Project for the "Local Volatility Model" course

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You have a file `MarketData_XY.xls` that contains several sets of market data consisting in

- a market volatility surface $\{\sigma_{mkt}(T_i, K_{i,j})\}$ for $i \in \{1, \ldots, N\}$, $j \in \{1, \ldots, M\}$
- market expiry dates $(T_1, \ldots, T_N)$
- market strikes $\{K_{i,j}\}$ (or deltas $\{\Delta_j\}$ for FX assets)
- forwards $(F_0(T_1), \ldots, F_0(T_N))$ and discount factors $(D_0(T_1), \ldots, D_0(T_N))$ at market expiry dates

so that the market price of a call option with expiry T_i and strike $K_{i,j}$ is obtained by Black formula $Bl(F_0(T_i), K_{i,j}, T_i, \sigma_{mkt}(T_i, K_{i,j}), D_0(T_i))$.

Assume that the local volatility function is parameterized by a local volatility matrix $\{v_{i,j}\}$ at the nodes $\{K_{i,j}\}$ with piecewise constant interpolation in time and spline interpolation with flat extrapolation in strike.

Solve the following exercises:

1 Write a function in Matlab that calibrates a local volatility model using fixed-point algorithm. Code must be optimized so that at each iteration of the algorithm only one Dupire equation is solved

2.1 Consider market data of the asset E CORP, contained in file `MarketData_XY.xls`, and calibrate a local volatility model

2.2 With the model found above, price two call options with expiry $T = 0.5$ and strikes $K = \kappa \cdot S(0)$ with $\kappa = 0.9, 1.1$, by solving Dupire equation or by Monte Carlo simulation. Compute the relevant implied (spot) volatilities.

2.3 Using a Monte Carlo simulation, price two forward starting option with start date $T_1 = 2$, expiry $T_2 = 2.5$ and strikes $\kappa = 0.9, 1.1$. Compute the implied forward volatilities.

2.4 Using the model implied volatilities above, compute the skew of the spot and forward smile, namely slope of the spot/forward implied volatility as a function of strike κ. What typical feature of the Local Volatility model do you observe?

2.5 Now, suppose that you find market quotes for the same forward starting options considered above and realize that are very different from the model prices. Is this difference between model and market prices a symptom of bad calibration?

3 Consider market data of the asset FAIL contained in file `MarketData_XY.xls` and try calibrate a local volatility model. Verify that the calibration procedure fails and explain why (hint: compute market prices $C_0(T_i, K_{i,j})$ of call options via Black's formula and observe the convexity of the map $K_{i,j} \mapsto C_0(T_i, K_{i,j})$ for fixed i)

4.1 Consider market data of an FX asset Y, contained in file `MarketData_XY.xls`. Forwards at the expiry dates are given in terms of the spot $Y(0)$, the domestic discount factors $\{D_0^d(T_i)\}$, the foreign discount factors $\{D_0^f(T_i)\}$ according to the formula $F_0(T_i) = Y(0)\frac{D_0^f(T_i)}{D_0^d(T_i)}$. Find market strikes $\{K_{i,j}\}$ from the quoted deltas and calibrate a local volatility model.

4.2 Consider the same example but with a different value of the spot $Y'(0) = Y(0)\cdot 1.01$. Recomputed forwards with the new spot using formula $F_0(T_i) = Y'(0)\frac{D_0^f(T_i)}{D_0^d(T_i)}$. Calibrate a local volatility model and compare the model parameters found now with the ones found in exercise 4.1. Is this local volatility model *sticky-delta* or *sticky-strike*?

The project has to be done in groups of 3-4 persons. As soon as the composition of a group is communicated by email, the group will receive the file containing market data.
Each group will have to send <u>all</u> Matlab code used to solve the problems and a document (pdf, word, powerpoint, etc..) discussing the results.
Deadline: February 6, 2020.
Please use the following email address: `riccardo.longoni@bancaimi.com`