Boston House Price Prediction - Linear Regression

Objective

This project's objective is to **predict the housing prices of a town or a suburb based on the features of the locality provided to us**. In the process, we need to **identify the most important features affecting the price of the house**. We must use data preprocessing techniques and construct a linear regression model.

Dataset

Each record within the database describes a house in Boston suburb or town in Boston. The data was drawn from the Boston Standard Metropolitan Statistical Area (SMSA) in 1970. Detailed attribute information can be found below:

Attribute Information:

- **CRIM:** Per capita crime rate by town.
- **ZN:** Proportion of residential land zoned for lots over 25,000 sq.ft.
- **INDUS:** Proportion of non-retail business acres per town.
- **CHAS:** Charles River dummy variable (= 1 if tract bounds river; 0 otherwise).
- **NOX:** Nitric Oxide concentration (parts per 10 million).
- **RM:** The average number of rooms per dwelling.
- **AGE:** Proportion of owner-occupied units built before 1940.
- **DIS:** Weighted distances to five Boston employment centers.
- **RAD:** Index of accessibility to radial highways.
- **TAX:** Full-value property-tax rate per 10,000 dollars.
- **PTRATIO:** Pupil-teacher ratio by town.
- **LSTAT:** % lower status of the population.
- **MEDV:** Median value of owner-occupied homes in 1000 dollars.

Importing the necessary libraries and overview of the dataset

In [1]:

```
# Import libraries for data manipulation
import pandas as pd

import numpy as np
```

```
# Import libraries for data visualization
import matplotlib.pyplot as plt

import seaborn as sns

from statsmodels.graphics.gofplots import ProbPlot

# Import libraries for building linear regression model
from statsmodels.formula.api import ols

import statsmodels.api as sm

from sklearn.linear_model import LinearRegression

# Import library for preparing data
from sklearn.model_selection import train_test_split

# Import library for data preprocessing
from sklearn.preprocessing import MinMaxScaler

import warnings
warnings.filterwarnings("ignore")
```

Loading the data

In [2]:

```
df = pd.read_csv("Boston.csv")

df.head()
```

Out[2]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	LSTAT	MEDV
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	4.98	24.0
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	9.14	21.6
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	4.03	34.7
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	2.94	33.4
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	5.33	36.2

Observation:

- The price of the house indicated by the variable MEDV is the target variable and the rest of the variables are independent variables based on which we will predict the house price (MEDV).

Checking the info of the data

In [3]:

```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 13 columns):
```

```
 #   Column   Non-Null Count  Dtype  
---  ------   --------------  -----  
 0   CRIM     506 non-null    float64
 1   ZN       506 non-null    float64
 2   INDUS    506 non-null    float64
 3   CHAS     506 non-null    int64  
 4   NOX      506 non-null    float64
 5   RM       506 non-null    float64
 6   AGE      506 non-null    float64
 7   DIS      506 non-null    float64
 8   RAD      506 non-null    int64  
 9   TAX      506 non-null    int64  
 10  PTRATIO  506 non-null    float64
 11  LSTAT    506 non-null    float64
 12  MEDV     506 non-null    float64
dtypes: float64(10), int64(3)
memory usage: 51.5 KB
```

Observations:

- Each of the columns contains a total of **506 non-null observations**. This shows that the data has **no missing values**.
- The dataset contains **13 columns**, all of which are of the numeric data type.

Exploratory Data Analysis and Data Preprocessing

Summary Statistics of this Dataset

In [4]:

```
df.describe().T
```

Out[4]:

	count	mean	std	min	25%	50%	75%	max
CRIM	506.0	3.613524	8.601545	0.00632	0.082045	0.25651	3.677083	88.9762
ZN	506.0	11.363636	23.322453	0.00000	0.000000	0.00000	12.500000	100.0000
INDUS	506.0	11.136779	6.860353	0.46000	5.190000	9.69000	18.100000	27.7400
CHAS	506.0	0.069170	0.253994	0.00000	0.000000	0.00000	0.000000	1.0000
NOX	506.0	0.554695	0.115878	0.38500	0.449000	0.53800	0.624000	0.8710
RM	506.0	6.284634	0.702617	3.56100	5.885500	6.20850	6.623500	8.7800
AGE	506.0	68.574901	28.148861	2.90000	45.025000	77.50000	94.075000	100.0000
DIS	506.0	3.795043	2.105710	1.12960	2.100175	3.20745	5.188425	12.1265
RAD	506.0	9.549407	8.707259	1.00000	4.000000	5.00000	24.000000	24.0000
TAX	506.0	408.237154	168.537116	187.00000	279.000000	330.00000	666.000000	711.0000
PTRATIO	506.0	18.455534	2.164946	12.60000	17.400000	19.05000	20.200000	22.0000
LSTAT	506.0	12.653063	7.141062	1.73000	6.950000	11.36000	16.955000	37.9700
MEDV	506.0	22.532806	9.197104	5.00000	17.025000	21.20000	25.000000	50.0000

Observations:

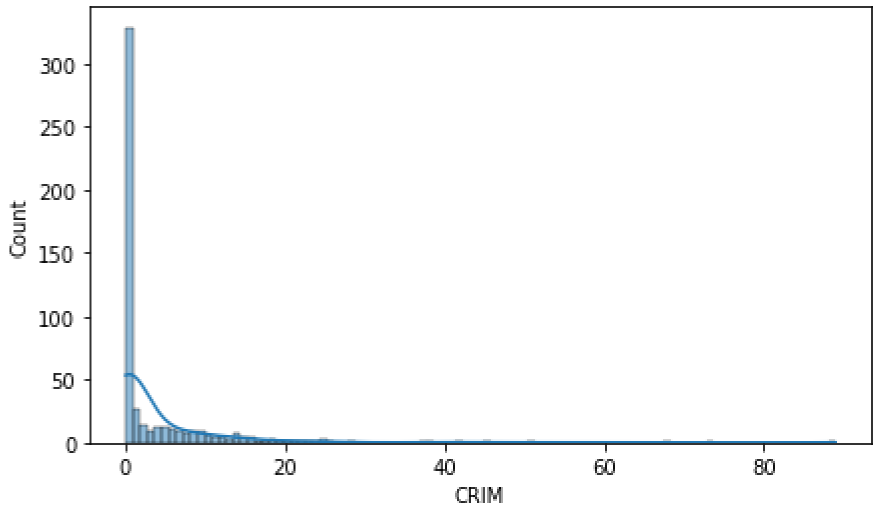
- **CRIM:** Per capita crime rate by town
    - Around 75% of the crime rate falls between ~0-4 with a max of 88 suggesting a possible **outlier**
- **ZN:** Proportion of residential land zoned for lots over 25,000 sq.ft.
    - Over 50% have 0% have residential land zoned for lots over 25,00sq.ft with the max 100%, suggesting this is **perhaps a rare commodity**.
- **INDUS:** Proportion of non-retail business acres per town
    - Ranges from 0.4-27% with an average of 11%, suggesting most towns have some industrial businesses.
- **CHAS:** Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
    - With a mean of 0.069 only ~7% of houses bound the Charles River.
- **NOX:** Nitric Oxide concentration (parts per 10 million)
    - Ranges from 0.38-0.87 with a average of 0.55. Distribution does looks nominal.
- **RM:** The average number of rooms per dwelling
    - Ranges from 3.5-8.7 with an average of 6.2. Distribution does looks nominal.
- **AGE:** Proportion of owner-occupied units built before 1940
    - Ranges from 2.9-100y with an average of 68 years. Distribution does looks nominal.
    - **Min age of 2.9y indicates that no houses in the database are newly built**
- **DIS:** Weighted distances to five Boston employment centers
    - Ranges form 1.1-12.1 with an average of 3.7. Distribution does looks nominal.
- **RAD:** Index of accessibility to radial highways
    - Ranges from 1-24 with over 75% being the max 24.
    - There is a **large jump from the 50th percentile (5) and 75th percentile (24)**. Speculating that perhaps there are 2 cathegories of houses, those in rural areas and those more urban.
- **TAX:** Full-value property-tax rate per 10,000 dollars
    - Ranges from 187-711 with and average of 408. Distribution does looks nominal.
    - **That range suggests these are mid to high income houses.**
- **PTRATIO:** Pupil-teacher ratio by town
    - Ranges from 12.6-22 with an avergage of 18.4. Distribution does looks nominal.
- **LSTAT:** % lower status of the population
    - Ranges from 7-37.9% with an average of 12%. This indicates that most areas have little lower socio-economic class.
    - **The jump from 75th percentile (16.9%) to the max (37%) is indicative of a lower socio-economic area or less likely an outlier**
- **MEDV:** Median value of owner-occupied homes in 1000 dollars
    - Ranges from 5k-50k with an average of 22. Distribution does looks nominal.

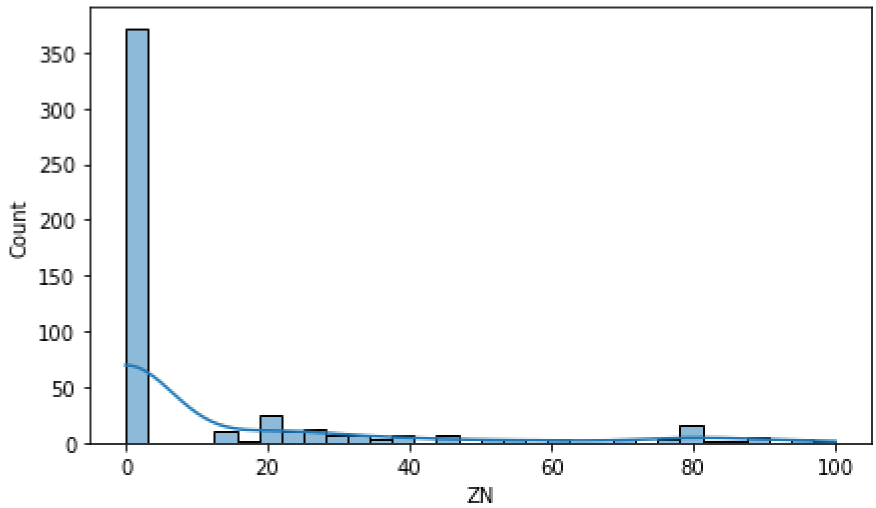
Univariate Analysis

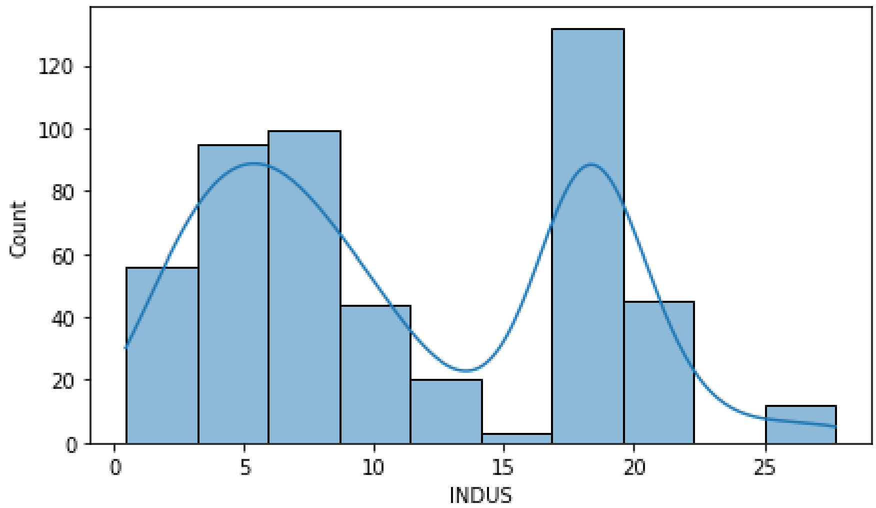
In [ ]:

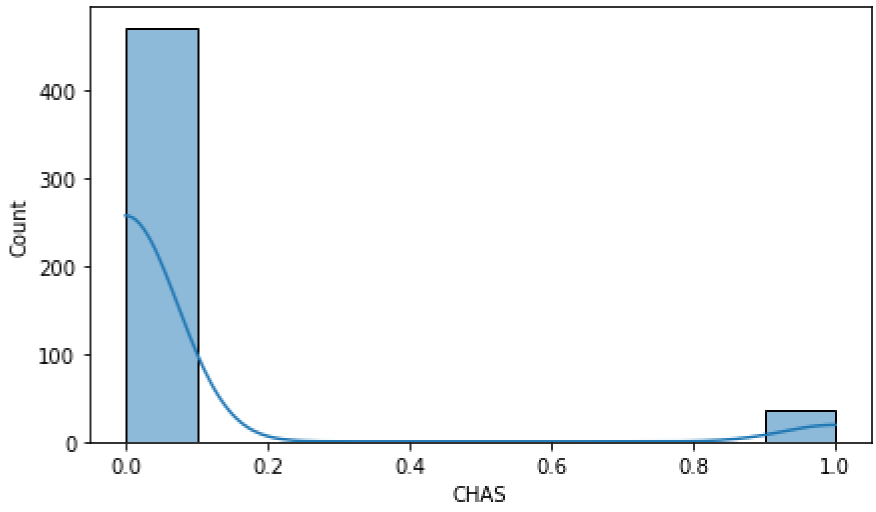
```
# Plotting all the columns in order to look at their distributions.
for i in df.columns:

    plt.figure(figsize = (7, 4))

```

```
    sns.histplot(data = df, x = i, kde = True)

    plt.show()
```

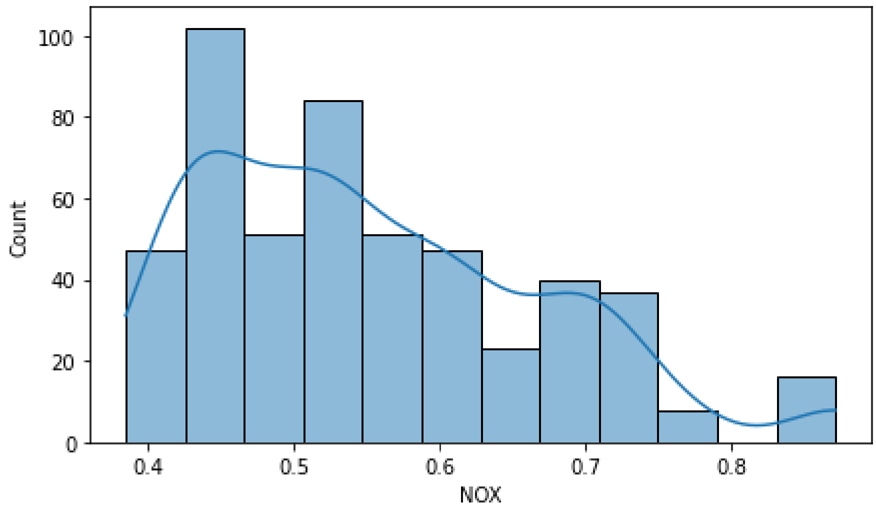




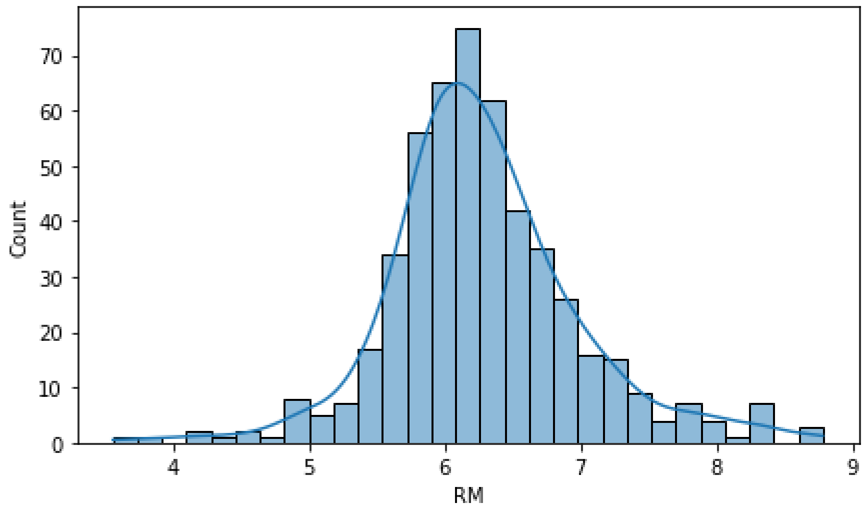




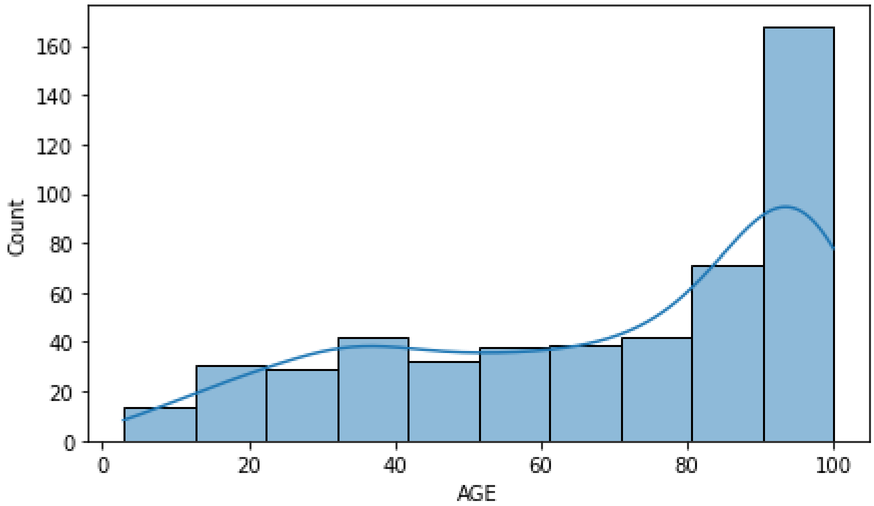
Count
CHAS
0
100
200
300
400
0.0
0.2
0.4
0.6
0.8
1.0

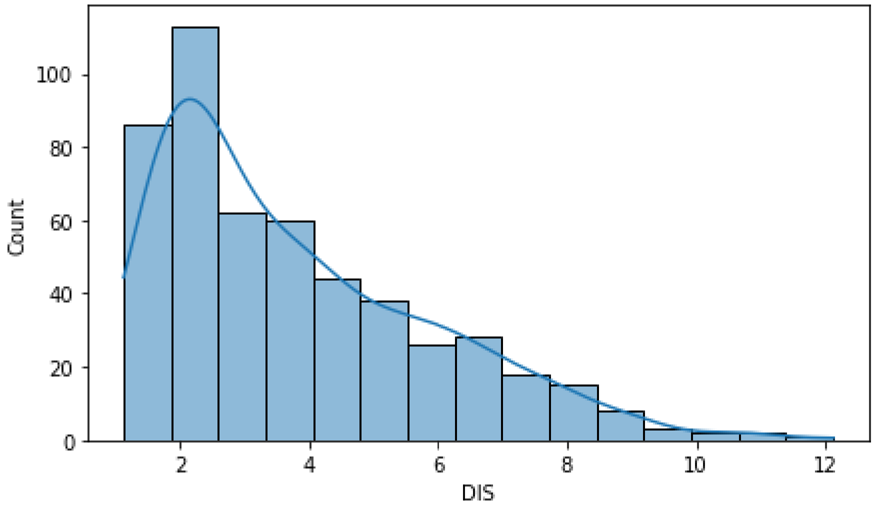


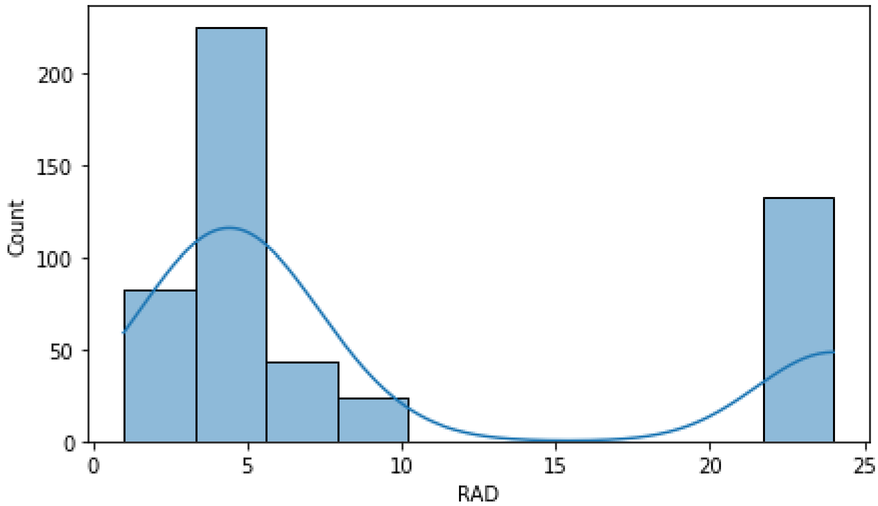
Count
NOX
0
20
40
60
80
100
0.4
0.5
0.6
0.7
0.8

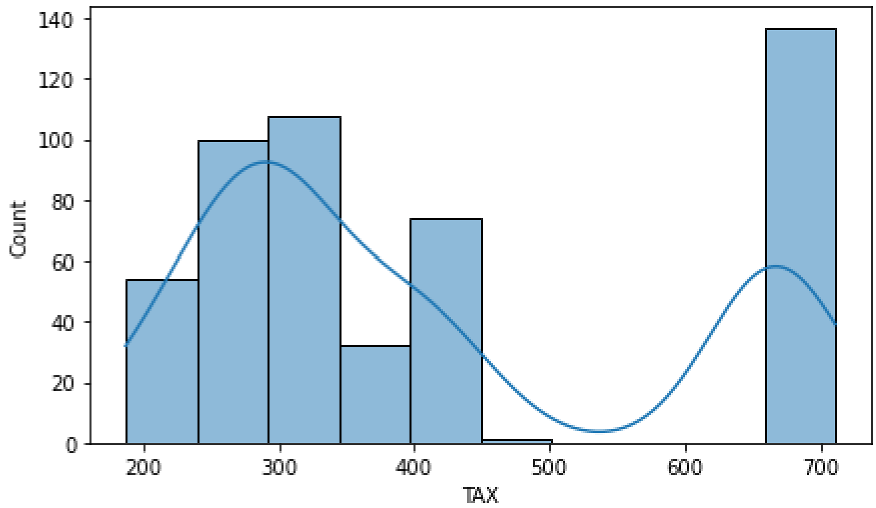


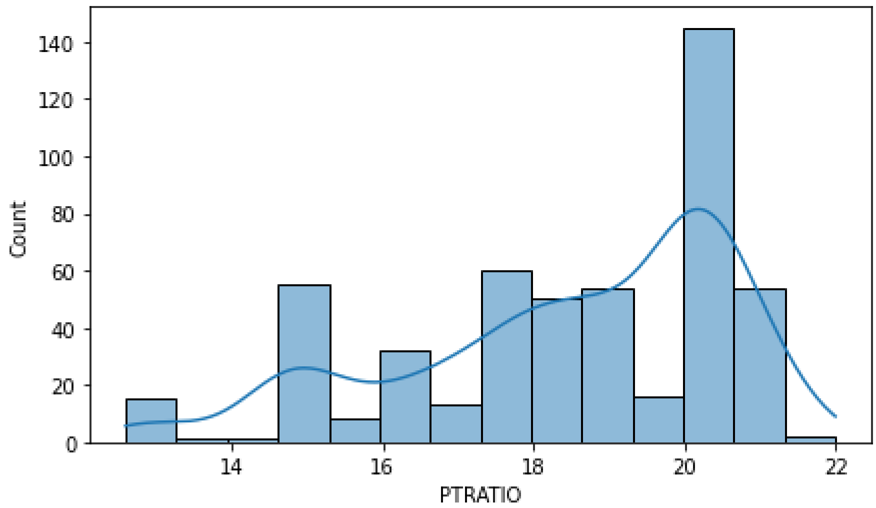
Count
RM
0
10
20
30
40
50
60
70
4
5
6
7
8
9

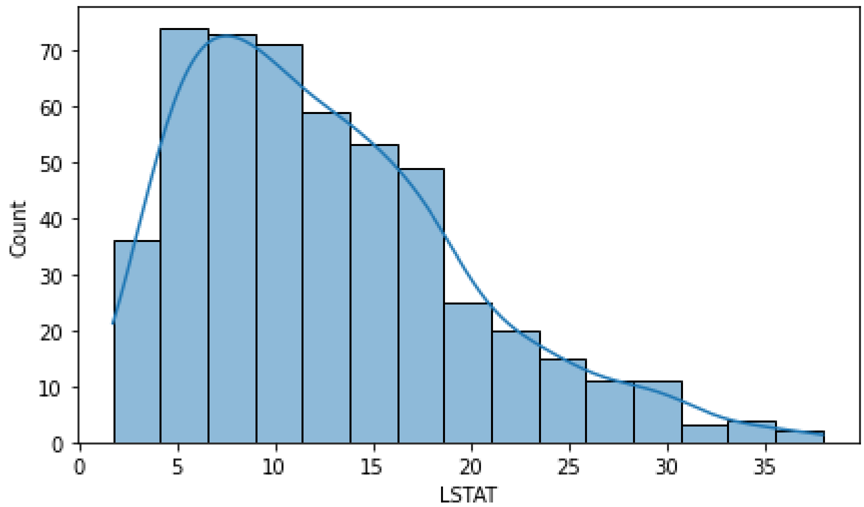

Count
AGE
0
20
40
60
80
100
120
140
160
0
20
40
60
80
100

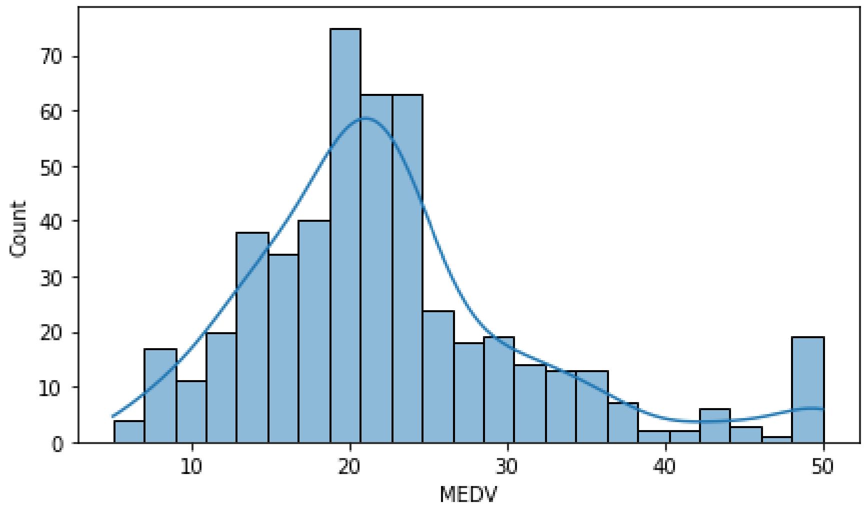

Count
DIS
0
20
40
60
80
100
2
4
6
8
10
12


Count
RAD
0
50
100
150
200
0
5
10
15
20
25


Count
TAX
0
20
40
60
80
100
120
140
200
300
400
500
600
700


Count
PTRATIO
0
20
40
60
80
100
120
140
14
16
18
20
22


Count
LSTAT
0
10
20
30
40
50
60
70
0
5
10
15
20
25
30
35



Observations:

- **CRIM:** Per capita crime rate by town.
  - Heavily right skewed with most values being 0.
- **ZN:** Proportion of residential land zoned for lots over 25,000 sq.ft.
  - Most residential areas have 0 ZN, followed by a near uniform distribution from 10-100%.
- **INDUS:** Proportion of non-retail business acres per town.
  - Apears to be 2 peaks centered at 5% and 17%.
- **CHAS:** Charles River dummy variable (= 1 if tract bounds river; 0 otherwise).
  - Very few houses tract river.
- **NOX:** Nitric Oxide concentration (parts per 10 million).
  - Right skewed.
- **RM:** The average number of rooms per dwelling.
  - Reletively normal distribution around 6.2.
- **AGE:** Proportion of owner-occupied units built before 1940.
  - Heavily left-skewed, **suggesting most houses are older**.
- **DIS:** Weighted distances to five Boston employment centers.
  - Heavily right-skewed.
- **RAD:** Index of accessibility to radial highways.
  - Reiterrates our above observation, like **two categories of houses (rural and urban)**.
- **TAX:** Full-value property-tax rate per 10,000 dollars.
  - Again looks like a similar representation to RAD of **two categories of houses (rural and urban)**.
- **PTRATIO:** Pupil-teacher ratio by town.
  - Left-skewed.
- **LSTAT:** % lower status of the population.
  - Right-skewed suggesting there are fewer overall lower socio-economic people.
- **MEDV:** Median value of owner-occupied homes in 1000 dollars.
  - Slightly skewed. **As this is our dependent variable will need to take action to normalize it**.

Least squares regression models assume the residuals are normal, and a non-normal dependent variable will produce non-normal residual errors. Therefore, as the dependent variable is sightly skewed, we need to apply a **log transformation on the 'MEDV' column** and check the distribution of the transformed column.

Note: Using methods like quantile regression and robust regression can use non-normal dependent variables.

In [ ]:

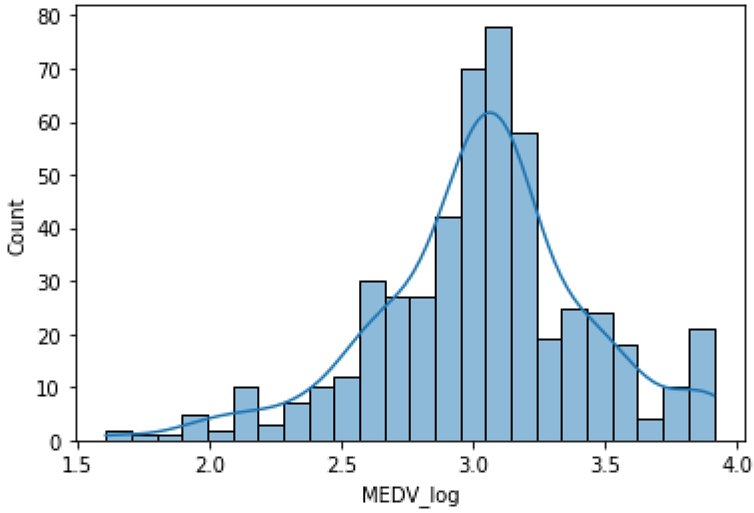
```
df['MEDV_log'] = np.log(df['MEDV'])
```

In [7]:

```
sns.histplot(data = df, x = 'MEDV_log', kde = True)
```

Out[7]:

```
<AxesSubplot:xlabel='MEDV_log', ylabel='Count'>
```



Observation:

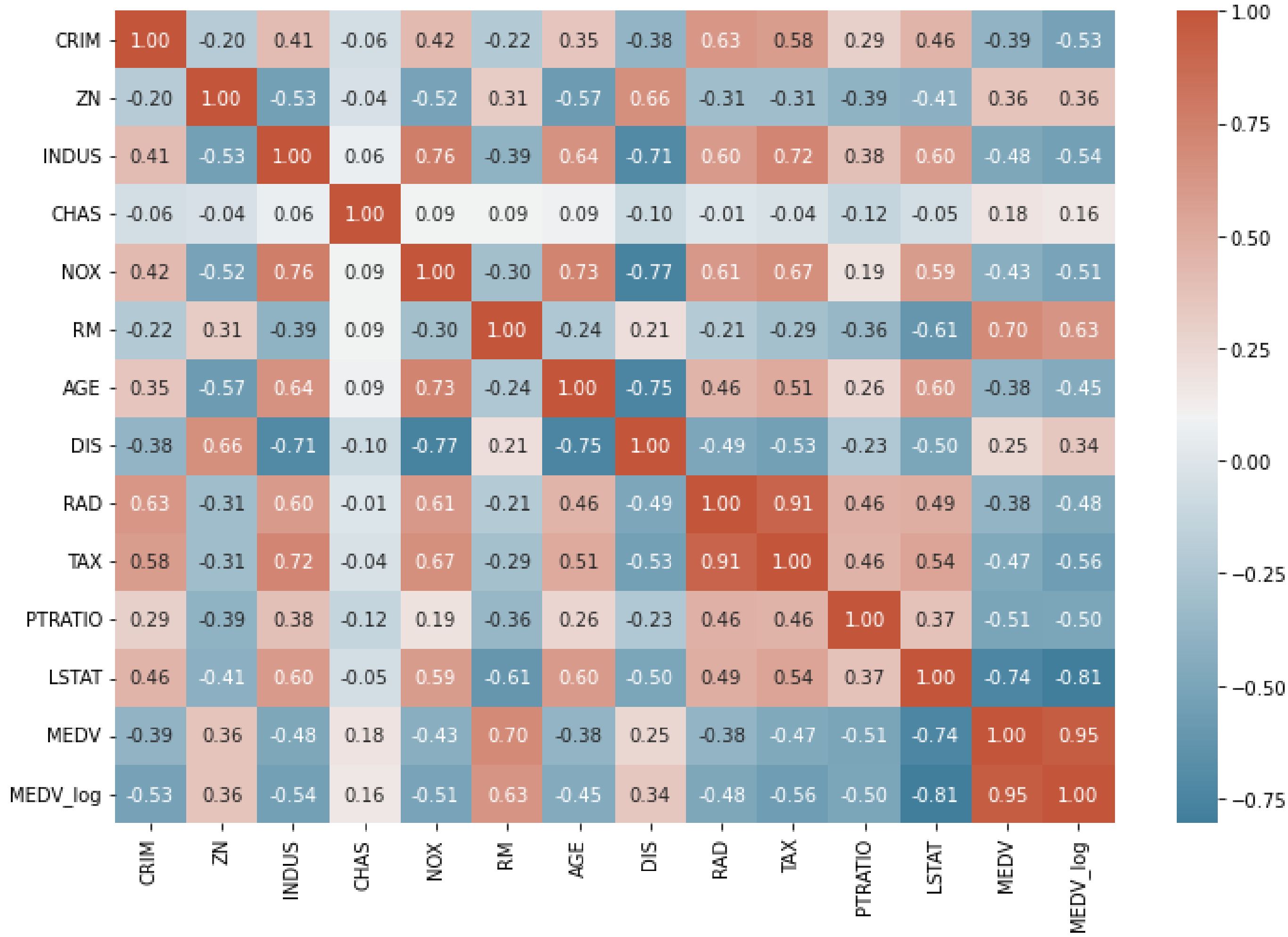
We can move forward as the log-transformation (**MEDV_log**) seems to have a **nearly normal distribution without skew**.

Bivariate Analysis

Check For correlations using heatmap

In [8]:

```
plt.figure(figsize = (12, 8))

cmap = sns.diverging_palette(230, 20, as_cmap = True)

sns.heatmap(df.corr(), annot = True, fmt = '.2f', cmap = cmap)

plt.show()
```



Observations:

Correlations involving dependent variable:

- The highest possitive correlating feature for `MEDV_log` is `RM` (average number of rooms).
  - This makes sense as more rooms typically indicates a larger home
- The highest negative correlating feature for `MEDV_log` is `LSTAT` (% lower status of the population).
  - This makes sense as cities often have lower income areas.
- It is note worthy that 8/12 of our features have negative correlations with `MEDV_log`, this means **most of them are measuring undesirable factors**.

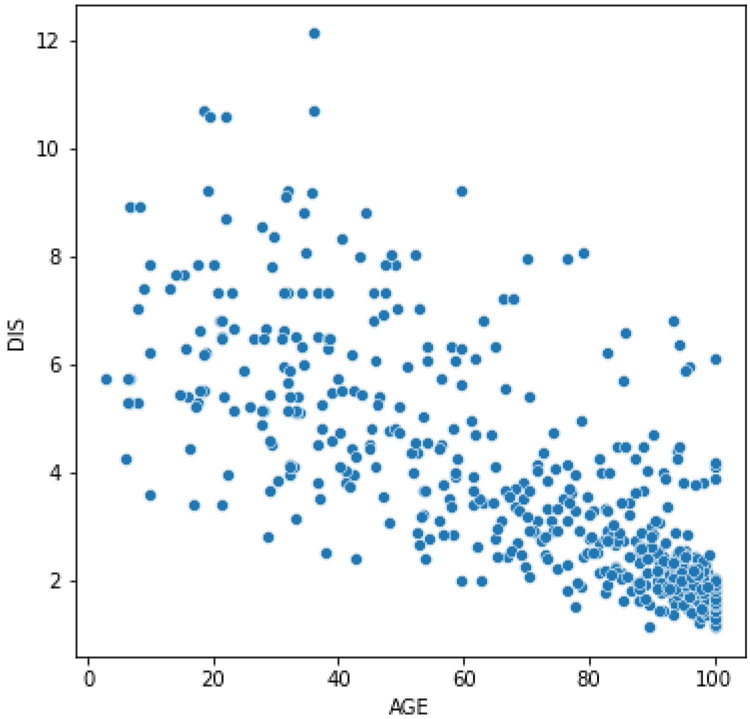
Other strong correlations (>= 0.7 or <= -0.7) not involving our dependent variable:

- Positive Correlation between `NOX` and `INDUS`, makes sense as more industrial areas would produces more Nitric Oxide
- Positive Correlation between `NOX` and `AGE`, perhaps indicating that the older areas are more industrialized?
- Neggative Correlation between `DIS` and `INDUS`, `DIS` and `NOX`, `DIS` and `AGE`.
  - Distance to Boston employment centers seems to indicate a more modern area seperate from the older industrial areas that produce more nitric oxide.
- Positive Correlation between `TAX` and `INDUS`
- Very high Positive Correlation between `TAX` and `RAD`

Visualizing the relationship between the features having significant correlations (>= 0.7 or <= -0.7)

In [9]:

```
# Scatterplot to visualize the relationship between AGE and DIS
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'AGE', y = 'DIS', data = df)

plt.show()
```

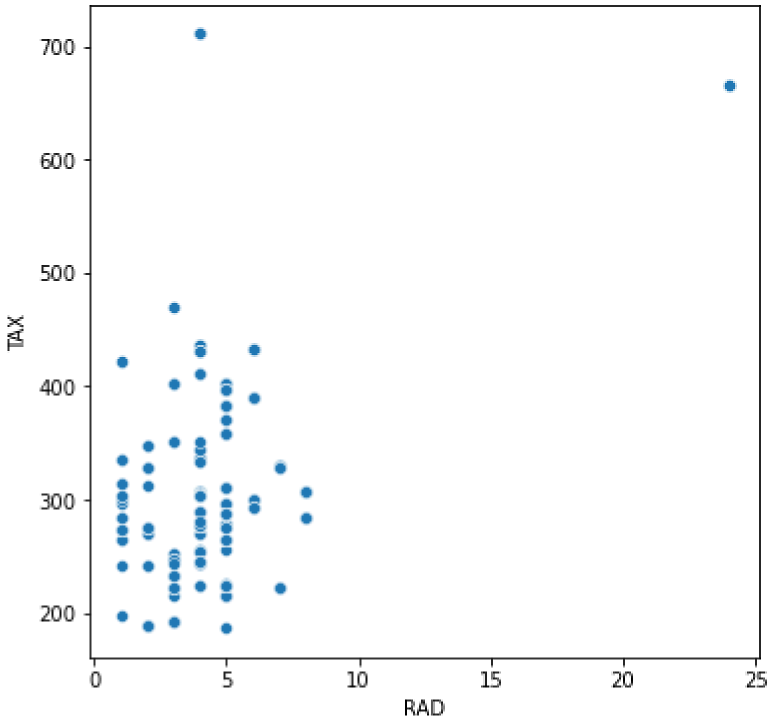


Observations:

- As the percentage of older homes in the town rises, the residences' distance from Boston's employment centers seems to decline somewhat. The employment centers in Boston might be found in established communities with a relatively high percentage of owner-occupied units constructed before 1940.

In [10]:

```
# Scatterplot to visulaize the relationship between RAD and TAX
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'RAD', y = 'TAX', data = df)

plt.show()
```



Observations:

- The correlation between RAD and TAX is very high. However, there is no discernible pattern between the two variables.
- **The strong correlation might be due to outliers.**

Check the correlation remains after removing the outliers.

In [11]:

```
# Remove the data corresponding to high tax rate
df1 = df[df['TAX'] < 600]

# Import the required function
from scipy.stats import pearsonr

# Calculate the correlation
print('The correlation between TAX and RAD is', pearsonr(df1['TAX'], df1['RAD'])[0])
```

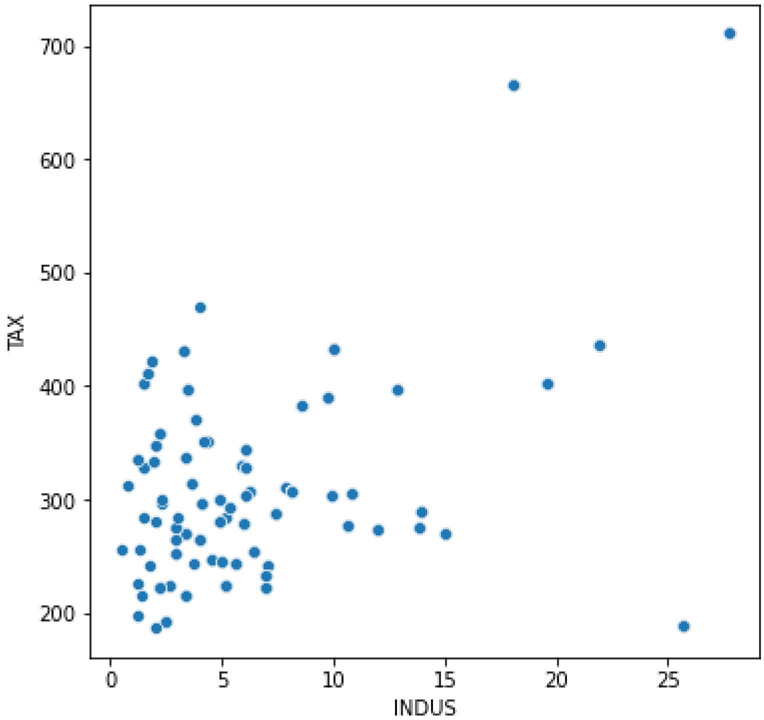
```
The correlation between TAX and RAD is 0.24975731331429196
```

Observation:

- Therefore, the outliers are the cause of the high connection between TAX and RAD. Some homes may have a higher tax rate for other reasons.

In [12]:

```
# Scatterplot to visualize the relationship between INDUS and TAX
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'INDUS', y = 'TAX', data = df)

plt.show()
```

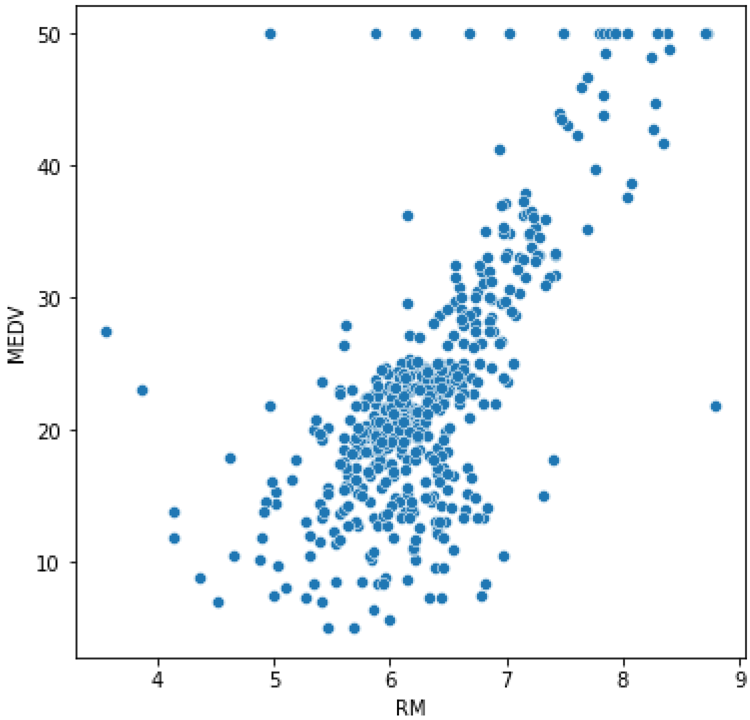


Observations:

- The percentage of non-retail business acres each municipality seems to rise in tandem with the tax rate. The relationship between the variables TAX and INDUS and a third variable may be the cause of this.

In [13]:

```
# Scatterplot to visulaize the relationship between RM and MEDV
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'RM', y = 'MEDV', data = df)

plt.show()
```

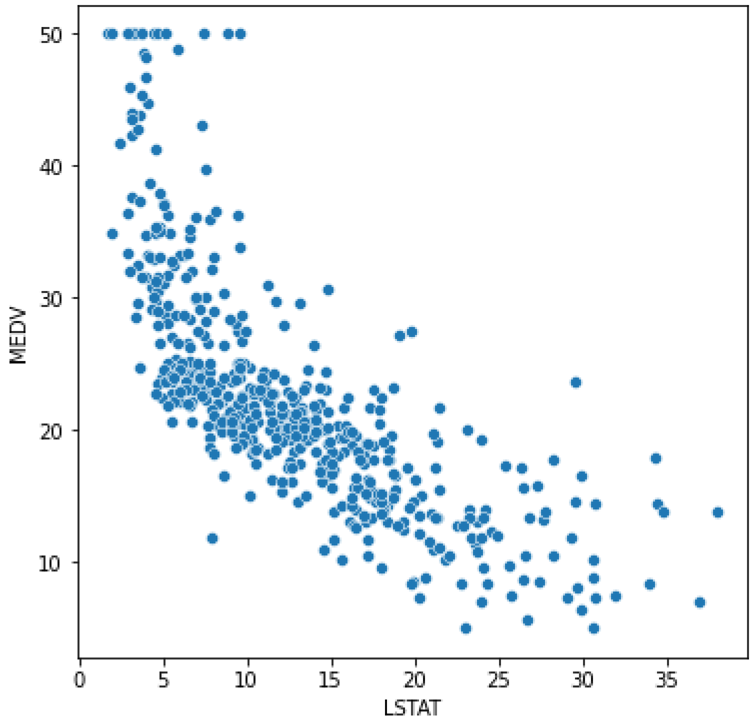


Observations:

- As the value of RM rises, the house's price appears to rise as well. Since bigger rooms typically cost more, this is to be expected.
- Since the MEDV number appears to be restricted at 50, there are a few outliers in a horizontal line.

In [14]:

```
# Scatterplot to visulaize the relationship between LSTAT and MEDV
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'LSTAT', y = 'MEDV', data = df)

plt.show()
```

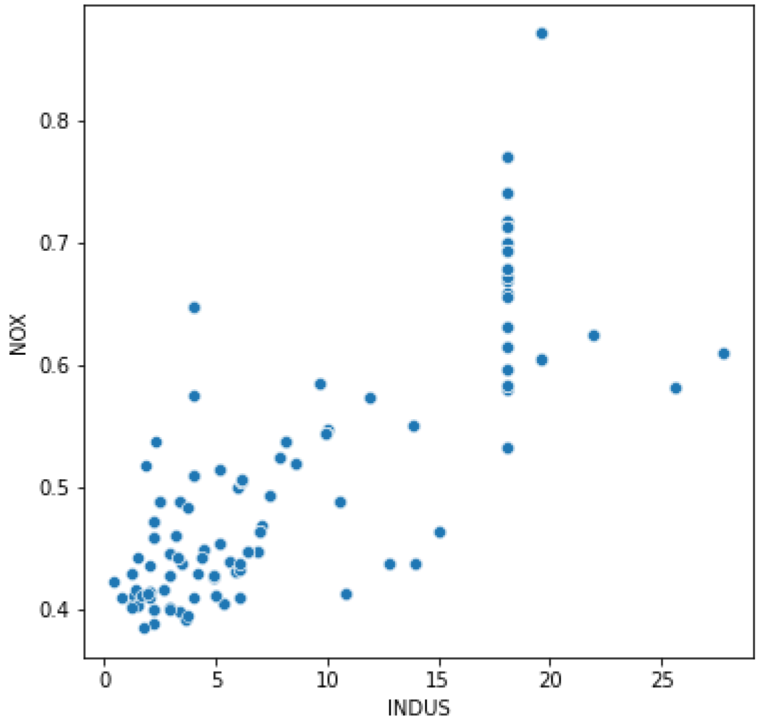


Observations:

- An increase in LSTAT typically results in a fall in the house's price. This is also feasible because housing costs are lower in neighborhoods with lower socioeconomic level. The data appears to be limited to 50, and there aren't many outliers.

In [15]:

```
# Scatterplot to visualize the relationship between INDUS and NOX
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'INDUS', y = 'NOX', data = df)

plt.show()
```

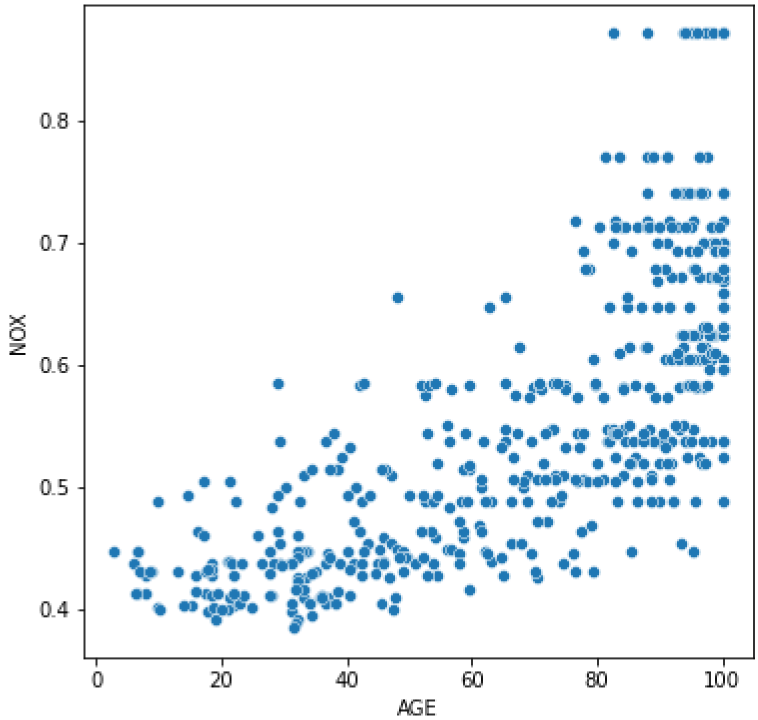


Observations:

- Nitric Oxide increase with industrial areas.
- No obvious outliers are present.

In [16]:

```
# Scatterplot to visualize the relationship between AGE and NOX
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'AGE', y = 'NOX', data = df)

plt.show()
```

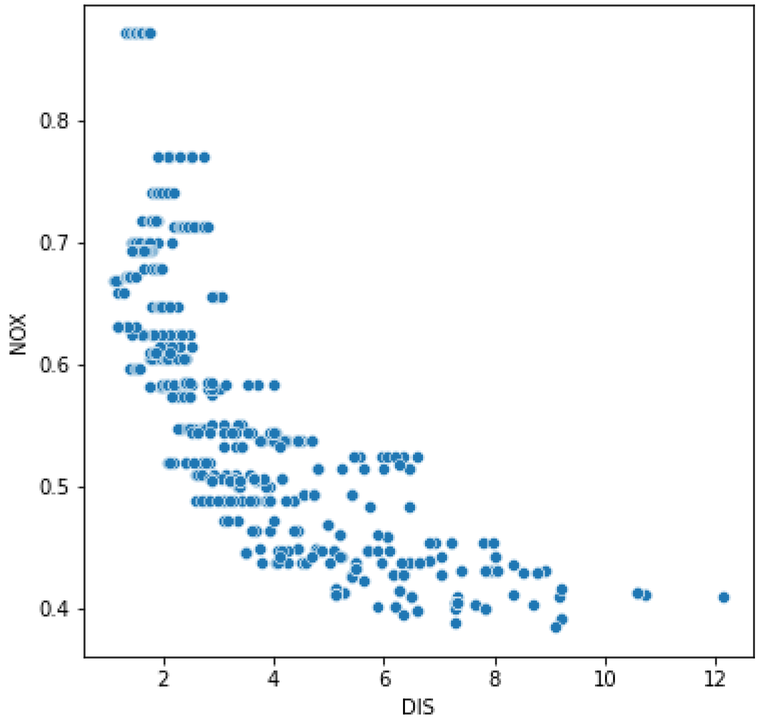


Observations:

- Nitric Oxide levels slightly rise with housing age, supporting the idea that those are more industrial regions.
- A cluster of the highest NOX values may be outliers.

In [17]:

```
# Scatterplot to visualize the relationship between DIS and NOX
plt.figure(figsize = (6, 6))

sns.scatterplot(x = 'DIS', y = 'NOX', data = df)

plt.show()
```



Observations:

- The farther one is from work centers, the lower the nitric oxide level. It's possible that those centers are situated in Boston's more recent, less hardworking neighborhoods.

LSTAT and RM have a linear relationship with the dependent variable MEDV. Also, there are significant **relationships among few independent variables, which is not desirable for a linear regression model**.

Let's first split the dataset.

Split the dataset

Let's split the data into the dependent and independent variables and further split it into train and test set in a ratio of 70:30 for train and test sets.

In [18]:

```
# Separate the dependent variable and indepedent variables
Y = df['MEDV_log']

X = df.drop(columns = {'MEDV', 'MEDV_log'})

# Add the intercept term
X = sm.add_constant(X)
```

Intercept Term

Allows the regression line to be shifted up or down on the y-axis to better fit the data. The value of the intercept term can be interpreted as the expected value of the dependent variable when all independent variables are set to zero.

In [19]:

```
# splitting the data in 70:30 ratio of train to test data
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size = 0.30, random_state
```

check the multicollinearity in the training dataset.

Check for Multicollinearity

Using the Variance Inflation Factor (VIF), to check if there is multicollinearity in the data.

Features having a VIF score > 5 will be dropped / treated till all the features have a VIF score < 5

In [20]:

```
from statsmodels.stats.outliers_influence import variance_inflation_factor

# Function to check VIF
def checking_vif(train):
    vif = pd.DataFrame()
    vif["feature"] = train.columns

    # Calculating VIF for each feature
    vif["VIF"] = [
        variance_inflation_factor(train.values, i) for i in range(len(train.columns))
    ]
    return vif


print(checking_vif(X_train))
```

```
   feature        VIF
0    const  535.372593
1     CRIM    1.924114
2       ZN    2.743574
3    INDUS    3.999538
4     CHAS    1.076564
5      NOX    4.396157
6       RM    1.860950
7      AGE    3.150170
8      DIS    4.355469
9      RAD    8.345247
10     TAX   10.191941
```

```
11  PTRATIO    1.943409
12    LSTAT    2.861881
```

Observations:

- There are two variables with a high VIF - RAD and TAX (greater than 5).
- Let's remove TAX as it has the highest VIF values and check the multicollinearity again.

Drop the column 'TAX' from the training data and check if multicollinearity is resolved.

In [21]:

```
# Create the model after dropping TAX
X_train = X_train.drop(columns = 'TAX')

# Check for VIF
print(checking_vif(X_train))
```

```
    feature         VIF
0     const  532.025529
1      CRIM    1.923159
2        ZN    2.483399
3     INDUS    3.270983
4      CHAS    1.050708
5       NOX    4.361847
6        RM    1.857918
7       AGE    3.149005
8       DIS    4.333734
9       RAD    2.942862
10  PTRATIO    1.909750
11    LSTAT    2.860251
```

VIF is less than 5 for all the independent variables, and we can assume that multicollinearity has been removed between the variables.

Model Building

Linear Regression Model1

In [22]:

```
# Create the model using ordinary least squared
model1 = sm.OLS(y_train,X_train).fit()

# Get the model summary
model1.summary()
```

Out[22]:

OLS Regression Results

Dep. Variable:	MEDV_log	**R-squared:**	0.769
Model:	OLS	**Adj. R-squared:**	0.761
Method:	Least Squares	**F-statistic:**	103.3
Date:	Thu, 15 Dec 2022	**Prob (F-statistic):**	1.40e-101
Time:	17:37:29	**Log-Likelihood:**	76.596

No. Observations:	354	**AIC:**	-129.2
Df Residuals:	342	**BIC:**	-82.76
Df Model:	11		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	4.6324	0.243	19.057	0.000	4.154	5.111
CRIM	-0.0128	0.002	-7.445	0.000	-0.016	-0.009
ZN	0.0010	0.001	1.425	0.155	-0.000	0.002
INDUS	-0.0004	0.003	-0.148	0.883	-0.006	0.005
CHAS	0.1196	0.039	3.082	0.002	0.043	0.196
NOX	-1.0598	0.187	-5.675	0.000	-1.427	-0.692
RM	0.0532	0.021	2.560	0.011	0.012	0.094
AGE	0.0003	0.001	0.461	0.645	-0.001	0.002
DIS	-0.0503	0.010	-4.894	0.000	-0.071	-0.030
RAD	0.0076	0.002	3.699	0.000	0.004	0.012
PTRATIO	-0.0452	0.007	-6.659	0.000	-0.059	-0.032
LSTAT	-0.0298	0.002	-12.134	0.000	-0.035	-0.025

Omnibus:	30.699	**Durbin-Watson:**	1.923
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	83.718
Skew:	0.372	**Prob(JB):**	6.62e-19
Kurtosis:	5.263	**Cond. No.**	2.09e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.09e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

Observations:

- R-squared assessment is not bad at 76.9%, should be improved.

Examining the significance of the model variables

It is not enough to fit a multiple regression model to the data, it is necessary to check whether all the regression coefficients are significant or not. Significance here means whether the population regression parameters are significantly different from zero.

From the above it may be noted that the regression coefficients corresponding to ZN, AGE, and INDUS are not statistically significant at level α = 0.05. In other words, the regression coefficients corresponding to these three are not significantly different from 0 in the population. Hence, we will eliminate the three features and create a new model.

Model2 - Using significant variables

In [23]:

```
# Create the model after dropping columns 'MEDV', 'MEDV_log', 'TAX', 'ZN', 'AGE', 'INDUS
Y = df['MEDV_log']

X = df.drop(['ZN','AGE','INDUS'], axis=1)

X = sm.add_constant(X)

# Splitting the data in 70:30 ratio of train to test data
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size = 0.30 , random_stat

# Create the model
model2 = sm.OLS(y_train,X_train).fit()

# Get the model summary
model2.summary()
```

Out[23]:

OLS Regression Results

Dep. Variable:	MEDV_log	**R-squared:**	1.000
Model:	OLS	**Adj. R-squared:**	1.000
Method:	Least Squares	**F-statistic:**	9.064e+28
Date:	Thu, 15 Dec 2022	**Prob (F-statistic):**	0.00
Time:	17:37:29	**Log-Likelihood:**	11011.
No. Observations:	354	**AIC:**	-2.200e+04
Df Residuals:	342	**BIC:**	-2.195e+04
Df Model:	11		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	9.104e-15	1.72e-14	0.530	0.596	-2.47e-14	4.29e-14
CRIM	-1.431e-16	7.89e-17	-1.814	0.071	-2.98e-16	1.2e-17
CHAS	1.416e-15	1.52e-15	0.928	0.354	-1.58e-15	4.41e-15
NOX	-5.163e-15	6.96e-15	-0.741	0.459	-1.89e-14	8.53e-15
RM	-3.886e-16	8.42e-16	-0.462	0.645	-2.04e-15	1.27e-15
DIS	-1.11e-16	3.2e-16	-0.347	0.729	-7.41e-16	5.19e-16
RAD	3.123e-17	1.3e-16	0.241	0.810	-2.24e-16	2.86e-16
TAX	-2.602e-18	6.67e-18	-0.390	0.697	-1.57e-17	1.05e-17

PTRATIO	-3.053e-16	2.6e-16	-1.173	0.242	-8.17e-16	2.07e-16
LSTAT	-1.041e-17	1.08e-16	-0.097	0.923	-2.22e-16	2.02e-16
MEDV	-4.857e-17	1.86e-16	-0.261	0.795	-4.15e-16	3.18e-16
MEDV_log	1.0000	4.75e-15	2.1e+14	0.000	1.000	1.000

Omnibus:	141.725	**Durbin-Watson:**	0.086
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	666.558
Skew:	1.661	**Prob(JB):**	1.81e-145
Kurtosis:	8.844	**Cond. No.**	1.96e+04

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.96e+04. This might indicate that there are
strong multicollinearity or other numerical problems.

Now, we will check the linear regression assumptions.

Checking the below linear regression assumptions

1. **Mean of residuals should be 0**
2. **No Heteroscedasticity**
3. **Linearity of variables**
4. **Normality of error terms**

1. Check for mean residuals

In [24]:

```
residuals = model2.resid

np.mean(residuals)
```

Out[24]:

```
7.338135121519608e-15
```

Observations:

- The mean residuals is very close to 0, therefore **the assumption is satisfied.**

2. Check for homoscedasticity

- Homoscedasticity - If the residuals are symmetrically distributed across the regression line, then the data is said to be homoscedastic.

- Heteroscedasticity- - If the residuals are not symmetrically distributed across the regression line, then the data is said to be heteroscedastic. In this case, the residuals can form a funnel shape or any other non-symmetrical shape.

- We'll use `Goldfeldquandt Test` to test the following hypothesis with alpha = 0.05:
  - Null hypothesis: Residuals are homoscedastic
  - Alternate hypothesis: Residuals have heteroscedastic

In [25]:

```
from statsmodels.stats.diagnostic import het_white

from statsmodels.compat import lzip

import statsmodels.stats.api as sms
```

In [26]:

```
name = ["F statistic", "p-value"]

test = sms.het_goldfeldquandt(y_train, X_train)

lzip(name, test)
```

Out[26]:

```
[('F statistic', 12.474530449630052), ('p-value', 1.9164807503085753e-48)]
```

Observations:

- Since the p-value < 0.05, we reject the Null-Hypothesis hence residuals have heteroscedastic.
- **Therefore the assumption is not satisfied and our model will overall be less accurate.**
- We can try and solve this by further transforming Y.

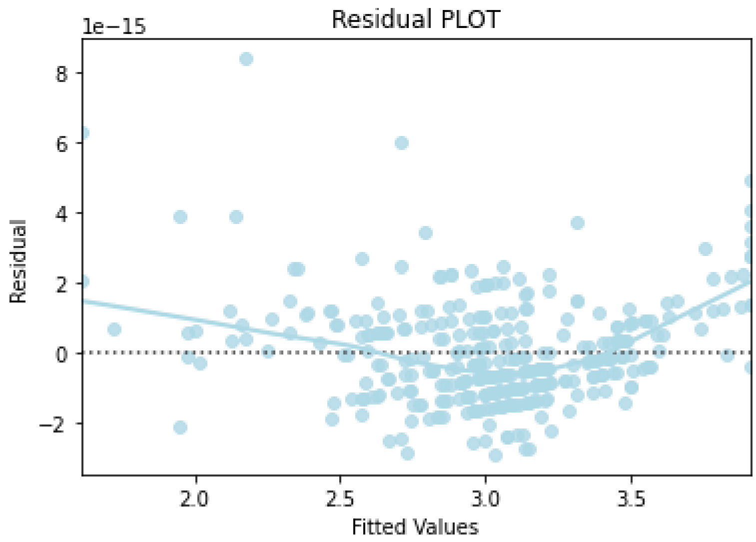
3. Linearity of variables

It states that the predictor variables must have a linear relation with the dependent variable.

To test the assumption, we'll plot residuals and the fitted values on a plot and ensure that residuals do not form a strong pattern. They should be randomly and uniformly scattered on the x-axis.

In [28]:

```
# Predicted values
fitted = model2.fittedvalues

# sns.set_style("whitegrid")
sns.residplot(x = fitted, y = residuals, color = "lightblue", lowess = True)

plt.xlabel("Fitted Values")

plt.ylabel("Residual")

plt.title("Residual PLOT")

plt.show()
```



Observations:

- There is no pattern in the residual vs fitted values, therefore **the assumption is satesfied**.

4. Normality of error terms

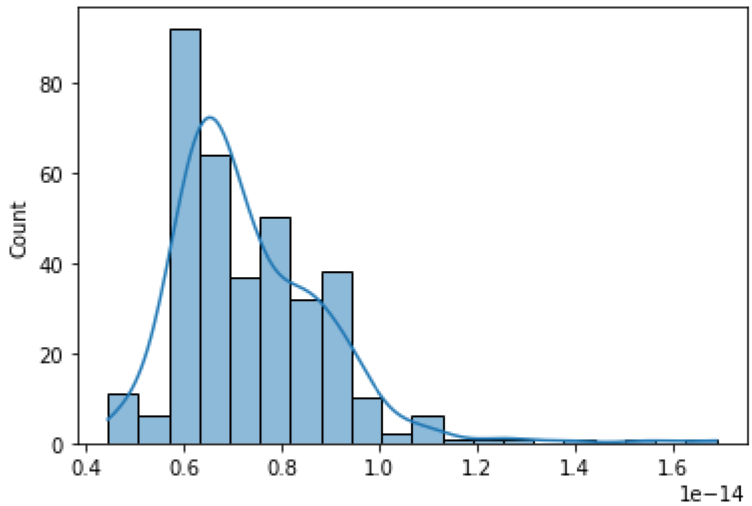
The residuals should be normally distributed.

In [29]:

```
# Plot histogram of residuals

sns.histplot(residuals, kde = True)
```

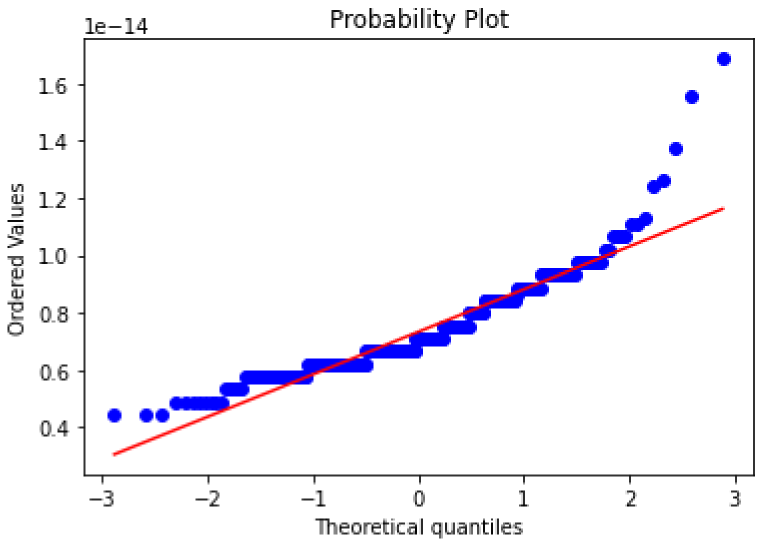
Out[29]:

<AxesSubplot:ylabel='Count'>



In [30]:

```
# Plot q-q plot of residuals
import pylab

import scipy.stats as stats

stats.probplot(residuals, dist = "norm", plot = pylab)

plt.show()
```



Observations:

- From the above plots, the residuals are skewed.
- **Therefore the assumption is not satisfied and our model will overall be less accurate.**

Check the performance of the model on the train and test data set

In [69]:

```
# RMSE
def rmse(predictions, targets):
    return np.sqrt(((targets - predictions) ** 2).mean())


# MAPE
def mape(predictions, targets):
    return np.mean(np.abs((targets - predictions)) / targets) * 100


# MAE
def mae(predictions, targets):
    return np.mean(np.abs((targets - predictions)))

## R2
from sklearn.metrics import r2_score


# Model Performance on test and train data
def model_pref(olsmodel, x_train, x_test):

    # In-sample Prediction
    y_pred_train = olsmodel.predict(x_train)
    y_observed_train = y_train

    # Prediction on test data
    y_pred_test = olsmodel.predict(x_test)
    y_observed_test = y_test

    print(
        pd.DataFrame(
            {
```

```
                "Data": ["Train", "Test"],
                "RMSE": [
                    rmse(y_pred_train, y_observed_train),
                    rmse(y_pred_test, y_observed_test),
                ],
                "MAE": [
                    mae(y_pred_train, y_observed_train),
                    mae(y_pred_test, y_observed_test),
                ],
                "MAPE": [
                    mape(y_pred_train, y_observed_train),
                    mape(y_pred_test, y_observed_test),
                ],
                "r2": [
                    r2_score(y_pred_train, y_observed_train),
                    r2_score(y_pred_test, y_observed_test),
                ],
            }
        )
    )


# Checking model performance
model_pref(model2, X_train, X_test)
```

```
    Data          RMSE           MAE          MAPE   r2
0  Train  7.504024e-15  7.338135e-15  2.509778e-13  1.0
1   Test  7.552803e-15  7.384444e-15  2.490217e-13  1.0
```

Observations:

- The train and test scores are very close, therefore our model **is not overfitted and generalizes well**.
- That the two scores are so close means there is likely little we can do to improve the model performance.

Apply cross validation to improve the model and evaluate it using different evaluation metrics

In [32]:

```
# Import the required function

from sklearn.model_selection import cross_val_score

# Build the regression model and cross-validate
linearregression = LinearRegression()

cv_Score11 = cross_val_score(linearregression, X_train, y_train, cv = 10)
cv_Score12 = cross_val_score(linearregression, X_train, y_train, cv = 10,
                             scoring = 'neg_mean_squared_error')


print("RSquared: %0.3f (+/- %0.3f)" % (cv_Score11.mean(), cv_Score11.std() * 2))
print("Mean Squared Error: %0.3f (+/- %0.3f)" % (-1*cv_Score12.mean(), cv_Score12.std()
```

```
RSquared: 1.000 (+/- 0.000)
Mean Squared Error: 0.000 (+/- 0.000)
```

Observations

- As predicted the model is already at peak performance and did not improve.

Get model Coefficients

Put model coefficients in a pandas dataframe with column 'Feature' having all the features and column 'Coefs' with all the corresponding Coefs. (4 Marks)

Hint: To get values please use coef.values

In [66]:

```
coef = model2.params

pd.DataFrame({'Feature' : coef.index, 'Coefs' : coef.values})
```

Out[66]:

	Feature	Coefs
0	const	9.103829e-15
1	CRIM	-1.431147e-16
2	CHAS	1.415534e-15
3	NOX	-5.162537e-15
4	RM	-3.885781e-16
5	DIS	-1.110223e-16
6	RAD	3.122502e-17
7	TAX	-2.602085e-18
8	PTRATIO	-3.053113e-16
9	LSTAT	-1.040834e-17
10	MEDV	-4.857226e-17
11	MEDV_log	1.000000e+00

In [67]:

```
# Let us write the equation of the fit

Equation = "log (Price) = "

print(Equation, end = '\t')

for i in range(len(coef)):
    print('(', coef[i], ') * ', coef.index[i], '+', end = ' ')
```

```
log (Price) =   ( 9.103828801926284e-15 ) *  const + ( -1.4311468676808659e-16 ) *  CRIM
+ ( 1.4155343563970746e-15 ) *  CHAS + ( -5.162537064506978e-15 ) *  NOX + ( -3.88578058
6188048e-16 ) *  RM + ( -1.1102230246251565e-16 ) *  DIS + ( 3.122502256758253e-17 ) *
RAD + ( -2.6020852139652106e-18 ) *  TAX + ( -3.0531133177191805e-16 ) *  PTRATIO + ( -
1.0408340855860843e-17 ) *  LSTAT + ( -4.85722573273506e-17 ) *  MEDV + ( 0.999999999999
9991 ) *  MEDV_log +
```

Conclusions and Business Recommendations

Conclusions

- We can forecast Boston's house values using this forecasting technique.
- With an r-squared of 1, the model accounts for 100% of the observed variation.
- The top 5 features that have the greatest impact on predicting housing prices are:
    - CRIM: Per capita crime rate by town - Where a lower crime rate results in a higher prices.
    - CHAS: Charles River dummy variable - Where being on the Charles River results in a higher prices.
    - NOX: Nitric Oxide concentration (parts per 10 million) - Where higher nitric oxide concentration results in higher prices
        - Note that NOX was heavily correlated to INDST and AGE which where dropped for that reason. Therefore it is likely that a higher NOX is acting as a stand in for the older and more industrial areas and that is key to increasing the price.
    - RM: The average number of rooms per dwelling - Where more rooms results in a higher price
    - DIS: Weighted distances to five Boston employment centers - Where a shorter distance to employment center results in higher prices.
        - We observed that lower DIS is likely representative of more urban areas of Boston

Recommendations

Our model would be a helpful tool in the banking, insurance, and real estate sectors and can anticipate Boston house values with high accuracy.

We were able to determine from our model that the main way to estimate value in Boston homes is via:

- Low-crime areas
- Having larger rooms
- Being close to more metropolitan areas
- Being on the Charles River's edge
- Being in older, more industrial neighborhoods

In [ ]:

```

```