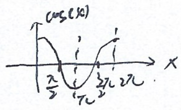
Group 26 Tu 16:00
Ming Qu
Liuhan Pan

Sheet 8

Assignment 1



$$f(x) = sgn(\cos(x)) = \begin{cases} 1 & 0 \geq x \geq \frac{\pi}{2} \text{ or } \frac{3}{2}\pi \leq x \leq 2\pi \\ -1 & \frac{\pi}{2} < x < \frac{3}{2}\pi \end{cases}$$

$$g(x) = \begin{cases} \frac{2}{\pi}x & 0 \leq x \leq \frac{\pi}{2} \\ -\frac{2}{\pi}x+2 & \frac{\pi}{2} < x < \frac{3}{2}\pi \\ \frac{2}{\pi}x-4 & \frac{3\pi}{2} \leq x \leq 2\pi \end{cases}$$

$$\langle f, g\rangle = \int_a^b f^*(x)\, g(x)\, dx = \int_0^{2\pi} f^*(x)\, g(x)\, dx$$
$$= \int_0^{2\pi} f(x)\, g(x)\, dx = \int_0^{\frac{\pi}{2}} \frac{2}{\pi}x\, dx + \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} (-1)\cdot\left(-\frac{2}{\pi}x+2\right) dx + \int_{\frac{3}{2}\pi}^{2\pi} \left(\frac{2}{\pi}x-4\right) dx$$
$$= \left[\frac{1}{\pi}x^2\right]_0^{\frac{\pi}{2}} - \left[-\frac{1}{\pi}x^2+2x\right]_{\frac{\pi}{2}}^{\frac{3}{2}\pi} + \left[\frac{1}{\pi}x^2-4x\right]_{\frac{3}{2}\pi}^{2\pi}$$
$$= \frac{1}{\pi}\cdot\frac{\pi^2}{4} - \left[-\frac{1}{\pi}\cdot\frac{9}{4}\pi^2+3\pi-\left(-\frac{1}{\pi}\frac{\pi^2}{4}+\pi\right)\right] + \left[\frac{1}{\pi}\cdot 4\pi^2-8\pi-\left(\frac{1}{\pi}\cdot\frac{9}{4}\pi^2-6\pi\right)\right]$$
$$= \frac{1}{4}\pi - 0 - \frac{1}{4}\pi = 0$$

$\Rightarrow$ the functions $f(x)$, $g(x)$ are orthogonal.

Assignment 2

$$\tilde{f}(t) = f(2t) = f(2t+T) = f\left(2\left(t+\frac{T}{2}\right)\right) = \tilde{f}\left(t+\frac{T}{2}\right) = \tilde{f}(t)$$

So, the period of $\tilde{f}(t)$ is $\frac{T}{2}$.

$$\tilde{a}_k = \frac{2}{\frac{T}{2}}\int_0^{\frac{T}{2}} \tilde{f}(t)\cos\left(\frac{2\pi kt}{\frac{T}{2}}\right) dt = \frac{4}{T}\int_0^{\frac{T}{2}} f(2t)\cos\left(\frac{4\pi kt}{T}\right) dt$$
$$= 2\cdot\frac{4}{T}\int_0^{\frac{T}{2}} f(2t)\cos\left(\frac{2\pi k(2t)}{T}\right) d(2t) \underset{\Downarrow}{=} \frac{2\cdot\frac{2}{\frac{T}{2}}}{2}\int_0^{T} f(t')\cos\left(\frac{2\pi k(t')}{T}\right) d(t') = a_k$$

set $t'=2t$

$$\tilde{b}_k = \frac{2}{\frac{T}{2}}\int_0^{\frac{T}{2}} f(2t)\sin\left(\frac{2\pi kt}{\frac{T}{2}}\right) dt = \frac{4}{T}\int_0^{\frac{T}{2}} f(2t)\cos\left(\frac{2\pi k(2t)}{T}\right) dt$$
$$= 2\cdot\frac{4}{T}\int_0^{\frac{T}{2}} f(2t)\cos\left(\frac{2\pi k(2t)}{T}\right) dt$$
$$= 2\cdot\frac{2}{\frac{T}{2}}\int_0^{T} f(2t)\cos\left(\frac{2\pi k(2t)}{T}\right) dt = b_k$$