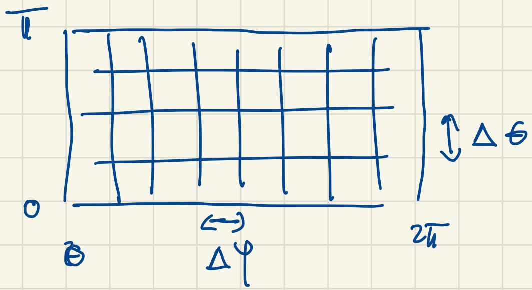
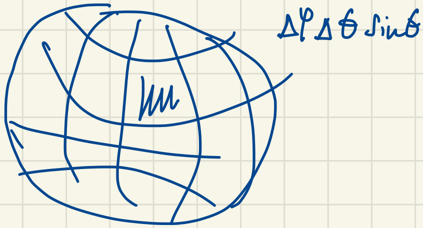
Scalar product on S^2:

$$(g,h)_{S^2} := \int_0^{2\pi}\int_0^{\pi} g(\varphi,\theta)\, h(\varphi,\theta) \sin\theta \, d\theta \, d\varphi$$





Properties of spherical harmonic basis functions:

- $\Delta_{S^2} P_n^i = -(n+1)\, n\, P_n^i$
- $f(\varphi,\theta) = \dots$ Gl. (3) von S. 7
- $(P_n^i, P_m^j)_{S^2} = 0 \quad \forall \; n \neq m, \; i \neq j$
- $(P_n^i, P_n^i)_{S^2} = 1$

Definition der P_n^i

. . .

Abbildungen einiger Basisfunktionen

Spectral method

~~Recall the Smochluchowski equation on S^2~~

$$\underbrace{\sin\theta\partial_t f}_{(1)} + \underbrace{\partial_\phi\left(a(\phi,\theta)f\right) + \partial_\theta\left(b(\phi,\theta)f\right)}_{(2)} = \underbrace{D_r\left(\partial_\phi\left(\frac{1}{\sin\theta}\partial_\phi f\right) + \partial_\theta\left(\sin\theta\partial_\theta f\right)\right)}_{(3)} \tag{4}$$

~~and the~~ ansatz

?

$$f(\phi,\theta,t) = f_0(t)\cdot P_0^0 + \sum_{n=1}^{\infty}\sum_{i=-n}^{n} c_{2n}^i(t)\cdot P_{2n}^i(\phi,\theta). \tag{5}$$

- Insert ansatz for f in (4)
- Multiply consecutively with all basis functions used in the ansatz and integrate the resulting equations over ϕ and θ

For $f(\varphi,\theta,t) = c_0^0(t) + \sum_{i=-2}^{2} c_2^i(t)\, P_2^i(\varphi,\theta)$ we obtain

$$\begin{pmatrix} c_0^0(t) \\ c_2^{-2} \\ c_2^{-1} \\ c_2^{0} \\ c_2^{1} \\ c_2^{2} \end{pmatrix}' - A \begin{pmatrix} c_0^0(t) \\ \vdots \\ c_2^2(t) \end{pmatrix} = \begin{pmatrix} 0 \\ -6D_r c_2^{-2}(t) \\ \vdots \\ -6D_r c_2^2(t) \end{pmatrix}$$

The matrix A has the form

$$A = \begin{pmatrix} \text{Matrix für allgemeinen } D_x \text{ angeben!} \end{pmatrix}$$