POLITECNICO DI MILANO

Insurance
Solvency II Project

Group 9

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Contents

1 Project presentation

Consider a simplified insurance company whose assets and liabilities sides are characterized as follows:

ASSETS

- there is a unique fund made of a bond combined with an equity
- at every time step t the value of the fund (before deducting the fees) is $F_t = B_t + S_t$
- at the beginning ($t = 0$) the value of the fund is equal to the insured capital $F_0 = C_0 = 1000$
- bond features:
  - AAA corporate zero coupon bond with maturity $T = 10$
  - $B_0 = 800$, face amount $N = 1000$
- equity features:
  - listed in the regulated markets in the EEA
  - $S_0 = 200$
  - No dividend yield
  - to be simulated with a Risk Neutral GBM ($\sigma = 20\%$) and a time varying instantaneous risk free rate r derived from the yield curve (EIOPA IT with VA 31.12.18), supposing linear interpolation of the zero rates and using the formula $DF_{t+dt} = DF_t * \exp(-r_t * dt)$

LIABILITIES

- Term Life policy with term $T = 10$
- the insured capital given in case of death/lapse and survivor at maturity is equal to
  - **CASE A:** guaranteed, $C_t = \max(C_0, F'_t)$
  - **CASE B:** not guaranteed, $C_t = F'_t$

  where
  - $F'_t = F_t - \text{fees}_t$
  - $\text{fees}_t = F_{t-1} \cdot 1.50\%$
- male insured aged $x = 60$
- mortality rates derived from the life table SI2017 (ISTAT website)
- flat annual lapse rates $l_x = 5\%$

Other specifications:

- the interest rates dynamic is deterministic, while the equity one is stochastic
- the default (credit) spread $s$has to be computed in the plain case (no IR stress) to match the zero coupon bond price $B_0 = 800$

QUESTIONS

1. For both cases A and B, code a Matlab script to compute the Basic Solvency Capital Requirement via Standard Formula and provide comments on the results obtained in A and B. The risks to be considered are:
   - Market Interest
   - Market equity
   - Market spread
   - Life mortality
   - Life lapse
   - Life cat
2. Calculate the duration of the liabilities in all the cases and provide comments on the results obtained
3. Replicate the same calculations in an Excel spread sheet using a deterministic projection. Do the results differ from 1? If so, what is the reason behind?
4. Open questions:
   - What happens to the asset and liabilities when the risk free rate increases/decreases? Describe all the effects
   - What happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

2 Summary tables

CASE A	Assets	Liabilities	BoF	dBoF	Duration
Base	1000.00	1072.29	-72.29	/	7.79
IR up	924.92	999.36	-74.42	2.15	7.75
IR down	1018	1089.82	-71.76	-0.51	7.81
Equity	922	1008.52	-86.51	14.24	7.74
Spread	944	1066.1	-122.07	49.8	7.82
Mortality	1000	1071.9	-71.88	-0.39	7.76
Lapse up	1000	1065.66	-65.63	-6.64	7.05
Lapse down	1000	1079.9	-79.91	7.64	8.64
Lapse Mass	1000	1051	-51.01	-21.27	5.38
CAT	1000	1072.2	-72.2	-0.09	7.79

CASE B	Assets	Liabilities	BOF	dBOF	Duration
Base	1000	1067.63	-67.63	/	7.82
IR up	924.92	986.03	-61.12	-6.51	7.82
IR down	1018	1087.2	-69.18	1.55	7.82
Equity	922	990.58	-68.57	0.95	7.83
Spread	944	1054.4	-110.37	42.74	7.88
Mortality	1000	1067.1	-67.14	-0.48	7.78
Lapse up	1000	1059.3	-59.32	-8.31	7.08
Lapse down	1000	1077.1	-77.12	9.50	8.66
Lapse mass	1000	1041	-41	-26.66	5.42
CAT	1000	1067.52	-67.52	-0.11	7.81

Case	A	B
BSCR	64.81	46.86

3 Theoretical Background

The aim of the project is to compute the **Basic Solvency Capital Requirement** ($BSCR$) of an insurance company according to the Solvency II regulation. To do this we use the formula:

$$BSCR = \sqrt{\sum_{i,j} C(i,j) \cdot SCR_i \cdot SCR_j} + SCR_{\text{intangible}} \tag{3.1}$$

where C is the correlation matrix between risks and SCR_i is the solvency capital requirement of the i-th risk factor.
In our specific case we consider the $SCR_{\text{intangible}}$ equals to zero and the risks considered are the Market and the Life ones. The SCR of a specific risk is computed as:

$$SCR = \max(BOF_{basic} - BOF_{shock}, 0) \tag{3.2}$$

where the BOF are the **Basic Own Funds**, defined as the difference between the actualized value of assets and liabilities. BOF is a measure of how much the company earns/loses from the business. Naturally, the first quantity we need to compute in order to study the impact of the stresses caused from each risk is the BOF in the basic case.

3.1 Assets

In the case study, the insurance company holds a fund composed by a AAA corporate zero coupon bond with maturity 10 years, $B_0 = 800$ and a face value N equal to 1000 and a stock traded in the regulated markets with $S_0 = 200$.

$$F_t = B_t + S_t \tag{3.3}$$

The instantaneous risk free rates used are taken from the EIOPA data, in particular the ones concerning the Italian yield curve with VA with the starting date 31/12/18. In the code are used the rates in exponential capitalization $R_t = ln(1 + r_t)$.
The value of the assets at time zero is equal to 1000.

3.1.1 Bond

We compute the bond value at each year i discounting the face value at that time. The discount factors used are forward discounts between the time i and the maturity T valued in 0.

$$B(t_0; t_i; T) = \frac{B(t_0, T)}{B(t_0, t_i)} \tag{3.4}$$

$$B(t_0, t_i) = e^{-(R_{t_i}+s)(t_i-t_0)} \tag{3.5}$$

It can be seen from the formula above that discount factors depend on the rates in exponential capitalization and by a spread. The spread is added to the computation in order to consider the default probability of the bond and it is computed, by definition, as the value that matches the discounted face value N in zero with B_0:

$$s = \frac{1}{T} ln\Big(\frac{N}{B_0}\Big) - R_T \tag{3.6}$$

3.1.2 Stock

The stock S_t is simulated with Risk Neutral **Geometric Brownian Motion**, considering a constant volatility σ equal to 20% and the risk free rate R, via a Monte Carlo algorithm with $M = 10^5$ simulations. We have divided the time interval [0,T] into $P = 10^2$ time steps in order to get a better approximation of the path of our stock. We compute the stock at time i with the formula:

$$S_i = S_{i-1} e^{(f_{i-1}-\frac{\sigma^2}{2})dt+\sigma\sqrt{dt}\cdot g} \tag{3.7}$$

where dt is the time step, g is a number drawn with a normal distribution with mean 0 and variance 1, and f_i is the forward rate between the time steps t_i and t_{i+1}, computed with the formula:

$$f_i = -\frac{t_{i+1}R_{i+1} + t_i R_i}{dt} \tag{3.8}$$

The interest rates not present in the EIOPA data are obtained through a linear interpolation.

Martingale testing for the stock value

In order to understand if the time grid chosen to replicate S_t is coherent we compute a Martingale Test on the simulated process. The martingale test is an average of the M simulations made for the behaviour of the stock S_t for each year. Since the vector g is a standard normal, the average over the simulations will yield a zero, hence giving birth to dynamics of average S_t as a risk-free bond. So the result of the test assures a martingale behaviour of the process S_t

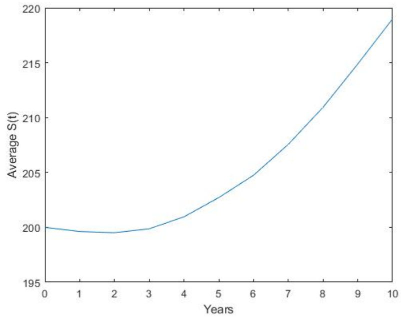


Figure 3.1: The graph shows the behaviour of the expected value of S_t, that behaves like a martingale.

3.2 Liabilities

3.2.1 Insured Capital

The liabilities of the insurance company depend on the insured capital C_t. The insured capital is the amount paid to the insured party at time t if he/she dies or lapses the contract, or at maturity T if he/she survives. The assumptions we are considering are the ones given from the text of the project plus the hypothesis that the insurance company cannot default. For this reason, it is not applied any spread in discounting the insured capital at time zero.
The insured capital C_t at time t depends on the value of the fund at the same time, and it is computed in two different possible contracts:

- **CASE A**: $C_t = \max(C_0, F'_t)$
- **CASE B**: $C_t = F'_t$

where F'_t is given by the formula:

$$F'_t = F_t - \text{fees}_t \tag{3.9}$$

$$\text{fees}_t = F_{t-1} \cdot 1.50\% \tag{3.10}$$

We can observe that the main difference between the two contracts is the presence of a guaranteed capital in the case A, not present in the other case.

In each simulation the value of the fund at time t is given by the sum of B_t and S_t at each year, where the bond value at a fixed time t is equal in every simulation.
The insured capital is computed as the mean of the simulations

3.2.2 Actuarial value of liabilities

The value at time t_0 of the liabilities is computed as the sum of all the cash flows the insurance company will pay to the insured party, discounted by both the financial discount and the demographic discount.
So the building blocks we need at each step of time are the discount factors, the insured capital, the probability to be inside the contract at the beginning of the year and the probability to leave it within the year.
The probability to be inside the contract in a certain time is the product between the survival probability p and the probability to had not lapsed the contract before l.
The probability to leave the contract within a year is the sum of the probability to die in that year

and the probability to be alive, but lapse the contract.
The resulting formula we have used is:

$$V(t_0) = \sum_t D(t_0, t) \cdot p_{t-1}\big(q_t + (1 - q_t) \cdot \text{LapseRate}\big) \cdot C_t l_{t-1} \tag{3.11}$$

where $D(t_0, t)$ is the discount factor between t_0 and t, q_t is the mortality at the t^{th} year, p_t is the probability to be alive at the beginning of t^{th} year, l_t is probability not to leave the contract before t, C_t is the Insured Capital at time t.

The mortality values have been extracted from the EIOPA database refer to men (or women) with an age included between 0 and 109 years old.

3.3 Shock cases

The calculations done in the shock cases are the same of the basic case with the input parameters changed. We assume that all the shocks happen at time 0^+, so it is important to notice that the value of the fund at time zero remains equals to the basic case.
When we compute the assets, we are interested in the real value of both the bond and the equity at the start of the period we are considering, so we take the value of the fund at time 0^+ (immediately after the shock).
Instead, when we compute the liabilities we need to take into consideration the quantities written on the contract, signed at time 0, so we consider as F_0 the original value of the fund (so the one computed before the shock).

3.4 Market Risks

The Market Risk defines the risk due to the level and the volatility of the financial instruments the company has in its portfolios. The risks we consider in our projects are:

1. **Interest risk**
   - upshift of the interest rates curve
   - downshift of the interest rates curve
2. **Equity risk**
3. **Spread risk**

	Interest	Equity	Spread
Interest	1	α	α
Equity	α	1	0.75
Spread	α	0.75	1

Where: $\alpha = 0$ if exposed to an upshift of the interest curve,
$\alpha = 0.5$ if exposed to a downshift of the interest curve.

3.4.1 Interest risk

The interest risk studies how a shift of the risk free instantaneous rates impact on the company business. We have computed first an *upward shift*, and then a *downward shift* in the rates curve in the two cases. The new computations are done using the shifted rates EIOPA data instead of the basic case ones.
As we can observe from the results, an upshift of the yield curve has a negative impact on the

business in case A and an irrelevant impact in case B. On the contrary, a downshift of the yield curve has a irrelevant impact on the business in case A and a negative impact in case B.
In the first open question we are going to explain with more details the causes of this behaviour.

3.4.2 Equity Risk

The equity risk arises from changes in the level of volatility of the equity. So we are interested in studying the variation of the SCR that results from a sudden decrease in the equity's value. According to Solvency II, since we are considering a Type 1 equity, the shock is obtained with a decrease of 39% in its current value. Both assets and liabilities are affected by this change.
The impact on the assets is quite simple, in fact we know that assets' value is just the sum of the current value of equity and bond; thus, a decrease in the equity value causes a direct impact on the assets, with a decrease exactly equal to the decrease observed in the equity value. That's what we expected, in fact if we compute the derivative of the assets with respect to the equity price, we obtain:

$$\frac{\partial A}{\partial S_0} = \frac{\partial (B_0 + S_0)}{\partial S_0} = 1 \tag{3.12}$$

So the variation in the equity is equal to the variation in the assets. Notice that this depends on the composition of our assets; if, for example, we had a equity derivative portfolio, the variation in the assets value would have been different from the variation in the equity.
The behavior of the liabilities is less trivial. First of all, we have to notice that the shock is applied in $t = 0^+$, so when we evaluate the insured capital as $C_t = \max(C_0, F_t')$ we have to consider as C_0 the original value of the assets (so $C_0 = 1000$), as we fix it before that the shock is applied. In general, we expect that a fall in the equity value causes a decrease in the liabilities, since the insured capital, which depends positively on the asset value, decrease. Obviously this depends on how the insured capital is defined. So we can compare the two cases.
In the case A we are considering a guarantee, so we have to pay to our insurers a minimum amount of money equal to the initial value of the fund, i.e. 1000 euros. This means that the effect of the fall of equity prices is not completely absorbed by the insured capital, but it's smoothed by the guarantee. Obviously this is worse from the insurance company point of view because it cannot fully benefit from the drop of equity prices
Instead in the B case the insured capital depends only on the fund value, so the insured capital is more sensitive to changes in the assets. This is obviously a benefit because it means that we have to pay less money to the policyholders. In order to understand better how the variation of the equity impacts on the liabilities, we can compute the derivative of the liabilities w.r.t the initial value of the equity. Consider the A case in the framework where we have a deterministic projection for our equity (just for simplicity); first of all we write the insured capital as a function of the equity initial value, so

$$C_t(S_0) = \begin{cases} \max\left[(B_0 + S_0'), B_1 + S_0 e^{r(t_1)t_1} - \left(B_0 + S_0'\right)1.5\%\right] & t = 1 \\ \max\left[(B_0 + S_0'), B_i + S_0 e^{r(t_i)t_i} - \left(B_0 + S_0 e^{r(t_{i-1})t_{i-1}}\right)1.5\%\right] & t \neq 1 \end{cases} \tag{3.13}$$

where S_0' is the value of the equity without the shock. Notice that we split the first case from the others because at the first year the fees are paid on the original value of the fund. Now we plug it into the formula of the liabilities and we obtain

$$V(S_0) = \sum_{i=1}^{10} D(t_0, t_i) p(t_0, t_{i-1}) \left[q(t_{i-1}, t_i) + \left(1 - q(t_{i-1}, t_i)\right)\text{Lapse Rate}\right] C_{t_i}(S_0) l_{i-1} \tag{3.14}$$

deriving w.r.t S_0 we obtain

$$\frac{\partial V}{\partial S_0} = D(t_0,t_1)\big[q(t_0,t_1) + \big(1-q(t_0,t_1)\big)\text{Lapse Rate}\big]e^{r(t_1)t_1}1_{X'>C_0} + \sum_{i=2}^{10} D(t_0,t_i)p(t_0,t_{i-1})\big[q(t_{i-1},t_i)+$$
$$+\big(1-q(t_{i-1},t_i)\big)\text{Lapse Rate}\big]l_{i-1}\big(e^{r(t_i)t_i} - e^{r(t_{i-1})t_{i-1}}1.5\%\big)1_{X>C_0}$$

with

$$X' = B_1 + S_0e^{r(t_1)t_1} - \big(B_0 + S_0'\big)1.5\%$$

and

$$X = B_i + S_0e^{r(t_i)t_i} - \big(B_0 + S_0e^{r(t_{i-1})t_{i-1}}\big)1.5\%$$

If we compute it with our numerical values, we observe that it's smaller than 1, so the liabilities are less sensitive to changes in the equity value than the assets. In the B case the result is similar, but the indicator function is absent, so the derivative increase.This means that the absence of guarantee makes the liabilities more sensitive to changes in the equity price.
Summing up, a drop in the equity price causes a decrease in both assets and liabilities.In presence of guarantee, the shock on the liabilities is weaker, so the insurance company has a lower value of the assets and liabilities have decreased way less proportional than the assets. This reflects into a lower BOF, which moves from -72.29 to -86.51, with an obvious increase of the dBOF. On the other side, the absence of guarantee is much more favourable because allows us to transfer the risk on the policyholders. In this case the BOF is quite similar to the basic case, with a dBOF=0.95.

3.4.3 Spread risk

The spread risk measures how the business of an insurance company, so its assets and liabilities, is affected by changes in the level or in the volatility of credit spreads over the risk free term structure. In particular in our case only the bond is affected by a spread increase. We compute the value of the new spread as the value that match the discounted face value and the market value in zero:

$$s_{new} = \frac{1}{T}ln\Big(\frac{N}{MV_{bond}}\Big) - r_T \tag{3.15}$$

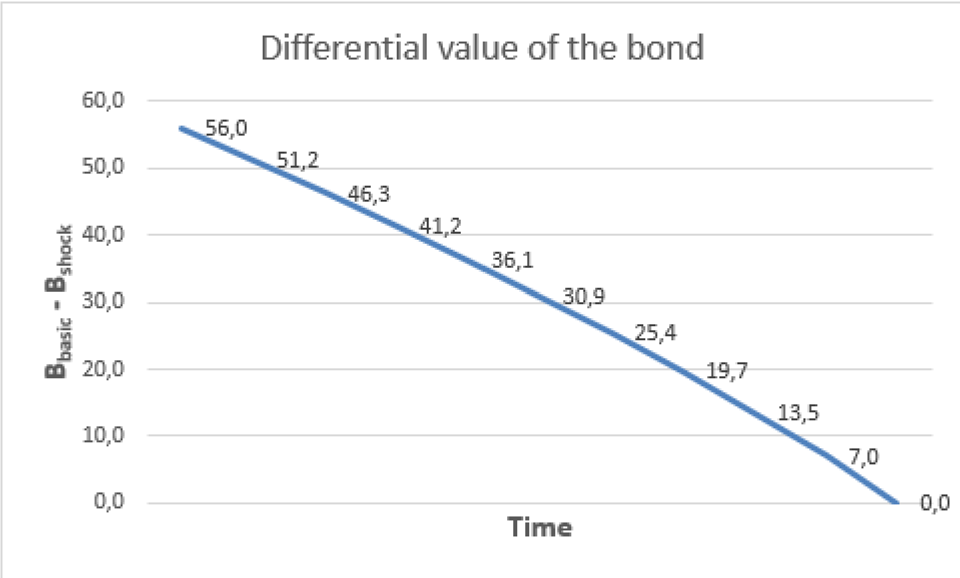
where T is the maturity, N the face value and MV_{bond} the market value of the bond, r_T the rates corresponding to the maturity. The MV_{bond} is computed according to Solvency II as:

$$MV_{bond} = B_0(1-(0.045+0.005(D_{bond}-5))) \tag{3.16}$$

B0 is the value of the bond at the starting date, D_{bond} the duration of the bond. The formula used is taken from the Solvency II table on spread risk for bonds. In particular we are in the case where the duration is included in 5 and 10 and the credit quality step is equal to 0, since the bond is AAA.

In our case the $dBOF$ obtained form the MATLAB computations is positive, so the shock has a negative effect on the insurance company business.
In the following plot is reported the difference between the value of the bond in basic case and in the shocked one over time

It is possible to observe that it is a monotone decreasing function, so at the starting dates the spread increase has a greater impact on the bond value. The value of the assets are computed with the value of the equity and of the bond at time 0^+ so at when the bond value is affected more. Although the liabilities are computed summing the discounted value of the insured capital we need to pay to the insurers over the time, in particular in our case the insurance company needs to pay most of the insured capital at maturity, date where the value of the bond is equals in the



two cases. For this reason the impact of the shock is greater on the assets than the one on the liabilities. This argument explains why the $dBOF$ is greater than 0.
Looking at the results obtained increasing the spread, we found out that this not affect the business of the insurance company since the $dBOF$ is equal to 0.

3.5 Life Underwriting Risks (LUR)

The life underwriting risk is the risk borne by the underwriter on the uncertainty of the life expectancy and the behaviour of the insured, united with uncontrollable risk factors such as the possibility of catastrophes.

In our project, for the computation of the SCRs associated with the LURs, we have considered three categories of risk:

1. **Mortality risk**
2. **Lapse risk**
3. **CAT (catastrophe) risk**

The description of the correlations between these three risks is provided by the correlation matrix below:

	Mortality	Lapse	CAT
Mortality	1	0	0.25
Lapse	0	1	0.25
CAT	0.25	0.25	1

3.5.1 Mortality risk

The mortality risk is the risk that an insurance company can suffer financially, because too many of their life insurance policyholders die before their expected lifespans. This may imply a loss, or an adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where usually an increase in the mortality rate leads to an increase in the value of insurance liabilities.

In order to measure the effect of this possible scenario, we apply a uniform increase of 15% in the mortality rates.
We first observe that changing the mortality rates doesn't affect the discount factors in the market and the value of the assets of the market, and by consequence neither the assets of the insurance company.
In particular, we observe that the insured capital C_t paid over the years doesn't change from the basic case. Hence, given that C_t is a monotone increasing function over time, the fact that mortality rates increase is an advantage for the insurance company, since it implies that we will have more payments in the first dates (when the capital is lower).
This fact is reflected with an increment of the BOFs after the mortality shock has been applied.
For this reason, in the two cases the Basic Capital Requirements associated with the Mortality Risk are zero.
Another effect in increasing the mortality rates is the reduced duration of the liabilities.

3.5.2 Lapse risk

Lapse risk is the risk of loss or adverse change in liabilities due to a change in the expected exercise rates of policyholder options (contractual policyholder rights to fully or partly terminate, surrender, decrease, restrict or suspend insurance cover or permit the insurance policy to lapse).
As we are dealing with a risk associated with the behaviour of the policyholders, we observe that even in this case the market situation remains untouched. That's why the insured capital C_t remains the same.
We notice that lapse and mortality risks are uncorrelated. In fact, these risks deal with independent phenomena. Moreover, a change on the lapse rate produces an effect similar to the one generated by a change in the mortality rate. Since from the company point of view the cash flow is equal in the two cases
We analyze three possibilities of lapse risk and select the worst situation for the policy insurance company.

- **Lapse Up**:
  We first analyze the effect of an instantaneous permanent increase of 50% in the assumed option exercise rates of the relevant option in all future years.
  As told above, this is comparable to a shift in the mortality rates. In fact, it produces a very similar effect: the duration is shorter and the BOFs increment, and for this reason the SCRs are zeros. The main difference with the mortality rate upward shift is in the absolute value of the dBOF, which in this case is bigger. That's because the shift now is 50% instead of 15% of before. For example, the payments of liabilities in the last year is decreased significantly compared with the mortality case.

- **Lapse Down**:
  We then analyze the effect of an instantaneous permanent decrease of 50% in the assumed option exercise rates of the relevant option in all future years.
  In this case, we expect the opposite phenomena, i.e. a longer duration and a decrease in the BOFs. The computations confirm this initial hypothesis: the duration increases of approximately 1 year, and the BOFs decrease, hence contributing in the global SCRs.

- **Lapse Mass**:
  Finally, we analyze the effect of a mass abandon of 40% in the first year, followed by a constant lapse rate of 5% in the following years.
  So the lapse mass is a significant change of the distribution of the insured capital in time

since many people exercise the option in the first year. So the insurance company will have to pay a large amount to the insured party during the first years. This is an advantage for the insurer, since as explained before, due to the C_t evolution, the insured capital is lower in early payments.

3.5.3 CAT risk

The CAT risk is the risk measure associated to the possibility of a catastrophic event, such as an epidemic or an earthquake.
Such catastrophes can be modelled though an absolute increment of 15 basis points in the rate of policyholders dying over the following year.
This shift will have a minor effect in an increase of the liabilities in the first year and a decrease in the ones from the second year until the end. Hence it results, analogously to the mortality risk, in an increase of BOFs and in a consequent zero effect in the SCRs. Moreover, the impact is small since we are shifting the mortality rate in only one year.

3.6 Duration

An indicator we take into consideration to understand the impact of a certain shock is the Duration of liabilities. In the project we have considered the **Macaulay Duration** D, that is defined by the following formula:

$$D = \frac{\sum_i V_i T_i}{\sum_i V_i} \tag{3.17}$$

where V_i is the discounted cash flow expected at time t_i. The denominator is equal to the Net Present Value of the liabilities.
Since the future payments are not known at time zero we need an indicator that gives information about the time distribution of them: this is the duration. Sooner we will pay back the capital to the policyholders, smaller will be the duration of liabilities.

As we have observed before, the company has benefits if more people die or leave the contract in the first years of the considered period, because the insured capital increases over time, so basically it has benefits in having a small duration.
Looking at the results in the summary tables, we observe that the shocks that have a significant increase in the duration value w.r.t. the basic case have a negative impact on the business of the company, while the ones that have a significant decrease have a benefit effect.

4 Deterministic projections

4.1 Theoretical introduction

We have reproduced the computations for the Basic Solvency Capital Requirements for the case in which the dynamics of the stock S_t are deterministic, i.e. when the volatility σ is equal to zero.

We first observe that the impact of volatility on the evolution of the stock value is quite low, given that the value of volatility in the stochastic problem is small ($\sigma = 20\%$). Hence we don't expect a big difference between the deterministic and the stochastic case.
Moreover, simulating S_t many times will yield an average evolution of the value of the stock equal to the deterministic evolution. The results confirm this view: below we have reported the BSCRs

for the deterministic evolution in the two cases. One can observe their similarity with the stochastic problem.

4.2 Summary Tables

CASE A	Assets	Liabilities	BoF	dBoF	Duration
Base	1000.00	1067.59	-67.59	/	7.82
IR up	924.92	994.33	-69.41	1.82	7.77
IR down	1018.05	1086.87	-68.82	1.23	7.82
Equity	922	1005.36	-83.36	15.77	7.75
Spread	944	1060.7	-116.7	49.11	7.84
Mortality	1000	1067.11	-67.11	-0.48	7.78
Lapse up	1000	1059.45	-59.45	-8.14	7.07
Lapse down	1000	1076.92	-76.92	9.33	8.66
Lapse Mass	1000	1043.03	-43.03	-24.56	5.41
CAT	1000	1067.49	-67.49	-0.1	7.81

CASE B	Assets	Liabilities	BoF	dBoF	Duration
Base	1000.00	1067.27	-67.27	/	7.82
IR up	924.92	985.67	-60.76	6.51	7.82
IR down	1018.05	1086.87	-68.82	1.55	7.82
Equity	922	990.35	-68.35	1.09	7.83
Spread	944	1054	-110	42.74	7.88
Mortality	1000	1066.78	-66.78	-0.48	7.78
Lapse up	1000	1058.98	-58.98	-8.28	7.08
Lapse down	1000	1076.73	-76.73	9.47	8.66
Lapse Mass	1000	1040.72	-40.72	-26.55	5.42
CAT	1000	1067.16	-67.16	-0.11	7.81

Case	A	B
BSCR	63.73	47.67

Table 4.1: BSCR in the two cases for the deterministic projection.

As we expect, implementing the computations of the deterministic case with different programming languages (e.g. with Matlab and with Excel) yields the same results.

4.3 Behaviour of the case A

We observe a curious behaviour of C_t in the case A, that can be explained by the formula for the computation of the capital. In fact, while in the deterministic case F'_t is always higher than C_0 after a given time t_k, in the stochastic case F'_t oscillates randomly and hence can go below or above the value C_0 in any time. Anyway, the stochastic value of the fund will normally behave as the deterministic one, resulting therefore in a similar BSCR.

5 Open Questions

5.1 What happens to the asset and liabilities when the risk free rate increases/decreases? Describe all the effects.

The impact of a shock in the interest rate is quite more complicate than the other shocks, since lots of terms in assets and liabilities depend on them. Let's start analyzing what happen to the assets. First of all, we notice that an increase in the interest rate causes a decrease in the value of the bond. Since the shift along the IR curve is not parallel (we have bigger shifts on shorter maturities), the initial values of the bond are more sensitive to this shift. Moreover, we are considering a zero coupon bond so it's duration is equal to the maturity of the bond (10 y). We know that the duration is a good measure of interest rate sensitivity, and the variation of the bond value can be expressed in term of duration as

$$\Delta B = -D \cdot B \cdot \Delta r \tag{5.1}$$

So since the duration is decreasing in time (at time t it's equal to (T-t)) the impact of the shift is greater on shorter maturities.
Instead, the equity value depends positively on the interest rates, so an increase in the IR causes an increase in the asset value. Moreover this effect is stronger on longer maturities since the stock is path dependent, so the shift at time t depends on the shifted value of the stock at time t-1, thus this effect intensifies during the life of the stock. Combining the two effects, we obtain that the value of our fund decrease almost all the years, except for the last two where the effect of the shock on the equity overcome the one on the bond (in particular at maturity the value of the bond is always equal to the face value, it's not influenced by the interest rate).
Since the insured capital depends on the value of the fund, even the liabilities are sensitive to changes in interest rates. Moreover all the cash flows are discounted by the discount factor which depends negatively on the interest rates. In the case A we observe that we have to guarantee to our insurer a minimum amount of money, so on shorter maturities we are not able to transfer the risk on our insurer. Instead in the B case what we have to pay depend only on the value of the fund, so the insured capital follows the evolution of the assets. In general in both the cases we observe a decrease in the value of the liabilities even in correspondence of the period where the fund value is higher, because we are using lower discount factors. The effects of these considerations can be seen on the value of the dBOF: in the case A it's positive, so the liabilities have decreased but not enough to compensate the variation of the assets, while in the B case the shock is completely absorbed by the liabilities with a relevant decrease of their value.
In case of downshift of the IR, our quantities moves in the opposite direction: the value of the bond increases and the equity decreases, with an overall effect of increase in the value of our fund up to when we are close to maturity, where the bond variation is null and the equity has decreased. Since the fund is increasing, we have to pay always an insured capital greater than the guarantee. So if we consider the deterministic case, the value of assets and liabilities computed in case A and B coincide (obviously the dBOF are different because they depends on the value of the BOF computed in the basic case), while in the GBM case they are slightly different.

5.2 What happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

In order to answer to this question, we have proceeded in the computation of the assets and the liabilities of the insurance company for both males and females, with the age of the insured varying from 61 to 75 years.
Clearly, assets won't change, being dependent only on market conditions, whereas liabilities depend on the insured conditions (i.e. the mortality rates) and therefore will be affected.

Age of the insured party

We observe that mortality rates of the insured increase as the insured grows older: the probability of death is higher as years pass. Being more exposed to the risk of death, older people will receive their capital earlier.

As a natural consequence, the duration of liabilities will decrease, having the effect of diminishing the costs for the insurance company. Hence the older is the insured, the better is for the insurance company in terms of profit.

Sex of the insured party

Similarly, we have analyzed the impact of the sex of the insured party in the liabilities of the company.
We again observe that the mortality rate of women is lower than the one of men, as women have a higher life expectancy. For this reason, normally women will receive the insurance capital later than men, given the same insured age.

Consequently, the impact will be a longer duration for liabilities, that converts in higher costs for the company.

Final results

The two graphs below describe the behaviour of the liabilities and their duration in function of the age of the insured party. The behaviour is just as expected: costs decrease with the age of the insured and are lower for women.

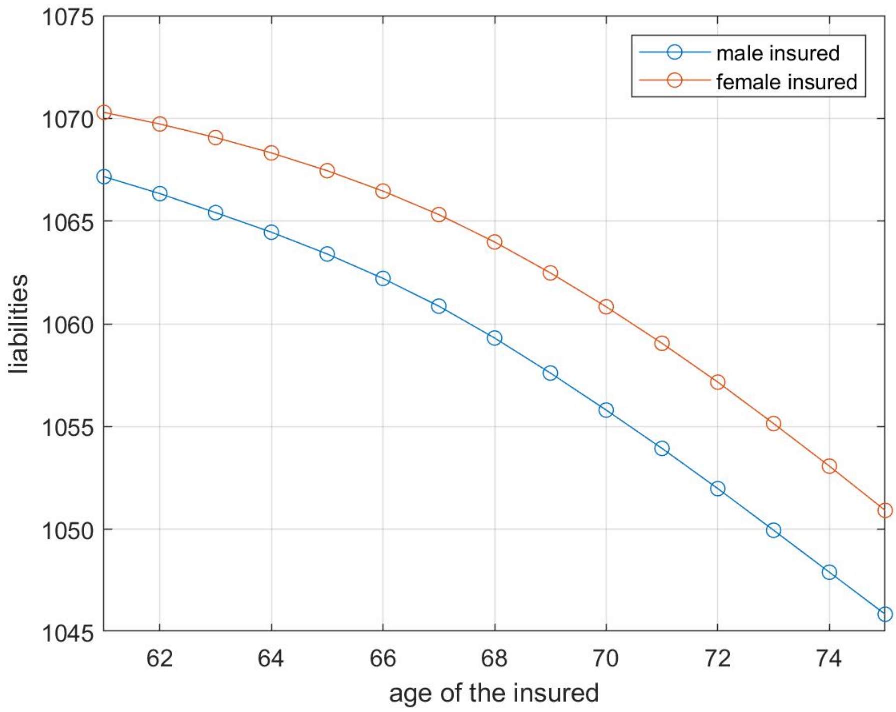


Figure 5.1: Case A: liabilities over the years for women and men.

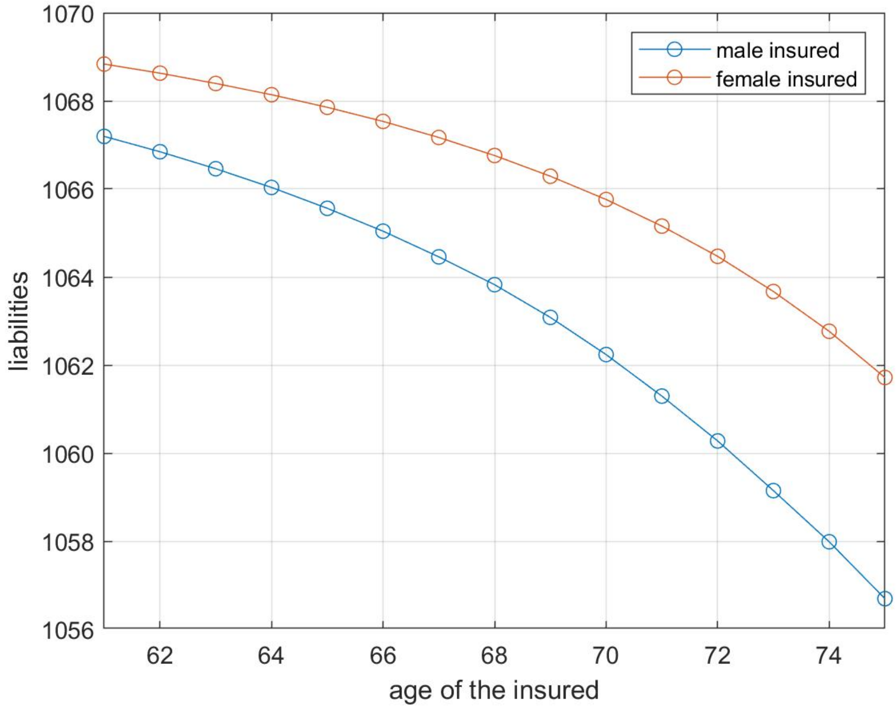


Figure 5.2: Case B: liabilities over the years for women and men.

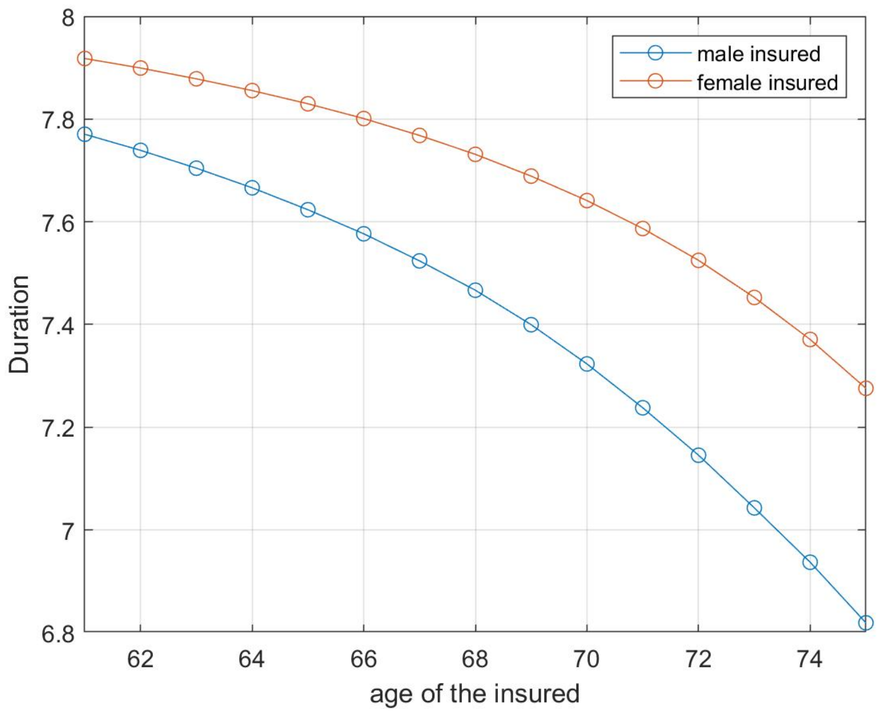


Figure 5.3: Case A: durations over the years for women and men.

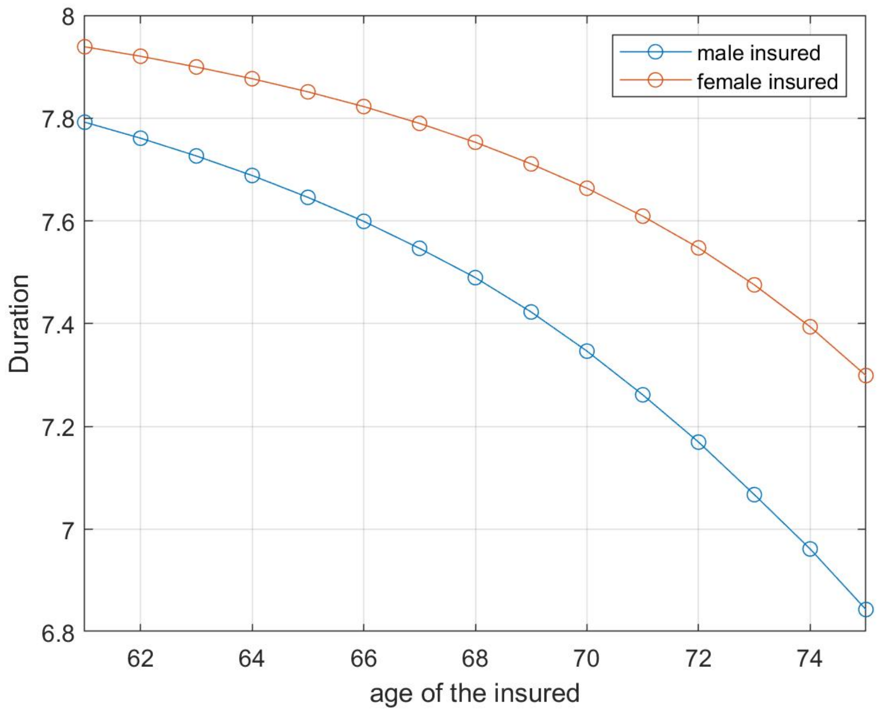


Figure 5.4: Case B: durations over the years for women and men.

5.2.1 Differences in cases A & B

We observe that the behaviour of the liabilities with respect to the age of the insured party is slightly different between the two valuation cases A & B.
In particular, for the case B we notice that the difference between liabilities of men and women increase over the age of the insured, while for case A it remains constant over time.

Moreover, liabilities are lower over the years if we select case A instead of case B, both for males and females.
Dealing with durations, their behaviour is the same given the two different cases.

6 Matlab

```
% Run of the assignment.

clear all
close all
clc

%% Read datas from the excel sheet:
r = xlsread('Final_Project_data_20181231.xlsx','c4:c18');
r_up = xlsread('Final_Project_data_20181231.xlsx','d4:d18');
r_down = xlsread('Final_Project_data_20181231.xlsx','e4:e18');

q = xlsread('Final_Project_data_20181231.xlsx','i3:i112');

```

```
%% data
T = 10;
N=1000;
B0=800;

%% Changing annual to continuous compounding:
R = log(1+r(1:T));
R_up = log(1+r_up(1:T));
R_down = log(1+r_down(1:T));

% spread:
s = 1/T*(log(N/B0))- R(10);

data = struct('T',T,'r',R,'N',1000,'S0',200,'s',s,'sigma',0.2,...
              'x',60, 'feeRate', 0.015, 'lapseRate', 0.05,'q',q);

%% Random simulation for computing S:
M = 100000; % number of MC simulation
P = 100;  % number of time step

g = randn(M,P); % std.n matrix

%% Flag to choose the case A or B
flagCase = 1; % case A: if flag = 1
              % case B:   otherwise
%% Basic case:
base = 0;
basic = SCR(data, base,g,flagCase);

%% a) Market interest: up & down
BOF_base = basic.BOF;
data.r = R_up;
IRup = SCR(data, BOF_base,g,flagCase);

data.r = R_down;
IRdown = SCR(data, BOF_base,g,flagCase);

capitalRequirements.ir = max(IRup.dBOF,IRdown.dBOF);
%% b) Market Equity
data.r = R;

data.S0 = data.S0*(1-0.39);
equity = SCR(data, BOF_base,g,flagCase);

capitalRequirements.equity = max(equity.dBOF,0);
%% c) Market spread

data.S0 = 200;
MV_bond = B0*(1 - (0.045+0.005*(10-5)));
data.s = 1/T*(log(N/MV_bond))- R(10);

spread = SCR(data, BOF_base,g,flagCase);

capitalRequirements.spread = max(spread.dBOF,0);

%% d) Mortality risk
data.s = 1/T*(log(N/B0))- R(10);
```

```
data.q = data.q*(1.15);
mortality = SCR(data, BOF_base,g,flagCase);

capitalRequirements.mortality = max(mortality.dBOF,0);

%% e) Lapse risk
data.q = data.q/(1.15);

data.lapseRate = data.lapseRate * (1.5);
lapseUp = SCR(data, BOF_base,g,flagCase);

data.lapseRate = 0.05*(0.5);
lapseDown = SCR(data, BOF_base,g,flagCase);

data.lapseRate = -1; %Flag for the case 'lapse mass':

%lapse apply at the first year
lapseMass = SCR(data, BOF_base,g,flagCase);

capitalRequirements.lapse = ...
    max([lapseUp.dBOF,lapseDown.dBOF,lapseMass.dBOF,0]);
%% f) Cat risk

data.lapseRate = 0.05;
data.q(data.x+2) = data.q(data.x+2) + 0.0015;

CAT = SCR(data, BOF_base,g,flagCase);

capitalRequirements.cat = max(CAT.dBOF,0);

%% Basic Solvency Capital Requirements:

BSCR = basiccapital(capitalRequirements,flagCase);
```

```
function [liabs,D]=liabilities(data,C)
%INPUTS
%   data: struct containing
% T:        Maturity
% x:        Current age
% r:        Rates
% fee_rate:      Fee rate
% lapse:   Lapse rate
%   q:         Mortality rates
%   F:         Funds
%
%OUTPUT
%V:         Current value of the liabilities

t = (0:data.T)';
discounts = exp(-data.r.*t(2:end));

% Selection of the mortality rates of interest:
q_x = data.q(data.x+2:data.x+data.T+1); %mortality rates ...
    between (ti,ti+1)
```

```
%% Computation liabilities

p = [1; cumprod(1 - q_x(1:end-1))];
l = (1-data.lapseRate).^t(1:end-1);
q_x(end) = 1;

if(data.lapseRate == -1)
    % CASE: lapse mass.
    lapse = 0.05;
    lapseMass = 0.4;
    l = (1-lapse).^t(1:end-1);
    V = zeros(size(discounts));
    V(1) = discounts(1)*p(1)*(q_x(1) + (1-q_x(1))*lapseMass)*C(2);
    V(2:end) = discounts(2:end).*p(2:end).*(q_x(2:end) + ...
        (1-q_x(2:end))*lapse).*C(3:end)*(1-lapseMass).*l(1:end-1);

else
    V = discounts.*p.*(q_x + (1-q_x)*data.lapseRate).*C(2:end).*l;
end

liabs = sum(V);
%% computation of the duration

D = dot(t(2:end),V)/liabs;

%% computation of the derivative wrt S0 in equity shock case

if(data.S0 == 122)
    V_der = zeros(10,1);
    g = (C(2:end)>1000);
    V_der(1) = discounts(1)*p(1)*(q_x(1) + ...
        (1-q_x(1))*data.lapseRate)* g(1)*exp(data.r(1));

    for i=2:10
        V_der(i) = (1-data.lapseRate).^(i-1) * ...
            discounts(i)*p(i)*(q_x(i) + ...
            (1-q_x(i))*data.lapseRate)* ...
            g(i)*(exp(data.r(i)*i)-exp(data.r(i-1)*(i-1))*0.015);
    end

    der = sum(V_der);
end
```

```
function [A,C_t] = assetvalue(data,g,flag)
%INPUT
% data: struct containing market data, such as:
%s:       spread
%N:       Face value of the bond
%S0:      Initial value of the stock
%r:       Spot rates
%T:       Time to maturity
%sigma:   Volatility of the stock
% g:      Monte Carlo Simulation for the stock values.
```

```
%
%OUTPUT
%bond:   Value of the bond year by year
%stock:  Value of the stock year by year
%% Value of the Bond


t =(0:data.T)';
B = zeros(data.T+1,1);

discount_ON = exp(-(data.r(end) + data.s )*t(end));
r = [0;data.r];

for i = 1:data.T+1

    discount_iN = discount_ON / exp(-(r(i)+data.s)*t(i));
    B(i) = data.N*discount_iN;
end
B(end) = data.N;

%% Computation of the stock with simulation matrix g
[P,M] = size(g);
dt = linspace(0,10,M+1)';

r_interp = zeros(M+1,1);
for i = 1:M+1
    r_interp(i) = interp1(t,r,dt(i));
end

step_t = 10/M;
fwd_rates = - ( (-dt(2:end).*r_interp(2:end) + ...
   dt(1:end-1).*r_interp(1:end-1))/step_t);

S_sim = zeros(P,M+1);
S_sim(:,1) = data.S0;

for i= 1:P
    for j = 1:M
        S_sim(i,j+1) = S_sim(i,j) * exp( (fwd_rates(j)- ...
           data.sigma^2/2)*step_t + data.sigma * ...
           sqrt(step_t)*g(i,j));
    end
end

index = 10*(0:10)+1;
S_t = S_sim(:,index);

%% martingale test

% test=mean(S_t); % test martingale
% plot(t,test)     % plot si nota il comportamento dello stock ...
   come un bond privo di rischio
% xlabel('Years')
% ylabel('Average S(t)')

%% Computation Fund value

```

```
F_t = B'.*ones(P,1) + S_t;
A = F_t(1,1);
F_t(:,1) = 1000;

F_prime = F_t(:,2:end) - F_t(:,1:end-1)*data.feeRate;
C_0 = mean(F_t(:,1));

if flag == 1 %case A

    C_t = [ C_0, mean(max(F_t(:,1).*ones(P,1), F_prime))]';

else    % case B

    C_t = [C_0, mean(F_prime)]';
end

end
```

```
function policyData = SCR(data, base,g,flagCase)

%% Compute assets values
[policyData.assets,C] = assetvalue(data,g,flagCase);

%% Liabilities & duration:

[policyData.liabs,policyData.duration] = liabilities(data,C);

%% BOF:
policyData.BOF = policyData.assets - policyData.liabs;

%% dBOF:
if base == 0
    policyData.dBOF = 0;
else
    policyData.dBOF = base - policyData.BOF;
end

end
```

```
function BSCR = basiccapital(capitalRequirements,flagCase)
% Computes the Basic Solvency Capital Requirements with the ...
   correlation
% tables provided by the regulation of Solvency II.
%
% INPUT:
%  - SCR: struct containing the solvency capital requirements ...
   associated to
%        the risks of interest.
%
% OUTPUT:
%  - BSCR.
%

%% Correlation Matrices:
```

```
% Market correlations:
if flagCase==1
mktCorr = [1, 0, 0;...
           0, 1, 0.75;...
           0, 0.75, 1];
else
    mktCorr = [1, 0.5, 0.5;...
               0.5, 1, 0.75;...
               0.5, 0.75, 1];
end

% Life correlations:
lifeCorr = mktCorr;
lifeCorr(1,2) = 0;
lifeCorr(2,1) = 0;

% Global correlations:
globalCorr = [1, 0.25; 0.25, 1];

%% Computations:
mktVector = [capitalRequirements.ir; ...
    capitalRequirements.equity; capitalRequirements.spread];
lifeVector = [capitalRequirements.mortality ; ...
    capitalRequirements.lapse; capitalRequirements.cat];

mktSCR = sqrt(dot(mktVector,mktCorr*mktVector));
lifeSCR = sqrt(dot(lifeVector, lifeCorr*lifeVector));

globalSCR = [mktSCR; lifeSCR];

BSCR = sqrt(dot(globalSCR, globalCorr*globalSCR));

end
```

```
% Run of the assignment.

clear all
close all
clc

%% Read datas from the excel sheet:
r = xlsread('Final_Project_data_20181231.xlsx','c4:c18');
r_up = xlsread('Final_Project_data_20181231.xlsx','d4:d18');
r_down = xlsread('Final_Project_data_20181231.xlsx','e4:e18');

qMen = xlsread('Final_Project_data_20181231.xlsx','i3:i112');
qWomen = xlsread('Final_Project_data_20181231.xlsx','m3:m114');

%% data
T = 10;
N=1000;
B0=800;

%% Changing annual to continuous compounding:
```

```
R = log(1+r(1:T));
R_up = log(1+r_up(1:T));
R_down = log(1+r_down(1:T));

% spread:
s = 1/T*(log(N/B0))- R(10);

%% Random simulation for computing S:
P = 100000; % numero di montecarlo
M = 100;  % numero di step considerati

g = randn(P,M);

%% Flag to choose the case A or B
flagCase = 1; % case A if flag = 1
              % case B    otherwise
%% Basic case
dataMan = ...
    struct('T',T,'r',R,'N',1000,'S0',200,'s',s,'sigma',0.2,...
                 'x',60, 'feeRate', 0.015, 'lapseRate', ...
                     0.05,'q',qMen);

dataWoman = ...
    struct('T',T,'r',R,'N',1000,'S0',200,'s',s,'sigma',0.2,...
                 'x',60, 'feeRate', 0.015, 'lapseRate', ...
                     0.05,'q',qWomen);

base = 0;

%% Choose the age interval
N = 15;
age = dataMan.x+1:dataMan.x+N;

%% Compute the liabilities
manLiabilities = zeros(N,1);
womanLiabilities = zeros(N,1);
manDuration = zeros(N,1);
womanDuration = zeros(N,1);


for i = 1:N
     dataMan.x = dataMan.x + 1;
     dataWoman.x = dataWoman.x + 1;
     manSCR = SCR(dataMan,base,g,flagCase);
     womanSCR = SCR(dataWoman, base, g, flagCase);

     manLiabilities(i) = manSCR.liabs;
     womanLiabilities(i) = womanSCR.liabs;
     manDuration(i) = manSCR.duration;
     womanDuration(i) = womanSCR.duration;

end

%% Plot
figure
plot(age,manLiabilities,'o-', age, womanLiabilities,'o-')
legend ('male insured','female insured')
```

```
grid on
xlabel('age of the insured')
ylabel('liabilities')
xlim([61, 75])

figure
plot(age,manDuration,'o-', age, womanDuration,'o-')
legend ('male insured','female insured')
grid on
xlabel('age of the insured')
ylabel('Duration')
xlim([61, 75])
```