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Performance Evaluation of Computer Systems and Networks

Epidemic Broadcast

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Introduction

In this work a study on the broadcast of an epidemic message is carried out.

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Appendix A

Given a scenario with N transmitter devices and a target device T in reach of all the transmitters, let us define the probabilities $P_1(j, N)$ as the probability of j devices out of N transmitting at the same time during slot 1 and $P_i(j)$ as the probability of j devices transmitting at the same time during slot i.
By specification, the successful reception of the message by device T happens if and only if **one** of the transmitters sends the message during the slot. Furthermore, the successful transmission of a device is "*a Bernoullian RV with success probability* p *on every slot, until it achieves success*"; therefore we can model $P_1(j, N)$ as follows:

$$\left.\begin{aligned}
&P_1(0, N) = (1-p)^N \\
&P_1(1, N) = Np(1-p)^{N-1} \\
&P_1(2, N) = \binom{N}{2} p^2 (1-p)^{N-2} \\
&P_1(3, N) = \binom{N}{3} p^3 (1-p)^{N-3} \\
&\ldots \\
&P_1(N-1, N) = \binom{N}{N-1} p^{N-1}(1-p) \\
&P_1(N, N) = \binom{N}{N} p^N
\end{aligned}\right\} P_1(j, N) = \binom{N}{j} p^j (1-p)^{N-j}$$

As for $P_i(j)$ we can model the system as if it was in the first slot, with N now being equal to $N - t$ transmitters, where t is the number of devices that have transmitted in the $(i-1)$-th slot.

Therefore, for $i = 2$ we have:

$$
\begin{aligned}
P_2(0) &= P_1(0,N)P_1(0,N) + P_1(2,N)P_1(0,N-2) + ... + P_1(N-1,N)P_1(0,1) = \\
&= P_1(0,N)P_1(0,N) + \sum_{k=2}^{N-1} P_1(k,N)P_1(0,N-k) = \\
&= \sum_{k=0}^{N-1} P_1(k,N-k)P_1(0,N) - P_1(1,N)P_1(0,N-1) \\
P_2(1) &= P_1(0,N)P_1(1,N) + P_1(2,N)P_1(1,N-2) + ... + P_1(N-1,N)P_1(1,1) = \\
&= P_1(0,N)P_1(1,N) + \sum_{k=2}^{N-1} P_1(k,N)P_1(1,N-k) = \\
&= \sum_{k=0}^{N-1} P_1(k,N)P_1(1,N-k) - P_1(1,N)P_1(1,N-1) \\
P_2(2) &= P_1(0,N)P_1(2,N) + P_1(2,N)P_1(2,N-2) + ... + P_1(N-2,N)P_1(2,2) = \\
&= P_1(0,N)P_1(2,N) + \sum_{k=2}^{N-2} P_1(k,N)P_1(2,N-k) = \\
&= \sum_{k=0}^{N-2} P_1(k,N)P_1(2,N-k) - P_1(1,N)P_1(2,N-1) \\
P_2(3) &= ... = \sum_{k=0}^{N-3} P_1(k,N)P_1(3,N-k) - P_1(1,N)P_1(3,N-1) \\
&... \\
P_2(N-2) &= \sum_{k=0}^{2} P_1(k,N)P_1(N-2,N-k) - P_1(1,N)P_1(N-2,N-1) \\
P_2(N-1) &= \sum_{k=0}^{1} P_1(k,N)P_1(N-1,N-k) - P_1(1,N)P_1(N-1,N-1) \\
P_2(N) &= P_1(0,N)P_1(N,N)
\end{aligned}
$$

which has the general form:

$$
P_2(j) = \begin{cases} \sum_{k=0}^{N-1} P_1(k,N)P_1(0,N-k) - P_1(1,N)P_1(0,N-1) & \text{j = 0} \\ \sum_{k=0}^{N-j} P_1(k,N)P_1(j,N-k) - P_1(1,N)P_1(j,N-1) & \text{0 < j < N} \\ P_1(0,N)P_1(N,N) & \text{j = N} \end{cases} \tag{3.1}
$$

where the term with the minus sign is due to the fact that, if only one device transmitted during slot i, the target device T will have correctly received the message and therefore, starting from slot $i+1$ onwards, it will not be listening for incoming messages any more but it will be itself transmitting instead.

For $i = 3$ we have:

$$
\begin{aligned}
&P_3(0) = \sum_{k=0}^{N-1} P_2(k)P_1(0, N-k) - P_2(1)P_1(0, N-1) \\
&P_3(1) = \sum_{k=0}^{N-1} P_2(k)P_1(1, N-k) - P_2(1)P_1(1, N-1) \\
&P_3(2) = \sum_{k=0}^{N-2} P_2(k)P_1(2, N-k) - P_2(1)P_1(2, N-1) \\
&... \\
&P_3(N-1) = \sum_{k=0}^{1} P_2(k)P_1(N-1, N-k) - P_2(1)P_1(0, N-1) \\
&P_3(N) = P_2(0)P_1(N, N)
\end{aligned}
$$

which has a general form similar to (3.1)

$$
P_3(j) = \begin{cases} \sum_{k=0}^{N-1} P_2(k)P_1(0, N-k) - P_1(1, N)P_1(0, N-1) & \text{j = 0} \\ \sum_{k=0}^{N-j} P_2(k)P_1(j, N-k) - P_1(1, N)P_1(j, N-1) & \text{0 < j < N} \\ P_2(0)P_1(N, N) & \text{j = N} \end{cases} \tag{3.2}
$$

We can further generalize formula 3.2 to obtain the probability $P_i(j)$ we introduced at the beginning:

$$
P_i(j) = \begin{cases} \sum_{k=0}^{N-1} P_{i-1}(k)P_1(0, N-k) - P_{i-1}(1, N)P_1(0, N-1) & \text{j = 0} \\ \sum_{k=0}^{N-j} P_{i-1}(k)P_1(j, N-k) - P_{i-1}(1, N)P_1(j, N-1) & \text{0 < j < N} \\ P_{i-1}(0)P_1(N, N) & \text{j = N} \end{cases} \tag{3.3}
$$