Sheet 3

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Assignment 1

a) $a = [r\cos t, r\sin t]^T$

$$V(t) = \int_{t_1}^{t_2} a(t)\,dt = [r\sin(t_2-t_1) + C,\ -r\cos(t_2-t_1) + C']^T$$

$$X(t) = \int_{t_1}^{t_2} v(t)\,dt = [X_0 + [-r\cos(t_2-t_1)] + C(t_2-t_1) + D,\ Y_0 + [-r\sin(t_2-t_1)] + C't + D']^T$$

b)

$$\vec{V_{n+1}} = \vec{V_n} + \vec{a_n}\,\Delta t$$
$$\vec{X_{n+1}} = \vec{X_n} + \vec{V_{n+1}}\,\Delta t$$
$$\vec{X_{n+1}} = \vec{X_n} + \vec{V_n}\Delta t + \vec{a_n}(\Delta t)^2$$
$$\underbrace{\vec{X_{n+1}} = \vec{X_n} + \Delta t\,\vec{V_n} + \frac{1}{2}\vec{a_n}\,(\Delta t)^2}_{\text{The important equation}}$$

The forward Euler integration is about the simplest possible way to do numerical integration. It works by treating the linear slope of the derivative at a particular value as an approximation to the function at some nearby value.

c) $X(t_1) = X(t_0) = [1, 0]^T \quad V(t_1) = [0, 0]^T \quad a = [\cos t, \sin t]^T$

$$\vec{X_{n+1}} = \vec{X_n} + \vec{V_n}\Delta t + \frac{1}{2}\vec{a_n}\Delta t^2$$

$$\vec{X_2} = [1 + 0\cdot 1 + \frac{1}{2}\cos\frac{\pi}{2}\cdot 1^2,\ 0 + 0\cdot 1 + \frac{1}{2}\sin\frac{\pi}{2}\cdot 1^2]^T = [1, \frac{1}{2}]^T$$

d)

$$\vec{X_{n+1}} = \vec{X_n} + \Delta t\,\vec{V_n} + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\vec{X_{t_1+h}} = \vec{X_{t_1}} + \vec{V_{t_1}}\Delta t + \frac{1}{2}a_{t_1}(\Delta t)^2 \qquad h = \Delta t = 0.25$$

$$\vec{V_{t_1+h}} = \vec{V_{t_1}} + \vec{a_{t_1}}\cdot\Delta t$$

$$\vec{a} = [\cos t, \sin t]^T, \quad \vec{V_{t_1}} = 0, \quad t_1 = \frac{\pi}{2}, \quad \vec{X_{t_1}} = [1, 0]^T$$

① $$\vec{V_{t_1+h}} = \vec{V_{t_1}} + \vec{a_{t_1}}\Delta t = 0 + 0.25[\cos\frac{\pi}{2}, \sin\frac{\pi}{2}]^T = [0, 0.25]^T$$

$$\vec{X_{t_1+h}} = \vec{X_{t_1}} + \vec{V_{t_1}}\cdot\Delta t + \frac{1}{2}\vec{a_{t_1}}(\Delta t)^2 = [1,0]^T + 0 + \frac{1}{2}\cdot(0.25)^2\cdot[\cos(\frac{\pi}{2}), \sin(\frac{\pi}{2})]$$
$$= [1, 0.03125]^T$$

$$\vec{V_{t_1+2h}} = \vec{V_{t_1+h}} + \vec{a_{t_1+h}}\Delta t = [0, 0.25]^T + \frac{1}{2}0.25[\cos(\frac{\pi}{2}+0.25), \sin(\frac{\pi}{2}+0.25)]^T = [-0.03, 0.37]^T$$

② $$\vec{X_{t_1+2h}} = \vec{X_{t_1+h}} + \vec{V_{t_1+h}}\cdot\Delta t + \frac{1}{2}\vec{a_{t_1+h}}\cdot(\Delta t)^2 = [1, 0.03125]^T + 0.25[0, 0.25]^T + \frac{1}{2}\cdot[\cos(\frac{\pi}{2}+0.25), \sin(\frac{\pi}{2}+0.25)]^T(0.25)^2$$
$$= [0.99, 0.124]^T$$

③ $$\vec{X_{t_1+3h}} = \vec{X_{t_1+2h}} + \vec{V_{t_1+2h}}\cdot\Delta t + \frac{1}{2}\vec{a_{t_1+2h}}(\Delta t)^2$$
$$= [0.99, 0.124]^T + [-0.03, 0.37]^T\cdot 0.25 + \frac{1}{2}[\cos(\frac{\pi}{2}+0.5), \sin(\frac{\pi}{2}+0.5)]^T\cdot(0.25)^2 = [0.96, 0.24]^T$$

④ $$\vec{V_{t_1+3h}} = \vec{V_{t_1+2h}} + \vec{a_{t_1+2h}}\cdot\Delta t = [-0.03, 0.37]^T + 0.25[\cos(\frac{\pi}{2}+0.5), \sin(\frac{\pi}{2}+0.5)]^T = [-0.15, 0.59]^T$$

$$\vec{X_{t_1+4h}} = \vec{X_{t_1+3h}} + \vec{V_{t_1+3h}}\cdot\Delta t + \frac{1}{2}\vec{a_{t_1+3h}}(\Delta t)^2$$
$$= [0.96, 0.24]^T + [-0.15, 0.59]^T\cdot 0.25 + \frac{1}{2}[\cos(\frac{\pi}{2}+0.75), \sin(\frac{\pi}{2}+0.75)]^T\cdot(0.25)^2 = [0.90, 0.41]^T$$

(e) $v(t_1) = [0, 0]^T$

$[r \sin\frac{\pi}{2} + C, -r\cos\frac{\pi}{2} + C']^T = [0, 0]^T$

$\therefore C = -r, \; C' = 0;$

$x(t_1) = [1, 0]^T = [1 + (-r\cos\frac{\pi}{2}) \cdot (-r)\frac{\pi}{2} + D, \; 0 + (-r\sin\frac{\pi}{2}) + D']$

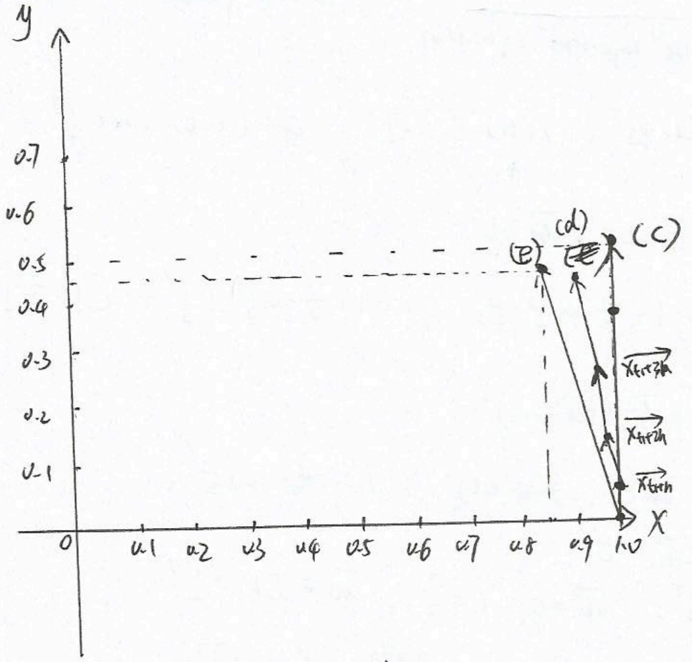
$\therefore D = \frac{\pi}{2}, \; D' = r$

$v(t) = [r\sin t - r, \; -r\cos t]^T$

$x(t) = [x_0 + (-r\cos t) - rt + \frac{\pi}{2}, \; y_0 - r\sin t + r]^T$

$= [1 - \cos t - t + \frac{\pi}{2}, \; -\sin t + 1]^T \qquad t = \frac{\pi}{2} + 1$

$= [0.84, 0.45]^T$



The integral method(e) is the most precise one.

Then, the smaller h, the more precise the result is.