

I don't really understand the problem. What we should at a given edge is

$$F = u_i \mathbf{n}_i + u_j \mathbf{n}_j$$

where u_i stands for the trace of the function u coming from triangle “ i ” and \mathbf{n}_i is the normal of the given edge, pointing outwards with respect to triangle “ i ”.

Say we choose to define $\mathbf{n} = \mathbf{n}_i = -\mathbf{n}_j$. Then, by definition, $u^+ = u_i$ and $u^- = u_j$, so we can express the flux F as:

$$F = u^+ \mathbf{n} + u^- (-\mathbf{n}) = (u^+ - u^-) \mathbf{n}$$

Say now that we have the other choice, $\mathbf{n} = \mathbf{n}_j = -\mathbf{n}_i$. In that case $u^+ = u_j$ and $u^- = u_i$, so the term becomes:

$$F = u^- (-\mathbf{n}) + u^+ \mathbf{n} = (u^+ - u^-) \mathbf{n}$$

It seems to me that the term:

$$[[u]]_{\mathbf{n}} = (u^+ - u^-) \mathbf{n} = u_i \mathbf{n}_i + u_j \mathbf{n}_j$$

does not depend on the choice of \mathbf{n} .