This particular structure does, however, illustrate some of the strong "classical" properties that will be possessed by models that satisfy the definition of the natural rate hypothesis advanced here. A major aim of the paper is to indicate how this definition of the natural rate hypothesis can be tested, and to present some test results.

This paper is organized as follows. In Section I a prototype of the model is described. However, no attempt is made here to rationalize in a deep way the equations comprising the model. Section I is designed to display the system briefly and to establish its classical nature. Section II then provides a definition of the natural rate hypothesis that is the cornerstone of the model. Statistical tests of the hypothesis are described. Section II also describes how the rational expectations theory of the term structure of interest rates is implemented in the model, and how its central implications can be tested. Section III implements the econometric tests described in Section II. Finally, Section IV contains estimates of the complete model. The casual reader not interested in econometrics can read only Sections I and IV and find there estimates of the model and a description of how it works.

Overview of the Model

I begin by describing a simple prototype of the model. It differs from the model finally estimated in some minor ways, but illustrates well the mechanics of the model.

The prototype consists of the following five equations:

(1.1) $$Un_t = \gamma[p_t - E_{t-1}p_t] + \sum_{i=1}^{n_1} \lambda_i Un_{t-i} + u_{1t} \quad , \gamma < 0$$

(A Phillips curve)

(1.2) $$n_{ft} = \beta[p_t - E_{t-1}p_t] + dUn_t + \sum_{i=1}^{n_2} w_i n_{ft-i} + u_{2t} \quad , \beta > 0, \; d < 0$$

(A labor force participation equation)

(1.3) $\quad y_t = \alpha_0 t + \alpha_1 [n_{ft} - Un_t + pop_t] + u_{3t}$, $\quad \alpha_1 \approx 1$

(A production function)

(1.4) $\quad R_t = R_{t-1} + \xi[Z_t - E_{t-1} Z_t] + u_{4t}$

(A martingale equation for the long-term interest rate)

(1.5) $\quad m_t - P_t = b_1 R_t + b_2 y_t + b_3(m_{t-1} - P_{t-1}) + u_{5t}$, $\quad b_1 < 0; \; b_2, b_3, \; > 0$

(A portfolio balance schedule)

The variables are defined as

Un_t = unemployment rate

P_t = log of GNP deflator

n_{ft} = log of labor force participation rate

y_t = log of real GNP

pop_t = log of population

R_t = long-term interest rate

m_t = log of money supply

Z_t = a vector of exogenous variables in the "IS" curve, including tax rates and government purchases.

u_{jt} = mutually and serially independent random terms with zero means so that $E_{t-1} u_{jt} = 0$, $j = 1, \ldots, 5$.

$E_{t-1} X_t$ = the mathematical expectation of X_t conditioned on information available at time t-1.

The variables Z_t, pop_t, and m_t are taken as exogenous.

Equation (1) is a "Phillips curve" that posits a direct supply side relationship between unemployment and the unexpected part of the current price level. The public's psychological expectation about the price level is supposed to be "rational," meaning that it equals $E_{t-1} P_t$.

The equation embodies the natural unemployment rate hypothesis since it asserts that unemployment does not depend on the anticipated part of the rate of inflation. Equation (1) is essentially Lucas's formulation of the Phillips curve [6].

Equation (2) is a labor force participation equation positing that the participation rate depends directly on the unexpected part of the price level and inversely on the unemployment rate (the "discouraged worker effect"). The presence of unemployment and the unexpected part of prices in equations describing labor-force participation is not unusual (for example, see Wachter [21] and the work cited by him).

Upon noting that the log of employment approximately equals $[n_{ft} - Un_t + pop_t]$, equation (3) is seen to be a Cobb-Douglas production function that excludes capital. The regressions reported by Lucas [5] and Bodkin and Klein [2] suggest that little violence is done to the data by omitting capital from (3). That is, time series regressions of the log of output against the logs of capital and employment typically display constant or increasing returns to employment and zero or slightly negative returns to capital. For my purposes, excluding capital from (3) permits the construction of a model in which there is no need to account for capital accumulation.

Equation (4) posits that the long-term interest rate is a "martingale". Fiscal policy and other aggregate demand variables influence the long rate in two ways. First, the unexpected components of Z_t influence the "innovation" in the long rate, i.e., the part of the long rate that can't be predicted from the past. Second, the foreseen or

expected part of Z_t is already reflected in R_{t-1}, and affects R_t in precisely the same way it affects R_{t-1}.

Equation (5) is a standard portfolio balance schedule.

The model is five equations in the five endogenous variables Un_t, n_{ft}, y_t, R_t, and p_t. The exogenous variables are Z_t, pop_t, and m_t.

To complete the model, the stochastic processes governing the exogenous variables m_t, Z_t, and pop_t must be specified. I will assume the autoregressive schemes

1.6a
$$m_t = \sum_{i=1}^{n_3} \xi_i m_{t-i} + \varepsilon_{1t}$$

1.6b
$$Z_t = \sum_{i=1}^{n_4} \Psi_i Z_{t-i} + \varepsilon_{2t}$$

1.6c
$$pop_t = \sum_{i=1}^{n_5} \omega_i pop_{t-i} + \varepsilon_{3t}$$

where the ξ_i's, Ψ_i's, and ω_i's are parameters, and the ε_t's are serially independent random variables with means of zero; they are assumed to be distributed independently of the u's in the structural equations (1)-(5). To solve the model and to forecast with it, expected values of the exogenous variables, e.g., $E_{t-1}m_t$ and $E_t m_{t+1}$, are required. These expected values are calculated using the above autoregressions for the exogenous variables. Partly, this is by way of imposing rationality, since the expected price $E_{t-1}p_t$ turns out to depend on $E_{t-1}m_t$, $E_{t-1}Z_t$, and $E_{t-1}pop_t$. Rationality amounts to requiring that the public's expectations of the exogenous variables m_t, Z_t, and pop_t equal the mathematical expectations computed from the appropriate objective probability distributions, i.e., the above autoregressions.

The model has a standard aggregate demand, supply representation in the p-y plane. Substituting (1) and (2) into (3) gives the aggregate supply schedule:

$$y_t = \alpha_0 t + \alpha_1 pop_t + [\alpha_1\beta + (\alpha_1 d - \alpha_1)\gamma](p_t - E_{t-1}p_t)$$

$$+ (\alpha_1 d - \alpha_1)\sum_{i=1}^{n_1} \lambda_i Un_{t-i} + \alpha_1 \sum_{i=1}^{n_2} w_i n_{ft-i}$$

$$+ (\alpha_1 d - \alpha_1)u_{1t} + \alpha_1 u_{2t} + u_{3t}.$$

Since $\alpha_1\beta_1 + (\alpha_1 d - \alpha_1)\gamma > 0$, the aggregate supply schedule is upward sloping in the p-y plane.

Substituting (4) into (5) gives the aggregate demand schedule

$$p_t = m_t - b_1 R_{t-1} - b_1 \xi(Z_t - E_{t-1}Z_t) - b_2 y_t$$

$$- b_3(m_{t-1} - p_{t-1}) - b_1 u_{4t} - u_{5t},$$

which slopes downward in the p-y plane. Increases in m_t and in the aggregate demand innovations $\xi(Z_t - E_{t-1}Z_t)$ cause the demand schedule to shift outward. The equilibrium p,y combination is determined at the intersection of the demand and supply curves.

While the model is clearly simultaneous in determining the current values of the five endogenous variables, for generating forecasts it is recursive. The one-period-ahead forecast of Un_t is determined by taking expectations in (1) conditional on data known at t-1:

$$(1.1') \quad E_{t-1}Un_t = \sum_{i=1}^{n_1} \lambda_i Un_{t-i}$$

which follows since $E_{t-1}[p_t - E_{t-1}p_t] = E_{t-1}p_t - E_{t-1}p_t = 0$. The forecast of n_{ft} is then given from (2) as

$$E_{t-1}n_{ft} = dE_{t-1}Un_t + \sum_{i=1}^{n_2} w_i n_{ft-i}.$$

Then from (3) we have the forecast of the log of GNP as

$$E_{t-1} y_t = \alpha_0 t + \alpha_1 [E_{t-1} n_{ft} - E_{t-1} Un_t + E_{t-1} pop_t].$$

From (4) the forecast of the long-term interest rate is simply

$$E_{t-1} R_t = R_{t-1} \; ,$$

which follows since $E_{t-1}[Z_t - E_{t-1} Z_t] = 0$. Finally, from the portfolio balance schedule, the forecast of p_t is

$$E_{t-1} P_t = E_{t-1} m_t - b_1 E_{t-1} R_t - b_2 E_{t-1} y_t - b_3 (m_{t-1} - P_{t-1}) \; .$$

To compute the forecasts of the endogenous variables, the forecasts $E_{t-1} m_t$ and $E_{t-1} pop_t$ of the exogenous variables are required.

The predictions of the model are obviously classical in spirit. The predictions of the "real" variables are all independent of the prediction of the money supply, which only influences the predicted price level. For predicting the long-term interest rate, predictions of the fiscal and other aggregate demand variables add no information to that in the current long rate since they are already properly embedded in the current long-term rate. Finally, the model implies that the monetary authority doesn't have the option of pegging the nominal interest rate R_t via some feedback rule by letting the money supply be whatever it must to guarantee portfolio balance at that interest rate.[4/] For suppose that the authority were to attempt to peg the interest rate via the feedback rule

(1.7) $R_t = F\theta_{t-1}$

where θ_{t-1} is a vector of observations on endogenous and exogenous

variables dated t-1 and earlier, and F is a vector of parameters conformable with θ_{t-1}. The predictions of R_t from (4) and (7) are clearly in general inconsistent, so that the interest rate is over-determined. Thus, this model is characterized by Wicksell's classical overdeterminacy of the interest rate (and indeterminacy of the price level) under a pegged interest rate.

It bears emphasizing that while for prediction the model has a very classical, recursive structure, it is a simultaneous model when it comes to determining current variables. Thus, money is not a "veil" in the model, since (random) increases in money can be shown to stimulate both GNP and the price level. So will (random) increases in the aggregate demand Z's. But it turns out that in this model it is best to predict as if money is a veil. The fact that variables are determined jointly simply can't be exploited in prediction; neither can it be exploited for control.

I have indicated that to generate forecasts of the endogenous variables, the exogenous variables should be set equal to the forecasts $E_t m_{t+1}$, $E_t Z_{t+1}$, and $E_t pop_{t+1}$, which are to be computed from the auto-regressions (6) that actually govern those exogenous variables. It seems that something more is possible in the way of forecasting, but it turns out not to be useful to the policy maker. In particular, it is possible to use the model to "predict" values of the endogenous variables in t+1, conditional on alternative assumed values for the exogenous variables m_{t+1}, Z_{t+1}, and pop_{t+1}, given values of $E_t m_{t+1}$, $E_t Z_{t+1}$, and $E_t pop_{t+1}$. For example, for a given $E_t m_{t+1}$, different values of m_{t+1} will be associated with different values of the real variables output and unemployment. The larger is $m_{t+1} - E_t m_{t+1}$, the larger will be the

"predicted" value of output, and the lower the "predicted" value of unemployment. But such "conditional" forecasts are of no use in forming policy. For example, it will not work to use the model to "forecast" unemployment for alternative values of m_{t+1}, given $E_t m_{t+1}$, and then to set m_{t+1} in order to achieve the unemployment rate desired by the monetary authority. Expecting that to work amounts to assuming that the public would continue to form its expectations about m_{t+1} by using (1.6a) even if the authority adopted the new and different rule for setting m implicit in the above procedure. That violates the assumption that expectations are rational. What affects unemployment and output is the gap $m_{t+1} - E_t m_{t+1}$, and there is no way that the authority can expect to set this gap at some desired nonzero level.

This completes the overview of the model. I now turn to the task of setting forth more precisely the nature of the key hypotheses underlying the model. In the process, statistical tests of those hypotheses will be described and implemented.

Section II
The Stochastic Model of Unemployment and Interest Rates

This section sets forth and describes tests of a naive but powerful formulation of the hypothesis that there is a natural rate of unemployment. The hypothesis formulated here is much stricter than the usual statement of the natural rate hypothesis, which posits that the government can persistently depress the unemployment rate below the "natural rate" only at the cost of accepting an accelerating inflation. In contrast, the present formulation implies that there is no way that the government can operate so that it can expect to depress the unemployment rate below the

natural rate, even in the short run. Among other things, that implies that policy makers face no "cruel choice" between inflation and unemployment over any relevant time frame.

The tests of the natural rate hypothesis implemented here differ substantially from the usual one, which involves testing the hypothesis that certain sums of distributed lag weights are unity or zero. This usual test has been harshly criticized on theoretical grounds,[5] and furthermore is subject to the purely econometric objection that economic time series data usually yield very little information about "long-run" magnitudes such as the sum of distributed lag weights.[6] The tests implemented here don't seem to depend on estimating any such long-run properties of lag distributions.

The present statement of the natural rate hypothesis is compatible with, but somewhat stronger than the one presented and tested by Lucas. The strategy that I use to test the hypothesis is more naive and purely "statistical" than was Lucas's procedure [6], which involved actually estimating a concrete structural model.

The Natural Rate Hypothesis

I begin with the univariate Wold representation of the unemployment rate, Un_t. Wold showed that if a variable, e.g., Un_t, is an indeter-

ministic, covariance-stationary process, it can be represented as a one-sided moving average of "white noise"

$$(2.1) \qquad Un_t = \sum_{j=0}^{\infty} a_j u_{t-j}, \qquad \sum_{j=0}^{\infty} a_j^2 < \infty$$

where the u's are serially uncorrelated with mean zero and finite variance σ^2. The model (2.1) is obviously intended to apply to deviations of unemployment from its mean and any deterministic components. To make things simpler without really altering the essentials, I shall assume that the u's and the other white noises to be introduced below are serially independent.[7/] I also assume that the roots of $\sum_{j=0}^{\infty} a_j \lambda^j = 0$ lie outside the unit circle, so that Un_t possesses the autoregressive representation

$$(2.2) \qquad Un_t = \sum_{j=1}^{\infty} g_i Un_{t-j} + u_t .$$

Even with these restrictions, equation (1) is a very general representation of a covariance-stationary, indeterministic process, the a_j's being chosen to enable the covariogram of $\Sigma a_j u_{t-j}$ to match that of Un_j. So far, then, I have not restricted the process for the unemployment rate very much.

Let the vector θ_t be the set of observations on all variables observed as of time t or earlier; θ_t includes observations on current and past GNP, interest rates, prices, and any other things, including unemployment itself, thought potentially to contribute to predicting unemployment. The following statement of the natural rate hypothesis can now be advanced: The unemployment rate Un_t is said to obey the natural rate hypothesis if in its (univariate) Wold representation (equation (1)), the innovation u_t obeys:

$$(2.3) \qquad E[u_t \mid \theta_{t-1}] = 0 ,$$

so that the innovation in the unemployment rate is statistically independent of each component of θ_{t-1} and so cannot be predicted on the basis of the information in θ_{t-1}. This means that taking into account components of θ_{t-1} other than lagged Un_t's does not, on the least-squares criterion, improve the forecast of Un_t that can be made on the basis of lagged Un's alone. The least-squares forecast of Un_t on the basis of Un_{t-1}, Un_{t-2}, ..., call it $\hat{Un}_t$, is given by

$$\hat{Un}_t = \sum_{i=1}^{\infty} g_i Un_{t-i} = \sum_{j=1}^{\infty} a_j u_{t-j} .$$

On our assumption that the u's are serially independent, $\hat{Un}_t = E[Un_t \mid Un_{t-1}, Un_{t-2}, ...] = E[Un_t \mid u_{t-1}, u_{t-2}...]$.

The statement that the best forecast of u_t conditional on all past data is simply its unconditional mean of zero amounts to a very strict version of the natural rate hypothesis. For θ_{t-1} includes past values of monetary and fiscal policy variables. Such variables are asserted to offer no aid in predicting the unemployment rate, once lagged unemployment rates are taken into account. Furthermore, (3) implies that the current value of any control variable that is determined via a deterministic feedback rule on θ_{t-1} is also of no use in predicting the unemployment rate. For example, suppose that the logarithm of the money supply at t, m_t, is determined according to the deterministic, very general feedback rule

$$(2.4) \qquad m_t = f(\theta_{t-1}) ,$$

where f is some (perhaps very complicated) function that determines the monetary authority's feedback rule. Then the above version of the natural rate hypothesis implies that once lagged Un's are taken into

account, current m_t is of no use in predicting Un_t, so that

$$E[Un_t \mid m_t, Un_{t-1}, Un_{t-2}, \ldots] = E[Un_t \mid Un_{t-1}, Un_{t-2}, \ldots].$$

This holds regardless of the nature of the function f or the particular
parameter values characterizing f. Now feedback rules of the form (4)
form the class of rules for government policy variables that control
theory indicates to be optimal ones for macroeconometric models (fixed
coefficient stochastic difference equations). The above statement of
the natural rate hypothesis implies that the choice of f has no effect
on the mean of the unemployment rate, conditional on past data. This is
a very strong implication about the conditional mean of the unemployment
rate, one that denies, for example, that policy makers have any scope to
trade a lower expected unemployment rate for a higher expected rate of
inflation.[8/] By way of contrast, the existing macroeconometric models, as
usually manipulated, all imply that the parameters of f and the feedback
rules for other government policy variables do help determine the
conditional mean of the unemployment rate, and that policy makers must
face up to a hard choice between the unemployment rate and the inflation
rate they can expect to achieve.

It is important to note that the above definition of the
natural rate hypothesis does not rule out the possibility that there are
correlations between the unemployment rate and other variables, such as
prices or wages or the money supply. It does imply, however, that any
such correlations that exist cannot be exploited in predicting the
unemployment rate. To take an example, Lucas' model of the "Phillips
curve" is

$$Un_t = \sum_{i=1}^{\infty} g_i Un_{t-i} + g_0 (X_t - EX_t \mid \theta_{t-1}) + u_t' ,$$

where X_t is the price level at time t, $EX_t|\theta_{t-1}$ is the mathematical

expectation of the price level at t, conditional on information avail-

able at time t-1, and u_t' is a well behaved disturbance term, one that

satisfies $Eu_t'|\theta_{t-1} = 0.$ [9/] The above equation posits a correlation

between the innovations of Un_t and X_t; but notice that

$$E[Un_t|\theta_{t-1}] = \sum_{i=1}^{\infty} g_i \, Un_{t-i} \, ,$$

so that such a correlation doesn't help in predicting the unemployment

rate. Obviously, the same sort of result would obtain were X_t inter-

preted as a vector of exogenous and endogenous variables.

As another example of correlation between unemployment and

another variable that does not aid in forecasting unemployment, consider

the system

$$Un_t = \sum_{i=1}^{m} g_i Un_{t-i} + u_t$$

$$X_t = \sum_{i=1}^{n} \lambda_i X_{t-i} + \sum_{i=1}^{q} \gamma_i Un_{t-i} + \varepsilon_t$$

where the g's, λ's, and γ_i's are parameters and u_t and ε_t are mutually

uncorrelated and serially independent random variables with finite

variances. In this system, unemployment helps predict X, even taking

lagged X's into account; but once lagged Un's are taken into account,

lagged values of X are of no aid in predicting unemployment.

Testing the Hypothesis

Granger [4] and Sims [18] have described the statistical

theory that can be used to construct tests of the natural rate hypoth-

esis as formulated above. According to Granger, "...We say that Y_t is causing X_t if we are better able to predict X_t using all available [past] information than if the information apart from [past] Y_t had been used." (p. 428) The above formulation of the natural rate hypothesis thus posits that the unemployment rate is <u>caused</u>, in Granger's sense, by <u>no</u> other variables. From Granger's paper, a direct statistical test of that hypothesis is available. Consider the unemployment rate Un_t and some other variable Y_t. Using the method of least squares, estimate the linear regression of Un_t on lagged Un's and lagged Y's,

$$(2.5) \qquad \hat{Un}_t = \sum_{j=1}^{m} \hat{\alpha}_j Un_{t-j} + \sum_{j=1}^{n} \hat{\beta}_j Y_{t-j} \;,$$

where the $\hat{\alpha}_j$'s and $\hat{\beta}_j$'s are least-squares estimates. On the null hypothesis that Y does <u>not</u> cause Un, the parent parameters β_j, $j=1, \ldots, n$, equal zero. The natural rate hypothesis can then be tested by testing the null hypothesis $\beta_j = 0 \; \{\, j=1, \ldots, n \,\}$, for various choices of Y. Alternatively, lagged values of several variables can be added to the right side of (5). On the natural rate hypothesis, all such variables bear zero coefficients.

An alternative way of testing the natural rate hypothesis as posed here is to employ the test for Granger causality proposed by Christopher Sims [18]. Assume that Un and some other series Y are jointly covariance-stationary and that they are purely indeterministic. Then the generalization of Wold's representation theorem to n-dimensions implies that Un_t and Y_t have the moving-average representation

$$(a) \qquad Un_t = \sum_{i=0}^{\infty} a_i \varepsilon_{t-i} + \sum_{i=0}^{\infty} b_i \eta_{t-i}$$

(2.6)

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(b) $\quad Y_t = \sum\limits_{i=0}^{\infty} c_i \varepsilon_{t-i} + \sum\limits_{i=0}^{\infty} d_i \eta_{t-i}$,

where ε and η are serially uncorrelated and mutually uncorrelated with finite variances; (6) is a very general representation of the two processes Un_t and Y_t , the a's, b's, c's, and d's being chosen to make the cross-covariogram between the moving sums on the left-hand sides of the two equations match that between Un and Y. Sims showed that Y does not cause Un in Granger's sense if and only if either all of the a_i's or all of the b_i's in (6) are zero.[10,11] On the basis of this result, Sims showed that Y_t could be expressed as a one-sided distributed lag of Un_t with a disturbance uncorrelated with past, future, and current Un's if and only if Y fails to "cause" Un. Sims's test for exogeneity of Un is to regress Y on past, present, and future Un's, and then to test the null hypothesis that coefficients on future Un's are zero. That is, by least-squares estimate

$$Y_t = \sum\limits_{i=-n}^{n} \gamma_i Un_{t-i} + e_t \ ,$$

where e_t is a residual. On the null hypothesis that Y does not cause Un, $\gamma_i = 0$ for $i < 0$.

The Interest Rate

The equation for the long-term interest rate is motivated by

the rational expectations version of the expectations theory of the term structure.[12/] Let R_{nt} be the yield to maturity on an n-period bond at time t, where n is large in relation to unit increments in t. I approximate the rational expectations theory of the term structure as asserting

(2.7) $\qquad R_{nt} = \frac{1}{n} [R_{1t} + E_t R_{1t+1} + \dots + E_t R_{1t + n-1}]$,

so that the n-period rate is an average of the current short rate R_{1t} and expected future short rates $E_t R_{1t+j}$, j=1, ... , n-1. Expectations about future short rates are assumed to be rational. Subtracting R_{nt} from R_{nt+1} gives

$$R_{nt+1} - R_{nt} = \eta_{nt+1} + \frac{1}{n}[E_t R_{1t+n} - R_{1t}]$$

where

$$\eta_{nt+1} = \frac{1}{n}[(R_{1t+1} - E_t R_{1t+1}) + (E_{t+1} R_{1t+2} - E_t R_{1t+2})$$
$$+ \dots + (E_{t+1} R_{1t+n-1} - E_t R_{1t+n-1})] .$$

The term η_{nt+1} is of the nature of an "innovation," and as an implication of rationality obeys $E_t \eta_{nt+1} = 0$. Furthermore, for large n and well behaved (i.e., flat enough) yield curves, $1/n[E_t R_{1t+n} - R_{1t}] \simeq 0$. Consequently, for large n, there obtains the approximation

(2.8) $\qquad E_t R_{nt+1} = R_{nt}$,

which says that the n-period rate is a martingale process.

Suppose that the reduced form for the short term interest rate is

$$R_{1t} = \beta Z_t$$

where β is conformable to Z_t and where Z_t is a vector of exogenous variables including government expenditures, tax rates, the money supply, and other determinants of the real rate of interest and the expected rate of inflation. Then we have

$$E_t R_{1t+j} = \beta E_t Z_{t+j}.$$

Then (2.7) becomes

$$R_{nt} = \frac{1}{n} \beta [Z_t + E_t Z_{t+1} + \ldots + E_t Z_{t+n-1}].$$

So we have

$$(2.9) \qquad R_{nt+1} - R_{nt} = \frac{1}{n} \beta [(Z_{t+1} - E_t Z_{t+1}) + (E_{t+1} Z_{t+2} - E_t Z_{t+2}) + \ldots$$

$$+ (E_{t+1} Z_{t+n-1} - E_t Z_{t+n-1})]$$

$$+ \frac{1}{n} \beta [E_t Z_{t+n} - Z_t].$$

Supposing that Z_t is a vector autoregressive process, it is easy to show that[13/]

$$(2.10) \qquad (E_{t+1} Z_{t+j} - E_t Z_{t+j}) = \Gamma_{j-1}(Z_{t+1} - E_t Z_{t+1})$$

where Γ_{j-1} is a square matrix conformable with Z, one whose elements are functions of the parameters of the autoregression for Z. Substituting (2.10) into (2.9), we obtain

$$R_{nt+1} - R_{nt} = \frac{1}{n} \beta [I + \Gamma_1 + \Gamma_2 + \ldots + \Gamma_{n-2}](Z_{t+1} - E_t Z_{t+1})$$

$$+ \frac{1}{n} \beta [E_t Z_{t+n} - Z_t].$$

Upon imposing our flat yield curve approximation $1/n[E_t Z_{t+n} - Z_t] = 0$, the above equation becomes

$$(2.11) \qquad R_{nt+1} - R_{nt} = \xi[Z_{t+1} - E_t Z_{t+1}],$$

where $\xi = (1/n)\beta[I + \Gamma_1 + \ldots + \Gamma_{n-2}]$. This is a version of equation (1.5). As before, we have the implication

$$(2.8) \qquad E_t R_{nt+1} = R_{nt}.$$

According to (8) a regression of $R_{nt+1} - R_{nt}$ against any variables dated t or earlier ought to have coefficients of zero. For example, a regression of $R_{nt+1} - R_{nt}$ against prices or rates of inflation dated t or earlier ought to have zero regression coefficients. The reason is that R_{nt} already has built into it expectations of inflation over almost all of the horizon for R_{nt+1}, and that any revisions in those expectations between t and t+1 can't be predicted on the basis of information available at time t, by virtue of the rationality of those expectations.

Another way to test equation (7) is to note that it implies that R_{nt} is not "caused" by any variable. That can be tested by fitting two-sided distributed lags of causal candidates against R_{nt}, and testing the null hypothesis that the coefficients on future R_n's are zero.

For my purposes, the important implication of the theory is that R_n cannot be predicted better by taking into account other variables, once lagged values of R_n have been taken into account. So it would be perfectly acceptable to modify equation (8) to read

$$(2.8') \qquad E_t R_{nt+1} = \sum_{i=0}^{n} w_i R_{nt-i},$$

which carries the crucial implication that R_{nt} is "caused" by no other variables. Equation (8') should perhaps be preferred over equation (8) according to certain theories about the liquidity premiums that allegedly infest the term structure.[14]

The assertion that other variables, such as monetary aggregates and fiscal policy variables, contain no information, over and above that contained in lagged values of the long rate, that can be used to predict the long rate is one that contradicts the implications of all existing macroeconometric models, as they are usually manipulated.[15] Stochastic simulations of these models will in general generate data for which a variety of monetary, fiscal and other variables "cause" the long rate and thereby aid in its prediction.

Observations on the Tests

The restrictions imposed by the statistical models for unemployment and the interest rate outlined here are stricter than what is

really necessary to deliver the "classical" policy implications of the model. Thus, suppose that X_t is a vector of "real" economic aggregates at time t including variables such as real GNP, unemployment, layoffs, interest rates, and so on; X_t excludes variables measuring the composition of output, such as aggregate consumption and investment, and outputs of particular commodities. Let g_t be a list of monetary and fiscal policy variables at time t. Then a model in general will have "classical" policy implications if it satisfies

$$(2.12) \qquad E[X_t | X_{t-1}, X_{t-2}, \cdots ; g_{t-1}, g_{t-2}, \cdots] = E[X_t | X_{t-1}, X_{t-2}, \cdots],$$

so that as a block the aggregate real variables X are statistically exogenous with respect to (not caused by, in Granger's sense) the variables in g. For a system satisfying (2.12), movements in the components of g don't have predictable effects on subsequent values of the real variables in X. So (2.12) exhibits the same sort of neutrality of certain real variables with respect to monetary and fiscal policy as does the model in section I.

While the model of section I is an example of a system satisfying (2.12), (2.12) is more general. There are systems satisfying (2.12) that violate the hypothesis for the unemployment rate and the interest rate described here in section II, which are key hypotheses underlying the model of section I. Thus, (2.12) does <u>not</u> imply

$$E[Un_t | Un_{t-1}, Un_{t-2}, \cdots ; g_{t-1}, g_{t-2}, \cdots] = E[Un_t | Un_{t-1},$$
$$Un_{t-2}, \cdots]$$

even though Un_t is a component of X_t.

A simple example that illustrates this is a system satisfying (2.12) in which, say, layoffs help "cause" unemployment. Suppose that some components of g_t are set via a feedback rule on layoffs. Then even though g doesn't cause (help predict) Un when lagged unemployment and lagged layoffs are taken into account, components of g will help predict unemployment when only lagged unemployment is taken into account. This is because g contains some information about lagged layoffs. This is a "spurious" type of causality from g to Un in which an omitted variable (layoffs) is causing both g and Un (see Granger [4]); when layoffs are omitted, g only appears to cause Un because it is standing in for the omitted lagged layoff rates.

The possibility of such spurious apparent causality running from components of g to Un is noteworthy, since the above statement of the natural rate hypothesis is so very strict. In particular, it rules out even the possibility that other real variables (the components of X in (2.12)) cause unemployment. This seems too drastic, since it is easy to imagine structures in which there is extensive causality from, say, GNP and layoffs to unemployment that satisfy (2.12) and so are basically "classical" in nature. For such a system our tests might well reject the very strict version of the natural rate hypothesis adopted above.

While failure of monetary and fiscal policy variables to cause unemployment and other real variables is sufficient to deliver classical policy implications, it is not really necessary. One can imagine structures in which policy variables cause (help predict) unemployment and other real variables, but in which switching from one deterministic rule for setting the policy variable to another leaves the stochastic behavior of unemployment unchanged. As an example, consider the structural system

$$(2.13) \qquad Un_t = \sum_{i=1}^{n_1} \lambda_i Un_{t-i} + \beta_0(m_t - E_{t-1}m_t) + \beta_1(m_{t-1} - E_{t-2}m_{t-1}) + u_t$$

$$(2.14) \qquad m_t = \sum_{i=1}^{n_2} \delta_i m_{t-i} + \varepsilon_t ,$$

where ε_t and u_t are random variables, and $E_{t-1}\varepsilon_t = E_{t-1}u_t = 0$. For the above structure, it is easy to calculate

$$E[Un_t | Un_{t-1}, \ldots, m_{t-1}, m_{t-2}, \ldots] = \sum_{i=1}^{n_1} \lambda_i Un_{t-i} +$$
$$\beta_1(m_{t-1} - \sum_{i=1}^{n_2} \delta_i m_{t-i-1}).$$

It follows that m helps predict (causes) Un_t. But notice that according to (2.13), switching from one deterministic rule for m (i.e., a rule for which $m_t = E_{t-1}m_t$) to any other deterministic rule will leave the stochastic behavior of unemployment unaltered. Even though m causes unemployment in this system, it is true that one deterministic rule is as good as any other, so that there is no scope for countercyclical policy by way of "leaning against the wind."

The preceding observations suggest reasons for believing that this paper tests versions of classical hypotheses that are really stronger than what is necessary to deliver classical policy conclusions, so that the tests seem biased against the natural rate hypothesis and other classical hypotheses. However, it is important to note that the tests are not uniformly biased against classical hypotheses, since it is possible to concoct nonclassical systems that will mimic the classical characteristics that my tests look for. Thus, the tests might be fooled into failing to reject the natural rate hypothesis in a system for which that hypothesis is false. Suppose that the true reduced form for Un_t is

(2.15) $\quad Un_t = \sum_{i=1}^{2} \lambda_i Un_{t-1} + \alpha_0 m_t + \alpha_1 m_{t-1} + \varepsilon_t$

where $E[\varepsilon_t | Un_{t-1}, \ldots, m_t, m_{t-1}] = 0$ and where the λ's and α's are fixed

parameters. Suppose that the authority sets m_t according to the deter-

ministic feedback rule

$$m_t = \sum_{i=1}^{2} \delta_i Un_{t-i}.$$

Then clearly,

$$E[Un_t | Un_{t-1}, Un_{t-2}, \cdots ; m_t, m_{t-1}] = \sum_{i=1}^{3} (\lambda_i + \alpha_0 \delta_i + \alpha_1 \delta_{i-1}) Un_{t-i}$$

where $\delta_3 = \lambda_3 = \delta_0 = 0$.

Here Un is not caused by m, in Granger's sense, because the authority,

by making m_t an exact function of past Un's, eliminates any value from

the m series for predicting Un.

While the tests might be fooled by such a structure, that

structure itself seems unlikely to me. In particular, if the reduced

form were (2.15) and the authority were to set m_t by a feedback only on

lagged Un's, and not on other variables also, presumably the authority

would want to minimize the variance of Un_t, which it could accomplish by

eliminating any serial correlation in Un_t. That is, in our example, it

could minimize the variance in Un by setting $\lambda_1 + \alpha_0 \delta_1 = 0$, $\lambda_2 + \alpha_1 \delta_1 = 0$

and $\delta_2 = 0$. Then the variance of unemployment would equal the variance

of ε_t. But in reality variables like unemployment and the deviation of

GNP from trend are highly serially correlated. That makes it hard to

believe that any failure of, say, m to "cause" Un is due to the authority's

manipulating m in response to past movements in Un, since that requires

inputing to the authority a perverse objective, i.e., one tolerating

much serial correlation and variance in Un.

Section III

Empirical Results

Tables 1-6 report the results of performing tests along the

lines proposed by Granger and Sims for quarterly data on the dependent

variables spanning the period 1952 II - 1972 III. The unemployment rate for all civilian workers is used for Un, while Moody's Baa corporate bond index is taken for the long-term interest rate R. The variables used as candidates for the "causal" variables Y are the logarithm of the money supply, currency plus demand deposits, (m); the federal and state and local government surplus on the national income accounts basis in 1958 dollars (surp); the logarithms of the GNP deflator (p); a straight-time wage index in manufacturing (w); federal and state and local purchases of goods and services in 1958 dollars (g); and federal and state and local purchases in current dollars, (g$).

Each of the series has been seasonally adjusted by taking the Fourier transform of the series, setting its real and imaginary parts to zero in a band of width $\pi/12$ about the seasonal frequencies, and then taking the inverse Fourier transform to obtain a seasonally adjusted series.[17/] This method has the virtue of applying a seasonal adjustment filter with the same frequency response function to each series, thereby avoiding the distortions in estimating distributed lags between variables that can be caused where the series have been adjusted asymmetrically (see Wallis [22] and Sims [17]). Furthermore, the method reduces the spectral power of the series to zero at the seasonal frequencies, which Sims [17] has argued helps eliminate bias in the form of seasonal patterns showing up in estimated distributed lag coefficients.

Table 1 reports the results of implementing Granger's test for causality between Un and each of the Y candidates listed above. For

each Y, the test is run in both directions: First, Un is regressed on

lagged Un's and lagged Y's to permit testing the null hypothesis that Y

does not cause Un (i.e., that the coefficients on lagged Y's are zero).

Then Y is regressed on lagged Y's and lagged Un's to permit testing the

null hypothesis that Un does not cause Y (i.e., that the coefficients on

lagged Un's are zero). Regressions in both directions include a con-

stant and a linear trend. The regressions include four lagged values of

the dependent variable and six lagged values of the other variable. The

F-statistic pertinent for testing the null hypothesis that the dependent

variable is not caused by the other variable is reported in the last column.

The F-statistic for m as the causal variable influencing Un

is significant at the 95 percent confidence level, though not at the 99

percent level. Similarly, the F-statistic for w as a causal variable

for Un is significant at the 95 percent confidence level. None of the

other causal candidates obtains an F that would require rejecting the

null hypothesis that they don't cause unemployment. In particular,

notice that the GNP deflator doesn't appear to cause unemployment.

In the other direction, the F-statistics reveal that the

hypothesis that Un doesn't cause g or g$ can be rejected at the 95

percent confidence level. The hypothesis that Un doesn't cause the

other four variables can't be rejected.

Tables 2 and 3 report summary statistics for the regressions

implementing Sims' test for unemployment.[18] Two-sided distributed lags

were calculated in each direction, one with Un the dependent variable

and the causal candidate Y the "independent" variable, the other with Un

and Y reversed. The data were quasi-differenced by applying the filter

$(1-.75L)^2$. Each regression included a constant and a trend, with four lead variables and twelve lagged variables. The regressions were first estimated by the method of least squares. Then the Fourier transform of the distributed lag coefficients was calculated. The amplitude of the Fourier transform was inspected to see if peaks occurred at the seasonal frequencies. In those cases where a peak occurred, indicating a seasonal pattern in the coefficients, the regressions were recomputed using Theil's mixed estimator to incorporate weak, stochastic prior information stating that there is no seasonal pattern in the distributed lag. In particular, suppose the regression estimated is

$$Un_t = \sum_{i=-4}^{12} \hat{b}_i Y_{t-i} + residual_t ,$$

and that a seasonal pattern characterizes the b_i's. The regression was then recalculated by adding observations on the three constraints

$$b_{-4} + b_0 + b_4 + b_8 = b_{-3} + b_1 + b_5 + b_9 + U_1$$

$$b_{-4} + b_0 + b_4 + b_8 = b_{-2} + b_2 + b_6 + b_{10} + U_2$$

$$b_{-4} + b_0 + b_4 + b_8 = b_{-1} + b_3 + b_7 + b_{11} + U_3 ,$$

where the U's are random variables obeying $EU_1 = EU_2 = EU_3 = 0$. Theil's mixed estimator requires estimates of the standard error of the disturbances in the regression, and the standard errors of U_1, U_2, U_3. The former was taken equal to the standard error of the residuals in the original least-squares regression. The latter standard errors were taken equal to one another at σ_u, which was set at either $(\max_i \hat{b}_i - \min_i \hat{b}_i)$ or $(\max_i \hat{b}_i - \min_i \hat{b}_i)/2$, where the $\hat{b}_i$'s are from the original least-squares regression. The covariance of each U with all other

random variables was assumed to be zero. Estimation incorporating this prior information in most cases sufficed to eliminate the seasonal in the distributed lag coefficients.

Table 2 summarizes the F-statistics pertinent for testing the null hypothesis that the coefficients on future values of the right-side variable are zero, i.e., the null hypothesis that the left-side variable doesn't cause the right-side variable. For no causal candidate Y does the F-statistic indicate rejecting that Y does not cause Un at the 95 percent confidence level. In particular, notice that in contrast to the results from applying the direct Granger test, it is not possible to reject the hypothesis that m or w does not cause Un. In the other direction, the F-statistic reveals that the hypothesis that g is not caused by Un must be rejected at the 95 percent confidence level. The next highest F is for g$, though it is not significant at the 95 percent confidence level. Qualitatively, the overall pattern of the results is similar to that obtained by applying Granger's test, with the important exceptions of the different results rendered for whether m causes Un and for whether w causes Un.

Table 3 reports F-statistics pertinent for testing whether the coefficients on current and lagged right-hand side variables are zero in the one-sided regressions corresponding to those in Table 2. Only the F-statistics for the regression of Un on surp and Un on w are significant at the 95 percent confidence level.

Tables 4, 5, and 6 report the results of applying Granger's and Sims's tests to determine whether the long-term interest rate, as measured by the Baa yield index, is statistically exogenous as implied by our theory. Table 4 records the results of applying the direct

Granger test. The F-statistic is the one pertinent for testing that the coefficients on lagged values of the causal candidate Y are all zero, so that Y does not cause or help predict the dependent variable. Where RBaa is the dependent variable, w is the only causal candidate that obtains an F-statistic that is significant at the 95 percent confidence level. At that confidence level, the results are thus consistent with the implications of the theory, with the exception of the results for w, which indicate that w "causes" RBaa. In the reverse direction, the hypothesis that RBaa does not cause the money supply must be rejected at the 95 percent confidence level.

For Sims's test, Table 5 summarizes the F-statistics pertinent for testing the null hypothesis of no causality for the interest rate. The results are compatible with those obtained from applying Granger's test. The hypothesis that RBaa is not caused by the causal candidate can be rejected at the 95 percent confidence level only for w. In the reverse direction, the hypothesis that RBaa fails to cause m must be rejected at the 95 percent confidence level.

Table 6 reports the F-statistics pertinent for testing the null hypothesis that coefficients on current and lagged values of the causal candidates are zero in the one-sided regressions corresponding to those in Table 5. The F's for w on RBaa and RBaa on w are the only ones significant at the 95 percent confidence level, though a couple of others are marginal and may be understated because possibly too many lagged variables have been included.

Table 7 reports F-statistics pertinent for testing whether the labor force participation rate nf is exogenous with respect to various

causal candidates. The model implies that n_{ft} is exogenous with respect to all variables in the model, with the possible exception of the unemployment rate. The unemployment rate can "cause" the labor force participation rate, say through the "discouraged worker effect," while not destroying the "recursive" structure of the model which prevents monetary and fiscal policy variables from "causing" the real variables Un, nf, and y.

The F-statistics in Table 7 emerge from implementing Sims's test. The only F-statistic that is significant at the 95 percent confidence level is the one pertinent for testing the null hypothesis that Un fails to cause nf. At that significance level the null hypothesis must be rejected, which is compatible with the presence of a "discouraged worker" effect that is useful for predicting labor force participation. While none of the other F-statistics is significant, the regression of m_t against nf_t did obtain several large and statistically significant coefficients on leading values of nf. This indicates that one ought perhaps to be cautious about the null hypothesis that m doesn't cause nf, despite the insignificant F-statistic. With this possible exception, the regressions summarized in Table 7 are consistent with the causal structure imposed by the model upon nf.

Table 8 reports the results of applying Granger's test to nf and various causal candidates. At the 95 percent confidence level, nf appears to "cause" w, p, g, and Un, while only Un appears to "cause" nf.[19]

All in all, the empirical results provide some evidence that the causal structure imposed on the data by the classical model of section I is not obscenely at variance with the data. The evidence that m seems to be caused by RBaa means that the assumption that m is exogenous,

embedded in the assumed autoregression (1.6), must be abandoned. But this is not essential, since for the purpose that the model is intended (unconditional forecasting), the regression in Table 4 will do just as well. Findings that contradict the model are that w seems to cause both RBaa and Un, according to both Sims's and Granger's tests. Also, according to Granger's test, m seems to cause Un, but it does not according to Sims's test. This last discrepancy requires reconciling, as does the apparently general tendency of Granger's test to reject exogeneity more readily than does Sims's test.[20,21]

I do not believe that these results render a sufficiently negative verdict on the model of section I that I should quit now before presenting estimates of the model. The causal candidate that does the most damage to the hypotheses of the model is the money wage w, which does not appear itself as a variable in the model of section I. Causal candidates drawn from the list of variables actually appearing in the model usually don't seem to violate the hypotheses of the model, which gives some encouragement to the project of estimating the model.

Section IV

Estimates of the Model[22]

To estimate the model, a proxy for $E_{t-1}p_t$ was required.[23]
Following the same procedure used in [14], the proxy for $E_{t-1}p_t$ was
formed by regressing p_t against a list of variables dated t-1 and
earlier.[24] In each case, this list included all the predetermined
variables that appear on the right side of the equation in which $E_{t-1}p_t$
appears.

Since the model is a simultaneous one, an instrumental vari-
ables estimator was used to estimate the coefficients. Current endoge-
nous variables that appear on the right side of an equation were re-
placed by the systematic part of a regression of that variable on the
same variables that were used to form the proxy for $E_{t-1}p_t$ plus current
values of the exogenous variables.[25,26]

The estimates are reported in Table 9. The production function
includes current and four lagged values of $n_t \equiv (n_{ft} + pop_t - Un_t)$.

The estimates of the production function, equation (3), are compatible with increasing returns to labor in the short run and slightly decreasing returns to labor in the long run.

The estimates reported in Table 9 possess signs that agree with a priori expectations. Unexpected increases in the price level are estimated to increase the labor force participation rate and decrease the unemployment rate. Increases in the unemployment rate decrease the labor force participation rate, which is consistent with a "discouraged worker" effect.

In the estimates reported in Table 9, I have not included innovations in Z_t as determinants of R, so that the equation for R (the Baa rate) is simply an autoregression. Two pairs of equations for portfolio equilibrium are reported. The first pair regresses the reciprocal of the log of velocity (m - p - y) against current and lagged interest rates, one member including and the other excluding trend. Including trend is seen to increase the coefficients on current and lagged interest rates, and to make their sum positive. This is a common though widely ignored result: including a trend in postwar estimates of demand schedules for money for the U.S. tends to eliminate any inverse dependence of velocity on interest rates. The second pair of portfolio balance equations regresses m - p on current and lagged y's and R's, again with and without trend. Including trend again has important effects on the coefficients. For my purposes, any of these four or any other reasonable demand schedule for money is suitable. Notice also that the model will work in the same "recursive" way if a demand schedule for money is dropped and replaced by a regression of $p_t + y_t$ on current and lagged m, the sort of equation estimated by Sims [18] and Andersen and Jordan [1].

Section V

Conclusions

This paper has estimated and tested a macroeconometric model with "classical," or "monetarist" policy implications, even though it has "Keynesian" short-run properties. Some evidence for rejecting the model has been turned up, but it is far from being overwhelming and decisive. The evidence that seems most damaging to the model comes from the role that the money wage plays in apparently "causing" unemployment and the long-term interest rate. On the other hand, the tests have turned up little evidence requiring us to reject the key hypothesis of the model that government monetary and fiscal policy variables do not cause unemployment or the interest rate. The fact that such evidence has been hard to turn up ought to be disconcerting to users of the existing macroeconometric models, since as usually manipulated those models all imply that monetary and fiscal policy do help cause unemployment and the interest rate.

Models of the kind presented in this paper imply that there is no scope for the government to engage in activist countercyclical policy so that it might as well employ rules without feedback for fiscal and monetary policy, e.g., Friedman's x-percent growth rule for the money supply. In contradistinction, macroeconometric models as they are usually manipulated imply that it is optimal for the government to use rules with feedback, which may imply "leaning against the wind," contrary to Friedman's rule. If we are to have any reason to believe that rules with feedback are superior to rules without feedback, there should be empirical evidence in hand that some existing macroeconometric model can

outperform models of the class studied in this paper. It is my impression that such evidence doesn't yet exist.

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Table 1

Regression of $Y(t) = \sum_{s=1}^{4} \alpha(s)Y(t-s) + \sum_{v=1}^{6} B(v)X(t-v) + \delta_1 t + \delta_0$

1952.II – 1972.III

Y	X	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)	δ_1	δ_0	$\bar{R}^2$	Standard Error of Est. Adj.	Durbin-Watson Statistic	F-Statistic on all B F(6,60)
Un	m	1.317 (10.25)	-0.541 (-2.64)	-0.029 (-0.14)	0.113 (0.94)	-14.244 (-1.89)	7.787 (0.57)	7.578 (0.53)	-24.943 (-1.75)	26.849 (1.85)	-0.862 (-0.10)	-0.010 (-1.32)	-9.526 (-1.86)	.918	.346	1.973	2.627*
m	Un	1.279 (9.75)	-0.367 (-1.60)	0.022 (0.09)	0.063 (0.44)	-0.002 (-0.88)	0.003 (0.57)	0.0005 (0.13)	0.00002 (0.01)	-0.002 (-0.56)	-0.0009 (-0.44)	0.0002 (1.33)	0.022 (0.26)	.999	.006	1.991	1.506
Un	Surp	1.435 (10.84)	-0.540 (-2.33)	-0.121 (-0.52)	0.134 (1.02)	-0.020 (-1.32)	0.035 (1.58)	-0.035 (-1.56)	0.029 (1.33)	-0.002 (-0.07)	-0.007 (-0.57)	-0.0004 (-0.21)	0.439 (2.36)	.906	.372	1.995	0.954
Surp	Un	1.143 (8.60)	-0.399 (-2.02)	0.149 (0.75)	-0.248 (-1.90)	-0.319 (-0.26)	0.268 (0.13)	-0.848 (-0.56)	2.875 (1.36)	-3.330 (-1.73)	1.979 (1.77)	-0.017 (-1.04)	-3.200 (-1.84)	.814	3.323	1.992	1.034
Un	g	1.498 (11.99)	-0.641 (-2.87)	-0.027 (-0.12)	0.057 (0.40)	0.814 (0.75)	-1.085 (-0.82)	-0.037 (-0.40)	1.152 (1.36)	-1.336 (-1.14)	0.143 (1.77)	0.002 (0.20)	2.036 (0.44)	.902	.380	1.917	0.489
g	Un	0.685 (5.20)	-0.042 (-0.27)	0.069 (0.46)	-0.110 (-0.96)	-0.016 (-1.10)	-0.004 (-0.82)	0.024 (0.88)	-0.065 (-0.15)	0.034 (1.31)	0.004 (0.22)	0.003 (3.28)	1.840 (3.30)	.960	.043	1.979	4.554**
Un	g$	1.506 (12.28)	-0.614 (-2.76)	-0.134 (-0.57)	0.147 (1.09)	0.204 (0.07)	-0.004 (-0.03)	-0.006 (-0.63)	0.211 (0.05)	0.034 (0.22)	-0.003 (-0.55)	-0.002 (-0.19)	-0.022 (-0.02)	.908	.368	2.056	1.145
g$	Un	1.135 (9.84)	-0.066 (-0.37)	0.099 (0.57)	-0.292 (-2.99)	-0.014 (-2.87)	-4.415 (-0.95)	7.494 (1.73)	-6.820 (-1.61)	3.457 (1.56)	0.004 (0.43)	0.002 (3.99)	0.530 (3.84)	.999	.014	1.806	2.504*
Un	p	1.469 (11.55)	-0.610 (-2.73)	-0.120 (-0.54)	0.132 (1.04)	-15.845 (-1.34)	22.334 (1.23)	-0.608 (-0.03)	-12.832 (-0.71)	14.059 (-0.85)	-3.717 (-0.34)	-0.018 (-1.43)	-14.386 (-1.43)	.906	.372	2.063	0.936
p	Un	1.090 (8.42)	0.003 (0.02)	0.034 (0.17)	-0.120 (-0.87)	0.0004 (0.31)	-0.002 (-0.94)	0.002 (0.78)	-0.002 (-0.88)	0.001 (0.54)	-0.0004 (-0.31)	0.00002 (0.20)	-0.026 (-0.30)	.999	.004	2.019	1.222
Un	w	1.238 (8.02)	-0.437 (-1.94)	-0.204 (-0.92)	0.122 (0.95)	-18.346 (-1.76)	25.905 (1.81)	-20.746 (-1.44)	20.274 (1.46)	1.146 (0.08)	1.050 (0.10)	-0.084 (-2.44)	-2.469 (-2.36)	.917	.350	2.027	2.371*
w	Un	1.066 (7.05)	-0.347 (-1.66)	0.431 (2.03)	-0.074 (-0.43)	-0.003 (-1.50)	0.00007 (0.02)	0.00001 (0.00)	0.004 (1.08)	-0.006 (-1.88)	0.002 (1.15)	-0.0006 (-1.72)	-0.010 (-0.79)	.9996	.005	1.969	1.688

*Denotes significance @ 5 percent

**Denotes significance @ 1 percent

t-statistics for coefficients appear in parentheses below relevant coefficients

Table 2

F-Statistic -- Two-sided Tests

Independent variable

Y	(a) Un		(b) Y	
m	.401	[1]	.951	[0]
surp	1.09	[1]	.396	[0]
g	.344	[2]	2.854	[2]
g$	.374	[1]	1.255	[1]
p	.647	[2]	.479	[0]
w	1.472	[2]	1.238	[0]

All F's are $F(4,50)$; significance levels are 2.56 for .95 confidence
3.72 for .99 confidence.

Column (a) regressions: $\quad Y_t = \sum_{i=-4}^{12} w_i Un_{t-i}$

Column (b) regressions: $\quad Un_t = \sum_{i=-4}^{12} w_i' Y_{t-i}$

F-statistic in column (a) is pertinent for testing null hypothesis
$\quad w_{-4} = w_{-3} = w_{-2} = w_{-1} = 0.$

F-statistic in column (b) is pertinent for testing null hypothesis
$\quad w'_{-4} = w'_{-3} = w'_{-2} = w'_{-1} = 0.$

0 means no Theil constraint used;

1 means Theil constraint used with $\sigma_u = \max w_i - \min w_i$;

2 means Theil constraint used with $\sigma_u = (\max w_i - \min w_i)/2.$

Table 3

F-Statistics for Coefficients on Current and Lagged Variables

$$\text{I.} \quad Y_t = \sum_{\alpha=0}^{12} \alpha_i Un_{t-i} + \beta_0 + \beta_1 t$$

$$\text{II.} \quad Un_t = \sum_{\alpha=0}^{12} \gamma_i Y_{t-i} + \delta_0 + \delta_1 t$$

Variable Name	Independent Variable			
Y_t	Un		Y_t	
m	0.883	1	0.911	0
surp	1.685	1	1.999*	0
g	1.454	2	0.520	2
g$	1.785	1	1.441	1
p	0.571	2	1.347	0
w	1.003	2	2.355*	0

All F's are $F(13, 54)$ All data are filtered: $(1-.75L)^2$

*Denotes significant @ 5 percent level

For meaning of superscripts 0, 1, and 2, see Table 2.

Table 4

Regression of $Y(t) = \sum_{s=1}^{4} \alpha(s)Y(t-s) + \sum_{\ell=1}^{6} \beta(\ell)X(t-\ell) + \delta_1 t + \delta_0$

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Y	X	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$	$\beta(1)$	$\beta(2)$	$\beta(3)$	$\beta(4)$	$\beta(5)$	$\beta(6)$	δ_1	δ_0	$\bar{R}^2$	Standard Error of Est. Adj.	Durbin-Watson Statistic	F-Statistic on all β $F(6,60)$
RBaa	m	1.447 (11.54)	-0.734 (-3.26)	0.553 (2.41)	-0.378 (-2.87)	0.635 (0.17)	3.311 (0.50)	-2.633 (-0.39)	-4.584 (0.79)	5.248 (0.79)	-1.016 (-0.27)	-0.0004 (-0.11)	-4.251 (-1.29)	.992	.161	1.867	0.805
m	RBaa	1.337 (10.39)	-0.187 (-0.80)	-0.317 (-1.27)	0.194 (1.38)	-0.012 (-2.55)	0.005 (0.62)	0.017 (1.89)	-0.011 (-1.19)	-0.002 (-0.21)	0.001 (0.21)	0.00001 (0.14)	-0.129 (-1.30)	.999	.006	1.925	2.730*
RBaa	Surp	1.471 (11.45)	-0.715 (-3.16)	0.491 (2.14)	-0.290 (-2.12)	0.009 (1.48)	-0.011 (-1.15)	0.010 (1.06)	-0.007 (-0.74)	0.002 (0.19)	0.001 (0.19)	0.003 (1.61)	0.141 (1.58)	.992	.163	1.867	0.593
Surp	RBaa	1.107 (8.83)	-0.331 (-1.77)	0.156 (0.85)	-0.265 (-2.18)	0.889 (0.34)	-7.705 (-1.66)	9.335 (1.89)	-7.956 (-1.58)	5.698 (1.17)	-1.244 (-0.44)	0.053 (1.41)	2.155 (1.19)	.828	3.194	1.960	1.948
RBaa	g	1.485 (11.82)	-0.720 (-3.20)	0.423 (1.88)	-0.254 (-2.00)	-0.044 (-0.10)	0.110 (0.20)	0.421 (0.75)	-0.426 (-0.76)	-0.030 (-0.06)	0.276 (0.90)	0.002 (0.72)	-1.139 (-0.97)	.992	.164	1.894	0.462
g	RBaa	0.918 (7.15)	-0.080 (-0.46)	0.036 (0.21)	-0.096 (-0.76)	0.060 (1.49)	-0.085 (-1.15)	0.067 (0.86)	-0.070 (-0.88)	0.047 (0.63)	-0.025 (-0.59)	0.002 (2.51)	0.990 (2.92)	.946	.050	1.971	0.711
RBaa	g$	1.485 (11.75)	-0.758 (-3.40)	0.470 (2.11)	-0.282 (-2.25)	1.130 (0.90)	-1.303 (-0.67)	1.216 (0.64)	-1.598 (-0.84)	1.161 (0.62)	-0.143 (-0.15)	-0.002 (-0.50)	-1.640 (-1.27)	.992	.164	1.888	0.502
g$	RBaa	1.193 (10.38)	-0.151 (-0.82)	0.218 (1.22)	-0.352 (-3.61)	-0.003 (-0.21)	0.026 (1.19)	-0.039 (-1.63)	0.024 (0.98)	-0.012 (-0.53)	0.007 (0.52)	0.001 (3.60)	0.369 (3.31)	.999	.015	1.650	0.656
RBaa	p	1.388 (10.86)	-0.687 (-3.20)	0.375 (1.74)	-0.187 (-1.30)	5.516 (1.11)	-3.821 (-0.50)	3.812 (0.50)	6.530 (0.88)	-11.143 (-1.62)	-1.321 (-0.29)	0.007 (0.24)	2.144 (0.24)	.992	.156	1.962	1.589
p	RBaa	1.083 (8.24)	0.003 (0.02)	0.021 (0.11)	-0.106 (-0.78)	0.003 (0.99)	-0.004 (-0.75)	0.010 (1.57)	-0.008 (0.04)	0.0003 (0.04)	-0.0002 (-0.06)	0.00002 (0.09)	-0.004 (-0.02)	.999	.004	2.016	1.482
RBaa	w	1.531 (11.87)	-0.770 (-3.45)	0.267 (1.21)	-0.117 (-0.92)	7.898 (2.19)	-15.196 (-2.66)	15.141 (2.48)	0.587 (0.09)	-12.076 (-2.00)	3.635 (0.93)	0.005 (0.60)	0.154 (0.36)	.993	.146	2.017	3.199**
w	RBaa	1.148 (8.95)	-0.333 (-1.65)	0.361 (1.70)	-0.282 (-2.04)	0.004 (0.94)	0.002 (0.24)	-0.004 (-0.45)	-0.004 (-0.52)	0.005 (0.63)	0.002 (0.45)	0.001 (2.20)	0.038 (2.45)	.9996	.005	2.065	1.704

*Denotes significance @ 5 percent

**Denotes significance @ 1 percent

t-statistics for coefficients appear in parentheses below relevant coefficients

Table 5

F-Statistics -- Two-sided Tests

Independent Variable

Y	(a) RBaa		(b) Y	
m	.886	0	2.808	0
surp	.708	0	1.454	0
g	.285	2	.373	0
g$	.853	2	.661	0
p	1.339	2	.450	2
w	3.251	2	1.932	2

All F's are F(4,50); significance levels are 2.56 for .95 confidence,
3.72 for .99 confidence.

Column (a) regressions:
$$Y_t = \sum_{i=-4}^{12} w_i RBaa_{t-i}$$

Column (b) regressions:
$$RBaa_t = \sum_{i=-4}^{12} w'_i Y_{t-i}$$

F-statistic in column (a) is pertinent for testing null hypothesis
$$w_{-4} = w_{-3} = w_{-2} = w_{-1} = 0.$$

F-statistic in column (b) is pertinent for testing null hypothesis
$$w'_{-4} = w'_{-3} = w'_{-2} = w'_{-1} = 0.$$

For meaning of superscripts 0, 1, and 2, see Table 2.

Table 6

F-Statistics for Coefficients on Current and Lagged Variables

$$\text{I.} \qquad Y_t = \sum_{i=0}^{12} \alpha_i RBaa_{t-i} + \beta_0 + \beta_1 t$$

Seasonal Dummies included

$$\text{II.} \qquad RBaa_t = \sum_{i=0}^{12} \gamma_i Y_{t-i} + \delta_0 + \delta_1 t$$

Variable Name	Independent Variable			
Y_t	RBaa		Y_t	
m	1.854	0	0.511	0
surp	1.771	0	1.648	0
g	0.344	2	0.446	0
g$	0.798	2	0.408	0
p	1.751	2	1.025	2
w	2.554**	2	2.004*	2

All F's are F(13,54).   All data are filtered with $(1-.75L)^2$

For meanings of superscripts 0, 1, and 2, see Table 2.

*Denotes significance @ 5 percent

**Denotes significance @ 1 percent

TABLE 7

F-Statistics--Two Sided Tests

Independent Variable

Y	(a) nf	(b) Y
Un	3.060 [0]	.945 [2]
m	1.591 [2]	.390 [0]
surp	1.320 [0]	.514 [2]
g	1.471 [2,φ]	1.594 [0]
g$	.499 [0]	.819 [2,φ]
p	.901 [0]	1.487 [0]
w	0.586 [0]	1.498 [2,φ]

All F's are F(4,50); significance levels are 2.56 for .95 confidence, 3.72 for .99 confidence.

Column (a) regressions:
$$Y_t = \sum_{i=-4}^{12} w_i \, nf_{t-i}$$

Column (a) regressions:
$$nf_t = \sum_{i=-4}^{12} w'_i \, Y_{t-i}$$

F-statistic in column (a) is pertinent for testing null hypothesis
$$w_{-4} = w_{-3} = w_{-2} = w_{-1} = 0.$$

F-statistic in column (b) is pertinent for testing null hypothesis
$$w'_{-4} = w'_{-3} = w'_{-2} \, w'_{-1} = 0.$$

φ superscript means some seasonal remains in the distributed lag weights despite the imposition of Shiller smoothness prior.

For meanings of other superscripts, see Table 2.

Table 8

Regression of

$$Y(t) = \sum_{s=1}^{8} \alpha(s)Y(t-s) + \sum_{\varepsilon=1}^{6} \beta(\varepsilon)X(t-\varepsilon) + \Delta_1 t + \Delta_0$$

1952.II - 1972.III

(t-statistics appear in parentheses below relevant coefficients.)

Y	X	α(1)	α(2)	α(3)	α(4)	α(5)	α(6)	α(7)	α(8)	β(1)	β(2)	β(3)	β(4)	β(5)	β(6)	Δ_1	Δ_0	$\bar R^2$	Std. Error of Est., Adj.	Durbin-Watson Statistic	F-Statistic on all F(6,56)
nf	lhn	1.027 (8.14)	-0.151 (-0.83)	-0.063 (-0.34)	0.273 (1.50)	-0.227 (-1.24)	-0.147 (-0.89)	-0.101 (-0.52)	-0.115 (-0.89)	-0.0007 (-0.52)	-0.0005 (-0.20)	0.003 (0.97)	-0.007 (-2.62)	0.009 (3.54)	-0.004 (-2.81)	0.00003 (1.29)	-0.036 (-0.98)	.964	.004	2.143	2.886**
lhn	nf	1.540 (12.02)	-0.757 (-3.22)	-0.390 (-1.57)	0.159 (0.64)	0.430 (1.68)	0.017 (0.07)	-0.162 (-0.75)	0.069 (0.57)	-21.500 (-1.75)	22.961 (1.29)	-5.060 (-0.29)	21.901 (1.31)	-32.547 (-1.92)	24.041 (1.99)	-0.001 (-0.63)	5.604 (1.76)	.916	.151	2.097	2.223*
nf	m	0.919 (6.76)	-0.176 (-0.99)	0.090 (0.50)	-0.086 (-0.49)	0.190 (1.08)	-0.116 (-0.67)	-0.101 (-0.75)	0.028 (0.22)	-0.057 (-0.64)	0.002 (0.01)	0.262 (1.47)	-0.384 (-2.12)	0.312 (1.76)	-0.103 (-1.02)	-0.0002 (-1.74)	-0.288 (-2.31)	.853	.004	2.031	1.983
m	nf	1.441 (10.29)	-0.525 (-2.01)	-0.117 (-0.41)	0.465 (1.62)	-0.474 (-1.60)	0.352 (1.10)	-0.228 (-0.71)	0.121 (0.70)	-0.036 (-0.17)	-0.073 (-0.25)	0.041 (0.13)	0.075 (0.25)	-0.039 (-0.14)	-0.137 (-0.69)	-0.0001 (-0.59)	-0.258 (-1.25)	.999	.006	2.000	0.309
nf	surp	0.927 (6.92)	-0.086 (-0.48)	0.028 (0.16)	0.114 (0.64)	0.022 (0.12)	0.013 (0.08)	-0.131 (-0.74)	0.027 (0.20)	0.00009 (0.55)	-0.0001 (-0.51)	0.00003 (0.67)	0.0003 (1.06)	-0.0003 (-1.19)	0.000001 (0.01)	0.00002 (1.04)	-0.045 (-1.18)	.841	.004	2.026	1.132
surp	nf	1.089 (8.34)	-0.374 (-1.93)	0.163 (0.81)	-0.398 (-2.02)	0.326 (1.74)	-0.210 (-1.13)	-0.102 (-0.58)	-0.024 (-0.20)	-22.533 (-0.22)	-144.945 (-1.06)	91.625 (0.67)	128.847 (0.94)	-69.150 (-0.50)	-53.598 (-0.53)	-0.012 (-0.66)	-17.639 (-1.9)	.826	3.190	1.967	0.980
nf	R	0.943 (7.42)	-0.071 (-0.41)	-0.031 (-0.18)	0.143 (0.84)	0.081 (0.46)	-0.119 (-0.64)	-0.155 (-0.85)	0.105 (0.78)	0.013 (1.19)	-0.026 (-0.24)	-0.004 (-0.24)	0.020 (1.38)	-0.016 (-0.50)	0.016 (0.53)	-0.000006 (-0.11)	-0.073 (-1.58)	.852	.004	2.054	1.938
R	nf	0.931 (7.19)	-0.087 (-0.41)	0.052 (0.31)	-0.129 (-0.78)	0.140 (0.92)	-0.090 (-0.69)	-0.144 (-0.84)	0.052 (0.41)	1.917 (1.25)	1.033 (0.50)	-0.555 (-0.27)	-5.256 (-2.62)	5.180 (2.44)	-2.354 (-1.51)	0.002 (2.88)	1.066 (1.93)	.953	.047	2.081	2.578*
nf	g	0.969 (7.27)	0.014 (0.08)	0.037 (0.21)	0.022 (0.13)	0.182 (0.93)	-0.052 (-0.27)	-0.144 (-0.84)	0.052 (0.41)	0.046 (1.45)	-0.047 (-0.94)	0.046 (0.96)	-0.105 (-1.23)	0.073 (1.46)	-0.003 (-0.11)	-0.0001 (-1.21)	-0.113 (-1.58)	.845	.004	2.015	1.385
g	nf	1.082 (8.45)	-0.100 (-0.52)	-0.128 (-0.73)	-0.438 (-2.35)	0.182 (0.93)	-0.052 (-0.27)	-0.098 (-1.04)	0.052 (0.41)	0.440 (0.89)	0.553 (0.80)	0.028 (0.04)	-0.746 (-1.14)	-0.055 (-0.09)	0.192 (0.41)	0.002 (3.14)	0.750 (1.97)	.999	.015	1.692	1.343
nf	p	0.938 (6.73)	-0.185 (-1.02)	-0.011 (-0.06)	0.039 (0.22)	0.050 (0.28)	-0.085 (-0.47)	-0.179 (-0.99)	0.098 (0.66)	0.181 (1.18)	-0.078 (-0.37)	0.160 (0.74)	-0.372 (-1.75)	0.192 (0.97)	-0.033 (-0.26)	-0.0002 (-1.17)	-0.397 (-1.58)	.840	.004	2.060	1.072
p	nf	0.853 (6.61)	0.038 (0.22)	0.138 (0.79)	0.196 (1.15)	-0.185 (-1.14)	0.086 (0.51)	-0.145 (-0.84)	-0.048 (-0.45)	0.360 (3.10)	-0.107 (-0.68)	-0.089 (-0.56)	0.131 (0.83)	0.123 (0.82)	-0.223 (-1.86)	0.0004 (2.38)	0.410 (2.10)	.9995	.003	2.104	2.935*
nf	w	0.920 (6.93)	-0.175 (-0.95)	-0.020 (-0.11)	0.021 (0.11)	0.075 (0.40)	-0.091 (-0.50)	-0.173 (-0.96)	0.081 (0.57)	0.207 (1.85)	-0.073 (-0.42)	0.010 (0.06)	-0.167 (-0.95)	0.132 (0.78)	-0.060 (-0.56)	-0.0004 (-1.15)	-0.217 (-2.00)	.841	.004	2.002	1.125
w	nf	1.049 (8.08)	-0.302 (-1.51)	0.297 (1.47)	-0.058 (-0.30)	0.030 (0.16)	-0.065 (-0.34)	0.104 (0.57)	-0.143 (-1.18)	0.553 (1.62)	-0.235 (-1.11)	-0.125 (-0.59)	0.219 (1.04)	0.017 (0.08)	-0.053 (-0.32)	0.0008 (2.25)	0.242 (2.25)	.9996	.005	1.972	3.137**

* Denotes significance @ 5 percent

** Denotes significance @ 1 percent

t-statistics for coefficients appear in parentheses below relevant coefficients.

Table 9

Estimates of the Model (1951 I – 1973 III)

1. $$Un_t = -.287 \ [\hat{P}_t - E_{t-1}P_t] + .0043 + .0000007t + 1.47Un_{t-1} - .59Un_{t-2} - .03Un_{t-3} + .04Un_{t-4}$$
$$(2.0) \qquad\qquad (2.5) \quad (0.5) \qquad (12.8) \qquad (2.9) \qquad (0.7) \qquad (0.1) \qquad (0.3)$$

$$\overline{R}^2 = .908 \qquad\qquad SEE = .371 \qquad\qquad D-W = 2.07$$

2. $$nf_t = .149 \ [\hat{P}_t - E_{t-1}P_t] - .075\hat{U}n_t - .038 + .00004t + .94nf_{t-1} - .11nf_{t-2} - .02nf_{t-3} + .12nf_{t-4}$$
$$(0.9) \qquad\qquad (1.9) \qquad (1.3) \quad (2.1) \qquad (8.2) \qquad (0.7) \qquad (0.2) \qquad (1.0)$$

$$\overline{R}^2 = .867 \qquad\qquad SEE = .0040 \qquad\qquad D-W = 2.00$$

3. $$y_t = 1.09\hat{n}_t + .24n_{t-1} - .24n_{t-2} - .14n_{t-3} - .02n_{t-4} + .35 + .0009t$$
$$(3.5) \quad (1.0) \qquad (1.0) \qquad (0.6) \qquad (0.1) \qquad (1.8) \ (4.5)$$

$$\hat{n}_t \stackrel{\cdot}{=} (nf_t - \hat{U}n_t + pop_t)$$

sum of weights on n = +.93

$$\overline{R}^2 = .95 \qquad\qquad s.e. = .00964 \qquad\qquad D-W = 2.03 \qquad\qquad filter: (1-.6L)^2$$

4. $$R_t = 1.52R_{t-1} - .77R_{t-2} + .44R_{t-3} - .24R_{t-4} + .15 + .0034t$$
$$(13.1) \quad (3.7) \qquad (2.4) \qquad (2.1) \qquad (1.8) \ (2.0)$$

$$\hat{n}_t \stackrel{\cdot}{=} (nf_t - \hat{U}n_t + pop_t)$$

sum of weights = .95

$$\overline{R}^2 = .99 \qquad\qquad SEE = .158 \qquad\qquad D-W = 1.89$$

5.a. $$m_t - P_t - y_t = -.0004\hat{R}_t - .0004R_{t-1} - .010R_{t-2} + .021R_{t-3} + .007R_{t-4} + .015R_{t-5} - .012R_{t-6} + .007R_{t-7}$$
$$(.0) \qquad\qquad (.0) \qquad (1.4) \qquad (2.4) \qquad (.9) \qquad (2.1) \qquad (1.6) \qquad (1.0)$$

$$- .91 - 1.37 \times 10^{-3} \times t$$
$$(212.0) \ (14.7)$$

$$\overline{R}^2 = .93 \qquad\qquad s.e. = .00856 \qquad\qquad D.W. = 1.97 \qquad\qquad filter: (1-.6L)^2$$

Sum of coefficients on R = + .024

Table 9 (Cont'd.)

5.b.

$$m_t - p_t - y_t = -.032\hat{R}_t - .006R_{t-1} - .027R_{t-2} + .014R_{t-3} - .007R_{t-4} - .0003R_{t-5} - .018R_{t-6} - .004R_{t-7} -.22$$
$$\quad (2.0) \quad (.7) \quad (3.2) \quad (1.6) \quad (.9) \quad (.0) \quad (2.0) \quad (.5) \quad (59.2)$$

$\bar{R}^2 = .28$ s.e. = .01085   D.W. = 1.91   filter: $(1-.8L)^2$

Sum of coefficients on R = -.080.

5.c.

$$m_t - p_t = -.0060\hat{R}_t - .0059R_{t-1} - .0091R_{t-2} + .0143R_{t-3} + .0080R_{t-4} + .0107R_{t-5} - .0022R_{t-6} + .0042R_{t-7}$$
$$\quad (0.5) \quad (1.2) \quad (1.7) \quad (2.6) \quad (1.6) \quad (2.0) \quad (0.4) \quad (0.8)$$

$$+ .45\hat{y}_t + .16y_{t-1} + .19y_{t-2} + .09y_{t-3} - .06y_{t-4} + .02y_{t-5} + .06y_{t-6} - .22 - .0003t$$
$$\quad (3.3) \quad (1.9) \quad (2.3) \quad (1.1) \quad (0.9) \quad (0.3) \quad (0.8) \quad (1.1) \quad (2.6) \quad (2.1)$$

$\bar{R}^2 = .29$ s.e. = .00561   D.W. = 2.10   filter: $(1-.8L)^2$

sum of weights on R = +.014   sum of weights on y = +.96

5.d.

$$m_t - p_t = -.0023\hat{R}_t - .0075R_{t-1} - .0089R_{t-2} + .0088R_{t-3} + .0045R_{t-4} + .0043R_{t-5} - .0044R_{t-6} + .0003R_{t-7}$$
$$\quad (0.2) \quad (1.5) \quad (1.6) \quad (1.7) \quad (0.9) \quad (0.9) \quad (0.8) \quad (0.1)$$

$$+ .25\hat{y}_t + .06y_{t-1} + .09y_{t-2} - .01y_{t-3} - .14y_{t-4} - .02y_{t-5} + .01y_{t-6} + .04y_{t-7} - .052$$
$$\quad (2.5) \quad (0.9) \quad (1.3) \quad (0.1) \quad (2.2) \quad (0.3) \quad (0.2) \quad (0.6) \quad (1.7)$$

$\bar{R}^2 = .25$ s.e. = .00576   D.W. = 1.99   filter: $(1-.8L)^2$

sum of weights on R = -.005   sum of weights on y = +.28

T-statistics are in parentheses beneath coefficients.

Hatted variables (^) are systematic parts of regressions against instrumental variables.

Footnotes

1/See Andersen and Carlson [1].

2/Cooper and Fischer [3] have made this point.

3/Models with this property have previously been analyzed by Sargent and Wallace [12,14].

4/This is one of the options analyzed for a stochastic Keynesian model by William Poole [11].

5/See Lucas [7] and Sargent [13].

6/See Sims [19]. A lag distribution that embodies a wrong prior restraint on the sum of the lag weights, but is a flexible enough lag distribution, can usually achieve a fit arbitrarily close to what could be achieved if the erroneous constraint on the sum of the lag weights were removed. (This assumes that the spectral density of the independent variable has no spike at zero frequency.)

7/Dropping the assumption that the u's and other white noises are serially independent, but are only serially uncorrelated, would necessitate replacing conditional mathematical expectations with linear least-squares forecasts in the subsequent argument. With that replacement, the argument would go through. The statistical tests reported in the next section only utilize the assumption that the various white noises are serially uncorrelated.

8/It does not necessarily follow that the distribution of the innovation in unemployment is independent of the feedback rule for policy, only that its conditional mean is. The empirical tests reported in this paper are of neutrality-in-conditional-means propositions. Stronger neutrality propositions, asserting invariance of the entire

probability distribution of some real economic variables with respect to the policy rule, obtain in some macroeconomic models. For example, see Sargent and Wallace [12].

[9/] The notations $E_{t-1}X_t$ and $EX_t | \theta_{t-1}$ are alternatives denoting the same concept, so that $E_{t-1}X_t \equiv EX_t | \theta_{t-1}$.

[10/] It is assumed that the process $[\begin{smallmatrix} Un_t \\ Y_t \end{smallmatrix}]$ possesses an auto-regressive representation.

[11/] This is a very important result, since it establishes the coincidence between Granger causality and the econometrician's definition of statistical exogeneity.

[12/] For expositions of the rational expectations theory of the term structure and evidence that it performs acceptably well, see Shiller [16] and Modigliani and Shiller [10].

[13/] This is an implication of Wold's chain rule of forecasting. See, e.g., Modigliani and Shiller [10] and Shiller [16].

[14/] A more adequate approximation than (2.7) is available, one that doesn't ignore the term $1/n[E_t R_{1t+n} - R_{1t}]$. Notice that the term structure equation (2.7) implies that

$$(2.8'') \quad E_t n\, R_{nt+1} = (n+1)\, R_{n+1,t} - R_{1t}.$$

Equation (8'') implies that R_n is "caused" by (i.e., not exogenous with respect to) R_{n+1} and R_1, but is not caused by (i.e., is exogenous with respect to) any other variables once R_{n+1}, and R_1 are taken into account. Equation (8'') shares the "classical" character of the less adequate approximation equation (8). Essentially, equation (8'') asserts that as a block, the term structure of interest rates is statistically exogenous or not "caused" by other variables. This is enough to preserve the

"classical" nature of the model, but is weaker than requiring the interest rate on bonds of a given maturity to be statistically exogenous with respect to all other variables.

[15] The St. Louis model is no exception.

[16] The wage is an index of the straight-time manufacturing wage (w), which is seasonally adjusted and reported on a monthly basis in Employment and Earnings. The civilian unemployment rate (Un) seasonally unadjusted, on a monthly basis, was taken from Employment and Earnings. For population I used the civilian noninstitutional population 16 and over, constructed by subtracting armed forces from the total population 16 and over. The noninstitutional population 16 and over was interpolated from annual figures compiled by Current Population Survey and reported in Table 1, Handbook of Labor Statistics, 1973. Armed forces were obtained by averaging monthly numbers reported in Employment and Earnings. The civilian labor force 16 years and older was taken from Employment and Earnings and divided by pop_t to obtain the labor force participation rate. The money supply (m) is "M1," currency plus adjusted demand deposits taken from the Federal Reserve Bulletin. The Baa rate (R) was obtained from Moody's Investor's Service. For R, w, and Un, the monthly figures were averaged to obtain quarterly figures. The GNP deflator (p), federal and state and local purchases in current dollars (g$), and federal and state and local purchases in 1958 dollars (g) were all taken from the National Income Accounts; the federal and state and local government surplus in current dollars was also taken from the National Income Accounts, and then divided by the GNP deflator to obtain the surplus in 1958 dollars (surp).

[17]/A deterministic trend was extracted before taking the Fourier transform, and then added back in after taking the inverse transform. The degrees of freedom for the F-statistics have been adjusted for the loss of degrees of freedom due to setting the seasonal bands to zero. The appropriate correction is described by Sims [17].

[18]/To save space, the graphed lag distributions and various summary statistics of the two-sided tests have been relegated to a mimeographed appendix that is available from the author on request. The graphs and various statistics from the Hannan efficient regressions discussed below are also in this appendix.

[19]/While the Durbin-Watson statistics from most of the two-sided regressions are close to two, there is a possibility that the presence of higher than first-order serial correlation is making inappropriate the F-statistics in the text. For this reason, the two-sided regressions were recomputed using a version of Hannan's efficient estimator, which is asymptotically equivalent to generalized least squares allowing for high-order serial correlation in the disturbances. The results are reported in the mimeographed appendix to this paper. The general pattern agrees with the results in the text, though there are differences in details. For example, in the Hannan efficient results, w does not seem to cause the Baa rate or the unemployment rate. If anything, then, the Hannan efficient regressions seem more favorable to the exogeneity hypotheses imposed by the classical model than are the two-sided regressions reported in the text.

[20]/In implementing Granger's test, I specified a maximal number of lagged own terms, usually four, upon which a variable was

permitted to depend. If the variable in question is exogenous but follows a mixed moving average, autoregressive process so that its autoregression is of infinite order, this misspecification could lead to erroneous rejection of the hypothesis of exogeneity. With Sims's test, premature truncation of the lag distribution will lead to too frequent rejection of the hypothesis of exogeneity when it is true.

[21/] Christopher Sims points out to me that since the autoregressive part of the direct Granger regression whitens the residuals, thereby reseasonalizing them, it is not possible for the Granger test to "ignore" the seasonal bands, as the Sims test as applied here does. This could conceivably account for some of the differences in the results of the two tests.

[22/] The estimates of the model use the data seasonally adjusted by setting their Fourier transforms equal to zero in the seasonal bands, the same data used in the tests in Section III. Estimates of the model using officially seasonally adjusted data were also made. The results, which are qualitatively similar to those summarized here, are in the mimeographed appendix.

[23/] For population (pop), I took the civilian population over 16 years old, while for the labor force I used the civilian labor force over 16. The labor force participation rate nf was measured as the ratio of the latter to the former. The total civilian unemployment rate was used. Notice that $n_{ft} + pop_t - Un_t$ approximately equals civilian employment, so my production function views GNP as a function only of civilian employment.

[24/] To form the proxy for $E_{t-1}p_t$, p_t was regressed on a constant, trend, three seasonal dummies, and p, w, nf, and Un, each lagged one through four times.

[25/] The endogenous variables were replaced by the systematic part of a regression of themselves against pop_t, m_t, g_t, $surp_t$, the log of current government employment, and all of the variables reported in footnote 23.

[26/] The reader may wonder whether equations (1) and (2), which have lagged endogenous variables as regressors, can be consistently estimated by the technique employed. If the residuals are serially correlated, my estimates are not consistent. But it is straigthforward to show that, for example, the Un vs. p exogeneity tests of Section III can be viewed as tests for serial correlation of the disturbances in equation (1), failure to reject exogeneity of unemployment (p's failing to "cause" Un) being consistent with no serial correlation. In effect, then, some testing for the null hypothesis of no serial correlation has been carried out, with results favorable to the null hypothesis.