Useful Integrals and Identities

1. INTEGRALS

Indefinite Integrals

Note that in this section, constants of integration are taken to be equal to zero. Therefore, if taking indefinite integrals one must add constants of integration to the provided expressions.

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) \tag{1.1}$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) \tag{1.2}$$

$$\int x\sin(x)dx = \sin(x) - x\cos(x) \tag{1.3}$$

$$\int \sin(x)\cos(x)dx = \frac{\sin^2(x)}{2} = -\frac{\cos^2(x)}{2} \tag{1.4}$$

$$\int \sin^2(x)dx = \frac{x}{2} - \frac{\sin(2x)}{4} = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2} \tag{1.5}$$

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} = \frac{x}{2} - \frac{\sin(ax)\cos(ax)}{2a} \tag{1.6}$$

$$\int x\sin^2(ax)dx = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2} \tag{1.7}$$

$$\int x^2\sin^2(ax)dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right)\sin(2ax) - \frac{x\cos(2ax)}{4a^2} \tag{1.8}$$

$$\int xe^{ax}dx = e^{ax}\left(\frac{x}{a} - \frac{1}{a^2}\right) \tag{1.9}$$

$$\int x^2e^{ax}dx = e^{ax}\left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right] \tag{1.10}$$

Definite Integrals

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \qquad (n \text{ is a positive integer.}) \tag{1.11}$$

$$\int_0^\infty e^{-ax^2} dx = \left(\frac{\pi}{4a}\right)^{1/2} \tag{1.12}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^{n+1} a^n} \left(\frac{\pi}{a}\right)^{1/2} \qquad (n \text{ is a positive integer.}) \tag{1.13}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}} \qquad (n \text{ is zero or any positive integer.}) \tag{1.14}$$

2. Trig Identities

General Identities

$$1 = \cos^2\theta + \sin^2\theta \tag{2.1}$$

$$e^{\pm i\theta} = \cos\theta \pm i \sin\theta \tag{2.2}$$

Double- and Half-Angle Trig Identities

$$\begin{aligned}
\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\
&= 2\cos^2\theta - 1 \\
&= 1 - 2\sin^2\theta \\
&= \frac{1 - \tan^2\theta}{1 + \tan^2\theta}
\end{aligned} \tag{2.3}$$

$$\sin(2\theta) = 2\cos\theta\sin\theta \tag{2.4}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta} \tag{2.5}$$

$$\cot(2\theta) = \frac{\cot^2\theta - 1}{2\cot\theta} \tag{2.6}$$

$$\cos\left(\frac{\theta}{2}\right) = \operatorname{sgn}\left(\pi + \theta + 4\pi\left\lfloor\frac{\pi - \theta}{4\pi}\right\rfloor\right)\sqrt{\frac{1+\cos\theta}{2}} \tag{2.7}$$

$$\sin\left(\frac{\theta}{2}\right) = \operatorname{sgn}\left(2\pi - \theta + 4\pi\left\lfloor\frac{\theta}{4\pi}\right\rfloor\right)\sqrt{\frac{1-\cos\theta}{2}} \tag{2.8}$$

$$\begin{aligned}\tan\left(\frac{\theta}{2}\right) &= \csc\theta - \cot\theta \\ &= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \\ &= \frac{\sin\theta}{1+\cos\theta} \\ &= \frac{1-\cos\theta}{\sin\theta} \\ &= -1 \pm \sqrt{\frac{1+\tan^2\theta}{\tan\theta}} \\ &= \frac{\tan\theta}{1+\sec\theta}\end{aligned} \tag{2.9}$$

Product-Addition/Subtraction Trig Identities

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)] \tag{2.10}$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \tag{2.11}$$

$$\tan\alpha\tan\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{\cos(\alpha-\beta) + \cos(\alpha+\beta)} \tag{2.12}$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)] \tag{2.13}$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \tag{2.14}$$