UNIVERSITY OF PISA
School of Engineering

PERFORMANCE EVALUATION OF COMPUTER SYSTEMS AND NETWORKS

EPIDEMIC BROADCAST

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Contents

Chapter 1

Introduction

In what follows the study on the broadcast of an epidemic message is carried out. The specifications are detailed in the following:

> ### Epidemic broadcast
> Consider a 2D floorplan with N users randomly dropped in it. A random user within the floorplan produces a *message*, which should ideally reach all the users as soon as possible. Communications are *slotted*, meaning that on each slot a user may or may not relay the message, and a message occupies an entire slot. A *broadcast radius* R is defined, so that every receiver who is within a radius R from the transmitter will receive the message, and no other user will hear it. A user that receives more than one message in the same slot will not be able to decode any of them (*collision*). Users relay the message they receive *once*, according to the following policy (*p-persistent relaying*): after the user successfully receives a message, it keeps extracting a value from a Bernoullian RV with success probability p on every slot, until it achieves success. Then it relays the message and stops. A sender does not know (or cares about) whether or not its message has been received by its neighbors.
>
> Measure at least the broadcast time for a message in the entire floorplan, the percentage of covered users, the number of collisions.
>
> In all cases, it is up to the team to calibrate the scenarios so that meaningful results are obtained.

The work is organized as follows:

- Firstly, an initial overview and a presentation of the problem are given. Here, meaningful parameters to be tweaked and useful scenarios are identified and some considerations about them are made.

- Secondly, a graph-based modelling technique, commonly used in literature, is proposed and some simplified scenarios are analysed.

- Then, in Chapter 4 the complete model is considered and some assumptions about it are made, such as its performance when one or more parameters are set to extreme values or its asymptotic behaviour with different configurations of the parameters.
- In chapter 5 the development of the simulator is described and the results of its validation are presented.
- Chapter 6 concerns the full simulation and performance evaluation of the system.

Chapter 2

Overview

To perform the analysis of the broadcast of a message that should reach as many users as possible in a 2D floorplan, the following hypotheses were made:

- the floorplan always has a rectangular shape and it is empty, (i.e. there are no obstacles such as walls or pillars in it);
- each user is thought as a point mass and does not move inside the floorplan;
- the transmission of a message is instantaneous and it happens at the beginning of every time slot; the whole apparatus can therefore be considered a *Discrete Time System*.

Depending on the performance metrics to be analysed, different choices can be made about the parameters to be adjusted. According to the specifications, the main three metrics for this study are:

- the broadcast time `T` for a message to cover as many user as possible in the entire floorplan; the effective duration of the transmissions slots obviously has an effect on the total broadcast time[1], but it is only a scaling factor on the total number of slots. `T` can therefore be measured in terms of slots and converted to units of time accordingly.
- the percentage of covered users `U`;
- the number of collisions `C`; collisions are detected by nodes, when they try to receive more than one message in the same slot; in order to messure the number of collisions, we consider a single event of collision every time a node detect one of them; every statistical analysis is done taking this assumption.

The following parameters have been identified: the transmission range of the users, the *per-slot* transmission probability, the floorplan size and its shape, and the density of users in the floorplan per square metre.

[1]The total amount of time required for a broadcast message to cover the entire floorplan.

To be more in detailed:

- the radius of transmission `R`: it represents the maximum distance between two users such that the message sent from one is detected by the other; it is the same for every user on the floorplan; realistic values for R have been taken from Bluetooth Low Energy standard and range from a minimum of 5 metres to a maximum of 50 metres.
- the *per-slot* transmission probability `p`: it is the success probability for the Bernoulli random variable associated to the transmission; as a probability, it can assume values between 0 and 1;
- the width of the side of the floorplan rectangle `L`;
- the *aspect ratio* of the floorplan rectangle `a`, defined as the ratio between the longer side L and the shorter side W; because of the radial symmetry of the transmission phenomenon, there is no actual need to consider values for `a` lower than 1; this parameter is fixed at 1 in our simulation model, so we are considering only sqare floorplans; this will allow us to focus the analysis on other parameters, but still considering a quite common scenario;
- number of users `N`, defined as the number of users in the floorplan;
- users population density `d`, defined as the number of users per square meter. We choose to study two opposite density scenarios (high: $1/m^2$, low $0.1/m^2$).

The rationales behind the choices of the parameters were the following:

- the transmission radius clearly has a great impact on all the performance metrics; the greater the radius, the faster the message moves across the floorplan and the higher the number of users that can be reached; on the other hand, a greater radius is likely to cause more collisions than a smaller one;
- the higher the transmission probability, the faster a message "moves away" from a user; at the same time, a high transmission probability implies a high collision probability in a local area where two or more nodes are transmitting;
- a bigger floorplan area, all else being equal, will require a longer time to be covered entirely by the broadcast message;
- a very long and very narrow floorplan will probably cause less collisions than a square one with the same area, as the average number of users in the collision range of other users decreases; on the other hand, the performance of this type of scenario will be higly influenced by the position of the starting node;
- in order to exploit various values of population density `d` we choose to fix the total users `N`=100, then we choose the 2 values for L: 10 metres and 100 metres.

Chapter 3

System modelling

Wireless communication networks have been extensively studied in scientific literature and one of the most used mathematical tools to model them and their behaviour is *graph theory*. In this work the same approach was used.
If we think of the users involved in the broadcasting of the message as the nodes of a graph, as the broadcast propagation goes on and the message is transmitted among the nodes, the system goes through different states where each node could be either transmitting, listening or stopped. We can therefore think of the state of the entire system as a combination of the states of each node. The different nodes behaviours were modelled by means of three different states:

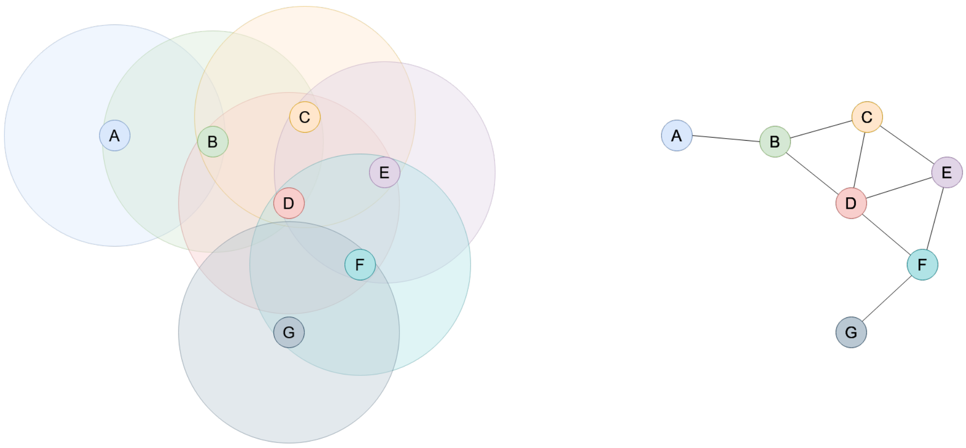
- `listening`: the node has not received the broadcast message yet and therefore it is still listening for incoming messages from other nodes;

- `transmitting`: the node has received the message and during each time slot it is trying to transmit it to adjacent nodes; in what follows, a node in `transmitting` state will be referred to as *active* node;

- `sleeping`: the node has already received the message and retransmitted it; once a node is in a `sleeping` state, it will remain in such state and will therefore have no effect on the system any more.

3.1 Graph model for wireless systems

The N users dropped on the floorplan make up the set of vertices V of a graph G, whose set of edges E is composed by all the connections between pairs of nodes one in reach of the other. Consider the following as a simplified scenario to exemplify this model. In figure 3.1 (a) devices A, C and D are within device B transmission radius. In the equivalent graph, there will be edges that connect B to A, to C and to D. The same goes for all the other vertices. The resulting graph is shown in figure 3.1 (b).



a) Ranges representation b) Equivalent graph representation

Figure 3.1

In general, the existence of an edge from vertex i to vertex j means that nodes i and j are within reach of each other. Two vertices connected by an edge are said to be *adjacent*. The set composed by the vertex v together with all its adjacent vertices form a subgraph called the *neighbourhood of v*.
During the broadcast, a node can only receive from and transmit to its neighbourhood.

Once a node has transmitted the message and has gone into `sleeping` state, it disappears, along with all the edges connected to it. Therefore, the set of vertices V changes with time. This is what in literature is called a *dynamic graph* or, more specifically, a *node-dynamic graph*[1].

Modelling the system with graphs also allows for a simple computation of the lower bound for the broadcast time `T`, which can be useful for the validation of the simulator. Given a graph $G(V, E)$ which represents the users in the floorplan, let node v^* be the starting point for the broadcast, i.e. the first node with the message. In the best case scenario, the system evolves with no collisions at all and the message moves along the paths of the graph, reaching all nodes. Let $d(u, v)$ be the *distance* between two vertices u and v, i.e. the length of the shortest directed path from u to v consisting of arcs, provided at least one such path exists. Then, the lower bound for the broadcast time is given by the highest distance between v^* and any other vertex. This quantity, in graph theory, is called *eccentricity* of the vertex v^*. More formally, the eccentricity of v^* is defined as follows:

$$\epsilon(v^*) = \max_{v \in V} d(v^*, v) \tag{3.1}$$

3.2 Simplified models

In what follows, incrementally more complex scenarios are presented showing the reasoning that led us to obtain the final complete model.

3.2.1 Single queue configuration

Let us consider a configuration where users are arranged in a line, as shown in figure 5.3. Each user only has two neighbours, except for the outer ones (nodes A and E) that only have one. Assuming node A the starting user for the broadcast message, this is the only node in `transmitting` state while all the other nodes are initially in `listening` state.

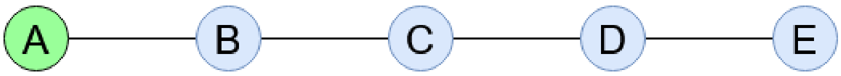


Figure 3.2: Graph of the Single queue configuration

It is clear that, with this configuration, each listening node has a maximum of one active node in its neighbourhood and thus cannot possibly receive the message from two different sources at the same time. This guarantees the absence of collisions. In such a scenario, 100% asymptotic coverage is ensured: during each slot, the active node extracts a Bernoulli RV with success probability p. The probability of the active node not transmitting for k consecutive slots is a geometric distribution:

$$P(X = k) = (1 - p)^k. \tag{3.2}$$

The successful transmission of the message from an active node to its neighbour is thus guaranteed since $\lim_{k\to\infty}(1 - p)^k = 0$. As for the total broadcast time `T`, on average it is equal to the mean value of the geometric distribution, $\frac{1-p}{p}$, times the number of hops needed to reach the last node:

$$E[T] = \frac{1 - p}{p} \cdot (N - 1). \tag{3.3}$$

3.2.2 Star configuration with one active node

Another useful simple configuration worth analysing is the star-shaped one. In this setup, there is a central node A connected to $N-1$ nodes, all of which are non-adjacent to each other. Let us suppose A to be the broadcast starter. The absence of collisions is ensured in this scenario as well, for the same reason as the previous configuration. At each time slot, A extracts a Bernoulli RV. When the extraction is successful, A broadcasts the message to all of its neighbourhood and total coverage is reached. Hence, 100% asymptotic coverage is ensured in this case too, as the probability of A not transmitting for k consecutive slots is given by Eq. 3.2 and goes to 0 as k goes to infinity.

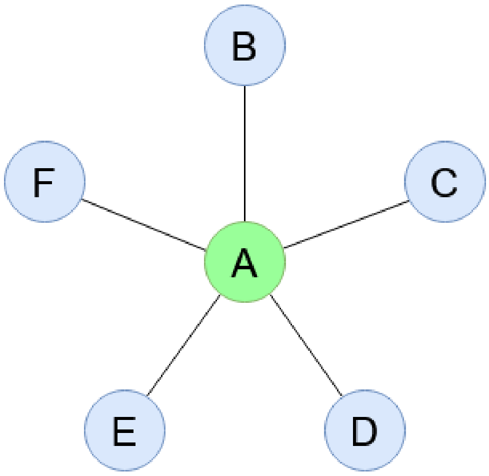


Figure 3.3: Graph of the Star configuration with one active node

In this case, the average total broadcast time $E[T]$ is simply equal to the mean value of the geometric distribution, $\frac{1-p}{p}$.

If the broadcast starter node was not the one placed in the center of the star, there would not be much difference: absence of collisions and total coverage would be ensured as well. As far as it concerns the total broadcast time `T`, its expected value would just be $2 \cdot \frac{1-p}{p}$, since the are now **two** hops involved in the broadcast: one from the starter node to the center node and the another from the center node to all the $N-2$ remaining ones.

3.2.3 Star configuration with all but one active node

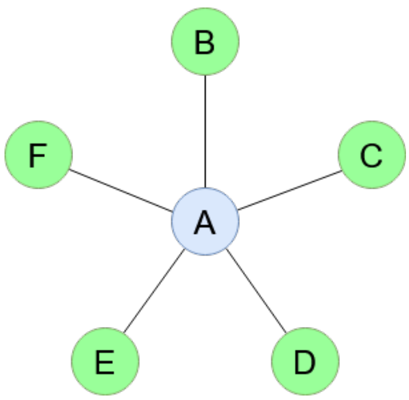


Figure 3.4: Graph of the Star configuration with all but one active node

This configuration can be seen as the complement of the previous one: each node on the ray of the star is active and trying to transmit the message to the center node. This is actually the first scenario where collisions might happen with the possibility that total coverage might never be reached. To simplify the analysis of this system and obtain some insights, it is useful to model it by means of a discrete-time Markov chain.

Discrete-time Markov chain model for N nodes transmitting to a target

Since the state of the system evolves only once per slot, it can be modelled with a discrete-time Markov chain (DTMC) as follows:

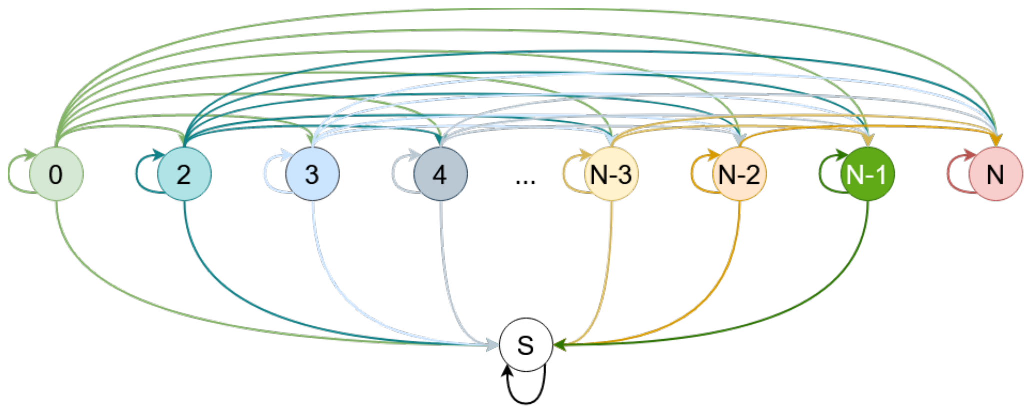


Figure 3.5: Discrete-time Markov chain for a scenario with N active devices

Figure 3.5 shows the DTMC for a generic configuration with N transmitters (transition probabilities are not shown for the sake of clarity).
State 0 and states 2 to N represent the number of `sleeping` devices, namely devices that have already sent the message and have stopped. The number of `sleeping` devices was chosen over the number of `transmitting` devices as the former only allows for non-decreasing sequences of states, since a device cannot become active again once it enters the `sleeping` state.
State S represents the successful transmission state, which the system transitions to when only one device has transmitted the message during the previous slot.
The initial state X_0 is 0. Any transition from a state i to a state j, $j \neq S$, means that more than one device has transmitted the message resulting in the target device detecting a collision. Both state S and state N are *absorbing states*, i.e. states that, once entered, cannot be left (as can be seen in Fig. 3.5, where the only outgoing arrow from each of the two goes back to the state itself).
If the system transitions to state N, it stays in it indefinitely since all the devices would be `sleeping` and they cannot become active again. This implies that there will never be total coverage since the target device will forever stay in a `listening` state without ever receiving the message. On the other hand, state S, although being an absorbing state as

well, should actually be considered as an "exit" state rather than a "sink" state: if the system transitions to this state, the target device has successfully received the message: total coverage has been reached.
Another interesting observation concerns state $N-1$: once the system reaches this state, it would be guaranteed that the target device will sooner or later receive the message, for the same reason set forth in 3.2.1.

Transition probabilities

Let us now address the challenging part: computing the transition probability.
During the first slot, the probability that j devices out of N transmit the message is the following:

$$P_1(j, N) = \binom{N}{j} p^j (1-p)^{N-j} \tag{3.4}$$

Derivation of 3.4 can be found in Appendix A.

As for the probability of j devices transmitting at the same time during slot k, be it $P_k(j)$, we can model the system as if it was in the first slot, with the total number of active devices now being equal to N - t, where t is the total number of devices that have transmitted up to the (k-1)-th time slot.
Carrying on with the computation of the transition probabilities this way, leads to unsatisfactory results which are difficult to interpret.

A better way to compute the probability of having j devices sleeping at slot k, is to use the *stochastic matrix* of the Markov chain. If the probability of moving from state i to j in one time slot is $Pr(j|i) = P_{i,j}$, the stochastic matrix P is given by using $P_{i,j}$ as the i-th row and j-th column element, e.g.

$$P = \begin{bmatrix} P_{0,0} & P_{0,S} & P_{0,2} & \dots & P_{0,j} & \dots & P_{0,N} \\ P_{S,0} & P_{S,S} & P_{S,2} & \dots & P_{S,j} & \dots & P_{S,N} \\ P_{2,0} & P_{2,S} & P_{2,2} & \dots & P_{2,j} & \dots & P_{2,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{i,0} & P_{i,S} & P_{i,2} & \dots & P_{i,j} & \dots & P_{i,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{N,0} & P_{N,S} & P_{N,2} & \dots & P_{N,j} & \dots & P_{N,N} \end{bmatrix}$$

Since S and N are absorbing states, $P_{S,j} = 0$ for j $\neq$ S and $P_{N,j} = 0$ for j $\neq$ N.
Moreover, all the possible transitions can only generate a non-decreasing sequence of states and there are no transitions between states that differ by one, hence $P_{i,j} = 0$ $\forall i > j$ and $\forall j = i + 1$.
All the other elements of the matrix can be computed using the following formula:

$$P_{i,j} = \binom{N-i}{j-i} p^{j-i} (1-p)^{N-j} \tag{3.5}$$

and $P_{i,S}$ is given by:

$$P_{i,S} = \binom{N-i}{1} p(1-p)^{N-i-1} \tag{3.6}$$

which is just a particular case of (3.5) when $j = i + 1$.

Therefore the stochastic matrix becomes:

$$P = \begin{bmatrix} P_{0,0} & P_{0,S} & P_{0,2} & \dots & P_{0,j} & \dots & P_{0,N} \\ 0 & 1 & 0 & \dots & 0 & \dots & 0 \\ 0 & P_{2,S} & P_{2,2} & \dots & P_{2,j} & \dots & P_{2,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & P_{i,S} & 0 & \dots & P_{i,j} & \dots & P_{i,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \dots & 1 \end{bmatrix}$$

Let x_0 be the *initial state vector*, i.e. an $N \times 1$ vector that describes the probability distribution of starting at each of the N possible states.
To compute the probability of transitioning to state j in **k** steps, it is now sufficient to multiply the initial state vector x_0 by the stochastic matrix raised to the k-th power, e.g.

$$P_k(j) = x_0 \cdot P^k \tag{3.7}$$

In our case, the system always starts in state 0, so we have

$$x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

which yields

$$P_k(j) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdot P^k = (P^k)_{0,j} \tag{3.8}$$

Calculating the k-th power of a matrix can be an intensive task from a computational point of view. To improve the complexity of the computation, rows and columns of P can be rearranged, moving the S row and the S column as penultimate, thus obtaining

$$P = \begin{bmatrix} P_{0,0} & P_{0,2} & P_{0,3} & \dots & P_{0,N-1} & P_{0,S} & P_{0,N} \\ & P_{2,2} & 0 & \dots & P_{2,N-1} & P_{2,S} & P_{2,N} \\ & & P_{3,3} & \dots & P_{3,N-1} & P_{3,S} & P_{3,N} \\ & & & \ddots & \vdots & \vdots & \vdots \\ & & & & P_{N-1,N-1} & P_{N-1,S} & 0 \\ & & & & & 1 & 0 \\ 0 & & & & & & 1 \end{bmatrix}$$

P is now an upper triangular matrix and this allows for faster computation of its powers in 3.8 .

Chapter 4

Complete Model

Chapter 5

Simulator

In order to obtain experimental results for the presented scenarios, a simulator was built using OMNeT++. This will allow us to reproduce the different scenarios with different values for the identified parameters.

5.1 Omnet++ and INET framework

OMNeT++ is an extensible, modular, component-based C++ simulation library and framework, primarily for building network simulators[1].
The INET Framework is an open-source model library for the OMNeT++ simulation environment. It provides protocols, agents and other predefined models for researchers and students working with communication networks. INET is especially useful when designing and validating new protocols, or exploring new or exotic scenarios[2].
OMNeT++ is a library and a framework, and can be used with the dedicated IDE. Not only it allows for development of the simulator itself, but also to export simulation results and to inspect simulation behaviour with a graphical user interface. Exploiting C++ compiler optimizations, it can achieve the lowest simulation duration possible. Networks are composed by modules; there are two types of modules: simple module and compound module (which can contain other modules itself). INET is an extension of OMNeT++, dedicated to recreating network simulation environments, with the capability of reproducing the activity of a wireless communication system across multiple nodes. It contains ready to use definitions and implementations of network related modules.

The choice was made to use the INET framework in order to avoid spending too much time on the coding side and therefore be able to focus more on other aspects such as the problem modeling and analysis.

[1] https://omnetpp.org/
[2] https://omnetpp.org/download-items/INET.html

5.2 Network architecture

The network based architecture is composed by an Array of Host modules, an Integrated visualizer (visualizer) and a UnitDiskRadioMedium (radioMedium);

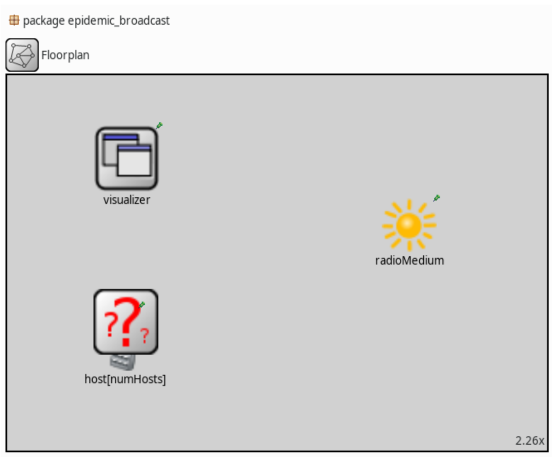


Figure 5.1: Floorplan.ned

- **UnitDiskRadioMedium** is a compound module provided by INET. This radio medium model provides a very simple but fast and predictable physical layer behavior. It must be used in conjunction with the `UnitDiskRadio` model. It can simulate the behaviour of the wireless communication channel with various levels of abstraction.

- **Integrated visualizer** is a compound module provided by INET. It's resposible for the visual representation of modules properties and events in the graphic user interface.

- **Host** is the compound module developer for representing a node in our network environment.

The `Host` module extends the NodeBase module defined by INET. This module contains the most basic infrastructure for network nodes that is not strictly communication protocol related. The following diagram shows usage relationships between types:

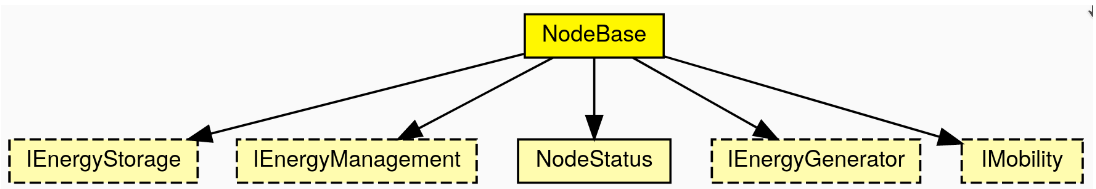


Figure 5.2: NodeBase Diagram.

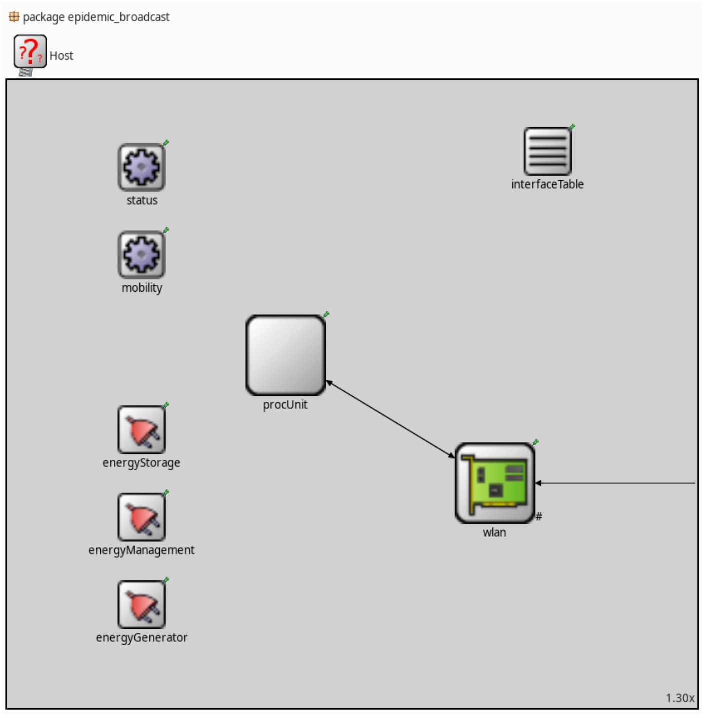


Figure 5.3: host.ned

- the `mobility` module privided by INET manages the position of the parent module `Host`; it allows various types of movements, but we are going to be using it only for the initial random placement of the nodes, then all nodes will be stationary.

- the `interfaceTable` module is provided by INET and is required for correct operation of the radioMedium module.

- the `wlan` module is the wireless interface that allows nodes to communicate with each others. It's an `AckingWirelessInterface` compound module, which is the simplest wireless interface provided by INET.

- the `status` module is provided by INET as well and is required to shutdown and restart network interfaces.

- the `procUnit` module is the custom made processing unit, that implements the node behaviour when a message arrives. It's connected to the `wlan` module in order to be able to receive and send messages.

- `energyStorage`, `energyManagement` and `energyGenerator` are modules inherited from `NodeBase` but they are not instantiated as we don't need to model energy related behaviours.

The `wlan` module is in charge of checking each and every message for collisions, and drop broken packets instead of forwarding them to the processing unit. The processing unit

ProcUnit implements the behaviours of the nodes; it handles the broadcast message when received, and then its retransmission when the random variable extraction results in a success. Finally it shuts down the network interface, preventing it from receiving any messages or provoking collisions. The network interface is turned off also for the entire duration of the RV extractions.

5.3 Parameters and Statistics

During the simulation, signals are used to collect the statistics. They are all collected by the Flooplan module:

- The wlan module emits a signal every time a collision is detected; this signal is collected by the packetDropIncorrectlyReceived statistic of the same module; we are interested in the total number of collisions detected by each node.
- The ProcUnit module emits 2 signals when initialized, hostX and hostY, collected, respectively, by the hostXstat and hostYstat statistics; those are the coordinates of the parent node in the floorplan.
- The ProcUnit module emits the timeCoverage singal as well, collected in the timeCoverageStat statistic; this is a vector containing, for each node that received the broadcast message, the number of the time slot when the broadcast message was actually received; at the end of the simulation, it's size represents the number of covered nodes.

Most significant parameters set up in the initialization file (floorplan.ini) are reported below:

- `Floorplan.host[*].procUnit.slotLength = 1`
- `Floorplan.host[*].procUnit.p = 1`

5.4 Design Choices and Optimizations

Using the INET framework for the development of the simulator allowed us to make use of pre-built modules for modeling wireless communications; for example, collision detection and statistics collection is already implemented by INET modules. During the development we choose for every aspect the optimal level of abstraction for our purposes, but it's possible to model other aspects just changing the types of INET modules used, or by adding new ones. We voluntarily avoided taking into account phenomena like pathloss and node movement, and we restricted our considerations to a discrete time scenario. However, modeling continuous time scenarios can be done easily by changing few INET modules types and attributes.
INET modules have also pre-built optimization structures, that become indispensable when the number of hosts become larger; in order to make the simulator ready for high

complex scenarios we used an the `neighborCache` structure offered by the `radioMedium` module. This module is in charge of storing proximity information of each and every node, in order to speed up message delivery. Setting the type of this module to `GridNeighborCache` it's possible to reduce the time needed for a simulaton with more than 2000 devices dropped on the floorplan, by a factor of 10; we also found that this type of cache (with the right value for the `cellSize` parameter) is the best trade-off between speed and memory occupancy, for this type of workload[3].

[3]https://doc.omnetpp.org/inet/api-current/neddoc/inet.physicallayer.contract.packetlevel.INeighborCache.html

Chapter 6

Simulation

As we previusly described, we decided to consider a population of 100 users in two different scenarios:

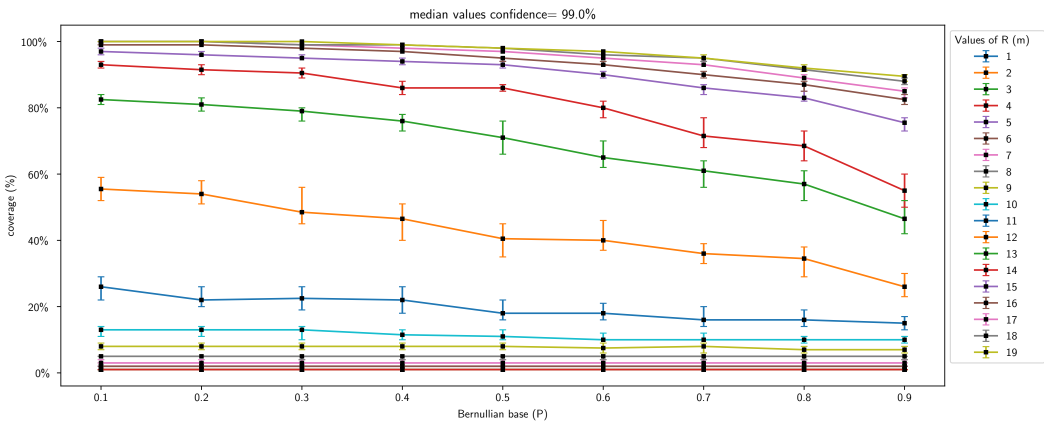
- **Small**: 10mx10m floorplan, with the transmission range going from 1m to 4.5m, with 0.5m steps, and the Bernullian base p going from 0.05 to 0.95 with 0.05 steps;
- **Big**: 100mx100m floorplan, with the transmission range going from 1m to 19m, with 1m steps, and the Bernullian base p going from 0.1 to 0.9 with 0.1 steps;

In order to acheive meaningfull results with a minumum accurancy of 90%, we decide to repeat the same scenario with the same parameters for 200 times, and than compute mean and median values from performance indexes, alogn with their confidence intervals.

6.1 Big

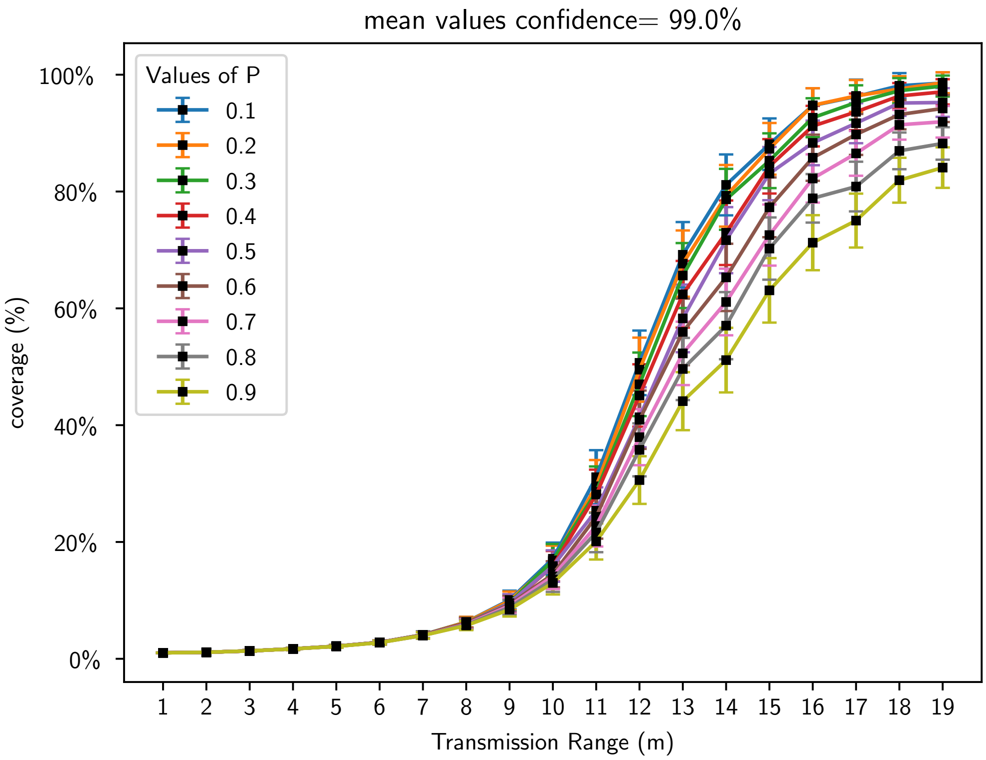
6.1.1 Coverage

This plot show the coverage achived in function of P, for different values of R.



As we can observe, the coverage we can reach is maximum when P is low; this was predictable by the fact that a low transmission probability minimize the number of collisions, increasing the number of correct transmissions. The acheiveble coverage decrease with the increasing of p, but this beahveour is quite slow and dont degenerate to 0, but it land at a value around (XX%) of the bmaximum. This shape is common for all the values of the transmission range R, but it's more evident for R>10.

This plot show the coverage achived in function of R, for different values of P.



As we can expect, and accordingly with the previus plot, the final coverage of the flooplan is very influenced by the transmission range; in this plot we can observe that the covreage increase exponentially in respect to the transmission range. As we can expect, when the transmission range become very large ($> 20m$), the coverage tend to 100%. We recognized the shape of a sigmoid[1] which can be express as an hyperbolic tangent (tanh); we found a good fit for this function:

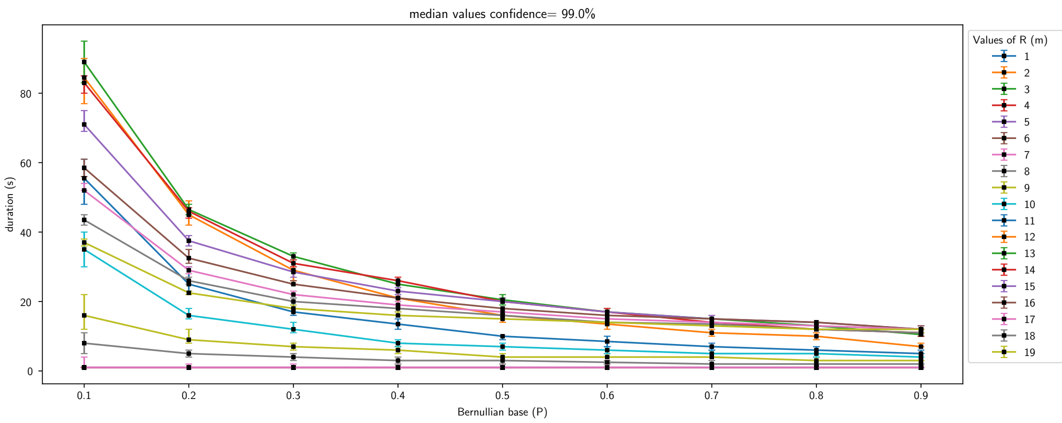
$$C = \frac{1 + \tanh(aR + b)}{2}$$

with C coverage, R transmission range, a and b depending on p as:

[1] https://mathworld.wolfram.com/SigmoidFunction.html

6.1.2 Duration

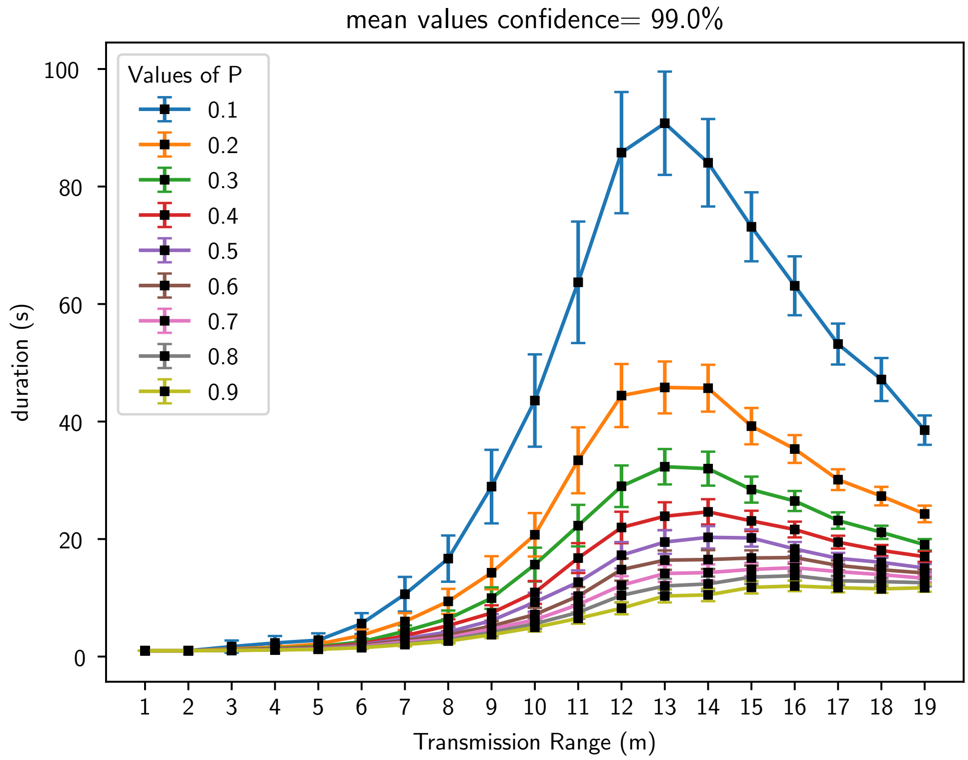
This plot show the Duration of the simulation (in slot) in function of P, for different values of R.



The simulation needs more slots to complete when P is low; this is a consequence of two factors:

- the probability of retransmission is low, so nodes will skip more slots without transmitting;
- with less collisions, we can reach more nodes, and we have to wait until they all retrasmit the message once.

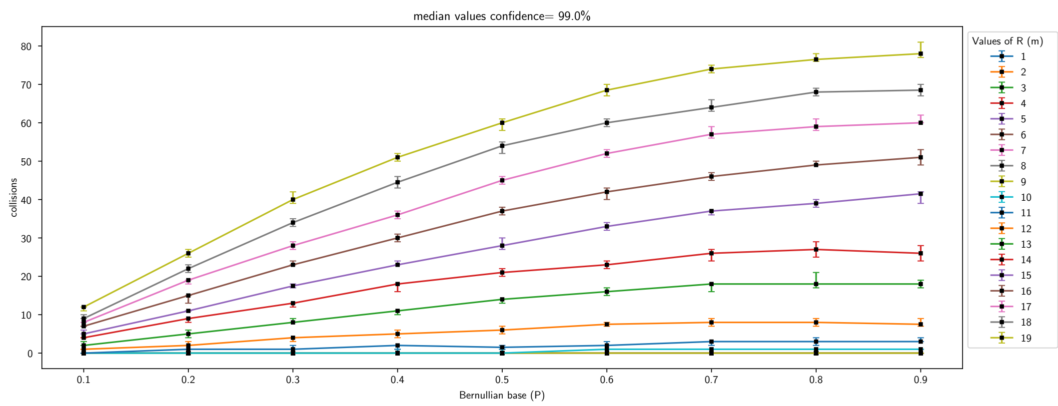
This plot show the Duration of the simulation (in slot) in function of R, for different values of P.



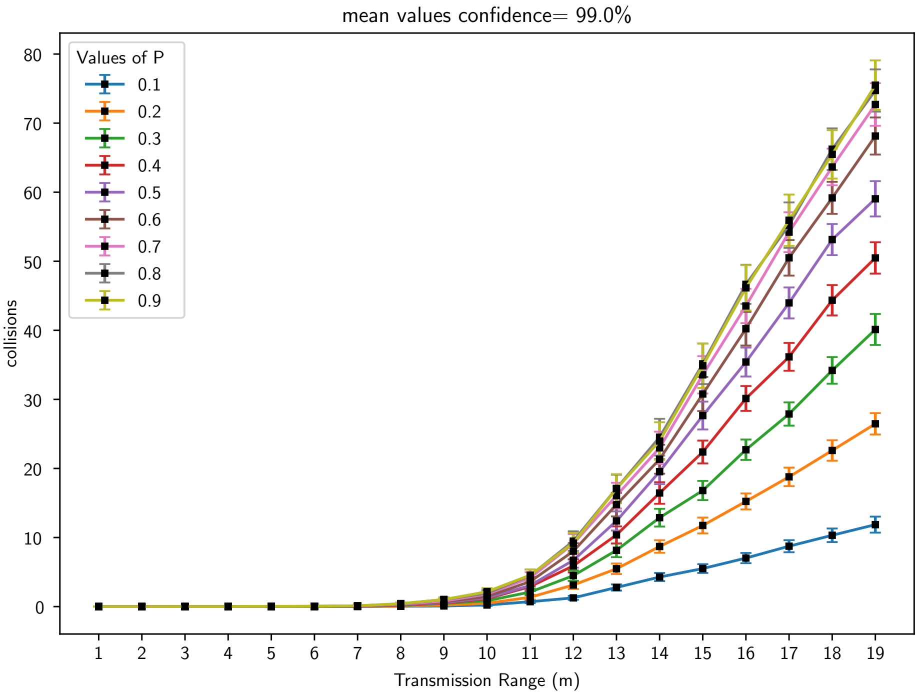
The duration of the simulation tend to increase with the transmission range reaching a peak around 13m; the maximum value depends on the probability of retransmission (p), but the tranmission range at which is reached do not. This is an intresting result, and can be explained by:?

6.1.3 Collisions

This plot show the Number of collisions in function of P, for different values of R.



This plot show the Number of collisions in function of R, for different values of P.



Chapter 7

Appendices

Appendix A

Given a scenario with N transmitter devices and a target device T in reach of all the transmitters, let us define the probabilities $P_1(j, N)$ as the probability of j devices out of N transmitting at the same time during slot 1 and $P_i(j)$ as the probability of j devices transmitting at the same time during slot i.
By specification, the successful reception of the message by device T happens if and only if **one** of the transmitters sends the message during the slot. Furthermore, the successful transmission of a device is "*a Bernoullian RV with success probability* p *on every slot, until it achieves success*"; therefore we can model $P_1(j, N)$ as follows:

$$\left.\begin{aligned}
&P_1(0,N) = (1-p)^N \\
&P_1(1,N) = Np(1-p)^{N-1} \\
&P_1(2,N) = \binom{N}{2} p^2 (1-p)^{N-2} \\
&P_1(3,N) = \binom{N}{3} p^3 (1-p)^{N-3} \\
&\ldots \\
&P_1(N-1,N) = \binom{N}{N-1} p^{N-1} (1-p) \\
&P_1(N,N) = \binom{N}{N} p^N
\end{aligned}\right\} P_1(j,N) = \binom{N}{j} p^j (1-p)^{N-j}$$

As for $P_i(j)$ we can model the system as if it was in the first slot, with the total number of active devices now being equal to $N - t$, where t is the number of devices that have transmitted in the (i-1)-th slot.

Appendix B

An example of stochastic matrix for a system with $N = 5$ and $p = 0.4$:

$$P = \begin{bmatrix} P_{0,0} & P_{0,2} & P_{0,3} & P_{0,4} & P_{0,S} & P_{0,5} \\ & P_{2,2} & 0 & P_{2,4} & P_{2,S} & P_{2,5} \\ & & P_{3,3} & 0 & P_{3,S} & P_{3,5} \\ & & & P_{4,4} & P_{4,S} & 0 \\ & & & & 1 & 0 \\ 0 & & & & & 1 \end{bmatrix}$$

Here with numerical values (rounded to 4 decimal places):

$$P = \begin{bmatrix} 0.0778 & 0.3456 & 0.2304 & 0.0768 & 0.2592 & 0.0102 \\ & 0.216 & 0 & 0.288 & 0.432 & 0.064 \\ & & 0.36 & 0 & 0.48 & 0.16 \\ & & & 0.6 & 0.4 & 0 \\ & & & & 1 & 0 \\ 0 & & & & & 1 \end{bmatrix}$$

Appendix C

Given a generic convess shape as a floorplan, we can compute it'area (A); if we imagine to put a (puntiform) transmitter (trx) in a random point of the floorplan, it's easy to compute the probability for the trx to be located in a given point x as:

$$P(x) = 1/A$$

Given that the trasmssion raduis in r, we can define a circle centered in x, as the transmission range of the first trx (R). If the trx in far enough from the border of the floorplan, all it's transmission range fall into the floorplan; in this case, the probability for another (random positioned) trx to be placed inside this transmission range is equal to:

$$P(x') = \int_R P(x) = \frac{2\pi r^2}{A}$$

If the firts trx is placed at a distance less than r from one or more borders of the floorplan, the probaility to randomly place another trx in it's range is less than the previous case; this is because part of the area of the transmission range will fall out of the floorplan, and, by construction, trxs can't be placed there. Depending on the floorplan' shape, we can identify the partition of the shape where the transmission range will fall out of the shape itself, and we will do specific computatin for this area; In general, by the middle value theorem for integrals, and the total probability theorem, the probailiy for 2 transmitters (let x and y be their coordinates) randomly placed to be close enough to communicate it's:

$$\rho = P(|x - y| < r) = \frac{1}{A} \int_A P(x')$$

Since we know that $P(x')$ is not constant for all the area of the shape, this integral might be difficult to compute, even for a square floorplan; in any case it's value, it's constant for a given floorplan and a given r. Taking in to consideration only one trx, we can compute the probability density function for the number of trx in its transmission range, in function of the number of trx in the entire floorplan $C(N)$. Every time we put a trx in the floorplan, the probability that it falls in the transmission range of the highlighted one is ρ. This is true for every $N > 1$, because of indipendance.

$$C(N) = \left(\frac{1}{p}\right)^{(N-1)}$$

This is a Bernullian distribution, and we can use this result only taking one trx at time; in other words we can't use this distrubution for all the trx in a random configured floorplan, because they are not indipendent. The distrubution for one trx will be influenced by the position of others. Anyway, we can apply this distribution to many transmitters, each one taken from a different floorplan; in this way we can validate the indipendanceof different random configurations.

Bibliography

[1] Frank Harary. *Graph theory*. Addison Wesley, 1969.