Bild Stäbchen

The particles are suspended in a fluid.

In an initially homogenous suspension, rods form clusters with higher particle concentrations which sediment faster than a single rod.

The clusters lead to downward streamers that are balanced by upward streamers.

Within a cluster, particles align in direction of gravity most of the time, occasionally they flip.

Mathematical Model

The second and fourth term describe the movement of the rods through a macroscropic velocity field. The third term relates to a rotation of rod-like particles due to a macroscopic velocity gradient and diffusion.

The term on the right hand side of the first equation models rotational and translational diffusion.

We consider the case where gamma=0, so we ignore two terms bc the buoyancy term is more important.

(in which the effects of translational Brownian motion of the rod-like particles are ignored.
So the effect of elastic forces in the flow equation is not that important compared to the buoyancy forces)

Status of project

There‘s a previous work from Dahm, Helzel and Giesselmann. They considered a simplified model for the kinetic equation on S^1. For the simpler model, they derived a hierachy of moment equations. And they also investigate accuracy of the moment system.

Model Shear Flow

We consider the velocity of this form u, the velocity depends on the x-direction and time.
The flow occurs in z-direction. And in this case f is also a function of one variable.

Properties Basis Functions

- we only consider the even order (bc the particles are symmetric)

Ansatz for the Moment Equations

We only consider the even order (bc the particles are symmetric).

Derivation of the Moment Equations

This is the general form of the hyperpolic system w. source term.

Derivation: A Closer Look

Term [1]:
Bc the function c doesnt depend on \phi and \theta, we take this out of the integral. And this expression is the L2-inner product.

For N=1, the matrix A has the form. The term $\cos \varphi \cos \theta \sin \theta$ is a multiple one of the second order basis functions. Let‘s consider the projection of products of spherical harmonics to spherical harmonic basis function

The first row means that we multiply the ansatz with P00. And we will get the entry [1,3]. We get here an entry bc we insert the ansatz w this term $\cos \varphi \cos \theta \sin \theta$ and project it to P00.

Assume: Set f^N to P00. Then multiply this ansatz with the basisfct up to order two, we will get this entry [3,1].

For N=2:

Block A_{1,2}: If we use in our ansatz the basis functions up to order 4, and then multiply them by the basis functions of up to order 2, we will get the Block A_{1,2}.

Block A_{1,2}^T : If we use up to order 2 in order Ansatz and multiply it w basis function order 4, we get the Block A_{1,2}^T, which is the transport of A_{1,2}.

If more base functions are used, we always have three additional blocks.

Moreover the Matrix A is symmetric, therefore the system is hyperbolic.

Remaining terms:

We follow the same procedure. We insert our Ansatz and then project onto all the basis functions used in the Ansatz. In this case, however, the calculations are more extensive because we need to compute the derivative of \theta. This leads to the matrix D*Q.

Splitting Alg

Step 1, 5 and 9, we use a Strang Splitting Algorithm for our moment equations and between these steps we use the Strang Splitting for the second equation. For the hyperbolic system in step 5 we use wave propagation.

Numerical Simulation

We consider here the numerical solution for a fixed w_x.

For a larger Dr=1, by using N=1 moment equations we can see that the solution structure has already a good approximation as N=6.

N = 1: 6 moment equations
N = 6: 91 moment equations

Accuracy Study

Here we focus on the coupled system.

In the following example we use N=6 moment equations for the reference solution.
This accuracy study fit in with what we have shown previously for the first subproblem.

Cluster Formation

We started with almost constant density at some random noise and with these pictures I want to illustrate how the cluster form.

Current work

Gleichung (6): Here the source term didnt change, on the left hand side we get additional a Matrix B.

Current Work Bilder

The terms dx and dy (red term) describe the transport along the x and y directions. These terms mean that particles are transported into space at different speeds depending on their orientation (\phi and \theta).

Over time (t=200), this piecewise constant flow velocity causes the particles located on the left side of the system ($x<50$) to drift to the right and upwards, while the particles on the right side ($x\geq50$) drift to the left and downwards. As the velocities in the two areas are opposite, two separate clusters are formed as the particles in the two areas move in opposite directions.

The splitting into two clusters is caused by the piecewise constant velocity distribution. The particles located in the areas $x<50$ and $x\geq50$ have different and opposite velocities, which causes the particles to move in opposite directions and ultimately form two separate clusters.

Die Strömungsgeschwindigkeiten w_x und w_y beeinflussen, wie die Partikel entlang der x- und y-Achsen transportiert werden. Durch die Terme dx und dy (roter Term) wird der Transport entlang der x- und y-Richtung beschrieben. Diese Terme bedeuten, dass Partikel je nach ihrer Ausrichtung (\phi und \theta) mit unterschiedlichen Geschwindigkeiten in den Raum transportiert werden.

Über die Zeit hinweg (t=200), bewirkt diese stückweise konstante Strömungsgeschwindigkeit, dass die Partikel, die sich auf der linken Seite des Systems (x<50) befinden, nach rechts und oben abdriften (da die Partikel eine konstante positive Geschwindigkeit in der x- und y-Richtung haben), während die Partikel auf der rechten Seite (x≥50) nach links und unten driften (da die Geschwindigkeiten negativ sind.). Da die Geschwindigkeiten in den beiden Bereichen entgegengesetzt sind, bilden sich zwei separate Cluster, da sich die Partikel in den beiden Bereichen in entgegengesetzte Richtungen bewegen.