ROLLING THE DICE – Random Walk game!

Let's say our friend suggested a game. The game is: I will role the dice. If dice shows number 1 or 2, you go step back. If it shows number 3,4 or 5 you go step forward, otherwise you throw the dice again. Then our friend says, „we bet in 100$ you won't make 75 steps forward in 150 throws!". The question is will you take the bet?

Solution to the game = Mathematical way! We'll simulate the game and calculate the probability of reaching more than 75 steps in 150 throws. The game we play is modeled as random walk.

First, we intruduce the random variable X_i which models the step in i-th throw. Than $X_i, \forall i = 1, .., n$, where n is number of throws, has the following distribution:

$$X_i \sim \begin{pmatrix} -1 & 0 & 1 \\ 1/3 & 1/6 & 1/2 \end{pmatrix}$$

Now, when we have the random variable that models the steps, we can define random variable Y_t as the of steps moved forward after t throws. For example: Let say the dice rolled the following sequence of numbers $d = (6, 3, 5, 2, 6, 5, 5, 4, 5, 2, 1, 3)$. If we translate the numbers into the steps we get $s = (0, 1, 1, -1, 0, 1, 1, 1, 1, -1, -1, 1)$, than the value of random variable $Y_{12} = 4$. Interpretation is: after 12 dice throws we made 4 step forward. So the formal definition is would be:

$$Y_t = \sum_{k=1}^{t} X_k \tag{1}$$

First idea would be to calculate the expeced value of Y_{150} to see expected number of steps moved forward after 100 throws. The expetation would be:

$$E(Y_{150}) = E\left(\sum_{k=1}^{150} X_k\right) = 150\, E(X_1) = 150 \cdot \frac{1}{6} = 25 \tag{2}$$

So, the expected value is only 25 steps. What will we do, is to simulate the game. And see the distribution of end steps. First, let's simulate the game 10 times and visualise Y_{150} for every game.

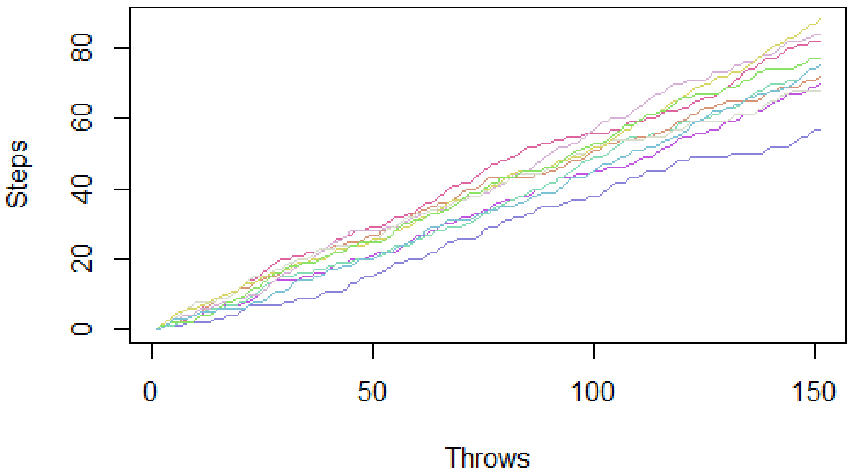

Figure 1: 10 simulates of the game

From the picture we see that there is a chance to make more than 75 steps. Let's now do more simulations, so we can estimate the probability more accurate. We'll take $n = 500$ repeats of the game. Now we take last number of every simulation end define vector *ends*. Vector *ends* contains 500 numbers. Histogram of *ends* is shown below:

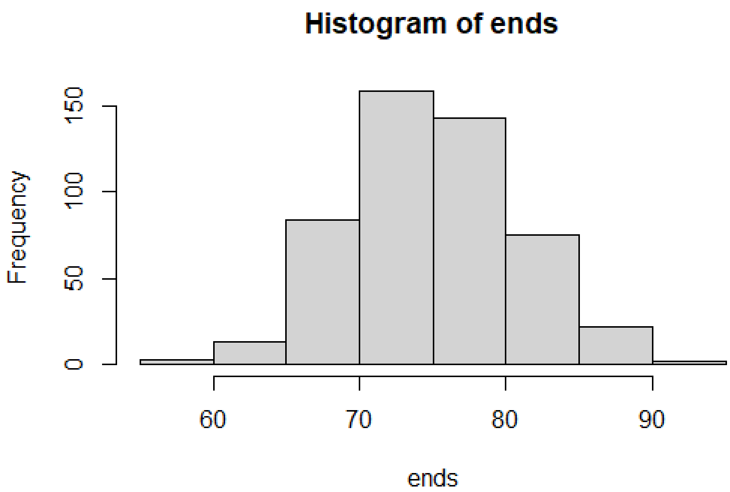

Figure 2: Histogram of *ends*

From histogram we can see that there is some chance to make 75 or more steps. Now me can estimate the probability of making 75 or more steps as:

$$\hat{p} = \frac{1}{500} \sum_{k=1}^{500} \mathbb{1}_{\{ends_{(k)} \geq 75\}}$$

The estimated probability is $P(Y_{150} \geq 75) = 0.56$. So, we have estimated chance of 56%. Is it worth to take the risk? Decide for yourself!