COMP3170 Assignment 2 Transformations

Objective

The purpose of this assignment is to test your knowledge of:

- 2D & 3D transformations: translation, rotation, scale, and shear
- Homogeneous coordinates & affine matrices
- Nested coordinate frames & the scene graph

Instructions

Throughout this assignment, all diagrams that *you* draw should be carefully drawn to scale **using graph paper** that you can download from the Internet. Sketches can be done by hand (using a ruler) and scanned (using a traditional scanner, a smartphone, or any other device that can legibly capture your sketches). You can also create your sketches using a suitable drawing program. Make sure there is always a clear distinction between the object and the axes in your diagrams. Marks will be deducted for poorly legible work.

You will be submitting your solution as a PDF, so ensure that you allow time for scanning your work to prepare your final PDF submission. Also, make sure that your scanned document is clearly legible: easy to read and clear for marking.

We will use the following notation for basic transformations. Please also **use this notation** in your answers. Incorrect notation may lose you marks.

2D Transformations

- $T(dx, dy)$ – translate by dx units in the x direction and dy units in the y direction.
- $R(\theta)$ – rotate anticlockwise by angle θ.
- $S(sx, sy)$ – scale by sx in the x direction and sy in the y direction.
- $Sh_h(h)$ – shear by h units horizontally.
- $Sh_v(v)$ – shear by v units vertically.

3D Transformations

- $T(dx, dy, dz)$ – translate by dx units in the x direction, dy units in the y direction and dz units in the z direction.
- $R_x(\theta)$ – rotate about the x axis by angle θ.
- $R_y(\theta)$ – rotate about the y axis by angle θ.
- $R_z(\theta)$ – rotate about the z axis by angle θ.
- $S(sx, sy, sz)$ – scale by sx units in the x direction, sy units in the y direction and sz units in the z direction.

Note: All points should be expressed as **column vectors**, i.e.:

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} \text{ for 2D points, and}$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ for 3D points.}$$

This means that vector multiplication should be done **on the right**.

E.g., transforming point $P = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ by translation matrix $T(dx, dy) = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix}$ is represented by the equation:

$$Q = T(dx, dy)P$$

$$Q = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$Q = \begin{pmatrix} x + dx \\ y + dy \\ 1 \end{pmatrix}$$

Question 1. Sketching Transformations in 2D [30 marks]

Figure 1 below shows a flag made up of four vertices with coordinates given in a model coordinate frame. Draw the result of applying each of the following transformations M to the flag. Label the new coordinates for the four vertices in world coordinates. Show values to 1 decimal place. [5 marks each]

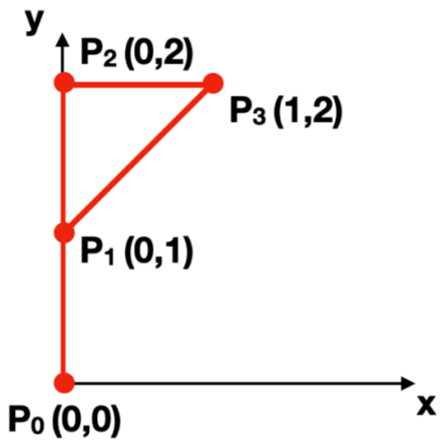

Figure 1: A triangular flag with vertices (0,0), (0,1), (0,2), (1,2)

E.g. For $M = T(-1{,}0)$ the resulting diagram would be:

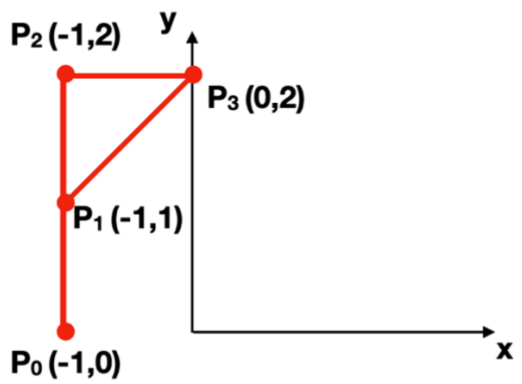

Figure 2: The flag above after the transformation M=T(-1,0) is applied.

Note: These transformations do not stack. Apply each transformation to the original flag in Figure 1, not the previous question's result.

a) $M = R(180°)$
b) $M = S(1, 0.5)$
c) $M = S(-1, 1)T(1, 0)$
d) $M = T(1, 0)R(90°)$
e) $M = R(45°)T(0, -1)$
f) $M = S(1, 0)R(-45°)$

Question 2: 2D Homogeneous Matrices [30 Marks]

For each of the homogeneous matrices in (a)-(e) below:

i. Draw (to scale) the inner (model) and outer (world) coordinate frames for the transformation represented by M, [2 marks]

ii. Write the decomposition of M into **two** simple 2D transformations. Answers should be specified in T, R, Sh, S order. [4 marks]

[6 Marks each]

For example, the matrix:

$$M = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Can be drawn as:

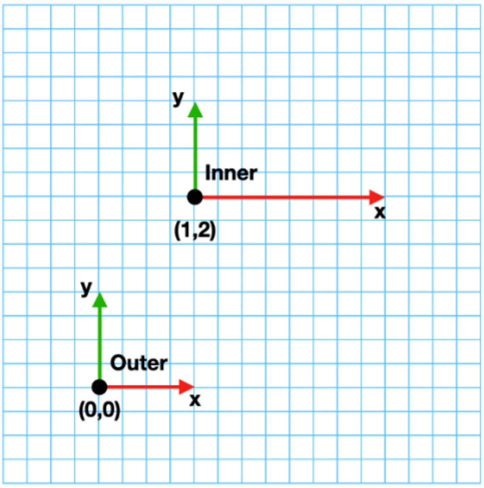

Figure 3: Example sketch of inner and outer coordinate frames for the matrix above.

And written as:

$$M = T(1,2)S(2,1)$$

Note: Some questions have multiple correct answers. Any one correct answer is enough.

a) $M = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b) $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

d) $M = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e) $M = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

Question 3: 3D Transformations [40 Marks]

In the question below, give your answers as the **product of simple transforms**, e.g.

$$M = R_y(90°)T(0,100,0)R_x(45°)S(2,1,1)$$

Transforms can be given in whatever order best suits the question.

Show your working. Incorrect answers with correct working may receive partial marks.

Do **not** calculate matrix values unless specifically requested (i.e. in part (g)).

Consider the following model aeroplane with origin and coordinate frame as indicated in Figure 4:

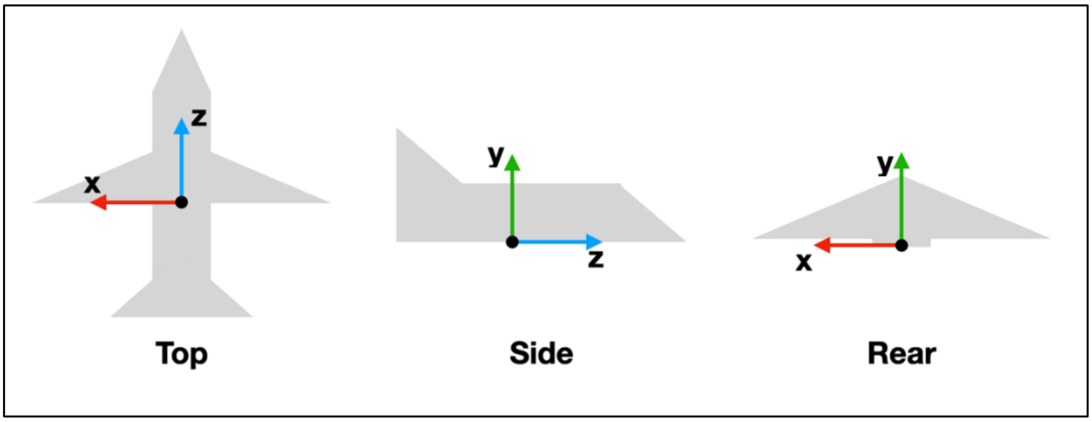

Figure 4: Model aeroplane, viewed from the top, side and rear.

a) Is this a right-handed or left-handed coordinate system? [2 marks]

Two copies of this model (**P1** and **P2**) are placed in a scene, as shown in Figure 5 below.

- **P1** is 200m north of the world's origin point and 100m in the air. It is heading west.
- **P2** is 300m west of the world's origin point and 200m in the air. It is heading north-east.

Neither plane has been scaled.

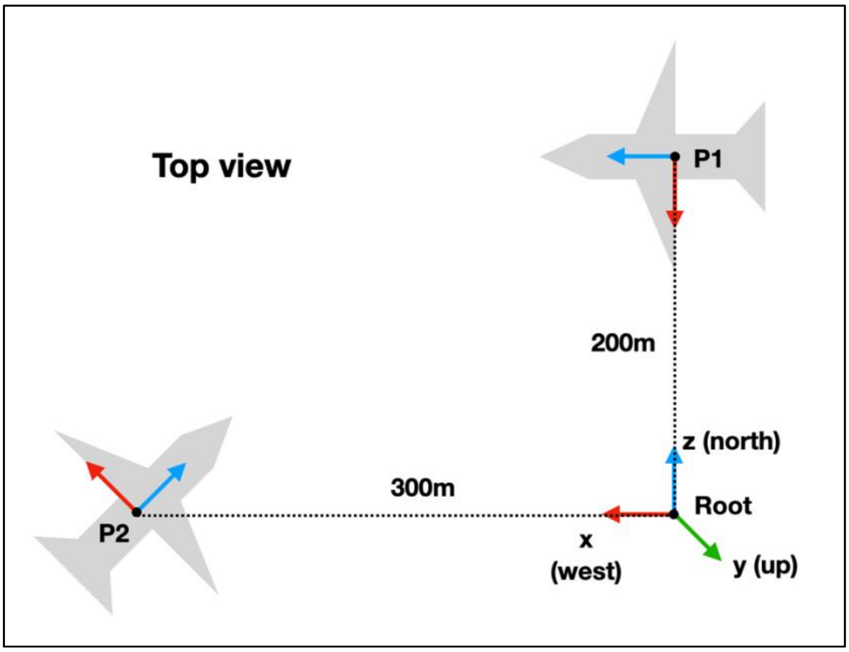

Figure 5: A top-down view of a scene containing two planes P1 and P2.

b) What is the model matrix $M_{P1 \to Root}$ representing **plane P1's** coordinate frame relative to the world, expressed as a product of simple 3D transformations (T, R_x, R_y, R_z, S)? [3 marks]

c) What is the model matrix $M_{P2 \to Root}$ representing **plane P2's** coordinate frame relative to the world, expressed as a product of simple 3D transformations (T, R_x, R_y, R_z, S)? [3 marks]

Plane P2 flies forwards in a north-east direction for 200m without changing its altitude.

d) What is the new model matrix $M_{P2 \to Root}$ representing **plane P2's** resulting coordinate frame relative to the world after this movement, expressed as a product of simple 3D transformations (T, R_x, R_y, R_z, S)? [3 marks]

A **camera C** is mounted on the right wing of the **plane P1** positioned 20 metres directly to the right of the origin, as shown in Figure 6. The z-axis of the camera points to the right side of the plane.

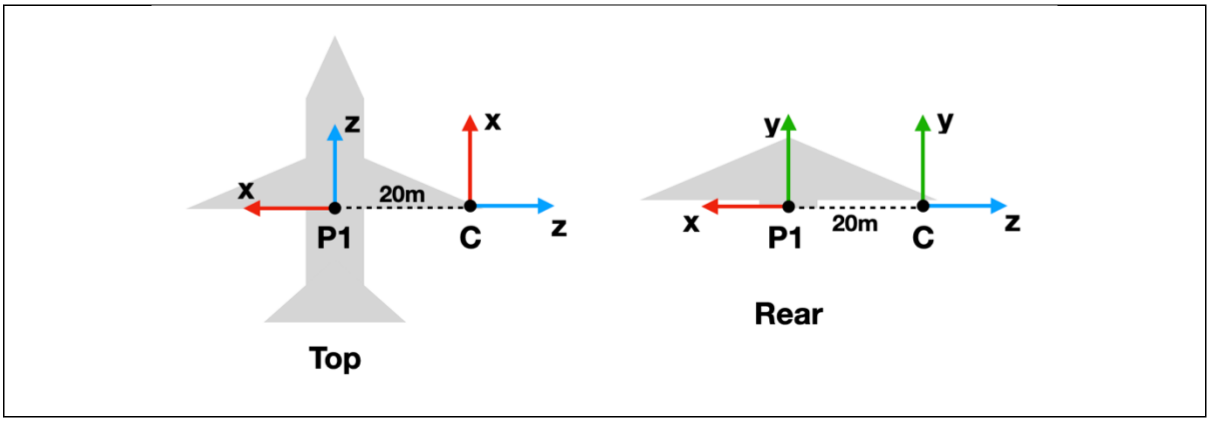

*Figure 6: Top and side views showing the camera **C** on the wing of plane **P1**.*

e) Draw a scene graph including **P1, P2** and **C**, with the world coordinate frame as the root. [4 marks]

f) What is the matrix $M_{C \to P1}$ representing the **camera C's** coordinate frame relative to the **plane P1**, expressed as a product of simple 3D transformations (T, R_x, R_y, R_z, S)? [3 marks]

The plane **P1** pitches up 30 degrees, as shown in Figure 7:

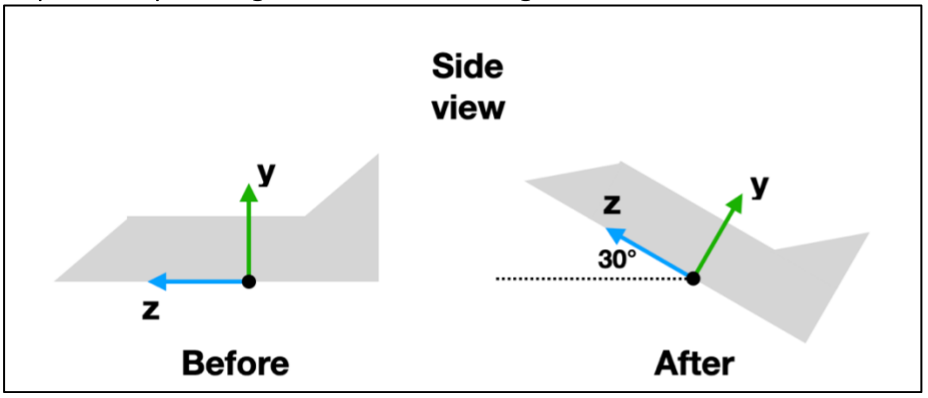

Figure 7: Side view showing the plane P1 pitching up 30° (before and after).

g) What is the **homogeneous matrix** M representing this 30° rotation in pitch? Write the matrix in full, using the trig functions sin() and cos(). You do **not** need to calculate a numerical result. [6 marks]

Taking the movements in (d) and (g) into account:

h) What is the model matrix $M_{C \to Root}$ representing the **camera C's** coordinate frame relative to the world, expressed as a product of simple 3D transformations (T, R_x, R_y, R_z, S)? [6 marks]

i) What is the model-view matrix $M_{P2 \to C}$ representing the **plane P2's** coordinate frame relative to the **camera C**, expressed as a product of simple 3D transformations (T, R_x, R_y, R_z, S)? [10 marks]