Università di Pisa
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Performance Evaluation of Computer Systems and Networks

Epidemic Broadcast

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Chapter 1

Introduction

In this work a study on the broadcast of an epidemic message is carried out.
The specifications are the following:

> ### Epidemic broadcast
>
> Consider a 2D floorplan with N users randomly dropped in it. A random user within the floorplan produces a *message*, which should ideally reach all the users as soon as possible. Communications are *slotted*, meaning that on each slot a user may or may not relay the message, and a message occupies an entire slot. A *broadcast radius* R is defined, so that every receiver who is within a radius R from the transmitter will receive the message, and no other user will hear it. A user that receives more than one message in the same slot will not be able to decode any of them (*collision*). Users relay the message they receive *once*,according to the following policy (*p-persistent relaying*): after the user successfully receives a message, it keeps extracting a value from a Bernoullian RV with success probability p on every slot, until it achieves success. Then it relays the message and stops. A sender does not know (or cares about) whether or not its message has been received by its neighbors.
>
> Measure at least the broadcast time for a message in the entire floorplan, the percentage of covered users, the number of collisions.
>
> In all cases, it is up to the team to calibrate the scenarios so that meaningful results are obtained.

The work is organized as follows:
firstly, an initial overview and a presentation of the problem are given. Here, meaningful parameters to be tweaked and useful scenarios are identified and some considerations about them are made.
Secondly, a graph-based modelling technique, commonly used in literature, is proposed and some simplified scenarios are analysed.

Then, in chapter [TODO] the real model is considered and some assumptions about it are made, such as its performance when one or more parameters are near the ends or such as its asymptotic behaviour with different configurations of the parameters.
In chapter [TODO] the construction of the simulator is described and the results of its validation are presented.
Chapter [TODO] concerns the full simulation and performance evaluation of the system.

Chapter 2

Overview

Chapter 3

System modelling

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Appendix A

Given a scenario with N transmitter devices and a target device T in reach of all the transmitters, let us define the probabilities $P_1(j, N)$ as the probability of j devices out of N transmitting at the same time during slot 1 and $P_i(j)$ as the probability of j devices transmitting at the same time during slot i.
By specification, the successful reception of the message by device T happens if and only if **one** of the transmitters sends the message during the slot. Furthermore, the successful transmission of a device is "*a Bernoullian RV with success probability* p *on every slot, until it achieves success*"; therefore we can model $P_1(j, N)$ as follows:

$$\left.\begin{aligned}
&P_1(0,N) = (1-p)^N \\
&P_1(1,N) = Np(1-p)^{N-1} \\
&P_1(2,N) = \binom{N}{2} p^2 (1-p)^{N-2} \\
&P_1(3,N) = \binom{N}{3} p^3 (1-p)^{N-3} \\
&\ldots \\
&P_1(N-1,N) = \binom{N}{N-1} p^{N-1} (1-p) \\
&P_1(N,N) = \binom{N}{N} p^N
\end{aligned}\right\} P_1(j,N) = \binom{N}{j} p^j (1-p)^{N-j}$$

As for $P_i(j)$ we can model the system as if it was in the first slot, with N now being equal to $N-t$ transmitters, where t is the number of devices that have transmitted in the $(i-1)$-th slot.

Therefore, for $i = 2$ we have:

$$
\begin{aligned}
P_2(0) &= P_1(0,N)P_1(0,N) + P_1(2,N)P_1(0,N-2) + ... + P_1(N-1,N)P_1(0,1) = \\
&= P_1(0,N)P_1(0,N) + \sum_{k=2}^{N-1} P_1(k,N)P_1(0,N-k) = \\
&= \sum_{k=0}^{N-1} P_1(k,N-k)P_1(0,N) - P_1(1,N)P_1(0,N-1) \\
P_2(1) &= P_1(0,N)P_1(1,N) + P_1(2,N)P_1(1,N-2) + ... + P_1(N-1,N)P_1(1,1) = \\
&= P_1(0,N)P_1(1,N) + \sum_{k=2}^{N-1} P_1(k,N)P_1(1,N-k) = \\
&= \sum_{k=0}^{N-1} P_1(k,N)P_1(1,N-k) - P_1(1,N)P_1(1,N-1) \\
P_2(2) &= P_1(0,N)P_1(2,N) + P_1(2,N)P_1(2,N-2) + ... + P_1(N-2,N)P_1(2,2) = \\
&= P_1(0,N)P_1(2,N) + \sum_{k=2}^{N-2} P_1(k,N)P_1(2,N-k) = \\
&= \sum_{k=0}^{N-2} P_1(k,N)P_1(2,N-k) - P_1(1,N)P_1(2,N-1) \\
P_2(3) &= ... = \sum_{k=0}^{N-3} P_1(k,N)P_1(3,N-k) - P_1(1,N)P_1(3,N-1) \\
&... \\
P_2(N-2) &= \sum_{k=0}^{2} P_1(k,N)P_1(N-2,N-k) - P_1(1,N)P_1(N-2,N-1) \\
P_2(N-1) &= \sum_{k=0}^{1} P_1(k,N)P_1(N-1,N-k) - P_1(1,N)P_1(N-1,N-1) \\
P_2(N) &= P_1(0,N)P_1(N,N)
\end{aligned}
$$

which has the general form:

$$
P_2(j) = \begin{cases} \sum_{k=0}^{N-1} P_1(k,N)P_1(0,N-k) - P_1(1,N)P_1(0,N-1) & \text{j = 0} \\ \sum_{k=0}^{N-j} P_1(k,N)P_1(j,N-k) - P_1(1,N)P_1(j,N-1) & \text{0 < j < N} \\ P_1(0,N)P_1(N,N) & \text{j = N} \end{cases} \tag{7.1}
$$

where the term with the minus sign is due to the fact that, if only one device transmitted during slot i, the target device T will have correctly received the message and therefore, starting from slot $i+1$ onwards, it will not be listening for incoming messages any more but it will be itself transmitting instead.

For $i = 3$ we have:

$$
\begin{aligned}
&P_3(0) = \sum_{k=0}^{N-1} P_2(k)P_1(0, N-k) - P_2(1)P_1(0, N-1)\\
&P_3(1) = \sum_{k=0}^{N-1} P_2(k)P_1(1, N-k) - P_2(1)P_1(1, N-1)\\
&P_3(2) = \sum_{k=0}^{N-2} P_2(k)P_1(2, N-k) - P_2(1)P_1(2, N-1)\\
&...\\
&P_3(N-1) = \sum_{k=0}^{1} P_2(k)P_1(N-1, N-k) - P_2(1)P_1(0, N-1)\\
&P_3(N) = P_2(0)P_1(N, N)
\end{aligned}
$$

which has a general form similar to (7.1)

$$
P_3(j) = \begin{cases} \sum_{k=0}^{N-1} P_2(k)P_1(0, N-k) - P_1(1, N)P_1(0, N-1) & \text{j = 0} \\ \sum_{k=0}^{N-j} P_2(k)P_1(j, N-k) - P_1(1, N)P_1(j, N-1) & \text{0 < j < N} \\ P_2(0)P_1(N, N) & \text{j = N} \end{cases} \tag{7.2}
$$

We can further generalize formula 7.2 to obtain the probability $P_i(j)$ we introduced at the beginning:

$$
P_i(j) = \begin{cases} \sum_{k=0}^{N-1} P_{i-1}(k)P_1(0, N-k) - P_{i-1}(1, N)P_1(0, N-1) & \text{j = 0} \\ \sum_{k=0}^{N-j} P_{i-1}(k)P_1(j, N-k) - P_{i-1}(1, N)P_1(j, N-1) & \text{0 < j < N} \\ P_{i-1}(0)P_1(N, N) & \text{j = N} \end{cases} \tag{7.3}
$$